

Hedge effectiveness testing for hedge accounting

Methods analysed

Jasper Wijnands

Internship paper

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Preface

The internship is the final stage of the Master Business Mathematics and Informatics. The goal of the internship is to apply the theoretical knowledge of the academic study in a business context.

The problem formulation for my internship is: *How do hedge effectiveness testing methods perform in hedge effectiveness testing for common hedge relationships and do changes in the methods enhance comparability of the results?*

I performed my internship at Ernst & Young within the department Financial Services Risk Management. For supervising my internship I would like to thank Sander de Ruiters, senior manager at Ernst & Young, who spent multiple hours reading my internship paper and has provided suggestions on the subjects I should investigate. Next to this, I would like to thank Ernst & Young Financial Services Risk Management for giving me the possibility to perform my internship.

I would like to thank Bert Kersten for reading my internship paper and providing comments and I would also like to thank Auke Pot.

Executive summary

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1 Ernst & Young FSRM

1.1 Company introduction

I have done my internship at Ernst & Young Financial Services Risk Management (FSRM). FSRM is an advisory group within Ernst & Young Accountants. It provides clients with advises on for example credit risk, market risk and operational risk. Next to advisory an important function of FSRM is providing audit support to Ernst & Young Accountants. This support varies from the valuation of (simple as well as complex) financial products and derivatives to support on topics as hedge accounting. The label for these kinds of task is DVC: Derivatives Valuation Centre. Team members are people within FSRM with a quantitative background, including myself. The number of employees of FSRM is approximately 20.

Figure 1 shows the services of FSRM. The external framework is formed by the outside circle. This mainly consists of rules. The circle one step to the centre shows the solutions of FSRM for the specific topic in the external framework.

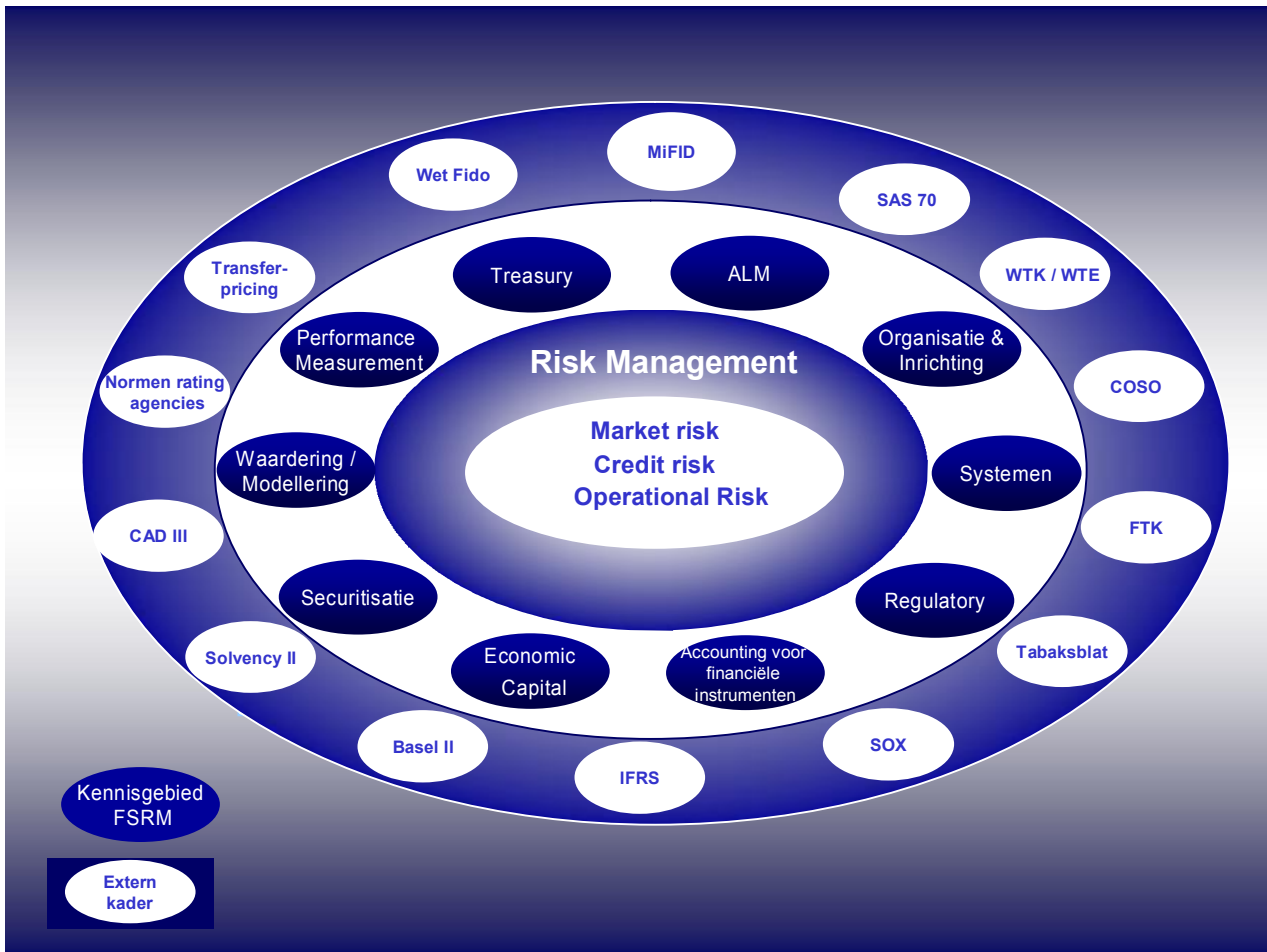
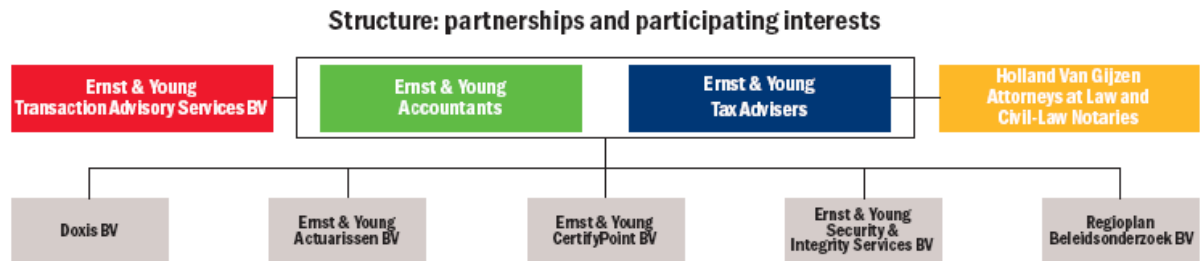


Figure 1. FSRM services diagram

Banking & Capital markets is one of the sectors FSRM focuses on. Other sectors are for example Insurance (related to Solvency II) and Asset Management.

In figure 2 the organizational structure of Ernst & Young is presented. FSRM is placed under Ernst & Young Accountants as specialist service provider.

Organization chart Ernst & Young (Netherlands)



Services: Ernst & Young offers specialized expertise and services in the following fields

Ernst & Young	Ernst & Young Transaction Advisory Services BV	Ernst & Young Accountants	Ernst & Young Tax Advisers	Holland Van Gijzen Attorneys at Law and Civil-Law Notaries
Information management	Transaction Support	Assurance	General Corporate Tax	Corporate & Commercial
Policy reviews	Mergers & Acquisitions Advisory	Technology and Security Risk Services	Tax Compliance	Employment
	Valuations & Financial Modelling	Actuarial Services	Multinational Corporate Tax	Finance
	Real Estate Advisory Services	Accounting	Indirect Tax (VAT and Customs)	Intellectual Property and ICT
		Advisory Services	Transaction Tax	Real Estate
		Internal Audit Services	International Tax Services	Mergers & Acquisitions
		Environmental Audit	Human Capital	
		Financial Services Risk Management	Tax Accounting & Risk Advisory Services	
		Security & Integrity Services	Personal Client Services	

Figure 2. Organization chart Ernst & Young

Competitors of Ernst & Young are mainly the other accounting firms of the Big Four: PricewaterhouseCoopers, Deloitte and KPMG.

1.2 Financial Instruments Working Group (FIWG)

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2 Introduction

In this chapter I will provide an introduction to hedge accounting and also a general introduction to my internship.

2.1 Introduction to hedge accounting

During the last decades the use of financial instruments has grown significantly. The markets for derivatives have had a considerable growth. Large use of derivatives by companies required accountancy to develop rules for accounting for these derivatives. The original accounting rules for derivatives were very important for the development of a hedge accounting framework. This hedge accounting framework is described in International Accounting Standard (IAS) 39 *Financial Instruments: Recognition and Measurement*, which is part of the International Financial Reporting Standards (IFRS).

The most important terms in IAS 39 are the hedged item and the hedging instrument. Here follow the definitions of a hedged item and a hedging instrument.

“A hedged item is an asset, liability, firm commitment, highly probable forecast transaction or net investment in a foreign operation that (a) exposes the entity to risk of changes in fair value or future cash flows and (b) is designated as being hedged.”

“A hedging instrument is a designated derivative or (for a hedge of the risk of changes in foreign currency exchange rates only) a designated non-derivative financial asset or non-derivative financial liability whose fair value or cash flows are expected to offset changes in the fair value or cash flows of a designated hedged item.”

The hedge accounting framework is about recognizing profits and losses on hedged items and hedging instruments in the same place in the financial statements or / and in the same period in the financial statements. Situations where the original accounting rules create such mismatches in places or time are:

- Measurement differences
 - o Consider a hedged item that is an asset or liability. Assume this hedged item is valued at cost price. Assume the hedging instrument is an option, which is valued by calculating the fair value. The different types of measuring the value cause differences in the financial statements even when the hedge is economically perfect.
- Performance reporting differences
 - o In this case profit and loss on the hedged item is recorded at a different place in the financial statements than profit and loss on the hedging instrument. For

example, consider the scenario where the hedging instrument is recorded in equity and the hedged item in the profit and loss account. The result is that recording fair values of hedged item and hedging instrument at different places cause variation in the profit and loss account even when the hedge is economically perfect.

- Recognition differences
 - o Assume the hedged risk in the hedged item is currently not recognized in the financial statements. Adding a financial instrument will in this case cause variability in the financial statements, even when the hedge is economically perfect.
- Existence differences
 - o Assume the hedged item is a transaction that is probable, but not yet contracted. So, the hedged item is not present in the financial statements. Adding a financial instrument will in this case cause variability in the financial statements, even when the hedge is economically perfect.

When using hedge accounting the hedged item and hedging instrument could be recorded in the same place and at the same period in the financial statements. This solves the problems above and reduces the variability of the profit and loss account.

Hedging itself is a much wider topic than hedge accounting. For example, the International Financial Reporting Standards allow hedge accounting only for certain hedge relationships. The main requirements are:

- Formal documentation of the hedge relationship is required. In this documentation the risk management objective of the company and the strategy for undertaking the hedge is also required.
- At inception of the hedge relationship the hedge is expected to be highly effective.
- The effectiveness of a hedge relationship should be tested at the end of each period (on an ongoing basis) and the hedge relation should be highly effective. For example, the length of the testing periods can be one month or three months. The hedge relationship should be tested each financial reporting period. If the hedge relationship is assessed as ineffective, hedge accounting is no longer permitted.
- The fair value or cash flows of the hedged item and the fair value of the hedging instrument can be measured reliably.
- For cash flow hedging (explained later on) the future cash flow(s) must be highly probable and must have an exposure to variation of the future cash flow(s).

For a hedge relationship to qualify for hedge accounting, *all* requirements above have to be satisfied. One of the requirements, which states that a hedge relation should be *highly* effective (and not 100% effective), implies that there could be a certain amount of ineffectiveness in a period. This ineffectiveness should be recognized in the profit and loss account.

To test if a hedge relation is highly effective (at inception or on an ongoing basis), different methods exist. One of these is the critical terms method. If all the terms of the hedging instrument and the hedged item match, each change in fair value of the hedged item will be perfectly offset

by an opposite fair value change of the hedging instrument. So, hedge effectiveness is always 100% in this case. If critical terms can be applied, no effectiveness testing is needed because the hedge relation is always assumed to be effective. If critical terms cannot be applied, the effectiveness of a hedge relationship should be assessed in a different way. Therefore, multiple hedge effectiveness testing methods are available. For accounting purposes it is necessary to determine if effectiveness testing as performed by a client is allowed. This is the subject of my internship.

2.2 Problem formulation

The problem formulation for my internship is: *How do hedge effectiveness testing methods perform in hedge effectiveness testing for common hedge relationships and do changes in the methods enhance comparability of the results?*

To be able to formulate an answer to this question, a detailed investigation of the most common methods for hedge effectiveness testing is required. Next to this, I will describe the process of hedge effectiveness testing in detail.

2.3 Structure of internship paper

In this chapter I will handle some definitions and describe the process of hedge effectiveness testing. In chapter 3 I will focus on one of the options in the testing process: including or excluding the passage of time. In chapter 4 I will describe the methods used for hedge effectiveness testing and in chapter **Error! Reference source not found.** I will investigate these methods in depth.

To provide an answer to the problem formulation I will then compare the different hedge effectiveness testing methods. This is described in chapters **Error! Reference source not found.**, **Error! Reference source not found.** and **Error! Reference source not found.**

In chapter **Error! Reference source not found.** I will suggest adjustments to the hedge effectiveness testing methods.

I will look at hedge effectiveness testing from a business perspective in chapter **Error! Reference source not found.**

Finally, chapter **Error! Reference source not found.** states the conclusions and recommendations.

2.4 Definitions: cash flow hedges and fair value hedges

The terms *cash flow hedge* and *fair value hedge* are inextricably bound up with hedge accounting. I will first provide the definitions of IAS 39 for these terms. Then, I will explain the cash flow hedging and fair value hedging further with two examples. This will make the differences between the two types of hedge relationships clear.

Definitions of IAS 39:

“A fair value hedge is a hedge of the exposure to changes in fair value of a recognised asset or liability or an unrecognised firm commitment, or an identified portion of such an asset, liability or firm commitment, that is attributable to a particular risk and could affect profit or loss.”

“A cash flow hedge is a hedge of the exposure to variability in cash flows that (i) is attributable to a particular risk associated with a recognised asset or liability (such as all or some future interest payments on variable rate debt) or a highly probable forecast transaction and (ii) could affect profit or loss.”

2.4.1 Example fair value hedge

To hedge a financial debt multiple strategies exist. One of them is hedging the financial debt with an interest rate swap. The fixed rate of the financial debt is then offset by an equivalent fixed rate of the interest rate swap and only the floating rate payments of the swap remain. In this way the interest rate risk in the financial debt has been hedged.

For example, consider a financial debt with a 5% annual coupon. To hedge this position, company Y has entered into an interest rate swap with a 5% annual coupon fixed leg and a EURIBOR floating leg. If the payment dates of the financial debt and the interest rate swap match, the fixed coupon payments cancel out. The only cash flows that remain are the EURIBOR based cash flows of the floating leg of the swap. These remaining payments are not subject to interest rate risk and the fair value of the financial debt has been hedged.

2.4.2 Differences in fixed rates of hedged item and swap

Now consider the case that the EURIBOR floating leg of the swap includes a spread of 25 basis points. Although the fixed rates of the financial debt and the swap are the same, the fixed parts of the cash flows do not match this time. This is due to the 25 basis points spread in the floating leg of the swap.

The payment rates of both financial instruments are summarized in table 1.

Table 1. Payment rates

	Swap	Debt
Fixed leg	+5%	-5%
Floating leg	-(EURIBOR + 0.25%)	

This is equivalent to the payment rates in table 2.

Table 2. Payment rates

	Swap	Debt
Fixed leg	+4.75%	-5%
Floating leg	- EURIBOR	

From table 2 it is clear that the fixed cash flows are not the same for the hedging instrument and the hedged item.

In order to improve the effectiveness of the hedge relation, it would be better to hedge just a part of the financial debt. According to IAS39 paragraph 81 it is allowed to adopt a partial hedge. In this particular case it would be best to hedge only 4.75% of the total coupon of the debt with the interest rate swap. Then, the fixed parts of the swap and the financial debt match and effectiveness is higher.

2.4.3 Example cash flow hedge

A cash flow hedge is ‘the other way around’. Consider company Z has a contracted transaction for the delivery of 10 tonnes of pork bellies 1 year from now. At that time the company will deliver the pork bellies at the market price on that moment. So, the absolute value of the transaction is unknown.

This transaction could be hedged with a cash flow hedge. Company Z enters into a pork belly futures contract with a nominal amount of 10 tonnes. The payoff of the futures contract (1 year from now) is the difference between the market price for pork bellies at that time and the fixed price in the futures contract.

So, 1 year from now the company will receive the market price for 10 tonnes of pork bellies, plus the difference between the market price and the fixed contract price (this could be negative). Hence, in all scenarios the price that will be received is the fixed contract price in the futures contract. Therefore, company Z has no risk of variation in cash flows. There are costs to buy a futures contract. However, this price is also fixed.

2.4.4 Accounting rules

The rules for cash flow hedges and fair values hedges are:

- For cash flow hedges the effective profit and loss of the hedging instrument is initially recorded in equity. The profit and loss is transported to the profit and loss account as the hedged item affects profit or loss.

- In fair value hedges the hedged item and hedging instrument are both recorded in the profit and loss account. Changes in fair value of the hedged item and hedging instrument affect the profit and loss account directly. Because of the structure of a fair value hedge, these fair value changes are expected to be highly offsetting.

2.5 Hedge effectiveness testing

For both cash flow hedges and fair value hedges effectiveness can (and should) be tested. The way to apply hedge effectiveness tests to these different types of hedges has a lot of similarities. In figure 3 I present the process of hedge effectiveness testing.

Hedge effectiveness testing

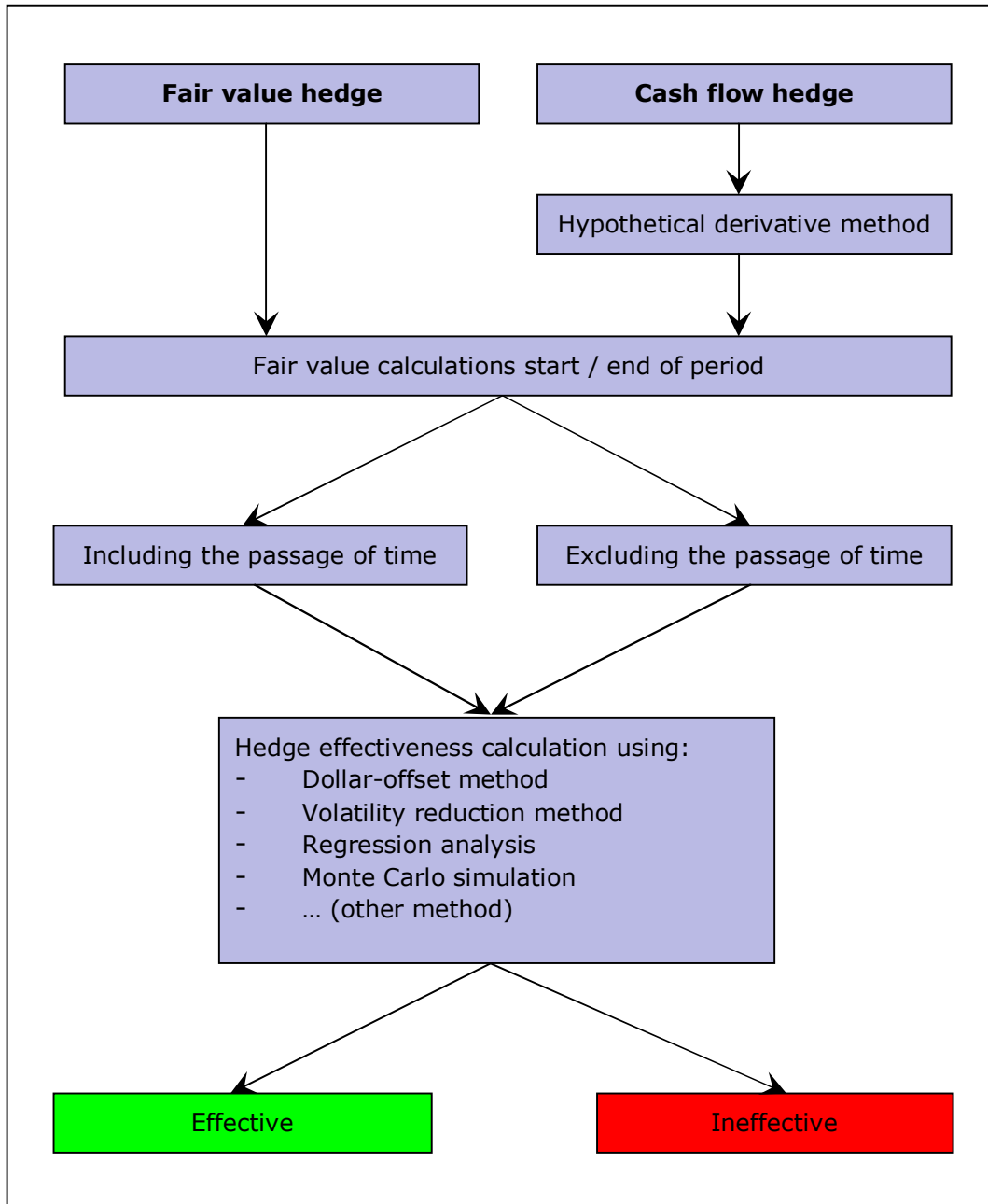


Figure 3. Hedge effectiveness testing process

For cash flow hedges the hypothetical derivative method should be used. This method is described in paragraph 2.6.

When calculating the fair values there are different options. One of them is to include the passage of time; the other is to exclude the passage of time. These methods are described in detail in chapter 3. In short, including the passage of time involves ‘normal’ fair value calculations at the

start and end of the period. Excluding the passage of time means the fair value at the start of the period is calculated with the remaining cash flows at the end of the period. The fair value at the end of the period is calculated normally. So, only the hedged risk (i.e. interest rate risk) is measured and the risk of fair value changes due to the passage of time is excluded.

After the fair value calculations at the start and end of the period, hedge effectiveness can be calculated. Therefore, multiple methods exist. An entity is free to use the method they want, as long as it is documented in the hedge documentation and the method is not overly optimistic in assessing hedges as highly effective. The most common methods include the Dollar-offset method, the Volatility reduction method and Regression analysis.

The process specified above is used for prospective testing as well as retrospective testing. The goal of prospective testing is to prove if the hedge is expected to be highly effective. If so, the hedge relationship qualifies for hedge accounting. Retrospective testing is applied at the end of a period (on an ongoing basis) and assesses if the hedge relationship can be recorded in the financial statements using hedge accounting.

I will describe each of the phases in the hedge effectiveness testing process, starting with the hypothetical derivative method. Then, I will discuss including the passage of time and excluding the passage of time. Thereafter, I will describe the most common methods for hedge effectiveness testing and describe how they can be used for prospective and retrospective testing.

2.6 Hypothetical derivative method

The hypothetical derivative method is only applicable for cash flow hedges.

The problem for cash flow hedges is that a hedge effectiveness test using fair value calculations is inappropriate. For example, consider a floating rate note is hedged with an interest rate swap (cash flow hedge). Assume the fair values of the floating rate note and the interest rate swap are calculated at the start and end of a period. Furthermore, assume that there were interest rate movements during this period. The fair value of the interest rate swap changed because of the interest rate movements. However, the fair value of the hedged item (a floating rate note) is insensitive to interest rate changes. If a hedge effectiveness test uses these fair value changes to calculate hedge effectiveness, the result will be that the hedge is assessed as ineffective. This is also the case when the cash flow hedge is a perfect hedge.

The hypothetical derivative method makes this comparison of fair value changes possible. This is achieved by designing a hypothetical derivative for the hedged item. The hypothetical derivative will have terms that exactly match the critical terms of the hedged item. Next to this, the hypothetical derivative could contain extra (fixed) cash flows in comparison to the hedged item.

At inception of the hedge relationship the fair value of the hypothetical derivative should be 0. For example, consider a floating rate note that is hedged with an interest rate swap (cash flow hedge). The hypothetical derivative is created by adding a fixed leg to the floating rate note. The resulting interest rate swap should have a fair value of 0 at inception of the hedge. This is attained

by setting the fixed rate of the interest rate swap at that rate that makes the fair value of the swap at inception equal to 0.

The hypothetical derivative is used, instead of the hedged item, to calculate fair value changes from the start until the end of a period. Fair value calculations for the hedging instrument are unchanged.

3 Including / excluding the passage of time

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4 Hedge effectiveness testing

First, if you have a cash flow hedge, a hypothetical derivative has to be designed which has the effect that you can calculate comparable fair value changes for the hedged item and hedging instrument. The next step is, for a fair value hedge and a cash flow hedge, choosing between the including the passage of time method and the excluding the passage of time method, as discussed above.

Then, you are able to calculate the effectiveness of the hedge relationship. For this purpose different methods are available. These will be discussed in this chapter in general.

4.1 Prospective / Retrospective

In hedge accounting both prospective and retrospective hedge effectiveness testing are important. For example, at initiation of the hedge relationship it has to be proven that it is highly effective. This is where prospective hedge effectiveness testing begins. The goal of prospective testing is to conclude if the hedge is *expected to be highly effective*. If the hedge relationship is not expected to be highly effective the application of hedge accounting is not allowed.

The precise procedure for prospective hedge effectiveness testing was quite unclear in the beginning. The International Accounting Standards Board (IASB) commented as follows:

“Additionally, comments made in response to the Exposure Draft Fair Value Hedge Accounting for a Portfolio Hedge of Interest Rate Risk demonstrated that it was unclear how the prospective effectiveness test was to be applied. The Board noted that the objective of the test was to ensure there was firm evidence to support an expectation of high effectiveness. Therefore, the Board decided to amend the Standard to clarify that an expectation of high effectiveness may be demonstrated in various ways, including a comparison of past changes in the fair value or cash flows of the hedged item that are attributable to the hedged risk with past changes in the fair value or cash flows of the hedging instrument, or by demonstrating a high statistical correlation between the fair value of cash flows of the hedged item and those of the hedging instrument. The Board noted that the entity may choose a hedge ratio of other than one to one in order to improve the effectiveness of the hedge as described in paragraph AG100.”

Prospective testing using historical data about fair values of the hedged item and hedging instrument is the most easy to use with the methods I will describe in the next paragraphs.

Retrospective hedge effectiveness testing is performed afterwards. The conclusion of a retrospective hedge effectiveness test is the assessment of a hedge relationship as effective or ineffective for a specific period. If the assessment of the hedge relation is ineffective, the fair value changes in the hedged item and hedging instrument cannot be excluded from the profit and

loss account. So, prospective hedge effectiveness testing is necessary for assessing the performance of the hedge relationship beforehand and retrospective hedge effectiveness testing is necessary to determine where the fair value changes of hedged item and hedging instrument should be recorded.

For all hedge effectiveness testing methods the fair values of the hedging instrument and hedged item need to be calculated. Formula (3) states how to calculate the fair value of a series of n yearly cash flows. In this formula CF_t is cash flow t , r_t is the interest rate at time t and t is the time until the cash flow in years.

$$FV = \sum_{t=1}^n \frac{CF_t}{(1+r_t)^t} \quad (3)$$

In the next paragraphs I will describe the most common hedge effectiveness testing methods and investigate how they are applied for prospective and retrospective testing. These methods are:

- Dollar-offset method
- Volatility reduction method
- Regression analysis
- Monte Carlo simulation

4.2 Dollar-offset method

4.2.1 Retrospective hedge effectiveness testing

The Dollar-offset method calculates hedge effectiveness using a ratio of fair value changes.¹ The easiest case is for retrospective hedge effectiveness testing. Therefore, fair values for the hedged item and hedging instrument are calculated for the start of the period and the end of the period. The change in fair value of the hedging instrument should be approximately opposite to the change in fair value of the hedged item. For a perfect hedge the fair value changes would exactly offset. Then, the ratio between the fair value change of the hedging instrument and the fair value change of the hedged item is calculated. This ratio is multiplied by -1. The result is the Dollar-offset ratio. In formula:

$$\text{Dollar-offset ratio} = - \left(\frac{\Delta_{FV_instrument}}{\Delta_{FV_item}} \right) \quad (4)$$

¹ Finnerty, J.D. and D. Grant (2002), *Alternative Approaches to Testing Hedge Effectiveness under SFAS 133*, June edition of Accounting Horizons.

For a perfect hedge the Dollar-offset ratio is equal to 1, which will be stated as 100%. If the Dollar-offset ratio for a hedge relationship is larger than 80% and smaller than 125%, the hedge is considered effective. The logic behind the [80%, 125%] interval is that the Dollar-offset method is independent of the arbitrary choice of the numerator and denominator because 80% is 4/5 and 125% is 5/4. So, the Dollar-offset ratio could also be calculated using formula (5).

$$\text{Dollar-offset ratio} = - \left(\frac{\Delta_{FV_item}}{\Delta_{FV_instrument}} \right) \quad (5)$$

This would result in a different Dollar-offset ratio, but the classification of the hedge relation as effective or ineffective will be the same. However, the formula for the Dollar-offset ratio that is first mentioned is used most often.

In the following example a hedge relation is analyzed from the time point t_0 until the time point t_1 . Assume the hedged item is a fixed rate bond with fair value 100 at t_0 . This fixed rate bond is hedged with an interest rate swap which has a fair value of 0 at t_0 . However, it is not a perfect hedge, because the fixed rate of the interest rate swap is slightly different from the fixed rate of the bond.

Consider what happens when at time point t_1 the hedged item has value 95 and the hedging instrument has value 6. See table 26.

Table 26. Example Dollar-offset method

Time	Hedged item	Hedging instrument
0	100	0
1	95	6
Difference	- 5	+ 6

In this example the Dollar-offset ratio yields $-\frac{6}{-5} = 1.2$, or 120%. Because 120% is in the interval [80%, 125%], the hedge is assessed as effective from time period 0 to 1 by the Dollar-offset method.

The Dollar-offset ratio is very sensitive to small changes in the fair values of the hedged item or the hedging instrument over a period. To illustrate this, consider the example above again. Consider a period where the change in the fair value of the hedged item is just -0.01 . For the hedge relation to be effective, the fair value change of the hedging instrument must be exactly $+0.01$. Otherwise, the Dollar-offset ratio will not be in the interval [80%, 125%]. For example, if the change in fair value of the hedging instrument is $+0.03$, the Dollar-offset ratio is equal to

300%. Hence, the hedge relation is assessed as ineffective by the Dollar-offset method. However, the absolute ineffectiveness is very small. This problem is known as the *small number problem*. If the change in fair value of the hedging instrument is 0, the denominator of the Dollar-offset ratio is zero and the Dollar-offset ratio is therefore undefined. This is a special case of the small number problem.

4.2.2 Prospective hedge effectiveness testing

For prospective hedge effectiveness testing the impact of a change in the underlying yield curve could be calculated. For example, value both the hedged item and the hedging instrument at inception with the yield curve with a 100 basis points parallel shift. Now, calculate the fair value change of the hedged item and the fair value change of the hedging instrument with respect to the valuation with the current yield curve. Apply the Dollar-offset method on these fair value changes.

Strict rules don't exist on how to apply prospective hedge effectiveness testing. For example, the fair value changes of a 100 basis point shift in the yield curve 3 months ahead could also be used to calculate hedge effectiveness. In this way, the passage of time is included.

4.3 Volatility reduction method

A different approach to evaluating the effectiveness of a hedge is to calculate the degree of volatility reduction. This involves a comparison of the volatility (standard deviation) of the combined portfolio (hedged item and hedging instrument), which will be called the package, with the volatility of the hedged item. Therefore, a Volatility reduction measure (VRM) exists; see formula (6).²

$$\text{VRM} = 1 - \frac{\sigma_{\text{package}}}{\sigma_{\text{hedged item}}} \quad (6)$$

For a highly effective hedge the volatility of the package should be much lower than that of the hedged item. In case of a perfect hedge the Volatility reduction measure is equal to one (or 100%).

For example, assume a fixed rate bond is hedged with an interest rate swap (fair value hedge). Assume the notional amount of the bond is EUR 1 million and the notional amount of the interest rate swap is also EUR 1 million. Every month the fair value of the bond and the interest rate swap is calculated and recorded. The total period is equal to 1 year.

² Kalotay, A. and L. Abreo (2001), *Testing Hedge Effectiveness for FAS 133: The Volatility Reduction Measure*, Volume 13 Number 4 of Winter edition of Journal of Applied Corporate Finance.

For each month the fair value change of the hedged item and hedging instrument is calculated. Furthermore, the fair value change of the package is calculated by summing these changes. Table 27 presents the fair value changes of the hedged item, hedging instrument and package for each month.

Table 27. Fair value changes hedged item, hedging instrument and package

Time	Δ Item	Δ Instrument	Δ Package (item + instrument)
1	+6000	-7000	-1000
2	-3000	+3500	+500
3	-2000	+1000	-1000
4	-500	+1000	+500
5	0	0	0
6	-6250	+5000	-1250
7	+3000	-4000	+1000
8	+8000	-6750	+1250
9	-1000	+500	-500
10	0	+1000	+1000
11	+500	-500	0
12	-2500	+2000	-500
Standard deviation	3913		859

As shown in table 27, the standard deviation of the changes in value of the item alone is 3913; that of the package is 859. Hedging with the interest rate swap has reduced the volatility to $(859 / 3913 * 100\% =) 21.97\%$ of what it originally was. In other words, a volatility reduction of 78.03% has been achieved. This is exactly the result of the Volatility reduction measure, as

$$1 - \frac{\sigma_{\text{package}}}{\sigma_{\text{hedged item}}} = 1 - \frac{859}{3913} = 0.7803.$$

There is no clear boundary on when to assess the hedge relation as effective and when to assess the hedge relation as ineffective. I will investigate this further later on.

4.3.1 Prospective hedge effectiveness testing

For prospective testing historical fair value changes should be used. On these data points the volatility of fair value changes of the hedged item could be calculated. Also use these historical fair value changes to calculate the standard deviation of the package. There are no hard rules on the number of data points that should be used.

4.3.2 Retrospective hedge effectiveness testing

For retrospective hedge effectiveness testing the fair value changes of the current period should be used. Historical fair value changes of former periods should be added as extra data points. There are no formal requirements on the minimum number of data points. However, for the first period there is only one data point. The Volatility reduction method is not applicable for one data point. So, historical data points could be used or simulation could be used to create extra data points. If a new period has passed, there is an extra data point available which replaces one of the simulated or historical data points.

4.4 Regression analysis

4.4.1 Prospective hedge effectiveness testing

Regression analysis is a statistical technique to describe linear relations between variables. In a hedge effectiveness context, we can use regression to estimate the parameters in the following relations:³

$$y = \alpha + \beta x + \varepsilon \quad (7)$$

Some assumptions are related to this regression equation. For example:

- The error terms ε are normally distributed with expectation 0 and constant variance σ^2
- The error terms ε are uncorrelated ($\text{Cov}(\varepsilon_i, \varepsilon_j) = 0$ for all $i \neq j$)

Another regression function is stated below.

$$y = \beta x + \varepsilon \quad (8)$$

For this equation some of the assumptions are not adhered to. For example, the error terms do not have an expectation of 0. Furthermore, the error terms are not independent.

In these equations y equals the changes in fair value of the hedged item and x equals the changes in fair value of the hedging instrument. The difference between the two equations is that the second determines the line of best fit that passes through the origin. The intercept α measures the expected change in fair value of the hedged item when there is no change in the fair value of the hedging instrument. The slope β measures the magnitude of the change in fair value of the hedging instrument relative to that of the hedge item. A perfect hedge should have a slope of 1 (or -1, this depends on the definition of hedge effectiveness testing) and an intercept of 0.

The regression analysis in figure 4 is performed with equation (8).

³ Kawaller, I.G. and R.B. Steinberg (2002), *Hedge Effectiveness Testing Using Regression Analysis*, September/October edition of AFP Exchange.

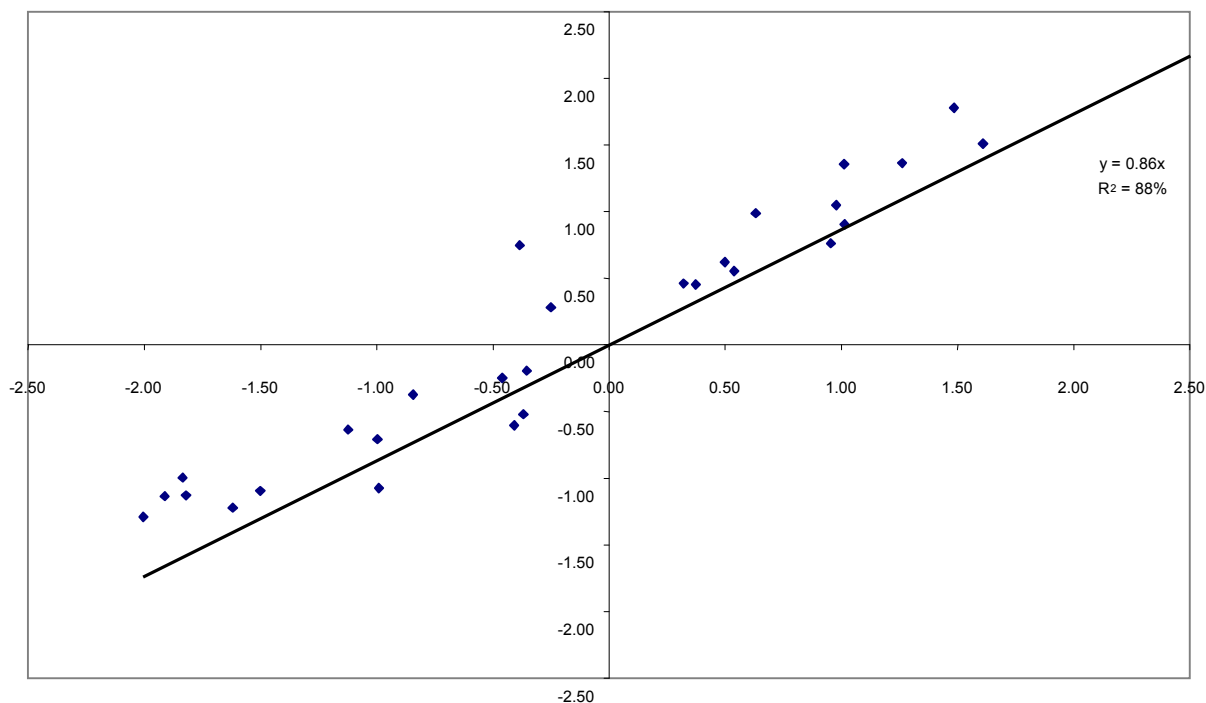


Figure 4. Example regression analysis

Regression analysis is very useful for prospective testing, as multiple regression points are needed. That is, multiple observations for fair value changes of the hedged item and corresponding fair value changes of the hedging instrument. These can be gathered from historical data.

Regression analysis can be applied to predict how the fair value of the hedged item is expected to change according to changes in fair value of the hedging instrument. So, the optimal ratio between hedging instrument and hedged item could be estimated.

An important step when using regression analysis is validation. For example, assume the slope is very close to zero, say 0.05. This would imply that the optimal ratio between hedged item and hedging instrument is 20 to 1.

However, a much better conclusion would be that the hedging instrument is not suitable to hedge the hedged item.

Another example is that the slope estimate of the regression analysis equals 1, but the variance of the measurement errors is enormous. In this case, the regression analysis is also not a reliable measurement for the quality of the hedge relationship.

The validity of a linear regression analysis can be checked with the R^2 statistic. This is sometimes called the ‘coefficient of determination’. With formula (9) R^2 can be calculated.

$$R^2 = \frac{\text{Regression sum of squares}}{\text{Total sum of squares}} = \frac{\sum_{i=1}^n (\hat{Y}_i - \bar{Y})^2}{\sum_{i=1}^n (Y_i - \bar{Y})^2} \quad (9)$$

In this equation n equals the number of data points, \bar{Y} is the average y value and \hat{Y}_i is the fitted regression value for measurement i.

The R^2 statistic belongs to the standard output for regression analysis in most statistical software packages. A regression analysis with an R^2 equal to or larger than 80% is seen as valid.

Note that the formula to calculate R^2 is based on regression analysis where all the assumption as mentioned before are adhered to. For regression analysis according to formula (8) (with the alpha parameter forced to 0) the formula to calculate R^2 is not applicable. J.G. Eisenhauer suggests in his paper “Regression through the Origin” to use the square of the sample correlation between observed and predicted values instead.

After validating the regression analysis, the results of the regression analysis should be used to assess the hedge as effective or ineffective. There are many possible boundaries (for example for β) which determine when a hedge is assessed as effective. The determination of these boundaries is a difficult exercise and is subject to subjectivity. I will elaborate on this later on in this paper.

4.4.2 Retrospective hedge effectiveness testing

For retrospective testing the focus is usually on one period. It is questioned if the fair value changes of the hedged item and the fair value changes of the hedging instrument are of the same order of magnitude, such that the hedge could be assessed as effective. However, to test the hedge relation retrospectively with regression analysis, this amount of data is not enough. To be able to apply regression analysis more than one point is needed. Therefore, for retrospective testing with regression analysis historical data about fair value changes is used. Hence, retrospective testing has a lot of similarities with prospective testing for this method.

Consider a hedge relationship is prospectively tested to be highly effective, using 30 data points of historical fair value changes. Then, assume 3 months pass and the fair values of both the hedged item and the hedging instrument have changed. For this period the effectiveness of the hedge should be assessed. The retrospective hedge effectiveness test for the first period of this hedge relation is performed on the same data points *plus* the data point that is obtained from the fair value changes in these 3 months.

4.5 Monte Carlo simulation

Another way of hedge effectiveness testing is using Monte Carlo simulation for scenario analysis. Monte Carlo simulation is used to simulate the fair value of the hedged item and hedging instrument over a predefined period. Next to these fair values the fair value of the package of hedged item and hedging instrument is also calculated. A measure for effectiveness is the change in fair value of the package in relation to the initial fair value of the hedged item. If the change in fair value of the package is very large in comparison with the initial value of the hedged item, the hedge relation is assessed as ineffective prospectively. In advance a maximum should be set on the change in fair value of the package (hedged item and hedging instrument together) in relation to the initial value of the hedged item.⁴ The hedge is assessed as effective if this maximum is exceeded in only a small fraction of the scenarios. The different scenarios that are simulated must include a realistic mix of stable conditions and extreme market moves. This method is used for prospective testing.

Monte Carlo simulation can also be applied as support for the other hedge effectiveness testing methods. With Monte Carlo simulation fair value changes of the hedged item and hedging instrument are simulated. To the generated fair value changes one of the methods defined above (Dollar-offset method, Regression analysis or Volatility reduction method) could be applied.

It is possible to generate fair value changes in various ways. There are no formal requirements on how to generate the fair value changes. In my opinion the most beautiful method is the one suggested by Andrew Kalotay and Leslie Abreo.⁵ Within this method interest rate movements are simulated and the fair values of hedged item and hedging instrument at the end of the testing period are calculated with each simulated yield curve. The steps of the methods are as follows if the testing period is equal to 3 months:

- 1) From the available set of historical yield curves, calculate daily ratio vectors. The ratio vector of day t contains for each maturity a ratio which equals $\frac{\text{interest rate at day } t}{\text{interest rate at day } t-1}$. The different maturities in the ratio vector vary from a few months to 40 years. So, the change of the whole yield curve in one day is described by one ratio vector. For example, a ratio vector could look like: $\left[\frac{3.45}{3.5}, \frac{3.6}{3.7}, \dots, \frac{4.75}{4.6} \right]$
- 2) For one simulation step draw randomly 62 of these daily ratio vectors and multiply these vectors. The testing period of 3 months is assumed to contain approximately 62 trading days. Now, one yield curve transformation vector is obtained.

⁴ Kawaller, I.G. and P.D. Koch (2000), *Meeting the "Highly Effective Expectation" Criterion for Hedge Accounting*, p. 84

⁵ Kalotay, A. and L. Abreo (2001), *Testing Hedge Effectiveness for FAS 133: The Volatility Reduction Measure*, p. 99

- 3) Multiply the yield curve at the start of the period with the transformation vector. In this way a simulated yield curve at the end of the period is obtained.
- 4) Value the hedged item and hedging instrument with the simulated yield curve and calculate the fair value changes compared to the start of the period.
- 5) Repeat step 2 – 4 until a reasonable number of fair value changes is obtained

As support for other hedge effectiveness testing methods Monte Carlo simulation is primarily used for prospective testing. Sometimes it is used for retrospective testing when extra data points are required besides the fair value changes of the current period.

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