Towards value-based ALM

Applying risk-neutral valuation techniques in the pension fund industry

Master thesis Business Mathematics and Informatics

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Towards value-based ALM: Applying risk-neutral valuation techniques in the pension fund industry

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ABSTRACT. This master thesis describes the investigation towards the possibilities of value-based Asset and Liability Management (ALM) and the additional value of this approach with respect to the traditional ALM method. Value-based generational methods give insight how the modifications in the financing setup can lead to the value transfers between generations. The goal of this research is extending of the existing system with risk-neutral scenarios for real interests and inflation. This thesis evaluates the policy alternatives and analyzes value transfers between a pension fund and the pension fund members by computing the value of the embedded options in a pension deal. Two case studies represent the results where it is clarified which party gains and which party loses in economic value terms from changes in the funding strategy of risk allocation rules. The advanced technique enables the risk management institutions to analyze their balance sheet risks in much more realistic and dynamic way.

Key words: value-based ALM, embedded options, risk-neutral valuation technique, intergenerational risk sharing.

Contents

PREFACE

The final part of the study program of Business Mathematics and Informatics (BMI) at the VU University in Amsterdam consists of an obliged internship. The purpose of such a working period is to gain experience in a business area and to solve a real business problem by using the knowledge acquired over the years. The problem needs to contain economical, mathematical and IT aspects.

This master thesis describes my master project within Pension Funds/Asset Managers Systems, one of the Business Units of ORTEC Finance. I have been working within the ALM-studies group that is responsible for support to the Pension plans in establishing an optimum strategic policy.

I have enjoyed working at ORTEC and I want to thank my colleagues for a valuable working experience. My special thanks go to Henk Hoek for sharing his excellent knowledge and expertise. To David van Bragt who helped me through the first part of my internship. I want to thank Marnix Engels for always excellent clear explanations and help with technical questions. I also want to thank Pim van der Stoel for supervising and motivating me. Finally, to my supervisors from Vrije University, Sandjai Bhulai and Harry van Zanten, for the comments and careful reading.

CHAPTER 1

INTRODUCTION

This chapter sketches the business area in which the company of my internship active. The description of the problem and the goal of the project is given. Furthermore, I give a general description covering the main points of the thesis.

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1. Business Environment

Demographers and the pension policy makers for the last 30 years underestimated the further increasing life duration of the population. The ageing and the shrinking of the working population are the main causes of high pension contributions and low pension payments. This leads to the question how the pension costs must be realized. Pension funds play of course an important role in the economy and have particularly a responsibility as a longterm investor. According to Theo Kocken, the aging society will put pressure on labor markets, health care and pension provisions for the forthcoming three decades. Decreasing birth rates, less workers, and an ageing population are asking, however, for firm reforms in the short term. Western countries have gradually moved into a situation where managing the indexation ambition and the related interest rate risk became a challenge. The Netherlands works actively on adapting the pension sector to this new era. (The current pension system is based on rebalancing of assets and risk.)

New developments in option theory (Kocken, 2006) change at present the structure of the financial markets and risk management analysis and tools. Using techniques from option theory makes possible to design a new pension system that is firm, righteous and economic situation durable. This new pension system can anticipate on the consequences of a rapidly ageing population. The topic about the need of improvement and innovation in the pension fund area has moved from a peripheral issue to a major source of concern around the world. At a financial level, the area of market risk management is becoming increasingly important.

Problem Description
Pension funds use ALM analysis to evaluate the pension deals in operation and to explore the performance of alternative pension deals. Usually the Asset and Liability management of a pension fund (referred to as *Classical ALM*) uses an economic model to simulate relevant variables, like inflation, interest rates, and returns on assets. For each of those scenarios the policy and the policy alternatives are calculated and analysed. In this evaluation the probability (like probability of underfunding) and an average (like average contributions rate) are most important.

The disadvantage of Classical ALM is the impossibility to discover the value transfers between shareholders and the impossibility to indicate hidden value transfers after the policy adjustment. Besides the traditional ALM the pension fund experts are currently investigating a *Value-based* generational accounting method, which gives insight into how the modifications in the financing set-up can lead to the value transfers between generations. Value-based ALM essentially uses the same output of scenario analysis as classical ALM, however, the future outcomes are discounted back to the present with an appropriate risk adjusted discount rate. Within the same pension scheme, the alternative asset mix for the different interest-groups will have different consequences. It can depend on the individual investment horizon. During the internship I will support the development and use the value-based ALM technique to evaluate the policy alternatives and to analyze the value transfers between the young, old, and future participants.

2. Goal of the project

In first part of my project I will conduct a literature study about pensions. In this study, special attention to the question about value transfers among generations will be given. Especially, I will give attention to the PhD thesis of Theo Kocken, who explained the balance sheet of a pension fund in terms of embedded options, and to the several works of N. Kortleve and E. Ponds. During the internship my tasks are to support the developing and testing the the economic scenario generator using inflation-linked products. The model that will be used in the project is a combined two-factor Hull-White Black-Scholes model. The insurers use Monte Carlo simulation to value embedded options. The same valuation will be used for pension funds. The existing system should be extended with risk neutral scenarios for real interests and inflation. The value-based ALM techniques should be applied and their additional value investigated and compared to classical ALM. Research about the possibility of the extension of the system with a single currency should be done.

3. Outline of this report

This report is structured as follows. Part 1 consists of summary of important basic knowledge of pensions and ALM, and focuses on the embedded options and risk sharing in particular. This part consist of 4 chapters and offers a theoretical foundation on value-based ALM and some relevant information about the policies, instruments, and interested parties involved in the decision making process to find the best possible pension deal. Part 2 consists of the chapters which give some background information about the risk-neutral valuation and mathematical background of the arbitrage free scenario simulations where the valuation techniques are based on. This part focuses on the research of the theoretical and Hollandia pension funds, where embedded options and intergenerational value transfers are investigated. The Summary & Recommendations chapter represent the main points

of the investigation and the topics for the future research.

Part 1

PART I: Literature Study

CHAPTER 2

UNDERSTANDING PENSIONS

In this chapter I sketch what a pension is, which types of pensions exist and the various ways in which pension rights are accumulated. Further, I describe various aspects which are directly related to ALM problems: interested parties, instruments which are at disposal of the boards of pension funds and the supervisors.

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1. Pensions and pension types

A *Pension* is a regular payment, which replaces the former salary, made to an unemployed person. This monthly income, usually associated with the period after retirement, can also be provided in case of disability or death of an employee. There are many types of pensions. A pension is a general term used to describe an investment built up during the working life and used at retirement to purchase an annuity to provide a continuing income. Many types of pension funds exist. Moreover, several ways to build up pension rights exist.

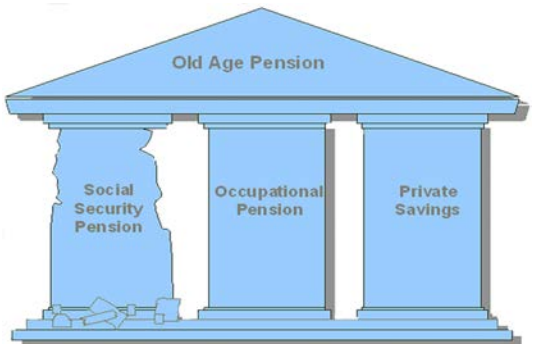

FIGURE 1. Three Pillars of the Pension System

In the Netherlands there are three pillars concerning pensions. The first pillar involves the public pension, provisioned by the government for everyone living in the Netherlands aged 65 and over. The first pillar is the state retirement pension or AOW, based on statutory national insurance. It is financed on a cost-allocation basis, which means no pension fund is formed as such.

- **State retirement pension AOW (Algemene Ouderdomswet)**
  This is a pension for the lifelong financial care of a person, after the pensionable age is reached. The retired person receives an AOW pension as a percentage of the net minimum wage.

The second pillar covers the pension scheme which depends on the labor relationship between the employer and the employee. The second pillar schemes are administered by pension funds and life insurers, who underwrite the pension payouts pledged by employers.
Descriptions of the most important types of pensions within the second pillar are as follows.

- **Old Age Pension (Ouderdomspensioen)**
  Up to the retirement age, everyone with a taxable income has to pay the Old Age Pensions contributions. This is a supplemental collective pension arrangement, which is part of the terms of employment agreed in negotiations between social partners. Depending on the pension system the participants built-up inferior or higher pension.

- **Widow's pension**
  This pension is payable to widows/widowers whose partner passed away, depending on his/her contribution record. Generally, this payment is made lifelong.

- **Partner pension**
  This is another survivors benefit which is equivalent to a widow's pension. This pension applies for people who live together without being married, and satisfy a number of conditions.

- **Orphan pension**
  This payment may be made to a person or family member who is taking care of an orphan. The person who supports a child is entitled to be paid an orphan's pension until the child has reached a pre-specified age.

- **Pension in case of disability WIA (Work and Income)**
  Since 1 January 2006 the WIA replaces the WAO (Invalidity Insurance Act) and applies to employees who have became incapacitated for work.

The third pillar consists of individual investments and life insurances, which everybody can contract with a life insurance company as an individual additional pension product. This pension has nothing to do with the relationship between employees and employers. There are two main forms of insurances in the third pillar. *Annuity*, the first form, entitles the holder to a fixed and periodic benefit that fluctuates with the value of the underlying investments. *Capital insurance*, the second form, is linked to the financing of a home and pays out a fixed amount of money against the risks of death or long life.

2. Pension fund types

The basis of the existence of pension funds is an agreement between generations and between various participants of a pension fund. If some participants die early, they will never profit from the contributions they made during the working period. On the other hand, other participants live longer than average and will receive more money from the fund than they actually saved by themselves. Because pension funds have a large number of participants, risks can be reduced.

- **Corporate pension fund**
  This type of pension fund has a single employer who contributes to the fund. Many employers, especially in the Netherlands, negotiate a fixed contribution for future years. This in fact leads to a system with the risk sharing only between employees, sleepers [1], and retirees. This system is called a collective defined contribution (CDC) system, where the employer only pays a defined amount but the participants remain collective in their risk sharing. Examples of Dutch pension funds of this category are the funds of Akzo Nobel, Phillips, Shell, and Unilever.

- **Industry-wide pension fund**
  A collective of many employers, often a mix of very small and very large firms, contribute to this kind of fund. Participating employees are employed in companies in the same branch of industry. Participation is mandatory. Indexation agreements are quite similar to those contained in corporate pension funds. In the Netherlands these are the pension fund for the building industry BPF Bouw and the Metal industry pension fund.

- **Public pension fund**
  This pension fund is similar to the industry-wide pension fund, although in this kind of fund there is sometimes only one or a limited number of employers. The possibility for financial injections in this fund are more flexible than in the industry-wide fund. Some public pension funds are directly related to the governmental authorities, and other indirectly, like university superannuation funds. Examples in the Netherlands are the Algemeen Burgelijk Pensioenfonds (ABP), and the Pensioenfonds voor de Gezondheidszorg, Geestelijke en Maatschappelijke Belangen (PGGM).

- **Specialists occupational pension fund**
  Participants in this fund are all professionals who have their own practice and all work in the same discipline, such as medical specialists, dentists, and lawyers. There is no employee-employer relationship and there is intergenerational risk sharing between active and retired participants in a savings system. The absence of a risk absorbing employer reveals the risk

[1] A sleeper or a former participant is entitled to a future pension benefit, but is no longer in service at the employer

aspects between the beneficiaries and increases the potential conflicts in the negotiation process.

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3. Pension systems

The pension systems in the Netherlands can be divided in two categories:

(1) **Defined Benefit Plan**
The main point of this system is *pension rights* which will be built-up in the service years and which will be acquired at the start of the pension date. According to this system, the employer pays a contribution to the pension fund, frequently including a contribution paid by the employee. The employee gets an additional pension right each year, let us say 1.75% of the pensionable salary. After 40 service years his/her pension will be equal to 70% of the salaries. Contributions that the participant pays during the service years depend on the interest rate. The pension sum is defined by the pension scheme, which is based on the final salary or on the average salary. I describe these two schemes and their variants.
Final pay scheme
In this scheme, the wage increase in the last year of service will affect the rights. The participant is guaranteed to receive benefits based on his pay in the last year of service and on the length of time he has been an employee.
Average pay scheme
In this scheme, every wage increase influences the pension that will be built-up in the remaining years of service.

(2) **Defined Contribution Plan**
A retirement plan wherein a certain amount or percentage of money is set aside each year by a company for the benefit of the employee. In this system, the employer yearly transfers money to purchase a part of the employee's pension. The level of the pension depends on the number of years the *pension contributions* have been paid, the realized return in the years the pension has been build up, and the interest rate at the moment of retirement. In other words, the employee gets a pension contribution from the employer and invests it, at own responsibility, to create the pension assets which are sufficient for his/her retirement. Nor the employer, nor the pension fund, carry responsibility for his/her pensions. There is no way to know how much the plan will ultimately give the employee upon retiring. The amount contributed is fixed, but the benefit is not. This system has fiscal consequences for the employee.

In the Netherlands, 91% of employees take part in the second pillar of the Dutch pension system. Of the second pillar schemes, 88% are defined benefit plans. The private sector provides also a voluntary early retirement scheme [**4**]. Eurostat, the European Commission's statistical office, reported in 2000 the pension assets in

different countries. Pension assets per person in European countries are presented in Table 1.

	Assets	Population	A/P		Assets	Population	A/P
AUS	€31 bn	8,1 mio	€3.827	ITA	302	57.7	5.234
BEL	33	10,3	3,204	NL	764	15,9	48.050
DK	204	5,3	38.491	POR	13	10,0	1.300
FIN	55	5,2	10.577	SPA	35	39,5	886
FRA	90	60,1	1.498	SWE	338	8,9	37.978
GER	316	82,1	3.849	UK	1542	59,7	25.829
IRL	54	3,8	14.211	*Average*	*291*	*28,2*	*14.995*

TABLE 1. Pension assets per person

CHAPTER 3

ALM FOR PENSION FUNDS

The ALM model is a financial model for pension funds which are developed to manage risks and to better understand them. With this model one can compute the impact of future capital market developments on their financial position. The ALM model focuses on the decision making problem of pension funds where flexibility plays an important role. The board of the funds can periodically change its decisions with respect to investments, contributions, and indexation. It makes possible to react on developments of financial markets and on wishes of the participants of the fund. To predict the possible future developments in uncertainties (like the future return on assets or inflations) a *scenario generator* is developed.

Before I describe the ALM model of ORTEC, first I consider the Asset Liability Management (ALM) problem for pension funds.

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1. Asset and Liability Management

Asset Liability Management for pension funds is a risk management approach, which takes into account the assets, the liabilities, and also the interactions between the different policies which the board of a pension fund can apply. The goal of an ALM study is to compose such an integral policy where a strategic investment mix, contribution and indexation policies are considered, with a view to stable and as low as possible pension costs.

The board of a pension fund should find acceptable policies that guarantee with large probability that the *solvency* of the fund is sufficient during the planning horizon. The solvency is the ability of the pension to fulfill all promised payments in the long-run and is measured as the *funding ratio* at the certain time moment. The funding ratio, which is the ratio of assets and liabilities, changes over time because of fluctuations in the liabilities and in the assets. Therefore, a pension fund rebalances its asset portfolio and adjusts its contribution rate regularly, in order to control changes of the funding ratio over time. In case of distress, the sponsor of the fund may have to help out with a remedial contribution.

If the funding ratio is less than one the *underfunding* occurs. Using other words you can characterize underfunding as a negative *surplus*. The surplus is the difference between the value of the assets and the value of the liabilities. The surplus is the part of the reserves of the pension fund that is not needed for paying benefit payments.

In the ALM decision process, conflicting interests exist. In the next section I will look in more detail at the interests of different parties and the instruments to control the funding ratio. Figure 1 shows objectives and constraints, risk drivers and policy instruments.

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2. Interested parties in the policy

At least five parties are involved in the decision making process by the board of the pension fund, or are interested in its results. First of all, the *active participants* (vakbond) are interested. They are especially concerned about the level of the contribution rate and their pension scheme (i.e., sufficient pension when they retire). Active participants make contributions on a regular basis to the fund to build up their rights concerning the type of pension described in Chapter 2. If the contribution rate increases, for example, the active participants have to make a larger contribution to the pension fund, which results in a lower disposable income. In particular, older active participants are also interested in the degree of indexation. They would like to be compensated for inflation in all years.

A second interested group consists of *retired persons* (Vereniging van gepensioneerden) and surviving relatives of them. For this group especially the indexation policy is important. Of course, they would like to receive full compensation for increases in prices or wages. Also this group is interested in the investing in less risky assets in order to keep the probability of underfunding low.

The *sponsor* of the fund (a company) is also involved. In situations of financial distress the sponsor plays an important role. If the funding ratio drops below a certain threshold, the sponsor of the fund may contractually be forced to restore the funding ratio. On the other hand, in case of financial prosperity, the sponsor may also benefit. However, not all pension funds have a sponsor. Every pension fund related to a single company has a sponsor. The government may act as a sponsor of the fund of civil servants. Funds related to companies in the same branch of industry, or funds for individuals with the same occupation, may not have a sponsor. Next to concerns about the level of contributions and restitutions, the sponsor is also interested in the costs associated with carrying out the pension administration.

Another party who takes a part in the discussion about the policy is the *supervisor* of the fund. Pension funds have to justify and report their activities to the supervisor. The role of the supervisor differs from country to country. The supervisor of the Dutch pension funds is De Nederlandsche Bank (DNB). Another authority, the Pensioenwet (PW) is a collection of laws and a number of instructions based thereupon, which protect pension benefits of employees. The DNB can give pension funds an instruction to bring their policies or execution of them into conformity with legislative provisions. The DNB also has the right to impose penalties and to report it if a fund is in breach of the law.

The greatest concerns of the *board of a pension fund* is the risk of underfunding: the risk that the value of the liabilities is higher than the value of the assets. In case of underfunding, the board of a pension fund has to inform the supervisor about this situation. The funding ratio should be sufficiently high within one year.

If the funding ratio is greater than one, but the buffers needed for investments are sufficiently large, the board should formulate a recovery plan.

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3. Policies and Instruments

The board of a pension fund has many instruments to control the funding ratio. The board should take into account the interests of all parties involved in the decision making process to find the best possible policy mix. I consider here the ALM process from the perspective of the pension fund. See Figure 1 for major policies and rules with which the fund can control the funding ratio.

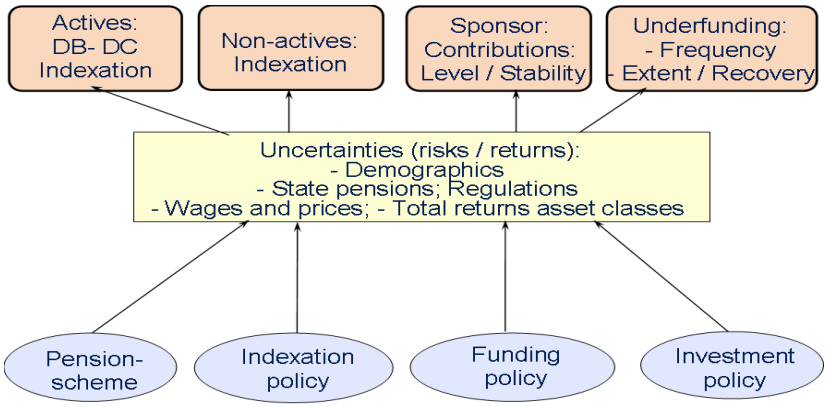

FIGURE 1. ALM System

- **Pension policy**
  The pension policy deals with decisions with respect to the different types of pensions that the fund includes in the pension regulation (Defined Benefit or Defined Contribution described in Chapter 2.3). The rules with respect to the benefit payments are registered in the pension rules. Especially the sponsor and active participants are interested in the pension rules, because they have to finance the system.

- **Indexation policy**
  The indexation policy is important in the valuation of the liabilities and (future) benefit payments. The board of the fund has to decide which base to use, for example, a consumer price index, or a wage index. Every year has to be decided whether the financial position of the fund suffices to give (full) compensation. An actuary plays a key role in this decision. Retired people and active participants would like to be compensated for increases in prices or wages. These are parties who benefit from indexing pension rights.

- **Funding policy**
  The board of a pension fund can not only manage its liabilities, also the assets can be managed. One of the instruments to manage the assets is by means of the funding policy. In the funding policy, the system is chosen on which the level of the contribution rate is determined. The levels of the lower and upper bounds that limit the funding level is part of the funding policy. Most pension funds use a dynamic contribution rate. In this system, the level of the contribution rate can be modified over the course of time. However, it is also possible that the different interested parties involved in the decision process agree about a fixed contribution rate. The active participants and the sponsor are the parties who are mainly interested in the level of the contribution rate, because they have to finance the system.

- **Investment policy**
  The value of the assets is also influenced by the investment policy. In this policy, the board of the pension fund decides in which asset classes the fund invests its assets. Also rules concerning rebalancing are part of the investment policy. For example, it is possible that investments are made in indices, or that assets are actively managed. Also investments to reduce risks, like currency hedging, are considered. The supervisor is concerned about the investment policy, because investments directly influence the risk of underfunding. Pension funds should invest their assets such that this risk is small. To do so, rules exist with respect to the levels of buffers which pension funds need if they invest in certain asset classes.

To be able to judge the financial position of pension funds well, not only the assets should be valued using observed market prices, but also a market value of the liabilities should be found. This is the result of discussions between pension funds, the supervisor and consultants.

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4. ORTEC

ORTEC Finance is one of the largest independent providers of financial solutions and professional consultancy. The company provides her clients policy advice and financial reports. In the institutional market ORTEC Finance integrates systems and consulting in the field of Asset Liability Management, Benchmark Construction, Portfolio Construction and Performance and Risk Management. The solutions are used by asset management companies, pension funds, insurance companies and housing corporations.

The ALM approach of ORTEC Finance has been further developed in the software program ALS (Assets and Liabilities Scenario-model). ALS was developed in order to provide pension plans with insight into the consequences of economic developments and proposed policy alternatives. In this way, the ALS helps to support pension plans in making strategic choices with regard to the pension, contribution, indexation, and the investment policy.

The ALM-software for pension plans is a professional Microsoft Windows application which is used by the ALM specialists (both internal and external) for:

- making actuarial costing forecasts,
- calculating open market valuations of pension liabilities,
- generating future liabilities forecasts,
- carrying out risk analysis,
- analyze consequences of local financial assessment frameworks,
- calculating valuations of embedded options.

The Pension Fonds/Asset Managers Systems is the business unit of ORTEC Finance where I am undergoing a supervised internship. ORTEC support the relationships with the academic world in the field of ALM and are, as a result, able to be constantly renew. With the advanced ALM systems ORTEC Finance is a forerunner at the social and scientific developments.

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5. Assets and Liabilities Scenario (ALS) model

The goal of the pension fund is to fulfill all obligations towards the participants. In this section, I describe the decision process in the way it is incorporated in the ALS model.

The Asset Liability Scenario-system (ALS) is a Windows-application designed to perform ALM-studies. The ALM-system is based on the concept of scenario analysis. ALS can be used to gain insight into the risks and returns of the current pension strategy. In addition, it gives insight into the effectiveness of alternative strategies.

The system can be divided into five modules:

- Actuarial module,
- Economics module,
- Financing module,
- Investment module,
- ALM-module.

I describe the functions of each module.

Actuarial module
The actuarial module is a flexible model for estimating the future developments of the fund-participants, the salaries, and the pension benefits. The main target of the actuarial module is to generate the mathematical benefits used in the ALM-analyses. Over the years, the benefits module has developed into a complete benefit prognosis system, which can also be used for: premium calculations, pension costs prognosis, examining the consequences of changes in the pension regulations, calculating the consequences of different discounting terms, etc. All of these aspects can be analyzed for the total population or for individuals.

Economics module
The economics module is used to generate the future scenarios of inflation, investment returns and other (macro-) economic series. The characteristics of these

scenarios can be based on an observed historic period or on prospective insights. This is done with a VAR model. The result of this module is a set of future scenarios for, for example:

- Wage- and price inflation,
- Interest rates of bonds with different maturities,
- Interest rates of cash,
- Total returns of stocks (yearly dividend plus market profit),
- Rental revenues and the increase in value of real estate.

Together they represent the uncertain extern economic environment for an ALM-context analyzed organization (pension fund, insurer, real estate corporation, bank, etc.) in the future.

Financing module
The Financing Module defines the agreements concerning (the height of) future contributions and indexation between sponsor(s) and pension funds. In this module, the actuarial variants will be selected, where the mathematical reserves will be based on. Further, the financing module offers the possibility to watch the development of the indexed reserves and cash flows. Finally, it is possible to simulate with fixed fund returns.

Investment module
The Investment module contains the definitions of the investment policy. A definition of the investment policy consists of the composition of the investment portfolio and the adjustment rules of this investment portfolio. The total investment portfolio consists of several different types of asset categories. The asset categories consist of specific input parameters that characterize the different categories. To smoothen the fluctuations in the investments it is possible to have an investment reserve. This acquires the investment reserve parameters to be filled in. Finally, the investment module contains the possibility to analyze the characteristics of the various asset categories.

ALM-module
ALM-policies are created in the ALM-field by combining an economic variant, a financing variant and an investment variant, the ALM variants are made, which are used for risk analysis. These ALM-variants can be calculated, and the results can be analyzed numerically and graphically.

CHAPTER 4

VALUE-BASED ALM

In the pension industry, Asset Liability management (ALM) is being used to come to an optimal funding strategy, indexation policy, and investment strategy for the fund. ALM analyzes the possible alternatives to form an optimal pension deal. Value-based ALM does not change the existing decision making, but adds an extra dimension by showing the present value - also called the economic value - of all decisions about the funding strategy, indexation policy and investment strategy. In this chapter I describe the technique, developed recently by Kortleve & Ponds, that can calculate the present value of contributions, benefits and shortfall/surpluses for the fund collectively and for various stakeholders.

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1. Value-based ALM vs. Classical ALM

Pension funds use ALM analysis to evaluate the pension deal in operation and to explore the performance of alternative pension deals. *Classical ALM* mainly uses items like the expected value of core variables and statistical models to simulate the relevant economic variables, like the returns on asset classes, inflation or interest. Classical ALM often makes use of techniques like Monte Carlo simulations to optimize the strategy of the fund. The output provides the insight in the distribution of the future possible policies. Policy variants are evaluated in terms of expected values (for example the funding ratio, the indexation rate, the contribution rate) and the probabilities, like a minimum probability of underfunding or the probability of a low indexation or no indexation at all. This give some idea about the sustainability of the pension deal in the long run. The problem with classical ALM in real life is the failure to make visible the hidden value transfers between stakeholders and the failure to indicate how the value transfer changes after the policy was adjusted.

Value-based ALM essentially uses the same output for scenario analysis as the classical ALM, however, the future outcomes are discounted back to the present with an appropriate risk adjusted discount rate. This is realized by discounting with either deflators, risk neutral valuation, or pricing kernels [**2**]. In the ORTEC system the outcomes are discounted with the risk neutral valuation.

The use of value-based ALM leads to at least two types of extra insights. The first new insight is the value the market currently attaches to future cash flows. ALM experts look at averages, but disregard information given by financial markets in the form of the present value of the future cash flows. Value-based ALM can calculate the present value of cash flows since these cash flows are linked to cash flows of financial titles like equities and bonds.

The second new insight is that one can look at the stakes of various parties and can see the impact of changing the pension deal on various stakeholders. This will help to avoid that one group member has to pay up for any shortfall but does not get compensated at the same time, and to avoid the negative impact on the present value for certain stakeholders. Ponds & Kortleve [**2**] employ the value-based approach to analyze transfers between old, young and future members within a pension fund where risks have to be born primarily by the plan members.

Recently the pension fund sector developed a method called *value-based generational accounting*. This method makes possible to identify if all interested parties (generations) get honest compensation for the risk allocated to them and to detect possible transfers of value in case of policy changes. In the following section I describe this method.

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2. Generational accounts in a pension fund

Pension funds can be considered as an insurance contract that is based on intergenerational risk sharing. Such a contract leads inevitable to hidden transfers of the value between generations, because real results can differ from the expectation. In this section, I focus on explaining the value-based generational accounting methodology developed by R. Hoevenaars and E. Ponds [**3**]. This method quantifies the value transfers between generations by computing path-dependent embedded options in the pension deal. Embedded options represent the fair price of risky payoffs in the pension deal. The uncertain cash flows from and to the participating cohorts, in particular contributions and benefits, are seen as embedded options which can be valued with stochastic discount factors.

The value-based approach reveals the zero-sum character of the deal. The zero-sum game in economic value terms means that the total economic value to be distributed amongst the generations is equal to the market value of the pension fund assets. Alternative funding and risk-allocation rules have no impact on the total economic value, however, this may lead to transfers of value between the age cohorts.

The balance sheet of the pension fund with market value of the assets A_t, total nominal liabilities L_t and the pension fund residue R_t is displayed below:

Activa	Passiva
A_t	L_t
	R_t

with $A_t = L_t + R_t$.

The participating age cohorts in a pension fund have a claim to these assets. For the cohort x this claim to the asset A_t^x is equal to the value of accrued liabilities of this cohort L_t^x *plus* the claim of this cohort to the residue R_t^x. We assume that the pension fund residue can be allocated amongst the cohorts at all times proportionally to the stake of the nominal liabilities of the cohort in total nominal

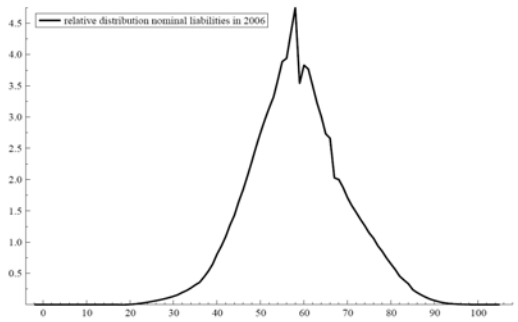

FIGURE 1. Relative distribution of the nominal liabilities in 2006 (y-axis) for different age cohorts with age in 2006 on the x-axis

liabilities: $R_t^x = \frac{L_t^x}{L_t} R_t$ [**3**]. The size of $\frac{L_t^x}{L_t}$ will decline in retirement as liabilities gradually are written off with the planned pension payments.

The junction of all balances of all cohorts is displayed in the balance of the pension fund below:

Activa	Passiva
A_t^x	L_t^x
	R_t^x

One period later, the pension fund balance is changed as follows (where all terms of $t+1$ are expressed in the economic value at t):

Activa	Passiva
A_t	L_{t+1}
C_{t+1}	PP_{t+1}
	R_{t+1}

with $A_t + C_{t+1} = L_{t+1} + PP_{t+1} + R_{t+1}$.

The economic value of the assets at the $t+1$ has to be equal to the assets A_t at time t. That is the reason why in the balance sheet the term A_t is unchanged. The term C_{t+1} is the economic value at t of contributions paid in $t+1$ and PP_{t+1} is the economic value at t of pension payments in $t+1$. The term L_{t+1} stands for the economic of accrued liabilities at the end of period $t+1$, being the sum of the accrued liabilities at the end of period t including indexation *minus* the liabilities written off in $t+1$ as they have been reserved for pension payments in $t+1$ *plus* the new accrued liabilities in $t+1$ due to one year of additional service of working members. The term R_{t+1} is the economic value at t of the pension fund residue at the end of period $t+1$.

Rearranging the equations mentioned above we have:

$$(L_{t+1} - L_t) + (PP_{t+1} - C_{t+1}) + (R_{t+1} - R_t) = 0$$

This expression says that the one year change in the value of the liabilities is backed by contributions and by either an increase or a decrease in the pension fund residue. This reflects the zero-sum nature of a pension fund, but this feature does not hold for the different age-cohorts. Splitting up this expression to the different

cohorts results in the following:

$$\Delta GA^x_{t+1} = (L^x_{t+1} - L^x_t) + (PP^x_{t+1} - C^x_{t+1}) + (R^x_{t+1} - R^x_t) \neq 0$$

The term ΔGA^x_{t+1} is the generational account option of cohort x, that is defined as the economic value at t of the change in the generational account of cohort x during $t+1$. The sum of all generational account options has to be necessarily equal to 0, reflecting that the pension fund is a zero-sum game in value terms:

$$\sum_{x \in X} \Delta GA^x_{t+1} = 0$$

CHAPTER 5

EMBEDDED OPTIONS IN PENSION FUNDS

This chapter explains the nature of the embedded options in the traditional DB pension scheme proposed by T. Kocken [**1**]. The relation between the corporate employer and the beneficiaries is central in this chapter. The focus will be on three kinds of embedded options in a corporate pension fund, which all depend on the funding ratio level.

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1. The relevance of embedded options in pension funds

Every agreement between two or more parties contains some rights that are not an obligation. Sometimes an agreement is a written contract with explicit mentioning of the obligations. However, often the exact details of an agreement are not clearly defined. Pension funds contain many examples of different kinds of "soft contracts" (for instance, the possibility to postpone an investment) or better termed the embedded options and therefore they should be valued as such.

Pension fund constructions are established via the process of negotiation between parties like pension fund board, employee, supervisor, etc. This negotiation process can generate a more complex agreement than the options known as a financial technique. This is the reason why embedded options are not easy to value.

Embedded option also has impact on society. Valuation and hedging of the risks captured in the asymmetric embedded options can create a possibility of risk sharing between different parties. In the graying population we have to think about the future that requires different planning and different pension structures. Clear insight in the value of embedded values and their behavior in a pension fund can be useful as a tool in optimal pension design, which will be used in the negotiation process.

In the next section the focus will be on the corporate defined benefit (DB) scheme, where the relation between "employer as a sponsor" and "employee/retiree as a beneficiary" will be highlighted. The analysis focuses on the various types of options embedded in DB pension fund contracts.

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2. The indexation option

Most of the time the pension entitlements of active members are linked to wage inflation (or to the individual career of the members of the pension fund), while the indexation rights of the non-active members are dependent on the price inflation.

In the Netherlands the conditional indexation is the most common practice. This kind of indexation depends on the wealth of the pension fund which is defined by the funding ratio. In other words, the members write an option on the funding ratio: If the funding ratio drops below a particular level, the members are obliged to (partially) waive indexation of their entitlements.

In a final salary system, in case of a funding shortfall only the inactive members will suffer from the indexation cuts. The final salary of active members will rise with their individual wage unconditionally. In an average salary system, the indexation for both active and inactive members can be postponed or reduced. This postponement will be abandoned when the pension fund reaches the safety level of the funding ratio. The active employees assume more risk in this system than they do in a final salary system.

Consider the example of the indexation option with a threshold level of the funding ratio of 120%. Assume a final salary scheme with the following indexation allocation. In case the nominal funding ratio exceeds 120%, full indexation is awarded that year. In case the funding ratio falls short of 120%, no indexation is granted that year. Each year the funding ratio will be compared to the 120 threshold to determine whether or not the indexation can be awarded that year.

Define A_t as the pension fund's assets and L_t as the liabilities at time t, the pay-off of the option in period t is defined as

$$\text{Payoff}_t IO = i_t(1 - \text{I}_{Index,t})L_t, \tag{2.1}$$

with the indexation indicator $\text{I}_{Index,t}$ defined as

$$\text{I}_{Index,t} = \begin{cases} 1 & \text{for} \quad \frac{A_t}{L_t} \geq 1.2, \\ 0 & \text{for} \quad \frac{A_t}{L_t} < 1.2, \end{cases} \tag{2.2}$$

and i_t the realized inflation in period t.

In words it means that the indexation option pays either zero or pre-agreed amount, depending on the stochastic underlying variable, which in this case is the funding ratio of the pension fund. Pension liabilities can be interpreted as real liabilities plus an indexation option written by retirees to the pension fund.

It is clear from the formulas that the indexation option is influenced by the volatility of assets and liabilities and also by the market variables at time t. For standard options like plain vanilla call or put options, it holds that the higher the volatility of the underlying variable, the higher the value of the option. However, the indexation option described in this section behaves differently. T. Kocken investigated that the indexation option holds that the value of the option decreases with rising volatility. For instance, if the funding ratio is low and there is no indexation grant expected, then the option is *in the money*. In this case the higher volatility will increase the probability of higher funding ratio, which results in heightening of probability of indexation and a lower option value as a consequence. Therefore the exposure to volatility is negative in case of low funding ratios and positive in case of high funding ratios. For more details see book T. Kocken [**1**] Chapter 3 and 4.

3. The parent guarantee option

The *parent (company) guarantee* option in the context of a corporate pension fund is defined as the explicit additional mandatory payment by a parent company to the pension fund in a situation when the funding ratio is lower than a pre-agreed level of this ratio.

The payoff of the parent guarantee at time t is defined as

$$\text{Payoff}_t PG = \max(L_t - X_t A_t, 0)(1 - \text{I}_{Default,t}), \tag{3.1}$$

with the default indicator $\text{I}_{Default,t}$ defined as

$$\text{I}_{Default,t} = \begin{cases} 1 & \text{default of the company occurred, } A_t < L_t, \\ 0 & \text{no default, } A_t \geq L_t, \end{cases} \tag{3.2}$$

with X_t the strike level of the parent guarantee. The strike level is the level below which the parent - in case of no default - is obliged to pay. The parent guarantee option can be illustrated by the following example. Consider different situations when the assets range from 85 to 120. Suppose, the liabilities are 100 and the strike level is 110%. It means that the option will pay off when the funding ratio lies between 100% and 110% as shown at Figure 1. In case of low funding ratio and

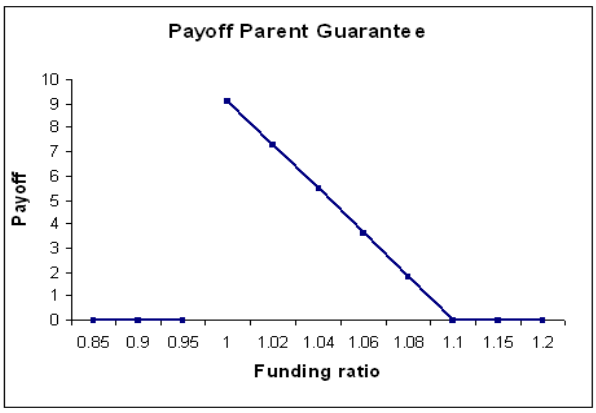

FIGURE 1. Payoff of parent guarantee option

non-default state of the parent company, the company will contribute extra funds. After the payment is made the funding ratio will rise. Compared to the indexation option which has an impact on the liability side, the parent guarantee has impact on the asset side. But as in the case of the indexation option, the parent guarantee depends on the volatility of the funding ratio between now and time t.

4. The pension put option

The pension put option is the option that is always present, as soon as the slightest (market) risk is taken and the company is not default-free. It entails the credit exposure of the beneficiaries (active members, sleepers and pensioners) to the pension fund possibly defaulting on its (conditionally indexed) commitments [**1**].

This is the option to trigger payments: the pension fund must be in a state of deficit (the assets value is lower than the value of the liabilities) and the parent company must be in a state of corporate default. Assume the corporate assets are fully available to cover pension shortfall, which means that the pension liabilities have the highest seniority of all debt on the balance sheet. Every country defines it self what the exact seniority of pension liabilities is.

The payoff, being the shortfall on the liabilities, at time t of the pension put is defined as

$$\text{Payoff}_t PP = \max(L_t - A_{PF,t} - D_{Corp,t}, 0)\mathrm{I}_{Default,t} \tag{4.1}$$

with $D_{Corp,t}$ the amount that can be retrieved by the pension fund from the defaulted parent company and $A_{PF,t}$ the pension fund's asset value at time t . The amount $D_{Corp,t}$ decreases with decreasing seniority of the debt. Sometimes, there is high correlation between $\mathrm{I}_{Default,t}$ and the pension fund solvency $L_t - A_{PF,t}$ when pension funds invest in their parent company's stocks, although it has been restricted in most countries.

Part 2

PART II: Case Studies

CHAPTER 6

RISK-NEUTRAL VALUATION

This chapter describes techniques for valuing cash flows generated within a stochastic projection model, on a market consistent basis. Traditionally actuaries value the cash flows using deterministic calculations and discounting at a risk-free rate. The risk-neutral valuation technique is consistent with the economic principles and will produce market-consistent valuations if the model is calibrated to the market.

First I give the definition of arbitrage and illustration of an arbitrage concept. Then a presentation of the binomial model will be given for the investment models with an unsure interest. Next, the valuation of the derivative using a replicating portfolio is shown. The section that follows, gives the interpretation of the formula obtained in preceding section. The name of the interpretation is a risk-neutral valuation.

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1. Arbitrage

Arbitrage can be defined as the purchase of securities on a market for immediate resale on another market in order to profit from a price discrepancy.

An arbitrage opportunity exists when an investor can construct two different portfolios of differing prices, which provide the same cash flows. Selling the more expensive portfolio and buying the cheaper portfolio with the proceeds produces unlimited return without a capital expenditure on the part of the investor.

Look at the next example from financial mathematics as a simple demonstration of arbitrage-free pricing, described by S. Jarvis, F. Southall and E. Varnell [**9**]. This example shows actions of two investors. Suppose we have a share with a current price of €1.20 with expected growth in share price of 10% and guaranteed return on cash over one year of 5%. The first investor is offering to buy or sell a forward [1] of this share at a certain price. The second investor will try to find an arbitrage and make a profit of it, and the first investor will look for the price for the forward contract to avoid an arbitrage opportunity.

The first investor makes a guess that the forward price might be the expected price of the share in one year $1.10\times$ €1.20 = €1.32. A second investor may choose for following strategy. He sells the forward thereby agreeing to sell the share for €1.32 in one year from now. At the same time he borrows €1.20 at 5% to buy this share. After one year he completes the forward by selling the share to the first investor for a agreed €1.32 and pays his creditors €1.26 back ($1.05\times$ €1.20). The

[1] The forward price of a stock is the price (agreed at time $t = 0$) at which two investors will trade the stock at a future time $t = n$.

second investor would make a profit of 6 cent per contract, at the expense of the first investor, and would sell as many contracts as possible to the first investor until he realized his mistake. This unlimited profit opportunity is an arbitrage because it creates profit from nothing. The second investor has not had to use any cent from his own capital.

Suppose now that the first investor is able to find an arbitrage. It turns out that the only price for a forward that avoids an arbitrage is the current share price with interest at the guaranteed return on cash, $1.05 \times €1.20 = €1.26$. The reason is that a hedge portfolio can replicate the forward contract. The second investor constructed the hedge portfolio, which contains a share and an obligation to repay the borrowing. At the end of the year, the cash flows of the forward contract and the hedge portfolio will be equal. Below the comparison of two strategies of the investor shown.

Contract	**Sell a Forward**	**Hedge a Forward**
Sell (buy) one share	(120)	(120)
Borrow (invest) cash in the market	120	120
Net cash flow now	0	0
Forward price receipts (payment)	132	126
Investment proceeds (repay borrowings)	(126)	(126)
Net cash flow at expiry	6	0

It is obvious that to avoid an arbitrage opportunity, the cash flows from the hedge-portfolio must be the same as the cash flows from the forward contract. Unfortunately, the arbitrage opportunities are not available to most investors. Specialist arbitrage players operate in most markets, seeking out arbitrage opportunities and taking advantage until the opportunity ceases to exist. In markets where a hedge portfolio can be constructed for a contract, the value of the contract is equal to the value of the hedge portfolio. This is the arbitrage-free pricing method.

> A model must provide a unique value for a unique set of the future cash flows regardless what portfolio is used to produce them. If this was not the case, the model would not produce consistent valuations. A valuation model which claims to be able to value cash flows must be therefore arbitrage-free to make any sense.[**9**]

Next section describes available techniques to valuate the securities and options. The using a simple model I show how the investment with unknown return can be modeled.

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2. Binomial model

This section gives a presentation of the binomial model for the investment models with unsure interest.

The main point in the binomial model is that there are two kinds of investments: the risky asset S and a risk-free asset B, like bonds. At the time $t = 0$ the rates of S and B are known from the financial newspaper.

At time $t = 1$ the value of the bond B is increased to value rB. However the value of risky asset S is unsure in the future. The uncertainty of the value of S in the binomial model is modeled as following. A positive development of the economy will result in the raising of the asset value to uS with probability p. There is also a probability $(1 - p)$ of the decreasing of the value to dS in case of the unfavorable development of economy. These two states of the economy are named up- and down state. Figure 1 shows the binomial model where also the value of a derivative f is noticed. The value of the derivative at time $t = 1$ depends on the value S at time $t = 1$. At state u the value of the derivative is f_u and in the down state the value of this derivative is f_d. These values can be present as an option of stock S.

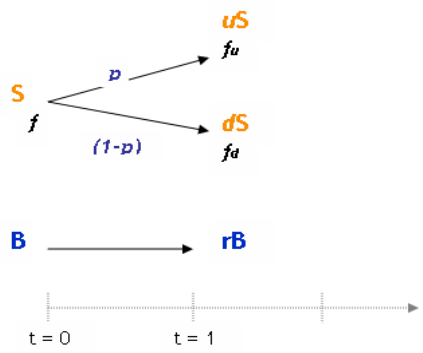

FIGURE 1. Binomial Model

3. Replicating portfolio

This section shows how to determine the value of derivative f at time $t = 0$. The fundamental idea of the valuation is based on the replication principle. Suppose, it is possible to construct the portfolio at time $t = 0$ with investments in assets S and B in such a way that in the up-state and in the down-state of the economy the value of the portfolio will be equal to the value of the derivative. In this case the value of the derivative at time $t = 0$ has to be equal to the value of the replicating portfolio.

We compose the replicating portfolio as following. Let ΔS be an amount of money to invest in S, and B an amount of money to invest in a risk-free asset. The initial investment at time $t = 0$ will be $\Delta S + B$. At the next point of time the portfolio can result in two possible values: value $u\Delta S + rB$ with probability p and $d\Delta S + rB$ in down-state d with probability $(1 - p)$.

The property of the replicating portfolio is that in state u the value of the portfolio will be equal to f_u and in state d the value will be f_d.

$$\begin{cases} u\Delta S + rB = f_u, \\ d\Delta S + rB = f_d. \end{cases} \tag{3.1}$$

The solutions of the system of Equations (3.1) give an expression for an amount ΔS in the risky investment and for an amount B in the risk-free investment.

The next step is to make the value of the derivative f at time $t = 0$ equal to the value of the replicating portfolio at time $t = 0$. Deriving ΔS from the system (3.1) we can express B in terms of u, d, r and f_u, f_d. After some simplifications the following expression for the valuation of the derivative can be found:

$$f = \Delta S + B = \frac{1}{r}\Big((\tfrac{r-d}{u-d})f_u + (\tfrac{u-r}{u-d})f_d\Big). \tag{3.2}$$

The fact of absence of the probabilities p and $(1-p)$ in this formula relies on the fact that the price of the derivative is made by replication. The replicating portfolio has to gage exactly the value of derivative in each state of the economy. Then it is settled that the risk that coincides with the derivatives can be completely neutralized by taking the contrary position in the replicating portfolio.

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4. Risk-neutral valuation

This section gives an interpretation of the formula obtained in the preceding section. This interpretation is named as a risk-neutral valuation.

Different interpretations of the formula (3.2) can be given, we give one of them. For the economy with $u < R < d$, the numbers $(R-d)/(u-d)$ and $(u-R)/(u-d)$ lie between zero and 1. These numbers can be interpreted as the *pseudo-probabilities* p^* and $(1-p^*)$. The formula (3.2) can be rewritten as following:

$$f = \frac{1}{r}\Big(p^* f_u + (1-p^*)f_d\Big). \tag{4.1}$$

The pseudo-probabilities actually are not the probabilities that say something about future, but just the numbers that help to interpret the valuation formula (4.1). Multiplying the value of the derivative with the pseudo-probability in each state, we can interpret further the formula (4.1) into the calculation of the pseudo-expectation.

$$f = \frac{1}{r}E^*\Big[f(1)\Big]. \tag{4.2}$$

The notation $E^*[f(1)]$ means that the pseudo-expectation is calculated for all possible realizations of the value of the derivative at time $t = 1$. Calculating the pseudo-expectation of the profit of investment S at time $t = 1$ we get following:

$$E^*\Big[S(1)\Big] = p^* uS + (1-p^*)dS = rS. \tag{4.3}$$

This result indicates that the pseudo-expected profit of the risky investment is equal to the profit r of the safe investment. This property of the valuation formula (4.2) is named as *risk-neutral valuation*.

It is important to emphasize that this interpretation is just a rewriting of the valuation formula (3.2) using the pseudo-expectation. This expectation says nothing about a real expectation of the interest of the investment S. Because we discuss here a valuating method for the derivatives, it should be possible to calculate the price of the trivial derivative $f \equiv S$ with formula (4.2). The correct price for S at time $t = 0$ can be calculated only if $\frac{1}{r}E^*[S(1)] = S$ holds. This is the reason why in the pseudo-expectation the risky investment also has interest r.

The only objection to the risk-neutral valuation is the omission of the probabilities in the model. Intuitively, it seems to be contradictable to use two different models to describe the same economy. But it is just an appearance invoked by the presence of the replication argument with which the value of the derivative is determined. To approach this intuitive objection another interpretation of the formula (3.2) can be considered, the deflator valuation technique. This technique gives the same market value for a derivative as the risk-neutral valuation method.[**9**]

CHAPTER 7

ARBITRAGE FREE SCENARIO GENERATOR

The valuation techniques applied are based on arbitrage free stochastic scenario simulations for all relevant market variables (yield curve, inflation, equity), resulting from adjusting the non-arbitrage free simulations in the ALM model used. The ALM model used is based on a Vector Auto-Regressive (VAR) model for the economic scenarios, which is built by stochastic differential equations (SDE). This chapter will give a short overview of the field of study concerning SDE's for stock prices and forward rates.

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1. Introduction

To be able to apply the value-based ALM for pension funds, ORTEC decided to extend the existing scenario generator with risk-neutral [1] scenario generator that handles real interest and inflation. Starting points for developing this model are given by Jarrow & Yildirim [**5**](2003) and Mercurio [**6**] (2005). I describe here how to implement Mercurio expressions for processes that are inferred from Jarrow& Yildirim and how to calculate the statistical characteristics of the model. Notice that the formulas concern a *linear* form, like a logarithm of the price index and stocks.

Using results obtained by Jarrow & Yildirim Mercurio notices that when the nominal [2](N) and real (R) instant forward rates and price index (I) follows undermentioned processes

$$\begin{aligned} df_N(t) &= \alpha_N(t,T)dt + \beta_N(t,T)dz_N(t) \\ df_R(t) &= \alpha_R(t,T)dt + \beta_R(t,T)dz_R(t) \\ dI(t) &= I(t)\mu(t)dt + \sigma_I I(t)dz_I(t), \end{aligned} \tag{1.1}$$

where (z_N, z_R, z_I) are the correlated $(\rho_{N,R}, \rho_{N,I}, \rho_{R,I})$ Wiener [3] processes for nominal and real interest rates and price index; and the instant forward rate volatilities σ taken from 1 factor Hull-White model are

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[1] In the risk-neutral world, the expected return on all securities is equal to the return on the so-called money-market account, which increases at the instantaneous short rate.

[2] The nominal interest rate R_N is a rate of interest before adjustment for the inflation π and can be calculated as $1 + R_R = (1 + R_N)/(1 + \pi)$, where R_R is the real interest rate.

[3] The Wiener process $W = (W(t))_{t \geq 0}$ is a continuous-time stochastic process with $W(0) = 0$ and increments $W(t) - W(s)$ normally distributed with mean zero and variance $(t - s)$ for all $0 \leq s < t$. It follows that $dW(t) \sim \mathcal{N}(0, \sigma^2 dt)$.

$$\begin{aligned} \beta_N(t,T) &= \sigma_N e^{-a_N(T-t)}, \\ \beta_R(t,T) &= \sigma_R e^{-a_R(T-t)}. \end{aligned} \tag{1.2}$$

The following holds: under the risk-neutral probability measure of the nominal interest rate, the instant nominal and real interest rate and price index follows the processes as mentioned below:

$$\begin{aligned} dr_N(t) &= [\theta_N(t) - a_N r_N(t)]dt + \sigma_N dz_N(t), \\ dr_R(t) &= [\theta_R(t) - \rho_{R,I}\sigma_R\sigma_I - a_R r_R(t)]dt + \sigma_R dz_R(t), \\ dI(t) &= I(t)[r_N(t) - r_R(t)]dt + \sigma_I I(t) dz_I(t), \end{aligned} \tag{1.3}$$

with $x \in \{N, R\}$.

$$\theta_X(t) = \frac{\partial f_X(0,t)}{\partial t} + a_X f_X(0,t) + \frac{\sigma_X^2}{2a_X}(1 - e^{-2a_X t}).$$

$f_X(0,t)$ is the instant forward rate for runtime t and comes from the initial nominal and real zero coupon yield curve; the drift parameter a_X and volatilities σ_X are constant.

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2. Two factor Hull-White model

To get insight into the statistical characteristics of the scenarios generated by the model and to calibrate the model by historical data, it is important to calculate these statistical characteristics. The knowledge about Hull-White equations is needed to gain inside to the model. The two-factor Hull-White (2HW) model [**8**] is a stochastic model that describes the instantaneous short rate. It has many functional qualities. The second factor makes the variability of the rates more market coherent. The 2HW model is perfect for calibrations to correlations-based products like swaptions. The model is called a no-arbitrage model because this model is on all times consistent with today's term structure. This structure is used as input of the model.

It is possible now to construct the 2HW model by starting with a combined set of two stochastic differential equations in risk-free world for real economy. The first equation defines the short rate $r(t)$. The second factor $u(t)$ is a theoretical factor, used to describe the volatility and correlation of the interest rate more precisely.

$$\begin{aligned} dr_R(t) &= [\theta_R(t) + u_R(t) - a_R r_R(t)]dt + \sigma_{R1} dz_{R1}(t), \\ du_R(t) &= -b_R r_R(t)dt + \sigma_{R2} dz_{R2}(t). \end{aligned} \tag{2.1}$$

The risk-free numeraire for the real economy is a bank account $B_R(t)$.

$$dB_R(t) = B_R(t) r_R(t) dt. \tag{2.2}$$

Now we make a passage from the risk-neutral probability measure of the real economy (Q_R) to the probability measure of the nominal economy (Q_N). This gives us the possibility to define the nominal as real interest rates with respect to the same (nominal) probability measure. Mercurio shows that the change of

probability measure is equivalent to the change of the numeraire from $B_R(t)$ to $B_N(t)/I(t)$. B_N is the state of the current bank account in the nominal economy and I is the measure for cumulative inflation. The stochastic differential equations for this numeraire under measure Q_N are:

$$\begin{aligned} dB_N(t) &= B_N(t)r_N(t)dt, \\ dI(t) &= (r_N(t) - r_R(t))I(t)dt + \sigma_I I(t)dz_I(t). \end{aligned}$$

Taking into account the corrections we made, the 2HW equations for real interest rates under Q_N are as follows:

$$\begin{aligned} dr_R(t) &= [\theta_R(t) - \rho_{R1,I}\sigma_{R1}\sigma_I + u_R(t) - a_R r_R(t)]dt + \sigma_{R1}dz_{R1}(t), \\ du_R(t) &= [-\rho_{R2,I}\sigma_{R2}\sigma_I - b_R u_R(t)]dt + \sigma_{R2}dz_{R2}(t). \end{aligned}$$

The complete the 2HW model with inflation and the stock process $S(t)$ in logarithm form results in the following:

$$\begin{aligned} dr_N(t) &= [\theta_N(t) + u_N(t) - a_N r_N(t)]dt + \sigma_N dz_N(t), \\ du_N(t) &= [-b_N r_N(t)]dt + \sigma_{UN}dz_{UN}(t), \\ dr_R(t) &= [\theta_R(t) - \rho_{R,I}\sigma_R\sigma_I + u_R(t) - a_R r_R(t)]dt + \sigma_R dz_R(t), \\ du_R(t) &= [-\rho_{UR,I}\sigma_{UR}\sigma_I - b_R r_R(t)]dt + \sigma_{UR}dz_{UR}(t), \\ dLnI(t) &= \left(r_N(t) - r_R(t) - \tfrac{\sigma_I^2}{2}\right)dt + \sigma_I dz_I(t), \\ dLnS(t) &= \left(r_N(t) - \tfrac{\sigma_S^2}{2}\right)dt + \sigma_S dz_S(t), \end{aligned} \tag{2.3}$$

where $(z_N, z_{UN}, z_R, z_{UR}, z_I, z_S)$ are correlated Wiener processes with correlations ρ and volatilities σ. The model has 25 parameters: 4 mean reversion, 6 volatilities and 15 correlations.

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3. Model implementation

To find a solution for a general SDE $dX_t = \mu(X_t, t)dt + \sigma(X_t, t)dW_t$ a numerical scheme can be used to simulate paths of X. For a small time of step Δt holds: $X_{t+\Delta t} = X_t + \mu(X_t, t)\Delta t + \sigma(X_t, t)\Delta W_t$. Applied to the 2HW, this results in the following scheme:

$$\begin{aligned} r_{t+\Delta t} &= r_t + (\theta(t) + u_t - a r_t)\Delta t + \sigma_1 \Delta z_1(t) \\ u_{t+\Delta t} &= u_t - b u_t \Delta t + \sigma_2 \Delta z_2(t) \end{aligned} \tag{3.1}$$

For the implementation we use the version of the described model in discrete time with Δt small enough in years, like 0.01. The derived representation is equal to the following special VAR(1)[4] model:

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[4]Vector auto regression (VAR) is an econometric model used to capture the evolution and the interdependencies between multiple time series. The evolution of a set of n variables, collected in a $n \times 1$ vector y_t, measured over the same sample period is a linear function $y_t = c + A_1 y_{t-1} + A_2 y_{t-2} + \cdots + A_p y_{t-p} + e_t$, which is a p-th order of VAR denoted as VAR(p). A matrix form of a VAR(1) in two variables can be written as: $\binom{y_{1,t}}{y_{2,t}} = \binom{c_1}{c_2} + \begin{pmatrix} A_{1,1} & A_{1,2} \\ A_{2,1} & A_{2,2} \end{pmatrix} \binom{y_{1,t-1}}{y_{2,t-1}} + \binom{\epsilon_{1,t}}{\epsilon_{2,t}}$.

$$\begin{pmatrix} r_N(t+\Delta t) \\ u_N(t+\Delta t) \\ r_R(t+\Delta t) \\ u_R(t+\Delta t) \\ LnI(t+\Delta t) \\ LnS(t+\Delta t) \end{pmatrix} = \begin{pmatrix} \theta_N(t)\Delta t \\ 0 \\ (\theta_R(t) - \rho_{R,I}\sigma_R\sigma_I)\Delta t \\ -\rho_{UR,I}\sigma_{UR}\sigma_I\Delta t \\ -\frac{\sigma_I^2}{2}\Delta t \\ -\frac{\sigma_S^2}{2}\Delta t \end{pmatrix} +$$

$$+ \begin{pmatrix} 1-a_N\Delta t & \Delta t & 0 & 0 & 0 & 0 \\ 0 & 1-b_N\Delta t & 0 & 0 & 0 & 0 \\ 0 & 0 & 1-a_R\Delta t & \Delta t & 0 & 0 \\ 0 & 0 & 0 & 1-b_R\Delta t & 0 & 0 \\ \Delta t & 0 & -\Delta t & 0 & 1 & 0 \\ \Delta t & 0 & 0 & 0 & 0 & 1 \end{pmatrix} + \begin{pmatrix} \epsilon_N(t) \\ \epsilon_{UN}(t) \\ \epsilon_R(t) \\ \epsilon_{UR}(t) \\ \epsilon_I(t) \\ \epsilon_S(t) \end{pmatrix},$$

with

$$\begin{pmatrix} \epsilon_N(t) \\ \epsilon_{UN}(t) \\ \epsilon_R(t) \\ \epsilon_{UR}(t) \\ \epsilon_I(t) \\ \epsilon_S(t) \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_N^2\Delta t & & & & & \\ \rho_{N,UN} & \sigma_{UN}^2\Delta t & & & & \\ \rho_{N,R} & \rho_{N,R} & \sigma_R^2\Delta t & & & \\ \rho_{N,UR} & \rho_{UN,UR} & \rho_{R,UR} & \sigma_{UR}^2\Delta t & & \\ \rho_{N,I} & \rho_{UN,I} & \rho_{R,I} & \rho_{UN,I} & \sigma_I^2\Delta t & \\ \rho_{N,S} & \rho_{UN,S} & \rho_{R,S} & \rho_{UR,S} & \rho_{I,S} & \sigma_S^2\Delta t \end{pmatrix} \right).$$

The correlation $\rho_{i,j}$ between ϵ_i and ϵ_j, mean reversion parameter a and volatilities of the first and second factor can be specified by the user. Using this discrete model the stochastic scenarios can be generated. The starting point of every scenario (at time zero) is given by the current short rate $r(0)$ and current equity price $S(0)$; $u(0) = 0$. Now we can determine $r(\Delta t)$, $u(\Delta t)$ and $S(\Delta t)$. By repeating this procedure the new samples ϵ will be used and the different scenario's will be generated.

If a great number of realizations is generated for r and u (each time by drawing different paths of the Wiener processes z), then the average of these realizations will be equal to the expectations of $r(t)$ and $u(t)$. It will give insight in the volatility of the future course of $r(t)$. This method is called *Monte Carlo simulation*. The Monte Carlo approach is used to price the embedded options in pension fund products.

The parameters for the two-factor Hull-White model have to be determined. To establish correct values the calibration is used, where the values correspond to the historical data. Calibration is a method to determine the settings of the model, by comparing the results of the model with known data. The purpose of the calibration is to replicate the dataset and to set the model in such way, that it would be optimal to value a pension fund cash flows with risk free scenarios.

The quantities resulting from the model introduced in this chapter, like interest rate volatilities, have significant value and can be compared to the market or the historical values. In my research I use the historical data, which includes data about the statistics (i.e., average, volatility, and auto-correlation) of interest rates.

When we calibrate the 2HW Black-Scholes model, we will first set the number of parameters of the 2-HW model and then try to find the optimal values for the remaining parameters. This must be done to historical data or to statistics from another model on the return on equity $s(t)$. In practice, the volatility of the return on equity is the most important quantity, since the expected value is for great parts

determined by the interest rate and further more the volatility is usually many times bigger than the expectation.

4. Test of the Arbitrage Free scenario set

In order to test the arbitrage free scenario set, I represent here the analysis of the capability of the valuation technique based on arbitrage-free stochastic scenario simulations resulting from the ALM model. The analysis is applied to the pension fund which consists of inactive and retired members only.

For the analysis in this section a "sleeping" pension fund is modeled. The term *sleeping* indicates that the pension fund has no active participants who pay contributions and provide cash inflow. There is also no inflow of new members in the pension fund. During the years the number of the participants in the fund will decrease until the last participant dies. The accumulated assets of the existing participants are corrected for the inflation and the interest rates.

Some assumptions are made for simplicity. An important assumption is that all participants are of the same sex and die at the age of 80. After 55 years the pension fund has no participants. The cash flow of the pension fund consists of the benefit payments. Assets of the pension fund in year $t = 0$ are known.

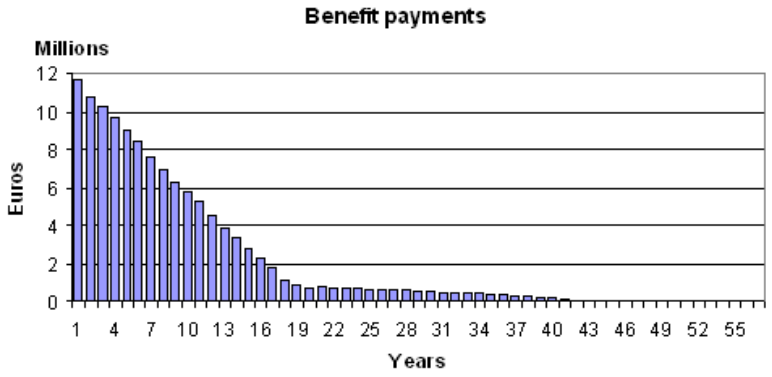

FIGURE 1. Pension benefits payments by the "sleeping" pension fund.

Based on above assumptions the pension fund is projected into the future. To represent the future developments of the assets and the benefits 500 simulation scenarios of 55 years are generated. In Figure 1 the benefit payments per year (averages over 500 scenarios) are shown. The benefits decrease over time right up to zero.

4.1. Analysis of "Sleeping" Pension Fund. In order to verify the arbitrage-free generator, which generates the discount factors for the risk-neutral valuation technique, I calculate the value which is based on cash flows scenarios discounted with "money banking account". If the arbitrage free scenarios are "correct", this value is expected to be equal to the value based on the initial real term structure.

The latter value is calculated by an actuary by calculating the present value of the expected cash flows with a real yield curve at $t = 0$.

For the calculation of the value based on cash flow scenarios I use the following strategy. After the future developments of the assets and benefits are simulated for each year in every scenario, the cash flows are discounted back to year $t = 0$. Discount factors and cash flows are acquired using the arbitrage-free generator. After applying the valuation technique, the present value of the cash flows of the pension fund is expected to be the same as the assets at year $t = 0$ as we assume an initial real funding ratio of 100%.

The present value of the cash flows of the "sleeping" pension fund expresses how much a future cash flow of the pension fund is worth today. The expected present value at time $t = 0$ is given by the average value over N scenario's of the sum values of the discounted assets (A_t) and the cumulative sum of the discounted benefits (B_i):

$$PV(sleepingPF) = \frac{1}{N} \sum_{n=1}^{N} \Big(A_t \times DF_t + \sum_{i=0}^{t} B_i \times DF_i \Big), \tag{4.1}$$

where DF_t denotes the discount factor calculated cumulative as $\frac{1}{(1+r)^t}$ for each time t, and acquired from the arbitrage-free economic variant. Notice that at any moment in the time t the following should hold: $PV_0 = PV_t$.

The discounted assets and the cumulative sum of the benefit payments are represented in Figure 2. As pensions are paid, the value of the assets decreases, but the present value of the cash flows remains unchanged in each year.

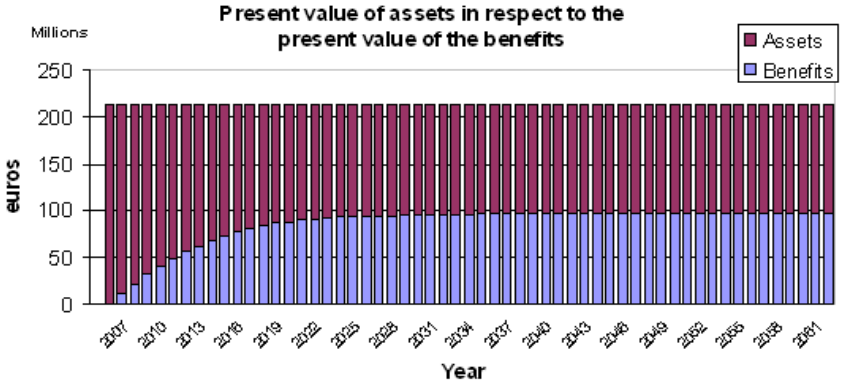

FIGURE 2. Present value of the cash flows in the "sleeping" pension fund.

The result of the calculations gives the same value of the cash flows as the value of the assets (A_0) at time $t = 0$, which confirms correct implementation of the valuation technique in the ALM model.

CHAPTER 8

MODEL SETUP

The framework for my investigation is based upon the article of Cui et al. (2007) [**7**], which serves as a benchmark for this thesis. This chapter describes the economic framework, the contribution, investment, and indexation policy alternatives, and the mathematics behind the calculations of the value transfers. Among others the description of the framework for the analysis using classic and value-based method is given.

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1. Introduction

This analysis is based on the method of value-based generational accounting [**3**]. Some important assumptions are made in this chapter in order to focus on the intergenerational value transfer. There are no sponsors in the modeled collective schemes and there are no obligations to capture the losses. In the economy there is a constant real interest rate. It is assumed that the wage inflation is identical to the price inflation, and I model a flat real wage profile for all plan participants, without modeling real wage uncertainties.

In this section I model returns on investment of the assets and the evolution of the liabilities in DB scheme. The framework as described in the next section and as proposed by Cui et al (2007) is used. In the case with the theoretical fund it is assumed that inflow and outflow of the members guarantees stable age compositions of the population. All individuals are of the same sex.

---

2. Economic framework

The following description of individual pension arrangements will serve as the benchmark for the collective schemes with intergenerational risk sharing (IRS) which I will present in the next chapter. This is the model as proposed by Cui et al. (2007), but not implemented in the ALS model I use in my thesis.

In the financial market, two asset classes are traded: risky stocks and real risk-free assets. All variables are expressed in real terms, i.e., scaled by the price level. The index linked bonds (ILB) with all maturities are available. I assume now that real interest rates r are non-stochastic. Real stock prices follow a geometric Brownian motion with drift:

$$dE_t/E_t = \mu_E dt + \sigma_E dZ_{E,t}$$

The real ratio of the investment portfolio equals:

$$\lambda_E = \frac{\mu_E - r}{\sigma_E}$$

The investment portfolio is a mix of equity and ILB's which the real value dynamics are given by

$$dA_t/A_t = [r + \omega(\mu_E - r)]dt + \omega\sigma_E dZ_t,$$

where ω is a portfolio weight for equities. The expected real return and the volatility increase linearly with the fraction ω.
The stochastic discount factor M_t in this economy is the deflator for real risky cash flows which in the model evolves according to

$$dM_t/M_t = -rdt - \lambda_E dZ_t.$$

As an alternative environment, we consider portfolios of equities and nominal bonds, which economy structure is adopted from the model of Brennan and Xia (2002). The instantaneous expected rate of inflation π follows an Ornstein-Uhlenbeck process:

$$d\pi_t = q(\pi - \pi_t)dt + \sigma_\pi dZ_{\pi,t}.$$

The price level changes according to

$$d\Pi_t/\Pi_t = \pi_t dt + \sigma_u dZ_{u,t}.$$

The stochastic discount factor M_t in this economy evolves according to

$$dM_t/M_t = -rdt - \kappa_E dZ_t - \kappa_\pi dZ_{\pi,t} - \kappa_u dZ_{u,t},$$

where $\kappa_t \equiv [\kappa_E, \kappa_\pi, \kappa_u]' = \rho^{-1}\lambda$ and λ is the vector of real market prices of risk. □

In stead of the stochastic discount factor M_t defined above, I use discount factors generated by the risk-neutral valuation technique described in Chapter 7.

The default values for the model parameters used in the calculations are based on estimates reported in Brennan and Xia (2002) and calibrations made by Cui et al. (2006). The expected real return on equity is assumed to be 6%, implying an equity premium of 4% and the volatility of equity returns is 15%. Furthermore, it is assumed that the real interest rate is constant and equals $r = 2\%$, and the subjective discount rate equals $\delta = 4\%$. The expected consumption b in the individual pension scheme is assumed to be equal over time, such that $b = 1 - p$, where p is the annual real cost contribution.

---

3. Transfer value of the age cohorts

The value of pension fund assets A_t is equal to the value of total pension fund nominal liabilities L_t plus the pension fund surplus S_t:

$$A_t = L_t + S_t. \tag{3.1}$$

The balance sheet next period $(t+1)$ expressed in present value terms at t is

$$A_t + PV(C_{t+1}) - PV(B_{t+1}) = PV(L_{t+1}) + PV(S_{t+1}), \tag{3.2}$$

where the term $PV(C_{t+1})$ denotes the economic value of the contributions paid in time $(t+1)$ and $PV(B_{t+1})$ stands for the economic value of the pension payments in time $(t+1)$. The term $PV(L_{t+1})$ is the economic value of accrued liabilities at the end of period $(t+1)$.

Using (3.1), we can rearrange (3.2) as:

$$PV(S_{t+1})^x = S_t^x + PV(C_{t+1})^x - PV(B_{t+1})^x - (PV(L_{t+1})^x - L_t^x), \tag{3.3}$$

where x refers to cohort x. The expression (3.3) is split up by age cohort. It is assumed that S_t^x at time $t = 0$ is equal to zero and the present value of the excess return on investments is zero. This results in the expression below:

$$PV(S_{t+1})^x = PV(C_{t+1})^x - PV(B_{t+1})^x. \tag{3.4}$$

The term $PV(S_{t+1})^x$ is the generational account option of cohort x, that is defined as the economic value of the change in the generational account of cohort x during $(t+1)$. In the next chapter the generational transfers that result from policy changes will be explored. The transfer value is the net present value of the difference between the received contributions and the pension benefits per generation.

Equation (3.4) can be rewritten as:

$$PV(Transfer^x) = \sum_{t=0}^{T} \Big(PV(C_t)^x - PV(B_t)^x \Big) \tag{3.5}$$

Since all cohorts together form the total pension scheme, the following must hold

$$\sum_x PV(Transfer)^x = 0. \tag{3.6}$$

This reflects the zero-sum nature of a pension fund.

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4. Framework of analysis

The analysis of the policy and its impact on a pension fund will be represented by two methods: the classic method, which is used in daily practice by managers, and the value-based method, amplifying value which is being examined in this thesis.

4.1. Classical analysis. The classic method is represented by a selected number of the risk and yield measures.

Average of the funding ratio: This measure indicates the average solvability position of the pension fund.

Standard deviation of the funding ratio: is a risk measure of the spread of the funding ratio about the mean. If the funding ratios in most of the scenarios are close to the mean, then the standard deviation is small.

Probability of underfunding: is a probability that the pension fund will be in the situation of underfunding during the time horizon.

Average of the contributions: is the "middle" or "expected" value of the contribution data set acquired from the simulated scenarios.

Standard deviation of the contributions: is a risk measure of the spread of contributions values around the mean. If the funding ratios in most of the scenarios are close to the mean, then the standard deviation is small.

95% VaR of the contributions: (Value at Risk) means that 5% of the contributions is at least equal to this level.

Probability of the 5% contribution increase: is a probability that the value of the contributions will increase more than 5% in one year. This measure shows a certain amount of the fluctuation of the contribution payments.

Probability of the indexation cuts: is a probability of the omission in the indexation in the situation where the value of the assets is more than the value of the nominal liabilities and less than the value of the indexed liabilities.

Average of the purchasing power: is an average value of all values of the purchasing power calculated in the simulation scenarios. Purchasing power is a ratio between the pension liabilities with the current indexation policy and the pension liabilities after adjusting for full indexation. If the liabilities stay constant while the prices and the wages increase, then purchasing power of the liabilities will decrease. Decreasing purchasing power points out the inflation. This measure shows whether there is equal treatment of pensioners and active members of the pension fund with regard to inflation. In particular, I calculated the purchasing power as proportion between the current indexation policy and the unconditional indexation policy.

95% VaR of the purchasing power: means that 5% of the purchasing power is at most this number.

4.2. Value-based analysis. The value-based method is represented by a balance sheet. The balance sheet of a pension fund can be displayed in the economic value expressions as presented below.

Assets	**Liabilities & Equity**
A_0	$L_{t=T}$
C_T	B_T
$OD_{t=T}$	$OS_{t=T}$

where

A_0 value of the assets at time $t = 0$,

C_T economic value of the contributions during the period $t = 0$ to $t = T$,

$L_{t=T}$ value of the liabilities at time $t = T$, discounted back to $t = 0$,

B_T economic value of the benefits during the period $t = 0$ to $t = T$,

$OD_{t=T}$ economic value of the deficit at the end of the year T, or "option deficit" price at $t = 0$ of surplus at time $t = T$,

$OS_{t=T}$ economic value of the surplus at the end of the year T, or "option surplus" price at $t = 0$ of surplus at time $t = T$.

This is not a formal pension fund balance sheet, but more theoretical interpretation of a pension fund's balance sheet capturing two kinds of embedded options. This is seen from the pension fund's perspective.

Calculations of the option surplus and the option deficit are presented by Formula (3.5) and represent the positive and the negative transfers, respectively. The terms in the balance sheet can be rearranged to get the fundamental expression below, reflecting the nature of a pension fund of being a zero-sum game in the economic value terms:

$$A_0 + C_T + OD_{t=T} - L_{t=T} - B_T - OS_{t=T} = 0. \tag{4.1}$$

The sum of the two options gives the economic value of the funding residue at time T:

$$R_{t=T} = OS_{t=T} - OD_{t=T}. \tag{4.2}$$

In case of a deficit the risk-bearing stakeholders have to make up for the funding shortfall. This may be interpreted as if these stakeholders have written a put with an exercise price for the residue of zero. Future surpluses will be distributed among the risk bearing stakeholders. So, the economic value of the surplus may be seen as the option contribution for a call on the surplus at the end of period T.

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5. Risk allocation

The risk allocation specifies who of the stakeholders, when, and to what extent is taking part in risk bearing. Risk allocation is presented using the contribution policy, indexation policy and investment portfolio.

Contribution policy

In the defined benefit scheme (DB) the benefits are guaranteed, but contributions are flexible. Contributions vary in order to absorb the funding shortage. The contribution policy can be formulated as

$$P_t = \begin{cases} P^* - \alpha S_t & FR < FR_{low} < FR_{upp}, \\ P^* & FR_{low} < FR < FR_{upp}, \end{cases}$$

where S_t is the aggregate surplus or deficit in the fund, P^* is the targeted contribution and FR is the real funding ratio. The slope coefficient $\alpha \in [r, 1]$ determines the absorbing speed of the funding risks. It holds that $r \leq \alpha \leq 1$. $\alpha = 1$ implies that funding surplus is immediately fully absorbed. A low value of α implies shifting the funding mismatch into the future, and eventually shares across generations. [**7**] In my research, the pension alternatives are evaluated with α equal to 10%, 20% and 100%.

A contribution charge is levied when the assets fall short of the indexed liabilities. The charge will increase when the deficit is increasing. The contribution policy is illustrated in Figure 1.

If the funding ratio drops below the threshold level of 100%, then α% of the shortage will be restored by asking participants for extra contributions. If the threshold level is 120%, then α% of the surplus with respect to 120% will be paid

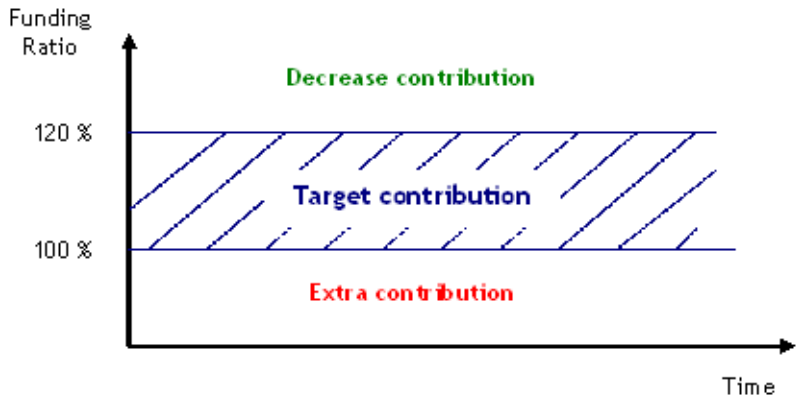

FIGURE 1. Contribution policy with respect to the real funding ratio.

back. In the theoretical pension fund there are no restrictions laid on the contribution jumps.

Indexation policy
In the pension fund scheme with the conditional indexation policy the indexation will be granted between pre-agreed levels of the funding ratio. The strategies with the certain lower and upper bounds will be evaluated, both stand for the percentage of the real funding ratio. In case of the conditional indexation, the indexation is granted linearly between the upper and the lower levels. The basic idea of the indexation policy ladder is illustrated in Figure 2.

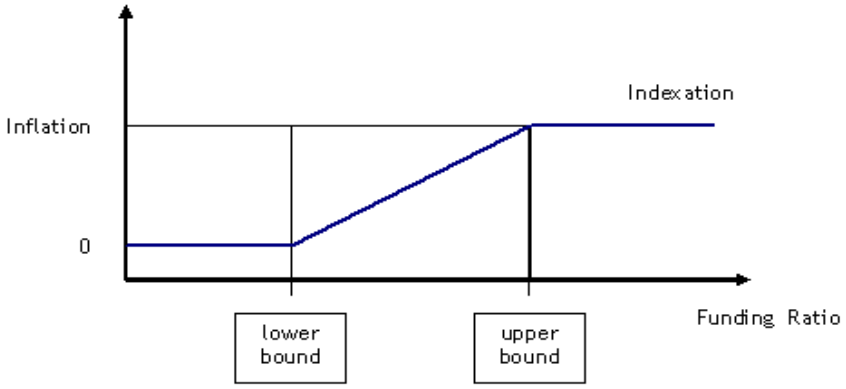

FIGURE 2. Indexation policy ladder with respect to the real funding ratio.

The upper bound of 100% denotes the situation when the assets exactly match the value of the indexed liabilities. The difference between the upper and lower

bound is the necessary indexation reserve needed to pay for the future indexation of the accrued liabilities. The indexation reserve can be also expressed as the difference between the value of indexed liabilities (based on the real yield curve) and the value of nominal liabilities. The indexation cut takes place between two bounds, where the size of the cut is related to the actual deficit in the indexation reserve. If the value of the assets exceeds the value of indexed liabilities, additional indexation will be provided until a full catching-up of previously missed indexation occurs.

CHAPTER 9

THEORETICAL FUND

To see th extended ALS system working, I calculate the market value of the transfers between generations for a pension fund based on theory. The framework for this investigation is based upon the article of Cui et al. (2007) [**7**]. The aim of this investigation is to obtain the same results by using the ALS system as the authors did in their research using their model as a benchmark.

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1. Introduction

The pension fund under investigation in this chapter imitates the characteristics of the pension fund from the article of J. Cui et al. "Intergenerational risk sharing within funded pension schemes" and the basic outcomes obtained by the authors of this article.

Main choices for pension accrual
The theoretical pension fund has one pension type, namely, Old Age Pension (OP). The pensions are build-up according to the defined benefit plan (DB) and the pension sum is defined by the pension scheme, which is based on the average salary (see Chapter 2). The maximum pension benefit that a participant can get is 82.4% with an accrual percentage of 2,06%, the age of the youngest entrant is 25 years and the pension age is 65 years, the age when the contribution payment stops. It is assumed that all participants (only active participants and retirees) die at the age of 80, this is the age when the benefits are stopped. The annual salary of the youngest participant is €25000, which increases by 1% each year, deduction from franchise is €12000, and the annual increase of the pension entitlements is 2%.

Main choice for the indexation policy
The fund pursues the complete indexation of the pension entitlements with the inflation of the pensions. In case of conditional indexation, the indexation is granted linearly between the upper and the lower bounds of the funding ratio.

Main choices for the contribution policy
The gross basic contribution is stipulated as a static contribution based on 16.6% of the salary sum. The cost-recovery premium is a nominal contribution which is composed of a comingservice, backservice indexation, backservice career and a risk premium, based on a fixed discount rate of 4% plus a replenishment for the required solvency level. The following agreements are made concerning the refund and additional payment of the net premium. If the real funding ratio drops below

the threshold level of 100%, then 10% of the shortage will be restored by asking participants for extra premiums. If the real funding ratio is above 120%, then 10% of the surplus will be paid back. No restrictions are laid on the premium jumps.

Main choices for the investment policy
The fund has a limited number of investment categories and only for one type of currency (euro). There has been invested in shares, nominal bonds and index linked bonds (ILB), static mix. The investments are steered on market value basis and the hedge-strategy is not used. The ILB compensates the inflation of the main sum and pays the inflation according to the wage inflation. The inflation expectation is based on the real interest according to the Nelson-Siegel real interest curve. I use the calibrated arbitrage-free economic variant in the investment module.

All pension alternatives will be evaluated for five different investment portfolios. The asset mixes of the equities, the nominal bonds and the index linked bonds are composed as following:

- 100% equities, 0% bonds and 0% ILB
- 0% equities, 100% bonds and 0% ILB
- 0% equities, 0% bonds and 100% ILB
- 50% equities,50% bonds and 0% ILB
- 50% equities, 0% bonds and 50% ILB

2. Options in the pension fund

In this section I illustrate the value transfer between the interested parties and the influence of the policy modifications. I consider the market value of the expected cash flows in the future using a technique from the option market. This technique uses a risk neutral valuation obtained from the Black-Scholes model. By translating cash flows in the market value, it becomes possible to assess the effects of the policy in the balance sheet. Remarkable of this approach is that we can see the future policy expressed in euros instead of in more difficult to interpret stochastic quantities. I start the chapter with the explanation of the term options in the pension fund.

2.1. Options in the pension fund. Consider a following agreement between the pension funds and the participant.

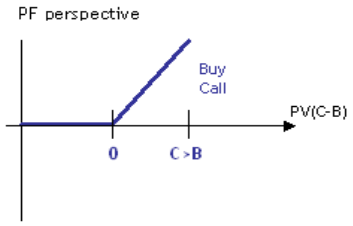

FIGURE 1. Pension fund (PF) buys a call option from a participant.

The pension fund buys a call option from the participant. This option gives the pension fund the right to keep the positive surplus which a participant may leave behind after death. The call option has a significant value when the present value of the contributions ($PV(C)$) is larger than the present value of the benefits ($PV(B)$).

Consider the period of 55 years for an entrant of 25 years old. If it appears that at the end of his life the participant has paid more premiums than the benefits available for him, then the difference in value of $(C-B)$ will remain in the possession of the pension fund. I.e., the surplus $(C - B)$ is the optional amount that pension funds could have after the participant's death. N. Kortleve names this amount the *option surplus*.

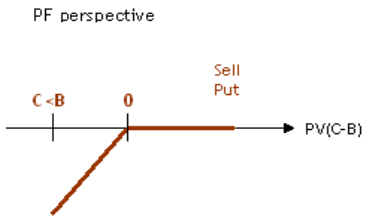

FIGURE 2. Pension fund (PF) sells a put option to a participant.

If at the end of the policy horizon it appears that the participants have paid less to premiums than the pension benefits they would receive, then we can speak of a negative surplus or deficit. By selling the put option the pension fund takes an obligation to cover all possible shortages in the pension liabilities of the participant. N. Kortleve calls this amount an *option deficit*. The put option from the pension fund perspective illustrated is in Figure 2.

2.2. Option agreements between the cohorts. Option contracts are a way to estimate the value of the transfers. A pension contract of a participant involves writing a call option and buying a put option from the pension fund, in the sense that the economic value can be both positive and negative.

All members of the pension fund can be split up in different age cohorts. This section illustrate the trade of the embedded options between different cohorts.

There are two situations that can appear. In the first situation the participant (or the age cohort) pays little contributions, but because of the rising funding ratio level and better economy, the participant nevertheless can get a high pension. In the other situation the participant can pay his contributions properly for 40 years, but because of the recession and underfunding only few benefits will be available to him. The participant would prefer to insure his benefits against the second situation at a certain price. On the other hand he can share his rising potential with the other cohorts in exchange for other benefits.

In other words, the cohort sells a part of its rising potential to the other generations, in exchange for a partial protection of its pension benefits. In terms of options the following can be said. The cohort writes a call option to the other generations and holds a put option from the other generations.

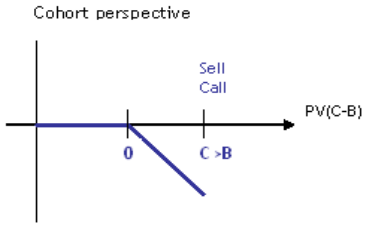

FIGURE 3. Cohort sells a call option to other cohorts

$(C - B)$ is the optional amount that the cohort transfers to the other generations, which suffers from the shortage. (From a pension fund point of view this amount is a positive surplus which a cohort may leave in the pension fund after death.)

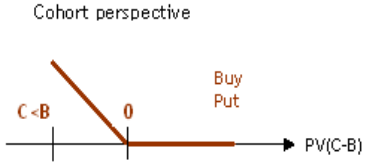

FIGURE 4. Cohort buys a put option from other cohorts

$(B - C)$ is the optional amount that the cohort could get from the other cohorts to cover the shortages in its pension benefits. (From a pension fund point of view this is an amount (negative surplus or deficit) which has to be transferred by the pension fund to a cohort suffering from a deficit.)

2.3. Put and Call in the theoretical fund. From the previous sections we can see that the positive and negative transfers can be interpreted differently from different angles. The analysis of the value transfers in this section is from the pension fund perspective. Here I examine the behaviour of the transfers with respect to the initial nominal funding ratio ranging from 60% to 170% [1]. The transfers of two different age cohorts are presented: 25 years old cohort which consists of the active members of the pension fund, and 65 years old cohort, the members of which are retirees. (See also Figures 5 and 6).

The figures reveal the nature of the embedded options hidden in the agreements between the pension fund and different age cohorts. The pension fund has different expectation from the retired participants who do not make payments anymore and from the young participants who contribute to the pension fund's assets.

[1]For the investigation in this section the following alternative is chosen: the contribution policy with $\alpha = 0.2$, the indexation policy with 100% lower bound and 120% upper bound, and the investment policy with 75% equities and 25% bonds.

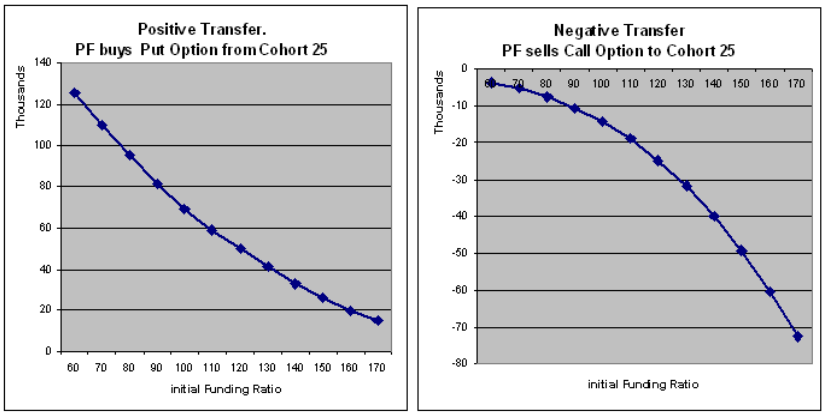

FIGURE 5. Transfers of the 25 years old cohort.

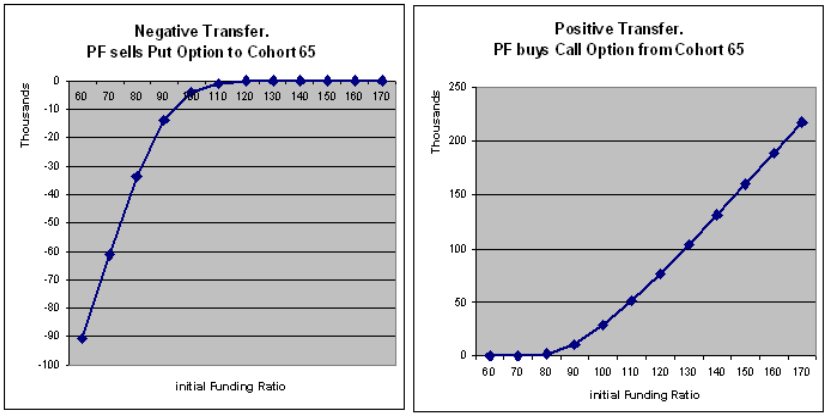

FIGURE 6. Transfers of the 65 years old cohort.

First consider the value that the young generation will leave in the pension fund (positive transfers in the left picture of Figure 5). If the young participant enters the pension fund with a low (initial) funding ratio he would leave behind a high value which is profit for the pension fund. The value of positive transfers converges to 0 for high funding ratios while the value of the negative transfers rises in proportion. For the young participant entering the pension fund with a high funding ratio means paying less contributions and getting heightened entitlements, which can result in short assets.

The case with the old generation (65 age cohort, see Figure 6) throws the opposite view. The higher the initial funding ratio of the pension fund, the higher the value that the retiree would leave behind. For the retiree in the fund with a low funding ratio the pension fund is obliged to cover the shortages in assets for this participant. A low funding ratio ($\leq 100\%$) implies that the retiree has paid too little contribution in the past. Detailed analysis will be presented in the next section. The summary picture of the studying policy alternative with the funding ratio 100% for all members of the theoretical pension fund is presented in Figure 8.

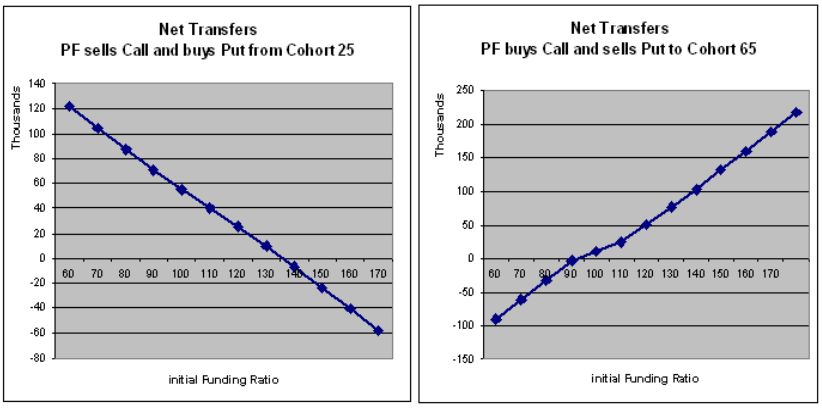

FIGURE 7. Net transfers per cohort.

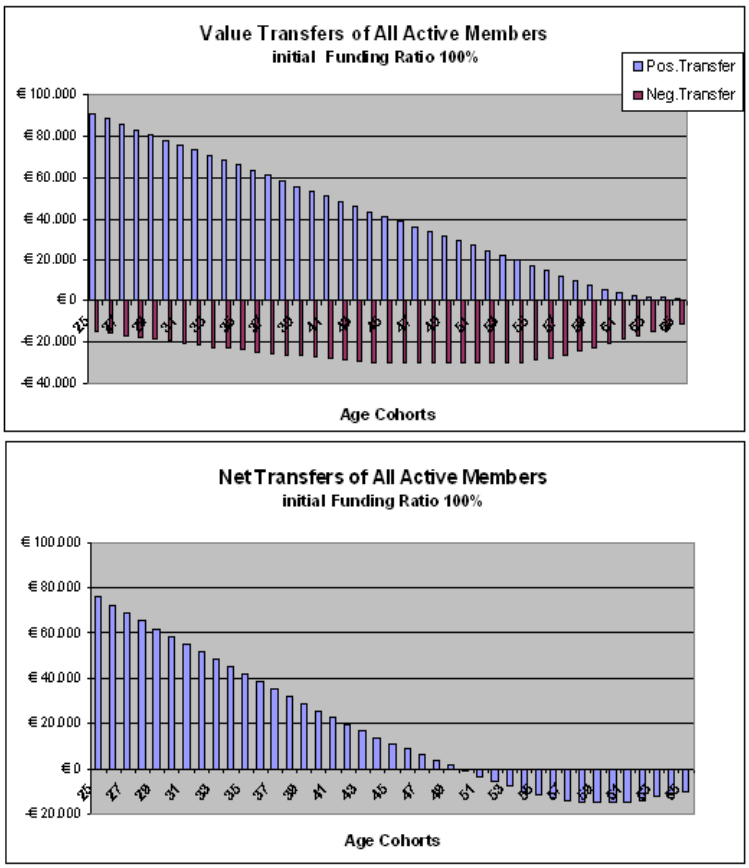

FIGURE 8. Value transfers of all pension fund members.

In the patterns of the agreements between the pension fund and the two age cohorts discussed above, the call and the put options can be recognized. The

developments of the value transfers in both graphs are similar to the buying/selling put option and buying/selling call option to an age cohort considering from the pension funds viewpoint. The interpretation of the selling a put option is simply the 'residual risk' of assets falling short of liabilities.

Figure 7 shows the net value of the transfers per age cohort (in respect to initial funding ratio), which is a straight line on the graphs. It is now clear that if the nominal initial funding ratio is high (more then 100%) the old generation will have paid more than they would receive. It is exactly the picture that we can expect when the cohort buys one kind of option and sells another one.

In the next section I examine the embedded options in the theoretical fund by composing the different policy alternatives and using the balance sheet to represent the cash flows 55 years in the future. The length of the 55 years horizon reflects the length of youngest generation.

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3. Results

In my research I looked at the cohorts from the perspective of the pension fund. Possible future cash flows of the generations in 55 years from now were analysed. For the notation of the call and put options I use the notations of N. Kortleve. The term for the optional amount that the cohort could receive from the pension fund to cover possible shortages is the *option deficit*, and the term for the optional amount that the cohort could leave to share with the other cohorts, is the *option surplus*.

3.1. Analysis of the options in theoretical fund. Here I examine three alternatives with the different investment policy (100% equities and 0% equities), with the conditional indexation, and different initial funding ratios. The starting point of the analysis of the first alternative as a current strategic policy and then two other alternatives, which differ from the first one on investment portfolio and on the initial funding ratio. The aim of this analysis is to examine the impact of changing policy on the present value of the cash flows comparing two generations.

3.1.1. *Alternative 1.*

- Contribution policy: $\alpha = 0.2$
- Indexation policy: lower bound 100% and upper bound 120%
- Investment policy: 100% equities and 0% bonds
- Initial real funding ratio is 100%

25 age cohort			
Assets	€3.7	Benefits	€62.5
Contributions	€115.0		
Option Deficit	€26.7	Option Surplus	€83.0
	Residue	*€56.3*	

65 age cohort			
Assets	€304.9	Benefits	€279.6
Contributions	€0.0		
Option Deficit	€4.2	Option Surplus	€29.4
	Residue	*€25.2*	

TABLE 1. Alternative 1 (x 1,000 euros).

The balance sheets of two age cohorts reflect economic values from a pension fund point of view. The 100% equity strategy may lead to an underfunding which implies a high risk profile for the pension fund to cover the liabilities. When the funding ratio will drop below the threshold of 100%, the 20% of the asset shortages in the liabilities of the pension fund will be restored by the active cohorts by paying extra contributions. It will results in much higher contribution payments with respect to the future benefits, the remainder of the payments will be left in the pension fund. The value of the remainder is presented as the option surplus value of 83 thousand euros. Because of unstable funding ratio over time the possibility of the liability shortages is high. The present value of the option deficit of the 25 age cohort is 26.7 thousand euros.

The risk-neutral valuation method attaches a high present value to the outcomes in bad economic times. It is very expensive to hedge the situation of underfunding which typically occurs in bad times. When a retired participant in the pension fund with 100% equity strategy, his entitlements will not be replenished in the situations of underfunding which results in a lower value of future benefits (279.6 thousand euros) and a high option surplus of 29.4 thousand euros. The economic value of the funding residue of the young cohort is more than twice as residue value of the retirees.

3.1.2. *Alternative 2.*

- Contribution policy: $\alpha = 0.2$
- Indexation policy: lower bound 100% and upper bound 120%
- Investment policy: 0% equities and 100% bonds
- Initial real funding ratio is 100%

25 age cohort			
Assets	€3.7	Benefits	€88.5
Contributions	€145.2		
Option Deficit	€0.0	Option Surplus	€60.4
	Residue	*€60.4*	

65 age cohort			
Assets	€304.9	Benefits	€291.9
Contributions	€0.0		
Option Deficit	€3.1	Option Surplus	€16.1
	Residue	*€12.9*	

TABLE 2. Alternative 2 (x 1,000 euros).

It is possible to decrease the probability of the underfunding by changing the investment policy by increasing amount of bonds to 100% and decreaseing risky equities to 0%. The 100% bond strategy lead to lower probability of the underfunding which results in higher benefit payments for the both generations. Unfortunatly, for the stability the young cohort will pay 30 thousand euros higher contributions comparing with the first alternative. For the pension fund this situation is most profitable because the fund will not need to transfer the value to the young cohort to cover the shortages (option deficit is zero) and the value that the cohort can leave behind is even greather than in the previous alternative. The values of the option surplus and the option deficit of the 65 age cohort are both decreased, the residue value is now 12.9 thousand euros which is most advantageous for this generation.

3.1.3. *Alternative 3.*

- Contribution policy: $\alpha = 0.2$
- Indexation policy: lower bound 100% and upper bound 120%
- Investment policy: 100% equities and 0% bonds
- Initial real funding ratio is 140%

25 age cohort			
Assets	€5.2	Benefits	€63.2
Contributions	€49.9		
Option	€55.6	Option	€47.5
Deficit		Surplus	
	Residue	€*-8.1*	

65 age cohort			
Assets	€426.8	Benefits	€290.4
Contributions	€0.0		
Option	€0.0	Option	€136.5
Deficit		Surplus	
	Residue	€*136.5*	

TABLE 3. Alternative 3 (x 1,000 euros).

The policy alternative differs from alternative 1 only by the initial nominal funding ratio of 140%. If the participant enters the pension fund with a high funding ratio, then indexation will be fullfilled and the assets will be replenished which lead to low contribution payments. The young cohort will pay almost twice as few contributions (*49.9*) compared to the young cohort from alternative 1 (*115.0*). By a full investment in the risky equities and receiving less premiums, the pension fund is susceptible to the danger of not meeting the liabilities of the young cohort. It results in a high possibility of the pension fund transfering more value to the young participants than receiving from them (the option deficit is 55.6 thousand euros). Obviously this cohort will have left much less value in the pension fund. The proportion between the two options has considerably changed in comparison to the first balance in this section. The economic value that the young generation receives from the pension fund is 8.1 thousand euros and the old generation the other way around, will leave 136 thousand euros behind which is profitable for the pension fund. The value transfers in this alternative are less honest than in the first one.

If we look at the balance sheets from the cohorts point of view, then it can be concluded that the 25 age cohort prefers a 100% equity strategy (rather then a high initial funding ratio) and the 65 age cohort prefers a 100% bond strategy.

3.2. Analysis of the policies. The comparison of the transfer values is displayed in the graphs in Appendix A. All policy variants are calculated with an initial real funding ratio of 100%. The description of the policy is given in the header of the graph. The investment portfolio consists of the equities and the nominal bonds only. Their proportion in the portfoilio is given on the x-axis. The contribution policy is represented by α, which stands for the percentage of the shortage to be restored by participants (in case the funding ratio falls below the threshold of 100%) or, in case the funding ratio is higher than 120%, it stands for the persentage of surplus to paid back to the participants. In case of an unconditional indexation

policy, the indexation is fully granted, in other cases the indexation is granted linearly between the lower and the upper bounds. I compare different policy variants for the three generations: 25, 40 and 65 years old cohorts. Each graph shows a positive and a negative transfer value per cohort per variant, meaning the value that will be left behind by cohort in the pension fund, and the value that will be received by cohort, respectively.

It is remarkable that in all cases the positive values are greater than the negative values and the active members of the pension fund leave much more value behind than the retirees. Figure 1 and Figure 2 (Appendix A) show that the modifications in the indexation and the contribution policies have no significant influence on the transfer values. The modifications in the investment portfolio in the variant with the conditional indexation give perceptible results: the higher the percentage of the risky equities the higher the value that cohorts will leave behind in the pension fund. But when the indexation policy is unconditional the risky investment policy becomes advantageous for the retired generation (see Figure 3 and Figure 4): the older the participants are, the less value they will leave in the pension fund.

3.3. ALM Analysis of the pension deals. Five distinctive pension deals will be discussed here. For the evaluation of these deals both traditional and value-based approaches of ALM are used. Value-based ALM will show who will gain or loose from changing the current pension deal. In contradiction to the classical approach, the value-based approach uses arbitrage free economic variants to produce the scenario's output. The future outcomes are discounted back to the present with an appropriate risk adjusted discount rate, which is realized by a risk neutral valuation (see Chapter 6).

This analysis enables us to show the impact of the alternative pension deals on the value for the stakeholders. All policy variants are calculated with an initial real funding ratio of 100%. These deals differ in the contribution policy, indexation policy and asset mix.

The term *pension deal* can be rendered as the contract between the pension fund and the stakeholders. This contract emphasizes the nature of the pension promise, the funding of this promise, and how the risks in the funding process are allocated among the stakeholders. A pension deal has clear rules prescribing who has to pay, when and to what extent in a deficit situation. These rules also set down who will benefit, when, and to what extent in a surplus situation. [**2**]

3.3.1. *Deal 1.*

- contribution policy: $\alpha = 1$,
- indexation policy: unconditional.

Pension deal 1 is characterized by the unconditional indexation policy and the contribution rate that is adjusted yearly in order to absorb the risk in the pension fund. This deal has 100% absorbtion speed of the risk. The target funding ratio is defined as the 100% funding ratio, this is the situation where the assets are equal to the value of the indexed liabilities. Table 4 reflects the core results in terms of the expected values and riskiness of the variables for five investment mixes. This is the usual classical ALM output.

In the situations of the overfunding a contribution cut takes place and the additional charge is asked in case of underfunding. Full adjustments of the funding

ratio back to the target level in one year will lead to extreme adjustments in contributions. Deviations of the contributions displayed in the classical ALM results are quite high, especially when the investment portfolio consists of a risky equity. The higher percentage of the equities in the portfolio, the higher the fluctuations in the contribution payments. The contribution rate in the equity strategy displays on average a downward trend (-10.52), which is caused by cuts in the contribution rate. The probability measure that the contribution increase will be greater than 95% is lower for the equity mix, which can be explained by a majority of the contribution cuts. The strong fluctuations of contribution payments (see Figure 1, Appendix B) in deal 1 results in a very high 5% Value at Risk. With the probability of 5% the total net contribution is at least 99.90.

The bond strategy delivers no excess return, so the real funding ratio fluctuates less and the average is (99.61) for 100% nominal bonds (see Figures 2 and 3, Appendix B). This leads to a higher probability of underfunding for 100% bonds. The funding ratio of the low risky bond strategy remains stable over time. Because of unconditional indexation the purchaising power is 100%. The results of the 100% nominal bonds and 100% index linked bonds (ILB) strategy are very close to each other, although the latter is less risky and the average of the contribution rate is lower. Table 5 shows the results of value-based ALM. The balance sheets reflect economic values. It is remarkable that the investment policy has no impact on the overall net transfers. However, there are likely transfers between generations. The funding residue in all variants are almost the same. Note that economic values of the funding residue in five investment variants are negative. This is to be explained by the higher economic value transfered to the participants than received from them.

	100% equities	100% bonds	100% ILB	50% equities 50% bonds	50% equities 50% ILB
Funding Ratio					
average (real)	122.55	99.61	99.61	102.22	102.26
average (nominal)	157.99	129.60	129.79	132.92	133.09
risk (st dev)	26.00	6.19	5.55	13.70	13.62
prob underfunding (real)	39.88	54.80	58.50	44.61	44.54
prob underfunding (nominal)	12.89	7.68	9.00	8.36	8.95
Contributions					
average	-10.52	19.12	19.03	4.94	4.89
risk (st dev)	88.41	38.92	32.67	77.44	76.49
95% VaR net premium	99.90	60.41	55.10	99.90	99.90
prob increase $> 5\%$	37.54	45.28	39.82	48.05	48.13
Indexation					
prob indexation cuts	0.00	0.00	0.00	0.00	0.00
average purchasing power	100.00	100.00	100.00	100.00	100.00
95% VaR purchasing power	100.00	100.00	100.00	100.00	100.00

TABLE 4. Classic ALM results Deal 1

			100% equities
Assets (A_0)	7.79	Liabilities	3.21
Contributions	10.57	Benefits	15.21
Option Deficit	3.30	Option Surplus	3.24
	Residue	*-0.06*	
			100% bonds
Assets (A_0)	7.79	Liabilities	3.21
Contributions	10.59	Benefits	15.21
Option Deficit	0.78	Option Surplus	0.73
	Residue	*-0.05*	
			100% ILB
Assets (A_0)	7.79	Liabilities	3.21
Contributions	10.60	Benefits	15.21
Option Deficit	0.30	Option Surplus	0.28
	Residue	*-0.02*	
			50% equities, 50% bonds
Assets (A_0)	7.79	Liabilities	3.21
Contributions	10.58	Benefits	15.21
Option Deficit	2.03	Option Surplus	1.97
	Residue	*-0.06*	
			50% equities, 50% ILB
Assets (A_0)	7.79	Liabilities	3.21
Contributions	10.59	Benefits	15.21
Option Deficit	1.72	Option Surplus	1.67
	Residue	*-0.05*	

TABLE 5. Value-based ALM results Deal 1 (x 1,000,000 euros).

3.3.2. *Deal 2.*

- contribution policy: $\alpha = 0.1$,
- indexation policy: unconditional indexation.

Pension deal 2 is characterized by slow a risk absorption in the contribution policy. The indexation policy is unconditional. This deal has 10% absorbtion speed of the risk. The merits of the modification in the contribution policy are apparent in Table 6, illustrating the effect on the contribution risk. All variants result in a decline of the risk of extreme contributions, being measured by the 5% VaR of the net contributions. Figure 4 and Figure 5 in Appendix B show the net contributions in two investment strategies. In particular, variant with the 100% bond mix results in a strong decline of the contribution deviation. Also the contribution jumps are dumpened. Because the shortfall in the assets will be restored by only 10%, the average funding ratio is lower for the variants with 100% bonds strategy (see Figure 6, Appendix B). Note that the variants with risky investments in the portfolio in deal 2 lead to a higher average funding ratio and a lower probability of underfunding. Despite the improved average funding ratio, the value of the funding residue in deal 2 is lower than in deal 1. The negative residue means that a mismatched risk is shifted towards the future generation. When we compare the averages of the contributions, we note that the quantities in the classical ALM vary strongly while the economical values of these payments will have almost no difference (see Table 7). Similar to deal 1, in deal 2 the investment policy has little influence on the funding residues in five variants.

	100% equities	**100% bonds**	**100% ILB**	**50% equities 50% bonds**	**50% equities 50% ILB**
Funding Ratio					
average (real)	223.51	96.14	96.51	121.34	121.29
average (nominal)	287.24	124.79	126.09	157.32	157.79
risk (st dev)	46.10	4.08	3.68	12.19	12.17
prob underfunding (real)	30.10	79.16	89.60	29.64	28.91
prob underfunding (nominal)	15.05	10.50	13.88	8.05	8.63
Contributions					
average	-13.95	19.12	18.97	3.45	3.47
risk (st dev)	14.92	2.92	2.90	7.76	7.76
95% VaR net premium	41.54	25.40	25.09	30.90	30.53
prob increase $>$ 5%	30.82	5.19	5.34	23.94	23.84
Indexation					
prob indexation cuts	0.00	0.00	0.00	0.00	0.00
average purchasing power	100.00	100.00	100.00	100.00	100.00
95% VaR purchasing power	100.00	100.00	100.00	100.00	100.00

TABLE 6. Classic ALM results Deal 2

			100% equities
Assets (A_0)	7.79	Liabilities	3.21
Contributions	10.27	Benefits	15.21
Option Deficit	3.13	Option Surplus	2.76
	Residue	*-0.37*	
			100% bonds
Assets (A_0)	7.79	Liabilities	3.21
Contributions	10.31	Benefits	15.21
Option Deficit	0.89	Option Surplus	0.56
	Residue	*-0.33*	
			100% ILB
Assets (A_0)	7.79	Liabilities	3.21
Contributions	10.33	Benefits	15.21
Option Deficit	0.46	Option Surplus	0.16
	Residue	*-0.30*	
			50% equities, 50% bonds
Assets (A_0)	7.79	Liabilities	3.21
Contributions	10.28	Benefits	15.21
Option Deficit	1.99	Option Surplus	1.63
	Residue	*-0.93*	
			50% equities, 50% ILB
Assets (A_0)	7.79	Liabilities	3.21
Contributions	10.29	Benefits	15.21
Option Deficit	1.71	Option Surplus	1.37
	Residue	*-0.34*	

TABLE 7. Value-based ALM results Deal 2 (x 1,000,000 euros).

3.3.3. *Deal 3.*

- contribution policy: $\alpha = 0.1$,
- indexation policy: 100% lower bound and 120% upper bound.

Pension deal 3 is characterized by a slow risk absorption in the contribution policy and a restricted indexation policy. The magnitude of the indexation is related proportionally to the size of the available indexation reserve, this is the difference between assets and nominal liabilities. In deal 3 the upper bound is the situation when the real funding ratio is 120% and the lower bound is the situation where the real funding ratio is 100%. The basic idea of the policy ladder is explained in Chapter 8. This deal has 10% absorbtion speed of the risk. From the classical ALM results in Table 8 we make up that the averages of the funding ratio in all variants are increased as result of the indexation cuts. The improved situation in the classical ALM is reflected in value-based ALM. The economic value of the funding residue is substantial by increased and has a positive value now, which means that the participants will leave value behind in the pension fund. This is advantageous for the future generations. Notice that comparable to previous deals the asset policy in deal 3 makes a difference on the balance sheet. The variations of the expected contributions from the classical ALM are seen in the value-based ALM. The economic value of the pension benefits is decreased, which can be expected as result of the indexation cuts. The latter is also the cause of the decline the average purchasing power.

	100% equities	100% bonds	100% ILB	50% equities 50% bonds	50% equities 50% ILB
Funding Ratio					
average (real)	270.75	114.98	113.46	144.67	144.19
average (nominal)	350.24	151.21	149.65	189.13	189.06
risk (st dev)	56.41	5.24	4.24	14.72	14.60
prob underfunding (real)	19.65	4.60	5.13	11.72	11.57
prob underfunding (nominal)	8.04	2.15	4.63	2.70	3.18
Contributions					
average	-18.14	15.86	16.85	-0.45	-0.25
risk (st dev)	14.57	1.70	1.44	7.72	7.67
95% VaR net premium	32.52	17.49	17.49	22.13	22.07
prob increase $>$ 5%	27.05	3.13	3.34	20.57	20.47
Indexation					
prob indexation cuts	27.91	64.13	74.56	27.97	27.21
average purchasing power	82.81	79.62	83.54	84.54	85.26
95% VaR purchasing power	54.96	66.92	77.15	61.04	62.90

TABLE 8. Classic ALM results Deal 3

			100% equities
Assets (A_0)	7.79	Liabilities	2.41
Contributions	6.22	Benefits	11.06
Option Deficit	2.78	Option Surplus	3.33
	Residue	*0.55*	
			100% bonds
Assets (A_0)	7.79	Liabilities	2.83
Contributions	8.08	Benefits	12.59
Option Deficit	0.54	Option Surplus	0.98
	Residue	*0.44*	
			100% ILB
Assets (A_0)	7.79	Liabilities	2.93
Contributions	8.71	Benefits	13.31
Option Deficit	0.18	Option Surplus	0.44
	Residue	*0.26*	
			50% equities, 50% bonds
Assets (A_0)	7.79	Liabilities	2.54
Contributions	6.77	Benefits	11.50
Option Deficit	1.64	Option Surplus	2.16
	Residue	*0.52*	
			50% equities, 50% ILB
Assets (A_0)	7.79	Liabilities	2.61
Contributions	7.10	Benefits	11.78
Option Deficit	1.35	Option Surplus	1.86
	Residue	*0.51*	

TABLE 9. Value-based ALM results Deal 3 (x 1,000,000 euros).

3.3.4. *Deal 4.*

- contribution policy: $\alpha = 0.1$,
- indexation policy: 75% lower bound and 120% upper bound.

Pension deal 4 differs from the previous pension deal by the lower indexation bound. Because the lower bound is 75% the liabilities will be partially indexed even in the situation of underfunding. It results immediatly in a decrease of the funding ratio on average and an increase of the underfunding probability compared to deal 3. Because of broader bounds the probability of indexation cuts increase. Classical ALM results show an increased purchasing power on average. This improved situation is reflected on value-based ALM by increased economic value of the pension benefits. Further we can see that the contributions are declined on average while they gain in economic value. Decreased benefits and increased contributions results in lower economic value of the funding residue.

	100% equities	**100% bonds**	**100% ILB**	**50% equities 50% bonds**	**50% equities 50% ILB**
Funding Ratio					
average (real)	255.31	107.88	106.61	138.38	137.87
average (nominal)	330.22	141.65	140.82	180.55	180.52
risk (st dev)	53.37	4.70	3.76	13.95	13.83
prob underfunding (real)	26.31	18.25	17.44	19.13	18.88
prob underfunding (nominal)	13.43	4.77	8.68	5.62	6.26
Contributions					
average	-16.01	16.21	16.65	0.75	0.97
risk (st dev)	13.27	0.74	0.25	6.92	6.86
95% VaR net premium	25.13	16.60	16.60	16.60	16.60
prob increase $>$ 5%	2.88	2.23	2.05	17.07	16.97
Indexation					
prob indexation cuts	33.70	80.02	88.03	34.99	34.26
average purchasing power	84.86	83.06	85.48	87.77	88.39
95% VaR purchasing power	54.83	69.00	75.57	62.39	64.52

TABLE 10. Classic ALM results Deal 4

			100% equities
Assets (A_0)	7.79	Liabilities	2.39
Contributions	6.00	Benefits	11.18
Option Deficit	2.67	Option Surplus	2.88
	Residue	*0.21*	
			100% bonds
Assets (A_0)	7.79	Liabilities	2.81
Contributions	8.16	Benefits	12.91
Option Deficit	0.63	Option Surplus	0.85
	Residue	*0.22*	
			100% ILB
Assets (A_0)	7.79	Liabilities	2.83
Contributions	8.38	Benefits	13.38
Option Deficit	0.34	Option Surplus	0.29
	Residue	*-0.05*	
			50% equities, 50% bonds
Assets (A_0)	7.79	Liabilities	2.55
Contributions	6.83	Benefits	11.81
Option Deficit	1.63	Option Surplus	1.89
	Residue	*0.26*	
			50% equities, 50% ILB
Assets (A_0)	7.79	Liabilities	2.62
Contributions	7.19	Benefits	12.13
Option Deficit	1.37	Option Surplus	1.61
	Residue	*0.24*	

TABLE 11. Value-based ALM results Deal 4 (x 1,000,000 euros).

3.3.5. *Deal 5.*

- contribution policy: $\alpha = 0.1$,
- indexation policy: 100% lower bound and 140% upper bound.

Because the lower indexation bound is 100%, the funding ratio on average decreases compared to the deal 4 and the deals with unconditional indexation. The upper bound is high which means that indexation cuts occurs till the proportion between assets and liabilities become 140%. The situations when the indexation cuts occur lead quite always to the decrease of the probability of underfunding. In good times no additional charges will be asked, the decrease in contributions is a result of this. Classical ALM results show a decreased purchasing power on average. This worsened situation is reflected on value-based ALM by decreased economical value of the pension benefits. The decreased surplus and increased deficit results in improved economic value of the funding residue.

	100% equities	100% bonds	100% ILB	50% equities 50% bonds	50% equities 50% ILB
Funding Ratio					
average (real)	309.26	125.60	124.59	161.95	161.39
average (nominal)	400.24	165.77	165.21	211.91	211.83
risk (st dev)	64.86	5.70	4.55	16.40	16.24
prob underfunding (real)	16.83	3.21	3.48	9.07	2.53
prob underfunding (nominal)	7.09	1.30	3.05	2.16	8.97
Contributions					
average	-18.86	15.95	16.73	-1.43	-1.25
risk (st dev)	14.64	1.46	1.01	8.05	7.99
95% VaR net premium	31.39	16.62	16.83	20.85	20.76
prob increase $> 5\%$	23.98	2.95	2.77	19.31	19.19
Indexation					
prob indexation cuts	33.04	74.49	78.52	34.94	34.11
average purchasing power	81.53	74.54	77.24	82.33	83.01
95% VaR purchasing power	53.21	60.84	69.07	59.11	60.12

TABLE 12. Classic ALM results Deal 5

			100% equities
Assets (A_0)	7.79	Liabilities	2.40
Contributions	6.43	Benefits	10.95
Option Deficit	2.73	Option Surplus	3.59
	Residue	*0.86*	
			100% bonds
Assets (A_0)	7.79	Liabilities	2.81
Contributions	8.16	Benefits	12.91
Option Deficit	0.63	Option Surplus	0.85
	Residue	*0.22*	
			100% ILB
Assets (A_0)	7.79	Liabilities	2.92
Contributions	8.59	Benefits	12.76
Option Deficit	0.08	Option Surplus	0.77
	Residue	*0.69*	
			50% equities, 50% bonds
Assets (A_0)	7.79	Liabilities	2.56
Contributions	7.01	Benefits	11.35
Option Deficit	1.52	Option Surplus	2.41
	Residue	*0.89*	
			50% equities, 50% ILB
Assets (A_0)	7.79	Liabilities	2.63
Contributions	7.33	Benefits	11.59
Option Deficit	1.21	Option Surplus	2.11
	Residue	*0.90*	

TABLE 13. Value-based ALM results Deal 5 (x 1,000,000 euros).

4. Conclusions

This chapter analyzes the value-based ALM in comparison to the classical ALM approach for the theoretical pension fund. Various pension deals are formulated which differ by the contribution or indexation policy. The pension deals are evaluated for five proposed asset mixes. Outcomes in terms of the statistical measures and the economic values are compared. View the summary of the economic values of the funding residue per pension deal.

	Deal 1	Deal 2	Deal 3	Deal 4	Deal 5
100% equities	-0.06	-0.37	0.55	0.21	0.86
100% bonds	-0.05	-0.33	0.44	0.22	0.22
100% ILB	-0.02	-0.30	0.26	-0.05	0.69
50% equities, 50% bonds	-0.06	-0.93	0.52	0.26	0.89
50% equities, 50% ILB	-0.05	-0.34	0.51	0.24	0.90

TABLE 14. Funding residues in the theoretical pension fund

From the investigation of the pension deals it can be concluded that the investment strategy makes no difference in economic value terms for alternatives with an unconditional indexation policy. Pension deal 1 with an unconditional indexation policy has least spread between economic values of the deficit and the surplus. Note, the unconditional indexation results in the negative transfers (shortages) and conditional indexation in the positive transfers (surplus). This difference can be presented by calculating of the indexation option which is similar to the indexation option represented by Theo Kocken in Chapter 5. This option depends on the funding ratio: if the funding ratio drops below a threshold level, the members are obliged to waive indexation of their entitlements. Because the indexation has influence on the liabilities and benefit payments, I use the sum of these quantities to calculate the indexation option as a difference of the totals between the deals with unconditional and conditional indexation policy. Figure 9 reveals the value of the indexation option which participants write on the funding ratio of the pension fund by changing the pension deal. We see that a deal with 75% lower and 120% upper indexation bounds (deal 4) is the least valuable option in case of changing policy. The alternatives consisting 100% bonds in the investment mix has always a lower option value than alternatives with 100% equities. It means that the participants will loose the advantage in the future in case the sponsor pays the contributions.

Classical ALM shows in some alternatives optimistic results like an increase in the funding ratio on average, while the funding residue of the pension fund declines in the economic value terms. Because the investigation shows sometimes contradictory results from both ALM approaches this analysis demonstrates the usefulness of the value-based ALM next to the classical approach.

The new insight of the value-based ALM approach is that we can look at the stakes of various parties joining the pension fund and can see the impact of changing the pension deal on various stakeholders. This will help to setup a sustainable pension deal and avoid that one group has to pay for shortfalls of the another. The intergenerational analysis of the option deal will be presented in the next chapter.

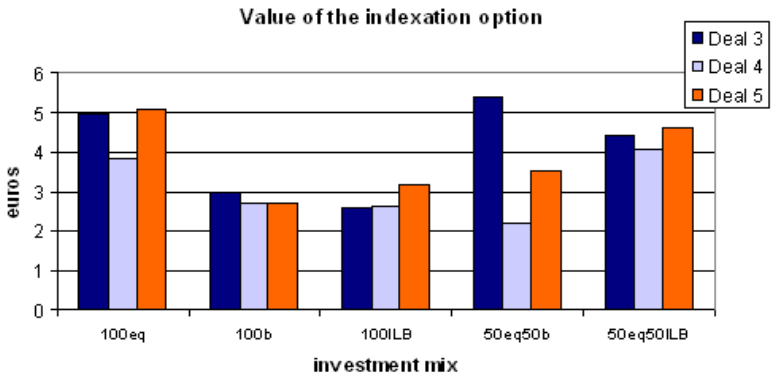

FIGURE 9. Value of the indexation option in the theoretical pension fund.

CHAPTER 10

HOLLANDIA FUND

This chapter investigates the possibilities of the value-based ALM in the pension fund close to an average Dutch pension fund. Data for this fund is obtained from several real pension funds to avoid the complexity and to create the "average" policy of a pension fund in the Netherlands.

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1. Introduction

The Hollandia fund can be interpreted as a "representative" Dutch pension fund with whom we regularly worked at the company to carry out the tests.

Main choices for the pension accrual
The Hollandia fund has two pension types, namely, Old Age Pension (OP) and the Widows/widower Pension (WP). Both types are built-up according to the defined benefit plan (DB) and the pension sum is defined by the pension scheme, which is based on the average salary (see Chapter 2). The maximum pension benefit that a participant can get is 70% on 40 years of service. The yongest entrant is 25 years old and the pension age is 65 years, the age when the contribution payment stops. For the WP pension type the maximum benefit is 49% on 40 participant years. The WP type has an possibility to exchange it for the other type of the pension.

Main choices for the indexation policy
The Hollandia fund pursues complete indexation of pension entitlements with the inflation of the pensions. In case of conditional indexation, the indexation is granted linearly between certain upper and lower bounds of the funding ratio. If it is chosen in the fund for the conditional indexation policy, then the arrear indexation will be caught up if the real funding ratio is larger than 100%.

Main choices for the contribution policy
The gross basic contribution is stipulated as a static contribution based on 100% NFTK (New Financial Test Framework) cost-recovery contribution. This contribution is the combination of the comingservice and the risk premium, based on a fixed discount rate of 3.5% plus a replenishment for the Required Solvency Level. This solvency level depends on the asset mix. The riskier the mix, the higher the solvency buffer. The initial real funding ratio is 100%.
Recovery contributions are asked in the following situations. In case of shortage in the assets (100% of the nominal market value of the Pension Liability Provision plus the Required Solvency Level) the additional contributions of the participants

are asked. The 1/15th part of the shortage must be deposited. If the assets are larger than 100% of the real value of the Pension Liability Provision, then 1/15th part of the surpluses will be returned to the participants, i.e., contributions are reduced. The contribution increases and falls are limited up to 2%. If the contribution amount is not defined, then the net contribution varies between 0% and 20% of the salary sum of a participant.

Main choices for the investment policy
For my research I have chosen for a limited number of investment categories and for one type of currency (euro). One has invested in equities, nominal bonds and index linked bonds (ILB), static mix. The investments are steered on the market value and no duration matching strategy is used. The ILB compensates the inflation on the principal sum and pays inflation according to the price inflation. The inflation expectation is based on the difference between the nominal and real interest according to the Nelson-Siegel real interest curve. I use the calibrated arbitration-free economic variant in the investment module.

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2. Results

In the research of the Hollandia fund all participants are divided in four age cohorts: the 25-34 cohort, the 35-44 cohort, the 45-54 cohort and the 55-64 cohort. Possible future cash flows of different generations in 20 years from are analyzed. The balance sheets of the cohorts represented in Section 2.1 should be looked from the pension fund perspective. This chapter uses the same notations as previous chapter.

2.1. ALM Analysis of the pension deals. Three distinctive pension deals are discussed here. For the evaluation of these deals both traditional and value-based approaches of the ALM are used. In contradiction to the classical approach, the value-based uses arbitrage free economic variant to produce the output of the scenario analysis. The future outcomes are discounted back to the present with an appropriate risk adjusted discount rate, which is realized by risk neutral valuation (see Chapter 6). The pension deals differ from each other by changing one or more policies. I give detailed description and analysis of the first pension deal and give explanation of the influences of the policy modifications.

2.1.1. *Deal 1.*

- contribution policy: fixed contributions, 12 % of the salary,
- indexation policy: unconditional.

Pension deal 1 is characterized by no active risk management at all. The funding ratio is not corrected by asking higher contributions from participants and the indexation is linked to the actual inflation and is always given. The alternative with an investment mix consisting of 100% equity is characterized by a high probability of bankruptcy of the pension fund. The results of this alternative are disregarded.

Table 1 shows the results of Classical ALM. The expected funding ratio of the alternatives with investment mixes consisting of equities will increase over time, but this policy has a higher risk profile compared to the alternatives consisting of

100% bonds or 100% index linked bonds. However, the investment policy has no influence at all in terms of value-based ALM, as can be read from Table 2.

	100% bonds	**100% ILB**	**50% equities 50% bonds**	**50% equities 50% ILB**
Funding Ratio				
average (real)	90.97	91.61	114.38	114.66
average (nominal)	126.53	129.08	159.23	160.65
risk (st dev)	4.02	2.10	12.07	12.01
prob underfunding (real)	90.70	100	43.96	43.59
prob underfunding (nominal)	10.93	16.40	13.79	14.93
Contributions				
average	12	12	12	12
risk (st dev)	0	0	0	0
95% VaR net premium	12	12	12	12
prob increase $> 5\%$	0	0	0	0
Indexation				
prob indexation cuts	0	0	0	0
average purchasing power	100	100	100	100
95% VaR purchasing power	100	100	100	100

TABLE 1. Classic ALM results Deal 1

			100% bonds
Assets (A_0)	2980	Liabilities	3148
Contributions	1705	Benefits	2030
Option Deficit	509	Option Surplus	16
	Residue	*-493*	
			100% ILB
Assets (A_0)	2980	Liabilities	3148
Contributions	1705	Benefits	2030
Option Deficit	509	Option Surplus	16
	Residue	*-493*	
			50% equities, 50% bonds
Assets (A_0)	2980	Liabilities	3148
Contributions	1705	Benefits	2030
Option Deficit	509	Option Surplus	16
	Residue	*-493*	
			50% equities, 50% ILB
Assets (A_0)	2980	Liabilities	3148
Contributions	1705	Benefits	2030
Option Deficit	509	Option Surplus	16
	Residue	*-493*	

TABLE 2. Value-based ALM results Deal 1 (x 1,000,000 euros).

The balance sheets reflects economic values. Note that the Option Surplus means the value transfer to the future and the Option Deficit means that the

future generations will be faced with a shortage in the assets. The residue is the difference between these two options. The alternatives with risky equities may be viewed as attractive because of the higher expected funding ratio as can be read from the classical ALM table, the value based ALM shows that taking risk makes no difference in the present value.

2.1.2. *Deal 2.*

- contribution policy: variable from 0 % to 30 % of the salary,
- indexation policy: unconditional.

Pension deal 2 is characterized by steering using the contribution policy in order to absorb the risk of the pension fund while benefit of the participants is defined and indexation is always given. The classical ALM results show a declined contribution rate in alternatives consisting of equities, which is result of the contribution cut. The strategies with 100% bonds and 100% index linked bonds delivers no excess return which results in the additional contributions for participants. The variability of the contributions is reflected in the value based ALM. Comparing deal 2 with deal 1, the value of contribution in the alternatives without equities in the investment mix is decreased while the option value of the surplus is almost unchanged. In the alternatives with equity strategy the option value of the surplus is significantly increased as result of which the value of residue is less than in other alternatives. A negative residue means that the mismatched risk is shifted towards the future active members. Note that the additional contributions are asked in economic bad times and will have a high present value. When the assets are larger than 100% of the real value of the pension liability provision, the contribution cuts will take a place and the value of the contributions will have a lower economic value. As can be expected, the most risky investment mix has high risk (30.77%) and the contributions are highly valuated (2151 mln euro).

	100% equities	100% bonds	100% ILB	50% equities 50% bonds	50% equities 50% ILB
Funding Ratio					
average (real)	148.59	91.17	93.24	113.92	114.79
average (nominal)	207.15	126.73	131.23	158.54	160.71
risk (st dev)	30.77	4.06	2.22	12.13	12.15
prob underfunding (real)	38.42	90.98	100	39.92	39.02
prob underfunding (nominal)	18.25	10.48	13.99	9.45	10.38
Contributions					
average	11.89	12.09	12.67	10.69	10.85
risk (st dev)	1.44	0.64	0.71	1.42	1.44
95% VaR net premium	20.00	14.55	16.07	18.99	19.39
prob increase $> 5\%$	0	0	0	0	0
Indexation					
prob indexation cuts	0	0	0	0	0
average purchasing power	100	100	100	100	100
95% VaR purchasing power	100	100	100	100	100

TABLE 3. Classic ALM results Deal 2

			100% equities
Assets (A_0)	2980	Liabilities	3148
Contributions	2151	Benefits	2030
Option Deficit	411	Option Surplus	364
	Residue	*-47*	
			100% bonds
Assets (A_0)	2980	Liabilities	3148
Contributions	1666	Benefits	2030
Option Deficit	546	Option Surplus	14
	Residue	*-532*	
			100% ILB
Assets (A_0)	2980	Liabilities	3148
Contributions	1768	Benefits	2030
Option Deficit	449	Option Surplus	19
	Residue	*-430*	
			50% equities, 50% bonds
Assets (A_0)	2980	Liabilities	3148
Contributions	1870	Benefits	2030
Option Deficit	495	Option Surplus	171
	Residue	*-324*	
			50% equities, 50% ILB
Assets (A_0)	2980	Liabilities	3148
Contributions	1909	Benefits	2030
Option Deficit	446	Option Surplus	157
	Residue	*-289*	

TABLE 4. Value-based ALM results Deal 2 (x 1,000,000 euros).

2.1.3. *Deal 3.*

- contribution policy: variable from 0% to 30% of the salary,
- indexation policy: conditional, 105% lower bound (minimal requirement) and Required Solvency level as upper bound.

Pension deal 3 is characterized by steering using both the contribution and the indexation policy. Due to the indexation policy we see at the classical results that because of the presence of the probability of the indexation cuts the average purchasing power significantly decreases. Classical results for the funding ratio and the contributions show no considerable difference with deal 2. The value-based ALM results shows that the percentage of the equities in the investment mix leads to extreme variability in the present value of the residue. As we can expect the conditional indexation policy results in a lower present value of the liabilities. In he alternative consisting of 100% equities, the participants are punished by extra payments to the pension fund and lower pension. 520 million euro is the present value that will be left by them in the pension fund. It is a too high price for taking a risk.

	100% equities	100% bonds	100% ILB	50% equities 50% bonds	50% equities 50% ILB
Funding Ratio					
average (real)	150.42	91.47	93.83	114.61	115.45
average (nominal)	209.82	127.07	131.94	159.49	161.59
risk (st dev)	30.86	4.09	2.28	12.11	12.13
prob underfunding (real)	36.30	90.30	99.85	39.18	38.13
prob underfunding (nominal)	15.25	9.75	12.98	7.91	8.99
Contributions					
average	11.70	12.05	12.61	10.53	10.69
risk (st dev)	1.48	0.63	0.68	1.42	1.45
95% VaR net premium	20.00	14.21	15.81	17.99	18.28
prob increase $> 5\%$	0	0	0	0	0
Indexation					
prob indexation cuts	33.38	6.51	9.65	17.74	18.59
average purchasing power	97.14	99.59	99.24	98.81	98.85
95% VaR purchasing power	84.55	97.49	96.40	92.70	93.07

TABLE 5. Classic ALM results Deal 3

			100% equities
Assets (A_0)	2980	Liabilities	2733
Contributions	2095	Benefits	1823
Option Deficit	376	Option Surplus	896
	Residue	*520*	
			100% bonds
Assets (A_0)	2980	Liabilities	3062
Contributions	1651	Benefits	2016
Option Deficit	497	Option Surplus	50
	Residue	*-448*	
			100% ILB
Assets (A_0)	2980	Liabilities	3122
Contributions	1763	Benefits	2025
Option Deficit	423	Option Surplus	19
	Residue	*-405*	
			50% equities, 50% bonds
Assets (A_0)	2980	Liabilities	2835
Contributions	1762	Benefits	1914
Option Deficit	459	Option Surplus	452
	Residue	*-6*	
			50% equities, 50% ILB
Assets (A_0)	2980	Liabilities	2919
Contributions	1832	Benefits	1953
Option Deficit	419	Option Surplus	358
	Residue	*-61*	

TABLE 6. Value-based ALM results Deal 3 (x 1,000,000 euros).

2.2. Analysis of the intergenerational transfers. All participants are divided in four age cohorts: the 25-34 cohort, the 35-44 cohort, the 45-54 cohort and the 55-64 cohort. The balance sheets are made for each generation from the pension fund's point of view. An Option deficit means the value the age cohort receives, the option surplus is the value that the age cohort leaves in the pension fund. So, the positive value of the residue is the value that the age cohort transfers to the other age cohorts.

Here I examine some alternatives with different investment, indexation and contribution policies for different age cohorts.

2.2.1. *Alternative 1.*

- Deal 1,
- Investment policy: all portfolios.

The pension deal is characterized by no influence of the investment policy on economic values in the balance. This deal requires value transfer to the participants from the pension fund.

25-34 age cohort			
Assets	€70	Liabilities	€606
Contributions	€449	Benefits	€7
Option	€95	Option	€0.13
Deficit		Surplus	
	Residue	*€-95*	

35-44 age cohort			
Assets	€399	Liabilities	€1006
Contributions	€457	Benefits	€29
Option	€179	Option	€0.39
Deficit		Surplus	
	Residue	*€-179*	

45-54 age cohort			
Assets	€719	Liabilities	€808
Contributions	€293	Benefits	€427
Option	€222	Option	€0.32
Deficit		Surplus	
	Residue	*€-222*	

55-64 age cohort			
Assets	€1091	Liabilities	€364
Contributions	€117	Benefits	€970
Option	€130	Option	€4.19
Deficit		Surplus	
	Residue	*€-125*	

TABLE 7. Alternative 1 (x 1,000,000 euros).

2.2.2. *Alternative 2.*

- Deal 2,
- Investment policy: 100% equities and 0% bonds.

Alternative 2 shows that the youngest cohort will share the transfer value with other age cohorts. The other three age cohorts need less value to receive compared to alternative 1. The contributions of the younger cohorts improve strongly from changing the contribution policy and having risky portfolio.

25-34 age cohort			
Assets	€70	Liabilities	€606
Contributions	€566	Benefits	€7
Option	€80	Option	€103
Deficit		Surplus	
	Residue	*€22*	

35-44 age cohort			
Assets	€399	Liabilities	€1006
Contributions	€578	Benefits	€29
Option	€122	Option	€64
Deficit		Surplus	
	Residue	*€-58*	

45-54 age cohort			
Assets	€719	Liabilities	€808
Contributions	€372	Benefits	€427
Option	€160	Option	€16
Deficit		Surplus	
	Residue	*€-144*	

55-64 age cohort			
Assets	€1091	Liabilities	€367
Contributions	€149	Benefits	€970
Option	€106	Option	€12
Deficit		Surplus	
	Residue	*€-94*	

TABLE 8. Alternative 2 (x 1,000,000 euros).

2.2.3. *Alternative 3.*

- Deal 2,
- Investment policy: 0% equities and 100% bonds.

Alternative 3 with 100% bonds is less risky strategy, so the present value of the contributions declines and as a result all generations are receivers. Note that in all alternatives in deals 1 and 2, the 45-54 age cohort receives the highest transfer value compared to other age cohorts.

25-34 age cohort			
Assets	€70	Liabilities	€606
Contributions	€438	Benefits	€7
Option	€106	Option	€0.29
Deficit		Surplus	
	Residue	*€-106*	

35-44 age cohort			
Assets	€399	Liabilities	€1006
Contributions	€447	Benefits	€29
Option	€190	Option	€0.39
Deficit		Surplus	
	Residue	*€-189*	

45-54 age cohort			
Assets	€719	Liabilities	€808
Contributions	€286	Benefits	€427
Option	€230	Option	€0.26
Deficit		Surplus	
	Residue	*€-230*	

55-64 age cohort			
Assets	€1091	Liabilities	€364
Contributions	€116	Benefits	€970
Option	€131	Option	€4.19
Deficit		Surplus	
	Residue	*€-127*	

TABLE 9. Alternative 3 (x 1,000,000 euros).

2.2.4. *Alternative 4.*

- Deal 3,
- Investment policy: 100% equities and 0% bonds.

Compared to alternative 2 (same investment mix) then conditional indexation policy has no influence on the present value of the pension benefits. The economic value of the benefits of the old generation decreases. Alternative 4 shows that the 55-64 age cohort transfers a higher value to the pension fund (remaining generations).

25-34 age cohort			
Assets	€70	Liabilities	€551
Contributions	€549	Benefits	€7
Option	€80	Option	€142
Deficit		Surplus	
	Residue	*€62*	

35-44 age cohort			
Assets	€399	Liabilities	€872
Contributions	€564	Benefits	€26
Option	€115	Option	€180
Deficit		Surplus	
	Residue	*€65*	

45-54 age cohort			
Assets	€719	Liabilities	€674
Contributions	€366	Benefits	€370
Option	€124	Option	€165
Deficit		Surplus	
	Residue	*€41*	

55-64 age cohort			
Assets	€1091	Liabilities	€296
Contributions	€148	Benefits	€862
Option	€68	Option	€149
Deficit		Surplus	
	Residue	*€81*	

TABLE 10. Alternative 4 (x 1,000,000 euros).

2.2.5. *Alternative 5.*

- Deal 3,
- Investment policy: 0% equities and 100% bonds.

By changing the investment mix to save the portfolio with 100% bonds, the value of the residue changes to a negative value, which means that the pension fund has to cover the shortages in the assets in the future.

25-34 age cohort			
Assets	€70	Liabilities	€592
Contributions	€433	Benefits	€7
Option	€99	Option	€2
Deficit		Surplus	
	Residue	*€-96*	

35-44 age cohort			
Assets	€399	Liabilities	€979
Contributions	€443	Benefits	€29
Option	€171	Option	€6
Deficit		Surplus	
	Residue	*€-166*	

45-54 age cohort			
Assets	€719	Liabilities	€782
Contributions	€285	Benefits	€421
Option	€204	Option	€5
Deficit		Surplus	
	Residue	*€-198*	

55-64 age cohort			
Assets	€1091	Liabilities	€352
Contributions	€116	Benefits	€963
Option	€118	Option	€9
Deficit		Surplus	
	Residue	*€-109*	

TABLE 11. Alternative 5 (x 1,000,000 euros).

I compare the effect of changing the investment strategy from risky to save the mix in Deal 2 and Deal 3. Figure 1 shows the differences in the value of the residue per age cohort. It is obvious that the policy with conditional indexation in Deal 3 is more sensitive to the choice of the investment mix. Changing the policy from alternative 4 to alternative 5 is very expensive for the pension fund. In particular, the 45-54 age cohort will claim the highest value to transfer.

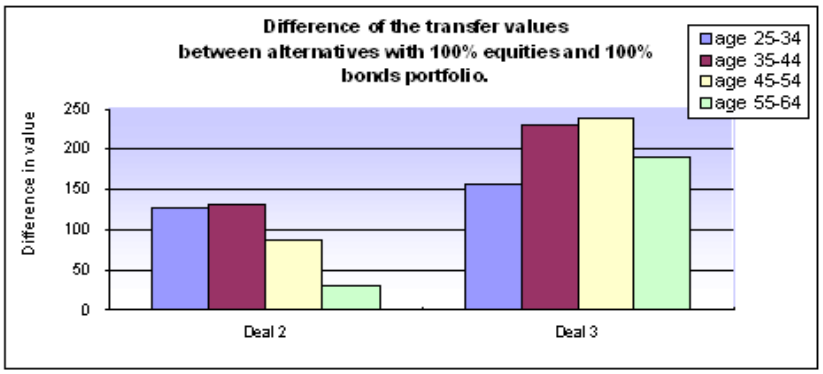

FIGURE 1. Difference in the value by changing the investment portfolio.

3. Conclusions

This chapter analyzed the value-based ALM with respect to the classical ALM approach for the Hollandia pension fund. The pension deals are evaluated for five proposed asset mixes. Outcomes in terms of the statistical measures and the economic values are compared.

The following table shows the economic values of the funding residue per pension deal. It is remarkable that alternatives consisting of equities are most manageable by changing policies. The higher the percentage of the equities in the mix, the higher the value that participants will transfer to the pension fund. (See also Figures 1 and 2 in Appendix B for intergenerational transfers.) As contributions and indexations are fixed in Deal 1, the present values should not depend on the assets. However, the 100% equity portfolio results in bankruptcies, and therefore in lower present values. This can be seen as a kind of pension put option (being a shortfall of the liabilities) described by Theo Kocken (see Chapter 5).

	Deal 1	Deal 2	Deal 3
100% equities	-	-47	520
100% bonds	-493	-532	-448
100% ILB	-493	-430	-405
50% equities, 50% bonds	-493	-324	-6
50% equities, 50% ILB	-493	-289	-61

TABLE 12. Funding residues in Hollandia pension fund

According to the classic measuring standards, changing the contribution policy from a fixed contribution rate to a variable contribution rate improves the average of the funding ratio and the probability of underfunding and declines the average contributions only for alternatives with a risky portfolio. According to the value-based measuring standards the value of the contributions in these alternatives increases, while benefits remain unchanged. This results in an improved value of the residue. Changing the indexation policy to the conditional one results in a lower average

percentage of the purchasing power while value-based ALM shows decreased value of the option deficit and an increased value of the option surplus.

The generational analysis of the generational transfers clarifies that young generations must pay high contributions, but will receive relatively little value of the benefits in comparison to older generations (see Figure 2).

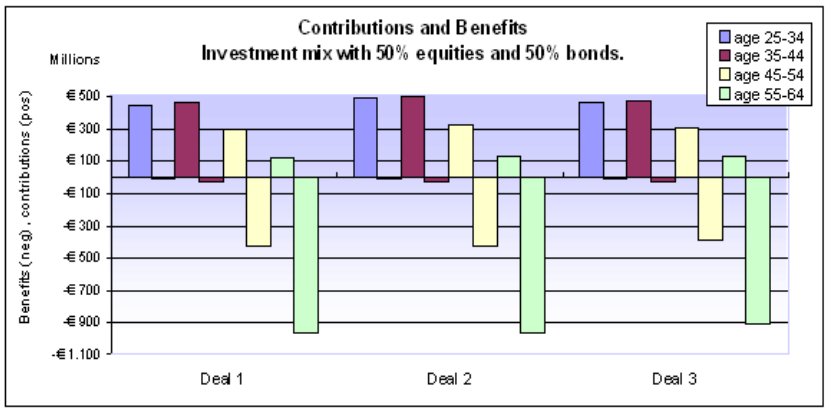

FIGURE 2. Comparison between the benefit payments and contributions.

Changing the pension deal has consequences for the generations in terms of economic values. Figure 3 in Appendix B demonstrates the difference of the values between each two deals. A positive value of the difference means deduction of the shortage value for the particular age cohort. It is remarkable that changing to a deal with conditional indexation will significant decrease the value of the shortages in the assets. Kortleve & Ponds [**2**] come to the same conclusion in their research.

I calculate the indexation option which is similar to the indexation option represented by Theo Kocken (see Chapter 5). Because the indexation has influence on the liabilities and benefit payments, I use the sum of these quantities to calculate the indexation option as a difference of the totals between the deals with unconditional and conditional indexation policy. Figure 4 in Appendix B presents the results and can be interpreted as how much risk the participants can run expressed in euros. The alternative with the most risky strategy has the most valuable indexation option. The parties involved in the decision making process would be interested in receiving these value-based results to choose the indexation policy. For retirees the indexation policy is very important, the sponsor is more concerned about the contributions and surplus while active participants are interested in low value of the contributions relative to the future benefit payments. For all stakeholders the value-based ALM can play an important role in the evaluation process of the pension deal.

CHAPTER 11

SUMMARY AND RECOMMENDATIONS

One of the most important economical and political issues nowadays is the ageing society that puts pressure on labor markets, health care, and pension provisions. Pension funds play an important role as a long term investor. A new pension system may be needed to anticipate on the consequences of a rapidly ageing population. The developments in the arbitrage free option theory reveal new possibilities in the risk management speciality.

The investigation in this master thesis focuses on the application of option theory, the investigation of the added value of the value-based ALM technique, and the comparison to the classical ALM.

Special attention in the thesis goes to discovering the intergenerational value transfers and indication of the hidden value transfers after the policy adjustments. Chapters 2 and 3 provide the basic knowledge of the pensions and give insight in Asset and Liability management. Also an ALM model is described which focuses on the decision making problem of the pension fund. This model makes possible to predict the possible future financial market developments and to react on wishes of the stakeholders interested in the pension policy adjustments. The model incorporated in the ALM-system of ORTEC, the company I where I did the internship, has many instruments, like indexation-, contribution- and investment policies, to control the funding ratio and to take into account the interests of all parties involved in the decision making process.

Chapter 4 explains the concept of the value-based ALM and the generational accounting methodology developed by R. Hoevenaars and E. Ponds. This method considers the embedded options in the pension deal as the uncertain cash flows from and to the participating generations. The options are calculated in terms of economic values and reflect the zero-sum nature of a pension fund. A zero-sum game means that if one generation will loose the value after the pension plan redesign, the other generation will receive it, or it will be distributed amongst the generations so that the market value of the pension fund assets remains unchanged. Chapter 5 explains the nature of indexation, parent guarantee, and pension put embedded options in the traditional Defined Benefit pension scheme, proposed by Theo Kocken in his PhD Thesis. The indexation and pension put option are used during the analysis of the theoretical and the Hollandia pension funds.

Chapter 6 explains and illustrates the concept of no-arbitrage and gives the interpretation of the derivative valuation using replicating portfolio. Risk-neutral valuation is characterized by the omission of the probabilities in the model. This is because the definition of the price of the derivative is made by replication. The risk is completely neutralized by taking a contrary position in the replicating portfolio.

Chapter 7 provides a mathematical background of the arbitrage free scenario simulations where the valuation techniques are based upon. The arbitrage free

scenario set is tested by modelling a pension fund and discounting the cash flows of it with the "money banking account". The obtained result appeared to be equal to the value based on the initial real term structure, which confirms that the arbitrage free scenario generator is implemented correctly.

Chapter 8 represents the economic framework for the investigation in the thesis and the mathematics behind the calculations of the transfer value of the age cohort. Classical measures and the balance sheet for the value-based method are described.

The investigation of the theoretical fund in Chapter 9 is based upon the framework from the article of Cui et al., the model which is used as a benchmark in this chapter. The option as an agreement between the age cohorts is considered from both pension fund perspective and cohort perspective. The behaviour of the put and call options in the pension fund with respect to the changing funding ratio is explained and illustrated. For two particular age cohorts I investigated the influence of the changing policies, investment strategy and initial funding ratio on the participants, and compared cohort balances in economic value terms for different alternatives. Further, five distinctive pension deals are discussed and evaluated using both traditional and value-based ALM approaches for the different investment strategies. Investigation of the pension deals, which differ by the contribution or indexation policy, shows that the investment strategy makes no difference in the economic value terms for alternatives with an unconditional indexation policy. Furthermore, the unconditional indexation results in negative transfers (shortages in the pension fund) and conditional indexation in positive transfers (surplus, the value transfered by participants to the fund). The indexation option, which participants write on the funding ratio by changing the pension deal, reveals that choosing a safe investment mix will reduce the value of the option. It means that the participants will lose the advantage in the future in case the sponsor pays the contributions. It is remarkable that the classical ALM shows in some alternatives the optimistic results like increasing funding ratio on average, while the surplus on the balance sheet of the pension fund declines in the economic value terms. This fact shows that value-based ALM reveals whether the modifications in the financing set-up can lead to value transfers.

Chapter 10 investigates the possibilities of value-based ALM in the Hollandia fund which is close to an average Dutch pension fund. Three distinctive pension deals are discussed and evaluated using both traditional and value-based ALM approaches for different investment strategies. Similar to the theoretical pension fund the investment strategy has no impact on results of the deal with unconditional indexation policy. Classical measures shows decreased average contributions and an improved average funding ratio in the deal where contribution payments are adjusted in order to absorb the risk (in the alternatives with a risky portfolio). In contradiction, the value-based measures shows increased contributions in economic value terms while the value of the benefits remains unchanged. This information would be interesting for the sponsor who is concerned about the contribution payments. The generational analysis of the generational transfers clarifies that the young generation pays a high contributions and restores the shortages in the assets of the pension fund. The indexation option shows how much risk the participants run expressed in euros if the indexation policy is changed to the conditional one. This option is most valuable in alternatives with a risky strategies and would be not appreciated by retired participants. Obviously, the classical ALM can not indicate

the hidden value transfers after policy adjustments. The value-based ALM can play an important role for all stakeholders in the evaluation process of the pension deal.

In short, by applying value-based ALM the current entitlements can be valuated (T. Kocken), the policy and the policy alternatives can be expressed in euros and be easily compared (Kortleve, Kocken), and the insight in the intergenerational value transfers can be given (Kortleve, Ponds). An important result of value-based ALM is that it shows that a pension fund is a zero-sum game in economic value terms. This insight suggests that any pension fund policy only implies transfers of value amongst the stakeholders. However, the value-based ALM tells nothing about the probability of underfunding which is an important measure for the De Nederlandsche Bank, the supervisor of the Dutch pension funds. The value-based approach does not change existing decision making, but adds a new dimension by showing the present value of all decisions about policies.

As recommendation for future studies I advise to do research about the possibility of the extension of the system with multiple currencies. Another issue that plays an important role in the pension fund industry, and that could be evaluated using value-based methodology, is the question whether it is necessary to use inflation hedging next to the interest rate hedging. Classical analysis without arbitrage free scenarios strongly depends on assumptions about the inflation risk premium, which will not effect the value-based outcomes.

APPENDIX A

Analysis of the policies. Theoretical Pension Fund.

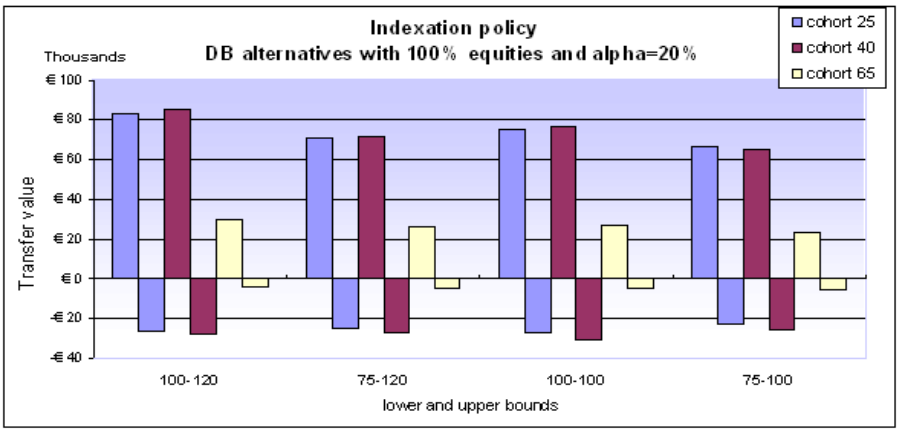

FIGURE 1. Indexation policy.

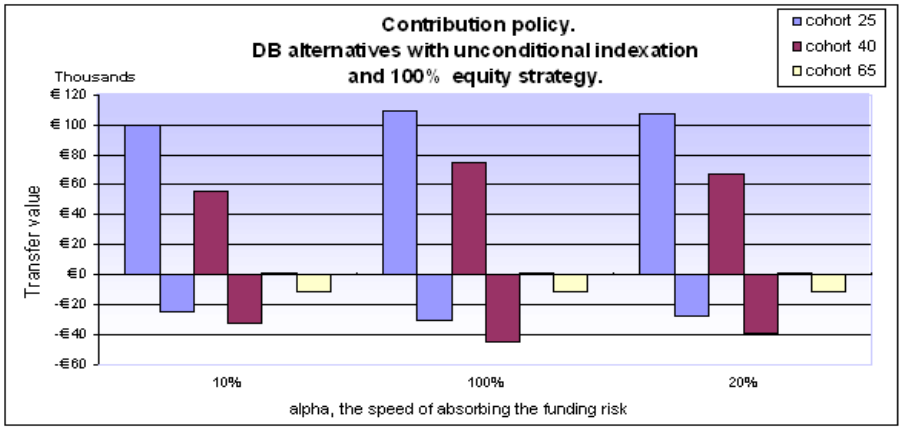

FIGURE 2. Contribution policy.

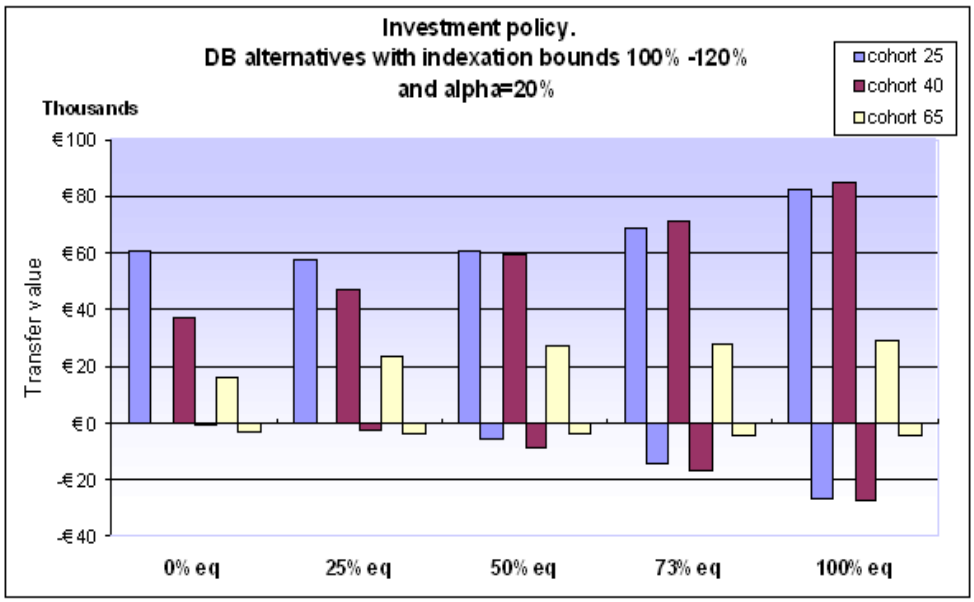

FIGURE 3. Investment policy.

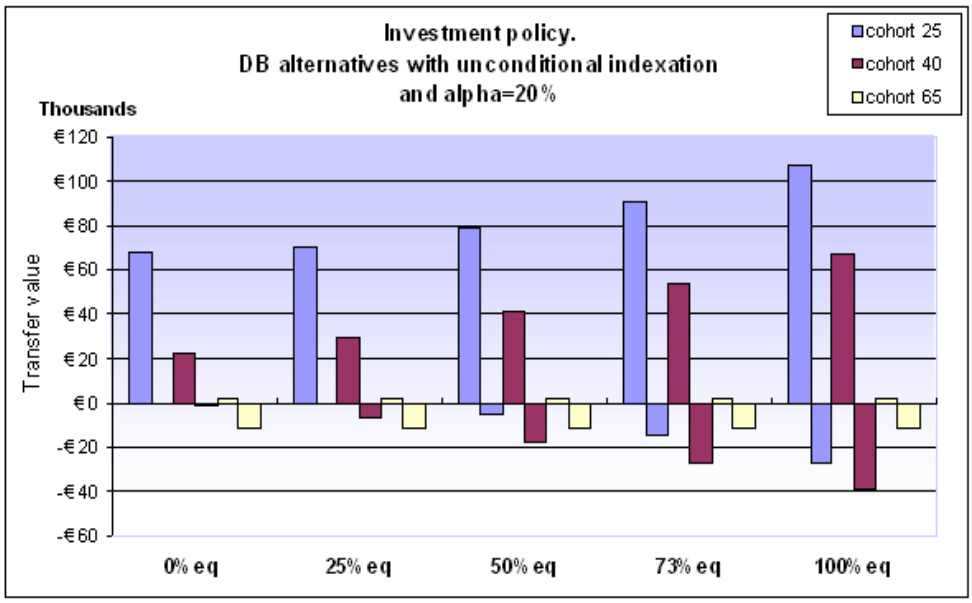

FIGURE 4. Investment policy.

APPENDIX B

Analysis of the policies. Hollandia Pension Fund.

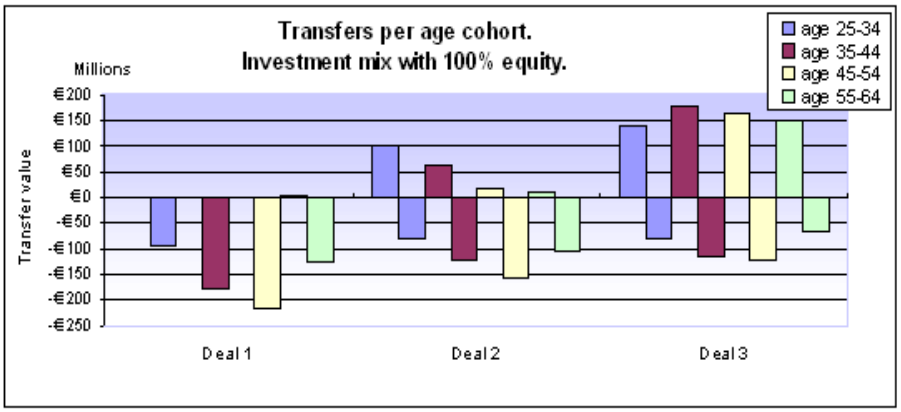

FIGURE 1. Value transfers with equity strategy.

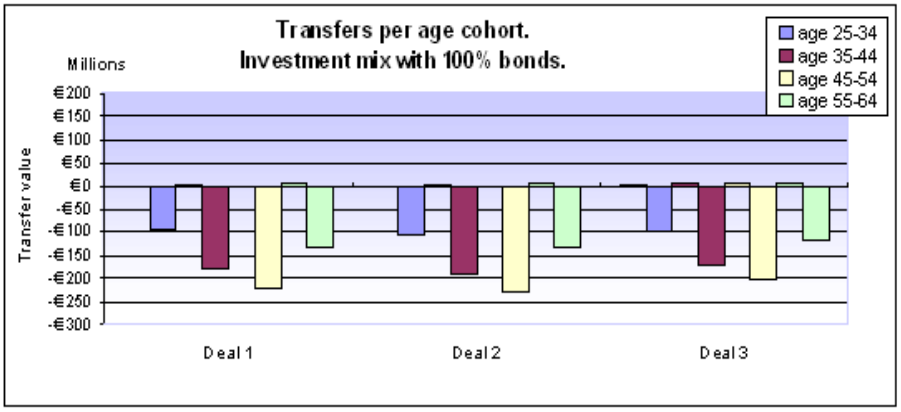

FIGURE 2. Value transfers with bonds strategy.

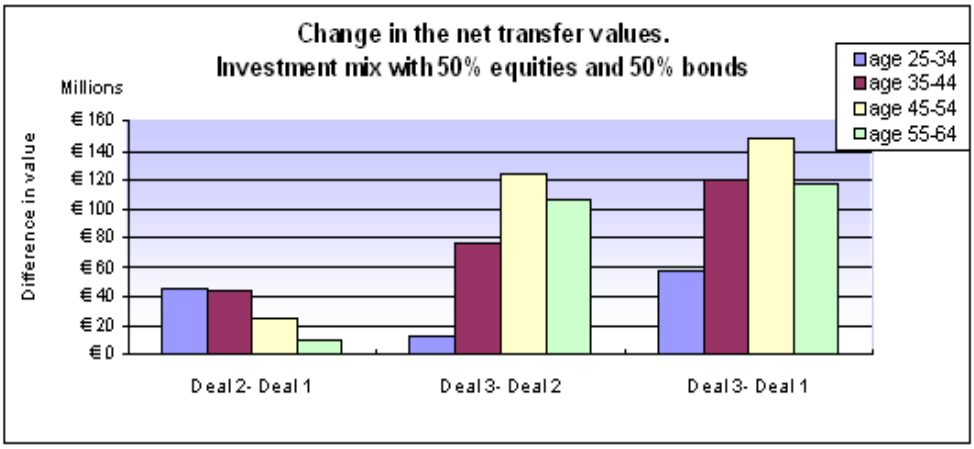

FIGURE 3. Difference of the net transfer values between deals.

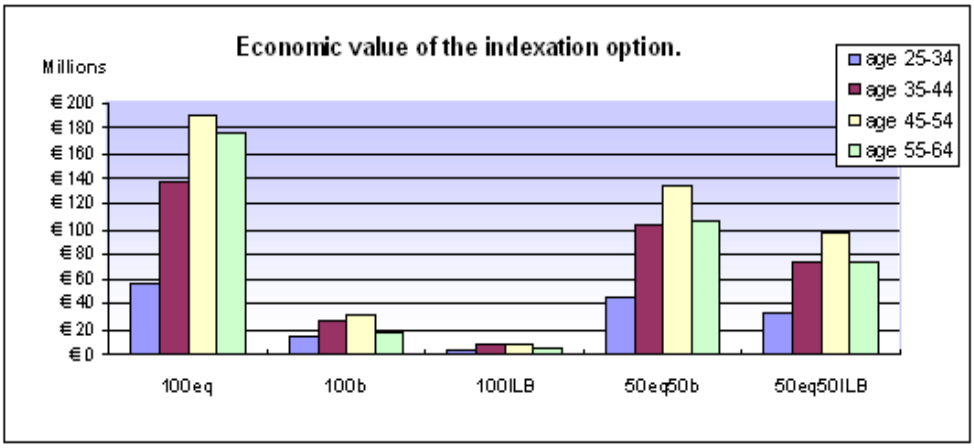

FIGURE 4. Economic value of the indexation option.

APPENDIX C

Graphs of classical measures. Theoretical fund.

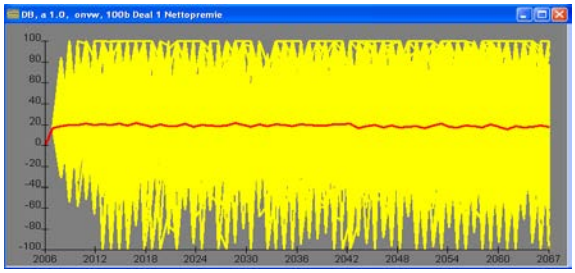

FIGURE 1. Net contributions in deal 1 with 100% nominal bonds.

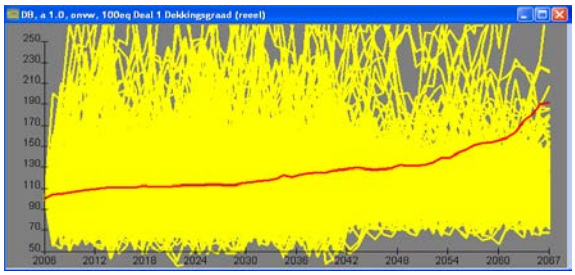

FIGURE 2. Real funding ratio in deal 1 with 100% equities.

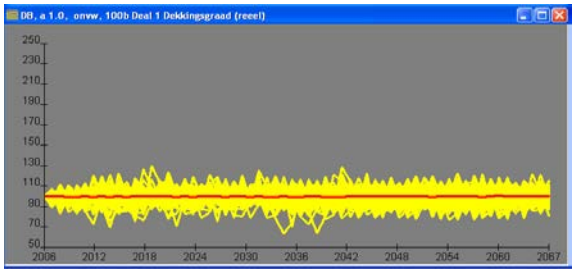

FIGURE 3. Real funding ratio in deal 1 with 100% nominal bonds.

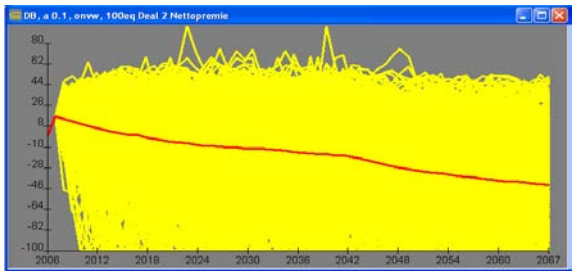

FIGURE 4. Net contributions in deal 2 with 100% equities.

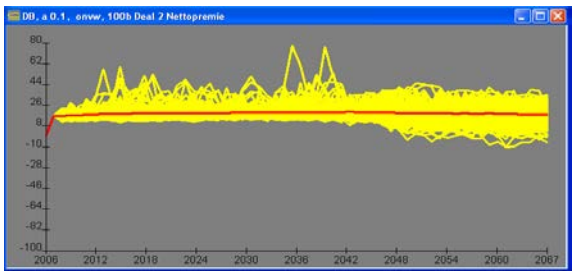

FIGURE 5. Net contributions in deal 2 with 100% nominal bonds.

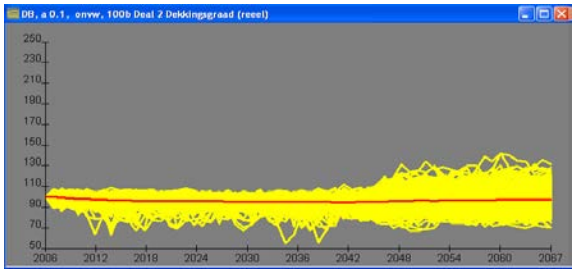

FIGURE 6. Real funding ratio in deal 2 with 100% nominal bonds.

APPENDIX D

Pension Vocabulary

- Arrangement = (pensioen) regeling
- Average wage scheme = middelloonregeling
- Defined Benefit scheme = aanspraken regeling
- Defined Contribution scheme = beschikbare premieregeling
- Entitlements = aanspraken
- Final pay scheme = eindloonregeling
- Funding ratio = dekkingsgraad
- Parent guarantee option = moedergarantie optie
- Pension put option = pensioenput optie
- Scheme = regeling
- Pension Liability Provision = Voorziening Pensioen Verplichtingen (VPV)
- Required Solvency Level = Voorziening Eigen Vermogen (VEV)
- Yield curve = rentetermijnstructuur

APPENDIX E

NOTATIONS

r	-	interest rate
γ	-	risk aversion parameter
δ	-	subjective discount rate
ρ	-	correlations
Z_t	-	Brownian motions
x	-	portfolio weight for equities
y	-	annual salary
y_t	-	(flat) real labor income
κ, λ	-	real market prices of risk
τ	-	term-to-maturity
b	-	consumption
b_t	-	accrued pension right
b_R	-	pension income during retirement
k	-	pension accrual rate (% over salary)
p_t	-	periodic contributions
m	-	fixed contributions
c_t	-	individuals consumption before and after retirement
M_t	-	stochastic discount factor
λ_E	-	ratio of investment portfolio
π_t	-	expected inflation
$P_t(\tau)$	-	real price for a nominal bond
$\frac{dP_t(\tau)}{P_t(\tau)}$	-	real return on the nominal bonds
$\frac{dA_t}{A_t}$	-	return on portfolios of nominal bonds and equities
α	-	slope coefficient for employees, $\in [r, 1]$. Speed of absorbing the funding risks. The lower α the higher the degree of IRS
β	-	slope coefficient for retirees, $\in [r, 1]$. Speed of absorbing the funding risks. The lower β the higher the degree of IRS
(α, β)	-	control of the speed of risk absorbtion
$\{m, x, \alpha, \beta, k\}$	-	decisions for scheme design parameters have to be made by pension fund
i_t	-	additional (adjustable) indexations
S_t	-	aggregate surplus in the fund
	-	
IRS	-	intergenerational risk sharing
ILB	-	index linked bond
CDB	-	collective defined benefit (scheme)
CDC	-	collective defined contribution (scheme)

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