Master Thesis

Capacity Allocation for the Reduction of Bottlenecks in the Amsterdam Elderly Care System

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Preface

This thesis is written for the completion of the Master of Science degree in Business Analytics at the Vrije Universiteit van Amsterdam (VU). The thesis was written during an internship at *Centrum voor Wiskunde & Informatica (CWI)*, the national research institute for mathematics and computer science in the Netherlands. The research at CWI aims to contribute to different societal fields, such as healthcare, energy, and security.

Within CWI, I joined the stochastics group, where I worked on a thesis in the Dolce Vita team, focusing on healthcare logistics for elderly care. I enjoyed working on such a relevant and interesting topic while learning a lot about different aspects of the healthcare system.

Acknowledgements

Working on the thesis had its challenges and difficulties, but I can say that I learned a lot from the whole process, especially from the following people who helped me a lot.

First, I want to thank drs. Rebekka Arntzen and drs. Tim de Boer for our useful weekly meetings, their support and guidance, and for answering all my questions throughout this process. Secondly, I want to thank prof. dr. Rob van der Mei and dr. René Bekker for their time as first and second readers, especially for their valuable ideas and feedback. Additionally, I want to thank the rest of the Dolce Vita team for allowing me to do this research and for their interest. And last but not least, I want to thank the CWI stochastics group for the enjoyable learning environment and good atmosphere.

Management Summary

Problem Statement An aging population combined with a decrease in available healthcare professionals results in growing waiting times for older people who require care. Consequently, people stay at home, and care availability seems to shift to acute care requests. One of the effects is an increase in extensive hospitalizations. This suggests that improving waiting times with the corresponding capacity requirements at one location could also help the excessive care requests at other sites. For this reason, an overarching perspective on the elderly healthcare system is needed.

Methodology We propose to model the elderly care system as a Jackson Network, a network of $M/M/c$ queues, or *stations*, allowing immediate insights into the station performance measures, such as the expected waiting time in the queue. The stations in this system are the healthcare types: long-term care, short-term care, geriatric care, home care, and hospitalizations. We show an application of the model with a case study of Amsterdam, which requires data analysis to obtain accurate parameters describing the length of stay at the station and the demand in terms of the average arrival rate. In addition, we propose an optimization method for this Jackson Network, providing insights into the preferred allocation of capacity to minimize waiting times. For this, we formulated and compared the exact Capacity Allocation Program (CAP), the greedy Marginal Allocation (MA), and a program using the rule of thumb Square-Root Staffing (SRS) principle.

Results The parameters of the Amsterdam case study are used to determine the preferred allocation using the MA method since this showed similar results to CAP within a more reasonable time. Most capacity should be allocated to stations with a high offered load, e.g., home care, since this station has a high demand and a reasonably long stay. For the case study, we also considered a *centralized model* and a *decentralized model*, where the latter assumes that long-term care and home care are arranged per city district. Comparing the results of these models showed us that (1) the centralized model requires 11 beds less than the decentralized model for the stability requirement, and (2) when allocating a similar budget to both models, the centralized model needs fewer beds while obtaining lower waiting times at all stations compared to the other model. These differences are because the centralized stations, i.e., home care and long-term care, require fewer beds while having lower waiting times. Consequently, more budget is available also to reduce waiting times in other care forms.

Recommendation This research shows the benefit of modeling the elderly care system as a Jackson Network, combined with an optimization method, as it shows bottlenecks in terms of waiting times and possible improvements with (1) the optimization method suggesting the best allocation of capacity, and (2) the comparison of two models showing the benefit of arranging care centrally as opposed to a decentralized setting.

Contents

Chapter 1

Introduction

The composition of Dutch society is changing, affecting one of the most vital parts of this society: the healthcare system. The percentage of elderly people (65+) in the Dutch population is increasing from about 12.8% in 1990 to 20% in 2022 (CBS, 2022). Figure 1.1 shows this will increase further in the coming years. More people will need aging-related care, while fewer people will provide this care and pay for the corresponding cost. This ratio between 65+ and 20 to 65-year-old people is called *grey pressure*, and the increase is shown in Figure 1.2, increasing from about 34% now to 50% in 2040 (CBS, 2022).

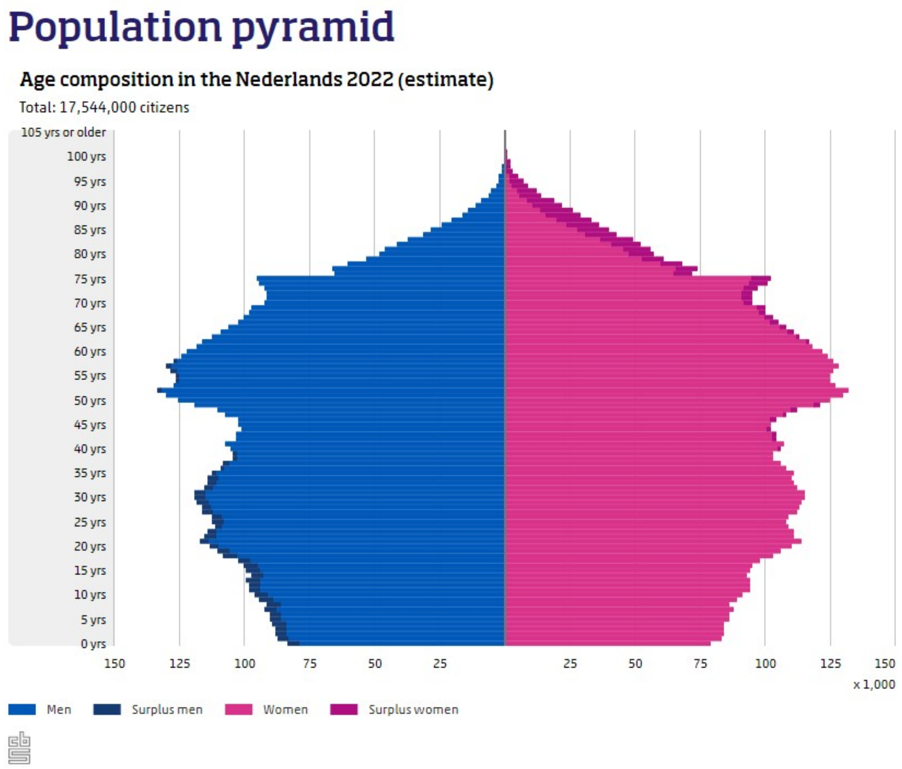


Figure 1.1: Population pyramid in 2022. CBS (2022)

Additionally, there is a growing shortage of healthcare professionals. According to a study commissioned by the Dutch Ministry of Health, Welfare and Sports (VWS), the need will be 135 thousand people in 2031 for the whole healthcare sector, with a lack of 51.9 thousand people in nursing homes (NOS, 2022). Thus, fewer professionals are available to care for an increasing group of older people.

Third, the high cost of care is an issue. The most expensive healthcare types for elderly care are long-term care, followed by the care provided in hospitals and care at home

(Vektis, 2018). Due to the relatively small number of workers caused by the *grey pressure*, as shown in Figure 1.2, the relative costs for care for older adults will be soaring.

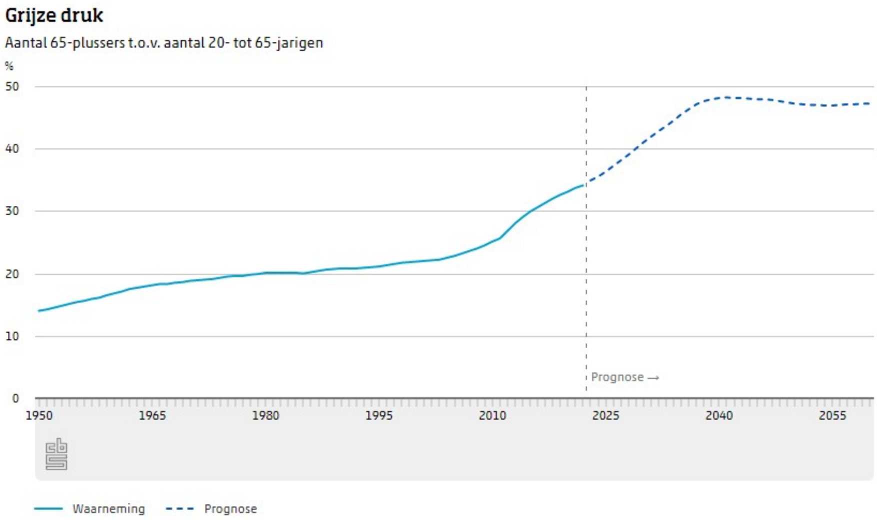


Figure 1.2: *Grey pressure*: The ratio between the 65+ and the 20 to 65 years old people. CBS (2022)

The aging population and the growing shortage of healthcare professionals lead to growing waiting lists for healthcare facilities. This holds especially for nursing homes, a much-needed care form. According to ActiZ (ActiZ, 2022), the organization representing around 400 care organizations, currently, about 21.7 thousand people are waiting for a place in a nursing home, almost four times as many people as in 2017. In addition, 10.8 thousand people are waiting as a precaution, obtaining some 'bridging care' while waiting for a place. These numbers show that the demand for long-term care is already rising.

A consequence of the waiting times and staff shortages is that care focuses on acute requests. In contrast, many elderly people have other geriatric syndromes prevalent when hospitalized, which are not considered (enough) in the consult, according to Buurman et al. (Buurman et al., 2011). These patients are more likely to return to the hospital within several months or a year or die because of this geriatric condition. This suggests that geriatric care waiting lists should be improved to prevent excessive hospitalizations.

Thus, the healthcare system for older aged people needs to change. A clear overview of the complexities, challenges, and bottlenecks that must be dealt with in the elderly care system is provided in a report named 'Krakende Ketens' (SIGRA, 2017). The document summarizes the healthcare forms that older people are likely to need, and the connections between these forms, showing the possible flows of patients in the elderly care system. This system is visualized as in Figure 1.3.

'Krakende Ketens' identifies the bottlenecks per chain, where a chain is a group of healthcare facilities working together on a similar problem. For example, 'prevention' is mainly done at home with home care or through the general practitioner. For these chains, the current bottlenecks and possible solutions are summarized. Some of the challenges discussed in 'Krakende Ketens' are:

- Early detection in primary care: detect vulnerable elderly people and provide suitable interventions.

- Prevent relapse after hospitalization by sufficiently organizing the care at home.

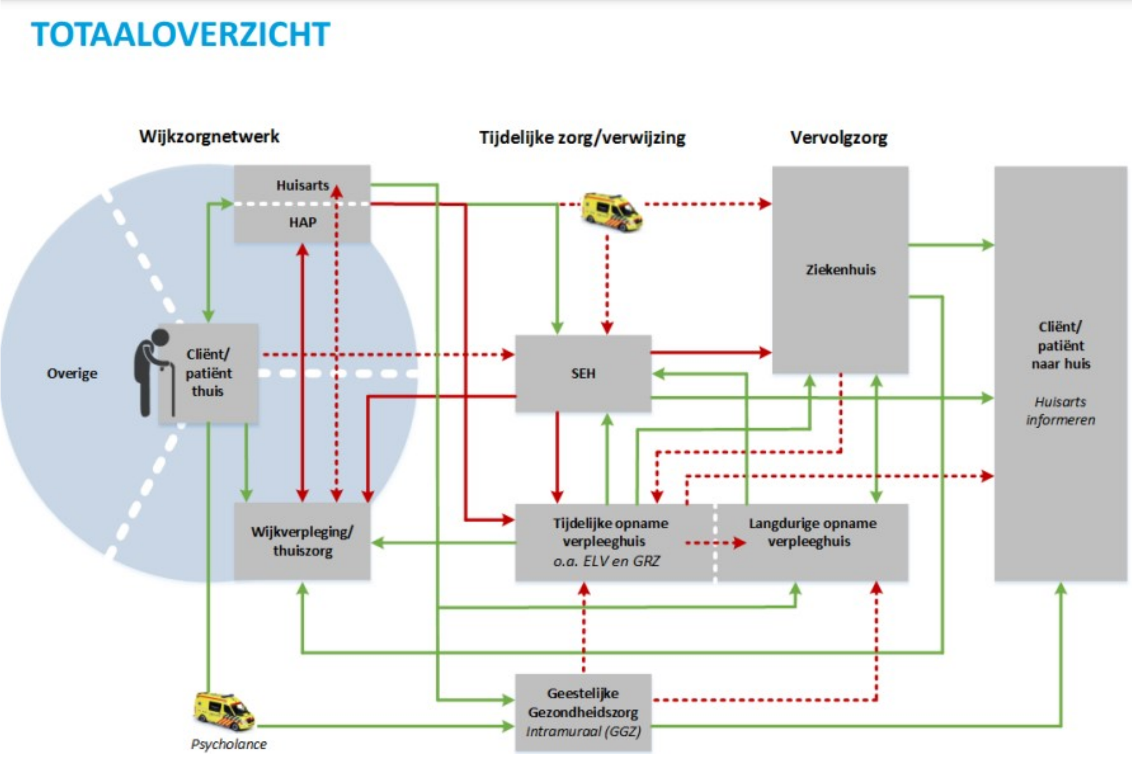


Figure 1.3: High-level overview of the different stations connected, the 'Krakende Ketens'. Red lines indicate flow problems, whereas green lines indicate no problems. SIGRA (2017)

- Allow temporary admissions to a nursing home as an alternative to the emergency department. People with medical issues go to the emergency department, whereas people with symptoms that mostly require nursing care should go to nursing homes.
- All domains require a good registration of supply and demand to quantify the bottlenecks and gain insight into the shortages. It should be decided for the whole chain where and how to register information.

Changes in the processes of the current healthcare system could reduce or remove bottlenecks. As all parts of the healthcare system are connected, a change in one entity can influence others. Thus, it is interesting to consider the whole healthcare system at once and find the consequences of possible structural or operational changes. This supports using a macro model, a connected network including the possible care paths in the healthcare system. A paper by Brailsford used *system dynamics* for this cause, with a case study of the emergency department in Nottingham, England (Brailsford, Lattimer, Tarnaras, & Turnbull, 2004). In this research, possible patient pathways were discovered, and patient flows were simulated, which enabled the discovery of system bottlenecks, where a reduction in admissions seemed to affect the reduction of bed occupancy substantially. Creating models with a similar approach will possibly justify decisions that can be made to improve a healthcare system, such as the elderly care system, which is of our interest.

Research Aim

This thesis aims to create a macro model of the elderly healthcare system and optimize the capacity allocation in the different healthcare stations to reduce congestion in terms of waiting times. To show the potential of this model in providing insights into bottlenecks and consequences of changes, we apply our model to the case study of Amsterdam. This includes data analysis to obtain parameters for our model based on accurate data. Additionally, we consider different aggregation levels for the macro model, providing insights into the effect of economies of scale. The final implemented model should identify the bottlenecks in the elderly care system and consider scenarios or impacts of policy changes. This thesis will show the process of estimating parameters and the different optimization methods used, together with an analysis of the performance of these methods for the optimization of the macro model.

Organization

The thesis is structured as follows. First, an overview is provided of previous research on the topic in Chapter 2. Next, Chapter 3 includes preliminary knowledge for the thesis. Chapter 4 will introduce the macro model together with the capacity allocation methods proposed, after which these are compared with a toy example in Chapter 5. We present the overall structure of the Dutch healthcare system in Chapter 6 before we start the case study of Amsterdam in Chapter 7. The case study's results are used for the optimization, and these results are provided in Chapter 8. We end with a conclusion and discussion in Chapter 9 and Chapter 10.

Chapter 2

Literature Review

Healthcare systems are complex, so mathematical models are helpful to gain insight into the dynamics within those systems. Some popular means to model a healthcare system are System Dynamics Models (SD), Agent-Based Models (ABM), and Discrete Event Simulation (DES).

SD are macro-level models describing the movement of aggregated entities in a system and how the system state changes over time. The aim is to examine the system's flows, trends, and behavior. SD was used by Brailsford to model the emergency department in Nottingham, UK, and to find system bottlenecks (Brailsford et al., 2004). A downside of the method is that it does not consider stochasticity.

ABM models are focused on micro-level system behavior, where individual agents have changing states; they learn from their own experiences and make decisions. Thus, this method is aimed at finding individual flows of activity. ABM allows the simulation of scenarios and identification of sensitive parameters in the system (Cassidy et al., 2019). Two downsides of ABM are extensive running times and the need for detailed information on individuals.

DES enables the simulation of dynamic behaviors of complex systems, with the comparison of potential practices or strategy options as a goal. However, DES is more advantageous for systems at the individual level (Zhang, 2018). Most realistic simulations require extensive data and excessive computational time, which is a downside for this method in a large complex system.

Queueing models do not require long running times and much data. Additionally, the performance measures of queues - related to delay, waiting, blocking, and throughput - make this an interesting and powerful tool for healthcare modeling, as the targets that need to be met in a hospital are summarized as minimizing the waiting time for patients that need care and maximizing the utilization of, mostly scarce, servers or resources, such as hospital beds and healthcare professionals (Lakshmi & Iyer, 2013).

Queueing Models

An overview of queueing theory in healthcare, written by Green, states its value for healthcare, as very little data is required and relatively simple formulas provide the results for the performance measures. Queueing models can be used to analyze the utilization for various supply and demand scenarios and to calculate the effects of, for example, different service capacities on performance measures. These measures provide insight into what is needed to reach specific service levels or delay targets. The difficulties of decision-making in healthcare include the constrained available resources and the severe consequences of delays in the system. An essential part of the queueing analysis is collecting data to obtain reasonable estimates for the parameter values. This may especially be difficult when determining the demand, as it can be challenging to detect the demand that has not been met (Green, 2006).

Zonderland and Boucherie also argue the use of queueing theory to model a healthcare system, because of its fast and flexible use in discovering bottlenecks and evaluating scenarios. The Jackson Network is a powerful modeling approach for exponential interarrival and service times, allowing product-form solutions. When exponential distributions are not appropriate, an alternative would be the Queueing Network Analyzer, from Whitt (Whitt, 1983). This model includes parameters for variability and can be solved with a decomposition method based on the product-form solution. The QNA is based on patient classes with fixed paths, but stochasticity is found in the arrival and service rates. This is also an interesting tool to model a system (Zonderland & Boucherie, 2012).

Some previous applications of queueing theory to model healthcare systems are the following. An article by Gorunescu et al. (Gorunescu, McClean, & Millard, 2002) models a department of geriatric medicine as a multi-server model with a maximum system capacity, according to a $M/PH/c/N$ queue. The service-time distribution is assumed to be of a phase type, with two phases because the patient can be divided into two forms of clinical activity: acute care and long-stay care. Performance measures are used to get insight into the rejection probability, the fraction of rejected patients, mean bed usage, and bed occupancy for different scenarios, where either the arrival rates, length of stay, number of beds, and extra waiting beds (places in the queue N) are modified. They conclude that a buffer of empty beds (N) allows for a more responsive and efficient service.

Another article studies a healthcare system as a closed queueing network (Chaussalet, Xie, & Millard, 2006) by modeling the process as a set of phases through which patients will flow. Patients go to the next phase with a certain probability or leave the system with a discharge probability. The authors assume that the system is always full, which they argue to be reasonable for healthcare systems with constrained bed allocations and waiting lists, such as geriatric medicine. The performance measures are the number of patients in each phase and the daily discharge rate. The authors study the effects of a changing length of stay and changing transfer probabilities between phases on the number of beds needed and the daily admission rate.

The authors extend this research by proposing a semi-open queueing network, where patients are admitted if any space is left or turned away. The performance measures are the loss probability of the system and the occupancy level per phase for the entire system. Changing parameters like bed capacity and arrival rate influence the occupancy level. There seems to be a trade-off between lowering the loss probability and a high occupancy level (Xie, Chaussalet, & Rees, 2007).

Another paper creates a queueing network of mental health institutions to find congestion in facilities, as patients spend too much time in intensive facilities. They study a queueing network with blocking to analyze the congestion. Blocked patients are forced to stay longer in their current facility. They use a decomposition algorithm to deal with blocking in an open queueing network system (Koizumi, Kuno, & Smith, 2005).

These queueing models can be used to make informed decisions about resource allocation, as discussed next.

Resource Allocation

Allocation of resources is an essential topic within healthcare. Bidhandi et al. (Bidhandi, Patrick, Noghani, & Varshoei, 2019) model a community care network as a network of queues, allocating capacity to different queueing stations by minimizing the sum of the blocking probabilities. First, they formulate a non-linear integer programming problem where the cost is minimized while imposing an upper bound on the sum of the blocking probabilities. Solving practical model instances is time-consuming, so a two-stage simulated annealing method is developed, which finds an initial solution in the first step, and improves this in the second step. The final capacity solution is tested in two different simulations, one with similar assumptions as the queueing network and the other with varying classes of patients, aiming to test the robustness of the allocation plan.

Another paper allocates beds in a public health care delivery system by using marginal analysis (Kao & Tung, 1981). They use a queueing model and forecasts of admission rates to approximate the patient population dynamics for services within a healthcare system and reallocate beds to minimize the expected overflows. The allocation consists of two stages: first, beds are allocated to services for proper server utilization, and second, the remaining beds are distributed to minimize the expected total average overflows, for which marginal analysis is used.

A paper by Fox also suggests using marginal analysis for capacity allocation, proposing an incremental allocation scheme where capacity should be allocated to the place with the highest return per spent dollar (Fox, 1966). Capacity allocation in a queueing network has also been done by Bretthauer, who presents an optimization model with as an objective the selection of capacity such that costs are minimized and the system performance constraints are satisfied. The capacity is controlled by the mean service rate at each node. The author also outlines the procedures to solve the capacity allocation problem when the objectives are convex or concave and for different types of constraints and variables (Bretthauer, 1995).

Research Opportunities

Within elderly care, the following research opportunities for the use of OR/MS models for elderly care planning were identified by Williams et al. (Williams, Gartner, & Harper, 2021).

1. Further research should focus on using methods such as queueing theory, or *combining multiple methods to create a more diverse model*. This is, for example, done by Bidhandi et al.(Bidhandi et al., 2019), as discussed above, who use queueing models to calculate the probability of blocking in a network of queues, which is used to optimize the capacity allocation by minimizing the sum of the blocking probabilities, with a simulated annealing approach.

2. Research should be done on using care planning across the complete patient journey of frail and elderly people. Elderly people have similar pathways to the rest of the population. However, they tend to have longer recovery times and other age-related issues. It is also stated that the research should *focus on the movement of the patients between care locations*, and not on the single care locations.

3. An interesting research opportunity is the *analysis of systems when there is a sudden increase in demand*. For example, there is an increased demand for certain care facilities due to Covid-19 or an increasing population.

A review article by Hulshof et al. underlines the importance of modeling and decision-making for a complete healthcare system. For different organizations, the care processes might be clear. However, patients have different perspectives on the delivery process of their needed care, as they possibly have to go through several stages of service. Instead of making optimal decisions for each healthcare process individually, it would benefit the patient if the whole journey is optimized. The review paper concluded that very few contributions were found on complete healthcare system interactions (Hulshof, Kortbeek, Boucherie, Hans, & Bakker, 2012). Thus this underlines this research opportunity to *model a whole care system*.

Our Contribution

In line with the identified research opportunity, we aim to design a model that:

1. Combines queueing theory with capacity allocation.
2. Is system-wide, i.e., considers all types of care facilities.
3. Can be used to answer what-if scenarios such as for an increase in demand.

For opportunities 1. and 2., queueing theory is used to create a macro model, for which we find a capacity allocation model to plan the care for the whole system at once. Our interest lies in the system's dynamics if something changes at one of the system's entities.

For the identification of the pathways in the system, from opportunity 2., a case study will be done on data from citizens of Amsterdam, in which we want to decide which parameters to use: how do elderly people move into the elderly care system, how much time do they spend at certain facilities and what paths do they take? The analysis also aims to find the correct aggregations to obtain the best reflection of the elderly population for the macro model.

And for opportunity 3., we use the created model to find the consequences of different scenarios, such as the increasing population of elderly people.

Chapter 3

Preliminaries

This section discusses some of the basics of queueing theory, the foundation for the modeling and optimization methods proposed in the next section.

Queueing Theory

Queueing theory is commonly used to model situations with stochastic elements, where observed values are random variables. A queueing model contains the following (Adan & Resing, 2017):

- Arrival process of customers: the inter-arrival times are independent, following a common distribution.
- Service times: the service times are independent and identically distributed, following a common distribution.
- Service capacity: a queue has a single or multiple server(s).
- Service discipline: customers can be served on a first-come-first-served (FCFS) basis, in random order, on a last-come first-served (LCFS) basis, with priorities, or by processor sharing.
- Waiting room: the number of customers waiting to start service can be constrained.
- Behaviour of customers: customers are either willing to wait or impatient.

The type of queueing model is specified by Kendall's notation: $a/b/c/d$, where a is the distribution of the inter-arrival time, b is the distribution of the service time, c is the number of servers in the system, d is the capacity of the system (if this capacity is finite, else d is excluded). Possible distributions for the inter-arrival time and service time are 1) M, exponential distribution – or Poisson Process for arrivals, due to its memory-less property), 2) G: general distribution, 3) D: deterministic times or 4) E_k: Erlang with k phases. This last distribution is a case of phase-type (PH) distributions, which expect a mixture of exponential distributions. The type of queue considered in this thesis is the $M/M/c$ queue: both inter-arrival times and service times are exponential, whereas the capacity consists of c servers. The waiting room is infinite.

We usually analyze one type of queue, considering its' performance given some parameters. Realistic or complex situations with multiple actors can require a complex approach, such as a queueing network. An example is a system where a customer has to go through several stages to complete a process. Customers following the same route through all queues results in a tandem queueing system. But when different routes are possible, this becomes a queueing network. This network can either be open, where external arrivals and customers can leave the system, or closed, with a fixed population within the system.

Jackson Networks

A specific queueing network is a Jackson Network if it meets the following assumptions (Kulkarni, 2011):

- The network consists of N service stations, $i = 1, .., N$.
- External arrivals λ_i at the stations $i = 1, .., N$ are from an independent Poisson process.
- The service times are exponentially distributed. The service rate is μ_i for station $i = 1, .., N$.
- Station i has c_i servers.
- Station i has an unlimited waiting room.
- When a customer completed service at station i this customer will go to station j with probability p_{ij} or leave the system with probability $1 - \sum_{j=1}^{N} p_{ij}$.

Thus, a Jackson Network consists of N connected $M/M/c$ queues, where all queues can have external arrivals and where routing between queues is dependent on the probability matrix P, which is of size $N \times N$, and where each element p_{ij} is the probability of going from station i to station j. The probability of leaving a station, $1 - \sum_{j=1}^{N} p_{ij}$, is usually more than 0 for at least some of the queues, such that all customers arriving in the system will eventually leave the system (Tijms, 2003).

All N stations must have a stable load for the arrivals in station i to be equal to the departures from station i (Kulkarni, 2011). In general, stability for an $M/M/c$ queue is calculated with the utilization formula in Equation (3.1), where the system is stable if the load ρ is less than one:

$$\rho = \frac{\lambda}{c\mu} < 1. \tag{3.1}$$

So, once the system is stable, the arrivals at station j (given by effective arrival rate γ_j) have, when they leave the station, a probability of p_{ji} of going to station i, meaning that $\gamma_j p_{ji}$ customers will arrive at station i from station j. This can be calculated for all stations connected to station i. This results in the traffic equation given in Equation (3.2), where γ_i is the *effective arrival rate* at station i.

$$\gamma_i = \lambda_i + \sum_{j=1}^{N} p_{ji}\gamma_j \tag{3.2}$$

The above equation can be rewritten in matrix form:

$$\gamma(I - P) = \lambda \tag{3.3}$$

Where, if $(I - P)$ is invertible, the equation has a unique solution given by

$$\gamma = \lambda(I - P)^{-1} \tag{3.4}$$

With this, we arrive at the theorem in (Kulkarni, 2011), which states that a Jackson Network with external arrivals λ and routing matrix P is stable if $I - P$ is invertible and

$\gamma_i < c_i\mu_i$, for all $i = 1,..,N$ where γ_i can be calculated with Equation (3.4). Once the system is stable, we can look at the limiting behavior of a Jackson Network (Kulkarni, 2011). Let $X_i(t)$ be the number of customers at station j at time t. $X_i(t)$ is a continuous-time Markov Chain, where the state at time t is $X(t) = [X_1(t), X_2(t), .., X_N(t)]$. If stability holds, as provided above, the process $\{X(t)\}$ has a unique limiting distribution in the product form

$$p(n_1, n_2, .., n_N) = p_1(n_1)p_2(n_2)...p_N(n_N) \tag{3.5}$$

for all $n_i = 0, 1, 2..$ and $i = 1, .., N$. Here $p_i(n)$ is the probability that n customers are at station i, which is calculated as

$$p_i(n) = p_i(0)\rho_i(n), \quad i \geq 0 \tag{3.6}$$

where

$$\rho_i(n) = \begin{cases} \frac{1}{n!}\left(\frac{\gamma_i}{\mu_i}\right)^n, & \text{if } 0 \leq n \leq c_i - 1 \\ \frac{c_i^{c_i}}{c_i!}\left(\frac{\gamma_i}{c_i\mu_i}\right)^n, & \text{if } n \geq c_i \end{cases} \tag{3.7}$$

and

$$p_i(0) = \left[\sum_{n=0}^{c_i-1} \frac{1}{n!}\left(\frac{\gamma_i}{\mu_i}\right)^n + \frac{(\gamma_i/\mu_i)^{c_i}}{c_i!}\frac{1}{1-\gamma_i/(c_i\mu_i)}\right] \tag{3.8}$$

This *product-form solution* makes the N stations independent of each other, such that we can look at all the N stations separately, simplifying the calculations of the corresponding performance measures. However, a strict criterion is that the system is stable, as shown in Equation (3.1). $M/M/c$ queues have the following performance measures:

Probability of Waiting or Delay Probability The probability that an arriving customer has to wait if all the servers are busy.

$$\Pi(c,\rho) = \frac{(c\rho)^c}{c!}\left((1-\rho)\sum_{n=0}^{c-1}\frac{(c\rho)^n}{n!} + \frac{(c\rho)^c}{c!}\right) \tag{3.9}$$

$$= \frac{1}{1 + (1-\rho_i)\frac{c!}{(c\rho)^c}\sum_{n=0}^{c-1}\frac{(c\rho)^n}{n!}} \tag{3.10}$$

Expected Waiting Time in the Queue The expected time a customer has to wait before starting service.

$$\mathbb{E}(W^Q) = \frac{\Pi(c,\rho)}{c\mu(1-\rho)} \tag{3.11}$$

Expected Number of People in the Queue The expected number of customers in the queue in steady-state.

$$\mathbb{E}(L^Q) = \Pi(c,\rho)\frac{1}{1-\rho}\frac{1}{c\mu} \tag{3.12}$$

Distribution of the Waiting Time The distribution of the waiting time of arriving customers.

$$\mathbb{P}(W^Q > t) = \Pi(c,\rho)e^{-(c\mu-\lambda)t} \tag{3.13}$$

An additional performance measure would be a service level, as described below.

Service Level For $M/M/c$ queues, the service level that needs to be met can be defined as the percentage of people that should wait at most t time units before starting service. This requires the waiting time distribution, as provided in Equation (3.13). To determine the number of servers to obtain a waiting time that is below a certain threshold, say α, use

$$\mathbb{P}(W^Q \leq t) = 1 - \Pi(c, \rho)e^{-(c\mu - \lambda)t} \tag{3.14}$$

And we add servers to the queue and recalculate the probability of waiting at most t until

$$\mathbb{P}(W^Q \leq t) > \alpha \tag{3.15}$$

Thus servers are added to the queue, and the waiting time is recalculated until this is below α. This gives that α% of the customers can be served within a waiting time of t time units.

Summary

When the assumptions for a Jackson Network are met, e.g., exponential interarrival and service times, and the system is stable, i.e., $\rho = \frac{\gamma}{c\mu} < 1$ (where γ are the effective arrivals calculated by the traffic equation, which uses the external arrivals λ and the transition matrix P), then the *product-form solution* can be used to analyze the stations independently. This simplifies the calculations of the corresponding performance measures, e.g., the probability of waiting and the expected waiting time in the queue.

Chapter 4

Capacity Allocation Methods

This chapter introduces the modeling and optimization methods used throughout this thesis. First, we model the macro model of the elderly healthcare system as a Jackson Network. After this, we define the three optimization methods used to allocate the budget over the different stations in the network. We start by proposing an exact method, which we call the Capacity Allocation Program (CAP). We expect the exact method to work up to a certain size of the input data, so we also introduce a heuristic approach, Marginal Allocation (MA). Finally, we use the Square-Root Staffing (SRS) principle, a rule of thumb to determine capacity for large systems based on the offered load in a queue. This seems a fast alternative to the other methods.

4.1 Macro Model as a Jackson Network

We model the healthcare system as a Jackson Network. Such a queueing network consists of multiple connected $M/M/c$ queues (or *stations*), meaning that people who finish service at one station can move to another. An example of a Jackson Network is provided in Figure 4.1. This idea coincides with the aim of the macro model of the elderly care system, in which healthcare facilities are connected to discover how people move around the system.

The Jackson Network is a simple but effective way to model such a complex system because it includes stochasticity, requires little data, and, most interestingly, a Jackson Network entails the *product-form solution*, which allows us to analyze stations in the system independently. This is useful because we are considering the healthcare system from a macro perspective, meaning that we look at transitions in the system in terms of flows of patients instead of individuals moving around. This allows us to describe the healthcare system with only a few parameters instead of excessive simulations.

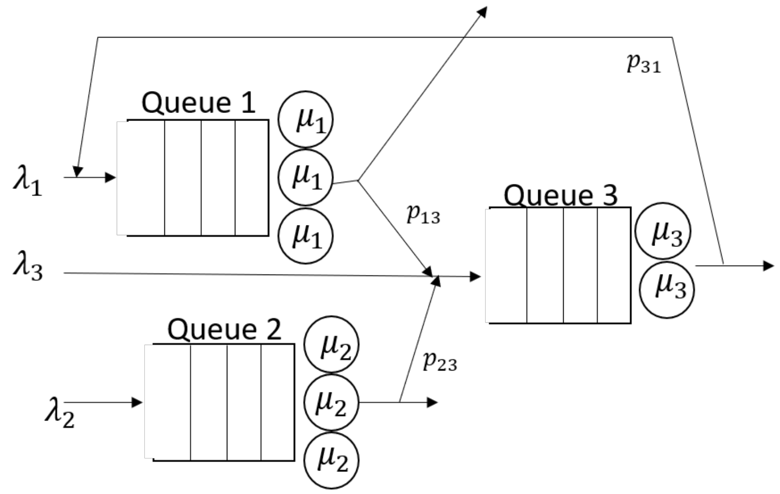


Figure 4.1: A visualization of an example of a Jackson Network. This Jackson Network has three stations, where queues 1 and 2 each have three servers, and queue 3 has two servers. The external arrival rate for queue i is λ_i. All servers in a station i have the same service rate μ_i. The transitions between queues are indicated with a p_{ij}: the probability of going from station i to station j. Further description of the notation is provided in the notation paragraph below.

The performance measures of the $M/M/c$ queues are very relevant to our cause, capacity allocation to reduce congestion, because of the focus on waiting times, capacity, and throughput.

We make some additional assumptions in this research:

- The *service times* of the station's servers follow *exponential distributions*. Exponential distributions have a constant hazard rate over time, meaning that the expected service duration is constant, despite the time that has already passed. The length of stay within a certain healthcare type might follow a different distribution, such as a Weibull or lognormal distribution. Still, we expect that the exponential distribution is a sufficient abstraction for the model since we expect the length of stay to average out in the long run. Hence this average would be enough to use as a parameter for the exponential distribution describing the service rates.

- The *external arrival rates* at stations follow an *independent Poisson process*. The customers at a station arrive independently of each other, meaning that one arrival does not affect another. This seems in line with people independently requiring care. For the assumption of a constant arrival rate, we have to assume that possible external influences, such as the seasons, are minimal and have no effect on the average arrival rate in the system.

- When a person leaves a station and joins the *waiting line* for another station, this person seems to be 'in between' stations. A Jackson Network consists of $M/M/c$ queues, where c stands for the number of servers in the station, and these queues do not consider blocking. So if a station has a waiting line, we expect people in the queue to wait somewhere else, probably at home.

- The people in the system are the same type; there are *no different patient classes* in the model. Due to the macro perspective of our model, we expect this to be sufficient to determine the general characteristics of the model. Additionally, we do not have 'urgent' patients, which is common in certain healthcare types; for example, some hospital beds are available for urgent arrivals. For the model, we do not distinguish any differences in patient urgency because we consider the system from a structural perspective, not on an operational (daily) level. We expect urgent or acute care to be arranged in addition to this model.

- Each place in the system contains *all the required resources to serve a patient*. The cost related to this bed is an average cost. Some patients might require more expensive care, whereas others might need less.
- The model is created from the *system's perspective* because we focus on the capacity of the healthcare stations. This means that a person can enter the system when the person requires care and leave the system when this person does not need any care from the system anymore. When a new demand arises, the patient can return as another external arrival at one of the stations. This differs from a patient's perspective, which could also include a station for no care, but this model focuses on the system.

Notation

To conclude, the following notation is used throughout the thesis:

- The network consists of N stations, $i = 1, .., N$. Each service station is a healthcare facility or healthcare type in the network.
- Each station $i = 1, .., N$ has an external arrival rate λ_i, a service rate μ_i and a cost per server (bed) per day c_i.
- The pathways between the stations are given as transition probabilities in a matrix P, where p_{ij} is the probability of going to station j after ending service in station i.
- The effective arrival rate γ_i for station i is calculated with the external arrival rate λ_i and the transition probabilities P.
- Station i has x_i servers, working in parallel.
- There is a budget C that should be divided over the system.
- $W_i(x_i)$ is the function that calculates the expected waiting time in the queue for station i, containing x_i servers.
- α_i is the service level per station, meaning the percentage of people waiting t days before obtaining service.

4.2 Capacity Allocation Program

We define the Capacity Allocation Program (CAP) as the problem that distributes a budget over different healthcare stations, such that the total average waiting time for a care station in the system is minimized while enforcing the stability constraint of the individual queues and the total budget constraint. We use a linear program formulation to search for the best value of decision variables, the allocations per station, such that the objective function is minimized. The objective and constraint functions should be linear to be solved faster.

The CAP program considers all stations i from the set $I = \{1, 2, .., N\}$. The decision variables are the number of servers, or beds, x_i at station i, for which $x_i \in \mathbb{Z}^+$ for all $i \in I$. The parameters needed in the model are, for station $i \in I$, the cost c_i per server, the effective arrival rate γ_i, and the service rate μ_i. In addition to this, we find the expected waiting time $W_i(x_i)$ at station $i \in I$ when this station has x_i beds, which is determined with an external function, and the weight that should be attached to this waiting time, w_i, which is based on the *arbitrary arrival*, which is the possibility that an arriving person arrives at location i, such that $w_i = \frac{\gamma_i}{\sum_{i=1}^{N} \gamma_i}$. Lastly, we have a parameter for the total budget C. An overview of the sets, parameters, and decision variables is provided in Table 4.1. The CAP is formulated in Equations (4.1) to (4.4).

Table 4.1: Sets, parameters, and decision variables in the initial CAP for the macro model.

Set	**Definition**
I	set of stations $\{1, 2, .., N\}$
Parameter	**Definition**
c_i	cost for a bed at station $i \in I$
γ_i	effective arrival rate at station $i \in I$
μ_i	service rate at station $i \in I$
$W_i(x_i)$	expected waiting time for x_i beds (servers) at station $i \in I$
w_i	weight of station $i \in I$. Here we use $w_i = \frac{\gamma_i}{\sum_{i=1}^{N} \gamma_i}$
C	budget
Variable	**Definition**
x_i	number of servers (beds) at station $i \in I$

$$\min_{x=(x_1,..,x_N)} \quad \sum_{i=1}^{N} w_i W_i(x_i) \tag{4.1}$$

$$\text{s.t.} \quad \sum_{i=1}^{N} c_i x_i \leq C \tag{4.2}$$

$$x_i > \frac{\gamma_i}{\mu_i} \qquad \forall i \in I \tag{4.3}$$

$$x_i \in \mathbb{Z}^+ \qquad \forall i \in I \tag{4.4}$$

The objective function in Equation (4.1) minimizes the sum of the weighted expected waiting times of the stations in I. The weight is the *arbitrary arrival*, such that the program focuses on minimizing waiting times of the locations with more demand regarding arrivals. The constraint in Equation (4.2) states that the total cost per station should not exceed the total budget. Equation (4.3) enforces stability for each station $i \in I$. The stability condition from Equation (3.1), which is $\frac{\gamma_i}{x_i \mu_i} < 1$, is rewritten as $x_i > \frac{\gamma_i}{\mu_i}$. Equation (4.4) is the integrality constraint.

Some modifications are made to the above program to be able to solve it. The first problem is that the waiting time function $W_i(x_i)$ is an external function that calculates the expected waiting time for station i given x_i servers. Some solvers have difficulty with these functions in which decision variables are used as input. To solve this, we extend the program as follows. For each station i, we create a set $S_i = \{1, .., M_i\}$, consisting of all the possible number of servers s_i that can be added to station i, restricted by the maximum value $M_i = \lfloor \frac{C}{c_i} \rfloor$. Thus, based on the budget C and the cost c_i per server, we calculate the maximum number of servers that can be assigned to station i. In addition to this, we introduce a binary variable n_{is_i}, which states for each possible number of servers s_i at station i whether this is used or not, such that $n_{is_i} = 1$ when the number of servers x_i at station i should be equal to s_i. This way, the $W_i(x_i)$ becomes $W_i(s_i)$, so no decision variable is included when calling the expected waiting function.

This modification requires additional computational time when solving the CAP, as it has to calculate a matrix of size $N \times M$ (where $M = \max\{M_1, .., M_N\}$) of waiting times before solving the program.

The extended version of the CAP is formulated in Equations (4.5) to (4.11), with sets, parameters, and decision variables as given in Table 4.2.

Table 4.2: Sets, parameters, and decision variables in the CAP for the macro model.

Set	**Definition**
I	set of stations $\{1, .., N\}$
S_i	set of the possible number of servers $\{1, .., M_i\}$, where M_i is calculated using the budget C: $M_i = \left\lfloor \frac{C}{c_i} \right\rfloor$
Parameter	**Definition**
c_i	cost for a bed at station $i \in I$
γ_i	effective arrival rate at station $i \in I$
μ_i	service rate at station $i \in I$
$W_i(s_i)$	expected waiting time for s_i beds at station $i \in I$
w_i	weight of station $i \in I$. Here we use $w_i = \frac{\gamma_i}{\sum_{i=1}^{N} \gamma_i}$, which shows an arbitrary arrival at station i
C	budget
Variable	**Definition**
x_i	number of beds at station $i \in I$
n_{is_i}	binary that shows how many beds s_i there are at station $i \in I$, for $s_i \in S_i$. $n_{is_i} = 1$ if station i has s_i beds; else $n_{is_i} = 0$

$$\min_{x=(x_1,..,x_N)} \quad \sum_{i=1}^{N} w_i \sum_{s_i=1}^{M_i} n_{is_i} W_i(s_i) \tag{4.5}$$

$$\text{s.t.} \quad \sum_{i=1}^{N} c_i x_i \leq C \tag{4.6}$$

$$x_i \geq \frac{\gamma_i}{\mu_i} + \epsilon \qquad \forall i \in I \tag{4.7}$$

$$x_i = \sum_{s_i=1}^{S_i} n_{is_i} s_i \qquad \forall i \in I \tag{4.8}$$

$$\sum_{s_i=1}^{S_i} n_{is_i} = 1 \qquad \forall i \in I \tag{4.9}$$

$$x_i \in \mathbb{Z}^+ \qquad \forall i \in I \tag{4.10}$$

$$n_{is} \in 0, 1 \qquad \forall i \in I, \forall s_i \in S_i \tag{4.11}$$

The objective function in Equation (4.5) minimizes the weighted sum of expected waiting times in the queue for $x = (x_1, .., x_N)$, where for station i the value of s_i, in combination with binary variable n_{is_i}, is used as input for the waiting time function W_i instead of decision variable x_i. This requires the additional summation over possible values of s_i. Equation (4.6) enforces the budget constraint. Equation (4.7) enforces stability for all stations. In the implementation, the strict inequality in Equation (4.3) became non-strict. Hence ϵ is added to the right-hand side, which is a very small number (e.g. $\epsilon = 0.001$). Equation (4.8) sets the value of decision variable x_i equal to the number of beds s_i for station i, by using the binary variable n_{is_i}. Equation (4.9) requires that only one of the values in S_i can be used as allocation for station i. Equations (4.10) and (4.11) include the integrality constraints.

4.3 Marginal Allocation

Marginal Allocation (MA) allocates capacity based on a marginal analysis of the system, in which the assignment is done step-wise, in each iteration choosing the 'best' station for the next allocation, where 'best' is defined by a specific formula. It is a greedy approach and is compared with CAP introduced previously.

Rolfe (Rolfe, 1971) describes the use of MA for multiple-server service systems as follows. The service system contains N facilities, each facility i consisting of c_i servers, serving customers in order of arrival. Customer arrivals follow a Poisson Process with parameter λ_i. The service times follow an exponential or constant distribution with service rate μ_i. The N facilities have an overall budget constraint $\sum_{i=1}^{N} c_i = S$, ensuring all servers S are distributed over the system. This constraint is slightly different when the stations have differing costs per server. a_i is the cost of assigning a server to facility i, resulting in an overall budget constraint $\sum_{i=1}^{N} a_i c_i \leq S$, where S is the budget. The aim is to allocate facilities to minimize the expected queueing time of customers in the system. Rolfe denotes the expected waiting time in the queue at facility i as $W_q^i(c_i)$, altogether resulting in the allocation problem provided in Equation (4.12), and a suggested MA procedure as in Algorithm 1.

$$\begin{aligned} \min_{c=(c_1,..,c_N)} \quad & \sum_{i=1}^{N} W_q^i(c_i) \\ s.t. \quad & \sum_{i=1}^{N} a_i c_i \leq S \\ & c_i \geq \frac{\lambda_i}{\mu_i} + 1 \qquad \forall i = 1,..,N \\ & c_i \in \mathbb{Z}^+ \qquad \forall i = 1,..,N \end{aligned} \tag{4.12}$$

Algorithm 1: Marginal Allocation procedure as suggested by Rolfe (1971).

1 Set capacity vector $c^{(0)}$, where $c_i^{(0)} = \frac{\lambda_i}{\mu_i} + 1$.
2 Set $k = 1$.
3 Set $c^{(k)} = c^{(k-1)} + e_i$ where e_i is the unit vector and i the index for which $\left(W_q^j(c_j^{(k-1)}) - W_q^j(c_j^{(k-1)} + 1)\right)/a_j$ is a maximum.
4 Stop at step k^* where $K(c^{(k^*)}) \leq S < K(c^{(k^*+1)})$. (Note that $K(c) = \sum_i a_i c_i \leq S$.) Otherwise, $k = k + 1$ and go to step 3.

Rolfe proves that, for constant service times, $W_q^i(c_i)$ is convex and strictly decreasing in the number of servers c_i. He conjectures the same for exponential service times with numerical analysis. Convexity of the waiting time function in the number of servers c_i of a $M/M/c$ queue is shown by Dyer and Proll (Dyer & Proll, 1977). They argue this shows that MA is a valid technique for solving the allocation problem since it will converge to the global optimum.

MA is used to allocate capacity in our Jackson Network. The process is visualized in the flowchart in Figure 4.2 and explained below. In the above theory, c_i was used for capacity and a_i for the cost per server at station i, whereas we use x_i for capacity and c_i for the cost of station i.

MA requires the Jackson Network parameters to iteratively calculate the performance measures, i.e., the load, and the expected waiting time $W_i(x_i)$ in the queue, which can all be found in chapter 3. The method consists of two phases. First is the *stabilization* phase, which step-wise assigns beds to the station with the highest unstable load. To calculate this load of station i, the effective arrival rate γ_i, the service rate μ_i, and the 'current' server capacity x_i are required. When all stations have a stable load, MA starts the *optimization* phase, in which the next bed is assigned to the station with the highest reduction in expected waiting time per invested unit of cost c_i (euro).

$$\frac{\gamma_i}{\sum_{j=1}^{N} \gamma_j} \frac{W_i(x_i) - W_i(x_i + 1)}{c_i} \tag{4.13}$$

Thus, in each step, Equation (4.13) is calculated for all stations $i \in I$, where MA assigns the next bed to station i with the highest waiting time reduction. This formula also contains a weight for the *arbitrary arrival* to include the importance of high-demand stations when deciding where to allocate the next bed. This optimization procedure is repeated until no budget is left for the next capacity assignment.

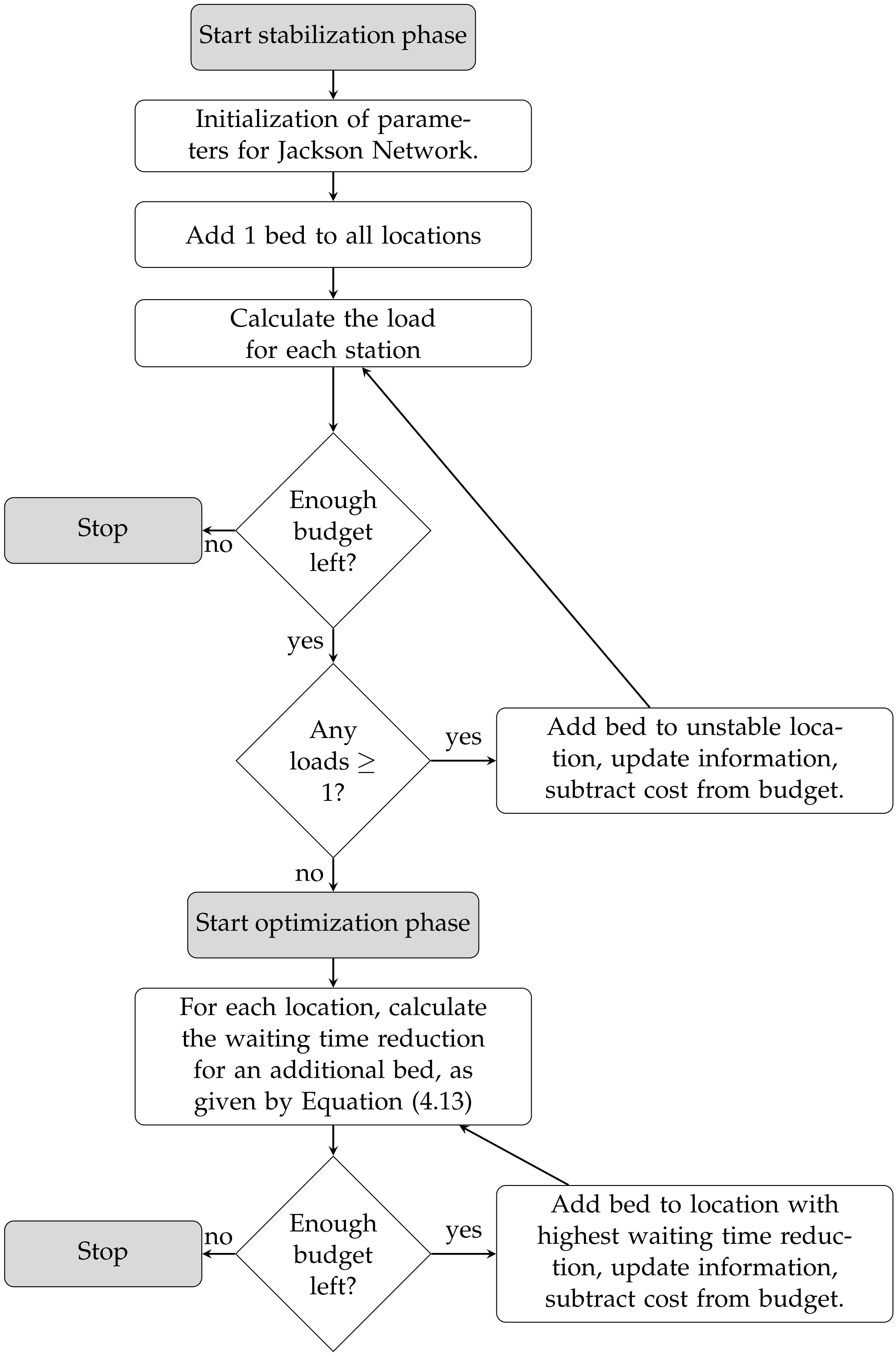


Figure 4.2: Flowchart of MA algorithm

4.4 Square-Root Staffing Principle

We also propose allocating capacity using the Square-Root Staffing (SRS) principle. This principle determines the number of servers needed based on the average offered load for a queue. It adds a *safety capacity* by taking the square root of the offered load multiplied by a quality measure. The formula for the SRS principle is provided in Equation (4.14), where offered load $a = \lambda\mathbb{E}(S) = \frac{\lambda}{\mu}$ and β is the Quality of Service (QoS) parameter. This QoS β is a constant that states how many times the square root of the offered load is added to the average offered load to decide on the needed number of servers. The higher the QoS, the more capacity is added to the station, and the better this station can handle sudden increases in offered load. The resulting s is the number of servers that should be placed at the station to handle the load by providing a specific QoS in terms of β. Note that s is a continuous value here and should thus be rounded if a discrete value is needed.

$$s = a + \beta\sqrt{a} \tag{4.14}$$

The SRS is obtained from the normal approximation: the number of customers in the system is approximately normally distributed if the offered load is not too small and the QoS parameter is high (Whitt, 2007). The number of busy servers follows a Poisson distribution with as mean the offered load a. For Poisson, the variance equals the mean. Thus, a also represents the variance, and $\sqrt{a}$ is the standard deviation. Therefore we set the capacity needed as $a + \beta\sqrt{a}$, where β shows the number of standard deviations from the mean we want to include. Considering the bell curve of the normal distribution, if we take $\beta = 2$, we have a safety margin of approximately 95%, meaning that variations in load within this range can be handled by the capacity s.

The QoS parameter β can be mapped in a delay probability α called the *Halfin-Whitt delay function* and shown in Equation (4.15), where Φ is the cdf and ϕ the pdf of the standard normal distribution. A higher QoS β results in a lower delay probability α, as seen in Figure 4.3.

$$P(\text{Delay}) \equiv \alpha \approx \textit{Halfin-Whitt}(\beta) \equiv \left(1 + \frac{\beta\phi(\beta)}{\Phi(\beta)}\right)^{-1}, \qquad 0 < \beta < \infty \tag{4.15}$$

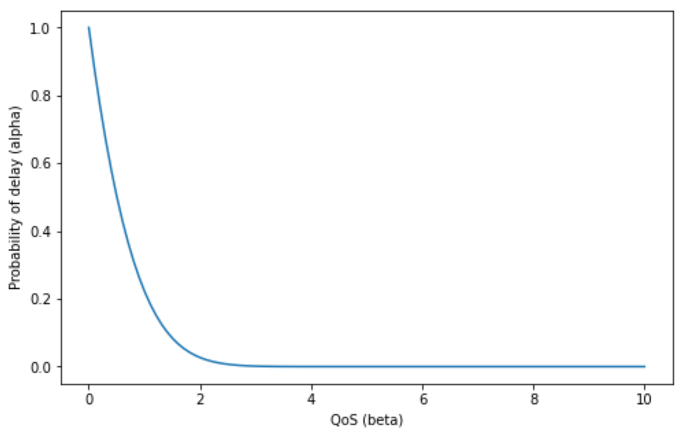


Figure 4.3: HW delay α for increasing QoS parameter β.

Because of its simplicity, we use this SRS principle to determine the capacity allocation for our macro model. For this, we decide on the number of servers s to add, for which the effective arrival rate γ, service rate μ, and QoS β are required. Our primary interest is to allocate capacity while minimizing the waiting time, which differs from SRS because this formula does not include the waiting time. But, since the waiting time depends on the delay probability, and a higher delay probability results in a lower QoS, we aim to maximize the QoS parameter. The number of beds is discrete, so we set the variable s_i for station i as an integer instead of a continuous value, as we have to consider both the offered load and the maximum budget. A MILP (Mixed-Integer Linear Program) model is created to solve the SRS for all locations simultaneously. Creating a system of equations is also possible, but we propose an exact program since the focus is to allocate beds to maximize the QoS with a constrained budget. The sets, parameters, and decision variables are mainly similar to previously proposed methods; however, we have the QoS parameter β_i for station i, the number of beds is called s_i for station i, and we use a dummy variable z to solve the system. Thus we provide an overview in Table 4.3. The model is formulated in Equations (4.16) to (4.22).

Table 4.3: Sets, parameters, and decision variables in the SRS program.

Set	Definition
I	set of locations $\{1, 2, .., N\}$
Parameter	**Definition**
c_i	cost for a bed at facility $i \in I$
γ_i	effective arrival rate at facility $i \in I$
μ_i	service rate at facility $i \in I$
C	budget
Variable	**Definition**
s_i	number of servers (beds) at facility $i \in I$
β_i	Quality of Service for facility $i \in I$
z	dummy variable

$$\max_{s=(s_1,..,s_N)} \quad z \tag{4.16}$$

$$\text{s.t.} \quad \beta_i > z \qquad \forall i \in I \tag{4.17}$$

$$\beta_i = \frac{s_i - \frac{\gamma_i}{\mu_i}}{\sqrt{\frac{\gamma_i}{\mu_i}}} \qquad \forall i \in I \tag{4.18}$$

$$\sum_{i=1}^{N} c_i s_i \leq C \tag{4.19}$$

$$s_i \in \mathbb{Z}^+ \qquad \forall i \in I \tag{4.20}$$

$$\beta_i \in \mathbb{R}^+ \qquad \forall i \in I \tag{4.21}$$

$$z \in \mathbb{R}^+ \tag{4.22}$$

The objective function in Equation (4.16) maximizes the QoS for each facility. A dummy variable z is introduced, equaling the lowest QoS β_i. The constraint in Equation (4.17) pushes the QoS values up. The constraint in Equation (4.18) chooses β_i and the number of beds s_i based on the SRS-formula. Equation (4.19) enforces that the allocation stays within the budget. Equations (4.20) to (4.22) are the integrality constraints.

The three methods proposed in this chapter are compared in the next chapter.

Chapter 5

Methods Comparison

This chapter compares the performance of the optimization methods explained in chapter 4, which are the Capacity Allocation Program (CAP) model, the Marginal Allocation (MA) model, and a program using the Square-Root Staffing (SRS) principle. We examine the behaviors and results of the methods with a toy example.

Toy Example

The toy example is visualized in Figure 5.1 and consists of four locations, $i = 1, 2, 3, 4$. For each station i, we have an external arrival rate λ_i, length of stay μ_i^{-1} and the cost per bed c_i, which are given in Table 5.1, and the transition probabilities P are provided in Table 5.2. We determine the effective arrival rate γ_i for station i with the traffic equations from Equation (3.2), provided in chapter 3, and the resulting values are shown in Table 5.1. The offered load $a = \frac{\gamma}{\mu}$ is also provided in the table, together with the minimum required number of servers x_{stable} that is needed for a stable throughput, as calculated with $x_{\text{stable}} > \frac{\gamma}{\mu}$. As Table 5.1 shows, this is 97, 46, 118, and 42, respectively, for locations 1 up to 4. The locations in the Jackson Network require at least this server capacity per station to calculate the performance measures.

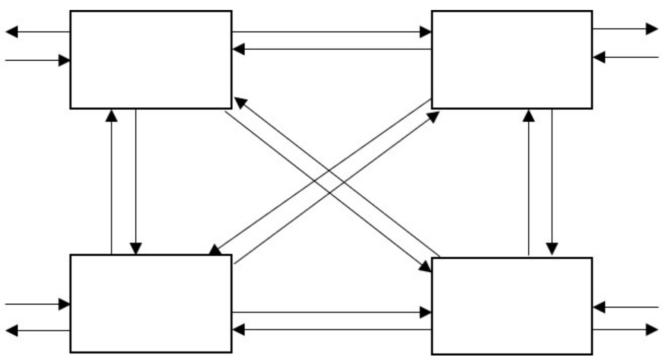


Figure 5.1: A visualization of the toy example. Each box is a queue with servers, as visualized in Figure 4.1. The lines between boxes are transition probabilities as given in Table 5.2, the service rates and external arrivals are given in Table 5.2.

Table 5.1: Initial parameter setting of external arrival rate λ, LoS μ^{-1} and cost per bed c, for locations $i = 1, .., 4$. On the right the effective arrival rate γ for locations $i = 1, .., 4$, calculated with λ and transition matrix P from Table 5.2. Together with the offered load a, based on γ and LoS. With a, we find the numbers of servers x_{stable} needed for stability.

Location	λ	μ^{-1}	c	γ	a	x_{stable}
1	6	5	1	19.20	96.00	97
2	1	3	2	15.20	45.80	46
3	3	7	3	16.80	117.60	118
4	8	2	4	20.80	41.60	42

Table 5.2: Initial parameter setting of the transition probabilities p_{ij} from location i to location j.

	Locations			
Locations	1	2	3	4
1	0	0.25	0.25	0.25
2	0.25	0	0.25	0.25
3	0.25	0.25	0	0.25
4	0.25	0.25	0.25	0

As shown by Table 5.1, location 3 has the highest offered load and requires most servers for a stable system.

5.1 Influence of Varying Budgets

We perform the allocation for a range of budgets. The minimum budget is calculated by summing the costs needed for a stable system, as given in Table 5.1. Stability is a constraint for CAP and a starting condition for MA before the actual optimization. SRS can only optimize anything when it can handle all offered loads. However, excessive budgets are likely to lead to random solutions for the exact programs and are unrealistic in our application, healthcare. Thus, as the maximum budget, we set the needed budget for a service level of 90% at all locations, meaning that 90% of the people do not have to wait at all $(t = 0)$. This is calculated as in Equation (3.15) in chapter 4.

Using the parameters in Tables 5.1 to 5.2, we obtain a minimum and maximum budget of 711 and 843, respectively, and we run the allocation with MA, CAP, and SRS for all budgets in between, with steps of 4, the price of the most expensive bed. The budget allocations, in terms of beds, are shown in Figure 5.3. The horizontal red line in each plot indicates the number of beds needed for a stable throughput at the location.

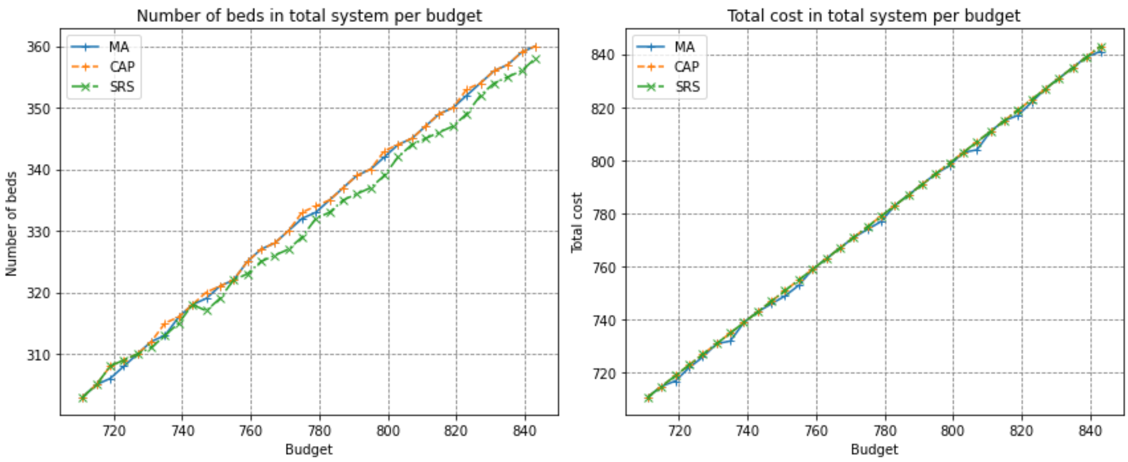


Figure 5.2: Total number of beds added to the system and total cost of all beds in the system.

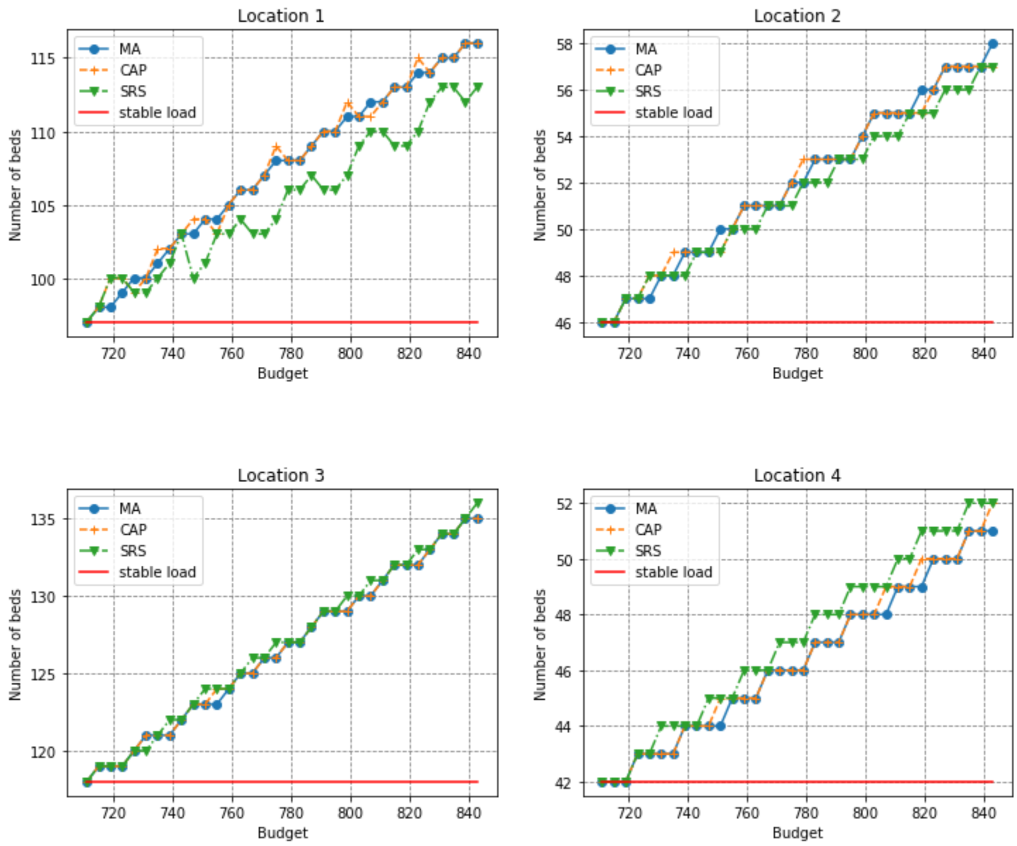


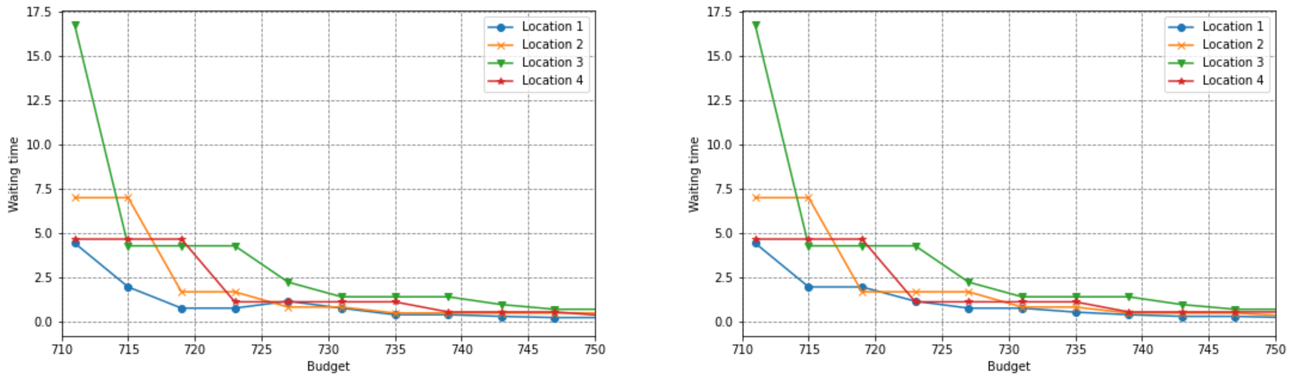
Figure 5.3: Bed allocation per budget per location for MA, CAP, and SRS. One figure per location, showing the final results. The red lines show the minimum number of beds required for a stable system.

Capacity Allocation Program

The allocation for CAP is visualized in Figure 5.3, showing an increased allocation for an increasing budget per location. The figure shows some spikes for location 1, so in some cases, the CAP method decides to decrease the number of beds for location 1 and to assign this budget somewhere else. In Figure 5.2 on the right, we see that these spikes are present because CAP tries to allocate the beds such that all budget is used.

Figure 5.4 (a) shows the corresponding decreasing waiting times for all locations in one figure. For this figure, we only look at the first allocations, up to a budget of 750. First, the method allocates to the location with the highest waiting time. This is location 3 for the first budget, after which the left-over budget is allocated to location 1. The figure also shows an increase in waiting time for location 1 around the budget of 727, meaning that this budget is allocated elsewhere for a slight increase in budget.

The decreasing structure of the waiting times suggests that the method will converge toward the global optimum; with each step, the decrease becomes less steep.



(a) CAP

(b) MA

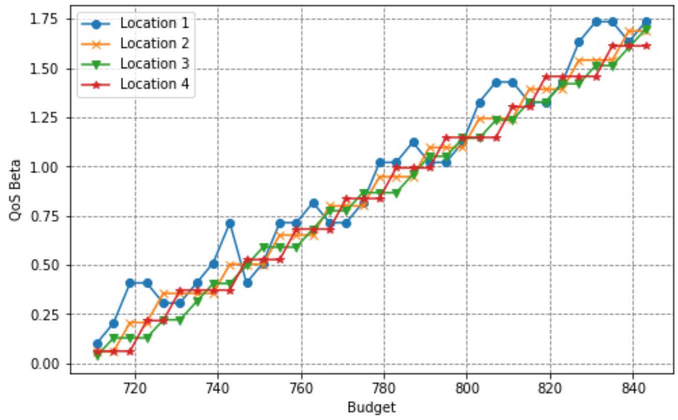
Figure 5.4: Expected waiting time per budget per location, for budgets between 710 and 750.

Marginal Allocation

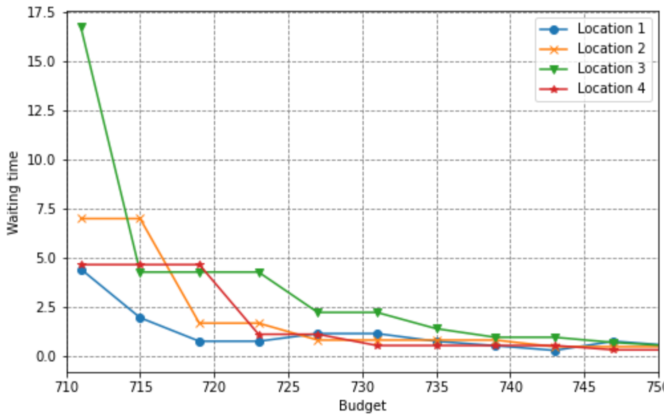
The allocation structure of the MA method is similar to CAP, as Figure 5.3 shows. However, for MA, the number of allocated beds increases more consistently, without any decreases, meaning no budget is reallocated for a higher budget. This is probably because of MA's greedy choice for the next assignment.

Figure 5.2 shows that MA does not always use the total budget. This is shown by the blue stars/dots below the *Total Cost*-line for budgets of approximately 720, 735, 750, 780, and 805. If the left-over budget is not enough for the location favored by MA, based on Equation (4.13) in chapter 4, then this final budget will not be used for any other location.

Figure 5.4 (b) shows the decreasing waiting times for the allocations, with all locations in one figure. The results look similar to CAP, with only a few differences, probably because of budget reasons. The decreasing waiting time, which becomes less steep for higher budgets, suggests that the method converges toward the global optimum. This was also expected since we found in chapter 4 that for station i, the waiting time function is convex in capacity c_i.



(a) Quality of Service per budget per location for SRS.



(b) Expected waiting time per budget per location for SRS.

Figure 5.5: SRS

Square-Root Staffing

Figure 5.3 shows that allocation with SRS deviates slightly from CAP and MA. This is best visible for location 1, which shows many peaks in the allocation structure. Possibly because this location has the lowest cost per bed and can be used to finish the budget, which might be moved to other locations for the next increased budget. The allocation for location 1 is always less than for CAP and MA, whereas SRS allocates more beds to location 4. Locations 2 and 3 overlap in allocation structure for the three methods. The differences in locations 1 and 2 results in fewer beds allocated to the total system when using SRS. Figure 5.2 underlines this, as seen in the figure on the left. For larger budgets, the number of beds allocated by SRS is always less than the numbers for CAP and MA.

The quality of the system increases linearly, as shown in Figure 5.5 (a). The increase in beds depends on $\beta\sqrt{a}$. Thus, each increase in the number of allocated beds leads to a rise in the Quality of Service $\beta = \frac{1}{\sqrt{a}}$. The higher the offered load, the lower the increase in QoS when an additional bed is added. This results in assigning most beds to location 3, followed by 1, 2, and 4.

The deviating allocations of SRS compared to CAP and MA are likely due to differing objectives. SRS aims to maximize the QoS for all stations evenly, and Figure 5.5 (a) shows that this QoS increases consistently for increasing budgets. Part of the budget is first assigned to location 1 and later reallocated somewhere else when an additional budget is available. SRS aims to allocate this to station i with the lowest β_i, which can only be done when enough budget is available. Figure (b) is the expected waiting time calculated after obtaining an allocation. This shows some differences with the waiting time for CAP and MA, e.g., for location 3, this is minimized later for SRS.

Service Level and Runtime

Additionally to the waiting times, Figure 5.6 shows the reached service levels (SL) per location, meaning that α% can be served within a waiting time of t (we have $t = 0$). For location 1, CAP and MA reach the SL earlier than SRS, which is the other way around for location 4. For the other two locations, this is similar. The figure shows which budget will result in which SLs per location.

Finally, the runtime in Figure 5.7 shows a significant difference in runtime for the CAP and the other two methods. The SRS is the lowest, and the runtime for MA also seems to increase when complexity in the data increases, in this case, an increase in budget.

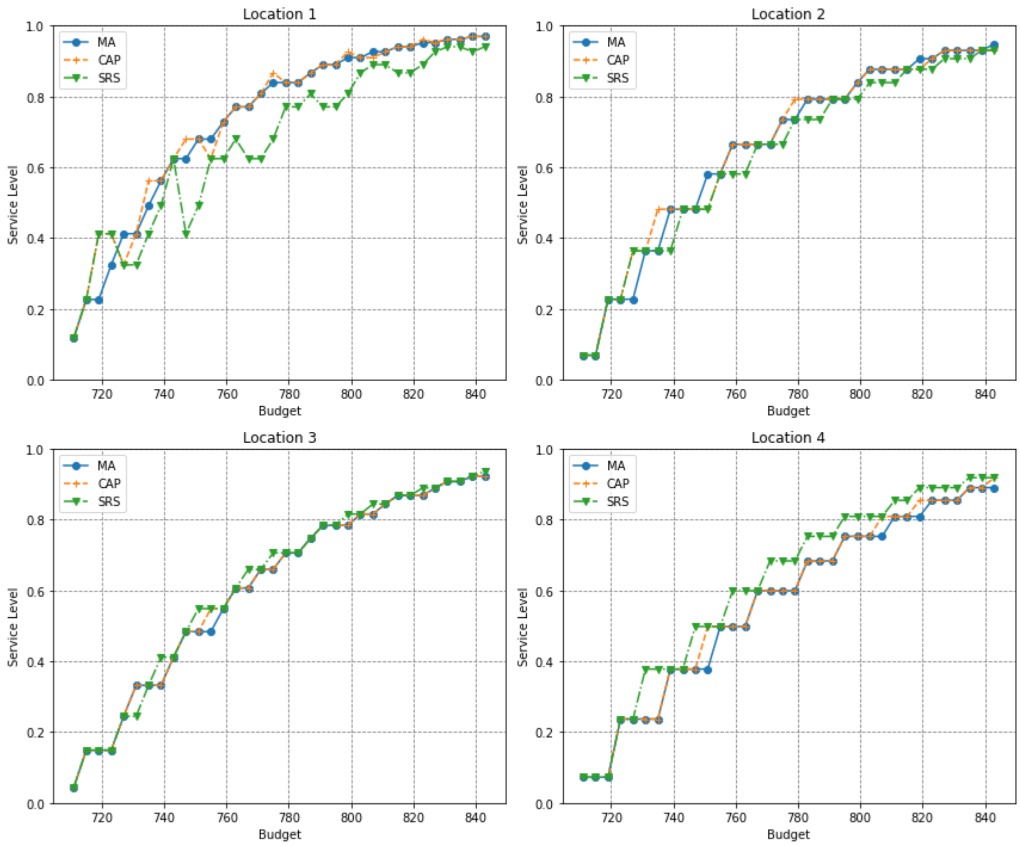


Figure 5.6: Reached service level per location. The maximum waiting time for the service level was set to $t = 0$.

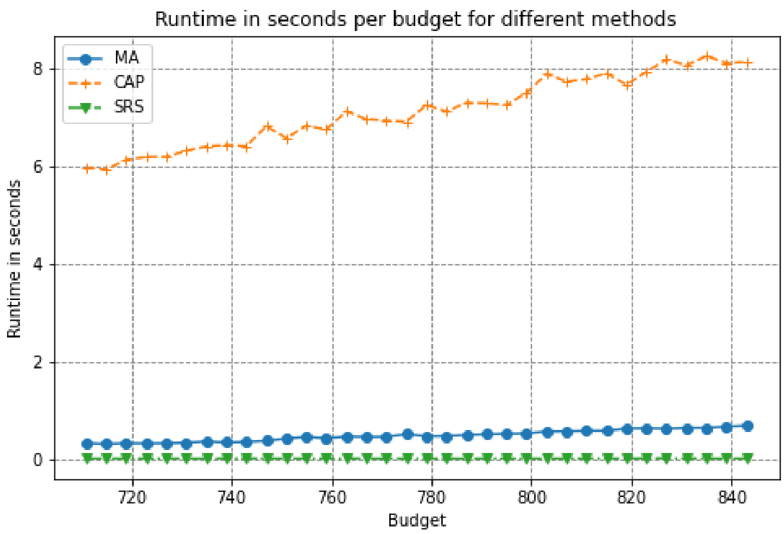


Figure 5.7: Runtime per budget per method.

5.2 Influence of Increasing Demand

This section investigates the influence of increasing demand on the system. An increase in demand is translated as an increase in arrivals. Due to transition probabilities, an increase in external rates also influences effective arrivals.

We modify external arrivals for location 1. For each step, we increase the external arrival rate, after which the effective arrival rates at the stations are recalculated. We optimize for different external arrivals, in a range of 6 up to 30, with a budget of 1238.

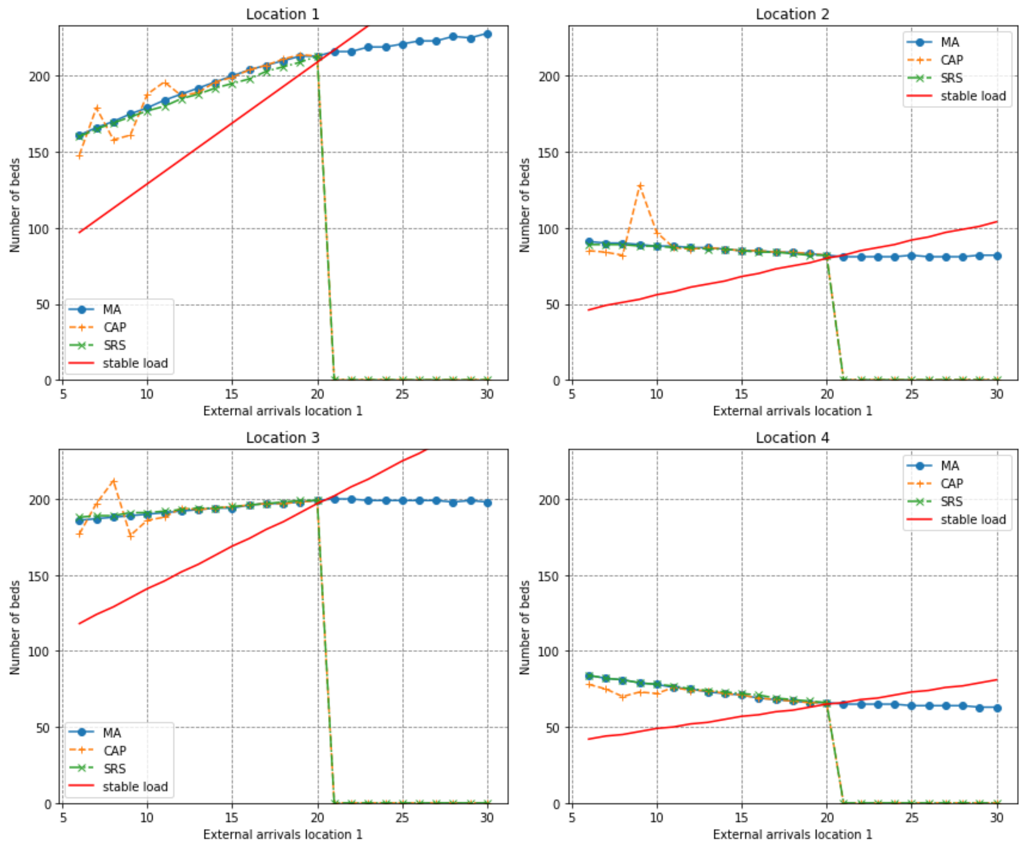


Figure 5.8: Bed allocation per budget per location for MA, CAP, and SRS. Per location, one figure shows the final results. The red lines show the minimum number of beds required for a stable system.

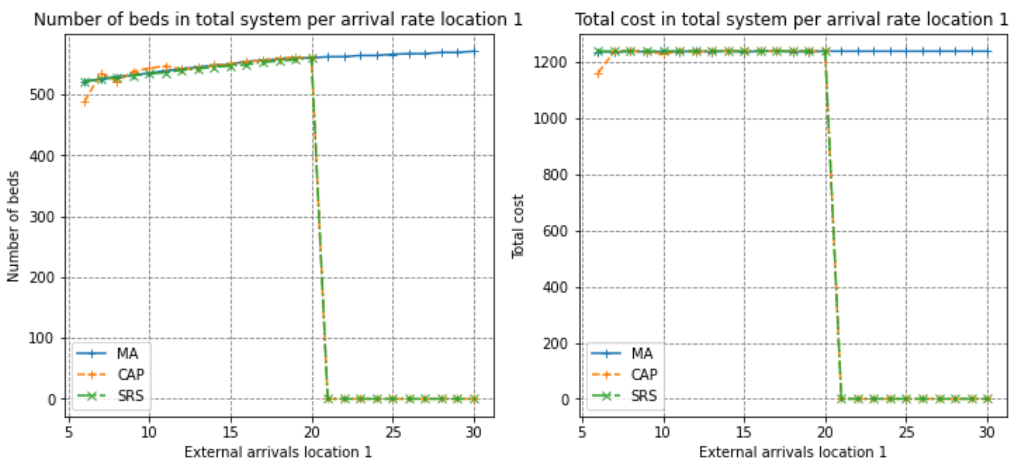


Figure 5.9: Total number of beds added to the system and total cost of all beds in the system.

Figure 5.8 shows the allocation per station for an increasing external arrival rate at location 1. The red lines indicate the minimum number of beds required for a stable load;

otherwise, CAP and SRS cannot find any feasible solution. In these unstable cases, MA can still propose a solution; however, the waiting times will also grow infinitely here. An increase in external arrivals at location 1 affects the effective arrivals everywhere, as 25% of the arrivals leaving station 1 go to other locations. Locations 1 and 3 receive more beds for increasing external arrivals. Location 1 has the most considerable increase, as this location has more effective arrivals due to the high external arrival rate and the increasing number of people returning from other stations. Fewer beds are assigned to locations 2 and 4. The offered loads for locations 1 and 3 are higher. Thus, they require more beds to obtain a stable load.

The random peaks for CAP for external arrivals between 5 and 15 might be due to the budget being too loose. These peaks disappear as the number of people in the system grows and the budget becomes tight.

Summary

From the methods comparison, we can conclude the following:

- The *Capacity Allocation Program (CAP)* focuses on assigning beds to locations with the highest waiting time, weighted by the probability of demand (arbitrary arrivals). The CAP seems to converge for higher budgets; with each step, the reduction in waiting time decreases. Nevertheless, the CAP method is computationally unattractive.

- *Marginal Allocation (MA)* makes greedy decisions on the allocation, giving preference to locations with the highest reduction in waiting time per invested cost. MA shows similar results to CAP and seems to converge; with each step, the reduction in waiting time decreases. Due to its greedy nature, MA does not use all the budget if the next preferred bed is too expensive. This example's computational time is fast but increases when complexity grows.

- The method based on the *Square-Root Staffing (SRS)* principle allocates beds based on the offered load. SRS assigns differently than CAP and MA, trying to improve the Quality of Service instead of minimizing the summed waiting time. Each additional bed is a fixed increase of QoS; thus, this increases linearly. More beds are required to increase the QoS for stations with a high offered load. The SRS appears to be computationally most attractive.

- An *increase in demand* for one location, considered an *increase in external arrivals*, also results in extra arrivals at other locations. To reduce waiting times and to satisfy the stability condition, capacity is removed from locations with lower offered loads and assigned to the sites with growing offered loads.

Chapter 6

Dutch Healthcare System

This section describes the general structure of the elderly care system and how we aim to model it. People require different types of care. Thus, the elderly care system consists of varying healthcare types.

Scope of the Model

'Krakende Ketens' (SIGRA, 2017) summarizes the elderly care system in four healthcare areas. Each area contains different care types; for example, in primary care, we have home care, the general practitioner, and the social support act (in Dutch WMO: Wet Maatschappelijke Ondersteuning), all aiming to help people to live healthily at home. In nursing homes, we see a division between short stays (geriatric rehabilitation and short-term residential care) and more extended stays, provided as long-term care. For hospitals, we can divide hospitalizations into different departments (usually planned) and the emergency department, aimed at acute requests for care.

Not all healthcare types are included in the model, i.e., the general practitioner (GP), emergency department (ED), social support act (SSA), and mental healthcare are not included. For the first two types, the duration of receiving care is in terms of minutes (for GP) or hours (for ED), and we require care types to be multiple days for the macro model. Additionally, these care types can be short interventions within another care type, for which the latter is not paused during the visit to GP or ED. The third care type, SSA, includes assistance for living at home and help in terms of adjustments in the house, and it is mainly provided in combination with other care, such as home care. Lastly, mental healthcare is not included because we focus on care related to aging. Psychogeriatric care is provided in nursing homes and differs from mental healthcare. And to mention last, elderly people can require other care, e.g., due to a disability. However, as mentioned before, the model focuses on care related to aging. One could argue that hospitalizations are also not merely for elderly people. However, older people tend to end up in extensive hospitalizations due to underlying geriatric conditions, as mentioned in the introduction of this thesis. Thus, we want to include this medical specialist care as well.

To conclude, for the macro model, we combine the following healthcare types: home care, the different hospital departments, geriatric rehabilitation, short-term care, and long-term care.

Healthcare Types in the Model

The healthcare types incorporated into the model are outlined here.

Home Care

Home Care (HC) contains personal care and nursing at home. This care can be needed after hospitalization or due to illness. The care is usually temporal, and the need for care is reconsidered after a while. The frequency of the care can differ for each person. Some people will receive weekly or daily care, usually for the activities of daily living. Other people will receive care only once or twice, such as checkups after a hospitalization. The aim is to help people live at home as long as possible.

In 2017 about 550 thousand people in the Netherlands received care from HC. Most people are 75-84 years old. However, relatively, from the group of 85+ years old, the percentage of HC users is the biggest, about 40%, and about 56% of this group require somatic care for over three months. The hours of care per week can vary greatly, but for 85+-year-old people, this average is 5.8 hours per week. However, for palliative care, this can be 22 hours a week. The 85+ group received HC for on average 6 months (Vektis, 2018). The Dutch Healthcare Authority (NZA) presented similar numbers for 2017-2019, ranging from 550 to 580 thousand people receiving HC (NZA, 2022b).

Long-Term Care

The main characteristic of Long-Term Care (LTC) is that this care is needed for the rest of a person's life. LTC care can be divided into several profiles describing the required type and amount of care. For example, LTC includes care aimed at problems related to aging ('nursing and personal care') but also for people with a disability. Here we only focus on the first type of care mentioned.

An indication is required for LTC care, which should be required by CIZ, an institution that decides who needs what type of intensive care. After receiving such an indication, the healthcare insurance (in Dutch *Zorgkantoor*) is responsible for arranging the placement in the preferred form. This can be placement at a nursing home or care at home provided by one or several organizations.

Within LTC, we look at the 'nursing and personal care'-profiles, abbreviated as VV, for which the following profiles are defined:

- VV 4: mostly aimed at personal care. Less intensive than the higher profiles.
- VV 5: aimed at psychogeriatrics (PG), such as dementia.
- VV 6: people with somatic (SOM) problems.
- VV 7: people with behavioral problems, usually related to psychogeriatric (PG) diseases.
- VV 8: people with severe somatic (SOM) problems.
- VV 9b: rehabilitation in a nursing home. The people already have an LTC indication or will need one after rehabilitation, whereas people obtaining geriatric rehabilitation (explained next) are expected to return home after rehabilitation.

Before 2013, receiving care from profiles VV 1, VV 2, and VV 3 was also an option, which contained less intensive care than the ones mentioned above. However, these profiles are eliminated because HC or SSA can provide similar care at home. On 1 January 2018, another profile was also eliminated: VV 10, used to indicate palliative care. Declarations for palliative care are made as a surcharge on the current healthcare profile.

LTC care is usually provided in nursing homes but can also be organized at home. This is arranged during the indication setting with CIZ. This LTC care at home is provided as *VPT* ('Volledig Pakket Thuis', the care from the profile is obtained at home, all provided by the same healthcare provider) or as *MPT* ('Modulair Pakket Thuis', the care at home is received as partial packages, possibly from different providers).

About 223 thousand older aged people received LTC in 2017. Approximately 92% of this is care from one of the *VV profiles*, mostly while living in a facility. According to Vektis, most people in a nursing home require care for dementia, in profile VV 5 (PG) (Vektis, 2018).

Geriatric Rehabilitation

Geriatric Rehabilitation (GR) is rehabilitation care for elderly people, providing them with intensive care to eventually return home. In contrast, if people are expected to require care for the rest of their lives, they are more likely to receive an LTC Rehabilitation (VV9b) indication.

Usually, GR care is needed after a hospitalization, e.g., after surgery for a fracture. Medical-specialist care is needed for these cases, registered using a *DBC*-healthcare trajectory. *DBC* is the dutch abbreviation for 'diagnose-behandel-combinatie', which translates as 'diagnostics-treatment-combination'. The DBC is a guideline to determine which care is needed for a treatment, in what quantity, and for what cost. For GR, the DBC trajectories estimate the needed length of stay per rehabilitation period, for example, after surgery. The duration of a DBC has a maximum of 120 days, but it can be extended.

In 2017, about 49 thousand people in the Netherlands received GR (Vektis, 2018), and about 53 thousand in 2019. Most treatments are required after a fracture and secondly after a stroke (Vektis, 2021).

Short-Term Care

Short-Term Residential Care (STC) is a temporal stay at a nursing home for people who, at that time, cannot care for themselves at home. The aim is to return home eventually. This is not medical-specialist care, as for hospitalizations and GR, but short medical care, which should help vulnerable people to recover and return home.

The type of STC care can be divided into different intensities: someone can receive high-complex care, low-complex care, or palliative care. The difference between the first two types lies in the complexity of the required care. The third type is care at the end of people's life.

This care is a separate form since 1 January 2017, as it was first funded through Long-Term Care. About 30 thousand people received this care in 2017, with as biggest group the 85+ year-olds. About 50% receives high-complex care, approximately 30% is low-complex care, and around 20% palliative care (Vektis, 2018). The Dutch Healthcare Authority (NZA) presented an increase in the number of people requiring this care, from about 33 thousand in 2017 to about 38 thousand in 2020 (NZA, 2022a).

Hospitalizations

Hospitalizations contain care that is required after the occurrence of an acute event. This is medical-specialist care, and elderly people are expected to need some other care after this, i.e., GR. The care forms outlined above are primarily aimed at elderly people, but this form is aimed at all people. The model focuses on care solely aimed at the elderly, but hospital departments are also included since this group tends to be hospitalized. Some examples of hospital departments are surgery, cardiology, and orthopedics. The departments included in the model are explained in the case study.

According to Vektis, around 300 thousand 65+-year-old people (9%) have an unplanned hospitalization yearly between 2017-2021 (Vektis, 2022).

Chapter 7

Case study: Amsterdam

This chapter includes a case study on Amsterdam. The first part consists of gathering and combining data, such that we have a dataset containing all healthcare paths of citizens of Amsterdam. The second part of this chapter includes the actual data analysis. We aim to discover some essential characteristics of different care forms: the *average length of stay* at each care type and the corresponding *demand* for these. This part also determines the parameters that are used for the Jackson Network describing the elderly healthcare system in Amsterdam, which is used for the allocation of capacity in the next chapter.

7.1 Data Description

In this case study, we use data from Netherlands Statistics (CBS) microdata, non-public data containing information about citizens in the Netherlands available for researchers. The datasets in CBS microdata are linkable at the level of individuals and addresses. We used (1) personal data combined with address data on a municipal level and (2) healthcare data consisting of declarations from the different healthcare forms. The healthcare datasets consist of declarations, where each instance is one declaration for a healthcare activity.

The steps in this process were selecting the target group, gathering and cleaning the data for the healthcare activities, and merging these into one dataset. The process is shown in Figure 7.1 and explained below in more detail.

Target Group Datasets

First, we define the target group for the analysis. This target group consists of people that lived in Amsterdam for more than 30 days, successively, between 1 January 2017 and 31 December 2019. These people are 65 years old or turn 65 years during this period. This time range of 2017-2019 is used because we only have data for all healthcare types available for 2017-2019. Besides this, the Covid-19 pandemic started in the first quarter of 2020 and greatly influenced the whole healthcare system.

We identify this group of elderly people in Amsterdam, starting with identifying their addresses. We have to select the care activities of people based on where the people lived because no information is provided on where the care activities took place. Thus, the period of living in Amsterdam is used to identify activities that have possibly happened in Amsterdam, thus in an Amsterdam healthcare facility. We obtain this data from datasets about the Dutch population, municipalities, and neighborhoods. The latter is helpful to determine the exact neighborhood per person since we want to aggregate some care types on the district level.

To select the elderly, we use the data with inhabitants of Amsterdam, calculate people's 65th birthdays, remove all people that turn 65 after 2020, and remove all people that passed away before 2017.

The next step is to identify and modify the datasets of the different healthcare types. We have data from Home Care (HC), Long-Term Care (LTC), Geriatric Rehabilitation (GR), Short-Term Residential Care (STC), and Hospitalizations (H). After this cleaning process, these are merged, and based on people's age and living dates in Amsterdam, these activities are included in the final dataset.

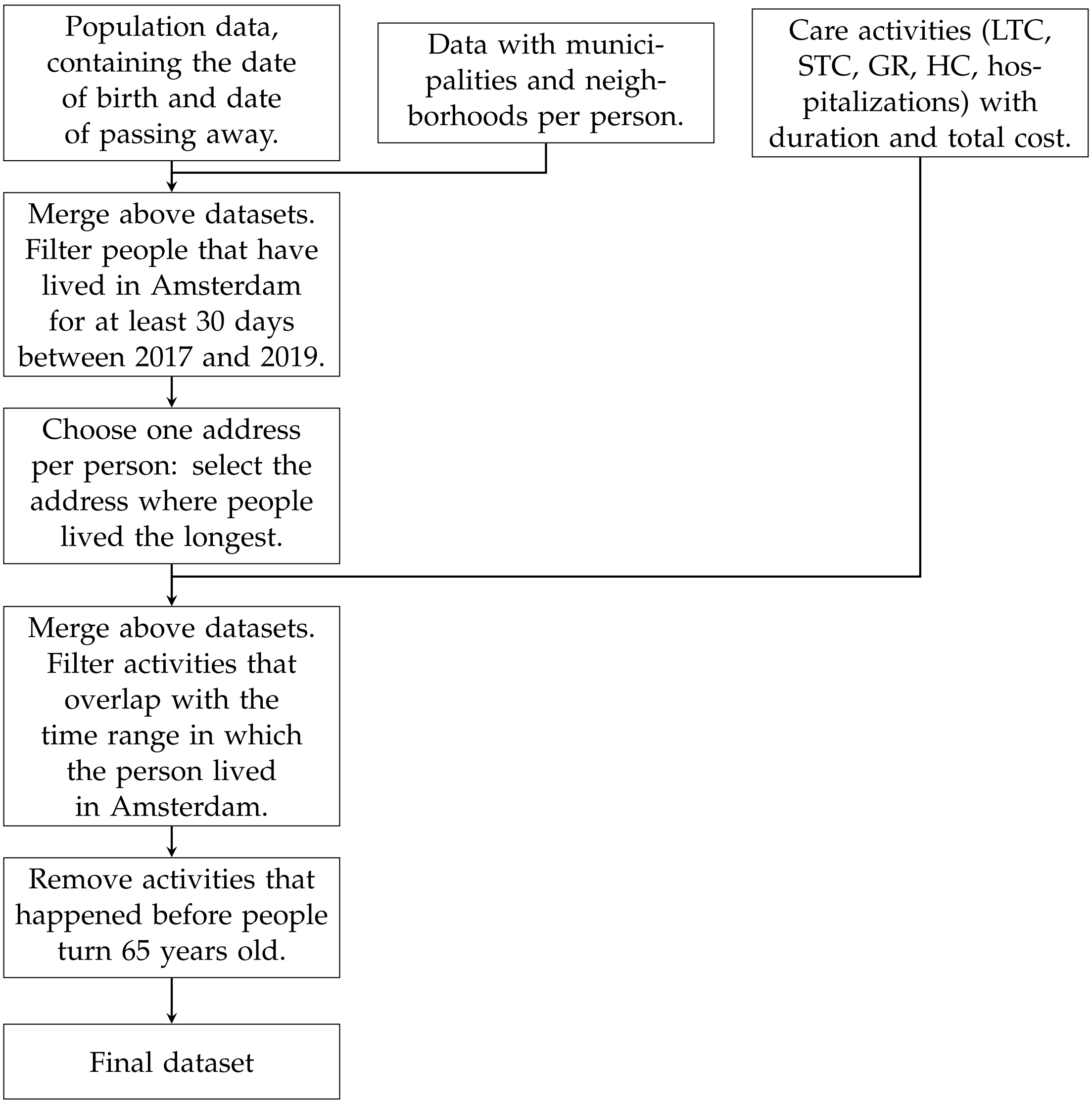


Figure 7.1: Selection process of target data.

Home Care Dataset

The HC data contains declarations of activities concerning personal care and nursing at home. However, the exact activities undertaken and the help duration are not included. These activities could range from daily care to aftercare after a hospitalization. The structure of declarations also differs; some seem to be for the whole month, whereas others are created per day, with possibly multiple single declared days in a month.

Because of these differences, we decide to look at the declarations for HC at a monthly level, which requires some modifications to the data. For all months, we checked if there was at least one declaration. If there are any consecutive months with declarations, these are merged. For this, we determine the final start and end dates, which mean 'between which dates did the person receive monthly care.' This process is shown in Table 7.1.

Table 7.1: Example of modifications for HC dataset with care activities. The example is made up.

Before modification					After modification				
Case	Activity	Start date	End date	Cost	Case	Activity	Start date	End date	Cost
1	HC	2019-02-10	2019-02-10	60	1	HC	2019-02-10	2019-03-02	180
1	HC	2019-02-20	2019-02-20	60	1	HC	2019-08-01	2019-10-31	1200
1	HC	2019-03-02	2019-03-02	60					
1	HC	2019-08-01	2019-10-31	1200					

Long-Term Care Dataset

The LTC dataset contains declarations of people who received intensive care at a healthcare institution or home. The declarations are made monthly, so consecutive declarations are merged into one activity.

LTC care is determined based on profiles, explained in Chapter 6. Each profile includes a fixed package of care to be provided, thus a somewhat fixed price. However, LTC nursing homes do not have a fixed number of places for each profile. Thus we model LTC into four stations:

- *LTC Somatic (SOM)*: somatic care in a nursing home, including profiles VV5 and VV7 received at nursing homes.
- *LTC Psychogeriatric (PG)*: psychogeriatric care in a nursing home, including profiles VV6 and VV8 received at nursing homes.
- *LTC at Home*: care at home includes all care provided at home; and care provided under VV1 up to VV4, which is now probably given at home.
- *LTC Rehabilitation*: rehabilitation in a nursing home, including profile VV9b. This is separate because of its temporal nature.

We perform extra modifications on some activities within 'LTC at Home'. This includes care delivered at home in 'modular packages', officially called *MPT* (explained in Chapter 6). These MPT declarations possibly include only a few days a month for consecutive months. We do the same as for HC (see Table 7.1); we merge all successive months with at least one declaration to obtain the time range in which people receive monthly care. We compare this to the indications provided by CIZ from an indications-dataset and only keep care activities that overlap with a CIZ indication.

Some overlap exists between declarations when people receive care from a new profile. This overlap is removed by using the start date of a new activity as the actual start date and changing the end date.

Geriatric Rehabilitation Dataset

As stated in Chapter 6, GR declarations are based on the *diagnosis-treatment combination (DBC)*, fixed 'packages' of care for a specific care request, containing all related care activities required for the desired treatment of a particular diagnosis. This DBC is delivered as a 'care package' (in Dutch, *zorgprestatie*), consisting of multiple activities required, such as physiotherapy, a stay at a facility, and an appointment with specialists. We determine the GR activity based on the 'stay at a facility'. As shown in Table 7.2, we combine the care packages with the care activities to determine the dates of the stay at the facility, with the corresponding cost of the GR treatment. The final selected activity is given in the table below.

A DBC package has a limit of 120 days, but it can be extended with a new DBC package. These consecutive GR stays are merged into the data.

Table 7.2: Example of merge process to obtain GR activities. Dataset 1 contains 'care packages' with the corresponding DBC code and cost. Dataset 2 contains the 'care activities' done within this package. The final row shows the data used for our final dataset. The example is made up.

Dataset 1: Care Package

Case	PackageID	DBC code	Start date	End date	Cost
1	12345	XXX-XX-XX-XXXX	2018-02-03	2018-04-05	3000

Dataset 2: Care Activities

Case	PackageID	Care activity	Date
1	12345	Stay at GR facility	2018-02-03
1	12345	Physiotherapy	2018-02-03
1	12345	Specialist geriatric medicine	2018-02-03
		...	
1	12345	Stay at GR facility	2018-04-03
1	12345	Appointment assistant	2018-04-05

Combine 1+2: Final GR activity

Case	Activity	Start date	End date	Cost
1	GR	2018-02-03	2018-04-03	3000

Short-Term Care Dataset

As mentioned before in Chapter 6, STC can be divided into *high complex*, *low complex*, and *palliative care*. High and low complex beds can be intertwined, so they are grouped as *STC*. *Palliative STC* can be provided in a hospice. Thus, this is a separate STC profile.

Hospitalizations Dataset

The hospital dataset contains information regarding hospitalizations. Almost 30 specialisms can be identified in the data, each with different groups and subgroups. For example, specialism surgery includes groups for traumatology and oncology, each with subgroups for the actual care request. We only look at general specializations.

The most commonly visited specialisms by the elderly in Amsterdam in our data are *internal medicine*, *surgery*, *cardiology*, *lung diseases*, *neurology*, *orthopedics*, *urology* and *gastroenterology (stomach, intestine, liver)*. These are considered for our model, and the other specialisms are summarized in *other departments*.

Hospitals provide different forms of care, in the CBS data separated into three types: day treatment, clinical hospitalization, and long-term observation. The first is only one day, requiring people to return home at the end of the day. In the second type, people stay at least one night. The last type, observation, is at least 4 hours, and the aim is to find a medical policy for the client. We only look at clinical hospitalization, from now on called hospitalization. The data also includes information about the urgency of an intake, but this is not used in this analysis. Additionally, there might be an overlap in the declarations, which are removed based on the start dates of an activity.

For this healthcare type, we also include activities of non-elderly people, as all people visit hospitals, and they are not primarily focused on the elderly. With this, we should obtain a more realistic throughput at the departments.

Healthcare stations in macro model

To summarize, the healthcare stations used for the macro model are shown in Table 7.3. We will call this the 'general model', as later on, we also incorporate city districts to aggregate on in the 'district model'.

Table 7.3: Healthcare stations included in macro model.

Healthcare form	Station
Long-Term Care	LTC Somatic (SOM)
Long-Term Care	LTC Psychogeriatric (PG)
Long-Term Care	LTC at Home
Long-Term Care	LTC Rehabilitation
Home Care	HC
Geriatric Rehabilitation	GR
Short-Term (Residential) Care	STC
Short-Term (Residential) Care	STC Palliative
Hospital departments	Neurology
Hospital departments	Surgery
Hospital departments	Internal Medicine
Hospital departments	Cardiology
Hospital departments	Orthopedics
Hospital departments	Urology
Hospital departments	Gastroenterology
Hospital departments	Lung Diseases
Hospital departments	Other departments

City Districts

For our 'city district' model, we assign a district to the care activities, assuming that the care activity occurred in a facility in this district. This division is made for activities from healthcare types LTC SOM, LTC PG, LTC at Home, and HC. We expect multiple nursing homes and multiple HC organizations in the city. For the other stations, we assume the care to be arranged more centrally. Thus, we only have one station per care type. This is also for simplicity reasons, even though we know that Amsterdam has multiple hospitals throughout the city.

Amsterdam consists of the following districts: *North*, *South*, *East*, *West*, *New-West*, *Southeast* and *Center*, all visualized in Figure 7.2. We can assign the district to people based on the known neighborhood in the data. People cannot move from a station in one district to a station in the other, as we made the generalization that people only live in one district during our period of interest (2017-2019).

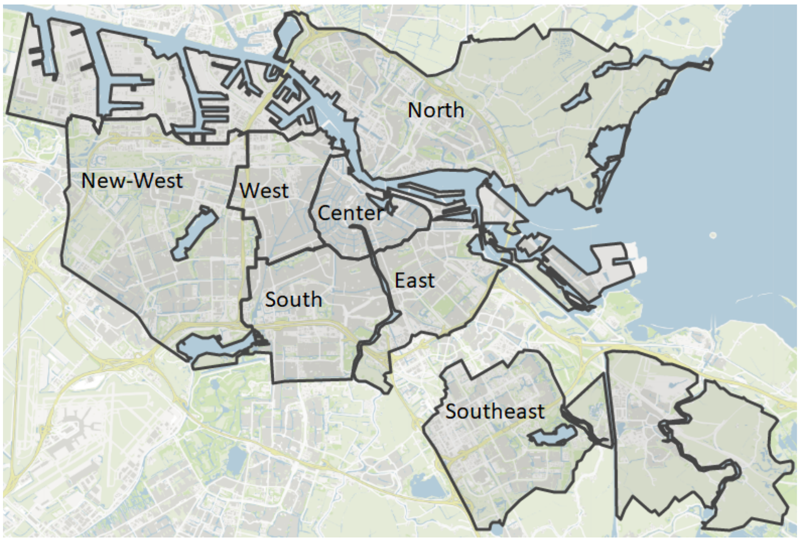
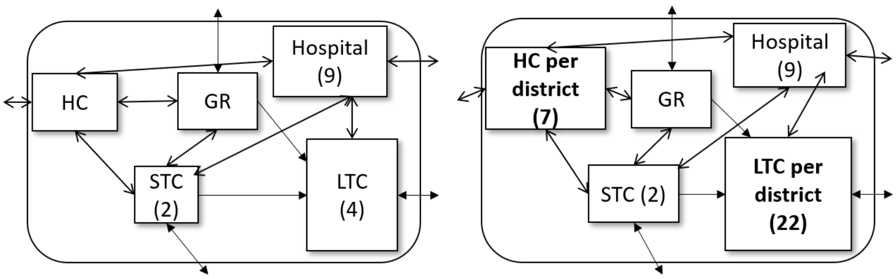


Figure 7.2: Image from `https://maps.amsterdam.nl/gebiedsindeling/` and added labels to the district incorporated in the model. Two districts are not included, the one in the far west (Westpoort) is merely an industrial district, and the one east from Southeast (Weesp) has only joined the Amsterdam municipality recently.

Thus for the city district model, we have multiple stations of the same healthcare type for each of the city districts. The stations in Table 7.3 still hold, only LTC SOM, LTC PG, LTC at Home, and HC are each replaced by stations per city district, meaning these four stations are replaced by $4 \times 7 = 28$ other stations.

Two Macro Models

With the divisions made above, we obtain two models: the general and district models. Abstractions of these models are provided in Figure 7.3. If there is a number behind the care form, then this type consists of that number of stations. Overall, the general model actually consists of 17 stations and the district model of 41 stations. But they are visualized together for simplicity.



(a) General Model. (b) District Model.

Figure 7.3: Global overview of the two macro models. If there is a number behind a care type, then this type is divided over this number of stations.

Final Dataset

For all the above datasets, we filter the healthcare activities of the people in our target group. Then we merge the datasets to obtain care paths per person. A person can have overlap in activities in the time ranges. We remove this because a person in a queueing network is in one place at a time. A ranking is created for the healthcare types to determine which care type to prefer when setting the dates correctly and removing overlap. The ranking starts at 1 (highest), up to 7, and it is as follows:

1. Passed away: activities cannot end after passing away. If this happens, the date is set to the day of passing away.
2. Hospitalization: hospitalizations are usually more expensive than other care types, and they happen after the occurrence of acute events.
3. GR: rehabilitation usually happens after hospitalization.
4. STC (STC/STC Palliative): STC can happen after GR or hospitalization, or it is used when someone temporarily needs more care.
5. LTC nursing home (SOM/PG/rehabilitation): LTC is quite expensive, and places in nursing homes are scarce.
6. LTC at Home: people choose to receive LTC care at home or wait for a place at their preferred nursing homes.
7. HC: HC for a person is less expensive than other care types.

We apply this ranking and merge the care activities with the target population dataset, containing information about people's age and living dates in Amsterdam. We use this information to remove activities that were finished before a person turned 65 years old and to remove activities that have no overlap with people's period of living in Amsterdam.

After applying all the above modifications, we obtained a final dataset containing all the care activities that the people in our sample population had within the period of our interest. We can identify the care path taken for each person, but we can also obtain all activities per care type. Table 7.4 shows an example of an instance in the final dataset. More information is provided per row, e.g., the date of turning 65, the dates of living in Amsterdam, and the date of passing away, but these are not provided in the example. The final dataset is used in the parameter estimation, discussed next.

Table 7.4: Example of the dataset with care activities. The example is made up.

Case	Activity	Start date	End date	Cost	District
1	Surgery	2019-01-13	2019-01-15	1500	South
1	GR	2019-01-16	2019-02-16	4800	South
1	HC	2019-02-17	2019-08-15	5400	South
2	LTC	2019-05-03	2019-12-31	24300	New-West

7.2 Parameter Estimation

We estimate the parameters needed for the macro model for each of the previously mentioned stations. These parameters are the average length of stay, the external arrival rates, the transition probabilities, and the cost per bed per day.

7.2.1 Length of Stay

When determining the LoS per station, we have two issues that should be taken into account: (1) we have censored data, meaning that not all activities have a known start and end date, and (2) the LoS during the Covid-19 pandemic (starting in begin 2020) might influence the LoS. The first note argues for using a method that considers censored activities, and the second issue requires us to set the last day of 2019 as the end day and to set activities ending after this day as censored.

Kaplan-Meier Estimation

Survival analysis allows the use of censored data. A commonly used method is the Kaplan-Meier (K-M) estimate, which creates a survival curve based on the data. We use this to fit a distribution to determine the average length of stay. The basics of the K-M method are shortly covered in Appendix B. The K-M formula calculates the joint probabilities for people having a LoS of at least t days.

$$S(t) = \prod_{i=1}^{N} (1 - \frac{d_s}{n_s}) \tag{7.1}$$

The formula is given in Equation (7.1), where d_s is the number of people that stayed s days before leaving the facility and n_s is the number of people that stayed at least s days at the facility, meaning that they were either discharged after s days or are still present. The K-M method is non-parametric. Thus, we must fit a parametric distribution to determine a reasonable estimate.

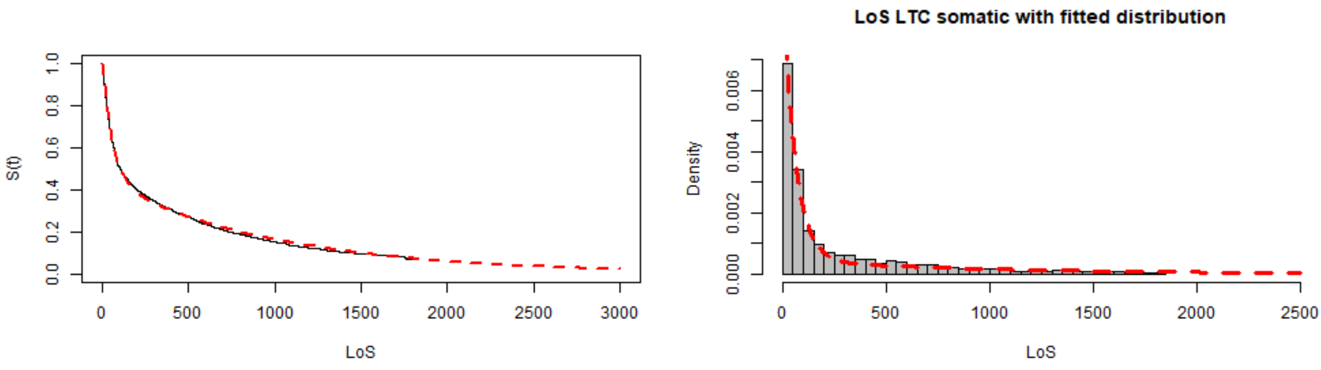
Estimating the Parameter

Fitting a parametric distribution to the K-M curve can be done, for example, by visual inspection or by using the MLE method. During the fitting process, we also proposed another way to estimate the average length of stay (*ALOS*) based on the K-M curve. Due to the censored data, we require this method to include the tail probabilities when determining this ALOS. Thus, the proposed formula will approximate the ALOS, but the calculation is not exact, since we estimate the tail distribution. This formula is given in Equation (7.2) below.

$$\mathbb{E}(S) = \int_0^\infty S(t) = \int_0^\tau S(t) + \int_\tau^\infty S(t) = \int_0^\tau S(t) + \mathbb{P}(S > \tau)\mathbb{E}(S_{\text{tail}}) \tag{7.2}$$

The first term in the formula can be obtained from the K-M curve, which is the area under this curve up to the maximum known LoS in the data. The second term entails the estimation of the tail for this K-M curve, which is unknown. We fit an exponential distribution, starting from the value at the end of the K-M curve, at $S(\tau)$. This should approximate the values in the tail, containing probabilities for the longer stays in a station. We assume that the curve $S_{\text{tail}}(t)$ for the tail is exponential because we do not expect a huge tail for the length of stay of each of the stations. Thus $S_{\text{tail}} \sim \exp(\mu_{\text{tail}})$, which makes it clear to obtain $\mathbb{E}(S_{\text{tail}})$: $P(S > \tau) = S(\tau) = \exp(-\mu_{\text{tail}}\tau)$, thus $\mu_{\text{tail}} = \frac{\log(S(\tau))}{-\tau}$ and for exponential distributions holds that $\mathbb{E}(S_{\text{tail}}) = \frac{1}{\mu_{\text{tail}}}$, which is used in Equation (7.2). Finally, we say that our estimate for the ALOS is $\mu^{-1} = \mathbb{E}(S)$ as calculated in Equation (7.2), which we use as a parameter for the service times.

The above methods are applied to the data for the different care stations. First, the LoS is determined for the data before merging the datasets and removing overlap to provide



(a) K-M Survival curve and fitted distribution. (b) Empirical and theoretical pdf of the LoS.

Figure 7.4: LTC Somatic

an estimated average LoS for the 'original data' to see if this is somewhat realistic. Then, we use the merged dataset with all care activities, without date overlap, to determine the parameter that should be used for the model.

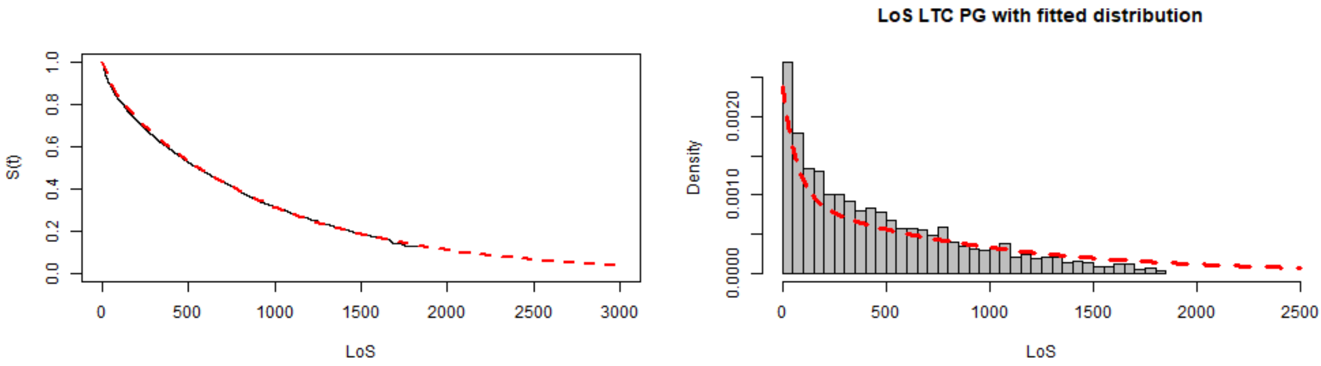
The general information on the probability distributions is provided in Appendix A.

Long-Term Care

People can stay at LTC for a long time, so we include more years of data to determine the LoS: the activities between 2015-2019 instead of activities between 2017-2019. No information is provided about start dates before 1 January 2015. Thus, activities starting from 2 January 2015 are included. The activities ending after 31 December 2019, or without a known ending, are set to end on 31 January 2019 and stored as censored data. With this data, we create the K-M survival curve for the LTC stations (LTC SOM, LTC PG, LTC at Home, LTC Rehabilitation).

First, we determine the parameters for the 'original dataset', i.e., the data where no overlap in activities has been removed. After visual inspection, we find the hyper-exponential with two phases (H-2 distribution) to be a good fitting distribution for *LTC Somatic*. An argument for this is that we expect two kinds of patient types: patients that stay for a short amount of time and those that stay longer. The first group has a rate of helping $\mu_1 = 1.67 \times 10^{-2}$ patients per bed per day, which is an ALOS of $\mu_1^{-1} = 60$ days on average. This is for 55% ($p = 0.55$) of the cases, according to the data. For the second group, we find the rate of helping $\mu_2 = 0.001$ patients per server per day, which is an ALOS of $\mu_2^{-1} = 1000$ days on average. This is for the other 45%. These parameters were found when fitting to the K-M survival curve, as shown on the right in Figure 7.4. The overall average for LTC SOM based on the fitted H-2 distribution is $\frac{p}{\mu_1} + \frac{1-p}{\mu_2} = 483$ days. This seems to fit best to the K-M curve and to the pdf of the empirical data in Figure 7.4. If we would use the method as explained in Equation (7.2), used to approximate the ALOS, we would obtain $\mathbb{E}(S) = \int_0^\tau S(t) + \mathbb{P}(S > \tau)\mathbb{E}(S_{\text{tail}}) = 402.73 + 0.07 \times 688.83 = 451.43$ days. This is not too far from our H-2 estimate, but still a difference of about 30 days. This could be due to differences in fitting, in the H-2 distribution or fitting of the tail probability in the used method. Overall, people stay on average 15-16 months in the somatic department in a nursing home.

We also fit an H-2 distribution to the *LTC Psychogeriatric* data because we expect short- and long-stay patients. For the short-stay patients, we find the rate $\mu_1 = 1.43 \times 10^{-2}$ patients per bed per day, thus an ALOS of $\mu_1^{-1} = 70$ days, and for the long-stay patients, we find rate $\mu_2 = 1.05 \times 10^{-3}$ patients per bed per day, thus an average of $\mu_2^{-1} = 950$ days. Here 10% ($p = 0.1$) belongs to the first short-stay group, whereas most seem to stay long. The fit can be seen in Figure 7.5. The overall ALOS is $\frac{p}{\mu_1} + \frac{1-p}{\mu_2} = 862$ days. The method from Equation (7.2) results in an approximated ALOS of $\mathbb{E}(S) = 729.90 + 0.13 \times 895.41 = 846.54$ days, only deviating 15 days of the H-2 average. This also suggests a reasonable estimate with this method. Thus, according to the data, people stay on average 28 months in the Psychogeriatric department in a nursing home.

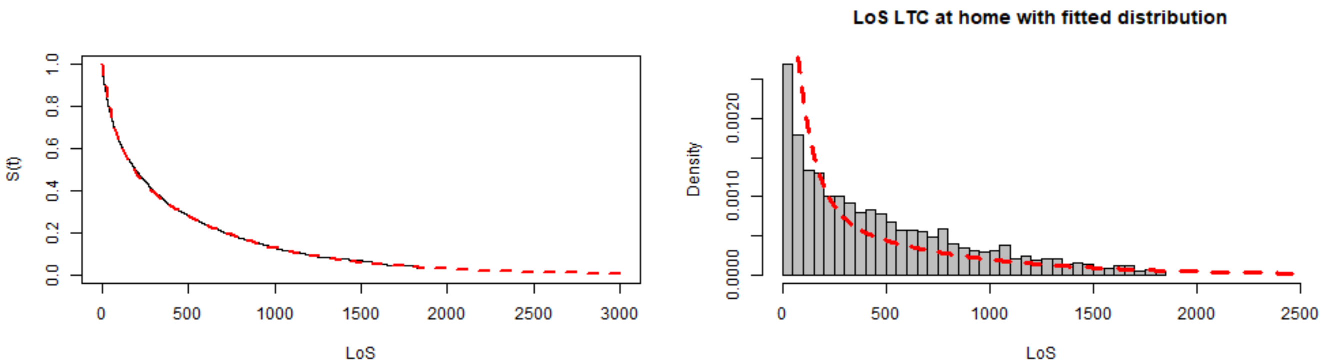


(a) K-M Survival curve and fitted distribution.

(b) Empirical and theoretical pdf of the LoS.

Figure 7.5: LTC Psychogeriatric.

The H-2 distribution also fits the *LTC at Home* data. For the short stay patients, we find rate $\mu_1 = 1.18 \times 10^{-2}$ patients per bed per day, thus the ALOS for this group is $\mu_1^{-1} = 85$ days, and for the long stay patients, we have rate $\mu_2 = 1.54 \times 10^{-3}$ patients per bed per day, meaning an ALOS of $\mu_2^{-1} = 650$ days. 40% ($p = 0.4$) belongs to the first group. The fit is visualized in Figure 7.6. The overall ALOS is $\frac{p}{\mu_1} + \frac{1-p}{\mu_2} = 424$ days. The method from Equation (7.2) results in an approximated ALOS of $\mathbb{E}(S) = 422$ days, almost the same average as for H-2. Thus, people require on average 14 months of LTC at home.



(a) K-M Survival curve and fitted distribution.

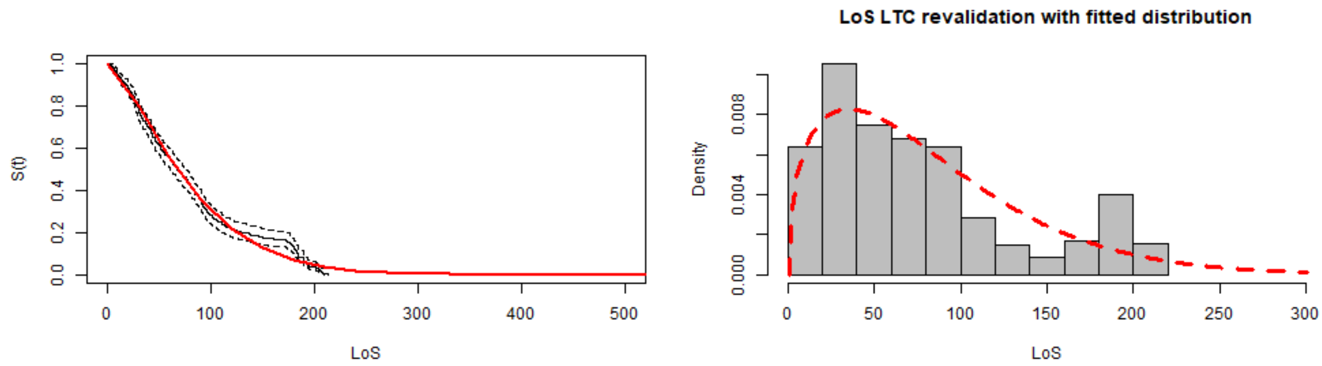
(b) Empirical and theoretical pdf of the LoS.

Figure 7.6: LTC at Home.

For *LTC Rehabilitation*, we do not expect two patient classes since the aim of this station is a short-term stay. Instead, we try to fit an exponential, Weibull, gamma, or lognormal distribution. We see that the Weibull distribution fits best on the data when looking at the fit to the K-M survival curve, the AIC, and log-likelihood (using MLE).

The K-M curve shows some interesting behavior around LoS-values of 180 days, as shown in the density plot on the right of Figure 7.7. This might be due to declaration behavior. People can stay at the LTC Rehabilitation ward for at most six months, with a possible extension of six months if required (this was found for in 2018[1] and 2019[2]) However, no evidence of this being the reason for the behavior in the density plot can be provided.

Overall, Weibull seems to have the best fit, with parameters scale $\lambda = 88.89$ and shape $k = 1.37$. This gives an ALOS of $\lambda\Gamma(1 + 1/k) = 81.26$ days for LTC Rehabilitation. The shape parameter k tells us that the failure rate is increasing over time (appendix Chapter A), meaning that the longer people are at the LTC Rehabilitation, the more likely they are to leave the station, which is also the intention of a rehabilitation period. With Equation (7.2), we obtain an approximated ALOS of $\mathbb{E}(S) = 81.30$ days, which corresponds with the Weibull obtained average. Thus, rehabilitation care at a nursing home takes on average 2.5-3 months.



(a) K-M Survival curve and fitted distribution.

(b) Empirical and theoretical pdf of the LoS.

Figure 7.7: LTC Rehabilitation.

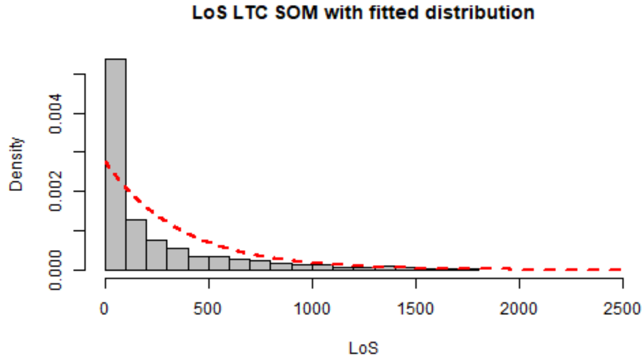
The next step is to determine the parameters for the macro model, for which we require the rate for the exponential distribution of the service time. In the fitting process above, we used the LTC data before it was merged with other care activities. In contrast, now, we will use the LTC data after merging with other activities, which means that the average length of stay might be lower due to the removal of overlap. However, this is probably balanced out by the arrival rate.

The previous results showed that the method from Equation (7.2) gave a reasonable estimated value for the ALOS compared to the *best fit* obtained visually or with MLE. Thus we use this method from Equation (7.2) to determine the macro model service rates.

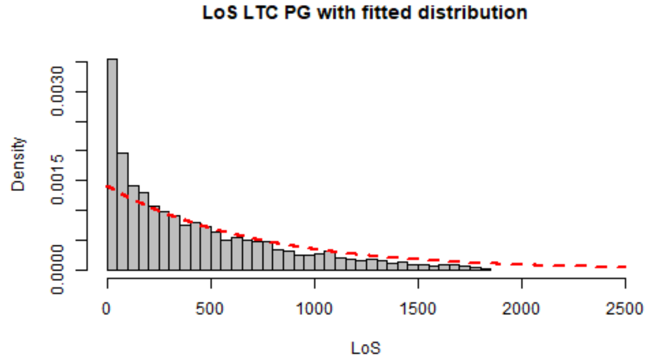
The parameters for the four sub-stations in LTC are $\mu^{-1} = 360.30$ days for LTC SOM, $\mu^{-1} = 711.25$ days for LTC PG, $\mu^{-1} = 73.19$ days for LTC Rehabilitation and $\mu^{-1} = 356.81$ days for LTC at Home. The fit of the exponential distribution with these parameters can be seen in Figure 7.8.

We also consider the LoS per city district for LTC SOM, LTC PG, and LTC at Home. A big city like Amsterdam usually has multiple nursing homes working independently, and we assume each district has one nursing home, helping all care demands from that district. Thus we use the method in Equation (7.2) to approximate the ALOS for these

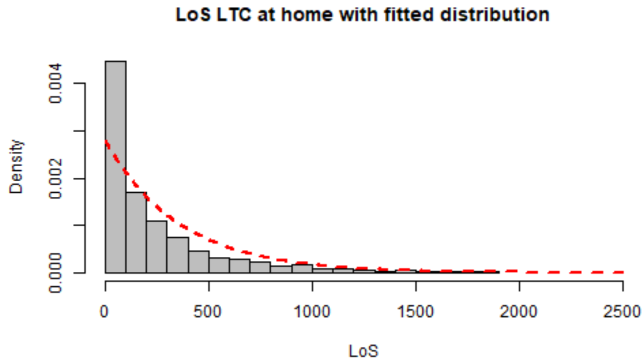
[1]https://wetten.overheid.nl/BWBR0040327/2018-01-01
[2]https://wetten.overheid.nl/BWBR0041631/2019-01-01/0



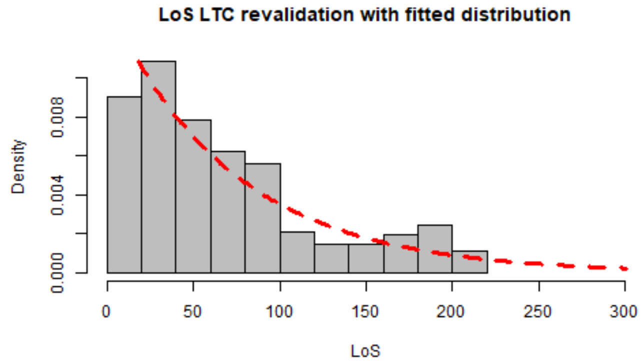
(a) LTC Somatic.



(b) LTC Psychogeriatric.



(c) LTC at Home.



(d) LTC rehabilitation.

Figure 7.8: Empirical pdf of LoS data with fitted exponential distribution.

as well, resulting in the parameters as given in Table 7.5. These values are similar, with some outlying values for LTC SOM in Amsterdam-East, LTC PG in Amsterdam South-East, and LTC at Home in Amsterdam Center.

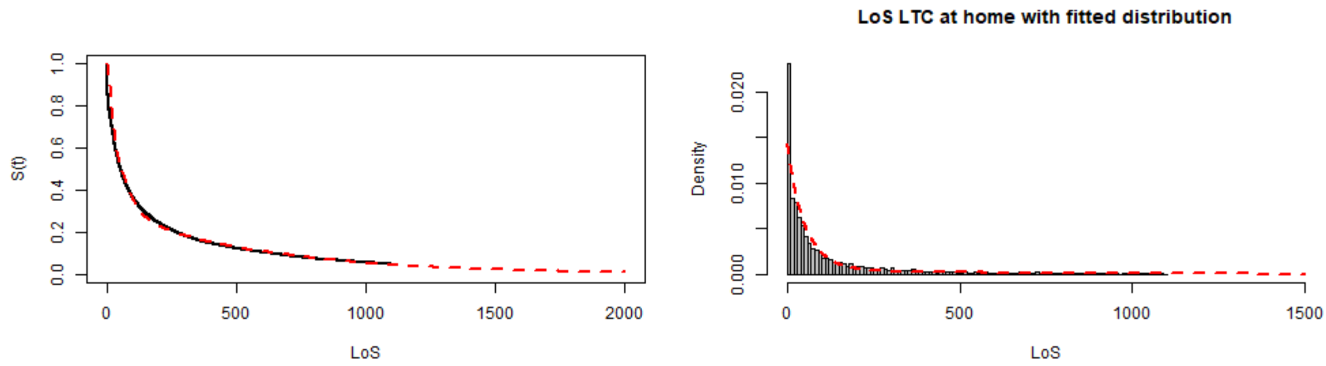
Table 7.5: ALOS obtained for LTC facilities per city district, parameters for the macro model.

District	LTC SOM	LTC PG	LTC at Home
North	375.00	707.78	356.29
South	324.19	663.37	345.57
West	364.96	704.48	351.74
New-West	347.15	712.19	375.92
Center	313.64	702.71	417.03
East	433.30	762.92	335.74
South-East	390.02	847.48	321.16

Home Care

For the previous section, activities with start dates between 2 January 2015 and 31 December 2019 were used to estimate the ALOS. However, for HC (and other non-LTC stations), we use activities starting from 2 January 2017. These are used in the K-M method.

We try to fit an H-2 distribution for HC, as we expect short- and long-stay patients. After visual inspection, the H-2 seems to fit well, with rate $\mu_1 = 0.02$ patients served per bed per day, which gives an ALOS of $\mu_1^{-1} = 50$ days for the short stay patients and rate $\mu_2 = 1.72 \times 10^{-3}$ patients per bed per day, which gives an ALOS of $\mu_2^{-1} = 580$ days for the long-stay patients. We expect people to be short-stay patients with a probability of 69% ($p = 0.69$). Thus the total ALOS is $\frac{p}{\mu_1} + \frac{1-p}{\mu_2} = 214.3$ days. The data and the fit are shown in Figure 7.9. The method from Equation (7.2) results in an approximation of the ALOS of $\mathbb{E}(S) = 182.94 + 0.05 \times 366.79 = 201.53$ days, deviating about 14 days from the H-2 estimate. Overall, people obtain an average of 6.5-7 months of home care.



(a) K-M Survival curve and fitted distribution.

(b) Empirical and theoretical pdf of the length of stay.

Figure 7.9: HC.

The method from Equation (7.2) is used to find the parameter for the HC station from the merged dataset, which is used for the exponential service distribution for the macro model. The ALOS is approximated as $\mu^{-1} = 162.06$ days for HC, and the fit is provided in Figure 7.10.

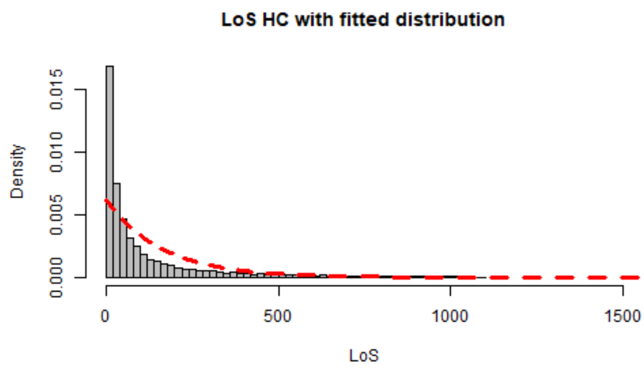


Figure 7.10: Empirical and theoretical pdf of the LoS for HC.

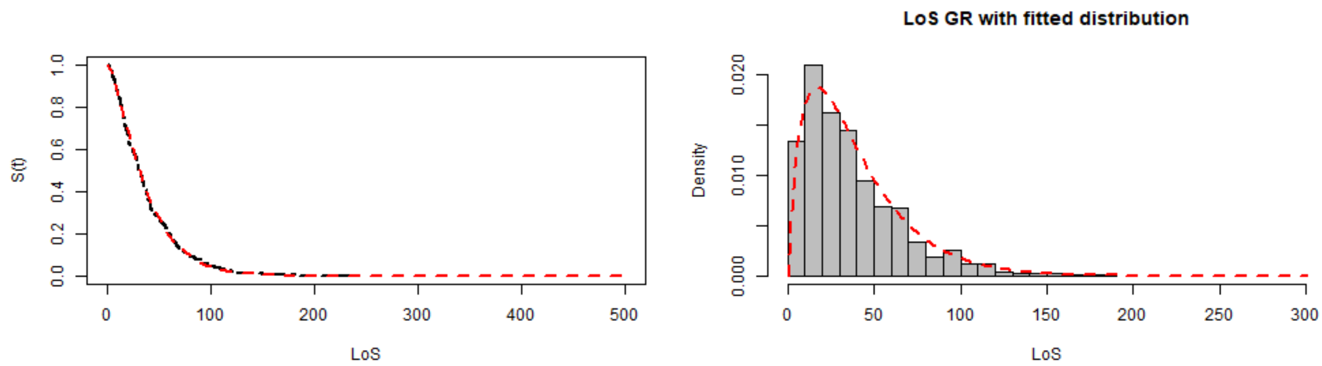
Finally, we also find the HC ALOS parameters per Amsterdam district, again with the method from Equation (7.2), given in Table 7.6. These seem much lower than the previously fitted distribution for the non-merged data. A likely reason for this is that HC ranks lowest when merging activities and removing overlap with other activities. Hospitalizations and visits to other care forms during an HC activity result in this HC activity being split up, resulting in a lower ALOS.

Table 7.6: ALOS in days obtained for HC per city district. Parameters for the macro model.

District	ALOS
North	158.41
South	162.11
West	155.34
New-West	172.36
Center	132.81
East	152.27
South-East	191.15

Geriatric Rehabilitation

The LoS for GR is estimated using the K-M method for the activities between 2 January 2017 and 31 December 2019, including some censored data points. We do not expect two patient classes due to the temporal use of GR. Thus, we fit an exponential, Weibull, gamma, or lognormal distribution. The gamma distribution fits the data best when visually investigating the fit to the K-M survival curve, along with the AIC and log-likelihood (using MLE). This can be seen in Figure 7.7. The gamma parameters are shape $\alpha = 1.71$ and rate $\beta = 4.44 \times 10^{-2}$, such that the ALOS is $\frac{\alpha}{\beta} = 38.74$ days for GR. Thus, patients at GR in Amsterdam have an ALOS for 38-39 days. With the method from Equation (7.2), we obtain an estimate of $\mathbb{E}(S) = 38.78$, similar to the gamma estimate.



(a) K-M Survival curve and fitted distribution.

(b) Empirical and theoretical pdf of the LoS.

Figure 7.11: Geriatric Rehabilitation.

The method from Equation (7.2) determines the parameter for the ALOS of GR for the macro model using the merged dataset. This is approximated as 36.53 days. The fit is visualized in Figure 7.12.

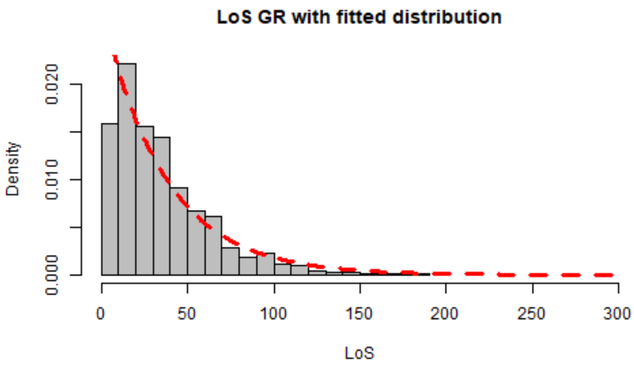
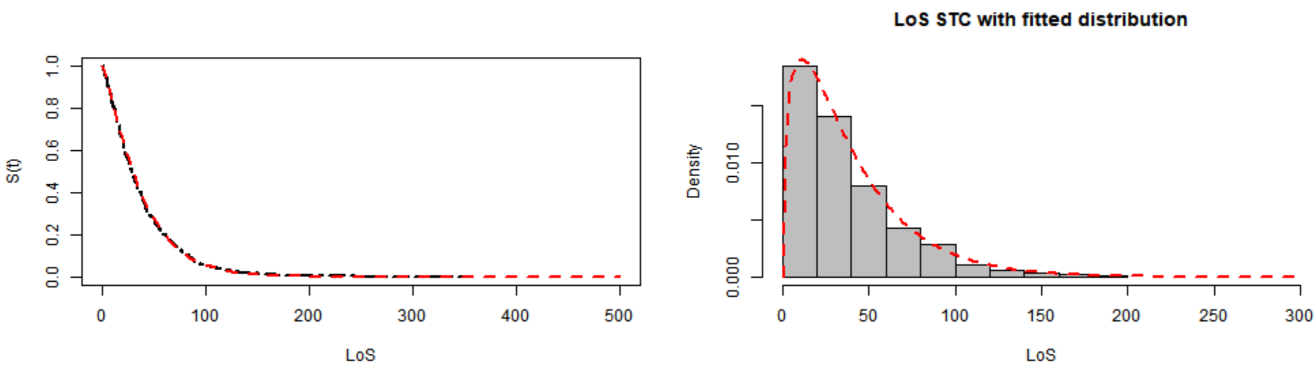


Figure 7.12: Empirical and theoretical pdf of the LoS determined by ALOS, for Geriatric Rehabilitation.

Short-Term Care

Using the K-M method, the STC and STC Palliative LoS are also estimated from activities starting between 2 January 2017 and 31 December 2019.

For *STC*, we have a similar approach as for GR, and we find that the gamma distribution fits best after visual inspection and comparing the AIC- and loglikelihood-scores. The parameters are shape $\alpha = 1.37$ and rate $\beta = 3.53 \times 10^{-2}$, so the ALOS is calculated to be $\frac{\alpha}{\beta} = 38.69$ days for STC. The K-M curve and fit on the empirical data are shown in Figure 7.13. The method from Equation (7.2) approximates the ALOS to be $\mathbb{E}(S) = 38.74$ days for STC, almost equal to the average found with the gamma distribution. This ALOS is also similar to GR.



(a) K-M Survival curve and fitted distribution.

(b) Empirical and theoretical pdf of the LoS.

Figure 7.13: Short-Term (Residential) Care.

The lognormal distribution fits best for *STC Palliative* after visual inspection and comparing AIC- and loglikelihood-scores. The data is skewed, such that a group of people with longer lengths of stay is in the distribution's tail. The parameters are meanlog $\mu = 2.65$ and sdlog $\sigma = 1.27$, leading to an overall average Los of $\exp(\mu + \frac{\sigma^2}{2}) = 31.59$ days for STC palliative. The K-M and fitted survival curves can be seen in Figure 7.14; unfortunately, the fitted distribution is missing due to privacy rules required by the CBS data. The method from Equation (7.2) gives an approximate ALOS of $\mathbb{E}(S) = 31.38$ days, the same as the lognormal estimate.

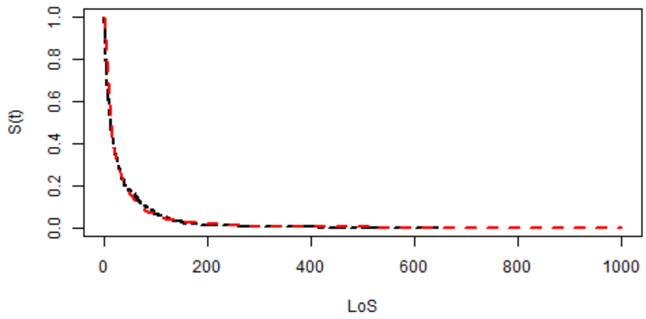


Figure 7.14: K-M Survival curve and fitted distribution of the LoS for STC palliative.

For the merged dataset, we also find the parameters for the exponential service distribution of the STC stations using Equation (7.2). The approximated ALOS for STC and STC palliative are 36.53 days and 30.95 days, respectively. The fit is visualized in Figure 7.15 for STC, not for STC Palliative, due to the privacy rules of CBS.

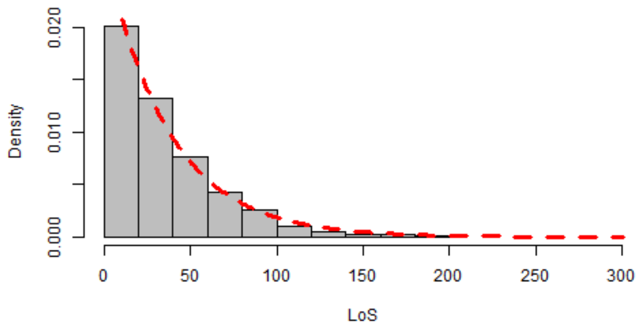


Figure 7.15: Empirical and theoretical pdf of the LoS for STC (not palliative).

Hospital departments

The LoS values of the hospital departments are estimated with K-M survival curves based on activities with start dates between 2 January 2017 and 31 December 2019. Activities of non-elderly people are included, next to those of 65+-year-old people. Thus, the estimated length of stay is not solely for the elderly. The following hospital departments are included: internal medicine, surgery, cardiology, lung diseases, neurology, orthopedics, urology, and Gastroenterology (stomach, intestine, liver).

We considered the original data and the merged dataset in the previous stations, LTC, GR, STC, and HC. However, we only use the merged datasets to estimate the LoS of hospital departments because hospitalizations were the highest in the ranking when removing overlap. Thus, no differences with hospitalizations' 'original data' should exist. Table 7.7 shows the parameters for the best fitting distribution to the data, which is the lognormal distribution. This means that most people have an LoS around the same value, whereas a few people have a higher LoS, explained by the tail of the distribution. The table also includes the calculated mean for the lognormal parameters. Table 7.7 provides the parameters for the exponential distribution, calculated using Equation (7.2), as was also done for the other stations. These parameters are similar to the estimated averages with the lognormal distribution but not exactly the same.

Table 7.7: Parameters (meanlog and sdlog) and calculated mean for the (best fitting) lognormal distributions for the LoS of hospital departments for elderly and non-elderly people mixed. On the right, the ALOS in days is provided, approximated by Equation (7.2), which is used for the exponential service time parameter for the macro model. Activities of elderly and non-elderly people are combined.

	Lognormal parameters			Parameters macro model
Station	meanlog	sdlog	mean	ALOS
Neurology	1.31	0.83	5.20	5.82
Surgery	1.45	0.79	5.82	6.27
Internal medicine	1.61	0.84	7.07	7.44
Cardiology	1.45	0.80	5.84	6.22
Lung diseases	1.61	0.73	6.57	6.83
Orthopedics	1.33	0.66	4.67	4.99
Urology	1.16	0.58	3.77	4.00
Gastroenterology	1.37	0.76	5.25	5.77
Other	1.27	0.72	4.62	5.36

7.2.2 Arrival Rates

The stations' external arrival rates λ are determined here and calculated using the *merged dataset* containing the care paths of older aged Amsterdam citizens. External arrivals are those from outside the system; thus, for each station, we select the activities with no leading activities. In our case, *no leading activity* means no other activity was discovered in the data in the *last four weeks*. The arriving patients are considered on a *weekly level*, as this aggregation removes the variation we might get for weekdays and weekends. We fit a Poisson distribution to this data, for which the mean should equal the variance. Thus the Variance-to-Mean-Ratio (VMR) should be approximately 1. With the arrival rate per week, we can find the arrival rate per day, our required λ.

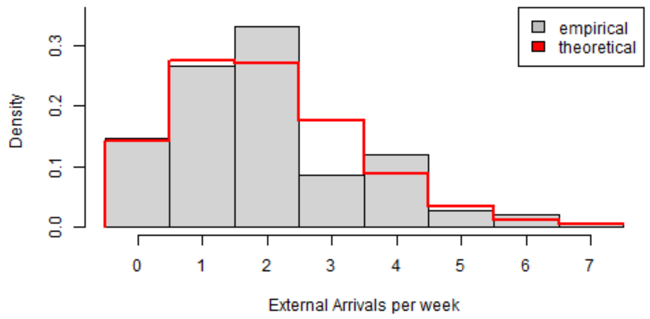
Long-Term Care

The empirical and fitted (Poisson) arrival distributions of the LTC SOM, LTC PG, and LTC at Home are visualized in Figure 7.16. For LTC SOM, LTC PG, and PTC at Home, some outliers were removed, respectively 1, 1, and 2, as these very high numbers seemed to be a deviation due to declaration behavior. No figures are provided to support this due to privacy rules for data visualization. Also, no figure is provided for LTC Rehabilitation due to the privacy rules of CBS for the visualization. The corresponding parameters are given in Table 7.8. For LTC SOM and LTC Rehabilitation, the VMR is close to 1, suggesting a good fit for Poisson. For LTC PG and LTC at Home, the VMR is slightly higher, as these stations have more variability in external arrivals. LTC at Home has the most external arrivals, followed by LTC PG. An argument could be that people with an LTC indication are expected to start with care from LTC at Home while waiting for a place at the preferred nursing home. The mean arrival rate for LTC Rehabilitation is low; a possible explanation is that most people arriving at this station come from hospitalization because they need to rehabilitate after surgery or another procedure.

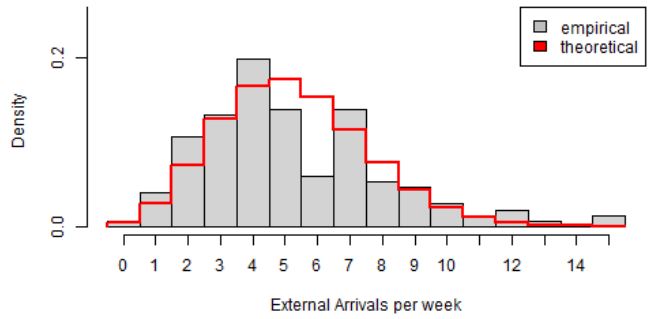
Next, the LTC stations are divided over city districts, leading to different arrival rates per LTC station per district (for SOM, PG, and Rehabilitation). These rates are provided in the appendix Chapter C in Tables C.1 to C.3.

Table 7.8: External arrival rate per week for LTC stations, with the mean, variance, and Variance-to-Mean-Ratio (VMR).

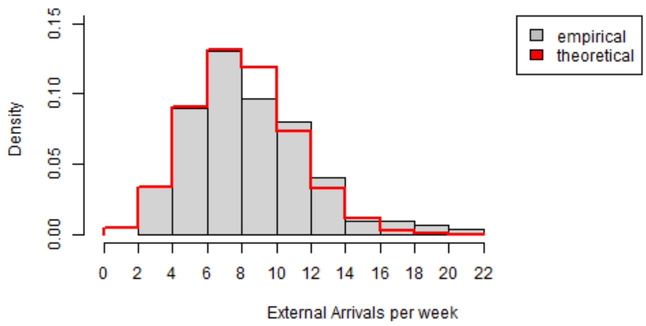
Station	mean	var	VMR
LTC SOM	1.96	2.15	1.09
LTC PG	5.25	8.17	1.56
LTC at Home	9.01	12.44	1.38
LTC Rehabilitation	0.11	0.34	1.01



(a) LTC Somatic.



(b) LTC Psychogeriatric.



(c) LTC at Home.

Figure 7.16: Empirical and theoretical pdf of external arrivals per week. The average of the empirical data is used to fit the theoretical Poisson distribution.

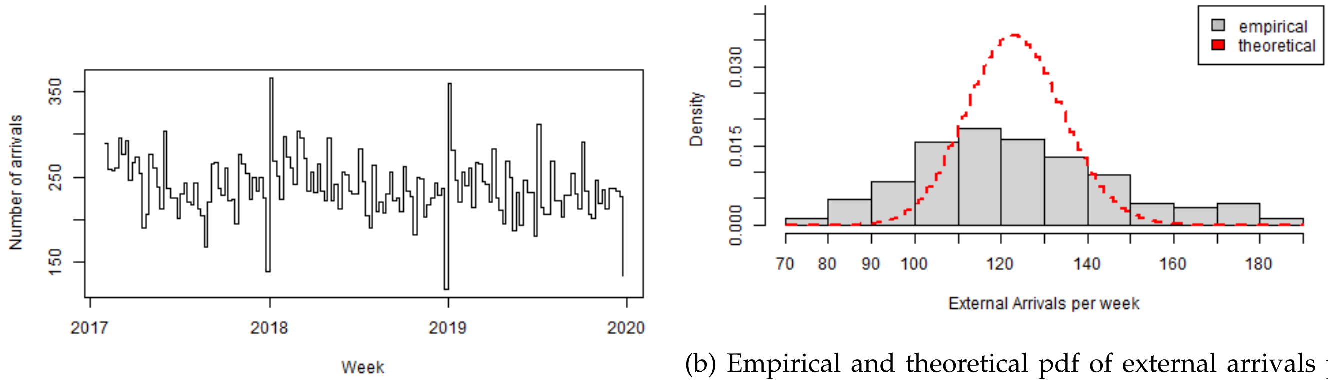
Home Care

Figure 7.17 shows the arrival distributions of the HC station. The figure on the left shows the weekly arrivals over time. Two deviating peaks are seen at the start and end of the years, possibly due to declaration behavior. Thus, two weeks with many external arrivals and two with only a few. These outliers are removed before calculating the mean arrival rate. This results in the arrival distribution as provided in the figure on the right, showing the empirical data and the fitted Poisson distribution. The corresponding parameters are given in Table 7.9. The VMR is 4.23, which suggests that this arrival process is not Poisson. Considering the arrivals per season does not improve the VMR. Possibly, there is some non-identifiable declaration behavior in the data. This might be due to different declaration styles by various HC organizations, but it cannot be verified. However, we will use the mean as a parameter for the Poisson distribution of HC arrivals.

Table 7.9 shows that, on average, 123.97 people arrive at HC per week from outside the system, meaning they did not receive any other care in the last four weeks. This seems high but could be explained by HC being a commonly used care type, as it should help people live at home as long as possible. The declaration style of HC activities could be a reason for this high arrival rate, as we decided to merge individual HC declarations on a monthly level, which means that people that have a gap of a month in their HC declarations are likely to leave and return to the system multiple times, and thus contribute to the high arrival rate. These are assumptions and have not been verified.

Table 7.9: External arrival rate per week for HC, with the mean, variance, and Variance-to-Mean-Ratio (VMR).

Station	mean	var	VMR
HC	123.97	524.97	4.23



(a) Arrivals per week over time.

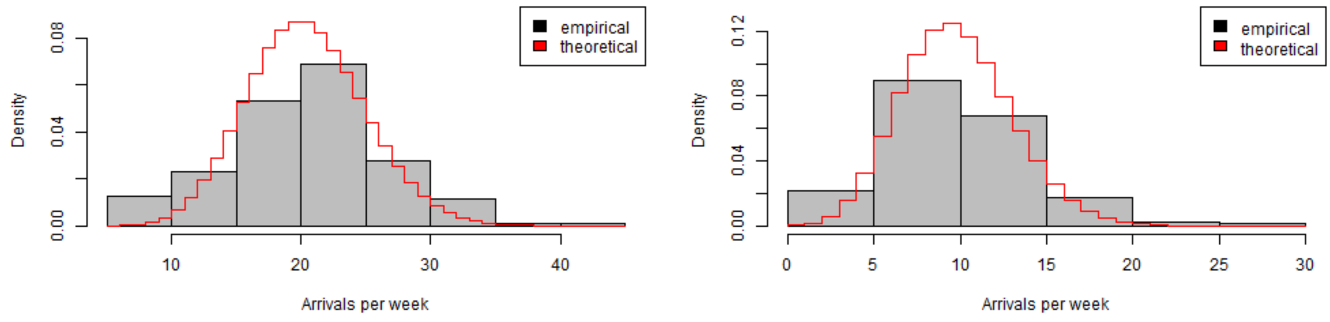
(b) Empirical and theoretical pdf of external arrivals per week. The average of the empirical data is used to fit the theoretical Poisson distribution.

Figure 7.17: Home Care.

Next, we consider the HC stations at the district level to obtain HC arrival rates for Amsterdam North, West, East, South, South-East, New-West, and Center. The parameters for the districts are provided in the appendix Chapter C in Table C.4. The results seem to have improved slightly since the VMR for all stations decreased compared to the VMR in Table 7.9. However, they are still around 2 and 3. Thus, there is still a lot of variability, which makes it hard to assume a Poisson distribution. Figure 7.18 shows figures for the external arrival processes for Amsterdam North and Center.

Geriatric Rehabilitation

The arrival distribution at the GR station is shown in Figure 7.19. The corresponding parameters are provided in Table 7.10. The fit is imperfect, as VMR is closer to 2 than 1; however, as shown by the plot, the data is reasonably estimated with the Poisson distribution. The difference can mostly be found in the bin (in the figure) for 0 arrivals, which is higher for the empirical data than for the theoretical fit. Most weeks have either 0, 1, or 2 external arrivals, possibly because most people receive GR care at a facility immediately after leaving the hospital, as they have to rehabilitate from a hospitalization. This could be underlined by the transition probabilities going to GR, as provided in the next section.



(a) HC in Amsterdam North.

(b) HC in Amsterdam Center.

Figure 7.18: Empirical and theoretical pdf of external arrivals per week. The average of the empirical data is used to fit the theoretical Poisson distribution.

Table 7.10: External arrival rate per week for GR, with the mean, variance, and Variance-to-Mean-Ratio (VMR).

Station	mean	var	VMR
GR	2.29	3.72	1.62

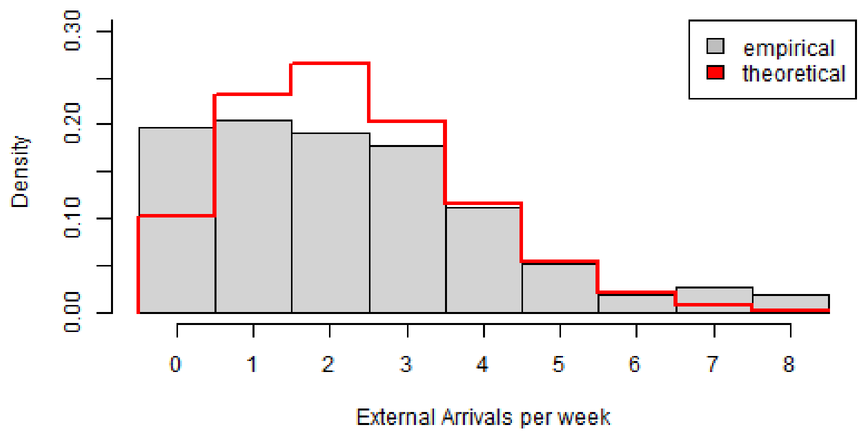


Figure 7.19: Empirical and theoretical pdf of external arrivals per week for GR. The average of the empirical data is used to fit the theoretical Poisson distribution.

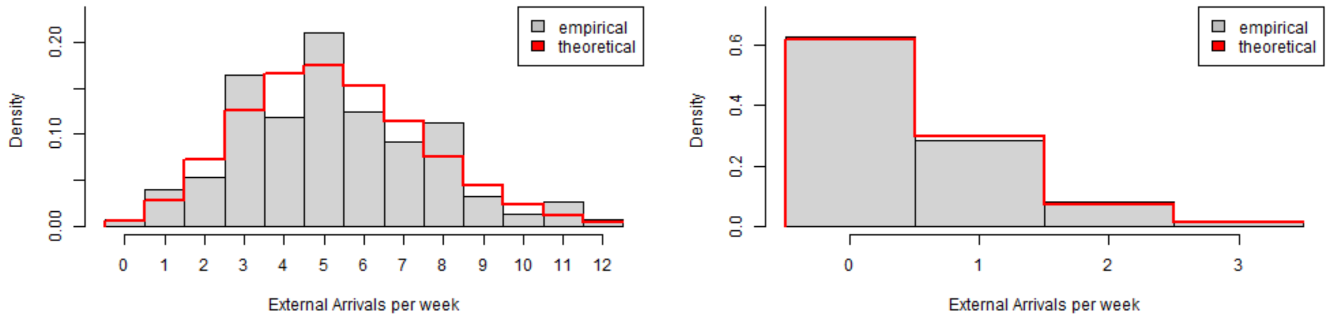
Short-Term Care

The arrival distributions of the STC stations are shown in Figure 7.20. The corresponding parameters are provided in table Table 7.11. The VMR values for both stations are very close to 1, suggesting a good fit of the Poisson distributions. The figures underline this, as the empirical and theoretical distributions are similar, which is the case for both STC stations.

The arrival rate is relatively low for STC Palliative, probably because it is a station with low demand. Palliative care is usually provided after other care forms. Thus, most arrivals should come from other stations and not as external arrivals. Besides this, palliative care for LTC patients differs from STC Palliative and is a surcharge in LTC care. Hence, the demand for palliative care can be found in different care types. This could be why this specific care has such low demand.

Table 7.11: External arrival rate per week for STC facilities, with the mean, variance, and Variance-to-Mean-Ratio (VMR).

Station	mean	var	VMR
STC	5.25	5.65	1.08
STC palliative	0.48	0.49	1.02



(a) STC.

(b) STC Palliative.

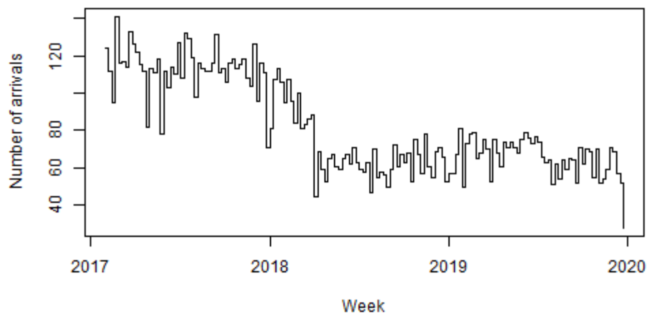
Figure 7.20: Empirical and theoretical pdf of external arrivals per week. The average of the empirical data is used to fit the theoretical Poisson distribution.

Hospital Departments

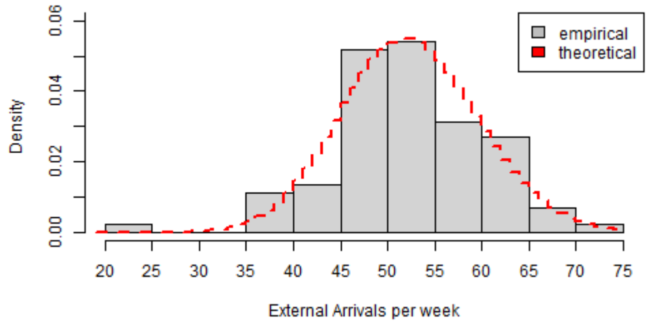
As explained in the LoS section, we include non-elderly people's activities for the hospital departments. The arrival distributions of the hospital departments are shown in Figure 7.21 for Neurology, and for some of the other departments in Appendix C in Figures C.1 to C.5. All the figures on the left show the weekly arrivals over time, where we identify multiple arrival processes. The process in the weeks until the first quarter of 2018 seems to differ from the process after this first quarter of 2018. A possible explanation for this could be a policy change in hospitalizations. However, no evidence is provided. We use the last process to estimate the average arrival rate for Neurology, Surgery, Internal Medicine, Lung diseases, Orthopedics, and 'Other' departments. The whole period is included when deciding on the arrival rates for the rest of the hospital departments, i.e., Cardiology, Urology, and Gastroenterology because we cannot identify multiple processes here. The empirical and fitted Poisson pdf can be seen on the right in Figure 7.21 and in Figures C.1 to C.5. The corresponding parameters are provided in Table 7.12. Some stations have a VMR of around 1; for others, this is about 2, e.g., Surgery, Cardiology, and Orthopedics. The closer the VMR is to 1, the better the fit to the Poisson distribution. Neurology, Internal Medicine, Urology, and Gastroenterology seem reasonable enough to assume a Poisson distribution. The other stations still have a bit too much variability.

Table 7.12: External arrival rate per week for the hospital departments, with the mean, variance, and Variance-to-Mean-Ratio (VMR).

Station	mean	var	VMR
Neurology	53.02	64.90	1.22
Surgery	128.73	290.51	2.26
Internal Medicine	87.23	101.39	1.16
Cardiology	76.43	149.70	1.96
Lung diseases	42.77	73.24	1.71
Orthopedics	41.57	93.24	2.24
Urology	39.70	52.17	1.31
Gastroenterology	27.43	34.55	1.26
Other	223.16	413.01	1.85



(a) Arrivals per week over time.



(b) Empirical and theoretical pdf of external arrivals per week. The average of the empirical data is used to fit the theoretical Poisson distribution.

Figure 7.21: Hospital department Neurology.

7.2.3 Transition Probabilities

The transitions between different stations are estimated as the transition probabilities, usually provided in a matrix as P, where each cell p_{ij} is the probability of going from station i to station j. A transition happens when a person starts at the next station at most four weeks later; else, the transition is set to 'leaving the system'.

For each activity, we create a subset in which we know the current activity i and the next activity j if the start state of j is at most four weeks later than the end date i. For each next activity j, we calculate how often people arrive here from i. Based on these quantities we calculate the transition probabilities from station i to j.

The transition probabilities are provided in Appendix D. The transitions for the general model are provided in Tables D.1 to D.2 and for the district model in Tables D.3 to D.6. In some cases, the counts underlying the probabilities for the district model were too small, so we had to round some cells up or down, giving slightly deviating results.

For the general model, some transitions that stand out are that 30% of GR return home to receive HC. Approximately the same holds for STC, where this is about 26%. From LTC at Home, around 24% seems to go to LTC PG, whereas only about 6% transition to LTC SOM. From LTC Rehabilitation, the transitions to LTC SOM, LTC PG, or LTC Rehabilitation are equally likely, around 18%.

7.2.4 Bed Cost

Finally, we find the average cost per bed per day for each station, provided in Table 7.13. These are estimates from the CBS data, where we calculate the daily cost per bed by dividing the total declaration cost for an activity by the number of days this activity took place. This is done for 2019, the most recent year in the data.

No declared costs were available for the hospitalizations. Thus we set it to 800, an estimation by Actiz[3], an organization that represents many healthcare institutions.

Table 7.13: Cost per bed per station.

Station	Cost	Station	Cost
LTC SOM	243	Neurology	800
LTC PG	242	Surgery	800
LTC at Home	91	Internal medicine	800
LTC rehabilitation	269	Cardiology	800
HC	61	Lung diseases	800
GR	563	Orthopedics	800
STC	227	Urology	800
STC Palliative	329	Gastroenterology	800
		Hospital Other	800

Summary

The stations in the macro model for Amsterdam are Long-Term Care, Short-Term Care, Geriatric Care, Home Care, and some Hospital Departments.
First, we determine the distributions for the length of stay. Due to censored data, the length of stay of each care form was estimated using the Kaplan-Meier survival curve. When fitting this curve, we discovered that a hyper-exponential distribution with two phases best describes the length of stay for LTC Somatic, LTC Psychogeriatric, LTC at Home, and HC. These stations have two types of patients: people with a short visit and people with an extended stay. The latter especially holds for Psychogeriatric patients. The other stations, i.e., STC, GR, LTC Rehabilitation, and hospitalization, are better described by a Weibull, gamma, or lognormal distribution.
The gathered data were merged into a dataset containing the care paths of elderly people. With this, we determined the parameters required for the macro model, i.e., the Jackson Network. Using the K-M curve, we approximated the average length of stay (ALOS), the external arrivals, and the transition probabilities.
Overall the arrival rate seemed to be approximately following a Poisson distribution. However, this was not plausible for HC due to a high VMR. Considering the HC on the district level already showed some improvements. The mean arrival rate is used for our cause, but additional analysis with possibly other data sources would be preferable.

[3]https://fluent.nl/publicatie/jaarlijks-ruim-300-000-ouderen-onnodig-in-ziekenhuisbedden/

Chapter 8

Results Case Study

This chapter determines the optimal capacity allocation over the elderly care system of Amsterdam based on the Jackson Network model. The parameters are obtained in the case study in the previous chapter. The assignment is performed with the optimization models proposed in chapter 4. The first method proposed is the Capacity Allocation Program (CAP), which is not used due to computational time. However, we determine the capacity based on Marginal Allocation (MA), and we also consider Square-Root Staffing (SRS) to show differences in allocation.

The section contains two analyses, one for the *general model* in which we consider the allocations for MA and SRS. And one for a comparison between the *general model*, in which the healthcare types are from a centralized perspective, and one for the *district model*, in which LTC and HC healthcare types are split up per city district.

8.1 General Model

With the estimated length of stay, arrival rates, transition probabilities, and cost per bed for the general model, we find the (daily) effective arrival rate, which is γ, together with the offered load per station a. These are provided in Table 8.1, and the transition probabilities can be found in Tables D.1 to D.2 in Appendix D. The minimum number of places required for stability is x_{stable}, for which we can determine the minimum needed total budget per station per day. The system requires a total of *11,369* beds for a stable situation.

Table 8.1 shows that HC has the highest offered load since the ALOS and the external arrival rates are relatively high. This coincides with HC's policy that this care form should help people living at home longer. Additionally, the transition probabilities in Table D.1 in Appendix D show that the percentage of people transitioning from GR, STC, and the hospital departments Internal Medicine, Lung Diseases, and Cardiology to HC is relatively high, respectively around 30%, 25%, and around 14-16% for the mentioned hospital departments. The hospital departments have the highest external arrival rate here, so we expect these to influence the HC effective arrivals the most. After HC, LTC PG has the second highest offered load, followed by LTC at Home and LTC SOM. These stations have a longer ALOS than the others, which is expected since people stay at LTC for the rest of their lives. STC Palliative has the lowest offer load, probably because the ALOS and the external arrival rates are relatively low, as are the transition probabilities to this station. It is specific care in the final stages of people's lives.

We define the budget that we wish to divide over the stations before running the capacity allocation programs. This budget should meet the stability requirement, for which we sum up the cost per day in Table 8.1, resulting in a total budget of *2,041,377* needed per day. Due to simplicity, we use a budget per day instead of per year because

Table 8.1: On the left, for each station, the estimated parameters external arrival rate λ per day, ALOS μ^{-1} in days, cost c per day. With the transition probabilities (Tables D.1 to D.2) we obtain effective arrival rates γ per day, offered load a, the required number of beds for stability x_{stable}, and the corresponding total daily cost.

Station	μ^{-1}	λ	c	γ	a	x_{stable}	cost p/day
GR	36.53	0.33	563	5.28	192.9	193	108659
STC	37.15	0.75	227	3.2	119.07	120	27240
STC Palliative	30.95	0.07	329	0.47	14.69	15	4935
HC	162.06	17.71	61	33.4	5412.61	5413	330193
LTC SOM	360.3	0.28	243	1.95	702.91	703	170829
LTC PG	711.24	0.75	242	3.48	2476.56	2477	599434
LTC at Home	356.81	1.29	91	4.53	1617.7	1618	147238
LTC Rehabilitation	73.19	0.02	269	0.28	20.23	21	5649
Cardiology	6.22	10.92	800	14.39	89.55	90	72000
Surgery	6.27	18.39	800	23.19	145.49	146	116800
Neurology	5.82	7.57	800	9.34	54.34	55	44000
Urology	4.0	5.67	800	7.19	28.77	29	23200
Orthopedics	4.99	5.94	800	6.96	34.76	35	28000
Internal Medicine	7.44	12.46	800	21.19	157.59	158	126400
Lung Diseases	6.83	6.11	800	9.51	64.93	65	52000
Gastroenterology	5.77	3.92	800	5.48	31.63	32	25600
Other (hospital)	5.36	31.88	800	37.0	198.26	199	159200

the relative proportions are still present. Multiply the daily budget by 365 days for the total yearly budget. The daily budget is created by adding 1% and 5% to the budget for stability. A multiple of budgets is used to identify differences between the methods.

The suggested capacity per location and the resulting waiting time in days are provided in Table 8.3 for both MA and SRS. For SRS, the waiting time was calculated for the allocation after the optimization, as the SRS method makes decisions with the Quality of Service parameter β. Table 8.2 shows the relative changes in the capacity allocation Δx compared to the required x_{stable} for the stable case, together with the decrease in waiting time ΔW compared to the waiting time for stability W_{stable}. Below we discuss the results by MA and SRS.

Marginal Allocation

Table 8.2 shows that the MA, when allocating the +1% budget, already improves the waiting times to a great extent, compared to the stable situation. The allocation of the +5% budget further improves the waiting times, but the difference is negligible. MA allocates most beds to locations with the highest waiting time W_{stable}, primarily to HC, LTC at Home, and LTC PG, which also seem to have the highest decrease in waiting time. In addition, GR seems to have a high reduction in waiting time with only a few additional beds. For larger budgets, +5%, MA also allocates beds to the hospital stations, where a slight increase in beds significantly influences the waiting time. However, most beds are assigned to HC, LTC at Home, and LTC PG, followed by LTC SOM.

Square-Root Staffing

We concluded in Chapter 5 that SRS differs from MA, as it overall allocates fewer beds. In Table 8.3, we can see that for the budget +1%, the beds are more spread out over all stations than MA. HC and LTC at Home receive three times as few beds as MA. This becomes a factor two difference for the higher budget +5%. SRS aims to equally increase the Quality of Service, which depends on the offered load of a station.

Last, the SRS method solved the instances much faster than MA. This was for +1% and +5%, respectively, 0.82 and 0.62 seconds, whereas MA took 462.16 and 633.28 seconds. However, the computational time is not the most important for this model, as it is not required to run this frequently or quickly.

Table 8.2: The increase in capacity Δx beds with the decrease in waiting time ΔW in days compared to capacity x_{stable} and waiting time W_{stable} for the stable system. This is done for budget for stability +1% and +5%, for MA and SRS.

Increase in capacity Δx and decrease in waiting time ΔW compared to x_{stable} and W_{stable}										
	Stability		+1%				+5%			
			MA		SRS		MA		SRS	
Station	x_{stable}	W_{stable}	Δx	ΔW	Δx	ΔW	Δx	ΔW	Δx	ΔW
GR	193	347.66	3	338.81	3	338.81	13	346.94	14	347.07
STC	120	35.9	2	26.91	2	26.91	13	35.51	10	35.1
STC Palliative	15	89.22	1	73.79	1	73.79	4	87.7	4	87.7
HC	5413	413.29	39	411.31	14	404.54	130	413.23	72	412.77
LTC SOM	703	4160.99	7	4125.05	6	4116.96	33	4159.4	26	4157.74
LTC PG	2477	1606.72	13	1569.52	10	1554.75	63	1605.19	49	1603.43
LTC at Home	1618	1166.48	18	1155.91	10	1141.66	65	1166.11	41	1164.61
LTC Rehabilitation	21	77.75	1	52.42	1	52.42	5	75.73	4	74.22
Cardiology	90	12.94	1	9.41	2	11.12	8	12.73	9	12.78
Surgery	146	11.71	1	8.15	2	9.78	10	11.53	12	11.6
Neurology	55	7.96	1	5.31	1	5.31	6	7.72	7	7.79
Urology	29	16.59	1	14.14	1	14.14	4	16.26	6	16.47
Orthopedics	35	19.4	1	16.31	1	16.31	5	19.12	6	19.22
Internal Medicine	158	17.2	2	14.79	3	15.67	10	16.98	12	17.06
Lung Diseases	65	101.83	1	96.4	2	99.44	7	101.55	8	101.63
Gastroenterology	32	14.5	1	11.38	1	11.38	4	14.04	6	14.32
Other (hospital)	199	6.83	2	5.3	2	5.3	11	6.69	13	6.74

Table 8.3: The capacity x beds with the waiting time W in days compared to capacity x_{stable} and waiting time W_{stable} for the stable system. This is done for budget for stability +1% and +5%, for MA and SRS.

Capacity x and waiting time W compared to x_{stable} and W_{stable}										
	Stability		+1%				+5%			
			MA		SRS		MA		SRS	
Station	x_{stable}	W_{stable}	x	W	x	W	x	W	x	W
GR	193	347.66	196	8.85	196	8.85	206	0.72	207	0.59
STC	120	35.9	122	8.99	122	8.99	133	0.39	130	0.8
STC Palliative	15	89.22	16	15.42	16	15.42	19	1.51	19	1.51
HC	5413	413.29	5452	1.98	5427	8.75	5543	0.06	5485	0.52
LTC SOM	703	4160.99	710	35.94	709	44.03	736	1.59	729	3.25
LTC PG	2477	1606.72	2490	37.2	2487	51.97	2540	1.53	2526	3.29
LTC at Home	1618	1166.48	1636	10.56	1628	24.82	1683	0.37	1659	1.87
LTC Rehabilitation	21	77.75	22	25.33	22	25.33	26	2.02	25	3.53
Cardiology	90	12.94	91	3.53	92	1.82	98	0.21	99	0.16
Surgery	146	11.71	147	3.56	148	1.92	156	0.17	158	0.11
Neurology	55	7.96	56	2.65	56	2.65	61	0.25	62	0.17
Urology	29	16.59	30	2.45	30	2.45	33	0.33	35	0.12
Orthopedics	35	19.4	36	3.09	36	3.09	40	0.28	41	0.18
Internal Medicine	158	17.2	160	2.41	161	1.54	168	0.22	170	0.14
Lung Diseases	65	101.83	66	5.43	67	2.39	72	0.28	73	0.2
Gastroenterology	32	14.5	33	3.12	33	3.12	36	0.46	38	0.18
Other (hospital)	199	6.83	201	1.53	201	1.53	210	0.14	212	0.09

Increasing Demand

To see the effect of an *increasing demand for care*, which translates as an *increase in external arrivals* into our system, we multiply all external arrivals by the same percentage increase for multiple scenarios of increased demand: +1%, +3%, +5%, and +10%. We take the allocation obtained with the 'budget of stability +5%', which can be seen in Table 8.3 above. The resulting waiting times for increased demand are provided in Table 8.4.

For an increase in external arrivals of +1% for all locations, the waiting times increase somewhat, but not too much. However, for the SRS method, station LTC PG, the waiting time becomes four times as large as the waiting time with the 'initial' number of arrivals. For arrivals +3%, multiple stations become unstable. These are HC and LTC PG for MA, whereas for SRS, also LTC at Home becomes unstable, with an exaggerated waiting time for LTC SOM. MA seems to have a favorable allocation regarding this increase in arrivals. And HC and LTC PG seem to be most sensitive to the rise in demand in the healthcare system. For arrivals +5%, the methods have the same unstable stations, although the stable stations' waiting times differ. For arrivals +10%, the MA and SRS allocations have the same number of unstable stations, where two differ: STC is stable for MA but not for SRS, and Lung diseases are stable for SRS, not for MA.

An increase in demand for the total care system seems to have the most effect on the facilities with longer average lengths of stays, such as HC and LTC. The hospital departments are less sensitive to increases in demand unless this increase is exaggerated.

Table 8.4: Waiting times for allocation with '+5%' budget, after an increased demand. The increase is +1%, +3%, +5% and +10%.

Waiting time after increasing the initial external arrivals λ.										
	λ		+1%		+3%		+5%		+10%	
Station	MA	SRS	MA	SRS	MA	SRS	MA	SRS	MA	SRS
GR	0.72	0.59	1.06	0.87	2.49	1.97	7.73	5.46	∞	∞
STC	0.39	0.8	0.52	1.08	0.94	2.04	1.74	4.15	14.69	∞
STC Palliative	1.51	1.51	1.68	1.68	2.06	2.06	2.54	2.54	4.31	4.31
HC	0.06	0.52	0.45	6.43	∞	∞	∞	∞	∞	∞
LTC SOM	1.59	3.25	3.28	6.93	16.53	56.82	∞	∞	∞	∞
LTC PG	1.53	3.29	6.1	14.76	∞	∞	∞	∞	∞	∞
LTC at Home	0.37	1.87	1.11	6.0	12.29	∞	∞	∞	∞	∞
LTC Rehabilitation	2.02	3.53	2.28	3.99	2.9	5.1	3.69	6.55	6.77	12.57
Cardiology	0.21	0.16	0.27	0.2	0.48	0.35	0.92	0.63	∞	11.73
Surgery	0.17	0.11	0.25	0.15	0.52	0.31	1.39	0.69	∞	∞
Neurology	0.25	0.17	0.3	0.21	0.47	0.32	0.74	0.49	3.92	1.82
Urology	0.33	0.12	0.38	0.14	0.53	0.19	0.74	0.25	2.2	0.55
Orthopedics	0.28	0.18	0.33	0.21	0.46	0.29	0.66	0.4	1.97	1.01
Internal Medicine	0.22	0.14	0.32	0.2	0.72	0.43	2.28	1.04	∞	∞
Lung Diseases	0.28	0.2	0.36	0.25	0.58	0.4	0.98	0.65	10.96	3.44
Gastroenterology	0.46	0.18	0.55	0.21	0.77	0.29	1.11	0.39	3.72	0.89
Other (hospital)	0.14	0.09	0.21	0.14	0.54	0.33	2.51	1.0	∞	∞

8.2 Comparison with District Model

We compare the general model to the district model, which contains HC and LTC stations per district instead of a central capacity arrangement.

Parameters District Model

We obtained the required parameters in the previous chapter. However, technical difficulties were encountered when using these parameters, making it hard to compare the general and district models. Some differences in transition probabilities and the estimated length of stay result in a different offered load for the district model compared to the general model.

Some numbers of the transition probabilities in the district model, in Appendix D Tables D.3 to D.6, had to be rounded up or down due to privacy rules from CBS. This leads to differences in effective arrivals compared to the general model. Hence, for comparison reasons, the transition probabilities of the general model are also used for the district model, where the transition probabilities from a station to a *decentralized station*, such as LTC or HC, are copied and divided by 7. With this, the probability of going from GR to HC, 29.63, becomes 4.23 for a transition from GR to HC North, HC West, etc.

Additionally, we had to use the length of stay estimates from the general model for each of the *decentralized stations*, instead of using the estimates found per district. The mean of the districts' LoS values, weighted by the population sizes, gave a different value than the estimated LoS for the stations in the general model. A possible reason for this difference could be the estimation of the tail distribution with the censored data. However, due to time constraints, no additional thorough analysis was done to find this difference.

No problems were found for the external arrivals because the sum of the external arrivals per decentralized station was equal to the external arrival of the central station.

After these modifications, we obtained a total offered load similar to the offered load from the general model. This is *11,361.96* for the district model and *11,361.99* for the general model, where the difference might be due to some differences in rounding.

Comparing Stability

For the district model, we use the parameters defined in Table 8.5. The table also shows the effective arrivals, the offered load, the minimum required number of beds for stability, and the corresponding cost for this stable situation. HC stations have the highest offered load for this district model, given as a. This is primarily due to the high (effective) arrival rate and the relatively long ALOS. After this, LTC PG has the highest offered load, which seems to be mainly due to the long ALOS. When we sum the required cost per station per day, we find a budget of *2,043,045* needed for stability. Again, for simplicity, we have a daily budget instead of a yearly budget. The district model requires *11,380* beds for this stable situation, *11* beds more than the general model. These differences lie in the beds needed for the district model for LTC SOM, LTC PG, LTC at home, and HC, which are 705, 2480, 1621, and 5416, respectively. For LTC SOM, the district model requires two more beds for a stable system than the general model. LTC PG, LTC at Home, and HC need three additional beds.

Comparing Allocations

The district model is used to compare the allocation of a budget when care is centralized, as in the general model, to decentralized care stations, as for LTC and HC in the district model. We use the Marginal Allocation (MA) algorithm and take a budget of *2,100,000* to divide over the stations, which is about 2.7-2.8% extra compared to the budget required for stability in both models. The resulting allocations for both models are provided in Table 8.6.

This table shows the differences in allocation, and what stands out first is that the general model obtained fewer beds for the whole system. In the general model, fewer beds are allocated to the stations of LTC and HC, while the waiting times are also generally low compared to the district model. Since fewer beds are needed to obtain lower waiting times at the centralized stations of LTC and HC, more budget is left to assign beds to other stations, lowering these waiting times as well. And a small increase in beds in those other stations seems to have a big influence on the waiting times.

Table 8.5: For each station the external arrival rate λ per day, ALOS μ^{-1} in days, cost c per bed per day, with calculated effective arrival rate γ per day, offered load a, the required number of beds for stability x_{stable}, and the corresponding total daily cost for a stable station.

Station	μ^{-1}	λ	c	γ	a	x_{stable}	cost p/day
GR	36.53	0.33	563	5.28	192.9	193	108659
STC	37.15	0.75	227	3.2	119.07	120	27240
STC Palliative	30.95	0.07	329	0.47	14.69	15	4935
HC North	162.06	2.99	61	5.23	848.13	849	51789
HC East	162.06	2.31	61	4.55	737.57	738	45018
HC West	162.06	2.17	61	4.41	714.73	715	43615
HC South	162.06	3.49	61	5.73	928.92	929	56669
HC New-West	162.06	3.34	61	5.58	904.51	905	55205
HC Southeast	162.06	1.94	61	4.18	677.09	678	41358
HC Center	162.06	1.47	61	3.71	601.65	602	36722
LTC SOM North	360.3	0.04	243	0.28	101.68	102	24786
LTC SOM East	360.3	0.03	243	0.27	96.57	97	23571
LTC SOM West	360.3	0.04	243	0.28	101.0	101	24543
LTC SOM South	360.3	0.06	243	0.3	107.82	108	26244
LTC SOM New-West	360.3	0.05	243	0.29	105.43	106	25758
LTC SOM Southeast	360.3	0.03	243	0.27	95.54	96	23328
LTC SOM Center	360.3	0.02	243	0.26	94.86	95	23085
LTC PG North	711.24	0.09	242	0.48	338.89	339	82038
LTC PG East	711.24	0.09	242	0.48	340.24	341	82522
LTC PG West	711.24	0.09	242	0.48	340.91	341	82522
LTC PG South	711.24	0.21	242	0.6	427.04	428	103576
LTC PG New-West	711.24	0.14	242	0.53	379.94	380	91960
LTC PG Southeast	711.24	0.06	242	0.45	319.38	320	77440
LTC PG Center	711.24	0.07	242	0.46	330.15	331	80102
LTC at Home North	356.81	0.25	91	0.71	253.48	254	23114
LTC at Home East	356.81	0.12	91	0.58	206.58	207	18837
LTC at Home West	356.81	0.16	91	0.63	223.23	224	20384
LTC at Home South	356.81	0.26	91	0.72	257.22	258	23478
LTC at Home New-West	356.81	0.27	91	0.74	262.65	263	23933
LTC at Home Southeast	356.81	0.12	91	0.58	207.94	208	18928
LTC at Home Center	356.81	0.12	91	0.58	206.58	207	18837
LTC Rehabilitation	73.19	0.02	269	0.28	20.23	21	5649
Cardiology	6.22	10.92	800	14.39	89.55	90	72000
Surgery	6.27	18.39	800	23.19	145.49	146	116800
Neurology	5.82	7.57	800	9.34	54.34	55	44000
Urology	4.0	5.67	800	7.19	28.77	29	23200
Orthopedics	4.99	5.94	800	6.96	34.76	35	28000
Internal Medicine	7.44	12.46	800	21.19	157.59	158	126400
Lung Diseases	6.83	6.11	800	9.51	64.93	65	52000
Gastroenterology	5.77	3.92	800	5.48	31.63	32	25600
Other (hospital)	5.36	31.88	800	37.0	198.26	199	159200

Table 8.6: Results of allocation with MA, in terms of beds x and waiting time W in days for the district and general models for a budget of 2,100,000.

	District model				General model	
Station		x		W	x	W
GR		197		6.08	201	2.04
STC		125		3.03	128	1.32
STC Palliative		16		15.42	17	6.17
HC North	*873*		*1.91*			
HC East	*761*		*2.0*			
HC West	*738*		*1.99*			
HC South	*955*		*1.81*			
HC New-West	*930*		*1.88*			
HC Southeast	*700*		*1.98*			
HC Center	*623*		*2.17*			
HC	*Total:*	5580	*Avg:*	1.95	5504	0.26
LTC SOM North	*107*		*33.56*			
LTC SOM East	*101*		*44.94*			
LTC SOM West	*106*		*37.26*			
LTC SOM South	*113*		*35.85*			
LTC SOM New-West	*110*		*43.93*			
LTC SOM Southeast	*100*		*44.42*			
LTC SOM Center	*100*		*34.78*			
LTC SOM	*Total:*	737	*Avg:*	39.20	723	6.15
LTC PG North	*348*		*40.2*			
LTC PG East	*349*		*43.0*			
LTC PG West	*350*		*40.42*			
LTC PG South	*437*		*37.4*			
LTC PG New-West	*389*		*42.16*			
LTC PG Southeast	*328*		*43.25*			
LTC PG Center	*339*		*41.8*			
LTC PG	*Total:*	2540	*Avg:*	41.04	2515	6.15
LTC at Home North	*265*		*11.31*			
LTC at Home East	*217*		*12.51*			
LTC at Home West	*234*		*12.18*			
LTC at Home South	*269*		*10.86*			
LTC at Home New-West	*275*		*9.93*			
LTC at Home Southeast	*219*		*11.03*			
LTC at Home Center	*217*		*12.51*			
LTC at Home	*Total:*	1696	*Avg:*	11.41	1662	1.52
LTC Rehabilitation		22		25.33	24	6.32
Cardiology		92		1.82	94	0.75
Surgery		149		1.23	151	0.62
Neurology		56		2.65	58	0.83
Urology		30		2.45	31	1.06
Orthopedics		36		3.09	38	0.75
Internal Medicine		161		1.54	163	0.78
Lung Diseases		67		2.39	69	0.86
Gastroenterology		33		3.12	34	1.42
Other (hospital)		202		1.02	205	0.42
Total beds		11739			11617	

Chapter 9

Conclusion

This thesis shows an application of modeling the elderly healthcare system using the Jackson Network, such that bottlenecks can be identified and reduced. The Jackson Network was combined with an optimization study, suggesting preferred capacity allocations to lower the system's waiting times. The proposed methods were applied to a case study of Amsterdam, resulting in two main findings.

First, we found that most capacity should be allocated to stations with the highest offered load, e.g., HC, which also coincides with the policy that people should live at home as long as possible. These stations with high offered loads are most sensitive to an overall increase in demand, which was translated as an increase in external arrivals to the system. Thus these stations require the most additional resources for growing demand.

Secondly, and most importantly, the case study showed the benefit of centralizing care as opposed to decentralized care. This was demonstrated by comparing two models, the *general model* and the *district model*. The difference between these two models can be found in the decentralized model, which contains a care facility per city district instead of one for the whole city. The decentralization of stations for the district model was applied to HC and LTC. First, to meet the stability requirements for the entire system, the general model required 11 fewer beds for the HC and LTC stations than the district model. Secondly, when allocating a similar budget to both models, the general model also required fewer beds for HC and LTC while having lower waiting times. With this, extra budget was left for other stations, e.g., GR and STC, reducing these waiting times as well.

These findings show the potential of the Jackson Network for the identification of bottlenecks in terms of waiting times and how these can be reduced. This is done by comparing multiple scenarios, i.e., the general model and the district model, and with an optimization study, suggesting the best allocation strategy to reduce bottlenecks.

Chapter 10

Discussion

This chapter discusses the limitations of the research and possible improvements that can be made in future research.

Limitations

During the case study, some difficulties were encountered with the declaration data concerning Home Care (HC). The declarations were either daily or monthly, but the exact duration, activities, and care period were not provided. This required some additional assumptions on the durations because we merged activities that happened in consecutive months. This enabled us to obtain an average length of stay which seemed somewhat reasonable compared to the statistic in the HC section in chapter 6. However, the arrival rate did not satisfy the assumption of the Poisson process due to the high Variance-to-Mean Ratio (VMR). This argues for an additional analysis of HC data, preferably obtained from HC organizations. While fitting the arrival rates, we investigated differences in arrival processes for different seasons, e.g., higher demand in winter, but clear patterns were not identified from the declaration data. Overall we assumed the average arrival rate to be sufficient for the macro model, but additional investigation with more precise data is preferred. Another solution would be the exclusion of HC from the model due to the high VMR. However, since HC is a vital part of elderly care, this was not as desired. This result adds some doubts to the created Jackson Network.

In addition, no statistical test is reported supporting the Poisson assumption for the found parameters in this thesis. The VMR ratio, combined with a visual inspection, was considered enough for the assumptions in the Jackson Network. However, we would be more confident with statistical tests.

When determining the average length of stay with the proposed method in Equation (7.2), based on the integral of the Kaplan-Meier (K-M) survival curve with an estimated tail distribution, we assumed all tail distributions to be exponential. However, fitting a tail distribution closer to the 'best-fitting' distribution would be preferred to obtain an estimate closer to the true underlying distribution. Overall, comparing the average of the method used to determine the parameters with the best fit seemed to give reasonable results.

In the results comparison of the district and general models, a difference was found in the offered load, so modifications had to be made to the parameters of the district model. The first problem was the rounding in the transition matrix due to the privacy rules of CBS. Secondly, we found some difficulty comparing the average length of stay for the general model to the (weighted) averages per district. This might be because of the method based on the K-M curve, which included an estimate of the tail probabilities. Not enough time was available to go into much more detail. Overall, the preference would be to compare the actual estimated parameters.

Lastly, the Jackson Network consists of $M/M/c$ queues, for which the performance measures focus on expected waiting times and service levels. When a person ends service at a station and requires care at another station, the Jackson Network assumes this person is in the waiting line, possibly waiting at home. However, this is not entirely realistic, as people ending service in a hospital and requiring a place at STC, GR, or LTC may wait until they can receive a place here. This is blocking, which is not incorporated into the model. This would require another type of queue, but then the Jackson Network and its product form solutions do not hold.

Future Research

The first point of future research is to get a better estimate of the actual demand for care, which can be used in a more complete scenario analysis. The parameters for the model were based on historical declaration data. The unsatisfied demand is not included in this model. This could be people on the waiting list or people receiving informal care but requiring more intensive care. To gain insight into what is needed to help this group of people, it would be of interest to estimate or predict demand while including this group. This does require more exact data, which was not available in this thesis.

The second point concerns the optimization method. In this research, the model solely focused on reducing waiting times given a specific budget to allocate. However, in an ideal and unrealistic situation, if an enormous budget is available, the optimization methods would suggest spending the entire budget so that all waiting times are as low as possible. But we should also consider the throughput in the stations and the cost-effectiveness of additional capacity investments. This would suggest the use of a multi-objective approach, which would either minimize both the waiting time and the cost or minimize the waiting time while maximizing the throughput, resulting in a Pareto frontier showing the trade-off between the ideal situation for the patients (low waiting times) and the ideal situation for the facilities (high throughput and low cost).

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Appendices

Appendix A

Probability Distributions

This section contains general information about the probability distribution functions used in this thesis.

Exponential Distribution

The exponential distribution with parameter μ has a probability density function:

$$f(x) = \mu e^{-\mu x}, \quad x > 0 \tag{A.1}$$

And as the distribution function:

$$F(x) = 1 - e^{-\mu x}, \quad x > 0 \tag{A.2}$$

The importance of the exponential distribution is that it has the memoryless property, as given by:

$$P(T > x + t | T > t) = P(T > x) = e^{-\mu x}, \quad x \geq 0, \qquad t \geq 0$$

Meaning that after a time t, the distribution of x is still the same; thus, this also has the same mean $\frac{1}{\mu}$.

For exponential distributions, we have:

$$\mathbb{E}(X) = \frac{1}{\mu}, \quad \sigma^2(X) = \frac{1}{\mu^2}$$

Poisson Distribution

The Poisson distribution is a discrete probability distribution, giving the probability distribution of how often an event will likely occur within a certain period. The Poisson distribution with parameter λ has the probability distribution:

$$\mathbb{P}(X = x) = \frac{\lambda^x}{x!} e^{-\lambda}, \quad x = 0, 1, 2, .. \tag{A.3}$$

And the following properties:

$$\mathbb{E}(X) = \lambda, \quad \sigma^2(X) = \lambda \tag{A.4}$$

Gamma Distribution

The gamma distribution is a continuous probability distribution with two parameters: shape $\alpha > 0$ and rate $\beta > 0$.

The probability density function is:

$$f(x) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x} \tag{A.5}$$

And the cumulative distribution function is:

$$F(x) = \frac{1}{\Gamma(\alpha)} \gamma(\alpha, \beta x) \tag{A.6}$$

And the mean and variance are:

$$\mathbb{E}(X) = \frac{\alpha}{\beta}, \quad \sigma^2(X) = \frac{\alpha}{\beta^2} \tag{A.7}$$

Weibull Distribution

The Weibull distribution is a continuous probability distribution with two parameters: shape $k \geq 0$ and scale $\lambda \geq 0$.

The probability density function is:

$$f(x) = \begin{cases} \frac{k}{\lambda}\left(\frac{x}{\lambda}\right)^{k-1} e^{(-x/\lambda)^k} & x \geq 0, \\ 0 & x < 0, \end{cases} \tag{A.8}$$

And the cumulative distribution function is:

$$F(x) = \begin{cases} 1 - e^{(-x/\lambda)^k} & x \geq 0, \\ 0 & x < 0, \end{cases} \tag{A.9}$$

And the mean and variance are:

$$\mathbb{E}(X) = \lambda\Gamma(1 + \frac{1}{k}), \quad \sigma^2(X) = \lambda^2\left[\Gamma\left(1 + \frac{2}{k}\right) - \left(\Gamma\left(1 + \frac{1}{k}\right)\right)^2\right] \tag{A.10}$$

When $k = 1$, the failure rate is constant. In this case, the Weibull distribution equals an exponential distribution.

When $k < 1$, the failure rate is said to decrease over time.

When $k > 1$, the failure rate is said to increase over time.

Lognormal Distribution

The lognormal distribution is a continuous probability distribution, with two parameters μ and σ^2.

The probability density function is:

$$f(x) = \frac{1}{x\sigma\sqrt{2\pi}} \exp\left(-\frac{(\ln(x) - \mu)^2}{2\sigma^2}\right) \tag{A.11}$$

And the cumulative distribution function is:

$$F(x) = \frac{1}{2}\left[1 + \text{erf}\left(\frac{\ln(x) - \mu}{\sigma\sqrt{2}}\right)\right] \tag{A.12}$$

And the mean and variance are:

$$\mathbb{E}(X) = \exp\left(\mu + \frac{\sigma^2}{2}\right), \quad \sigma^2(X) = \left[\exp(\sigma^2) - 1\right] \exp\left(2\mu + \sigma^2\right) \tag{A.13}$$

Hyper-Exponential Distribution

The hyper-exponential distribution is a continuous probability distribution, a mixture of multiple exponential distributions. The number of parameters depends on the number of exponential distributions in the mixture, for which each exponential distribution has its rate μ_i and probability p_i. The hyper-exponential distribution is a phase-type distribution in which the phases are parallel.

The probability density function is

$$f(x) = \sum_{i=1}^{N} p_i \mu_i e^{-\mu_i x} \tag{A.14}$$

And the cumulative distribution function is

$$F(x) = 1 - \sum_{i=1}^{N} p_i e^{-\mu_i x} \tag{A.15}$$

And the mean and variance are

$$\mathbb{E}(X) = \sum_{i=1}^{N} \frac{p_i}{\mu_i}, \quad \sigma^2(X) = \sum_{i=1}^{N} \frac{2p_i}{\mu_i^2} - \left(\sum_{i=1}^{N} \frac{p_i}{\mu_i}\right)^2 \tag{A.16}$$

Appendix B

Kaplan-Meier Estimator

The Kaplan-Meier (KM) estimation is a method to compute survival over time, in which censored data is allowed, which are cases for which the survival time is still unknown because they are either still alive or because the survival outcome is unknown. Survival is measured from a starting point (e.g., from when people took a particular medicine) until the time of death (usually called the *event*). The survival function is $S(t)$, and the equation is as follows:

$$S(t) = \begin{cases} 1, & \text{for } t = 0 \\ \prod_{i=1}^{N}(1 - \frac{d_s}{n_s}), & \text{for } t = 1, .., N \end{cases} \tag{B.1}$$

Here d_s is the number of cases with the event at time t, and n_s is the number of cases that have the event at least at time t, thus either at time t, after time t, or the event has not happened yet.

Appendix C

Arrival Rates

Table C.1: External arrival rate per week for LTC SOM, per city district, with the mean, variance, and Variance-to-Mean-Ratio (VMR).

District	mean	var	VMR
North	0.32	0.33	1.06
South	0.48	0.58	1.19
West	0.31	0.27	0.87
New-West	0.43	0.56	1.31
Center	0.21	0.22	1.05
South-East	0.21	0.28	1.36
East	0.23	0.22	0.95

Table C.2: External arrival rate per week for LTC PG, per city district, with the mean, variance, and Variance-to-Mean-Ratio (VMR).

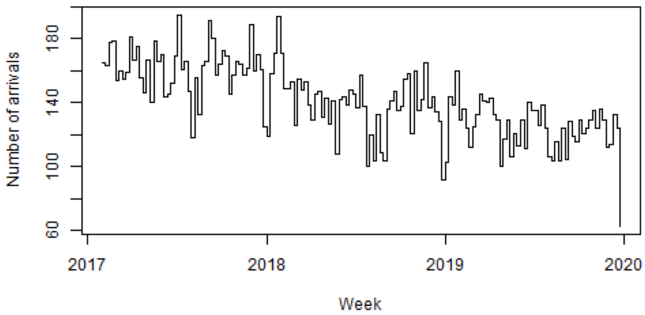
District	mean	var	VMR
North	0.67	0.84	1.25
South	1.55	1.89	1.22
West	0.67	0.69	1.03
New-West	1.07	1.20	1.12
Center	0.56	0.59	1.05
South-East	0.43	0.42	0.96
East	0.67	0.66	0.99

Table C.3: External arrival rate per week for LTC at Home, per city district, with the mean, variance, and Variance-to-Mean-Ratio (VMR).

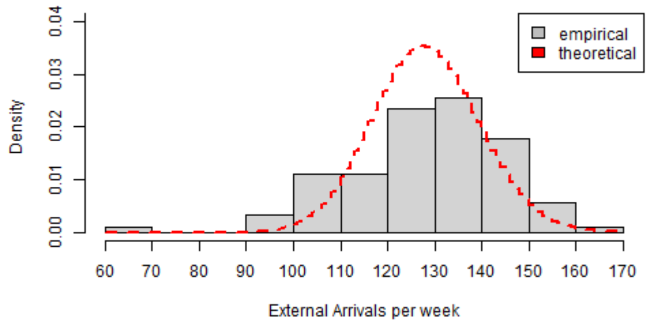
District	mean	var	VMR
North	1.82	2.47	1.36
South	1.95	2.62	1.34
West	1.22	1.37	1.12
New-West	2.05	2.86	1.40
Center	0.87	0.96	1.10
South-East	0.87	0.84	0.97
East	0.84	0.82	0.97

Table C.4: External arrival rate per week for HC, split by city district.

District	mean	var	VMR
North	20.91	42.03	2.01
South	24.62	55.17	2.24
West	15.27	25.65	1.68
New-West	23.51	62.35	2.65
Center	10.31	17.86	1.73
South-East	13.75	41.73	3.04
East	16.18	36.55	2.26

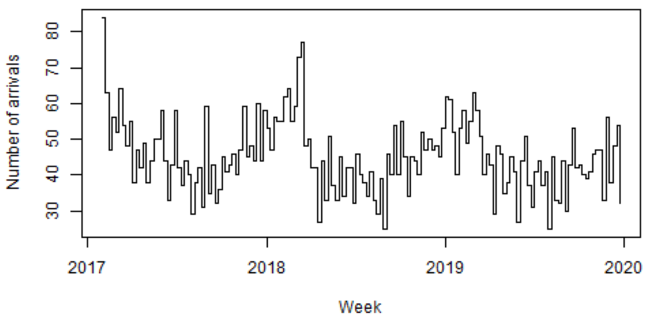


(a) Arrivals per week over time.

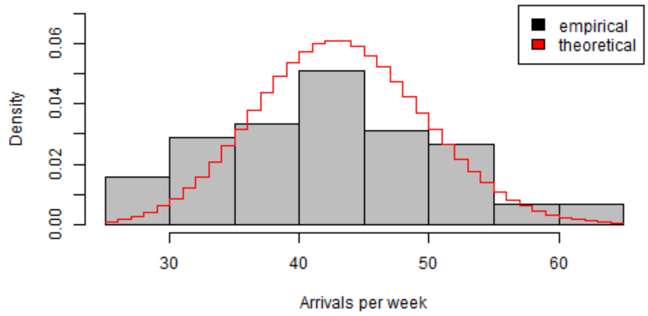


(b) Empirical and theoretical pdf of external arrivals per week. The average of the empirical data is used to fit the theoretical Poisson distribution.

Figure C.1: Hospital department Surgery.

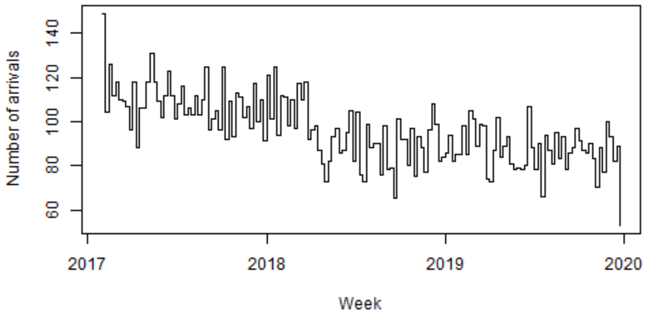


(a) Arrivals per week over time.

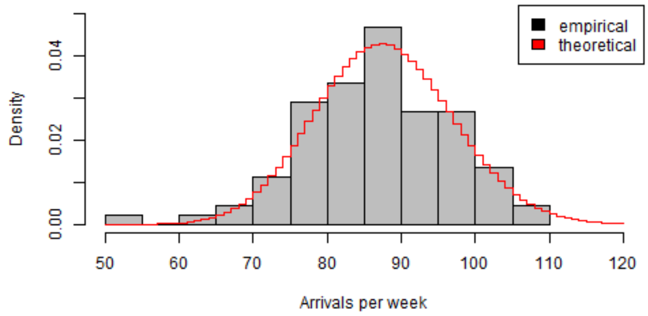


(b) Empirical and theoretical pdf of external arrivals per week. The average of the empirical data is used to fit the theoretical Poisson distribution.

Figure C.2: Hospital department Lung Diseases.

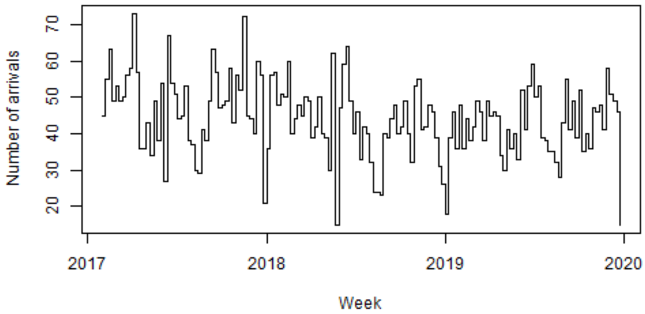


(a) Arrivals per week over time.

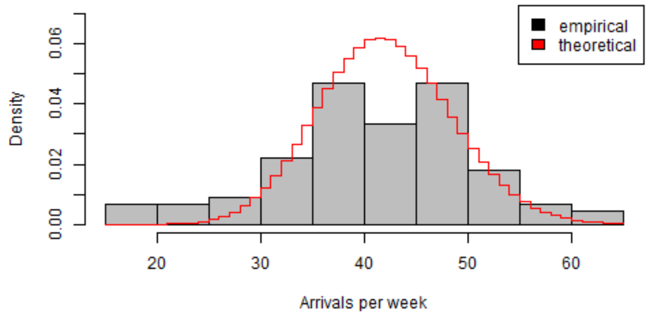


(b) Empirical and theoretical pdf of external arrivals per week. The average of the empirical data is used to fit the theoretical Poisson distribution.

Figure C.3: Hospital department Internal Medicine.

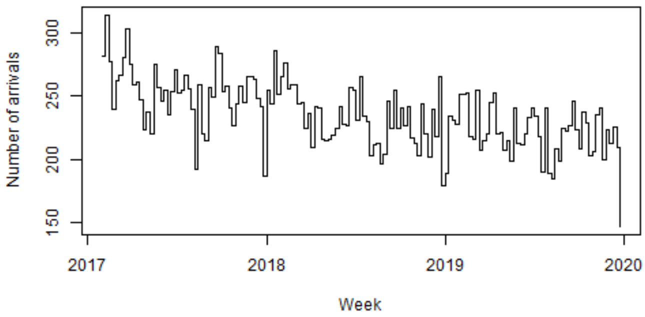


(a) Arrivals per week over time.

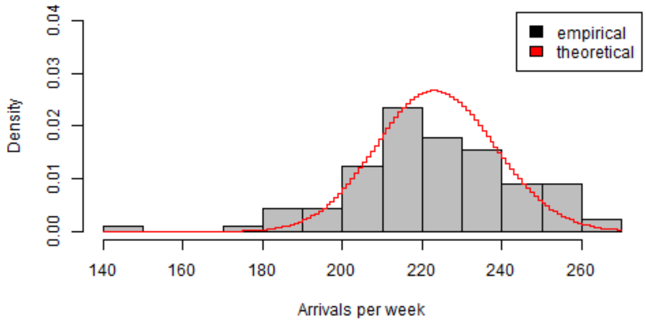


(b) Empirical and theoretical pdf of external arrivals per week. The average of the empirical data is used to fit the theoretical Poisson distribution.

Figure C.4: Hospital department Orthopedics.



(a) Arrivals per week over time.



(b) Empirical and theoretical pdf of external arrivals per week. The average of the empirical data is used to fit the theoretical Poisson distribution.

Figure C.5: Hospital Other departments.

Appendix D

Transition Probabilities

This section includes the transition probabilities calculated for the 'general model', in Tables D.1 to D.2. And for the 'district model', in Tables D.3 to D.6.

Table D.1: Transition probabilities in % for general model. Part 1 of 2.

From/To	GR	STC	STC Palliative	HC	LTC SOM	LTC PG	LTC at Home	LTC rehabilitation	Cardiology
GR	0.69	2.35	0.58	29.63	5.37	2.33	3.09	0.0	1.85
STC	2.06	1.16	1.65	25.95	8.76	11.34	6.2	0.0	1.16
STC Palliative	0.0	0.0	0.0	2.91	2.52	0.0	0.0	0.0	0.0
HC	0.49	2.5	0.33	0.08	0.07	0.44	3.84	0.0	4.79
LTC SOM	0.0	0.0	0.0	0.71	0.0	4.11	2.97	0.71	3.75
LTC PG	0.0	0.0	0.0	0.54	0.54	0.91	4.82	0.75	2.73
LTC at Home	0.3	0.0	0.0	1.79	5.95	23.39	2.84	0.3	6.22
LTC Rehabilitation	0.0	0.0	0.0	0.0	17.93	18.73	18.33	0.0	0.0
Cardiology	2.25	0.83	0.14	14.49	0.55	0.44	1.69	0.07	4.97
Surgery	5.8	1.3	0.09	8.81	0.43	0.96	0.75	0.39	0.31
Neurology	6.75	1.2	0.14	7.46	0.75	0.77	1.03	0.35	0.3
Urology	0.57	0.38	0.13	11.15	0.61	0.46	0.67	0.0	0.36
Orthopedics	14.21	2.01	0.0	10.54	0.37	1.07	0.55	0.4	0.19
Internal Medicine	3.29	2.14	0.37	16.13	1.2	1.13	1.47	0.11	0.7
Lung Diseases	3.49	1.04	0.41	16.86	0.88	0.43	1.39	0.11	0.8
Gastroenterology	1.37	0.9	0.33	12.71	0.68	0.49	1.06	0.0	0.45
Other (hospital)	0.65	0.43	0.04	2.86	0.11	0.28	0.26	0.04	0.48

Table D.2: Transition probabilities in % for general model. Part 2 of 2.

From/To	Surgery	Neurology	Urology	Orthopedics	Internal Medicine	Lung Diseases	Gastroenterology	Other (hospital)	Leave system
GR	3.6	1.42	0.43	1.37	4.86	1.78	1.03	1.46	38.14
STC	3.1	0.75	0.52	1.48	5.16	1.62	0.87	1.28	26.94
STC Palliative	0.0	0.0	0.0	0.0	1.94	0.0	1.94	0.0	90.7
HC	5.33	2.25	1.85	1.37	9.14	3.9	1.55	2.44	59.63
LTC SOM	5.17	3.12	2.76	1.27	15.37	4.82	2.27	1.84	51.13
LTC PG	7.92	1.87	1.02	2.41	7.44	1.5	0.96	2.62	63.97
LTC at Home	6.7	3.86	1.2	1.41	8.97	4.19	1.32	2.6	28.98
LTC Rehabilitation	3.98	3.98	0.0	0.0	3.98	3.98	0.0	0.0	29.08
Cardiology	0.49	0.36	0.15	0.07	1.04	0.57	0.29	1.43	70.21
Surgery	5.32	0.17	0.18	0.15	1.11	0.3	0.39	0.68	72.87
Neurology	0.72	2.06	0.08	0.08	0.91	0.27	0.1	1.76	75.28
Urology	0.62	0.13	7.54	0.13	1.25	0.13	0.14	0.22	75.49
Orthopedics	0.44	0.13	0.0	2.44	0.45	0.17	0.0	0.13	66.89
Internal Medicine	1.26	0.36	0.32	0.11	14.93	0.58	0.93	1.0	53.99
Lung Diseases	0.61	0.29	0.1	0.13	1.08	12.59	0.18	0.67	58.93
Gastroenterology	2.67	0.23	0.17	0.0	3.26	0.38	7.73	0.59	66.98
Other (hospital)	0.32	0.49	0.05	0.02	0.57	0.18	0.07	8.34	84.82

Table D.3: Transition probabilities in % for district model. Part 1 of 4.

From/To	STC	STC Palliative	GR	Cardiology	Surgery	Internal Medicine	Lung Diseases	Gastroenterology	Neurology	Orthopedics	Other (hospital)
GR	2.35	0.58	0.69	1.85	3.6	4.85	1.78	1.03	1.42	1.37	1.46
STC	1.16	1.65	2.06	1.16	3.1	5.16	1.62	0.87	0.75	1.48	1.28
STC Palliative	0.0	0.0	0.0	0.0	0.0	1.97	0.0	1.97	0.0	0.0	0.0
HC Center	2.13	0.37	0.37	3.55	5.08	8.41	3.36	1.61	2.06	1.31	2.47
HC New-West	2.63	0.31	0.81	5.47	5.57	9.17	4.06	1.72	2.27	0.94	3.38
HC Southeast	2.67	0.35	0.56	5.68	4.8	7.25	2.79	1.41	1.88	1.44	2.7
HC West	2.0	0.28	0.43	4.59	5.53	9.74	4.13	1.88	2.43	1.01	2.66
HC North	1.98	0.4	0.0	5.35	5.24	11.57	5.37	0.81	2.39	2.34	1.36
HC East	2.28	0.49	0.61	4.13	5.77	8.18	4.3	1.94	2.19	1.18	2.43
HC South	3.1	0.28	0.53	4.76	5.02	8.12	2.85	1.53	2.17	1.35	2.33
LTC SOM Center	0.0	0.0	0.0	7.52	11.28	15.79	7.52	0.0	7.52	0.0	0.0
LTC SOM New-West	0.0	0.0	0.0	4.06	5.54	9.59	3.69	4.06	3.69	0.0	3.69
LTC SOM Southeast	0.0	0.0	0.0	0.0	7.69	20.77	7.69	7.69	0.0	0.0	0.0
LTC SOM West	0.0	0.0	0.0	4.85	4.85	14.08	8.25	0.0	5.34	0.0	0.0
LTC SOM South	0.0	0.0	0.0	4.78	2.99	19.1	2.99	2.99	2.99	0.0	2.99
LTC SOM East	0.0	0.0	0.0	0.0	8.85	14.16	8.85	0.0	0.0	0.0	0.0
LTC SOM North	0.0	0.0	0.0	4.46	5.8	15.18	6.25	0.0	4.46	0.0	0.0
LTC PG Center	0.0	0.0	0.0	0.0	9.52	8.93	0.0	0.0	0.0	0.0	5.95
LTC PG New-West	0.0	0.0	0.0	2.69	7.26	4.57	0.0	0.0	2.69	2.69	3.76
LTC PG North	0.0	0.0	0.0	5.05	6.5	8.3	3.61	0.0	3.61	3.97	0.0
LTC PG East	0.0	0.0	0.0	4.74	10.9	8.53	0.0	0.0	0.0	4.74	0.0
LTC PG West	0.0	0.0	0.0	3.97	8.73	8.73	3.97	3.97	0.0	0.0	5.95
LTC PG South	0.0	0.0	0.0	2.38	8.57	6.43	0.0	0.0	2.38	3.33	2.38
LTC PG Southeast	0.0	0.0	0.0	0.0	8.2	13.93	0.0	0.0	0.0	0.0	0.0
LTC at Home Center	0.0	0.0	0.0	4.12	6.74	5.62	3.75	0.0	4.12	0.0	3.75
LTC at Home East	0.0	0.0	0.0	8.22	6.03	8.22	3.01	0.0	4.38	0.0	2.74
LTC at Home Southeast	0.0	0.0	0.0	4.93	5.28	4.93	3.52	0.0	3.52	3.52	3.52
LTC at Home New-West	0.0	0.0	0.0	4.93	5.7	6.63	4.93	2.16	4.93	1.54	4.62
LTC at Home West	0.0	0.0	0.0	7.27	9.25	12.33	5.07	0.0	2.64	2.2	2.64
LTC at Home South	0.0	0.0	0.0	5.61	5.31	10.02	3.49	1.97	3.03	1.52	1.67
LTC at Home North	0.0	0.0	0.0	7.42	8.01	11.06	4.95	1.46	4.37	2.47	1.75
LTC Rehabilitation	0.0	0.0	0.0	0.0	3.51	3.51	3.51	0.0	3.51	0.0	0.0
Cardiology	0.82	0.14	2.25	4.96	0.49	1.04	0.57	0.29	0.36	0.07	1.42
Surgery	1.3	0.09	5.8	0.31	5.31	1.11	0.3	0.39	0.17	0.15	0.68
Neurology	1.19	0.14	6.74	0.3	0.72	0.91	0.27	0.1	2.06	0.08	1.76
Urology	0.38	0.13	0.57	0.36	0.62	1.25	0.13	0.15	0.13	0.13	0.22
Orthopedics	2.01	0.0	14.23	0.19	0.44	0.45	0.17	0.0	0.13	2.44	0.13
Internal Medicine	2.14	0.37	3.29	0.7	1.26	14.93	0.58	0.93	0.36	0.11	1.0
Lung Diseases	1.04	0.41	3.5	0.8	0.61	1.09	12.59	0.18	0.29	0.13	0.67
Gastroenterology	0.9	0.33	1.37	0.45	2.67	3.26	0.38	7.74	0.23	0.0	0.59
Other (hospital)	0.43	0.04	0.65	0.48	0.32	0.57	0.18	0.07	0.49	0.02	8.33

Table D.4: Transition probabilities in % for district model. Part 2 of 4.

From/To	Urology	Leave system	LTC PG Center	LTC PG New-West	LTC PG North	LTC PG East	LTC PG West	LTC PG South	LTC PG Southeast	LTC Revalidation	LTC SOM Center
GR	0.43	38.12	0.17	0.5	0.29	0.22	0.33	0.72	0.17	0.0	0.29
STC	0.52	26.94	1.19	3.33	1.19	1.42	1.22	2.29	0.7	0.0	0.52
STC Palliative	0.0	92.13	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
HC Center	2.39	62.63	0.37	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
HC New-West	1.62	57.23	0.0	0.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0
HC Southeast	1.63	62.77	0.0	0.0	0.0	0.0	0.0	0.0	0.53	0.0	0.0
HC West	2.03	59.93	0.0	0.0	0.0	0.0	0.58	0.0	0.0	0.0	0.0
HC North	1.87	55.75	0.0	0.0	0.26	0.0	0.0	0.0	0.0	0.0	0.0
HC East	2.11	60.86	0.0	0.0	0.0	0.27	0.0	0.0	0.0	0.0	0.0
HC South	1.64	61.82	0.0	0.0	0.0	0.0	0.0	0.39	0.18	0.0	0.0
LTC SOM Center	7.52	35.34	7.52	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
LTC SOM New-West	0.0	57.93	0.0	4.06	0.0	0.0	0.0	0.0	0.0	0.0	0.0
LTC SOM Southeast	0.0	48.46	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
LTC SOM West	8.25	44.66	0.0	0.0	0.0	0.0	4.85	0.0	0.0	0.0	0.0
LTC SOM South	0.0	52.84	0.0	0.0	0.0	0.0	0.0	4.18	0.0	0.0	0.0
LTC SOM East	0.0	59.29	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
LTC SOM North	4.46	53.12	0.0	0.0	6.25	0.0	0.0	0.0	0.0	0.0	0.0
LTC PG Center	0.0	67.26	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
LTC PG New-West	0.0	72.58	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
LTC PG North	0.0	64.26	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
LTC PG East	4.74	57.82	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
LTC PG West	0.0	60.71	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
LTC PG South	0.0	65.24	0.0	0.0	0.0	0.0	0.0	2.38	0.0	2.38	0.0
LTC PG Southeast	0.0	69.67	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
LTC at Home Center	0.0	35.21	25.09	0.0	0.0	0.0	0.0	0.0	0.0	0.0	4.12
LTC at Home East	2.74	26.3	0.0	0.0	0.0	23.56	0.0	0.0	0.0	0.0	0.0
LTC at Home Southeast	0.0	27.46	0.0	0.0	0.0	0.0	0.0	0.0	29.58	0.0	0.0
LTC at Home New-West	0.0	31.43	0.0	22.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0
LTC at Home West	2.2	25.55	0.0	0.0	0.0	0.0	22.69	0.0	0.0	0.0	0.0
LTC at Home South	1.67	31.87	0.0	0.0	0.0	0.0	0.0	21.7	0.0	0.0	0.0
LTC at Home North	1.46	24.89	0.0	0.0	22.27	0.0	0.0	0.0	0.0	0.0	0.0
LTC Rehabilitation	0.0	25.61	0.0	3.51	3.51	3.51	3.51	4.21	0.0	0.0	0.0
Cardiology	0.15	70.1	0.07	0.07	0.1	0.07	0.07	0.08	0.07	0.07	0.07
Surgery	0.18	72.86	0.11	0.2	0.11	0.13	0.17	0.21	0.04	0.39	0.09
Neurology	0.08	75.19	0.08	0.2	0.12	0.1	0.13	0.1	0.08	0.35	0.08
Urology	7.55	75.6	0.0	0.15	0.0	0.13	0.0	0.0	0.0	0.0	0.13
Orthopedics	0.0	66.96	0.13	0.17	0.34	0.13	0.13	0.19	0.0	0.4	0.0
Internal Medicine	0.32	53.99	0.12	0.17	0.2	0.15	0.17	0.22	0.1	0.11	0.14
Lung Diseases	0.1	58.97	0.0	0.1	0.12	0.0	0.12	0.0	0.0	0.11	0.1
Gastroenterology	0.17	67.01	0.0	0.0	0.0	0.17	0.17	0.0	0.0	0.0	0.0
Other (hospital)	0.05	84.79	0.02	0.09	0.02	0.02	0.07	0.05	0.02	0.04	0.0

Table D.5: Transition probabilities in % for district model. Part 3 of 4.

From/To	LTC SOM New-West	LTC SOM North	LTC SOM East	LTC SOM West	LTC SOM South	LTC SOM South-east	LTC at Home Center	LTC at Home New-West	LTC at Home North	LTC at Home East	LTC at Home West
GR	1.27	0.87	0.6	0.72	1.23	0.38	0.31	0.6	0.62	0.53	0.22
STC	1.62	1.54	0.7	0.96	2.64	0.78	0.58	0.96	0.99	0.87	0.78
STC Palliative	1.97	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
HC Center	0.0	0.0	0.0	0.0	0.0	0.0	3.89	0.0	0.0	0.0	0.0
HC New-West	0.16	0.0	0.0	0.0	0.0	0.0	0.0	3.98	0.0	0.0	0.0
HC Southeast	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
HC West	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	2.76
HC North	0.0	0.19	0.0	0.0	0.0	0.0	0.0	0.0	5.13	0.0	0.0
HC East	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	3.24	0.0
HC South	0.0	0.0	0.0	0.0	0.11	0.0	0.0	0.0	0.0	0.0	0.0
LTC SOM Center	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
LTC SOM New-West	0.0	0.0	0.0	0.0	0.0	0.0	0.0	3.69	0.0	0.0	0.0
LTC SOM Southeast	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
LTC SOM West	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	4.85
LTC SOM South	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
LTC SOM East	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	8.85	0.0
LTC SOM North	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
LTC PG Center	0.0	0.0	0.0	0.0	0.0	0.0	8.33	0.0	0.0	0.0	0.0
LTC PG New-West	0.0	0.0	0.0	0.0	0.0	0.0	0.0	3.76	0.0	0.0	0.0
LTC PG North	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	4.69	0.0	0.0
LTC PG East	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	8.53	0.0
LTC PG West	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	3.97
LTC PG South	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
LTC PG Southeast	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
LTC at Home Center	0.0	0.0	0.0	0.0	0.0	0.0	3.75	0.0	0.0	0.0	0.0
LTC at Home East	0.0	0.0	8.77	0.0	0.0	0.0	0.0	0.0	0.0	3.29	0.0
LTC at Home Southeast	0.0	0.0	0.0	0.0	0.0	6.69	0.0	0.0	0.0	0.0	0.0
LTC at Home New-West	5.55	0.0	0.0	0.0	0.0	0.0	0.0	3.54	0.0	0.0	0.0
LTC at Home West	0.0	0.0	0.0	5.95	0.0	0.0	0.0	0.0	0.0	0.0	2.2
LTC at Home South	0.0	0.0	0.0	0.0	5.61	0.0	0.0	0.0	0.0	0.0	0.0
LTC at Home North	0.0	5.39	0.0	0.0	0.0	0.0	0.0	0.0	3.06	0.0	0.0
LTC Rehabilitation	3.51	3.51	3.51	3.51	3.86	3.51	3.51	6.67	3.51	3.51	3.51
Cardiology	0.12	0.1	0.07	0.07	0.13	0.07	0.11	0.23	0.41	0.18	0.32
Surgery	0.09	0.07	0.05	0.04	0.05	0.05	0.05	0.11	0.17	0.09	0.16
Neurology	0.17	0.09	0.08	0.12	0.17	0.08	0.08	0.28	0.24	0.13	0.13
Urology	0.0	0.13	0.0	0.22	0.0	0.13	0.0	0.13	0.13	0.0	0.13
Orthopedics	0.0	0.13	0.0	0.0	0.13	0.0	0.0	0.17	0.13	0.0	0.13
Internal Medicine	0.15	0.2	0.06	0.16	0.34	0.15	0.09	0.22	0.35	0.14	0.28
Lung Diseases	0.14	0.18	0.11	0.21	0.12	0.1	0.0	0.34	0.33	0.1	0.25
Gastroenterology	0.17	0.0	0.0	0.17	0.21	0.17	0.0	0.31	0.17	0.0	0.17
Other (hospital)	0.03	0.0	0.0	0.02	0.03	0.02	0.02	0.09	0.03	0.02	0.04

Table D.6: Transition probabilities in % for district model. Part 4 of 4.

From/To	LTC at Home South	LTC at Home South-east	HC Center	HC New-West	HC North	HC East	HC West	HC South	HC South-east
GR	0.5	0.31	2.37	6.29	4.99	3.62	3.46	6.7	2.18
STC	1.54	0.49	2.0	4.78	2.9	2.64	2.44	8.26	2.93
STC Palliative	0.0	0.0	0.0	0.0	0.0	1.97	0.0	0.0	0.0
HC Center	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
HC New-West	0.0	0.0	0.0	0.18	0.0	0.0	0.0	0.0	0.0
HC Southeast	0.0	3.52	0.0	0.0	0.0	0.0	0.0	0.0	0.0
HC West	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
HC North	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
HC East	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
HC South	2.53	1.19	0.0	0.0	0.0	0.0	0.0	0.11	0.0
LTC SOM Center	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
LTC SOM New-West	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
LTC SOM Southeast	0.0	7.69	0.0	0.0	0.0	0.0	0.0	0.0	0.0
LTC SOM West	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
LTC SOM South	4.18	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
LTC SOM East	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
LTC SOM North	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
LTC PG Center	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
LTC PG New-West	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
LTC PG North	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
LTC PG East	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
LTC PG West	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
LTC PG South	4.52	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
LTC PG Southeast	0.0	8.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0
LTC at Home Center	0.0	0.0	3.75	0.0	0.0	0.0	0.0	0.0	0.0
LTC at Home East	0.0	0.0	0.0	0.0	0.0	2.74	0.0	0.0	0.0
LTC at Home Southeast	0.0	3.52	0.0	0.0	0.0	0.0	0.0	0.0	3.52
LTC at Home New-West	0.0	0.0	0.0	1.54	0.0	0.0	0.0	0.0	0.0
LTC at Home West	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
LTC at Home South	2.88	0.0	0.0	0.0	0.0	0.0	0.0	3.64	0.0
LTC at Home North	0.0	0.0	0.0	0.0	1.46	0.0	0.0	0.0	0.0
LTC Rehabilitation	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Cardiology	0.3	0.13	0.9	3.27	2.53	1.64	1.82	2.53	1.78
Surgery	0.12	0.04	0.8	1.75	1.3	1.18	1.21	1.66	0.91
Neurology	0.13	0.08	0.64	1.26	1.07	1.07	0.93	1.84	0.64
Urology	0.17	0.13	1.18	1.97	1.81	1.52	1.52	2.17	1.02
Orthopedics	0.13	0.0	1.11	1.61	2.08	1.23	1.02	2.34	1.16
Internal Medicine	0.31	0.08	1.24	3.32	3.31	1.89	2.08	2.84	1.45
Lung Diseases	0.26	0.1	1.31	3.52	3.71	2.36	2.42	2.37	1.18
Gastroenterology	0.28	0.17	1.09	2.67	1.02	1.96	2.05	2.6	1.32
Other (hospital)	0.03	0.02	0.27	0.67	0.29	0.39	0.39	0.54	0.32