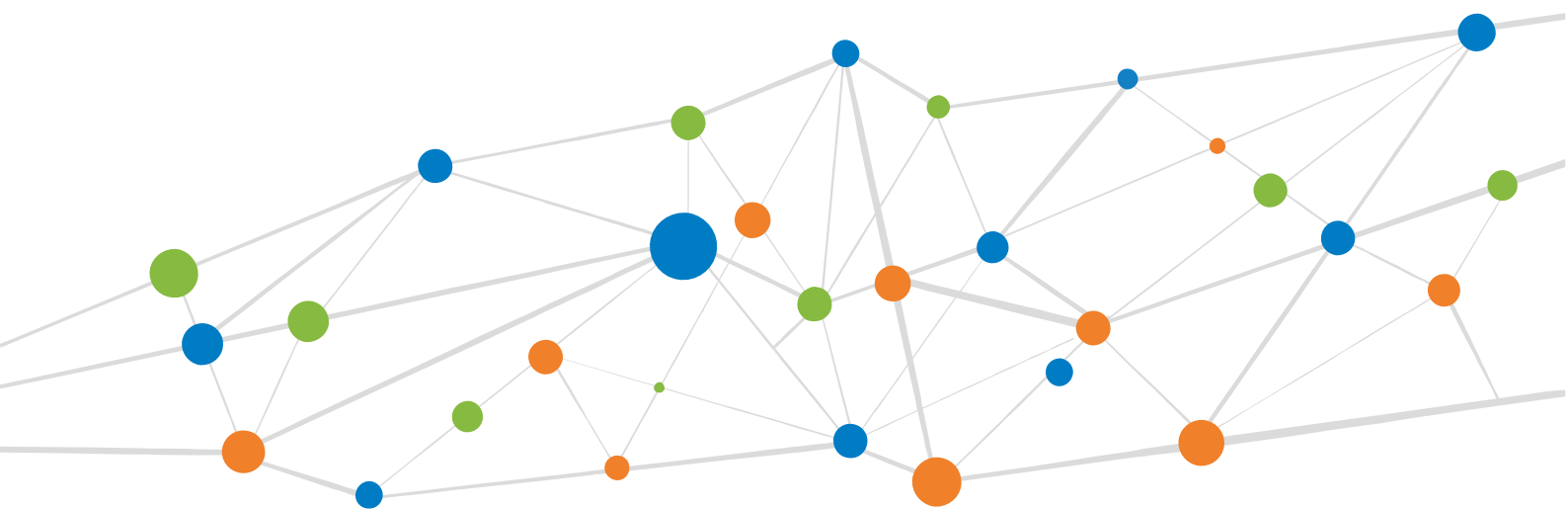


Duration Overlay Performance Attribution Model

Master project

07-10-2016



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Preface

In november 2015 I started with my master project at Ortec Finance. I remember being really excited, although in hindsight it was quite clear I did not fully comprehend the topic I would research. My background is business analytics, which mostly entails analyzing big data, optimizing business processes and predicting the future with statistics and data mining tools. So, whereas my fellow class mates did internships in this field, I was going to build a duration overlay performance attribution model. It is risk management, but not the risk management I expected.

After a few weeks it became clear that investment performance was a total new field for me. I started literally at the bottom. My only advantage was my knowledge of fixed income securities and portfolio management. Furthermore, the mathematics you will see in this project report is rather “easy”, but I know from the last year it is rather complex. Every calculation represents a decisions, where the outcome has to be correct, intuitive and tell the background story of where it came from.

Needless to say, it was a little overwhelming. But it also gave me the chance to research a whole new topic for me. Although my knowledge is still just scraping the surface, I am proud of the results and how far I have come. It also showed me the diversity of my education, my foundation is wide spread, from data mining to option theory. In summary, I had fun, expanded my horizon and profoundly satisfied to say that I did something different than expected.

I would also like to express my thanks to the people who were there for me during my internship. Firstly, I am most thankful to Bas Leerink, my supervisor at Ortec Finance. He spend hours answering my questions, expanded my knowledge and kept me on the right track regarding the topical overflow of information. Due to the fact this was a whole new topic for me, he probably spent more time on me than expected. Also, thanks for checking my English until to the point of boredom: writing reports is not one of my greatest strengths.

Secondly, thank you Teodor, for being my supervisor at the VU. Although I was from a different department, he still agreed. This is only accepted in exceptional circumstances and word had it that FEWEB professors would not agree that easily. In a nutshell: I was very lucky. Looking at his resume later on, I was actually quite honored.

Lastly, I also want to thank Marc Francke and Maarten Niederer. Marc also had the unfortunate task to check my grammar and he also supervised the progress of my internship. Maarten, on the other hand, was there for me during my first three weeks of my internship, when Bas was on holiday. He patiently answered all my questions. So thank you both for your time.

I also want to give special thanks to Vincent Mekking, who approved my visitation to the “Dutch Performance Management Round Table”. I went with Lucas Vermeulen and most people there were professionals in the field of investment performance. Me being there was clearly unique, I felt like a kid in a candy store. Thank you so much for this opportunity!

Abstract

As a result of a research to the interest rate risk management within pension funds, the DNB concluded that the decisions made regarding interest rate risk were insufficiently defined. Furthermore, pension funds did not have a clear insight in their interest rate risk exposure and the overlay manager had a free rein regarding implementing the overlay portfolio. Consequently, the ex-post overlay evaluation of a pension fund was also inadequate.

Therefore, we described how we build a duration overlay performance attribution model. The goal of the model was to capture the interest rate risk management process of a pension fund, so that a valid ex-post overlay evaluation could be achieved. Such a model did not yet exist, however.

In order to build such a model, we first investigated its validation requirements. By definition a duration overlay performance analysis evaluates the excess return of the overlay portfolio relative to a benchmark by decomposing the return into risk factors that follow both the decision- and investment process of the overlay portfolio. Therefore, we concluded that the model:

- should be based on the risk factors of the underlying securities;
- should be able to evaluate the decisions of the overlay manager relative to a benchmark;
- should follow the investment- and decision making process of the overlay portfolio.

The first requirement was met by a fixed income performance attribution model, because an overlay portfolio consists of fixed income instruments. For the other two requirements, we added the following features based on the investment- and decision making process of the overlay portfolio:

- The ability to define the interest rate risk exposure of the pension fund.
- The ability to define the hedge of the overlay benchmark and the overlay portfolio.
- The ability to evaluate the decisions made regarding the interest rate risk management
- The ability to define an overlay benchmark.

We tested our model with data provided by Ortec Finance. Based on the outcome, we conclude that our model has the needed requirements to act as a valid duration overlay performance attribution model. The outcomes gives a clear insight in the interest rate risk exposure, its distribution over the different maturities of the portfolio and we are able to derive the decisions made regarding its management. Also, it states the hedge decisions of the overlay manager and evaluates his decisions relative to the benchmark. Furthermore, the outcome allows for the board to monitor the overlay manager over time.

The model does, however, need some improvements. The focus of the model lies on the interest rate risk management process, therefore the fixed income performance attribution model used only consists of the most basic features.

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1. Introduction

Since the year 2008 a major part of the pension funds in the Netherlands lack the financial resources to meet their future liabilities, our pensions. The two main reasons for this development is the low interest rate and the longevity of the participants, which resulted in an increase of the value of the liabilities. This led to a decrease of the funding ratio, the ratio between the assets and liabilities that measures the ability of a pension fund to pay its future pensions.

As a result pension funds became more aware of their interest rate risk. In order to manage this risk, investing in duration overlay portfolios became more popular. A pension fund usually invests a part of its assets in a fixed income portfolio, due to its save character and to match the cash flows of the liabilities. Both the fixed income portfolio and the liabilities contain the same sensitive nature towards the yield curve, also called interest rate risk. However, generally there exists a mismatch between these sensitivities as a result of the long term horizon of the liabilities. By investing in an overlay portfolio, consisting of fixed income instruments, a pension fund is able to decrease this mismatch.

Due to these developments the Dutch central bank (DNB, De Nederlandsche Bank) administered an inspection to the interest rate risk management by pension funds. They concluded that the pension funds policy, as well as their ex-post evaluation reports, regarding their interest rate risk management were insufficient. As a result, the DNB urged the pension funds to improve in these areas.

Nonetheless, a framework for an ex-post evaluation for an interest rate overlay portfolio, also called an duration overlay performance attribution model, does not yet exist. There do exist fixed income performance attribution models, but although the overlay portfolio does consist of fixed income securities, these frameworks focus mostly on sources of return and neglect the part of risk attribution and the decision making process of the overlay portfolio. Popular fixed income performance attribution models were introduced by Campisi (2000), Lord (1997) and van Breukelen(2000).

Moreover, a high number of clientele of Ortec Finance are pension funds. Consequently, the question how to provide an ex-post overlay performance attribution that represents the position of their interest rate risk and overlay portfolio is relevant for Ortec Finance. Although Ortec Finance does own software to provide detailed ex-post performance analysis, this analysis does not extend to duration overlay portfolios yet.

Therefore the aim of this thesis is to develop an ex-post duration overlay portfolio performance analysis framework that captures the interest rate risk management of a pension fund. The outcome of this model should give a clear insight of the interest rate risk exposure, the decisions made regarding this risk and the outcome of these decisions. Consequently, the model can be used to evaluate and monitor the overlay portfolio.

The methodological approach taken to build this model is to begin with an existing fixed income performance attribution model and extend it to the decision making process of an overlay portfolio. We begin with a general introduction of a pension fund and some preliminary information about fixed income securities, interest rate risk and hedging this risk. Where after we will describe existing models and the choices for our model in more detail. Eventually we will test our model on actual portfolio data and discuss the results.

2. A pension fund

In this chapter we give a general framework of a pension fund, which leads to a better understanding of the interest rate risk a pension fund faces. We will see that the liabilities and a part of the assets of a pension funds both are prone to interest rate sensitivities. Therefore, we will describe the balance sheet of a pension fund more in depth. Other risk drivers that a pension fund may face are also mentioned in this chapter.

2.1. Dutch Pension Funds

The Dutch pension system is a combination of a state pension and an individual pension. The state pension, also known as AOW, is a pay-as-you-go pension, which means that the whole Dutch workforce contribute to these pensions. The individual pensions, on the other hand, are financed by their own contributions and returns on their investments.

The individual pensions are usually conducted by a pension fund, which is founded by a certain company, institution or union which provides retirement income for their retired employees. This means that participating in an individual pension scheme is mandatory when a certain sector or employee decide to provide it. Therefore, it is not surprising that more of 90% of the employees in the Netherlands are participants of a pension fund.

The pension fund schemes that are used are defined benefit schemes, defined contribution schemes or a combination of the two. Where the pension fund bears the investment risk in a defined benefit scheme and the participants the risk in a defined contribution scheme respectively. Obviously, the hybrid scheme divides the risk between the pension fund and the participants, which is also the scheme mostly used in the Netherlands.

2.2. Interest rate risk

Like we mentioned in the previous section, pension funds invest the contributions of their participants to be able to pay their future pensions. We also explained in the introduction that the funding ratio measures the ability of a pension funds to do so, where the funding ratio is the ratio between the assets and the liabilities, as defined below.

$$\text{Funding Ratio} = \frac{\text{Total Value Assets}}{\text{Present Value Liabilities}} \quad (2.1)$$

So in order to be able to establish the funding ratio, let us take a closer look at the assets and the liabilities by using the balance sheet of a pension fund in figure 2.1.

2.2.1. Liabilities

The liabilities of a pension fund consist of the future pensions. However, looking at figure 2.1 a small fraction is taken by a surplus or deficit, because the assets and liabilities are equal in the balance sheet. Consequently, a higher value of assets results in a surplus on the liabilities side and vice versa. A pension fund obviously strives for a surplus, because that would yield a higher funding ratio.

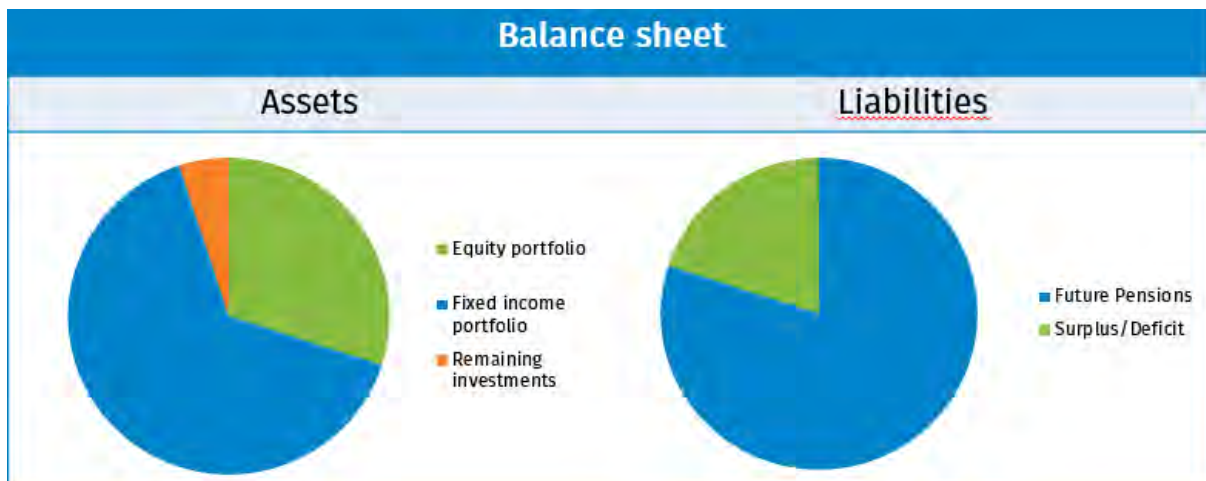


Figure 2.1 Balance sheet of a Pension fund

2.2.2. Assets

The assets side of the balance sheet are represented by the investments. Usually pension funds investment the contributions in equities and fixed income securities, where a small fraction goes to remaining investments. Both equity and fixed income investments have their own purpose:

- **Fixed income investments.** Fixed income investments are seen as safe investments. Furthermore, with the use of fixed income securities they want to match the cash flows and maturities of the liabilities. Also, the fact that fixed income securities have equal risk factors as the future pensions, gives the fixed income portfolio certain hedge capabilities regarding these risk factors. A more detailed explanation is given below.
- **Equity investments.** The future pensions are value as nominal pensions, which means that inflation is not taken into account. By investing in equities, the pension strive to be able to pay out real pensions to their participants.

Now that we know what both the assets and liabilities entail, to establish the funding ratio we only need to determine the present value of the future pensions, which we can do by applying the discounted cash flow method. By discounting the cash flows of the future pensions with the current yield, we are able to determine how much funding a pension fund needs to meet its future liabilities. The yield, however, changes over time, as such the value of the liabilities also fluctuates. This is also known as interest rate risk, which is an inherent risk for pension funds.

To be able to manage this risk, a pension fund wants to create the same risk exposures on the assets side. Since fixed income securities, such as bonds, hold an equal sensitive nature to yield changes, pension funds invest in fixed income portfolios, like we have seen in figure 2.1. In the next chapter we will describe interest rate risk and how to manage it in a more detail.

2.3. Other risk sources

Although interest rate risk is the main source of risk for a pension fund, they are also prone to other risks, which will be explained below:

Investment Risk

Investment risk occurs from being investment in certain markets/securities, in particular: market risk, counter party risk and currency risk. When a part of the assets consist of equities, then this part is also exposed to market risk, because market movements have a high influence on the market value. Counterparty risk exposure is the risk a counter party could default. Furthermore, currency risk is the exposure to exchange rate changes when a pension fund invests in securities based in another currency.

Longevity risk

The risk of an increase in the life expectancy of the customer is also known as longevity risk. This risk can increase the liabilities significantly, because pensions are payed until the participant passes away and therefore could decrease the funding ratio. It represents the risk that the pensioners outlive the pension funds' assets due to outliving the life expectancy.

Liquidity risk

The risk a pension fund is not able to pay their short term liabilities due to illiquidity of the assets. In this case the value of the assets is sufficient to pay their liabilities, but they are unable to do so, because their funds only consist of illiquid assets.

Outsourcing risk

By outsourcing certain processes, like asset management, the pension fund may be exposed to the risk of the decrease in quality, integrity or continuity of the outsourced activities. For direct orders of the pension fund could easily be ignored.

Operational risk

All existing processes are exposed to certain failures, like miscommunication, system failures, and unforeseen circumstances. This risk consists of every event that may happen due to deficiencies/failures which influence the performance of the pension fund and is called operational risk.

Compliance risk

A pension fund is like every other company subject to certain legislation. The risk the pension fund is not living up to this legislation or does not recognize timely changes in legislation, lie within compliance risk.

Reputation risk

The risk the pension fund incurs reputation damage due to own actions.

Some of these risk exposures have greater significance than others. To reduce these risks the board of the pension fund decides how they are individually handled.

3. Interest rate risk

In the previous chapter we described the composition of the balance sheet of a pension fund, where we explained that a part of the portfolio of a pension fund consists of fixed income portfolio, which hold the same interest rate risk exposure as the liabilities. In order to build an overlay performance attribution model that gives a clear insight in this risk, we need some preliminary information about the definition of a duration overlay performance attribution model. Also, knowledge about how to quantify interest rate risk in fixed income securities and how to hedge the interest rate risk, would give us a better understanding in how to formulate the framework. Therefore, we will describe the interest rate risk in fixed income securities in more detail in this chapter. In addition, we shall elaborate upon hedging procedures for this particular risk.

3.1. Interest rate risk in fixed income securities

We shall start this section with describing the definition of an overlay performance attribution analysis. We will see that based on this definition we will have a starting point to derive the interest rate risk within a fixed income security. Where after we shall describe various ways to quantify interest rate risk and which method we will adopt for our model.

3.1.1. Overlay performance attribution analysis

Like we stated, we start with the definition of a duration overlay performance attribution analysis: a duration overlay performance analysis evaluates the excess return of the overlay portfolio relative to a benchmark by decomposing the return into risk factors that follow both the decision- and investment process of the overlay portfolio¹. We know that one of these risk factors is interest rate risk. So, by decomposing the return of a single fixed income security we would be able to identify its interest rate risk exposure distinctively.

3.1.2. Fixed income securities

The return of a fixed income security is the gain or loss of the security over a certain time period. This can easily be determined by subtracting the value of the security at the beginning of the performance period from the value at the end of the period. So, before defining the return, let us first define the value of a fixed income security at time t as follows:

$$V(t, y) = \sum_{i=1}^I \frac{c_{t_i}}{(1+y)^{t_i-t}}, \quad (3.1)$$

where c_{t_i} is the cash flow at a certain future time t_i in the future, $\{c_{t_i} : i = 1, \dots, I\}$ and y is the yield to maturity. Besides the fact that we are now able to define the return, we can also conclude from equation 3.1 that a security consists of the following two risk factors: time and yield, where we define the latter as interest rate risk.

¹ In chapter 4 we will elaborate upon an overlay performance attribution analysis in more detail.

3.1.3. Taylor series expansion

With the use of the value equation 3.1, we can now define the percentage return of a security on interval $[t_{i-1}, t_i]$ as follows:

$$r(t_{i-1}, t_i) = \frac{V(t_{i-1}, y_{i-1}) - V(t_i, y_i) - \text{cash flows in} + \text{cash flows out}}{V(t_{i-1}, y_{i-1})}, \quad (3.2)$$

where the *cash flows in* and *cash flows out* are the in- and outgoing cash flows during the time interval. To determine explicitly how much each distinctive risk factor, the time and yield, contributed to the return, we would like to rewrite the equation into an additive form. To be able to do so we use a multivariate Taylor series, where we use the multivariate Taylor series because equation (3.1) is based on two arguments.

Theorem 3.2.1 (Multivariate Taylor series) *if $f : \mathbb{R}^n \rightarrow \mathbb{R}$ is $n + 1$ times continuously differentiable, $a = (a_1, \dots, a_n) \in \mathbb{R}^n$, and $x = (x_1, \dots, x_n) \in \mathbb{R}^n$, then*

$$\begin{aligned} f(x_1, \dots, x_n) &= f(a_1, \dots, a_n) + \frac{1}{1!} \sum_{i=1}^n \frac{\partial^1 f(a_1, \dots, a_n)}{\partial x_i} (x_i - a_i) \\ &+ \frac{1}{2!} \sum_{i=1}^n \sum_{j=1}^n \frac{\partial^2 f(a_1, \dots, a_n)}{\partial x_i \partial x_j} (x_i - a_i)(x_j - a_j) \\ &+ \frac{1}{3!} \sum_{i=1}^n \sum_{j=1}^n \sum_{v=1}^n \frac{\partial^3 f(a_1, \dots, a_n)}{\partial x_i \partial x_j \partial x_v} (x_i - a_i)(x_j - a_j)(x_v - a_v) \\ &+ \sum_{k=4}^n \left(\frac{1}{k!} \sum_{j_1}^n \dots \sum_{j_k}^n \frac{\partial^k f(a_1, \dots, a_n)}{\partial x_{j_1} \dots \partial x_{j_k}} (x_{j_1} - a_{j_1}) \dots (x_{j_k} - a_{j_k}) \right) \\ &+ \mathbf{R}_{k+1}(a_1, \dots, a_n). \end{aligned}$$

Moreover, if $\lim_{k \rightarrow \infty} \mathbf{R}_{k+1}(a_1, \dots, a_n) = \mathbf{0}$ then,

$$f(x_1, \dots, x_n) = f(a_1, \dots, a_n) + \sum_{k=1}^{\infty} \left(\frac{1}{k!} \sum_{j_1}^n \dots \sum_{j_k}^n \frac{\partial^k f(a_1, \dots, a_n)}{\partial x_{j_1} \dots \partial x_{j_k}} (x_{j_1} - a_{j_1}) \dots (x_{j_k} - a_{j_k}) \right)$$

By applying the Taylor series for small changes in x , we get:

$$\begin{aligned} V(t_i, y_i) &\approx V(t_{i-1}, y_{i-1}) + \frac{\partial V(t_{i-1}, y_{i-1})}{\partial t_{i-1}} (t_i - t_{i-1}) \\ &+ \frac{\partial V(t_{i-1}, y_{i-1})}{\partial y_{i-1}} (y_i - y_{i-1}) \\ &+ \frac{1}{2} \frac{\partial^2 V(t_{i-1}, y_{i-1})}{\partial y_{i-1}^2} (y_i - y_{i-1})^2 \\ &\approx V(t_{i-1}, y_{i-1}) + \frac{\partial V(t_{i-1}, y_{i-1})}{\partial t_{i-1}} \Delta t + \frac{\partial V(t_{i-1}, y_{i-1})}{\partial y_{i-1}} \Delta y + \frac{1}{2} \frac{\partial^2 V(t_{i-1}, y_{i-1})}{\partial y_{i-1}^2} (\Delta y)^2 \end{aligned} \quad (3.3)$$

Rewriting equation (3.3) to equation (3.2) then gives:

$$\begin{aligned}
 r(t_{i-1}, t_i) &= \frac{V(t_{i-1}, y_{i-1}) - V(t_i, y_i)}{V(t_{i-1}, y_{i-1})} \\
 &\approx \frac{1}{V(t_{i-1}, y_{i-1})} \frac{\partial V(t_{i-1}, y_{i-1})}{\partial t_{i-1}} \Delta t + \frac{1}{V(t_{i-1}, y_{i-1})} \frac{\partial V(t_{i-1}, y_{i-1})}{\partial y_{i-1}} \Delta y \\
 &\quad + \frac{1}{2} \frac{1}{V(t_{i-1}, y_{i-1})} \frac{\partial^2 V(t_{i-1}, y_{i-1})}{\partial y_{i-1}^2} (\Delta y)^2,
 \end{aligned} \tag{3.4}$$

where we find the following common terms:

- $y_{i-1} = \frac{1}{V(t_{i-1}, y_{i-1})} \frac{\partial V(t_{i-1}, y_{i-1})}{\partial t_{i-1}}$,
- **Modified Duration** $= \frac{1}{V(t_{i-1}, y_{i-1})} \frac{\partial V(t_{i-1}, y_{i-1})}{\partial y_{i-1}}$,
- **Modified Convexity** $= \frac{1}{2} \frac{1}{V(t_{i-1}, y_{i-1})} \frac{\partial^2 V(t_{i-1}, y_{i-1})}{\partial y_{i-1}^2}$.

Thus, we were able to decompose the return of a fixed income security into its risk factors time and yield distinctively by rewriting the return equation (3.2) to an additive form (3.4). Also, we found two commonly used and worldwide accepted fixed income security terminologies in the process. As such, we can simplify equation (3.4) to:

$$r(t_{i-1}, t_i) \approx y_{i-1} \Delta t - D_M \Delta y + \frac{1}{2} C_M (\Delta y)^2, \tag{3.5}$$

where,

- D_M is the modified duration of the bond
- C_M is the modified convexity of the bond

Figure 3.1 below graphically explains the relationship between the value of a security and the yield. From equation 3.5, we conclude that the modified duration and convexity together measure the impact of a yield change on the value of a security, which indicates that we can use them together to measure interest rate risk.

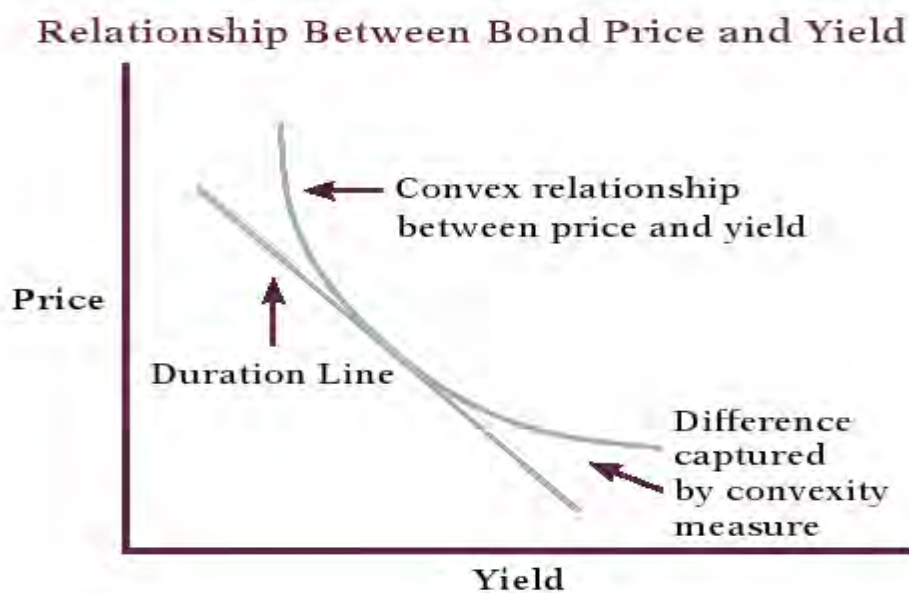


Figure 3.1 Relationship between bond price and yield: Duration and convexity, for small changes in the yield:-

3.1.4. Duration and convexity

In the previous section we derived a method to explicitly evaluate the bond risk factors time and yield, which led us to the common bond terms duration and convexity. However, there exist several different concepts of duration besides modified duration, like Macaulay duration, effective duration, key rate duration and money duration. We will describe each notion in more detail below. Note that we only will only elaborate upon duration, as convexity can be derived as the same matter as duration by using equation (3.4).

Macaulay duration

Frederic Macaulay was a Canadian economist, who was the creator of bond duration (Macaulay (1938)). He saw every payment related to the bond, the coupon payments and the redemption, as individual zero coupon bonds. This because zero coupon bonds with the same maturity were prone to the same interest rate risk, where bonds containing different coupon rates, but holding the same maturity, differ in interest rate risk exposure. Taking the proportional value of these zero coupon bonds to the value of the bond, gives the weighted average of each payment. These average values together form the effective maturity of the bond. Therefore, the Macaulay duration is expressed in years. Equation 3.6 below gives the expression to calculate the weighted average of the cash flows:

$$w_t = \frac{c_t}{V(t,y)}, \quad (3.6)$$

which we can use to calculate the Macaulay duration using the following equation:

$$D = \sum_{i=1}^I t_i * w_{t_i}, \quad (3.7)$$

where the outcome is expressed in years. Note that liabilities are fixed income securities with a single cash flow, better known as zero coupon bonds. So for the liabilities equation 3.6 would result in a weighted average of exactly one, resulting in the fact that the Macaulay duration is equal to the maturity of the liabilities.

Although this form of duration is widely used, we can't derive it from our Taylor expansion equation (3.4), which means that it is not advisable to use this form for evaluating interest rate risk. We can, however, determine the modified duration (equation 3.4) with the use of Macaulay duration as follows:

$$D_M = \frac{D}{(1+\frac{y}{k})}, \quad (3.8)$$

where k is the number of coupon payments each year.

Effective duration

Although modified duration could be used for all fixed income securities, it does assume that payments do not change over time. This means that we cannot use that form of duration for fixed income securities with embedded options. We can, however, use effective duration by representing the modified duration with the use of a partial derivative with respect to y:

$$\begin{aligned} D_E &= \frac{1}{V(t,y)} \frac{\partial V(t,y)}{\partial y}, \\ &= \lim_{h \rightarrow 0} \frac{1}{V(t,y)} \left(\frac{V(t,y+h) - V(t,y-h)}{2h} \right), \\ &\approx \frac{1}{V(t,y)} \left(\frac{V(t,y+h) - V(t,y-h)}{2h} \right), \end{aligned} \quad (3.9)$$

for small h . Choosing $h = 1\%$ for securities without an embedded option would result again in the modified duration.

Key rate duration

All cash flows of a security are usually discounted by the securities yield to maturity, like in equation (3.1). However, each cash flow actually has a distinct yield. Discounting each cash flow with its own yield, would result in the following changes in equation (3.2) and (3.4):

$$\begin{aligned}
 r(t_{i-1}, t_i) &= \frac{V(t_{i-1}, y_{1_{i-1}}, \dots, y_{n_{i-1}}) - V(t_i, y_1, \dots, y_n)}{V(t_{i-1}, y_{1_{i-1}}, \dots, y_{n_{i-1}})}, & (3.10) \\
 &\approx \frac{1}{V(t_{i-1}, y_{1_{i-1}}, \dots, y_{n_{i-1}})} \frac{\partial V(t_{i-1}, y_{1_{i-1}}, \dots, y_{n_{i-1}})}{\partial t_{i-1}} \Delta t + \\
 &\quad \frac{1}{V(t_{i-1}, y_{1_{i-1}}, \dots, y_{n_{i-1}})} \sum_{j=1}^n \frac{\partial V(t_{i-1}, y_{1_{i-1}}, \dots, y_{n_{i-1}})}{\partial y_{j_{i-1}}} \Delta y \\
 &\quad + \frac{1}{2} \frac{1}{V(t_{i-1}, y_{1_{i-1}}, \dots, y_{n_{i-1}})} \sum_{j=1}^n \frac{\partial^2 V(t_{i-1}, y_{1_{i-1}}, \dots, y_{n_{i-1}})}{\partial y_{j_{i-1}}^2} (\Delta y)^2, & (3.11)
 \end{aligned}$$

where $y_{j_{i-1}}$ is the yield for cash flow j at time $i - 1$, $\{y_{j_{i-1}} : j = 1, \dots, n\}$. Now, if we were to shock $y_{j_{i-1}}$ with 1%, we can define KRD_{M_j} as the key rate duration correspondent to this particular shock:

$$KRD_{M_j} = \frac{1}{V(t_{i-1}, y_{1_{i-1}}, \dots, y_{n_{i-1}})} \frac{\partial V(t_{i-1}, y_{1_{i-1}}, \dots, y_{n_{i-1}})}{\partial y_{j_{i-1}}} \Delta y. \quad (3.12)$$

Furthermore, we get the modified duration again when we sum up the key rate durations as follows:

$$D_M = \sum_{j=1}^n KRD_{M_j}. \quad (3.13)$$

This yields that key rate durations give an insight in the interest rate risk exposure distribution of a security. Note that we can also derive the effective key rate duration in the same manner with the use of equation 3.9.

Money duration

We defined the return of a single security in equation (3.2), but we could also look at absolute numbers, which yields the gain of the security:

$$g(t_{i-1}, t_i) = V(t_{i-1}, y_{i-1}) - V(t_i, y_i), \quad (3.14)$$

$$\approx \frac{\partial V(t_{i-1}, y_{i-1})}{\partial t_{i-1}} \Delta t + \frac{\partial V(t_{i-1}, y_{i-1})}{\partial y_{i-1}} \Delta y + \frac{1}{2} \frac{\partial^2 V(t_{i-1}, y_{i-1})}{\partial y_{i-1}^2} (\Delta y)^2, \quad (3.15)$$

where we already derived (3.14) in equation (3.3), from which we define the modified money duration:

$$MD_M = \frac{\partial V(t_{i-1}, y_{i-1})}{\partial y_{i-1}} \Delta y. \quad (3.16)$$

Looking at equation (3.15), we conclude that modified money duration is the value change of the security due to a change in y .

Clearly, effective money duration MD_e could be obtained in the same manner with the use of equation (3.9)

Portfolio duration

Now that we are able to construct the duration and convexity of a single security, we can also apply this to define the duration of a whole portfolio. We assume a portfolio with value V_P , duration D_M^P , convexity C_M^P , yield y_P and n fixed income securities, each with value $V(t_{k_{i-1}}, y_{k_{i-1}})$, duration D_{M_k} convexity C_{M_k} and yield y_k for $k \in \{1 \dots N\}$. This yields:

$$\begin{aligned}
 D_M^P &= \frac{1}{V_P} \frac{\partial V_P}{\partial y_P}, \\
 &= \frac{1}{V_P} \left[\sum_{k=1}^N V(t_{k_{i-1}}, y_{k_{i-1}}) \frac{1}{V(t_{k_{i-1}}, y_{k_{i-1}})} \frac{\partial V(t_{k_{i-1}}, y_{k_{i-1}})}{\partial y_{k_{i-1}}} \right], \\
 &= \sum_{k=1}^N \frac{V(t_{k_{i-1}}, y_{k_{i-1}})}{V_P} D_{M_k}, \\
 &= \sum_{k=1}^N w_k D_{M_k}, \tag{3.17}
 \end{aligned}$$

where w_k is the weight in the portfolio of security k .

Based on the described forms of duration, we know that key rate duration and money duration are merely extensions, which can be calculated with modified or effective duration. Considering both the modified and effective duration can be derived from the Taylor Series expansion, both durations could be used for our purposes. However, we advise to use the effective duration, also for the extensions, since it also can be used for fixed income securities with embedded options. Furthermore, like we mentioned before, choosing $h = 1\%$ for equation 3.9 would yield an equal modified and effective duration for securities without embedded options.

For simplifying reasons, we will use the terms D , MD , KRD from now on, which indicates or is based on the effective duration, unless stated otherwise.

3.2. Hedging interest rate risk

From equation 3.4 it became clear that modified and effective duration measures the sensitivity of a security to changes in yield in an easy and straightforward approach. Furthermore, it is worldwide accepted terminology in fixed income portfolio management. As such, we can use duration to quantify and hedge interest rate risk in portfolios.

3.2.1. Interest rate risk exposure

In order to explicitly evaluate the risk exposure of a pension fund, let us start with a scenario where there is no interest rate risk exposure: a funding ratio of 125% which does not change over time. So when we look at the return of both the liabilities (r_L) and the fixed income portfolio (r_A) on interval $[t_{i-1}, t_i]$, the following equation must hold:

$$r_A(t_{i-1}, t_i) = r_L(t_{i-1}, t_i),$$

which yields that the duration between the portfolio and the liabilities should be equal. We therefore define the interest rate risk of a portfolio as:

$$\text{Exposure} = D_P - D_L, \quad (3.18)$$

where,

- D_P is the duration of the fixed income portfolio
- D_L is the duration of the liabilities

But what does the exposure actually mean? Looking at the equation, we conclude that the exposure is either positive or negative, although the duration of the liabilities in a pension fund tend to be higher than of the fixed income portfolio. For example, when a pension fund has, an exposure of -10 D_E and the yield drops with 1%, the value of the liabilities will increase 10% more than the value of the fixed income portfolio.

3.2.2. Interest rate hedging

To immunize the interest rate risk for a pension fund, the return between the liabilities and the fixed income portfolio of a pension fund, and therefore the duration, should be equal. From the previous chapter we know that this is rarely the case, which means we have to hedge the exposure away by ensuring that the duration becomes equal.

Since a pension fund usually has a negative exposure to interest rate risk, we want to add duration to the fixed income portfolio to hedge it. Commonly the following three securities are used to do so:

- Bonds
- Swaps
- Swaptions

All securities together to hedge the interest rate risk are commonly known as the overlay portfolio.

From what we have learned in the previous sections, it is clear that bonds can be used to increase the duration of a portfolio. . To see that swaps and swaptions can achieve the same, we will describe the characteristics of swaps and swaptions below in more detail.

Swaps

Swaps are over-the-counter derivatives where two counterparties exchange predetermined cash flows based on their underlying instruments. Although swaps can be used in several different markets, we will only discuss the swaps based on interest rates as underlying instrument. Based on a predetermined notional amount, they will exchange the six-month Interbank Offered Rate (For example: the EURIBOR in Europe, the LIBOR in London and the Federal Funds rate in the USA) and a predetermined fixed rate during a predetermined amount of years. The swap curve at that moment will determine the fixed rate investors are willing to trade for the six-month IBOR, which is initially the same amount. This means, also for no arbitrage purposes, the value of a swap is initially zero. However, this can change when the counterparties agree to a spread. In figure 3.1 the idea of a swap is visualized. We can see that a swap has two legs, one fixed leg and one floating leg. Also, depending on the contract, an investor owns a payer or receiver swap.

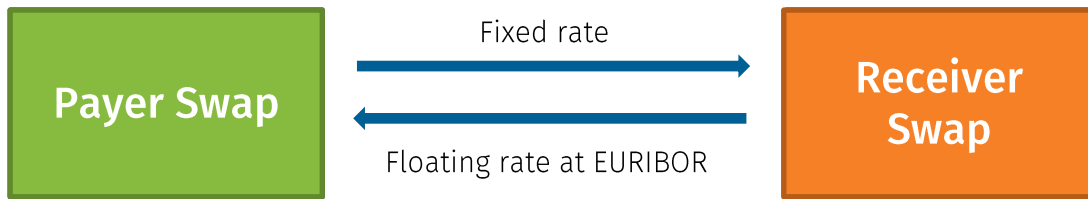


Figure 3.1 Swap exchange

Although the notional is never exchanged in a swap contract, we can see both the fixed and the floating leg as a bond. The only difference lies in the maturity. Initially, the bond of the fixed leg has a maturity of the contract horizon, whereas the bond of the floating leg has a maturity of six months (when the 6 month IBOR is exchanged).

Furthermore, we can see the floating leg as a new investment in a six month bond against the new six-month EURIBOR over and over again. This means, although the value of a swap is initially zero, it can get a positive or negative value over time. The value of a swap can therefore simply be determined by subtracting the value of the floating leg bond from the value of the fixed leg bond:

$$\text{Value receiver swap} = V_{\text{fixed}}(t, \text{fixed yield}) - V_{\text{float}}(t, \text{IBOR}), \quad (3.19)$$

$$\text{Value payer swap} = V_{\text{float}}(t, \text{IBOR}) - V_{\text{fixed}}(t, \text{fixed rate}),$$

where,

- V_{fixed} is the value of the fixed leg bond,
- V_{float} is the value of the floating leg bond

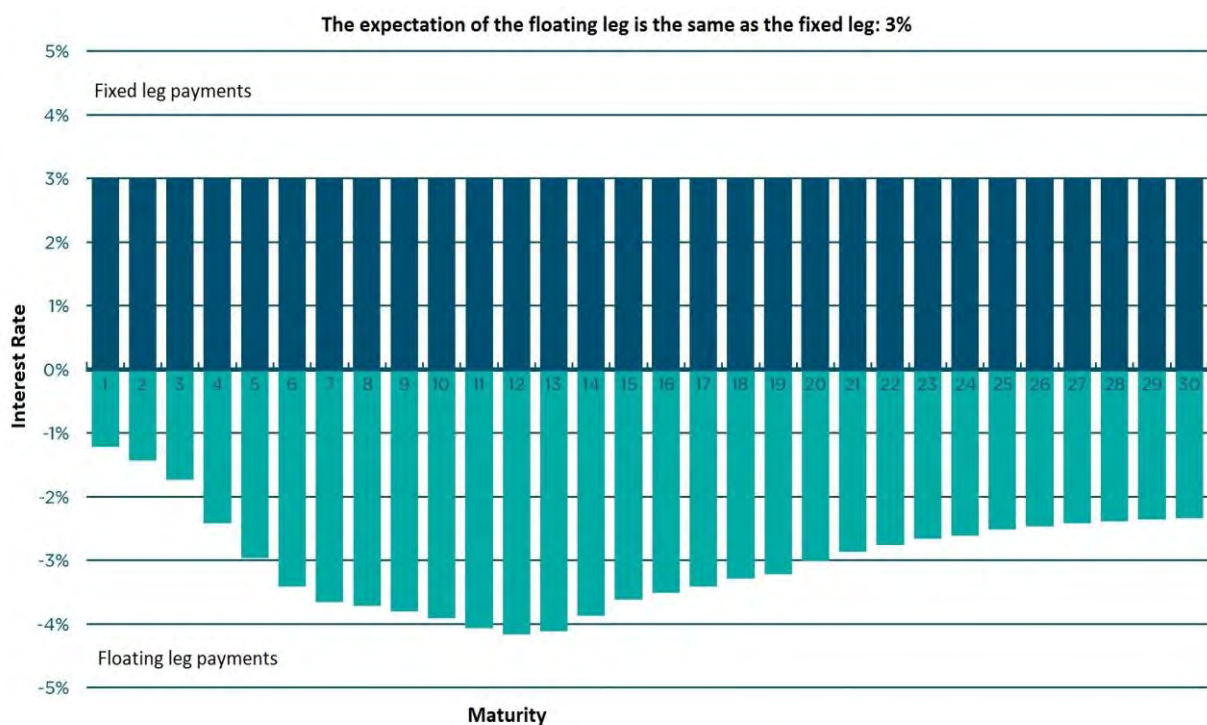


Figure 3.2 Swap payments during a horizon of 30 years

In figure 3.2 we can find an example of the payments of a swap with a maturity of 30 years. Together with the knowledge of the previous chapters we conclude the following about swaps:

- It can be used to exchange a floating rate for a fixed rate and vice versa.
- It is perfect to use for hedging interest rate risk: if the interest rate goes up, it generates a positive return and vice versa. So it levels the price changes in the liabilities.
- Because of the higher duration of the fixed leg, a swap can either decrease or increase the duration of the assets of a pension fund.

This means that swaps, like fixed income securities, can be used for hedging the interest rate risk, which is due to the fact that although swaps are derivatives, they hold the same properties as fixed income securities. Another benefit of swaps is the availability of these derivatives in high maturities in contrary to fixed income securities.

Swaptions

A swaption is an option on a swap. An option gives the owner the right to buy or sell a security for a predetermined price at a certain time in the future². A swaption, however, gives the owner the right to enter a swap for a predetermined notional and fixed rate, since a swap initially has no value. This option gives investors the chance to speculate about the course of the interest rate. Like for other options, the premium of a swaption can be determined with the use of the Black Scholes formula³.

The influence of a swap and swaption on the funding ratio of a pension fund is visualized in figure 3.3. We see that a swap levels the change in the funding ratio, when the interest rate increases or decreases. A swaption on the other hand protects against a decrease in the interest rate, but also gives the upside when the interest rate increases. Obviously, investors would prefer swaptions when interest rates are rising. Keep in mind the circumstances in figure 3.3 only hold during one period and represent a 100% hedge of the interest rate risk. After this period the value of the swap could have changed and thereby its effect on the funding ratio.

Although swaps and swaptions can be used to hedge interest rate risk, be advised that both derivatives come with their own individual sets of risks. Most of these risks we already know from chapter one:

- Counter party risk
- Operational risk
- Liquidity risk: depending on a payer/receiver swap, when the value of the swap becomes negative/positive, the pension fund should own enough cash to comply with the terms of the swaps

But also leverage arises as a new sort of risk: the risk losses can grow infinitely during the swap contract. Swaptions also hold this risk, but the loss is limited to the swaption premium.

² See DeMarzo P., Berk J., Corporate Finance, 2011, for more option theory

³ In lecture notes pages.stern.nyu.edu/~dbackus/3176/adlec4.pdf the pricing of swaptions is elaborated upon.

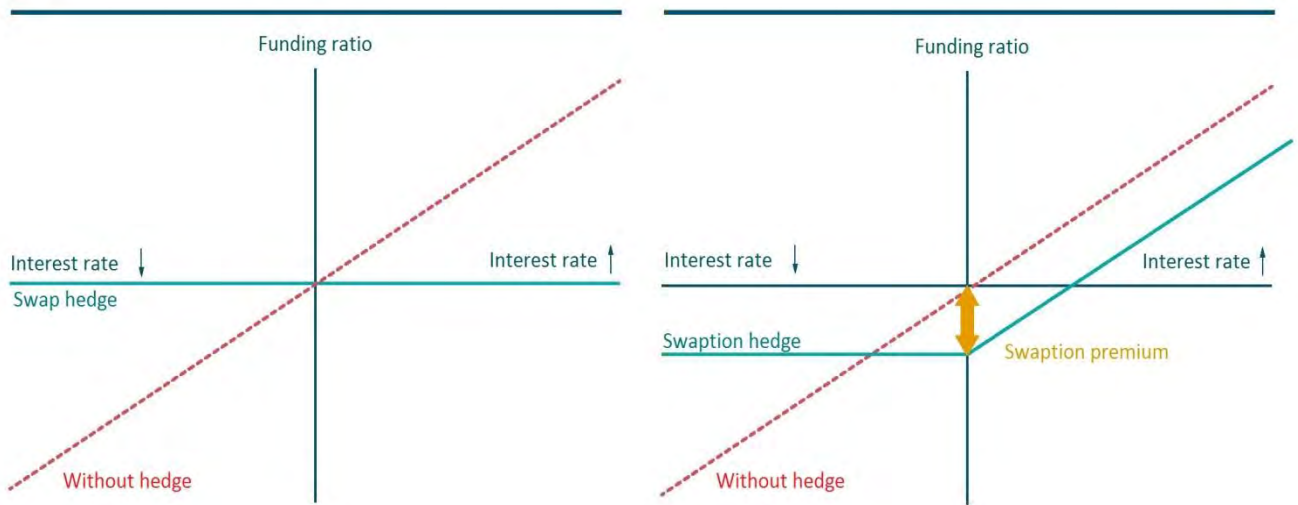


Figure 3.3 Influence of a swap/swaption on the funding ratio

4. Overlay Performance Attribution Model

In this chapter we will build the performance attribution model for an overlay portfolio. In the previous chapter we mentioned that an overlay performance analysis evaluates the excess return of the overlay portfolio relative to a benchmark by decomposing the return into risk factors that follow both the decision- and investment process of the overlay portfolio. Furthermore, the DNB also gave guidelines what the model should comprehend, based on their findings of their research about managing interest rate risk by pension funds. Therefore, we will first elaborate upon the findings of the DNB, where after we shall define the requirements of our model. Secondly, we will describe the decision making and investment process of an overlay portfolio. Thirdly, we take a look at the models that already exist. Where after we will conclude this chapter by describing our model correspondent to our findings.

4.1. DNB findings

In the introduction we mentioned that the DNB conducted a research to get insight into the interest rate risk management of pension funds. Their main finding was that the strategic decisions regarding interest rate risk are not clearly defined. The DNB found the following deficiencies in the interest rate risk policies:

- **Quantifying interest rate risk exposure.** Defining a hedge percentage is not enough, a method how interest rate risk exposure is quantified also has to be determined.
- **Deviate boundaries for the overlay manager.** Without these boundaries the overlay manager is free to implement the overlay portfolio as he pleases. Furthermore, these boundaries act as a reference for monitoring the overlay portfolio.
- **Interest rate risk distribution.** The board does not take into account that hedging interest rate risk does not have to be equally divided over all maturities, which could result in unwanted yield curve exposure
- **Securities.** It is not clear for the overlay manager which instruments he may use to implement the overlay portfolio.

One of the main consequences of these deficiencies lie in the unintentional given freedom to the overlay manager when implementing the overlay portfolio, which also leads to unwanted open interest rate risk exposure. Furthermore, due to the insufficiently defined hedge policies, the ex-post evaluation do not have enough foundation to evaluate the decisions of the board and the overlay manager properly. Based on their finding, the DNB gave an example of a proper ex-post report evaluation, which can be found in appendix A.

4.2. Model requirements

Based on the findings of the DNB and the definition of a duration overlay performance attribution model, we are able to define our model requirements as follows:

- It should be based on the risk factors of the underlying securities.
- It should be able to evaluate the decisions of the overlay manager relative to a benchmark.
- It should follow the investment- and decision making process of the overlay portfolio.

The last requirement will ensure that the decisions made regarding the interest rate management are sufficiently defined, which would resolve the identified deficiencies by

the DNB. To be able to meet these requirements, we will research them in more detail in sections 4.3-4.6.

4.3. Investment process

One of the requirements of our model lie in the fact that it should follow the decision and investment process of the overlay portfolio. Although a pension fund has several investment processes for their different portfolios, only the process regarding the overlay portfolio will be described here, which is displayed in figure 4.1 below. The process consists of four stages: An ALM study, implementing the strategic investment plan, implementing the overlay portfolio and monitoring the overlay portfolio.

The process begins with an ALM (Asset Liability Management) study, which provides the board of the pension fund with, amongst other outcomes, the possible financial consequences of their decisions regarding interest rate risk hedging. These decisions of the board are subject to the desired interest rate risk exposure of the whole portfolio. Depending on these financial consequences, the board defines the final mandates regarding interest rate risk. These mandates together form the strategic investment plan.

Besides these strategic decisions, some tactical decisions will be made regarding the overlay portfolio. These tactical decisions can be made by the board as well, but also the chief investment officer of the pension fund may have the responsibility to make these. An example of a tactical decision is defining the deviate boundaries of the overlay manager.

After that, the plan will be handed over to the overlay manager. The plan constitutes the “to-do” list for the manager to construct the overlay portfolio and certain margins in which he can deviate from it. Based on the “to-do” list and deviate margins, the manager will then implement the overlay portfolio including his duration bets.

The process ends with monitoring the overlay portfolio by ex-post evaluation to assess the manager’s performance and whether the portfolio still lies within the range of the mandates. Therefore, these ex-post evaluations have to be administered frequently, in order for the overlay manager to be able to rebalance the overlay portfolio when it starts to deviate from the strategic decisions of the board. The board, on the other hand, determines whether to adjust their decisions by monitoring the overlay portfolio and changes in the economy.

Consequently, this process obviously is a recurrent loop, where some stages will be revisited more often than others. An ALM study is usually done quarterly or semiannually, since market circumstance change over time, which will impact the desired risk strategy of the pension fund. Whereas the overlay manager may find himself rebalancing the portfolio more often to keep in line with the mandate requirements.

Also, looking closely at the process, we are able to notice another important detail. At the beginning of this chapter we already mentioned that the main goal of our model consists of evaluating the performance of the overlay manager relative to a benchmark. Furthermore, the mandates of the board regarding interest rate risk exposure dictate the desired risk profile of the board and certain boundaries to indicate how far the overlay manager is allowed to deviate from these mandates, enabling him to place certain duration bets. Therefore, we actually want to assess the performance of the overlay manager in comparison to the decisions of the board, which make these the perfect benchmark.

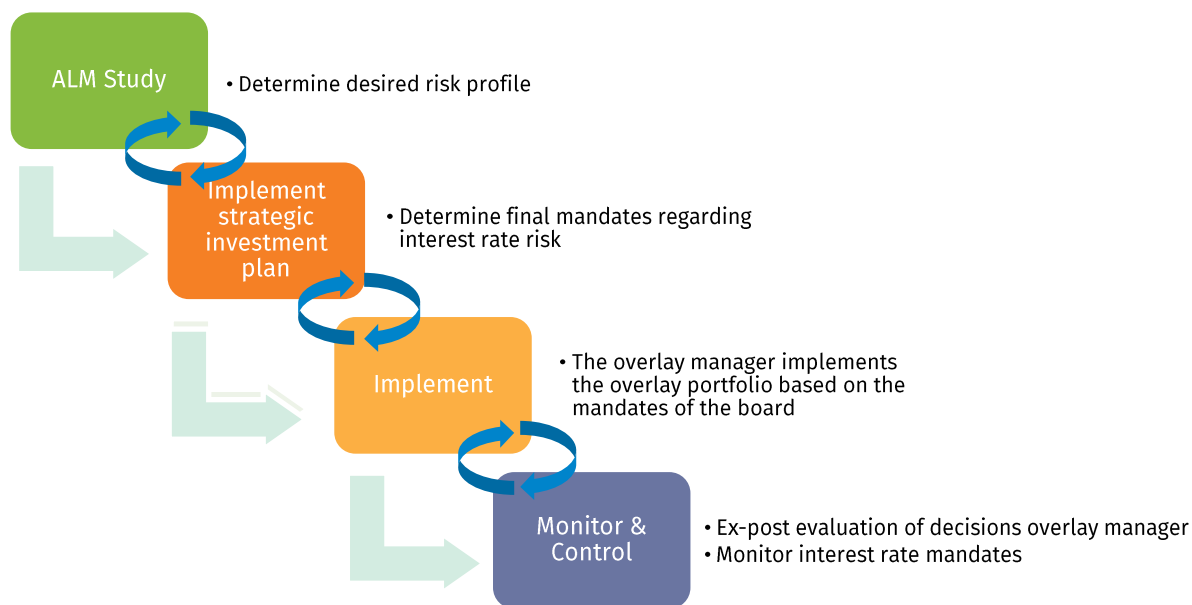


Figure 4.1 Investment process of an overlay portfolio

4.4. Decision making process

From the previous section, it became clear that several decisions are made during the investment process, where we can define the decisions of the board as strategic decisions and the decisions of the overlay manager as tactical decisions. In the first two stages of the investment process, the board makes the strategic decisions, based on the outcome of the ALM study, which are:

- The interest rate hedge strategy (d.1)
- The extend of the interest rate risk to be hedged (d.2)

These decisions will lead to a few tactical decisions, which will be very important to the overlay manager and are decided by the board or chief investment officer:

- Interest rate risk boundaries (d.3)
- Securities used in the overlay portfolio (d.4)

Although we elaborated about how to express interest rate risk, these decisions do have some implications, which we will describe below.

4.4.1. The interest rate hedge strategy (d.1)

From chapter three, we know that we are able to hedge the open interest rate risk exposure by matching the duration exposure. This exposure can be defined by the following two strategies:

- The “equal gain”-strategy.
- The “equal funding ratio”-strategy.

To understand the differences between these strategies and their implications, we shall derive the open interest rate risk exposure for both cases. Let us start by looking at the funding ratio at the beginning of the performance period:

$$Funding\ Ratio_{t-1} = \frac{Total\ Value\ Assets}{Present\ Value\ Liabilities} \quad (4.1)$$

For future reference we indicate the total value of the assets as V_P and the present value of the liabilities as V_L , where we assume the assets only consist of a fixed income portfolio. We are able to compare both strategies by indicating the funding ratio after the performance period:

Equal gain	Equal funding ratio
$Funding\ Ratio_t = \frac{V_P + x}{V_L + x} \quad (4.2)$	$Funding\ Ratio_t = \frac{V_P(1+y)}{V_L(1+y)} \quad (4.3)$

It is easy to see that x represents an amount and y is a percentage change. Therefore, the output for the “equal gain”-strategy is easier to interpret when money duration is used and effective duration is used for the equal funding ratio strategy.

In chapter 3 we explained that interest rate risk can be hedged by implementing an overlay portfolio which duration matches the exposure (equation (3.7)). So hedging the interest rate risk for 100% would yield:

$$D_P - D_L = 0 \Leftrightarrow D_P = D_L, \quad (4.4)$$

where $D_L = D_P + D_O$, the duration of the fixed income portfolio and the duration of the overlay portfolio respectively. Assuming that the yield would change for 1%, we can fill in x and y in equations as follows:

Equal gain	Equal funding ratio
$Funding\ Ratio_t = \frac{V_P + MD_P}{V_L + MD_L}$	$Funding\ Ratio_t = \frac{V_P(1+D_P)}{V_L(1+D_L)}$
$Funding\ Ratio_t = \frac{V_P + MD_P + MD_O}{V_L + MD_L}$	$Funding\ Ratio_t = \frac{V_P(1+(D_P + D_O))}{V_L(1+D_L)}$

Using 4.4 we can rewrite this to:

$$\begin{aligned} MD_P + MD_O &= MD_L & D_P + D_O &= D_L \\ D_O &= -(MD_P - MD_L) & D_O &= -(D_P - D_L) \end{aligned}$$

Since D_O is a 100% hedge, we can see this as the exposure, this yields:

$$\text{Exposure MD} = MD_P - MD_L \quad (4.5) \quad \text{Exposure D} = (D_P - D_L) \quad (4.6)$$

For a more flexible approach, we also want to be able to determine both exposures in duration and money duration:

$$\begin{aligned} \text{Exposure D} &= \frac{\text{Exposure MD}}{V_L} & (4.7) \quad \text{Exposure MD} &= \frac{(D_P - D_L)}{V_P} \\ \text{Exposure MD} &= \left(\frac{MD_L}{V_L} * V_P \right) - MD_P & (4.8) \end{aligned}$$

where MD_i is the money duration of portfolio i , $\{D_i : i = \{P, L, O\}\}$.

Now that we are able to quantify the interest rate risk exposure between the assets and the liabilities, which strategy do we want to use? Looking at equation 4.1, the funding ratio can either be positive or negative at the beginning of the performance period:

Equal gain	Equal funding ratio
$Funding\ Ratio_{t-1} = \frac{V_P}{V_L} < 1,$	$Funding\ Ratio_{t-1} = \frac{V_P}{V_L} > 1,$

This yields:

$$V_P < V_L \text{ and } \frac{MD_L}{V_P} > \frac{MD_L}{V_L} \qquad V_P > V_L \text{ and } \frac{MD_L}{V_P} < \frac{MD_L}{V_L}$$

Therefore, we conclude:

$$\frac{V_P + MD_L}{V_L + MD_L} > \frac{V_P}{V_L} \qquad \frac{V_P + MD_L}{V_L + MD_L} < \frac{V_P}{V_L}$$

This means that it would be most profitable when cash flow matching would be used when the funding ratio is below 100% and when it is higher than 100% the strategy for an equal funding ratio would be used.

4.4.2. The extend of the interest rate risk to be hedged (d.2)

Based on the interest rate risk exposure, the board will also decide which part of the exposure will be hedged. Suppose the board decides to hedge the interest rate risk for 70%, this decision can be implemented as follows :

- Hedge the duration for predetermined maturities for 70%,
- Hedge the duration in such a way the portfolio has an average duration matching of 70%,

where the duration is either the duration exposure derived in the previous section or the duration exposure of the liabilities. We will explain the ramifications of this duration decision in the following example for an “equal gain”-strategy on the performance interval $[t_{i-1}, t_i]$:

Duration Exposure	Duration Liabilities
-------------------	----------------------

$$V_L = \text{€ } 2000, MD_L = \text{€ } 200$$

$$V_P = \text{€ } 1500, MD_P = \text{€ } 100$$

This yields:

$$Funding\ ratio_{i-1} = \frac{V_P}{V_L} = \frac{\text{€ } 1500}{\text{€ } 2000} = 75\%$$

$$Exposure\ MD = MD_P - MD_L = \text{€ } 100 - \text{€ } 200 = -\text{€ } 100$$

Determining the hedge gives then:

$$\begin{aligned} hedge\ MD &= 70\% * -Exposure\ MD \\ &= \text{€ } 70 \end{aligned}$$

$$\begin{aligned} hedge\ MD &= (70\% * MD_L) - MD_P \\ &= \text{€ } 40 \end{aligned}$$

If the yield decreased for 1% during the performance period, we get:

$$Funding\ ratio_t = \frac{V_P + MD_P + hedge\ MD}{V_L + MD_L}$$

$$Funding\ ratio_t = \frac{V_P + MD_P + hedge\ MD}{V_L + MD_L}$$

$$\begin{aligned} Funding\ ratio_t &= \frac{\text{€ } 1500 + \text{€ } 100 + \text{€ } 70}{\text{€ } 2000 + \text{€ } 200} \\ &= 75,91\% \end{aligned}$$

$$\begin{aligned} Funding\ ratio_t &= \frac{\text{€ } 1500 + \text{€ } 100 + \text{€ } 40}{\text{€ } 2000 + \text{€ } 200} \\ &= 74,55\% \end{aligned}$$

Based on these findings we conclude that we can define the %-hedge of the overlay benchmark and overlay as follows:

$$\% - hedge = \frac{hedge\ MD}{-Exposure\ MD} \quad (4.9)$$

$$\% - hedge = \frac{hedge\ MD + MD_P}{MD_L}, \quad (4.10)$$

where we can rewrite (4.10) to make it also usable for the “equal funding ratio”-strategy:

$$\% - hedge = \frac{hedge\ MD + MD_P}{-Exposure\ MD + MD_P} \quad (4.11)$$

Note that we can switch between money duration and duration in equations in 4.9-4.11. Also, for a 100%-hedge, both the “exposure” hedge and the “liabilities” hedge is equal.

As we can see, both strategies have a different impact on the hedge and the funding ratio. To determine which strategy would yield the highest funding ratio at the end of the performance period, we look for the highest hedge:

$$\max(\text{hedge}\% * \text{Exposure}, (\text{hedge}\% * D_L) - D_P) = \text{hedge}\% * \text{Exposure} \forall \text{hedge}\%, D_L, D_P$$

We also mentioned that the board could define the hedge for certain maturities. This would prevent unwanted open interest rate risk due to unequal hedging for different maturities. Therefore, defining the hedge for maturities instead of the whole portfolio is preferred.

4.5. The overlay benchmark

Besides the fact that the previous described decisions of the board form the “to do”-list for the overlay manager, they also define the overlay benchmark. To determine an actual benchmark based on these decisions, we start with the most easy case: a 100% exposure hedge based on an “equal gain”-strategy. Then, the overlay benchmark would consist of the liabilities and the fixed income portfolio securities in the exact opposite position. Thus, the benchmark would be long the liabilities and short the fixed income portfolio. This is also the starting point from where we can derive the overlay benchmark for different decisions.

So for the “equal gain”-strategy and a hedge $h\%$ of the exposure, we recalculate each position as follows:

$$V_i(\text{short}) = V_i(\text{long}) * h\% \quad \text{and} \quad V_i(\text{long}) = V_i(\text{short}) * h\%, \quad (4.12)$$

where V_i is the value of security i . By keeping the other characteristics equal a perfect benchmark can be constructed. If we want to hedge the duration of the liabilities, however, we only take the opposite position in the pieces of the liabilities.

For the “equal funding ratio”-strategy the duration of the fixed income portfolio has to change. This has some indications due to the mismatch between the fixed income portfolio and the liabilities. Although the benchmark will reflect the board decisions, performances could differ from the “expected” performance due to choices regarding the duration in the benchmark.

To conduct the benchmark, we still use equation (4.19) for the liability securities in the benchmark, but for the fixed income securities take the following steps for each maturity bucket:

- Construct the benchmark for an “equal gain”-strategy based on a 100% hedge
- Choose a duration d_{xy} for maturity bucket $x - y$
- Determine the expected duration in bucket $x - y$ for a liabilities duration hedge with:

$$\text{Expected duration}_{xy} = \left(\frac{D_{Lxy}}{V_L} * h\% * V_P \right) - D_{Pxy}, \quad (4.13)$$

where D_{Lxy} is the money duration of the liabilities in bucket $x - y$ and D_{Pxy} the money duration of the fixed income securities in bucket $x - y$.

For the exposure duration hedge, use:

$$\text{Expected duration}_{xy} = h\% * -\text{Exposure}_{xy}, \quad (4.14)$$

where Exposure_{xy} is the exposure in maturity bucket $x - y$.

- Add the following value to each value in bucket $x - y$:

$$\frac{(Expected\ duration_{xy} - Actual\ Duration_{xy})}{(d_{xy}/100)} / Number\ of\ pieces_{xy}, \quad (4.15)$$

where *Actual Duration*_{xy} is the duration of the constructed benchmark in step 1 in bucket $x - y$ and *Number of pieces*_{xy} is the number of fixed income security pieces of the of the constructed benchmark in step one.

4.6. Existing models

In this section we are going to look at existing performance attribution models in the literature, which are able to identify the risk factors in our portfolio. The first acknowledged performance attribution model was founded by Brinson (1985). This model decomposed the difference between the return of the portfolio and the benchmark in the allocation and selection effect. These effects measures the decisions of the portfolio manager to select securities in different segments, like equities and fixed income securities, or select different securities then the benchmark respectively. The Brinson model became the standard methodology for performance attribution analysis.

The Brinson model, however, uses decomposition factors which correspond mostly to equity factors. Campisi (2005) and Lord (1997), amongst others, argued that fixed income portfolios need a different approach then the Brinson model, due to the following differences between equity and fixed income securities:

- **Liquidity.** Fixed income securities tend to be less liquid than equities. This is due to the fact that fixed income investors usually hold their security until maturity. This in contrast to the equities, which have a very active secondary market.
- **Homogeneity.** Fixed income investments are comparable to each other. Where there exists a large differentiation between equities from different companies, bonds tend to sell at the same price and have the same returns if they have similar characteristics.
- **Sources of risk.** The underlying risk sources of both securities are not comparable. Bonds are most sensitive to changes in the yield to maturity, where the main risk for equities lies in the beta of the particular security.
- **Sources of return.** The return for the securities also has different sources: fixed income return comes from coupon payments and equity return comes from value increase of the company or dividend.
- **Holding periods.** The possible holding period also differs. A bond can be held until its maturity; equities can be hold forever (or until bankruptcy of the company), since it does not have a maturity.

These differences indicate different return drivers for fixed income and equity portfolios. Where equity return attribution is driven by selection and allocation effects, fixed income portfolio return attribution is driven by coupon return and yield curve return. Coupon return is the income the investor receives for investing in a fixed income security and yield curve return is the price change of the security due to changes in the yield of maturity. Because the overlay portfolio only holds fixed income securities, we conclude that our model should be based on a fixed income performance attribution model. Furthermore, it should also reflect the decision making and investment process of an overlay portfolio.

4.6.1. Fixed income performance attribution models

Looking at the fixed income performance attribution models in the literature, the following three models are most commonly used:

- Duration models
- Key rate duration models
- Full-repricing models

Duration models

Most of the existing fixed income models in the literature are so called duration models. The total return of these models is approximated with equation (3.5), which also explains the name of the model. The total return is decomposed into the yield curve return and the carry return, the return due to a change in the yield curve of the fixed income security and the accrued interest respectively. The change in the yield curve can also be decomposed in spread return and duration or treasury return, where the duration return is often decomposed in the most common yield curve changes, like shift and twist changes. Figure 4.6 displays this decomposition.

A well-known duration model comes from Campisi (2000). His model is very simple and only holds the most necessary decompositions. Another familiar model comes from Lord (1997), which decomposes the price return further than Campisi and added a time component to the decomposition. Another model worth mentioning comes from van Breukelen (2000), who links the investment process of Robeco to the attribution of the model.

The advantages of the duration model lie in the fact that the calculations are easy to understand within bond terminology and are the same for every security in the portfolio. Furthermore, the accuracy of the model can be expanded by using a wider Taylor expansion, which would give a more accurate total return.

Key rate duration models

The terminology of key rate duration was first introduced by Ho in 1992. The models based on key rates follow the same structure as the previous mentioned duration models, but instead of decomposing the duration return into yield curve changes, it is decomposed in a predetermined set of key rate durations. In the previous chapter we described key rate durations as the duration at a particular yield, but since not all yields can be used for scope reasons, certain yields are chosen. This way a better understanding of the interest rate risk exposure distribution of the portfolio is conceived. Figure 4.2 displays the duration return decomposed into several key rates.

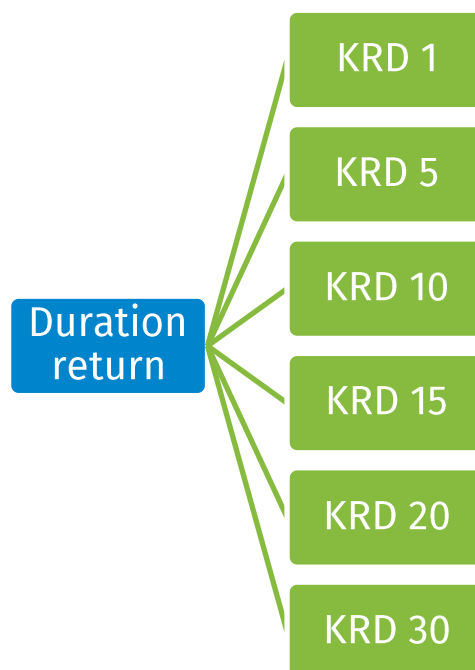


Figure 4.2 Duration return decomposed in several key rate durations.

Full-repricing models

For this model every security in the portfolio is priced with its own formula, by repricing the security by only changing one source. As a result, we would get the distinct values when only the time changed, the shift change in the yield curve and so on. These prices together would then follow a certain “price path”, like displayed in figure 4.1. It is obvious that the residual in this model is zero, because an exact formula is used to calculate the return of the securities instead of using an approximation, like the duration model.

For a better understanding, we look at how we can determine a price path of a single security by following the decomposition in figure 4.6. From chapter three we know that the return of a single security is defined as:

$$r(t_{i-1}, t_i) = \frac{V(t_i, y_i) - V(t_{i-1}, y_{i-1}) - \text{cashflows in} + \text{cashflows out}}{V(t_{i-1}, y_{i-1})}$$

Assuming the cash flows are zero, we obtain the start point:

$$V_{start} = V(t_{i-1}, y_{i-1}).$$

From where we can define the following path:

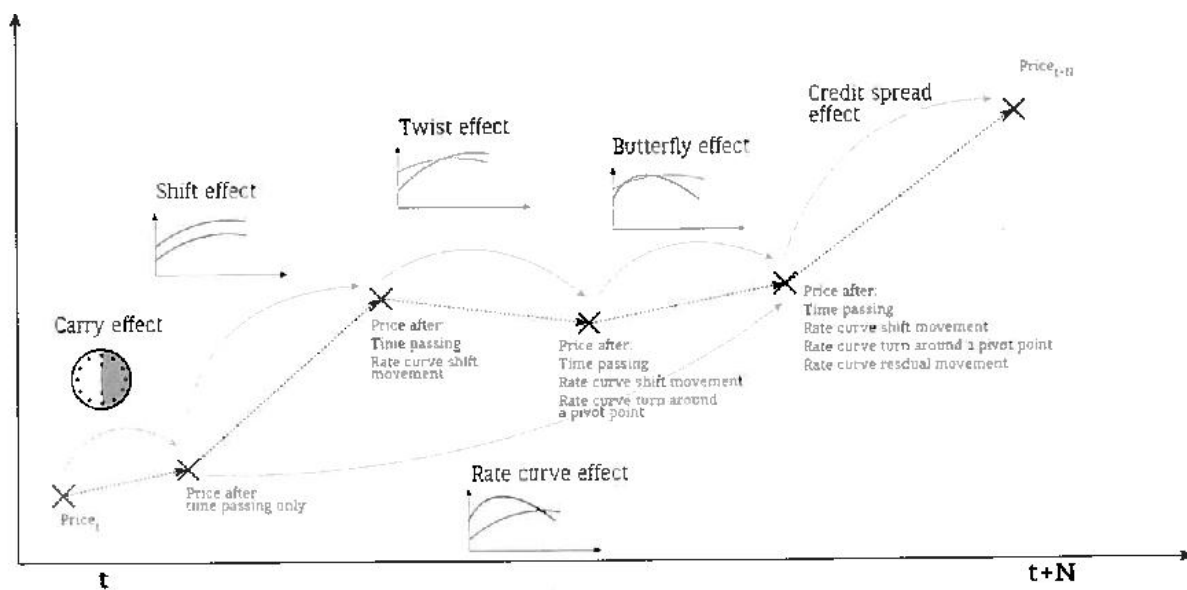
$$V_{carry} = V(t_{i-1} + \Delta t, y_{i-1}),$$

$$V_{shift} = V(t_{i-1} + \Delta t, y_{i-1} + \Delta y_{shift}),$$

$$V_{twist} = V(t_{i-1} + \Delta t, y_{i-1} + \Delta y_{shift} + \Delta y_{twist}),$$

$$V_{selection} = V(t_{i-1} + \Delta t, y_{i-1} + \Delta y_{shift} + \Delta y_{twist} + \Delta y_{selection}),$$

$$V_{end} = V_{selection}.$$



Source: BNP Paribas Securities Services

Figure 4.3 Full decomposition price path

Both key rate duration and full-repricing models seem to hold more accurate results than duration models. The full-repricing model, however, requires a lot of data and calculations, which makes this approach rather expensive. Furthermore, key rate model gives us more insight in the interest rate risk distribution of the pension fund, which would make this model an obvious choice.

Nonetheless, the key rates would only offer the interest rate exposure at certain yields. To be able to have a full insight in the interest rate risk distribution, we adopt a method from the performance attribution model of Ortec Finance, which also separates results to maturity buckets. In addition to the fact that this approach is more favorable above key rate durations, it also gives us the opportunity to use a duration model, which can be used with the least amount of input data and uses the most common terminology.

4.6.2. Yield curve decomposition

Besides the fact that there does not exist an accepted framework for a fixed income portfolio attribution model and several models exist, another challenge lies in the decomposition of the yield curve movements that occurred during the performance period. The literature commonly refers to shift, twist and butterfly movements in the yield curve, where:

- A shift movement is a parallel shift of the yield curve over the whole maturity horizon of the portfolio,
- A twist movement is the steepening or flattening of the yield curve for a certain maturity,
- A butterfly movement is a curvature movement of the yield curve.

These curve movements are also illustrated in figure 4.2 below. Also, a more descriptive interpretation can be found in figure 4.3.

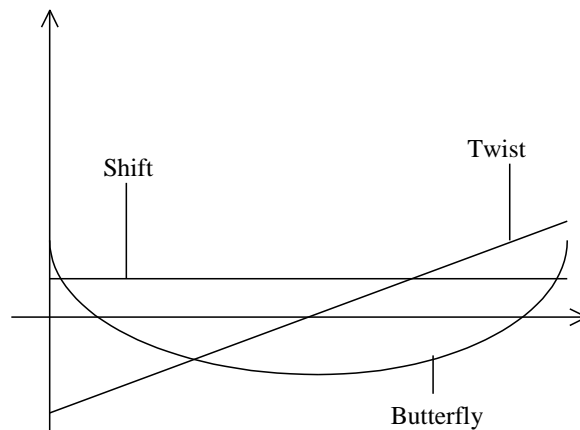


Figure 4.3 Shift, twist and butterfly movements

Although these terms are widely used in this subject, no distinct framework exists to define these movements. But most frameworks are based on a revaluation approach.

Revaluation approach

The revaluation approach is based on repricing the security using a yield to maturity based on a particular movement, where the yield curve changes are decomposed as follows:

$$\Delta Y = \Delta Y_{Shift} + \Delta Y_{Twist} + \Delta Y_{Butterfly},$$

where,

- ΔY is the total yield change
- ΔY_{Shift} is the change in yield due to shift
- ΔY_{Twist} is the change in yield due to twist
- $\Delta Y_{Butterfly}$ is the change in yield due to butterfly

From chapter two we know that we can approximate price changes with the use of duration for small yield changes, so instead of repricing the underlying security, we can use its duration to calculate the return for the different movements more easily:

$$Shift\ return = -MD * \Delta Y_{Shift},$$

$$Twist\ return = -MD * \Delta Y_{Twist},$$

$$Butterfly\ return = -MD * \Delta Y_{Butterfly},$$

where MD is the money duration.

This leaves us with obtaining the yield curve decomposition of these three movements, for which the following techniques are most frequently used:

- Fitting the yield curve to a term-structure model and decompose the curve into the three different curve changes for every maturity in the portfolio. One could use the Nielson-Siegel model, for example.
- A Principal Component Analysis (PCA), where the curve changes are obtained by finding the principal components of the yield curve through analyzing the underlying structure of the curve.
- Empirically obtain the decompositions based on a self-chosen pivot point on the yield curve.

According to a comparison made by Esseghaier et al. (2004) between these methods, the empirical approach holds the most explanatory value, by keeping the residual as little as possible. Furthermore, this approach is more flexible to the investment process due to the pivot point. By choosing the pivot point according to the bets made of the overlay manager or ex-post yield curve movements, the outcome of the model would be more accurate. Campisi (2000), for instance, adopted this method in his fixed income attribution model.

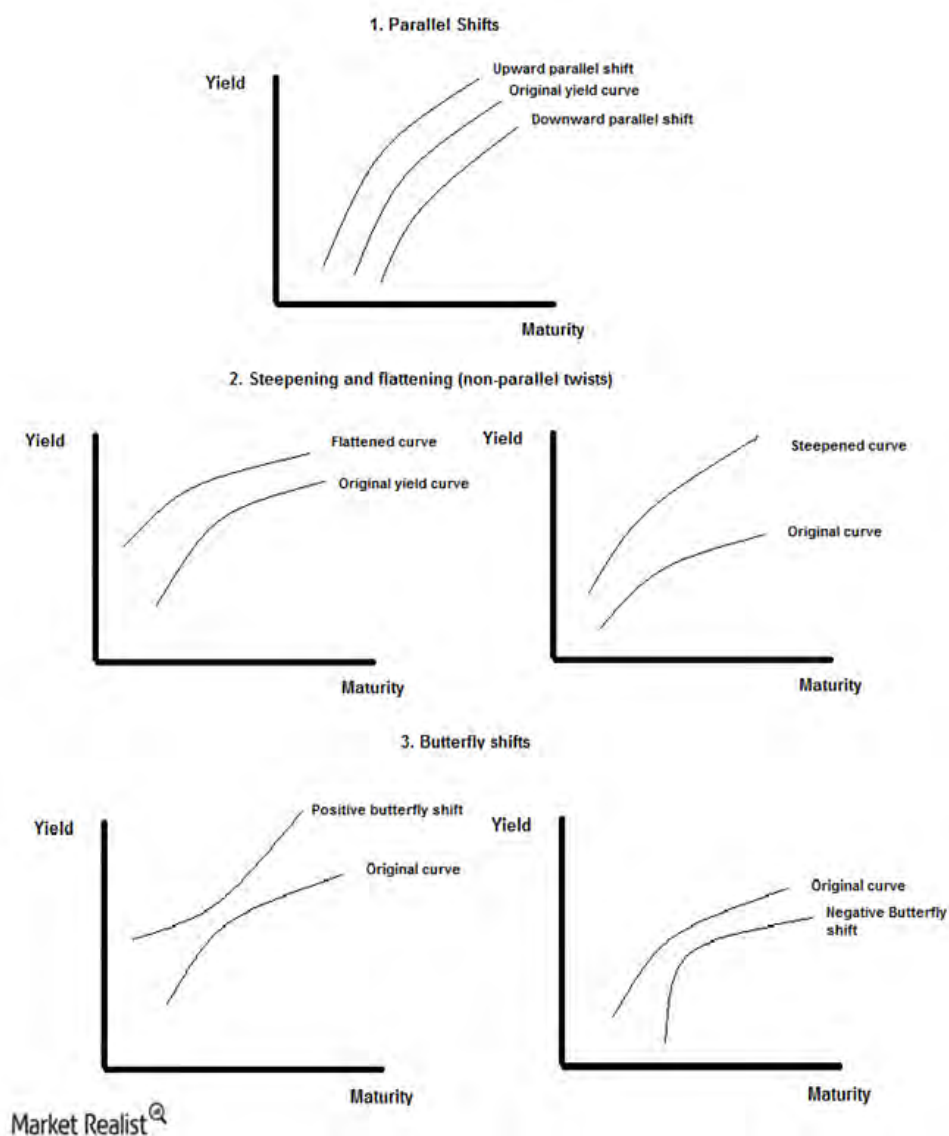


Figure 4.4 Yield curve changes

4.7. Model mathematics

In the previous sections we decided that our model will be based on a duration model extended with the use of maturity buckets. Furthermore, we will use the empirical approach to evaluate the shift and twist return of the yield curve. In addition, the model will be extended with the estimation of the different decisions made regarding interest rate risk and implementing the overlay portfolio.

In this section we elaborate upon the framework and mathematics of our model. First the return decomposition of a single security is described, which is easily translated to a performance attribution model. Where after we shall describe how the decisions can be evaluated. We conclude with input requirements of the model, the yield curve decision and defining the benchmark

4.7.1. Return decomposition

Like we mentioned in the previous chapter, performance attribution measures the sources of return. These sources of return are equal to evaluate the risk factors of a fixed income security. To be able to do so, we derived the Taylor Series expansion (3.14). With use of equation (3.5) we can rewrite (3.14) to:

$$g(t_{i-1}, t_i) \approx V(t_{i-1}, y_{i-1}) y_{i-1} \Delta t - D_d \Delta y + \frac{1}{2} C_d (\Delta y)^2, \quad (4.12)$$

where C_d is the money convexity. By going through the equation step by step, we will be able to identify and evaluate the distinct sources of return.

Total gain

From the previous chapter we already know that $g(t_{i-1}, t_i)$ is the total gain of a security during the interval $[t_{i-1}, t_i]$, which we defined as follows:

$$g(t_{i-1}, t_i) = V(t_i, y_i) - V(t_{i-1}, y_{i-1}) - \text{cashflows in} + \text{cashflows out} \quad (4.13)$$

Carry gain

The next part in equation (4.6) is $V(t_{i-1}, y_{i-1}) y_{i-1} \Delta t$, which depends on the risk factor time. We can define two sources of gain that depend solely on time:

- Accrued interest: accumulated interest for a coupon paying fixed income security during the performance period.
- Roll down gain: since fixed income securities are valued as the sum of its discounted cash flows, we conclude that the value of the security goes to its face value when it nears maturity, which is also referred to as roll down gain.

We capture both time related gains as in the Taylor Series Expansion with the term “Carry gain”:

$$\text{Carry gain} = V(t_{i-1}, y_{i-1}) y_{i-1} \Delta t. \quad (4.14)$$

We could decompose the carry gain further to the two resources, the roll down gain, however, is usually very small, so we chose to ignore this.

Price gain

The last two parts of equation (4.6) depend on changes in the yield. We know that changes in the yield cause a change in the value of the security, which we capture as follows:

$$\text{Price gain} = -D_d \Delta y + \frac{1}{2} C_d (\Delta y)^2, \quad (4.15)$$

which we can further decompose in duration and convexity gain:

$$\text{Duration gain} = -D_d \Delta y_{rf}, \quad (4.16)$$

$$\text{Convexity gain} = \frac{1}{2} C_d (\Delta y_{rf})^2, \quad (4.17)$$

where y_{rf} is the duration matched risk free rate. In our case the swap curve is used as the risk free rate. Looking at equation 4.17, we conclude that the convexity gain would be small. We will, therefore, not add convexity gain to our model.

In section 4.3.2 we described several methods to decompose the yield curve into shift, twist and butterfly changes, where we chose to use the empirical approach in our model.

However, since butterfly changes are usually negligibly small, we will not adopt this term in our model. Whereas shift and twist gain are evaluated as follows:

$$\text{Shift gain} = -D_d \Delta y_{Average}, \quad (4.18)$$

$$\text{Twist gain} = \text{Duration gain} - \text{Shift gain}. \quad (4.19)$$

For the fixed income portfolio, there exists another return component. Pension funds can also invest in high yield securities in comparison to government securities. Where government bonds are low in risk and priced with a risk free rate, high yield bonds are higher in risk and therefore are priced with a spread additional to the risk free rate. This spread is highly correlated with the risk of the security: the higher the risk, the higher the spread. Since the liabilities are valued with a risk free rate, duration hedging is also based on the risk free rate. So, for risk management purposes we are not particularly interested in this spread return, but to give an accurate decomposition of high yield securities, we capture it under “selection gain”:

$$\text{Selection gain} = -D_d (\Delta y - \Delta y_{rf}), \quad (4.20)$$

where y_{rf} is the duration matched risk free rate.

Residual gain

We have now been through all factors in equation (4.6), but we did not address the fact that it is an approximation. This yields that there is a certain residual between the actual total gain (4.7) and our return decomposition:

$$\text{Residual} = \text{Total gain} - \text{Carry gain} - \text{Price gain}, \quad (4.21)$$

Putting it all together then yields:

$$\text{Total gain} = \text{Carry gain} + \text{Duration gain} + \text{Selection gain} + \text{Residual}, \quad (4.22)$$

which is also displayed in figure 4.5.

You may have noticed that all equations yield gains or losses. In the previous section we mentioned, however, that the user can choose between money or a percentage output. Looking at equation (3.2) on the other hand, shows us that we can easily determine the return by dividing the term by $V(t_{i-1}, y_{i-1})$.

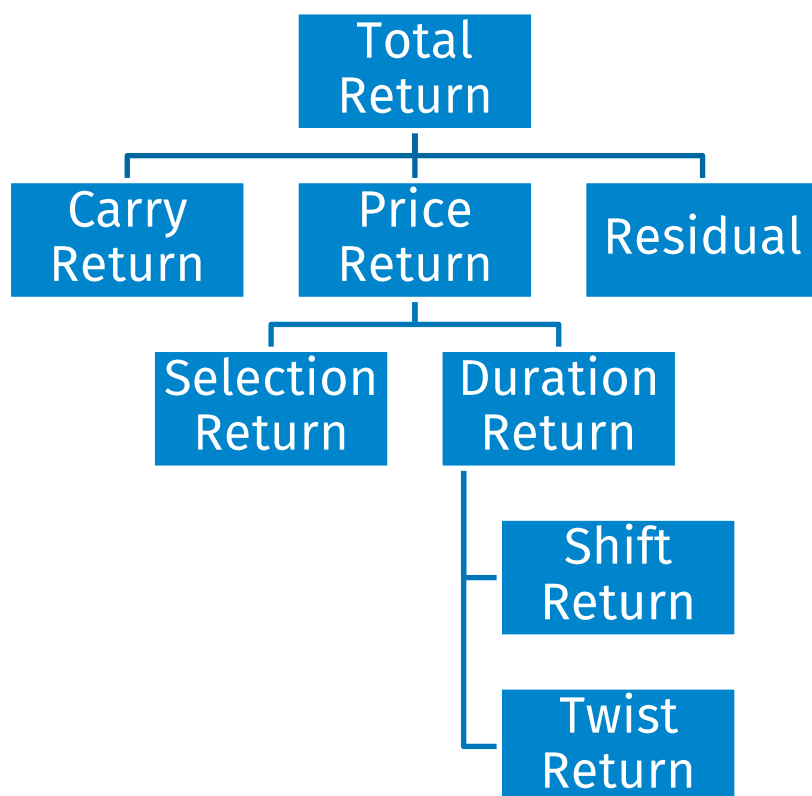


Figure 4.5 Return decomposition of our model

4.7.2. Performance attribution model

Every different source of return can now simply be added together to come to the total gain of that particular source. So, we determine the total gain of source k in a portfolio with N securities as follows:

$$Total_k = \sum_{n=1}^N k_n, \quad (4.23)$$

where k_n is the gain of source k of security n , $\{k_n : n = 1, \dots, N\}$. Obviously, equation (4.17) can also be used to calculate the total value of the portfolio. Furthermore, we can rewrite the equation to determine the value of a certain maturity bucket by defining N as N_{xy} , the number of securities in maturity bucket $(x, y]$.

Moreover, we mentioned that the performance attribution analysis evaluates the overlay portfolio by comparing it to a benchmark, which define simply by subtracting:

$$Effect_k = Total_{O_k} - Total_{B_k}, \quad (4.24)$$

where $Effect_k$ is the effect of source k , $Total_{O_k}$ is the total gain of source k of the overlay portfolio and $Total_{B_k}$ is the total gain of source k of the overlay benchmark.

4.7.3. Decisions

Now that we are able to decompose the return for the liabilities, fixed income portfolio, overlay benchmark and the overlay portfolio and determine the effects, we have arrived at the most important aspect of our model: evaluate the decisions. In section 4.2 we defined the decisions regarding the overlay portfolio:

- The interest rate hedge strategy (d.1)
- The extend of the interest rate risk to be hedged (d.2)

We also mentioned earlier that a pension fund chooses to invest in an fixed income portfolio to match the liabilities as far as possible. Therefore, we will start our decisions evaluation with the decision of taking a fixed income portfolio. In table 4.1 below the decisions and their evaluation methods are summarized. To explain our motivation, we will elaborate upon each evaluation distinctively:

- **No fixed income portfolio.** When there would be no fixed income portfolio, the board decided to let the pension fund completely exposed to its interest rate risk, so the value change of the liabilities would be the result of that full exposure.
- **Fixed income portfolio.** Holding a fixed income portfolio, however, would hedge the open interest rate partially and would yield the gain of the portfolio
- **Interest rate hedge.** If the board decided to hedge the interest rate risk, the gain of the benchmark would capture that decision, since the benchmark is the perfect implementation of the strategic decisions of the board.
- **Hedge strategy.** The evaluation of the chosen hedge strategy can be defined by its return relative to the return that would have been generated if the other strategy was chosen.
- **Duration hedge.** The evaluation of the chose duration to hedge can also be defined by its return relative to the return that would have been generated if the other strategy was chosen.
- **Overlay manager.** In the previous section we already mentioned that we can evaluate the overlay manager by comparing his results to the benchmark. Furthermore, by decomposing the return in different sources, we are able to capture distinctively which source generated an excess return.

Decision	Equation
No fixed income portfolio	$-g_L(t_{i-1}, t_i)$
Fixed income portfolio	$g_A(t_{i-1}, t_i)$
Interest rate hedge	$g_{OB}(t_{i-1}, t_i)$
Hedge strategy	$g_{OB}(t_{i-1}, t_i \text{Chosen strategy}) - g_{OB}(t_{i-1}, t_i \neg \text{Chosen strategy})$
Duration Hedge	$g_{OB}(t_{i-1}, t_i \text{Chosen Duration}) - g_{OB}(t_{i-1}, t_i \neg \text{Chosen Duration})$
Overlay manager	$g_O(t_{i-1}, t_i) - g_{OB}(t_{i-1}, t_i)$

Table 4.1 Evaluation different decisions

4.7.4. Input requirements

To finalize our model, we need to define the input needed, for both the beginning of the performance period and the end:

- **Swap curve**
- **Liabilities.** The maturity, value and duration for every piece.
- **Fixed income portfolio.** The maturity, value, duration, yield to maturity and income for every security
- **Overlay portfolio.** The maturity, value, duration, yield to maturity, coupon and income for both the fixed and floating leg for every piece
- **Board decisions**
- **Maturity buckets**

4.7.5. Yield curve decision

Like we mentioned in the previous section, one of the input requirements is the value of the liabilities. The liabilities can be valued with the yield curve, where one of the following yield curves is used:

- **The swap curve.** Equivalent to a yield curve, the swap curve captures the swap rate for different maturities. This means that the swap curve reflects the economic reality, which makes it probably the optimal approach to determine the actual pension payments on the long run.
- **The UFR curve (Ultimate Forward curve).** This curve was created in 2012 by the DNB for pension funds to determine their solvency, to prevent too much fluctuation for long term interest rates. For the first 20 years it follows the swap curve, after which it gradually rises to the UFR, 3,3% at this time.

Although the UFR curve is mandatory by determining the funding ratio, usually both these curves are used to compare their performances.

Besides the decision which yield curve is used to value the liabilities, the decision which yield curve to use for the performance analysis should also be made. When these curves are not the same in both cases, this has some implications for the outcome. Figure 4.6 below displays the difference between the two curves. So when the liabilities are valued with the UFR curve and the analysis is done with the swap curve the twist changes tend to increase, while the shift changes decrease and vice versa.

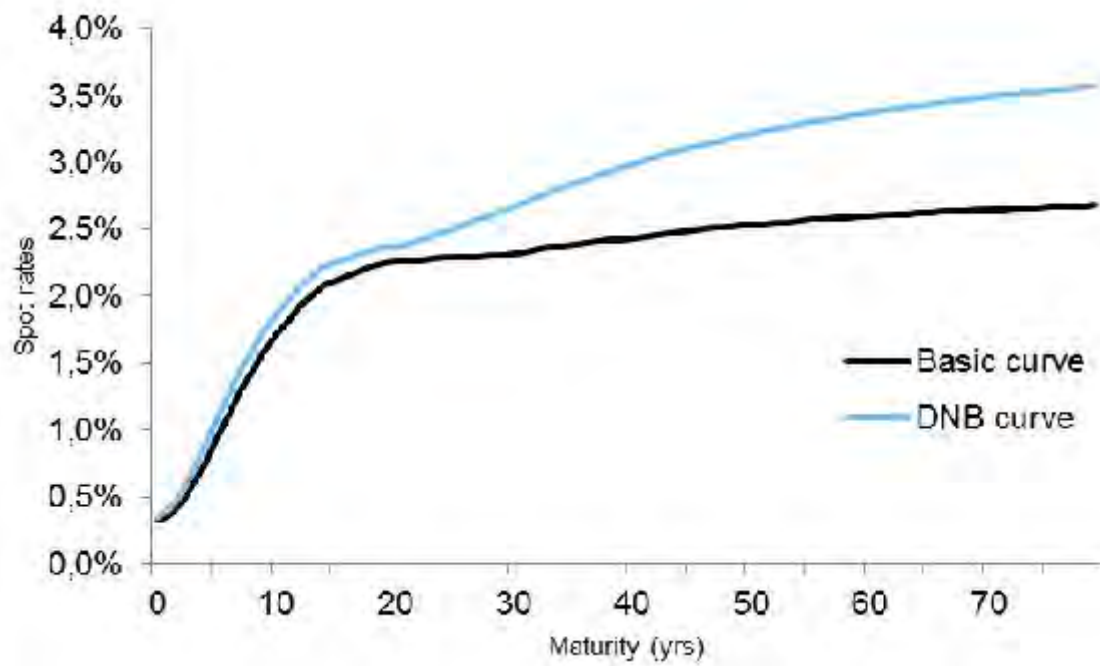


Figure 4.6 Basic curve vs UFR curve

5. Overlay Performance Analysis

We tested our model with provided data by Ortec Finance. In this chapter we will describe the input data, the decisions made regarding the interest rate risk and display the results.

5.1. Input data

Before we discuss the results, we will first elaborate upon the data provided by Ortec Finance. We got detailed data for the liabilities, fixed income portfolio, equity portfolio, overlay portfolio and the swap curve for a performance period of one day. However, the duration for the liabilities was missing, so we used the maturity instead. Moreover, the decisions made regarding interest rate risk are summarized in table 5.1.

Decisions made regarding interest rate risk	
Strategic Decisions	<ul style="list-style-type: none"> ▪ The interest rate exposure is measured by keeping the funding ratio equal ▪ The interest rate risk exposure is hedged for 70% for all maturities higher than 7 ▪ The overlay manager has 10% implement boundaries
Tactical Decisions	<ul style="list-style-type: none"> ▪ The output is in money duration ▪ Only Swap securities are used in the overlay portfolio ▪ The following maturity buckets will be used: <ul style="list-style-type: none"> ▪ 0-1 Years ▪ 1-7 Years ▪ 7-10 Years ▪ 10-15 Years ▪ 15+ Years

Table 5.1 Strategic and tactical decisions made regarding interest rate risk

5.2. Output

We will display the results of our model in this section. Since fixed income performance attribution analysis already exist, we will focus mainly on the results of the extensions we made. In the previous chapter we described, amongst other things, the requirements of our overlay duration performance attribution model. Based on these findings, we conclude that the outcome of the model should:

- give a clear insight in the interest rate risk exposure of the pension fund;
- follow the investment and decisions making process of the overlay portfolio;
- decompose the total return to the different risk factors;
- be able to evaluate the decisions made regarding risk management;
- allow for monitoring the overlay manager over time.

Based on these findings, we chose an output as displayed in table 5.2. We will look per requirement mentioned above, where we can find it in the output.

Begin position								
Value liabilities	Value FIP	Duration liabilities	Duration FIP	Exposure	Duration benchmark	%-hedge	Duration overlay	%-hedge
Total return								
Carry return	Price return						Residual	
	UFR effect	Selection return	Duration return					
			Shift return		Twist return			
Evaluation of the decisions								
End position								
Value liabilities	Value FIP	Duration liabilities	Duration FIP	Exposure	Duration benchmark	%-hedge	Duration overlay	%-hedge

Table 5.2 Output of our model

5.2.1. The interest rate risk exposure

If we look table 5.2, we can find the interest rate exposure in the begin position, which is partly displayed in table 5.3. Besides the interest rate risk exposure, the table also states the value of the liabilities, the value of the fixed income portfolio, the duration of the liabilities and the duration of the fixed income portfolio. The first thing that catches our eye is the fact that the value of the liabilities is larger than the value of the portfolio. Furthermore, there exists a rather large duration mismatch between the two. Looking at the maturity bucket distribution, the liabilities are mostly concentrated in the highest maturity bucket, whereas the securities in the portfolio are particularly present in the first two. This also explains the high duration of the liabilities in contrast to the duration of the portfolio. This resulted in a negative exposure, mostly concentrated in the highest maturity bucket.

Besides the mismatch between the fixed income portfolio and the liabilities, we are also able to derive the strategy the board chose to quantify interest rate risk. Since the difference between the duration of the liabilities and the portfolio does not equal the interest rate risk exposure, we conclude that the board decided to measure the interest rate risk by the “equal funding ratio”-strategy.

	Val. Liabilities	Val. Portfolio	Dur. Liabilities	Dur. Portfolio	Exposure
Total	2.700,14	2.274,37	565,44	71,41	-404,87
0-1 Yrs	109,68	424,41	0,97	0,02	-0,80
1-7 Yrs	590,18	1.557,29	25,29	39,61	18,31
7-10 Yrs	246,07	19,37	21,82	1,20	-17,18
10-15 Yrs	344,21	12,92	44,09	1,11	-36,02
15+ Yrs	1.410,00	260,38	473,27	29,46	-369,18

X €1.000.000

Table 5.3 Part of the begin position

5.2.2. Investment and decision making process

By stating the whole position of the pension fund at the beginning of the performance period, we are able to identify the investment and decisions making process of the pension fund. Tables 5.3 and 5.4 together form the start position of the pension fund. Based on the position we are able to find the following decisions of the board:

- The interest rate risk exposure is based on the “equal funding ratio”-strategy.
- The exposure duration is hedged, because we determine the exposure by multiplying the hedge with the exposure.
- The interest rate risk is hedged for 70% in the latest three maturity buckets.

Whereupon the overlay manager decided to slightly under hedge the third and last maturity bucket. Whereas he slightly over hedged the 10-15 years maturity bucket.

Furthermore, we can see due to the green marking that his duration bets lie within the boundaries given by the board.

	Exposure	Dur. Benchmark	%-hedge	Dur. Overlay	%-hedge
Total	-404,85	295,67	73,03%	287,02	70,89%
0-1 Yrs	-0,80	-	0,00%	-	0,00%
1-7 Yrs	18,34	-	0,00%	-	0,00%
7-10 Yrs	-17,18	12,03	70,00%	11,97	69,66%
10-15 Yrs	-36,02	25,22	70,00%	25,79	71,60%
15+ Yrs	-369,18	258,42	70,00%	249,26	67,52%

X €1.000.000

Table 5.4 Part of the begin position

5.2.3. Decomposition into different risk factors

In table 5.2 the total return decomposition reflects our decomposition described in chapter 4. Since this part is purely based on an already existing fixed income model, we will only display the performance decomposition of the overlay portfolio relative to the benchmark⁴. From the start position we know that the board decided to hedge the open interest rate risk exposure with 70% in the highest three maturity buckets. Whereupon the overlay manager decided to place his own duration bets. In figure 5.1 below the overall performance of the overlay portfolio relative to the benchmark is displayed. We also added the expected gain. From the previous chapter it became clear that determining the benchmark is not always an exact science. Although the benchmark does reflect the decisions of the board, the benchmark generated a higher gain than expected, due to duration choices made regarding the pieces of the benchmark.

⁴ The full decomposition can be found in the appendix

Furthermore, the duration bets of the overlay manager in the third and highest maturity bucket generated a positive gain. Moreover, his largest bet regarding the highest maturity bucket, also generated the highest return.

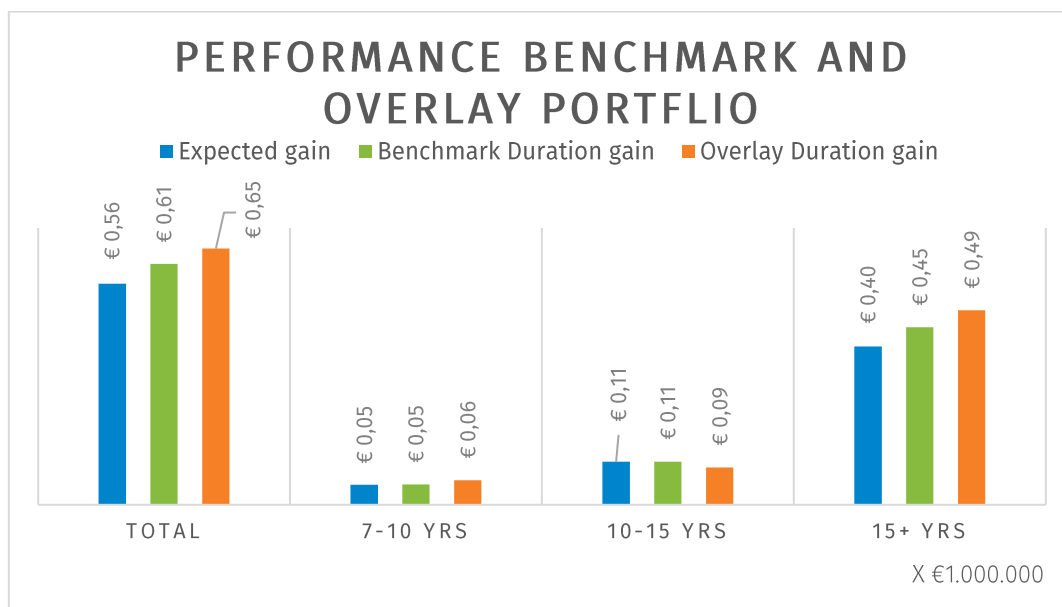


Figure 5.1 Performance overlay benchmark and overlay portfolio

To see from which risk factors the gain generated, we decomposed the gain in figure 5.1 in shift and twist gain. The result can be found in figure 5.2. We can see that both the shift and twist gain generated by the benchmark are quite high in the highest maturity bucket. This may indicate that the benchmark pieces are not well chosen. The duration gain for the overlay portfolio, on the other hand, is mostly generated by a shift change in the yield curve.

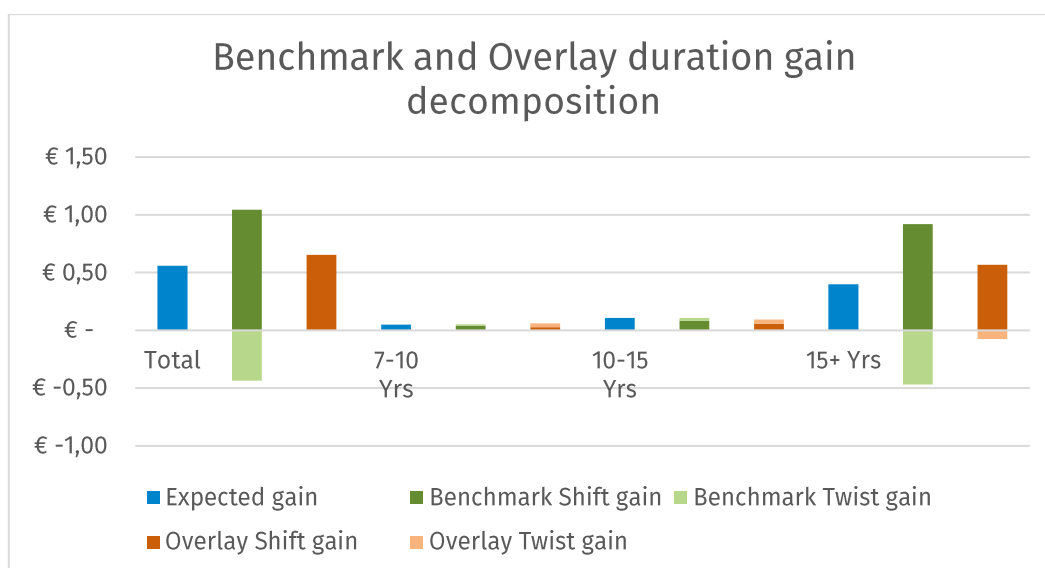


Figure 5.2 Duration gain decomposition of the benchmark and overlay portfolio

5.2.4. Evaluate decisions

In the previous sections we were able to derive the board and overlay manager decisions made during the investment and decision making process of the overlay portfolio. In table 5.5 we can find the value of these decisions. If the pension fund would not have chosen to hedge the interest rate risk, they would have lost € - 1,21 million. The decision to hedge it generated a gain of € 0,61 million. However, by choosing the “equal gain”-strategy would have gained another € 0,12 million. The fact that the decision to hedge the duration of the liabilities would have gained more, is another indication that the benchmark does not reflect the decisions very well. The last decision, the duration bets of the overlay portfolio, also generated a positive gain. Unfortunately, the pension fund suffered an overall loss, due to the selection choices of high yield securities.

Decision	Gain	Cumulative gain
No fixed income portfolio	- 1,21	
Fixed income portfolio	-0,10	-1,31
Interest rate hedge	0,61	-0,7
Hedge strategy	-0,12	
Duration hedge	-0,01	
Overlay manager	0,65	-0,66

x €1.000.000

Table 5.5 Value of the decisions

5.2.5. Monitoring the overlay portfolio

Another important requirement of the analysis is that it should allow for the monitoring of the overlay manager. To show that our model included this feature, we state the end position of the pension fund in table 5.6 below. The hedge for the overlay portfolio increased, which was expected due to its generated gain. Also, the green markers indicate that the overlay portfolio still lies within the boundaries chosen by the board. So, with the positioning and decomposition output of our model, the board is able to monitor and value the overlay manager over time. Furthermore, they can monitor whether the overlay manager complies with their decisions.

	Val. Liabilities	Val. Portfolio	Dur. Liabilities	Dur. Portfolio	Exposure
Total	2.700,14	2.274,37	565,44	71,41	-404,87
0-1 Yrs	109,68	424,41	0,97	0,02	-0,80
1-7 Yrs	590,18	1.557,29	25,29	39,61	18,31
7-10 Yrs	246,07	19,37	21,82	1,20	-17,18
10-15 Yrs	344,21	12,92	44,09	1,11	-36,02
15+ Yrs	1.410,00	260,38	473,27	29,46	-369,18

	Exposure	Dur. Benchmark	%-hedge	Dur. Overlay	%-hedge
Total	-404,87	295,91	73,09%	292,83	72,33%
0-1 Yrs	-0,80	-	0,00%	-	0,00%
1-7 Yrs	18,31	-	0,00%	-	0,00%
7-10 Yrs	-17,18	12,03	70,04%	12,68	73,81%
10-15 Yrs	-36,02	25,23	70,05%	26,80	74,40%
15+ Yrs	-369,18	258,64	70,06%	253,35	68,62%

x €1.000.000

Table 5.6 Position pension fund at the end of the performance period

6. Conclusion and discussion

In the previous chapter we have elaborated upon the outcome of our model based on input data provided by Ortec Finance. In this chapter we will discuss the results. First we will look at our output, whether the model gives a satisfying result. Where after we will elaborate upon the advantages and possible extensions of the model and how to deal with the latter.

6.1. The outcome of the model

From the previous chapter, we know that the model should

- give a clear insight in the interest rate risk exposure of the pension fund;
- follow the investment and decisions making process of the overlay portfolio;
- decompose the total return to the different risk factors;
- be able to evaluate the decisions made regarding risk management;
- allow for monitoring the overlay manager over time.

In the output section of chapter 5, we explained in detail how we displayed the position of the pension fund at the beginning of our performance period, which we also gave for the end of the performance period. Looking at these positions we were able to derive most of the decisions that were made by the board and the overlay manager regarding interest rate risk. The exposure was defined based on an equal funding strategy, because it was not simply the difference of the duration of the liabilities and the fixed income portfolio. The hedge extension by the board was represented by the hedge of the overlay benchmark. Finally, the hedge position of the overlay portfolio reflected the duration bets of the overlay manager. Where the latter also checks whether the overlay portfolio lies still within the hedge boundaries of the board.

Also, the positioning gives a clear insight in the interest rate risk exposure of the pension fund. Due to the use of maturity buckets, the risk distribution of over all maturities are explicitly displayed. Which gives a better understanding of the risk profile of the pension fund over time.

Furthermore, based on the total performance of the liabilities, fixed income portfolio, overlay benchmark and overlay portfolio, we were able to evaluate the decisions that were made during the investment and decision making process. These evaluations give a concise idea which decisions gave the most excess return. In addition, the performance of the overlay manager can be monitored over time.

The model also allows us to see which factors generated the highest excess return. Figure 5.2 states the decomposition of the overlay benchmark and the overlay portfolio. Due to different choices of the overlay manager, the decomposition of the overlay portfolio is different from the benchmark. So we are able to evaluate the choice of the overlay manager in more detail.

Moreover, from the small residual found in our outcome, we conclude that our model is also able to decompose almost all of the gain to a certain risk factor. So, despite of the fact that we used an approximation formula, the model gives valid results.

Based on our findings, we advise Ortec Finance to extend their own performance attribution model with our model. That way they are able to provide pension funds an overlay performance attribution analysis which is easy to understand, evaluates the decisions and which meets the requirements of the DNB. Furthermore, due to the fact that Ortec Finance would extend their model with ours, instead of completely adopting it, they can fully exploit the advantages below, where their own model would mostly take care of the disadvantages.

6.2. Advantages of the model

We will elaborate upon the main advantages of our model:

- **There now exists an overlay performance attribution model.** Although fixed income attribution models did exist, a model explicitly for overlay portfolio was not available yet. With our model a detailed performance attribution analysis can be performed which also evaluates decisions made during the investment and decision making process of the overlay portfolio.
- **It meets the requirements of the DNB.** We mentioned in chapter 4 that the DNB found several insufficiencies in pension funds about defining their interest rate risk management. Due to the fact that the decisions regarding interest rate risk exposure are required as input for the model, makes sure that the board policy is well defined.
- **It can be used by both the board and the overlay manager as a monitor reference.** The overlay manager can monitor if the overlay portfolio is still within the required boundaries, while the board can monitor their own decisions and the performance of the overlay manager.
- **It gives a clear insight in the risk exposure distribution.** Due to the use of maturity buckets within the decisions and the outcome of the model, monitoring the risk distribution is also possible.
- **It decomposes the return into risk factors from fixed income securities.** The model is based on the underlying risk factors of the securities.
- **The outcome of the model is intuitive.** Due to the fact that we used well-known terminology of fixed income securities, the model is easy to comprehend.
- **It does not need extensive input and is easy to calculate.** Most of the input is already in-house available for pension funds. So the model provides output based on minimum costs.
- **It is easy to extend the model.** The model allows easy alterations, so more accurate results could be obtained when desired.

6.3. Possible extensions

Although our model has many advantages due to the fact that we based our model on well-known terminology, the investment and decisions making process and the requirements of the DNB, it also could use some improvements:

- **The decomposition is based on an approximation.** For large changes in the yield curve, the model does not give a valid outcome any more
- **The model can only be used for one performance period.** To solve this, a multi-period model could be added to our model

- **The model is not able to handle “exotic” fixed income securities, like inflation linked bonds or swaptions.** Calculations for these securities should be added to the model, before these securities can be used as input.
- **The calculation of the benchmark is not an exact science when the equal funding ratio strategy is used.** Due to our model the overlay manager had an positive effect of only €0,04 million, which should have been €0,09 million. Due to the choices made regarding the overlay benchmark, it generated a higher return than expected. This begs the question whether the outcome represented the right effect. It should therefore be investigated whether expected returns should be used based on the decisions of the board instead of a benchmark based on calculated pieces.
- **We were not able to test our model on more portfolio data.** Which means that the robustness of the model could not bet tested.
- **We only used a linear model to extrapolate the yield curve point provided to us.** A more advanced model would have given better results
- **The model only exists of basic decomposition factors.** More factors, like the convexity, could be added for a more accurate result.

7. Conclusion

The DNB conducted an investigation to the interest rate risk management of pension funds. One of their main findings included that their ex-post overlay evaluation was not sufficient. Therefore we build an overlay portfolio performance attribution model. Current existing fixed income performance attribution models do not follow the investment- and decision making process of the overlay portfolio and therefore give not sufficient insight in the interest rate management. We extended such an existing model with the following features:

- the ability to quantify the interest rate risk exposure
- the ability to define the benchmark based on the decisions regarding interest rate risk
- the ability to evaluate the decisions made regarding interest rate risk

We tested our model with data provided by Ortec Finance. Based on the outcome, we conclude that our model has the needed requirements to act as a valid duration overlay performance attribution model. The fact that the model explicitly defines the start and end position of the pension fund results in a few benefits. First, it gives a clear insight in the interest rate risk exposure and its distribution. Second, the hedge positions of the overlay benchmark and overlay portfolio reflect the decisions made regarding the interest rate risk management. Third, the board is able to evaluate and monitor the decisions of the overlay manager over time. Whereupon the results of the decisions of both the board and the overlay manager are stated in the decomposition of the total return.

Other advantages of our model lie in the fact that the outcome is intuitive, because it is based on common fixed income instrument terminology. In addition, the input requirements are limited, which makes it cost-friendly. Furthermore, the model is easy to extend for more advanced results.

On the other hand, the model could use some improvements for more advanced futures. The decomposition of our model is very basic. Also, it is only build on basic coupon bonds and swaps. In addition, the model can only be used for one performance period. Moreover, additional research is needed regarding the construction of the overlay benchmark.

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Appendix

A Example DNB Overlay Performance Attribution Analysis

in miljoenen euro's	MW 2012K4	PD.1	PD.2	PD.5	PD.10	PD.15	PD.20	PD.25	PD.30+	Rente (totaal)	Spread	inflatie	(Implied) vol	Tijd	Overig	MW 2013K1
		-0,10%	-0,10%	-0,05%	-0,05%	0,00%	0,00%	0,10%	0,20%							
Delta risicodrijvers		-0,10%	-0,10%	-0,05%	-0,05%	0,00%	0,00%	0,10%	0,20%		-0,25%	0,10%	0,50%	-25%		
Staatsobligaties	390	0,0	0,1	0,1	0,2	0,0	0,0	-2,3	-3,9	-5,8	0,5	0,0	0,0	0,0	0,1	384,8
Index-linked obligaties	55	0,0	0,0	0,0	0,0	0,0	0,0	-0,1	-0,3	-0,4	0,1	0,4	0,0	0,0	0,0	55,2
Bedrijfsobligaties	55	0,0	0,0	0,0	0,1	0,0	0,0	0,0	0,0	0,1	0,3	0,0	0,0	0,0	-0,1	55,3
Hypotheken	100	0,0	0,1	0,0	0,0	0,0	0,0	0,0	0,0	0,2	0,4	0,0	0,0	0,0	-0,2	100,3
Rentederivaten	25	0,0	0,0	0,3	0,5	0,0	0,0	-0,5	-5,0	-4,8	0,0	0,0	0,0	0,0	0,0	20,3
(effectief)		0,0	0,0	0,1	0,3	0,0	0,0	-0,3	-2,5	-2,4	0,0	0,0	2,5	-0,3	0,5	25,4
Overige beleggingen	375										1,9	0,0	0,0	0,0	-5,0	371,9
Totaal beleggingen	1000	0,1	0,2	0,4	0,8	0,0	0,0	-3,0	-9,2	-10,7	3,2	0,4	0,0	0,0	-5,2	987,7
(effectief)		0,1	0,2	0,3	0,5	0,0	0,0	-2,7	-6,7	-8,3	3,2	0,4	2,5	-0,3	-4,7	992,8
Verplichtingen	1000	0,1	0,2	0,4	0,8	0,0	0,0	-3,5	-17,0	-19,1					6,0	986,9
(UFR)	950	0,1	0,2	0,3	0,7	0,0	0,0	-2,4	-3,8	-4,8					4,0	949,2
Gerealiseerde afdekking		100%	100%	100%	100%	100%	100%	86%	54%							
Ondergrens		90%	90%	90%	90%	80%	80%	80%	40%	73,0%						
Doelstelling		100%	100%	100%	100%	100%	90%	90%	50%	75,0%						
Bovengrens		110%	110%	110%	110%	100%	100%	100%	60%	77,0%						

B Full decomposition

	Liabilities		Assets		Liabilities		Assets		Liabilities		Assets	
	Total gain	Total gain	Carry gain	Carry gain	Shift gain	Shift gain						
Total	€ 1,25	€ -0,10	€ -	€ -0,31	€ 1,29	€ 0,16						
(0, 1)	€ -0,00	€ -0,12	€ -	€ -0,12	€ 0,00	€ 0,00						
(1, 7)	€ 0,12	€ -0,26	€ -	€ -0,16	€ 0,06	€ 0,09						
(7, 10)	€ 0,10	€ 0,00	€ -	€ -0,00	€ 0,05	€ 0,00						
(10, 15)	€ 0,19	€ 0,01	€ -	€ -0,00	€ 0,10	€ 0,00						
(15, 1000)	€ 0,84	€ 0,26	€ -	€ -0,03	€ 1,08	€ 0,07						
	Liabilities		Assets		Liabilities		Assets		Liabilities		Assets	
	Twist gain	Twist gain	Duration gain	Duration gain	Residual gain	Residual gain						
Total	€ -0,07	€ 0,07	€ 1,21	€ 0,24	€ 0,03	€ -0,07						
(0, 1)	€ -0,00	€ 0,00	€ 0,00	€ 0,00	€ -0,00	€ -0,00						
(1, 7)	€ 0,05	€ 0,01	€ 0,10	€ 0,10	€ 0,01	€ -0,24						
(7, 10)	€ 0,04	€ 0,00	€ 0,09	€ 0,01	€ 0,01	€ -0,01						
(10, 15)	€ 0,09	€ 0,00	€ 0,19	€ 0,00	€ -0,00	€ 0,00						
(15, 1000)	€ -0,25	€ 0,06	€ 0,83	€ 0,12	€ 0,02	€ 0,17						
	Liabilities		Assets		Benchmark		Overlay		Benchmark		Overlay	
	Duration gain	Dur. gain	Exposure	Duration gain	Duration gain	Dur. effect						
Total	€ 1,21	€ 0,24	€ -0,79	€ 0,61	€ 0,65	€ 0,04						
(0, 1)	€ 0,00	€ 0,00	€ -0,00	€ -	€ -	€ -						
(1, 7)	€ 0,10	€ 0,10	€ 0,01	€ -	€ -	€ -						
(7, 10)	€ 0,09	€ 0,01	€ -0,07	€ 0,05	€ 0,06	€ 0,01						
(10, 15)	€ 0,19	€ 0,00	€ -0,16	€ 0,11	€ 0,09	€ -0,01						
(15, 1000)	€ 0,83	€ 0,12	€ -0,57	€ 0,45	€ 0,49	€ 0,04						
	Benchmark		Overlay		Benchmark		Overlay		Benchmark		Overlay	
	Shift gain	Shift gain	Shift effect	Twist gain	Twist gain	Twist effect						
Total	€ 1,04	€ 0,65	€ -0,39	€ -0,43	€ -0,00	€ 0,43						
(0, 1)	€ -	€ -	€ -	€ -	€ -	€ -						
(1, 7)	€ -	€ -	€ -	€ -	€ -	€ -						
(7, 10)	€ 0,04	€ 0,03	€ -0,01	€ 0,01	€ 0,03	€ 0,03						
(10, 15)	€ 0,08	€ 0,06	€ -0,02	€ 0,03	€ 0,04	€ 0,01						
(15, 1000)	€ 0,92	€ 0,57	€ -0,35	€ -0,47	€ -0,08	€ 0,39						

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