An Analytical Approach to Capacity Analysis on the Dutch Railway Network

Master Thesis
MSc Business Analytics

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February, 2024

Abstract

This thesis resulted from a thesis-internship at Arcadis NL guided by the Vrije University of Amsterdam. In this thesis an analytical model for calculating the minimal headway per station was defined and it is then shown how this model can contribute to important insights to better the bottlenecks in the railway network. The model is specifically designed upon assumption for the Dutch railway network and is kept simple in order to have clear interpretations on the results. This thesis' findings were checked in a simulation tool called XANDRA, to validate the results are, at the very least, reasonable. With the use of parameter analysis it was concluded that for the case study capacity can increase by nearly 18% per hour, only from solving one single bottleneck.

Keywords: *headway, capacity, railway, analytical model, parameter analysis*

Acknowledgment

When this thesis project started in July 2023, the research topic was not entirely specified. While identifying the agreed topic, I discovered our ideas on the topic did not align with my interests and the company's needs. It then took some time before the topic was redefined and specified to match my interests and fields of study and the company's needs and interests. The thesis internship lasted a bit longer than expected because of this, but I had great support from both the company and the university to get to the current topic of this thesis. This rough start was a great learning experience, as was the rest of this project.

Next to everything I learned, I also experienced this project as enjoyable and a good preparation for my future career path(s). For the enjoyable eight months spent on this project, I would like to express my gratitude to many people for helping me whenever needed to make this project a great learning experience. My first special gratitude goes to Daniël Burggraaf for supporting me in this project and showing me how Arcadis works. He was always available for questions and explanations on the railway network. I was guided pleasantly and we had many enjoyable talks about the thesis topic and different topics related to our shared interests. I am also very grateful for the guidance from Johan Bekker, who ensured everything was taken care of at Arcadis and made me part of the amazing teams of data scientists to experience firsthand how things go in the team. Thank you for making my time at Arcadis extra special by including me in the team and helping me along the way of this project, especially in the final stages of my graduation. I would also like to thank Menno van Lint, Harm Loonstra and Martin van Essen for the time they spent on me and their explanations, help, and brainstorming sessions whenever I needed them. I want to thank all the other employees at Arcadis that made this internship very special. Thank you all for the pleasant time you gave me, for being open and flexible, and for inviting me to the fun activities. Also, thanks for all the interesting talks I had, which made my time at Arcadis very informative.

In addition, I would like to thank Ronald Meester and Annemieke van Goor for supporting me in this thesis project. Annemieke, you were of great help in helping me find the motivation to find this wonderful internship after being rejected multiple times and not knowing where to start anymore. Ronald, you were of great help, always open for discussion, and supported me in following my interests and adapting the plan. I am also thankful that Oliver Fabert is willing to be the second reader of this thesis project; Oliver, thank you for taking the time to read and grade this thesis project in order for me to be able to (hopefully) graduate. Lastly, I am grateful to my girlfriend, who supported me during my choice to start this master's and during this master's. I am especially grateful for her mental support in making it this far towards graduation.

Leiden, February 2023
Lisa de Jong

Table of Contents

List of Figures

List of Tables

1 Introduction

There are one million kilometers of railways worldwide, servicing over three trillion passenger kilometers annually. From urban public transit systems to regional railways to high-speed intercity trains, millions of people rely on railways for work, leisure and travel. Therefore, making sure that the trains get the passengers to their destinations safely, affordably, comfortably and on time is a complicated task (Janny M.Y. Leung and Lai, 2016).

In the Netherlands, one and a half million people take the train daily (ProRail, 2019), which, together with a substantial amount of freight trains on the rails as well, makes it the busiest railway network in Europe (Pascal Ramaekers and Pouwels, 2009) and the number of daily passengers will keep increasing in the upcoming ten years (ProRail, 2019). According to Pier Eringa (President-director ProRail at the time), the Dutch Railway expects an increase in demand of $30-40\%$ over the upcoming ten years, with an increase of $50-60\%$ in the Randstad area (ProRail, 2019). Pier Eringa adds that more trains than ever are on time, the number of delays and other inconveniences has decreased a lot, making traveling by rail a trustworthy alternative. This is great news, but on the other hand, it causes demand to rise way quicker than expected. To facilitate all this demand, the Dutch railway network needs to be adapted soon (ProRail, 2019).

Because it is expensive, both in money and in time, to expand or change the railway layout. It is important first to see if there are possibilities to facilitate the demand on the existing railway network and if there are no possibilities, it is important to look at where the bottlenecks of the railway are.

As is commonly done in studying railway networks. In this report, a capacity analysis is performed. Analytical formulas for minimal headway in the Netherlands are developed and then used in a small case study to show how these formulas can be of value for Arcadis.

1.1 Background Information on Arcadis

Before the research questions and the purpose of this thesis are addressed, it is important to look at some background information; because this research is performed at Arcadis, it is essential to include background information about the company.

Arcadis is a leading company delivering sustainable design, engineering and consultancy solutions for natural and built assets. It is dedicated to improving quality of life by creating livable places where people and communities thrive. They mainly focus on finding innovative and lasting solutions to some of the world's biggest challenges (Arcadis, n.d.).

In the Netherlands, a substantial amount of teams work on projects concerning the railway network in the Netherlands. Hence, Arcadis has many projects with companies like ProRail, ASSET Rail and NS Stations.

- ProRail is the economic owner of the rail infrastructure of the Netherlands (ProRail, n.d.).

- ASSET Rail is partly owned by Arcadis (and partly by Dura Vermeer), and it is responsible for more than 40% of all maintenance of the Dutch railway (ASSET Rail, n.d.).

- NS Stations is part of NS *(Nederlandse Spoorwegen)*, they are responsible for the design and maintenance of all train stations in the Netherlands (Nederlandse Spoorwegen, n.d.).

An example of a project Arcadis is working on is changing the layout of the Central station of Amsterdam. Arcadis has a lot of knowledge of railway networks, which is used to work on designing, repairing and managing the railway. However, they work mainly with simulation tools to show that their ideas work.

As a Business Analytics and previously Econometrics student, I proposed to take a look at the railway network with a more mathematical point of view. Taking some of the knowledge on the railway Arcadis has and transforming it into a mathematical formula that could assist in future

railway projects. For me, this is a great opportunity to get to know all the ins and outs of the train railway network in the Netherlands and for Arcadis this is a way to see things from a different angle and even work with some of the conclusions from this thesis.

1.2 Research Topic

The article by Yung-Cheng Lai and Jong, 2010 (See Section 2.10.2 for more information on the article itself) is what got me into the topic of this thesis project. As informative and useful as their research was, small steps on how the formulas used came to be as they are presented were unclear to me and hard to get an explanation to. This intrigued me to dive into such formulas myself and find a proper formula matching the needs of the Dutch railway system.

The topic for this thesis research will be a capacity analysis of the Dutch Railway network, where the main question: *What are bottlenecks in the Dutch Railway network and how can they be found?* will be answered. To be able to answer the research question, a few sub-questions were created to define the steps that must to be taken to come to a well-formed conclusion.

1. What model is/Which formulas are simple yet effective to calculate capacity on the Dutch railway network?

2. What parameters are important to the capacity of the railway and thus need to be used in the formulas?

3. What assumptions must be made to keep the model simple yet somewhat realistic?

4. How can the decided-upon model be used in real-life to identify bottlenecks?

1.3 Objectives of the Internship

This thesis report is the result of the Master Project Business Analytics, which is the graduation project for the master's program Business Analytics of the Vrije Universiteit Amsterdam. This thesis report builds on and integrates previous Business Analytics courses. It allows me to show that I am capable of independent research at an academic level on a specific topic in the field of Business Analytics under the supervision of the Faculty of Science, Vrije Universiteit.

As this thesis report results from more than six months of internship at Arcadis, the program's objectives are not only to apply the knowledge of Business Analytics courses practically but also to understand how Arcadis functions. Through the experiences obtained during this internship, I improved my social and communication skills. In addition, I got the chance to explore potential career options.

1.4 Report Outline

This thesis consists of seven sections. Section 2 discusses a literature study where some important terms for railway capacity will be discussed. Section 3 then discusses these terms on the Dutch railway network, and assumptions for the model are introduced. Next, Section 4 will give a detailed description of the formulas of the analytical model and how this model was built. Section 5 then uses the model from Section 4 to show its use. Section 6 concludes all findings and future research suggestions will be addressed in Section 7.

2   Literature research

In this Section, some interesting findings will be discussed that helped to understand the railway a lot better and that helped to come to the results of this thesis.

2.1   Analytical

Analytical methods describe the movement of trains by different mathematical formulas and algebraic expressions (Mikulcic and Mlinaric, 2021). They are usually straightforward models aimed at determining a preliminary solution and they usually obtain theoretical capacities (Abril et al., 2008); the ideal highest value with the maximum number of trains separated by the minimum headway allowed by the signaling system and block lengths (see definition *theoretical capacity* below) (Mikulcic and Mlinaric, 2021). More realistic practical capacity is obtained using assumptions close to reality and results in usually around $60 - 75\%$ of theoretical capacity (See definition *practical capacity* below) (Abril et al., 2008). Commonly used analytical methods are a former standard for line capacity determination based on the identification of the most critical track section. However, their main disadvantage is that they are unable to cover the capacity calculations for stations. Station capacity is important as often the busiest station in the double-track line is the bottleneck of the whole system due to the frequent stops and station dwell time. (Mikulcic and Mlinaric, 2021).

Analytical methods for computing railway (line) capacity are a good start for identifying bottlenecks and major constraints. However, analytical results vary from one method to another depending on the parameters considered in the model. Furthermore, analytical models are very sensitive to parameter input and train mix variations (Abril et al., 2008).

An analytical method that is applied on stations by Francesco Rotoli and Ricci, 2016 looks at haltering stations and passing stations separately and computes the capacity for single track (with balanced traffic) and double track line segments, considering very useful parameters. Another paper that covers a similar approach but adapted to the Taiwanese railway network and with a slightly different look on which parameters are to be included is the paper by Yung-Cheng Lai and Jong, 2010. A closer look will be taken at both papers in Sections 2.10.1 and 2.10.2 respectively.

2.2   Capacity

Capacity analysis is critical, especially for busy railway networks. In order to guarantee stable operations on the railway network, it is necessary to evaluate the capacity of the railway network and determine possible infrastructure bottlenecks. This capacity analysis requires specific and accurate microscopic models that include detailed infrastructure characteristics, signaling, train characteristics and interlocking logic (Besinovic and Goverde, 2018).

To be able to discuss railway capacity, some important and commonly used terminology of railway capacity needs to be defined:

- **Theoretical capacity** is the maximum number of train paths on the infrastructure in a given window and represents an upper limit for infrastructure capacity (Besinovic and Goverde, 2018).

- **Practical capacity** is the maximum number of train paths on the infrastructure in a given time window given the traffic pattern, operational characteristics or timetable structure (Besinovic and Goverde, 2018). It is the capacity that can permanently be provided under normal operating conditions (Abril et al., 2008).

- **Used capacity** is the actual traffic volume occurring over the network. It reflects actual traffic and operations that occur on the line. It is usually lower than the practical capacity (Abril et al., 2008).

As stated before, if the theoretical capacity represents the upper theoretical bound, the practical capacity represents a more realistic measure. Thus, practical capacity is calculated under more realistic assumptions related to the expected operating quality and system reliability (Abril et al., 2008).

2.3 Headway

Headway is the time between two consecutive trains. Minimal headway is needed to calculate the capacity of a railway network. Landex, 2009 states that the larger the minimum headway is, the less capacity there is on the railway network. The minimal headway is the minimal time interval needed for the station to accomplish necessary train arriving and departure operations, which is also the key to ensuring the safety of trains running in the section (Zhang, 2015). According to Abril et al., 2008, the headway time between consecutive trains depends mainly on two factors: the length of the blocks and the train speed. These two factors affect the travel time, the braking time and the release time. Generally, the headway time can be estimated by summing up these three times plus a constant operating time (Abril et al., 2008). There are many articles on calculating the minimum headway between consecutive trains and most articles focus on a specific part of the railway. A few examples are: as Zhang, 2015 introduces some formulas for the minimal headway of the high-speed railway in China, he found that the headway between two arriving trains is the bottleneck for the minimal headway. In addition, Ryus et al., 2013 calculate the minimum headway for single and double tracks and they state that the controlling headway will be the highest of all non-interference headway calculated in their paper (Ryus et al., 2013). Another example is Yung-Cheng Lai and Jong, 2010, who use their calculation of the minimum headway and the capacity of the railway network to identify bottlenecks at stations and use an optimization model to look at the trade-off of costs versus capacity increase (see Section 2.10.2).

2.4 Blocking time

Besinovic and Goverde, 2018 state that the concept of blocking times is closely related to capacity assessment. Most railway networks are split into smaller sections called blocks or interlocking segments. On the railway network, a block section or an interlocking segment (including one or more switches or crossings) can be exclusively allocated to a single train per given time unit. The blocking time of such a resource is the time during which the resource is solely dedicated to a single train and cannot be used by any other train. The blocking time includes setup and release times of the route and signals, as well as the driver's sight and reaction time before the approach signal. With the use of blocking times, a timetable can be converted into a blocking time stairway, which is a graphical depiction of the blocking times for a specific resource. It is applied by Besinovic and Goverde, 2018 to compute minimum headway. The resource that defines a minimum headway time is called a critical resource (Besinovic and Goverde, 2018).

2.5 Signaling

Mikulcic and Mlinaric, 2021 state that moving a signal, and therefore enlarging or shortening a block section, is often considered the most effective way of improving (line) capacity because they are more straightforward and less expensive than improving other aspects of infrastructure. Block section lengths, train speeds and train lengths are important parameters when looking at the performance of the signaling systems (Mikulcic and Mlinaric, 2021).

In maintaining safety, signaling relies on track detection systems that monitor whether a section of the track is occupied, interlocking, Automatic Train Protection (ATP) and appropriate communication connections (Mikulcic and Mlinaric, 2021). The ATP generally used in Europe at the moment is an example of an European Train Control System (ETCS), there are three operational levels of ETCS. Mikulcic and Mlinaric, 2021 explain that Level 1 and Level 2 share the philosophy for conventional signaling with track-side signals. Level 1 offers overlay for the existing signaling with additional protection in discrete points, whereas Level 2 represents a cab signaling system with wireless transmission of movement authority, continuous speed supervision linked with a dynamic braking system and an optional dependence on external signals. Level 3 is a moving block signaling system without classical blocks and aspects, which combines the benefits of continuous speed supervision and a train integrity monitoring system. The minimal distance between two consecutive trains depends on the speed and braking distance of the second train. In addition, there is also a

new 'level' still in the testing phase, which is called ERTMS, which is the hybrid ETCS Level 3 concept with virtual blocks ((Mikulcic and Mlinaric, 2021), (Abril et al., 2008)).

One way to meet the demand increase and transport more passengers is by increasing the frequency of trains, which is directly dependent on the headway limitations of the signaling systems (Mikulcic and Mlinaric, 2021).

Lai and Barkan, 2004 state that a drawback of fixed-block signal systems is that when a train is in any portion of a block, the system considers the entire block to be occupied and this causes restricting aspects to be displayed by signals in the following blocks, irrespective of the actual stopping distance of the particular following train. Trains are, therefore, often spaced at larger distances than required, which results in unused capacity (Lai and Barkan, 2004). This is especially a problem for passenger trains. Because of their shorter stopping distance, they can safely follow other trains more closely than freight trains can, but because of the long blocks needed to accommodate the safe-stopping distance of freight trains, they are separated by a greater distance than necessary (Lai and Barkan, 2004).

However, since ERTMS, which intends to solve this drawback by the use of moving block signaling, is still in the testing phase, the use of signals is still very much required. The signals help extend the train driver's visibility, which allows greater speeds (Abril et al., 2008).

2.5.1 Block Lengths

According to Abril et al., 2008, the length of blocks is very important, mainly on single-track sections. They state that, in general, if the length of the block increases, so does the travel time of trains, which results in less capacity on the railway network. This seems logical but note that the length of the railway does not change the length of the railway (see Figure 11). Abril et al., 2008 suggest that having smaller blocks will increase the traffic flow.

2.6 Single/Double Tracks

- *Single Track*: One railway track that is bidirectional and trains have to share this track to travel in both directions.

- *Double Track*: At least two railway tracks that are either bidirectional or directed, where at least one track is mostly dedicated to one direction and at least one other track is dedicated to the other direction. Trains do not necessarily have to share this track.

There are many differences between double and single-track railway lines, also regarding capacity. Double-track railway lines can generally operate significantly more than twice the number of trains than a single-track railway line. As each track of double track lines is normally operated in a single direction, the timetable can be planned for each direction (almost) independently (Landex, 2009). Whether a railway network layout uses single tracks or double tracks has a major impact on the network's capacity. Changing the layout from a single track to a double track and computing the effects is not as simple as multiplying the number of tracks: two tracks usually have around four times more capacity compared to a single track; however, a four-track line rarely increases capacity by more than 50% over a double line. Furthermore, adding a second track may not eliminate the problem because the station is the real bottleneck. In addition, side tracks at stations also increase the capacity of a single track because they allow trains to carry out crossings and over-takings (Abril et al., 2008, Ryus et al., 2013).

2.7 Speed Limits

The condition of the rails, ties, and ballast dictate the weight and type of equipment that can be used on the line, as well as the speeds allowed on the line. They have an important influence on the capacity. The speed limits are regulated utilising speed profiles, which take into account physics, safety, comfort, train types, etc. (Abril et al., 2008).

According to Abril et al., 2008, slow trains decrease the capacity because they need more travel time. Abril et al., 2008 also found that when speed decreases, the maximum line capacity decreases.

2.8 Trains

Railway capacity is very much dependent on traffic mix. The ideal case is when all trains are the same and have the same speed. As the train mix increases, so more and more different types of trains have to share the rail, more interference is generated. Thus, trains require over-takings and crossings, and the train mix reduces the traffic flow. Besides maximum speed, other rolling stock characteristics such as acceleration and deceleration are also important (Abril et al., 2008).

Burdett and Kozan, 2006 even try modifying their proposed formulas for theoretical (or, as they call it, "absolute") capacity by introducing a percentage train mix because they also recognize that this makes the outcomes more realistic.

2.9 Robustness

Robustness is also known as the quality of service or reliability. Since train operations do not occur in a perfect world, small and random disturbances and failures occur in the real management of trains during railway operations. This reduces the capacity and therefore, some buffer times should be considered to design a robust timetable. The stochastic effects of the imperfect world are often difficult to consider in (line) capacity evaluations. Still, it is worth mentioning as a balance between service reliability and maximum physical capacity is needed to find the economically optimal level of capacity. As Abril et al., 2008 already stated, introducing scheduled slacks among trains will decrease capacity but will increase reliability.

2.10 Into More Detail

In this subsection, the details of two papers that relate closely to this thesis are shortly discussed.

2.10.1 Francesco Rotoli and Ricci, 2016

Francesco Rotoli and Ricci, 2016 wrote an interesting article on the capacity of a railway network. They assume non-disrupted circulation within their proposal, which means a running train should always find the approaching signal 'clear' to avoid unnecessary acceleration/deceleration phases and disturbed circulation. They state that for a line segment, the capacity can be calculated by dividing the studied time period by the average minimal headway plus the expansion margin and some extra time based on the number of intermediate block sections on the line. As discussed in Section 3.4.1 later, this study assumes that these extra margins are equal to zero. Hence, capacity can be calculated by dividing the time period by the average minimal headway.

Furthermore, double track and single track with balanced traffic are discussed separately and formulas are proposed for haltering and passing stations.

Francesco Rotoli and Ricci, 2016 analyzed various combinations, combining $1.5m$ block length and $4m$ block length with travel speed limits of 90, 120 and $150km/h$. Using their computed formulas, they found that the smaller block length shows substantially higher daily capacity when the distance between two consecutive stations is small, and the higher the travel speed, the more capacity. These results hold for both single and double-track lines.

Zhang, 2015 found that two arriving trains is critical for the headway. It is therefore that this thesis will not look into passing stations like Francesco Rotoli and Ricci, 2016 do.

2.10.2 Yung-Cheng Lai and Jong, 2010

Yung-Cheng Lai and Jong, 2010 proposed an analytical approach of calculating the capacity of part of the Taiwanese railway network. It is because of this paper that the topic of this thesis was decided upon. Yung-Cheng Lai and Jong, 2010 state their formulas for arriving trains at Station Type I (See Figure 1(a)) as $T_{s,A1}$ as shown in Equation 1 and for arriving trains at Station Type II (See Figure 1(b)) as $T_{s,A2}$ as shown in Equation 2.

$$T_{s,A1} = \sqrt{\frac{2(L_i + s_x)}{K_a a_i(G_0)}} + \frac{v_j}{K_b b_j(G_i)} - \frac{v_y^2}{2K_a b_j(G_i)v_j} + \frac{B_1 + B_s - s_x}{v_j} + t_o + t_r + t_{di} \qquad (1)$$

$$T_{s,A2} = \frac{v_j}{K_b b_j(G_i)} - \sqrt{\frac{2s_e}{K_a a_i(G_0)}} + \frac{v_y^2}{2K_a b_j(G_i)v_j} + \frac{B_2 + B_1 + B_s - s_x}{v_j} + t_o + t_r + t_{dj} \qquad (2)$$

i	Index of the preceding train	v_y	Speed limit for yellow signal
j	Index of the following train	v_j	Cruising speed of train j
L_i	Length of train i	B_2	Length of 2nd block section before arrival station
s_x	Head-to-signal distance	B_1	Length of 1st block section before arrival station
s_e	Distance between tail of train to turnout	B_s	Length of the station block
K_a	Acceleration efficiency	t_o	Time for clearing the signal
K_b	Braking safety factor	t_r	Reaction time for driver and braking system
$a_i(G)$	Acceleration speed of train i on gradient G	t_{di}	Dwell time for train i
$b_j(G)$	Deceleration speed of train j on gradient G	t_{dj}	Dwell time for train j
G_0	Gradient before arriving at station	$T_{s,A1}$	Arrival headway at the same track in station
G_i	Gradient after departing from station	$T_{s,A2}$	Arrival headway at different tracks in station

Equations 1 and 2 and the parameter definitions are copied from Yung-Cheng Lai and Jong, 2010 their paper directly. As Zhang, 2015 found that arriving trains cause the critical headway, the formulas for that case are solely discussed. Equation 1 is the arrival headway of two consecutive trains at station Type I(See Figure 1(a)), and Equation 2 is the arrival headway of two consecutive trains at station Type II(See Figure 1(b)). As it is unclear from their paper how these formulas were developed and what is meant by certain parameters, it is hard to understand and work with the formulas. It is therefore, that for the Dutch railway network a similar model is developed where all the steps are explained such that working with the model will be easier and results can be interpreted. Section 4 will compare the model to Equations 1 and 2.

The paper of Yung-Cheng Lai and Jong, 2010 takes their results one step further by creating a capacity planning framework to determine the best capacity expansion policy. This is done by using an Alternatives Generator and an Investment selection model. Lastly, they perform a case study concluding that it is best to decrease block lengths to increase the railway network's capacity.
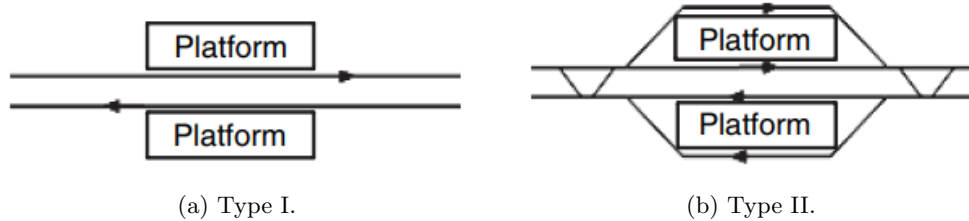

(a) Type I.   (b) Type II.

Figure 1: Station Types by Yung-Cheng Lai and Jong, 2010

3   The Dutch Railway

To be able to create and adjust formulas to fit the Dutch railway network, it is necessary to understand how the Dutch railway network operates and the assumptions made to make the railway network less complex.

ProRail is the owner of the railway network in the Netherlands and needs to make sure the entire network works safely and properly. It, therefore, has many rules to maintain a certain level of certainty that the railway is safe. These rules are assumed default in this research and capacity is only planned with all ProRail rules met (ProRail, n.d.).

3.1   Blocks and Signals

An important feature of the Dutch railway network and many other (European) railway networks is interlocking. The Dutch railway network is divided into smaller sections as Besinovic and Goverde, 2018 described (see Section 2) as well. These sections are 'guarded' by signals. Before diving into the formulas, it is necessary to explain the basics of the signals alongside the Dutch railway network. The formulas will be based on these basics.

The Dutch railway network is split up into blocks; each block starts with a signal that can take on three different colors (red, yellow, green). When a train crosses a green signal, all is clear, and the train can keep traveling as it is. When a train crosses a yellow signal, the train should start to break, anticipating a potential upcoming red signal. This yellow signal can come with a number attached; the number may be 4, 6, 8 or 10, which means the train needs to start braking to the speed limit of the given number times $10km/h$. There are various other signals, but these are irrelevant to discuss within this research. Lastly, when a train driver sees a red signal, it must stop immediately (Sverre, n.d.). Figure 2 shows a simple representation of this signal system.
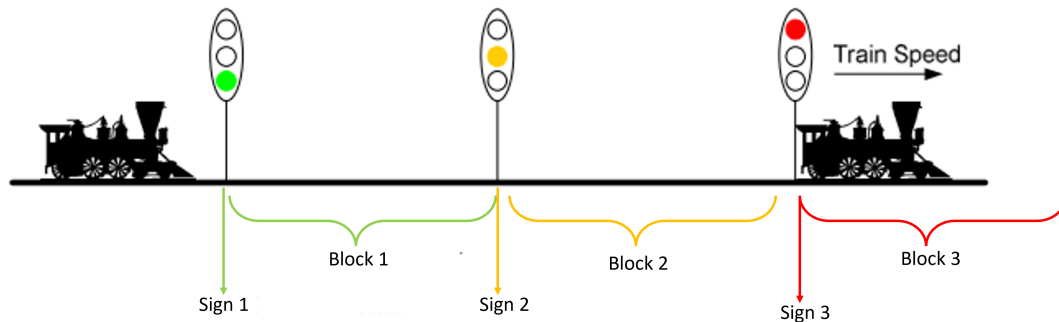
Figure 2: Edited version of *Headway Between Consecutive Trains* by Abril et al., 2008

On the other side, a signal turns red whenever the block it is connected to is occupied. The signal turns yellow whenever the block after the block that it is connected to is occupied and the signal is green otherwise. This last statement is sometimes not true; where some signals are manually handled due to complicated turnout sections, these are sections containing multiple switches/crossings. These signals are red by default and only turn green when the rail-traffic-leader knows the block is save and all switches and crossings in the block are set in the right position for the train to travel through.

3.2   Speed Limits

Next to the yellow signals which can be combined with a number to indicate a speed limit, there are signs along the Dutch railway that tell the train driver what the speed limit is. These speed

limits are based on the layout of the railway. Switches, sharp corners, stations and other potentially dangerous parts have lower speed limits according to ProRail's safety rules.

Signs are simple square and triangle boards that indicate the train should start braking to the local speed limit. Furthermore, switches and crossings can cause the local speed limit to be lower. Switches and crossings are special railway layouts allowing trains to switch rails. Table 1 shows the speed limit at a few of the most common switches.

	deflection	straight-ahead
Switch 1:9	40 km/h	160 km/h
Switch 1:12	60 km/h	160 km/h
Switch 1:15	80 km/h	200 km/h
(English) Crossing	40 km/h	80 km/h

Table 1: Speed Limits in Switches (ProRail, 2018a)

3.3 Automatic Train Protection

In the Netherlands, as well as most other countries nowadays, railway safety relies on Automatic Train Protection (ATP)(Mikulcic and Mlinaric, 2021). This helps the driver follow all speed limits and other traffic rules to prevent unsafe situations.

3.4 Methodology

In the next Sections, an analytical model will be developed for the Dutch railway network. As discussed previously in Section 2, an analytical model describes the movement of trains by different mathematical formulas and algebraic expressions (Mikulcic and Mlinaric, 2021). These models are quite straightforward and have proven to be of use when identifying bottlenecks. Analytical models usually give theoretical capacity and can also give practical capacity when considering realistic assumptions.

The model for this thesis research will be based on the information discussed above. The analytical model will be developed for two different types of stations and will consist of three states for each station type. Before developing the analytical model, assumptions need to be set in place.

3.4.1 Assumptions

Assumptions are necessary to make the model less complex and to be able to clearly interpret the results. The assumption below are valid throughout the rest of this research and teh model that is developed holds under these assumptions.

- It is assumed that reaction time and system operation time, for the time being, are insignificant and, therefore, equal to 0. Note that it is rather easy to add this to the headway later on.

- It is assumed that the gradient of the railway tracks is 0.

- Confirm the ProRail rules all trains are planned in such a way that no train is constrained by a yellow signal. This is called "conflict free planning" (ProRail, 2018b). Note that Francesco Rotoli and Ricci, 2016 also followed this assumption.

- Deceleration and acceleration are linear.

- Trains are assumed to travel at the maximum speed allowed.

- It is assumed that a perfect world, in which drivers do not make mistakes and dwell time is constant (ProRail, 2018b).

- It is assumed that accelerating/decelerating when going straight through a switch/crossing is possible, but this is not the case when going at an angle through the switch/crossing.

- It is assumed that the Dutch signals and signs are placed in such a way that the trains do not need more than one block to reach a desired speed limit.

4 The Model

In this section the model is developed, using simple illustrations, parameter definitions and discussion of the movements the train makes while arriving to a station, haltering at that station and leaving that station. Parameters are introduced first, after which the formulas for headway are determined.

4.1 Parameters

- a_i = acceleration of train i (m/s^2)

- B_1 = length of the first block before arriving at the station (m)

- B_n = length of the first block after leaving the station (m)

- B_s = length of the station block (m)

- $\hat{B}_1$ = length of the switch block, going to the side platform (m)

- $\hat{B}_n$ = length of the switch block, coming from the side platform (m)

- b_1 = first block before arriving at the station

- b_2 = first block before arriving at the station

- b_n = first block before arriving at the station

- b_s = first block before arriving at the station

- d_j = deceleration of train j (m/s^2)

- $\Delta t_{\{a,b\}}$ = time it takes to accelerate from a to b (sec)

- Δx = distance the train travels during acceleration from 0 to v_{b_n} (m)

- $\Delta y_{\{a,b\}}$ = distance needed to decelerate from a to b (m)

- e = minimal distance between the train head and the next signal, in such a way that the train driver cannot see the signal yet.

- h = head-to-signal distance when the train is at a stop at a station (m)

- i = index of the preceding train

- j = index of the following train

- L_i = length of train i (m)

- T_{S1} = time needed for one cycle (*Platform Type I*) = time needed between two trains such that no train experiences scheduled delays due to another train (sec)

- $T_{s2,c}$ = time needed for one cycle (*Platform Type II*), specifically case c = time needed between two trains such that no train experiences scheduled delays due to another train (sec)

- T_{M_q} = the needed time to travel the distance defined by Movement q (sec)

- t_{s,d_i} = dwell-time of train i at station s (sec)

- v_b = speed limit in block b (m/s)

$\rightarrow a_i, B_1, B_n, B_s, \hat{B}_1, \hat{B}_n, d_j, \Delta t_{\{a,b\}}, \Delta x, \Delta y_{\{a,b\}}, e, h, L_i, T_{S1}, T_{s2,c}, T_{M_q}, t_{s,d_i}, v_b \geq 0.$
$\rightarrow$ speed limits need to be whole numbers in km/h with steps of 5, e.g. $v_{b_b} \in \{0km/h, 5km/h, 10km/h, 15km/h, 20km/h, ..., 130km/h, 135km/h, 140km/h\}$

4.1.1 Parameter Comparison with Yung-Cheng Lai and Jong, 2010

It is worthy to note that some parameters in the list above match with parameters defined by Yung-Cheng Lai and Jong, 2010, shown in Section 2.10.2. there are also parameters that differ or are new. A short comparison of similar parameters is shown in Table 2.

Parameters thesis model		Parameters *Yung-Cheng Lai and Jong, 2010*
a_i	$\leftrightarrow$	$a_i(G)$ with G equals 0.
B_1	$\leftrightarrow$	B_1
B_s	$\leftrightarrow$	B_s
b_1	$\rightarrow$	is not defined, but would be similar in both papers
b_2	$\rightarrow$	is not defined, but would be similar in both papers
b_s	$\rightarrow$	is not defined, but would be similar in both papers
d_j	$\leftrightarrow$	$b_j(G)$ with G equals 0.
h	$\leftrightarrow$	s_x
i	$\leftrightarrow$	i
j	$\leftrightarrow$	j
L_i	$\leftrightarrow$	L_i
T_{S1}	$\leftrightarrow$	$T_{s,A1}$
$T_{s2,c}$	$\rightarrow$	$T_{s,A2}$
t_{s,d_i}	$\rightarrow$	t_{di}
v_b	$\rightarrow$	v_j
Note that		$G_0 \rightarrow$ equals 0 according to assumptions made for this thesis
		$G_i \rightarrow$ equals 0 according to assumptions made for this thesis
		$t_o \rightarrow$ equals 0 according to assumptions made for this thesis
		$t_r \rightarrow$ equals 0 according to assumptions made for this thesis

Table 2: Similar Parameters Yung-Cheng Lai and Jong, 2010 and This Thesis

Parameters that are not shown in Table 2 are in no clear way similar to the parameters defined by Yung-Cheng Lai and Jong, 2010. Also, through the assumptions made in Section 3.4.1, some of the parameters of Yung-Cheng Lai and Jong, 2010 are equal to 0 in the model that is determined in a moment. After the model is determined a further comparison will be done between the model and the formulas of Yung-Cheng Lai and Jong, 2010.

4.2 Headway

As introduced earlier, headway is the time needed between two consecutive train sin order for both trains to not crash into each other or be bothered by each other in any unforeseen way in theory.

4.2.1 Platform Type I

To determine a model for calculating the minimal headway, the minimal headway for a small station, with a platform along the main rail and no options to go around it (Figure 1(a)), is first looked at.

A train either stops at the station or passes the station, both at the main rail. In the most time-consuming case, both trains need to make a stop. this case is therefore considered.
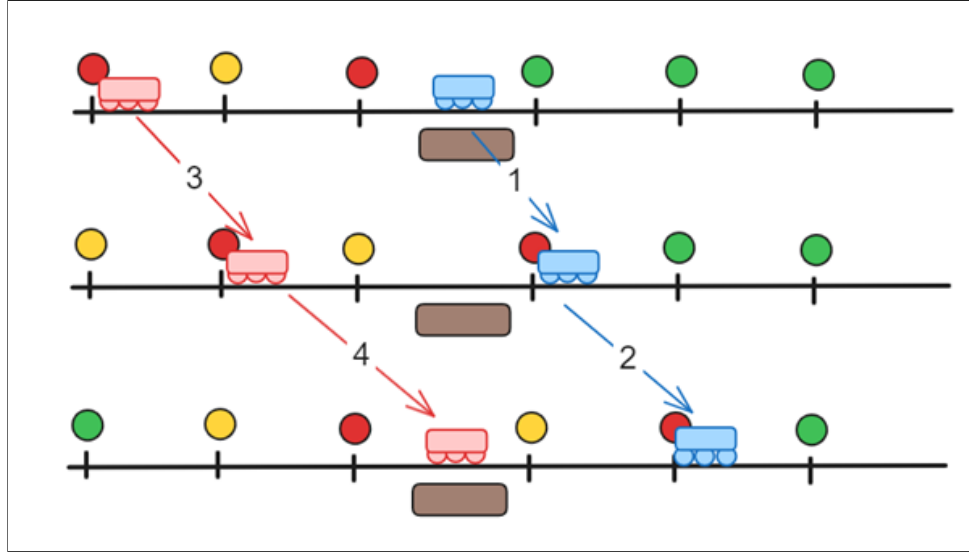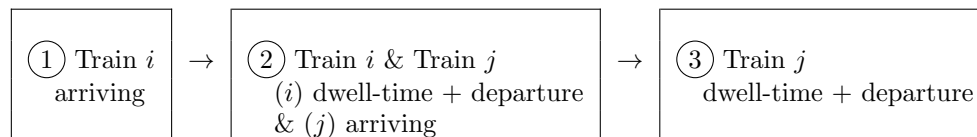

Figure 3: Train Movements Platform Type I

First, Figure 3 depicts train j (in red) following train i (in blue) and a platform where both trains need to make a stop of t_{s,d_i} and t_{s,d_j} seconds respectively. Figure 3 is to be read from top to bottom.

A start-up process will be discussed after which the "steady-state" cycle drawn in Figure 3 will start, where it is assumed train i just arrived and is starting its dwell-time and after that train i will leave the station, simultaneously train j will start arriving at the station to start its dwell-time and restart the cycle.

Since the "steady-state" cycle starts when train i has arrived and starts its dwell-time, the start-up process consists of train i arriving. Note, however, that the steady-state cycle ends when the following train starts its dwell time. So for a whole day of cycles, there will also be a finishing state where train j is doing dwell-time and leaving the station without a new train arriving.

This said, there are three states:

① Train i arriving	→	② Train i & Train j (i) dwell-time + departure & (j) arriving	→	③ Train j dwell-time + departure

Looking at Figure 3, 4 Movements are identified, denoted by the numbers 1 to 4. Movements 1 and 2 belong to train i doing dwell-time and departing, and Movements 3 and 4 belong to train j arriving. A formula will be created for all four Movements to calculate the time needed to complete

the Movement. From these formulas, the minimal headway can then be computed for this situation using Equations 3-5.

$$\text{State } \textcircled{1} \rightarrow \quad T_{M3} + T_{M4} \tag{3}$$

$$\text{State } \textcircled{2} \rightarrow \quad T_{M1} + Max\{T_{M2}, T_{M3}\} + T_{M4} \tag{4}$$

$$\text{State } \textcircled{3} \rightarrow \quad T_{M1} + T_{M2} \tag{5}$$

Calculating T_{M1}

First, it is essential to realize that there are two cases that can be considered for Movement 1. Without specific data, it is not known whether or not the train needs more than $L_i + h$ meters to accelerate to the speed limit in the first block after department (v_{b_n}). If it is assumed that the train needs more than its length plus the head-to-signal distance $(L_i + h)$ to accelerate, this means train i will be moving the entire distance of Movement 1 at the speed during acceleration (a_i). This will give Equation 6. Assuming that the train needs less than $L_i + h$ meters to accelerate to v_{b_n}, results in Equation 7. Where the total distance the train needs to accelerate is defined as Δx. It is more realistic to assume the latter, when considering how the Dutch railway blocks are designed. Hence, T_{M1} is defined by Equation 7.

$$T_{M1} = t_{s,d_i} + \frac{h + L_i}{a_i}, \quad \text{assuming } \Delta x > h + L_i \tag{6}$$

$$T_{M1} = t_{s,d_i} + \frac{h + L_i - \Delta x}{v_{b_n}} + \frac{\Delta x}{a_i}, \quad \text{assuming } \Delta x \leq h + L_i \tag{7}$$

Δx could be calculated in various ways, depending on the critical factors. Especially for trains, factors like gradient and angle might play an essential role in these calculations. Multiple factors of the train braking time and a proposed tool are described by Barney et al., 2001, but to keep it simple, Δx will be calculated as below.

To calculate the distance the train travels during acceleration from 0 to v_{b_n} (Δx), the two basic physics formulas are used. Where v_{final} is the speed after acceleration and where v_{start} is the speed before accelerating.

$$a_i = \frac{v_{final} - v_{start}}{\Delta t_{\{v_{start}, v_{final}\}}} \tag{8}$$

$$a_i = \frac{2(\Delta x - v_{start} \cdot \Delta t_{\{0, v_{b_n}\}})}{(\Delta t_{\{0, v_{b_n}\}})^2} \tag{9}$$

Rewrite Equation 8 with $v_{final} = v_{b_n}$ and $v_{start} = 0$:

$$\Delta t_{\{0, v_{b_n}\}} = \frac{v_{final} - v_{start}}{a_i} = \frac{v_{b_n}}{a_i} \tag{10}$$

Then Equation 10 is used to plug into Equation 9 like:

$$a_i = \frac{2(\Delta x - v_{start} \cdot (\frac{v_{b_n}}{a_i}))}{(\frac{v_{b_n}}{a_i})^2} = \frac{2\Delta x}{(\frac{v_{b_n}}{a_i})^2} \tag{11}$$

Simplify as:

$$2\Delta x = a_i \cdot ((\frac{v_{b_n}}{a_i})^2) = a_i \cdot \frac{v_{b_n}^2}{a_i^2} = \frac{v_{b_n}^2}{a_i}$$

Concluding,

$$\Delta x = \frac{v_{b_n}^2}{2a_i} \tag{12}$$

Using this to plug into Equation 7:

$$T_{M1} = t_{s,d_i} + \frac{h + L_i - \Delta x}{v_{b_n}} + \frac{\Delta x}{a_i}$$

$$= t_{s,d_i} + \frac{h + L_i - (\frac{v_{b_n}^2}{2a_i})}{v_{b_n}} + \frac{(\frac{v_{b_n}^2}{2a_i})}{a_i}$$

$$= t_{s,d_i} + \frac{h + L_i}{v_{b_n}} - \frac{v_{b_n}^2}{2a_i v_{b_n}} + \frac{v_{b_n}^2}{2a_i^2}$$

$$= t_{s,d_i} + \frac{h + L_i}{v_{b_n}} - \frac{v_{b_n}}{2a_i} + \frac{v_{b_n}^2}{2a_i^2} \tag{13}$$

Calculating T_{M2}

For train i to make Movement 2, the assumption stated before, that the train does not need more than $h + L_i$ meters to accelerate still holds. T_{M2} is as shown in Equation 14

$$T_{M2} = \frac{B_n}{v_{b_n}} \tag{14}$$

Calculating T_{M3}

For train j to make Movement 3, the train needs to decelerate from the speed limit in the second block before the station (v_{b_2}) to the speed limit in the first block before arriving at the station (v_{b_1}), it is assumed here that the train has enough meters to do so. After which, the train will travel at v_{b_1} at the remaining meters. The distance needed to decelerate ($\Delta y_{\{v_{b_2}, v_{b_1}\}}$), calculated similar to Δx below, is used and then come to Equation 19.

Recall from Equation 8 and 9 what the acceleration formulas are. $d_j = -1 \cdot a_i$ is used to make sure deceleration is a positive number.

Hence deceleration of train j is equal to:

$$d_j = -1 \cdot a_i = -1 \cdot \left(\frac{v_{final} - v_{start}}{\Delta t_{\{0, v_{b_n}\}}}\right) = \frac{v_{start} - v_{final}}{\Delta t_{\{v_{start}, v_{final}\}}}$$

Rewrite as:

$$\Delta t_{\{v_{start}, v_{final}\}} = \frac{v_{start} - v_{final}}{d_j} \tag{15}$$

Similarly;

$$d_j = -1 \cdot a_i = \frac{-2\Delta y + 2v_{start} \cdot \Delta t_{\{v_{start}, v_{final}\}}}{(\Delta t_{\{v_{start}, v_{final}\}})^2} \tag{16}$$

Equation 15 is plugged into Equation 16 to get:

$$d_j = \frac{-2\Delta y + 2v_{start} \cdot \left(\frac{v_{start} - v_{final}}{d_j}\right)}{\left(\frac{v_{start} - v_{final}}{d_j}\right)^2}$$

For Movement 3, $\Delta y_{\{v_j, v_{b_1}\}}$ is needed:

$$d_j = \frac{-2\Delta y_{\{v_{b_2}, v_{b_1}\}} + 2v_{b_2} \cdot \left(\frac{v_{b_2} - v_{b_1}}{d_j}\right)}{\left(\frac{v_{b_2} - v_{b_1}}{d_j}\right)^2}$$

Rewrite as:

$$\Delta y_{\{v_{b_2}, v_{b_1}\}} = \frac{v_{b_1}^2 - v_{b_2}^2}{-2d_j} = \frac{v_{b_2}^2 - v_{b_1}^2}{2d_j} \tag{17}$$

19

For Movement 4, $\Delta y_{\{v_{b_1},0\}}$ is needed :

$$d_j = \frac{-2\Delta y_{\{v_{b_1},0\}} + 2v_{b_1} \cdot (\frac{v_{b_1}-0}{d_j})}{(\frac{v_{b_1}-0}{d_j})^2}$$

Simplify as:

$$\Delta y_{\{v_{b_1},0\}} = \frac{v_{b_1}^2}{2d_j} \tag{18}$$

Now that the formulas for $\Delta y_{\{v_{b_2},v_{b_1}\}}$ $\Delta y_{\{v_{b_1},0\}}$ are defined, T_{M3} can be calculated and later T_{M4}. For T_{M3}, it is assumed that train j has enough time to decelerate; this can be assumed due to how the signals are placed in the Dutch network. Whenever there is not enough space to decelerate, the speed limit in the block is lowered to a point where there is sufficient space to decelerate.

$$
\begin{aligned}
T_{M3} &= \frac{\Delta y_{\{v_{b_2},v_{b_1}\}}}{d_j} + \frac{B_1 - \Delta y_{\{v_{b_2},v_{b_1}\}}}{v_{b_1}} = \frac{(\frac{v_{b_2}^2-v_{b_1}^2}{2d_j})}{d_j} + \frac{B_1}{v_{b_1}} - \frac{v_{b_2}^2 - v_{b_1}^2}{2d_j v_{b_1}} \\
&= \frac{v_{b_2}^2 - v_{b_1}^2}{2d_j^2} + \frac{B_1}{v_{b_1}} - \frac{v_{b_2}^2 - v_{b_1}^2}{2d_j v_{b_1}}
\end{aligned}
\tag{19}
$$

Calculating T_{M4}

Similar to Movement 3, for Movement 4, train j needs to decelerate, but this time from v_{b_1} to 0 and, if the distance needed for that is less than the meters to be traveled until the train needs to come to a stop, the train can first travel a bit at v_{b_1} before decelerating. The formula for the time needed to do Movement 4 is shown in Equation 20. It is assumed the distance needed to come to a stop is smaller or equal to the distance traveled in Movement 4, according to the same assumption as explained for Movement 3.

$$
\begin{aligned}
T_{M4} &= \frac{B_s - h - \Delta y_{\{v_{b_1},0\}}}{v_{b_1}} + \frac{\Delta y_{\{v_{b_1},0\}}}{d_j} \\
&= \frac{B_s - h - (\frac{v_{b_1}^2}{2d_j})}{v_{b_1}} + \frac{(\frac{v_{b_1}^2}{2d_j})}{d_j} = \frac{B_s - h + e}{v_{b_1}} - \frac{v_{b_1}}{2d_j} + \frac{v_{b_1}^2}{2d_j^2}
\end{aligned}
\tag{20}
$$

Concluding T_{S1}

Looking back at Equations 3 to 5, the formulas for the different Movements can now be plugged in:

State $\textcircled{1}$:

$$
\begin{aligned}
T_{S1} &= T_{M3} + T_{M4} \text{ (for train } i) \\
&= \frac{v_{b_2}^2 - v_{b_1}^2}{2d_i^2} + \frac{B_1}{v_{b_1}} - \frac{v_{b_2}^2 - v_{b_1}^2}{2d_i v_{b_1}} + \frac{B_s - h + e}{v_{b_1}} - \frac{v_{b_1}}{2d_i} + \frac{v_{b_1}^2}{2d_i^2} \\
&= \frac{v_{b_2}^2}{2d_i^2} - \frac{v_{b_2}^2}{2d_i v_{b_1}} + \frac{B_1 + B_s - h}{v_{b_1}} \\
&\to \sqrt{\frac{v_{b_2}^2}{2d_i^2} - \frac{v_{b_2}^2}{2d_i v_{b_1}} + \frac{B_1 + B_s - h}{v_{b_1}}}
\end{aligned}
\tag{21}
$$

State ②:

$$T_{S1} = T_{M1} + Max\{T_{M2}, T_{M3}\} + T_{M4}$$

$$= t_{s,d_i} + \frac{h + L_i}{v_{b_n}} - \frac{v_{b_n}}{2a_i} + \frac{v_{b_n}^2}{2a_i^2} + \frac{B_s - h + e}{v_{b_1}} - \frac{v_{b_1}}{2d_j} + \frac{v_{b_1}^2}{2d_j^2}$$

$$+ Max\{\frac{B_n}{v_{b_n}}; \frac{v_{b_2}^2 - v_{b_1}^2}{2d_j^2} + \frac{B_1}{v_{b_1}} - \frac{v_{b_2}^2 - v_{b_1}^2}{2d_j v_{b_1}}\} \tag{22}$$

So, either $T_{M2} > T_{M3}$:

$$T_{S1} = t_{s,d_i} + \frac{h + L_i}{v_{b_n}} - \frac{v_{b_n}}{2a_i} + \frac{v_{b_n}^2}{2a_i^2} \frac{B_s - h + e}{v_{b_1}} - \frac{v_{b_1}}{2d_j} + \frac{v_{b_1}^2}{2d_j^2} + \frac{B_n}{v_{b_n}}$$

$$= t_{s,d_i} + \frac{B_n + h + L_i}{v_{b_n}} - \frac{v_{b_n}}{2a_i} + \frac{v_{b_n}^2}{2a_i^2} + \frac{B_s - h + e}{v_{b_1}} - \frac{v_{b_1}}{2d_j} + \frac{v_{b_1}^2}{2d_j^2}$$

$$\rightarrow t_{s,d_i} + \frac{B_n + h + L_i}{v_{b_n}} - \frac{v_{b_n}}{2a_i} + \sqrt{\frac{v_{b_n}^2}{2a_i^2}} + \frac{B_s - h + e}{v_{b_1}} - \frac{v_{b_1}}{2d_j} + \sqrt{\frac{v_{b_1}^2}{2d_j^2}} \tag{23}$$

Or, $T_{M3} > T_{M2}$:

$$T_{S1} = t_{s,d_i} + \frac{h + L_i}{v_{b_n}} - \frac{v_{b_n}}{2a_i} + \frac{v_{b_n}^2}{2a_i^2} + \frac{B_s - h + e}{v_{b_1}} - \frac{v_{b_1}}{2d_j} + \frac{v_{b_1}^2}{2d_j^2} + \frac{v_{b_2}^2 - v_{b_1}^2}{2d_j^2} + \frac{B_1}{v_{b_1}} - \frac{v_{b_2}^2 - v_{b_1}^2}{2d_j v_{b_1}}$$

$$= t_{s,d_i} + \frac{h + L_i}{v_{b_n}} - \frac{v_{b_n}}{2a_i} + \frac{v_{b_n}^2}{2a_i^2} + \frac{B_1 + B_s - h + e}{v_{b_1}} + \frac{v_{b_2}^2}{2d_j^2} - \frac{v_{b_2}^2}{2d_j v_{b_1}}$$

$$\rightarrow t_{s,d_i} + \frac{h + L_i}{v_{b_n}} - \frac{v_{b_n}}{2a_i} + \sqrt{\frac{v_{b_n}^2}{2a_i^2}} + \frac{B_1 + B_s - h + e}{v_{b_1}} + \sqrt{\frac{v_{b_2}^2}{2d_j^2} - \frac{v_{b_2}^2}{2d_j v_{b_1}}} \tag{24}$$

State ③:

$$T_{S1} = T_{M1} + T_{M2} \text{ (for train } j)$$

$$= t_{s,d_j} + \frac{h + L_j}{v_{b_n}} - \frac{v_{b_n}}{2a_i} + \frac{v_{b_n}^2}{2a_i^2} + \frac{B_n}{v_{b_n}} = t_{s,d_j} + \frac{B_n + h + L_j}{v_{b_n}} - \frac{v_{b_n}}{2a_i} + \frac{v_{b_n}^2}{2a_i^2}$$

$$\rightarrow t_{s,d_j} + \frac{B_n + h + L_j}{v_{b_n}} - \frac{v_{b_n}}{2a_j} + \sqrt{\frac{v_{b_n}^2}{2a_j^2}} \tag{25}$$

4.2.2 Quality Check of Formulas Type I

As a quick sanity check to test these formulas the following tests are performed. The first test will be filling in a reasonable test sample and see if the headway that the formulas give is a reasonable number of seconds. Secondly, it is checked what happens to the headway given by the formulas when one of the parameters is increased/decreased and it is checked if this has the same effect as the relationship of headway and the particular parameters have on each other by logic. If the formulas pass both tests, it is assumed they are reasonable and they can then be used during the capacity analysis of the Dutch railway network.

4.2.2.1   Formula Quality Check 1

For a test sample, take a Type I station and trains i and j with the following parameters:

Parameter		Sidenotes
t_{s,d_i}	$54s$	
h	$10m$	
L_i	$410m$	
a_i	$1.1m/s^2$	
d_j	$1.3m/s^2$	
v_{b_2}	$100km/h$	Equals 27.77 m/s
v_{b_1}	$60km/h$	Equals 16.67 m/s
v_{b_n}	$80km/h$	Equals 22.22 m/s
B_1	$1500m$	
B_s	$800m$	
B_n	$1600m$	
e	$250m$	$\approx 9 \cdot v_{b_2}$

Table 3: Test Sample Type I

T_{S1} is calculated according to Equations 23 and 24. Starting with T_{M2} and T_{M3}, it is found that T_{M2} is equal to 72.01 and T_{M3} equals 90.68. Concluding that $T_{M3} > T_{M2}$, so Equation 24 can be used to calculated T_{S1}. Filling in the numbers from Table 3, this results is a headway of 226.76 seconds, which is 3 minutes and 47 seconds.

So, the headway in this sample test given by the formulas constructed in Section 4.2.1 is 3 minutes and 47 seconds. This is a reasonable headway time, as ProRail calculated with a headway time of 3 minutes up until 2020 (Treinreiziger.nl, 2020). It is not known to the public what ProRail based the 3 minute headway time on, but with the formula for this thesis giving 3 minutes and 47 seconds as a result, it is concluded the formula works quite well.

It is also observed that $T_{M3} > T_{M2}$, which was also found by Mikulcic and Mlinaric, 2021 and Abril et al., 2008.

4.2.2.2   Formula Quality Check 2

It is now evaluated what happens to the headway when one of the parameters is altered.

- $t_{s,d_i} \uparrow$: When t_{s,d_i} increases, train i has a longer dwell time; this means Movement 1 will increase and thus, the headway is expected to be higher than before because train j has to travel further away from train i to avoid having to wait at a red stop signal due to train i.

- $h \uparrow$: If h increases, there is less room for train j to decelerate; it is therefore required to start deceleration a little earlier and it will therefore travel a little slower, as this takes away a small part that was first traveled at v_{b_1}. However, on the other side, train i has a little more space to travel after dwell-time, which makes train i travel at v_{b_n} earlier than before as acceleration can be done in the extra meters of h. So train j will be a little slower, and train i will be a little faster, hence headway is expected to be a little smaller if $v_{b_n} > v_{b_1}$ and a little bigger otherwise (See Figure 4). As h is a relatively small number of meters compared to the length of blocks, the decrease/increase in headway is expected to be very small.
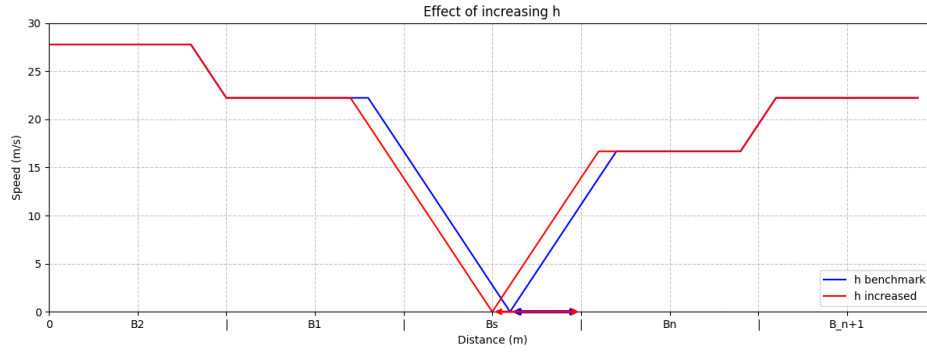
Figure 4: h Increase, T_{S1}

- $L_i \uparrow$: If the train length of train i is longer, it will take longer to empty a block for train i. This increases the headway as train j needs to be further away from train i, so it does not have to stop due to train i.

- $a_i \downarrow$: If train i is able to accelerate at a slower speed, it will take train i longer to accelerate, which leaves fewer meters to travel at v_{b_n}, this will therefore increase headway (See Figure 5).
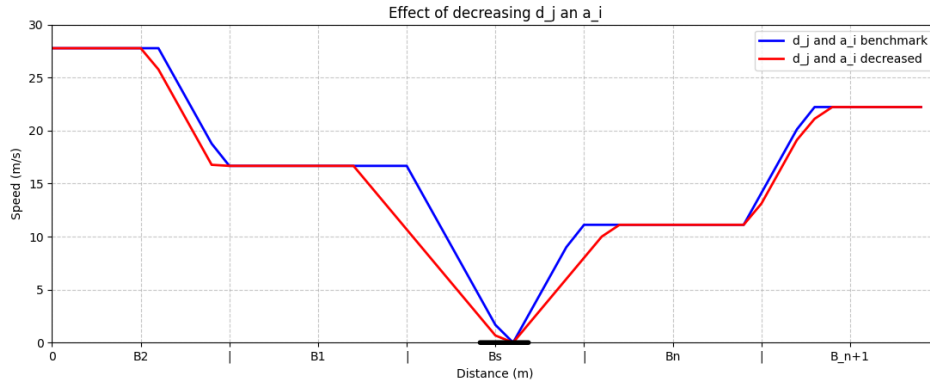

Figure 5: a_i Decrease and d_j Decrease, T_{S1}

- $d_j \downarrow$: When train j is able to decelerate at a slower speed, it will take train j longer to decelerate. Headway could go up or down depending on whether deceleration starts or ends at the same point. When deceleration begins at the same point (as is the case going from v_{b_2} to v_{b_1}), train j will take longer to reach v_{b_1}, which means it travels longer at a higher speed than v_{b_1}, which makes headway go down (see Figure 6). But when deceleration ends at the same point (as is the case when stopping at a platform), train j needs to start decelerating at an earlier point in order to be done by the same time as before. This causes train j to travel at a lower speed earlier; thus, headway goes up in this case (see Figure 5). It is observed that a decrease in deceleration causes the headway to increase if $T_{M2} > T_{M3}$ and decrease otherwise.
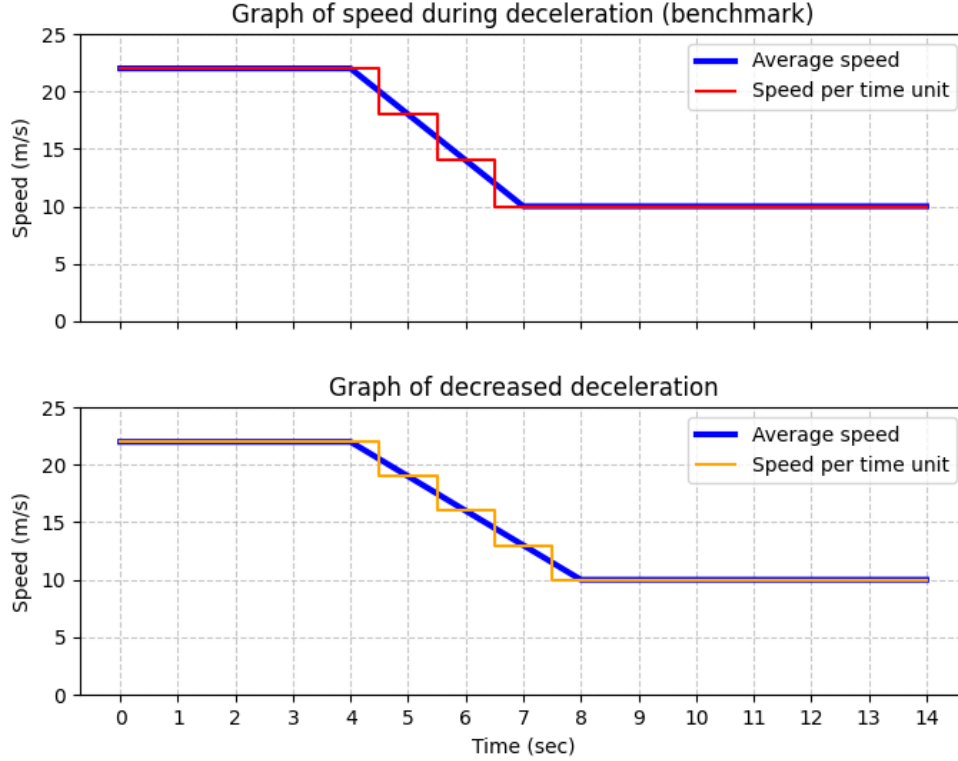
23

Figure 6: Consequence of Decreasing Deceleration

> $\rightarrow$ when deceleration is done in 3 seconds, the traveled distance is
> $(4x22m + 1x20m + 1x16m + 1x12m + 7x10m) = 206m$,
> which is on average $206/14 = 14.71m/s$.
> When deceleration takes 4 seconds, the traveled distance is
> $(4x22m + 1x20.5m + 1x17.5m + 1x14.5m + 1x11.5m + 6x10m) = 212m$,
> which is on average $15.14m/s$

- $v_{b_2} \downarrow$: When the speed limit in block b_2 decreases, this means train j will need less time to decelerate to v_{b_1} because block b_2 is not part of the formula, this makes travel time slower as train j has a lower average speed during deceleration and longer at v_{b_1} and thus headway increases. Note that when $T_{M2} > T_{M3}$, T_{M3} is not determining the headway and since v_{b_2} is only present in T_{M3}, it will not influence the headway.

- $v_{b_1} \downarrow$: If the speed limit in block b_1 decreases, this means train j will need more time to travel the same distance. This means the headway will increase.

- $v_{b_n} \downarrow$: When the speed limit in block b_n decreases, train i needs less time to accelerate, but more time to travel over the same distance. If v_{b_n} rises high enough it might cause $T_{M2} > T_{M3}$ which increases the headway. If v_{b_n} does not increase enough or decreases, as in this case, T_{M3} will still be bigger than T_{M2} and nothing changes.

- $B_1 \downarrow$: If the length of block b_1 decreases, train j has to cover a smaller distance to have traveled the block, this means train j will be faster and closer to train i, this makes headway decrease. By the same reasoning as v_{b_2}, B_1 does not influence the headway if $T_{M2} > T_{M3}$.

- $B_s \downarrow$: Similar to the reasoning for B_1, when B_s decreases, both trains need to cover a smaller distance to complete the block and can travel closer to each other without experiencing any delay from another train.

- $B_n \downarrow$: As explained for B_1 and B_s the headway will also decrease by similar reasoning when B_n decreases. However, if B_n decreases, T_{M3} will still be bigger than T_{M2} and this will not affect the headway; so if B_n decreases, headway stays the same, if B_n increases, headway may go up.

- $e \downarrow$: Similar to the reasoning for block lengths, when e decreases, train j needs to cover a smaller distance in Movement 4. Hence, headway will decrease.

Parameters	(226.76) T_{S1} (%)	Expectations	
$t_{s,d_i} \uparrow 5\%$	229.46 ($\uparrow$ 1.19)	$T_{S1} \uparrow$	
$h \uparrow 5\%$	226.76 ($\downarrow$ 0.003)	——	$T_{S1} \downarrow$
$L_i \uparrow 5\%$	227.69 ($\uparrow$ 0.41)	$T_{S1} \uparrow$	
$a_i \downarrow 5\%$	226.98 ($\uparrow$ 0.10)	——	$T_{S1} \uparrow$
$d_j \downarrow 5\%$	226.62 ($\downarrow$ 0.06)	$T_{S1} \downarrow$	
$v_{b_2} \downarrow 5\%$	227.74 ($\uparrow$ 0.43)	——	$T_{S1} \uparrow$
$v_{b_1} \downarrow 5\%$	233.85 ($\uparrow$ 3.12)	$T_{S1} \uparrow$	
$v_{b_n} \downarrow 5\%$	227.55 ($\uparrow$ 0.35)	——	$T_{S1} \uparrow$
$B_1 \downarrow 5\%$	222.26 ($\downarrow$ 1.98)	$T_{S1} \downarrow$	
$B_s \downarrow 5\%$	227.36 ($\downarrow$ 1.06)	——	$T_{S1} \downarrow$
$B_n \downarrow 5\%$	226.76 ($=$ 0.00)	$T_{S1} =$	
$e \downarrow 5\%$	226.01 ($\downarrow$ 0.33)	——	$T_{S1} \downarrow$

Table 4: Formula Quality Check 2 T_{S1}

Table 4 shows what happens to T_{S1} when the parameters (in the first column) is in-/decreased by 5%. the second column shows that the original headway was 226.76 seconds and the in-/decrease is shown for the changed parameter as *new headway(% in-/decrease)*. The third column summarizes what was discussed through the bullet-points above.
It is observed from Table 4 that all parameter changes lead to expected changes in the headway. Hence, it is assumed the formulas are valid.

4.2.2.3 Comparison with Yung-Cheng Lai and Jong, 2010

As mentioned in Table 2 T_{S1} of this research and $T_{s,A1}$ from the paper by Yung-Cheng Lai and Jong, 2010 are defined the same. the formulas are therefore compared to see in what ways they differ. Recall:

$$
T_{S1} = \begin{cases} t_{s,d_i} + \frac{B_n+h+L_i}{v_{b_n}} - \frac{v_{b_n}}{2a_i} + \sqrt{\frac{v_{b_n}^2}{2a_i^2}} + \frac{B_s-h+e}{v_{b_1}} - \frac{v_{b_1}}{2d_j} + \sqrt{\frac{v_{b_1}^2}{2d_j^2}}, & \text{if } T_{M2} \geq T_{M3} \\ t_{s,d_i} + \frac{h+L_i}{v_{b_n}} - \frac{v_{b_n}}{2a_i} + \sqrt{\frac{v_{b_n}^2}{2a_i^2}} + \frac{B_1+B_s-h+e}{v_{b_1}} + \sqrt{\frac{v_{b_2}^2}{2d_j^2}} - \frac{v_{b_2}^2}{2d_j v_{b_1}}, & \text{otherwise} \end{cases}
\tag{26}
$$

$$
T_{s,A1} = \sqrt{\frac{2(L_i+s_x)}{K_a a_i(G_0)}} + \frac{v_j}{K_b b_j(G_i)} - \frac{v_y^2}{2K_a b_j(G_i)v_j} + \frac{B_1+B_s-s_x}{v_j} + t_o + t_r + t_{di}
$$

Table 2 already stated that G_0, G_i, t_o and t_r are equal to zero in this research due to the assumptions made in Section 3.4.1. $T_{s,A1}$ is edited with the information stated in Table 2 to get:

$$
T_{s,A1} = \sqrt{\frac{2(L_i+h)}{K_a a_i}} + \frac{v_j}{K_b d_j} - \frac{v_y^2}{2K_a d_j v_j} + \frac{B_1+B_s-h}{v_j} + t_{s,di}
\tag{27}
$$

25

It seems that Yung-Cheng Lai and Jong, 2010 take $T_{M3} \geq T_{M2}$ as a given so it is left to compare Equations 27 and 26(II). A few similar terms are observed. For example when setting $v_j = v_{b_1}$ the term of $\frac{B_1 + B_s - h}{v_{b_1}}$ is present in both formulas. An other example is when setting $K_a = 1$ and $v_y = v_{b_2}$, the term of $-\frac{v_{b_2}^2}{2d_j v_{b_1}}$ is also found in both formulas. The same dwell time is also present in both formulas, but all other terms are different. As said before Yung-Cheng Lai and Jong, 2010 does not provide elaborate explanation of how they got to their formulas and thus we can only speculate that the differences in formulas might be due to the differences in design and/or plan rules for the Taiwanese railway network versus the Dutch railway network.

4.2.3 Platform Type II

Next, a model is determined for calculating the minimal headway for a bit of a more complicated situation. A station with two platforms, one along the main rail and one to go around it (Figure 1(b)) will now be considered.

For this station type, there are four different cases that can happen: (1) both train i and train j pass the station on the main rail, (2) train i passes the station at the main rail, while train j passes the station though the side rail, (3) train i passes the station at the side rail, while train j passes the station though the main rail, (4) both train i and train j pass the station at the side rail.

According to the Dutch railway timetable, it barely happens that two consecutive trains pass at the side station; this situation is therefore not considered. In addition, case (1) has already been covered in Section 4.2.1, so what is left to consider for this type of station is case (2) and case (3).

4.2.3.1 Station Type II Case (2)

Case (2) for station Type II is depicted in Figure 7. We see train j (in orange) following train i (in purple) and a platform with the main and side rails. As before, both trains need to make a stop of t_{s,d_i} and t_{s,d_j} seconds respectively. As in Figure 3, Figure 7 is to be read from top to bottom. A start-up process is discussed again, after which the "steady-state" cycle drawn in Figure 7 will start, where it is assumed that train i just arrived and is starting its dwell-time and after that train i will leave the station. Simultaneously, train j will start arriving at the station in order to start its dwell time and restart the cycle, where restarting the cycle is now a little different because it is not really a restart when train j starts its dwell time as it is stopping at a different platform. Later it is observed that the restart of the cycle will only occur after one cycle of case (3) is added to one cycle of case (2), but this will be discussed later. A model for case (2) will first be defined.
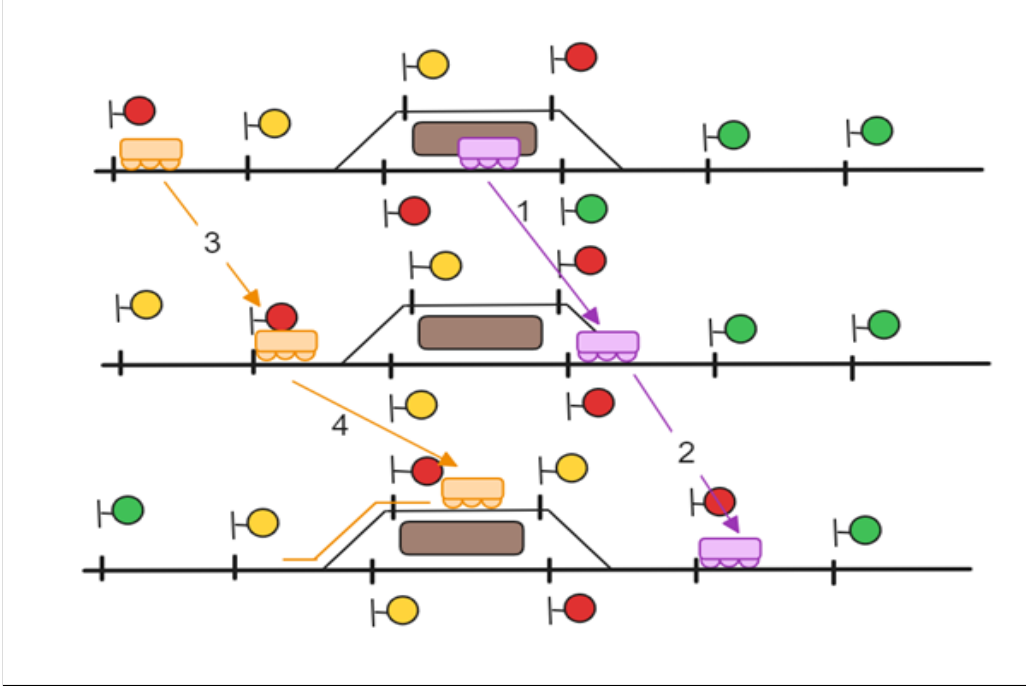
Figure 7: Train Movements Platform Type II (2)

Again, four Movements as shown by the numbers 1-4 in Figure 7 are defined, and the three states are still defined by Equation 3-5.

Calculating $\mathbf{T}_{M1}$

For Movement 1, train i needs to have dwell-time, then leave the station block. As the departure of train i is similar to the first case discussed, it is again assumed that train i does not need more than $h + L_i$ meters to accelerate and thus, train i will be accelerating a part of the distance traveled in Movement 1 and will be traveling at $v_{b_{ni}}$ the other part. Note that the speed limit in the switch block after leaving the station is denoted as $v_{b_{ni}}$ instead of v_{b_n} so that different speed limits for trains leaving the main platform or the side platform can be filled in.

$$T_{M1} = t_{s,d_i} + \frac{h + L_i - \Delta x}{v_{b_{ni}}} + \frac{\Delta x}{a_i} \quad \text{assuming } \Delta x < h + L_i \tag{28}$$

Recall that Δx was defined before in Equation 12, so Equation 28 and Equation 12 can be combined to come to Equation 29.

$$
\begin{aligned}
T_{M1} &= t_{s,d_i} + \frac{h + L_i - \Delta x}{v_{b_{ni}}} + \frac{\Delta x}{a_i} \\
&= t_{s,d_i} + \frac{h + L_i - \left(\frac{v_{b_{ni}}^2}{2a_i}\right)}{v_{b_{ni}}} + \frac{\left(\frac{v_{b_{ni}}^2}{2a_i}\right)}{a_i} \\
&= t_{s,d_i} + \frac{h + L_i}{v_{b_{ni}}} - \frac{v_{b_{ni}}^2}{2a_i v_{b_{ni}}} + \frac{v_{b_{ni}}^2}{2a_i^2} \\
&= t_{s,d_i} + \frac{h + L_i}{v_{b_{ni}}} - \frac{v_{b_{ni}}}{2a_i} + \frac{v_{b_{ni}}^2}{2a_i^2}
\end{aligned}
\tag{29}
$$

Calculating $\mathbf{T}_{M2}$

For Movement 2, train i travels further at $v_{b_{ni}}$, just like in the first case T_{M2} can be shown as:

$$T_{M2} = \frac{B_n}{v_{b_{ni}}} \tag{30}$$

27

Calculating $\mathbf{T}_{M3}$

For Movement 3 train j (in orange, Figure 7) is traveling the sight of block b_2, and block b_1 without its sight space (e). In addition, it is decelerating to $v_{b_{1j}}$ in order to go through the switch.

$$T_{M3} = \frac{e + (\hat{B}_{1j} - e) - \Delta y_{\{v_{b_2}, v_{b_{1j}}\}}}{v_{b_{1j}}} + \frac{\Delta y_{\{v_{b_2}, v_{b_{1j}}\}}}{d_j} \tag{31}$$

Equation 17 is used and plugged into Equation 31 to get Equation 32.

$$\begin{aligned}
T_{M3} &= \frac{e + (\hat{B}_{1j} - e) - \Delta y_{\{v_{b_2}, v_{b_{1j}}\}}}{v_{b_{1j}}} + \frac{\Delta y_{\{v_{b_2}, v_{b_{1j}}\}}}{d_j} \\
&= \frac{\hat{B}_{1j} - (\frac{v_{b_2}^2 - v_{b_{1j}}^2}{2d_j})}{v_{b_{1j}}} + \frac{(\frac{v_{b_2}^2 - v_{b_{1j}}^2}{2d_j})}{d_j} \\
&= \frac{\hat{B}_{1j}}{v_{b_{1j}}} - \frac{v_{b_2}^2 - v_{b_{1j}}^2}{2d_j v_{b_{1j}}} + \frac{v_{b_2}^2 - v_{b_{1j}}^2}{2d_j^2}
\end{aligned} \tag{32}$$

Calculating $\mathbf{T}_{M4}$

Lastly, for Movement 4, train j will travel the last bit of block b_1 onto the side rail and, of course, onto the platform in order to start dwell time.:

$$T_{M4} = \frac{e + B_{sj} - h - \Delta y_{\{v_{b_{1j}}, 0\}}}{v_{b_{1j}}} + \frac{\Delta y_{\{v_{b_{1j}}, 0\}}}{d_j} \tag{33}$$

Equation 18 is used to plug into Equation 33 as $\Delta y_{\{v_{b_{1j}}, 0\}}$.

$$\begin{aligned}
T_{M4} &= \frac{e + B_{sj} - h - \Delta y_{\{v_{b_{1j}}, 0\}}}{v_{b_{1j}}} + \frac{\Delta y_{\{v_{b_{1j}}, 0\}}}{d_j} \\
&= \frac{e + B_{sj} - h - (\frac{v_{b_{1j}}^2}{2d_j})}{v_{b_{1j}}} + \frac{(\frac{v_{b_{1j}}^2}{2d_j})}{d_j} \\
&= \frac{e + B_{sj} - h}{v_{b_{1j}}} - \frac{v_{b_{1j}}^2}{2d_j v_{b_{1j}}} + \frac{v_{b_{1j}}^2}{2d_j^2} \\
&= \frac{e + B_{sj} - h}{v_{b_{1j}}} - \frac{v_{b_{1j}}}{2d_j} + \frac{v_{b_{1j}}^2}{2d_j^2}
\end{aligned} \tag{34}$$

Concluding $\mathbf{T}_{S2,2}$

Looking back at Equations 3 to 5, the different Movements defined in Equations 28 to 34 can now be plugged in.

State ①:

$$\begin{aligned}
T_{S2,2} &= T_{M3} + T_{M4} \text{ (for train } i) \\
&= \frac{v_{b_2}^2 - v_{b_{1i}}^2}{2d_i^2} + \frac{\hat{B}_{1i}}{v_{b_{1i}}} - \frac{v_{b_2}^2 - v_{b_{1i}}^2}{2d_i v_{b_{1i}}} + \frac{e + B_{si} - h}{v_{b_{1i}}} + \frac{v_{b_{1i}}^2}{2d_i^2} - \frac{v_{b_{1i}}}{2d_i} \\
&= \frac{v_{b_2}^2}{2d_i^2} - \frac{v_{b_2}^2}{2d_i v_{b_{1i}}} + \frac{\hat{B}_{1i} + e + B_{si} - h}{v_{b_{1i}}} \\
\rightarrow \quad &= \sqrt{\frac{v_{b_2}^2}{2d_i^2}} - \frac{v_{b_2}^2}{2d_i v_{b_{1i}}} + \frac{\hat{B}_{1i} + e + B_{si} - h}{v_{b_{1i}}}
\end{aligned} \tag{35}$$

State ②:

$$T_{S2,2} = T_{M1} + Max\{T_{M2}; T_{M3}\} + T_{M4}$$

$$= t_{s,d_i} + \frac{h + L_i}{v_{b_{ni}}} - \frac{v_{b_{ni}}}{2a_i} + \frac{v_{b_{ni}}^2}{2a_i^2} + \frac{e + B_{sj} - h}{v_{b_{1j}}} - \frac{v_{b_{1j}}}{2d_j} + \frac{v_{b_{1j}}^2}{2d_j^2}$$

$$+ Max\{\frac{B_{ni}}{v_{b_{ni}}}; \frac{\hat{B}_{1j}}{v_{b_{1j}}} + \frac{v_{b_2}^2 - v_{b_{1j}}^2}{2d_j^2} - \frac{v_{b_2}^2 - v_{b_{1j}}^2}{2d_j v_{b_{1j}}}\} \tag{36}$$

So, either $T_{M2} > T_{M3}$

$$T_{S2,2} = t_{s,d_i} + \frac{h + L_i}{v_{b_{ni}}} - \frac{v_{b_{ni}}}{2a_i} + \frac{v_{b_{ni}}^2}{2a_i^2} + \frac{e + B_{sj} - h}{v_{b_{1j}}} - \frac{v_{b_{1j}}}{2d_j} + \frac{v_{b_{1j}}^2}{2d_j^2} + \frac{B_{ni}}{v_{b_{ni}}}$$

$$= t_{s,d_i} + \frac{B_{ni} + h + L_i}{v_{b_{ni}}} - \frac{v_{b_{ni}}}{2a_i} + \frac{v_{b_{ni}}^2}{2a_i^2} + \frac{e + B_{sj} - h}{v_{b_{1j}}} - \frac{v_{b_{1j}}}{2d_j} + \frac{v_{b_{1j}}^2}{2d_j^2}$$

$$\rightarrow t_{s,d_i} + \frac{B_{ni} + h + L_i}{v_{b_{ni}}} - \frac{v_{b_{ni}}}{2a_i} + \sqrt{\frac{v_{b_{ni}}^2}{2a_i^2}} + \frac{e + B_{sj} - h}{v_{b_{1j}}} - \frac{v_{b_{1j}}}{2d_j} + \sqrt{\frac{v_{b_{1j}}^2}{2d_j^2}} \tag{37}$$

Or, $T_{M3} > T_{M2}$:

$$T_{S2,2} = t_{s,d_i} + \frac{h + L_i}{v_{b_{ni}}} - \frac{v_{b_{ni}}}{2a_i} + \frac{v_{b_{ni}}^2}{2a_i^2} + \frac{e + B_{sj} - h}{v_{b_{1j}}} - \frac{v_{b_{1j}}}{2d_j} + \frac{v_{b_{1j}}^2}{2d_j^2} + \frac{\hat{B}_{1j}}{v_{b_{1j}}} + \frac{v_{b_2}^2 - v_{b_{1j}}^2}{2d_j^2} - \frac{v_{b_2}^2 - v_{b_{1j}}^2}{2d_j v_{b_{1j}}}$$

$$= t_{s,d_i} + \frac{h + L_i}{v_{b_{ni}}} - \frac{v_{b_{ni}}}{2a_i} + \frac{v_{b_{ni}}^2}{2a_i^2} + \frac{\hat{B}_{1j} + e + B_{sj} - h}{v_{b_{1j}}} + \frac{v_{b_2}^2}{2d_j^2} - \frac{v_{b_2}^2}{2d_j v_{b_{1j}}}$$

$$\rightarrow t_{s,d_i} + \frac{h + L_i}{v_{b_{ni}}} - \frac{v_{b_{ni}}}{2a_i} + \sqrt{\frac{v_{b_{ni}}^2}{2a_i^2}} + \frac{\hat{B}_{1j} + e + B_{sj} - h}{v_{b_{1j}}} + \sqrt{\frac{v_{b_2}^2}{2d_j^2}} - \frac{v_{b_2}^2}{2d_j v_{b_{1j}}} \tag{38}$$

State ③:

$$T_{S2,2} = T_{M1} + T_{M2} \text{ (for train } j)$$

$$= td_j + \frac{h + L_j}{v_{b_{nj}}} - \frac{v_{b_{nj}}}{2a_j} + \frac{v_{b_{nj}}^2}{2a_j^2} + \frac{B_{nj}}{v_{b_{nj}}}$$

$$= td_j + \frac{B_{nj} + h + L_j}{v_{b_{nj}}} - \frac{v_{b_{nj}}}{2a_j} + \frac{v_{b_{nj}}^2}{2a_j^2}$$

$$\rightarrow td_j + \frac{B_{nj} + h + L_j}{v_{b_{nj}}} - \frac{v_{b_{nj}}}{2a_j} + \sqrt{\frac{v_{b_{nj}}^2}{2a_j^2}} \tag{39}$$

4.2.3.2 Station Type II Case (3)

Next, Figure 8 depicts case (3) where it shows train j (in pink) following train i (in green) and a platform similar to the previously discussed case with Figure 7. As before, both trains need to make a stop of t_{s,d_i} and t_{s,d_j} seconds respectively. Figure 8 is to be read from top to bottom and again

a start-up process (State ①) will be discussed first, after which the "steady-state" cycle drawn in Figure 8 will start, where it is assumed that train i just arrived at the side platform to start its dwell-time and after that train i will leave the station. Simultaneously, train j will start arriving at the station on the main rail platform in order to start its dwell time and complete the cycle.
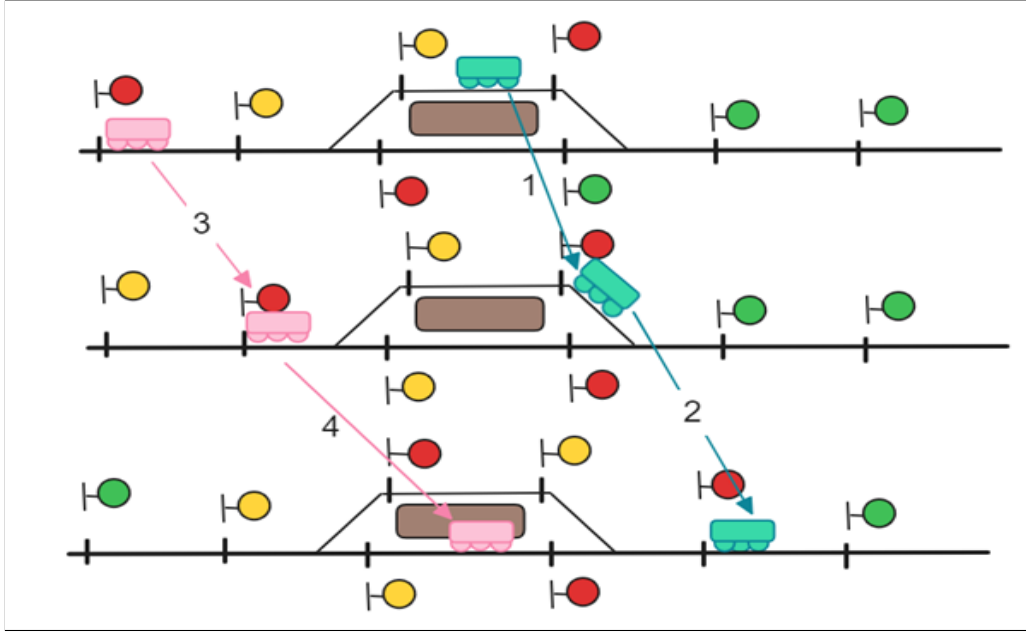

Figure 8: Train Movements Platform Type II (3)

As briefly mentioned before, case (2) and case (3) together form a full cycle, whereas in case (1), the cycle is just one arrival and one departure. Case (2) and case (3) are therefore discussed separately and then get the maximum headway of the two cases to get a safe value for the headway needed. This is also taken into account when looking at the case study in Section 5.

Now, a model for case (3) will be developed first. Again, four Movements are defined as shown by the numbers $1 - 4$ in Figure 8. The three states are still defined by Equation 3-5. Hence, we move on to defining the Movements needed once more.

Calculating T_{M1}

First of all, T_{M1} is looked at again, It is safe to assume that train i is able to accelerate to $v_{b_{ni}}$ in less than $h + L_i$, because this was already assumed in the previous cases, where $v_{b_{ni}}$ was higher due to going straight instead of through the switch at an angle. Therefore, T_{M1} can be described as in Equation 40. It is important to note that $v_{b_{ni}}$ is now a lower speed limit that relates to $\hat{B}_{ni}$.

$$T_{M1} = t_{s,d_i} + \frac{h + L_i - \Delta x_{\{0,v_{b_{ni}}\}}}{v_{b_{ni}}} + \frac{\Delta x_{\{0,v_{b_{ni}}\}}}{a_i} \tag{40}$$

Recall that $\Delta x_{\{0,v_{b_{ni}}\}}$ was defined before in Equation 12, so it can be plugged into Equation 40:

$$T_{M1} = t_{s,d_i} + \frac{h + L_i - \Delta x_{\{0,v_{b_{ni}}\}}}{v_{b_{ni}}} + \frac{\Delta x_{\{0,v_{b_{ni}}\}}}{a_i}$$

$$= t_{s,d_i} + \frac{h + L_i - \left(\frac{v_{b_{ni}}^2}{2a_i}\right)}{v_{b_{ni}}} + \frac{\left(\frac{v_{b_{ni}}^2}{2a_i}\right)}{a_i}$$

$$= t_{s,d_i} + \frac{h + L_i}{v_{b_{ni}}} - \frac{v_{b_{ni}}}{2a_i} + \frac{v_{b_{ni}}^2}{2a_i^2} \tag{41}$$

Calculating T_{M2}

In this scenario it is assumed that accelerating, while going through the switch is not possible, the train therefore passes through the switch at the speed limit reached in Movement 1 and the train will further accelerate when it passed the switch block, this is not part of the leading Movements of the current cycle, so this will not be taken into account at this point. Therefore, the time needed to do Movement 2 (T_{M2}) is as shown in Equation 42.

$$T_{M2} = \frac{\hat{B}_{ni}}{v_{b_{ni}}} \tag{42}$$

Calculating T_{M3}

For Movement 3, train j (in pink) will be decelerating to the speed limit in the first block before arriving at the station, its formula is similar to Equation 19 and for completeness shown in Equation 43.

$$T_{M3} = \frac{\Delta y_{\{v_{b_2}, v_{b_{1j}}\}}}{d_j} + \frac{B_1 - \Delta y_{\{v_{b_2}, v_{b_{1j}}\}}}{v_{b_{1j}}} \tag{43}$$

Again, recall that $\Delta y_{\{v_{b_2}, v_{b_{1j}}\}}$ was defined before in Equation 17, so it can be plugged into Equation 43.

$$
\begin{aligned}
T_{M3} &= \frac{\Delta y_{\{v_{b_2}, v_{b_{1j}}\}}}{d_j} + \frac{B_1 - \Delta y_{\{v_{b_2}, v_{b_{1j}}\}}}{v_{b_{1j}}} \\
&= \frac{\left(\frac{v_{b_2}^2 - v_{b_{1j}}^2}{2d_j}\right)}{d_j} + \frac{B_1 - \left(\frac{v_{b_2}^2 - v_{b_{1j}}^2}{2d_j}\right)}{v_{b_{1j}}} \\
&= \frac{v_{b_2}^2 - v_{b_{1j}}^2}{2d_j^2} + \frac{B_1}{v_{b_{1j}}} - \frac{v_{b_2}^2 - v_{b_{1j}}^2}{2d_j v_{b_{1j}}} \\
&= \frac{v_{b_2}^2 - v_{b_{1j}}^2}{2d_j^2} + \frac{B_1}{v_{b_{1j}}} - \frac{v_{b_2}^2}{2d_j v_{b_{1j}}} + \frac{v_{b_{1j}}}{2d_j}
\end{aligned} \tag{44}
$$

Calculating T_{M4}

Lastly, train j (in pink) needs to go straight through the switch and further towards the platform; this is defined as Movement 4.

$$T_{M4} = \frac{e + B_{sj} - h - \Delta y_{\{v_{b_{1j}}, 0\}}}{v_{b_{1j}}} + \frac{\Delta y_{\{v_{b_{1j}}, 0\}}}{d_j} \tag{45}$$

Earlier $\Delta y_{\{v_{b_{1j}}, 0\}}$ was defined by Equation 18. Equation 18 can again be used and plugged into Equation 45

$$
\begin{aligned}
T_{M4} &= \frac{e + B_{sj} - h - \Delta y_{\{v_{b_{1j}}, 0\}}}{v_{b_{1j}}} + \frac{\Delta y_{\{v_{b_{1j}}, 0\}}}{d_j} \\
&= \frac{e + B_{sj} - h - \left(\frac{v_{b_{1j}}^2}{2d_j}\right)}{v_{b_{1j}}} + \frac{\left(\frac{v_{b_{1j}}^2}{2d_j}\right)}{d_j} \\
&= \frac{e + B_{sj} - h}{v_{b_{1j}}} - \frac{v_{b_{1j}}^2}{2d_j v_{b_{1j}}} + \frac{v_{b_{1j}}^2}{2d_j^2} \\
&= \frac{e + B_{sj} - h}{v_{b_{1j}}} - \frac{v_{b_{1j}}}{2d_j} + \frac{v_{b_{1j}}^2}{2d_j^2}
\end{aligned} \tag{46}
$$

Concluding $T_{S2,3}$

Looking back at Equations 3 to 5 the Movements defined for case (3) can no be used and plugged in.

State ① :

$$T_{S2,3} = T_{M3} + T_{M4} \text{ (for train } i)$$

$$= \frac{v_{b_2}^2 - v_{b_{1i}}^2}{2d_i^2} + \frac{B_1}{v_{b_{1i}}} - \frac{v_{b_2}^2 - v_{b_{1i}}^2}{2d_i v_{b_{1i}}} + \frac{B_{si} - h + e}{v_{b_{1i}}} - \frac{v_{b_{1i}}}{2d_i} + \frac{v_{b_{1i}}^2}{2d_i^2}$$

$$= \frac{v_{b_2}^2}{2d_i^2} - \frac{v_{b_2}^2}{2d_i v_{b_{1i}}} + \frac{B_1 + B_{si} - h + e}{v_{b_{1i}}}$$

$$\rightarrow \sqrt{\frac{v_{b_2}^2}{2d_i^2} - \frac{v_{b_2}^2}{2d_i v_{b_{1i}}}} + \frac{B_1 + B_{si} - h + e}{v_{b_{1i}}} \tag{47}$$

State ② :

$$T_{S2,3} = T_{M1} + Max\{T_{M2}; T_{M3}\} + T_{M4}$$

$$= t_{s,d_i} + \frac{h + L_i}{v_{b_{ni}}} - \frac{v_{b_{ni}}}{2a_i} + \frac{v_{b_{ni}}^2}{2a_i^2} + \frac{B_{sj} - h + e}{v_{b_{1j}}} + \frac{v_{b_1}^2}{2d_j^2} - \frac{v_{b_{1j}}}{2d_j}$$

$$+ Max\{\frac{\hat{B}_{ni}}{v_{b_{ni}}}; \frac{v_{b_2}^2 - v_{b_{1j}}^2}{2d_j^2} + \frac{B_1}{v_{b_{1j}}} - \frac{v_{b_2}^2 - v_{b_{1j}}^2}{2d_j v_{b_{1j}}}\} \tag{48}$$

So, either $T_{M2} > T_{M3}$:

$$T_{S2,3} = t_{s,d_i} + \frac{h + L_i}{v_{b_{ni}}} - \frac{v_{b_{ni}}}{2a_i} + \frac{v_{b_{ni}}^2}{2a_i^2} + \frac{B_{sj} - h + e}{v_{b_{1j}}} + \frac{v_{b_1}^2}{2d_j^2} - \frac{v_{b_{1j}}}{2d_j} + \frac{\hat{B}_{ni}}{v_{b_{ni}}}$$

$$= t_{s,d_i} + \frac{h + L_i + \hat{B}_{ni}}{v_{b_{ni}}} - \frac{v_{b_{ni}}}{2a_i} + \frac{v_{b_{ni}}^2}{2a_i^2} + \frac{B_{sj} - h + e}{v_{b_{1j}}} + \frac{v_{b_1}^2}{2d_j^2} - \frac{v_{b_{1j}}}{2d_j}$$

$$\rightarrow t_{s,d_i} + \frac{h + L_i + \hat{B}_{ni}}{v_{b_{ni}}} - \frac{v_{b_{ni}}}{2a_i} + \sqrt{\frac{v_{b_{ni}}^2}{2a_i^2}} + \frac{B_{sj} - h + e}{v_{b_{1j}}} + \sqrt{\frac{v_{b_1}^2}{2d_j^2}} - \frac{v_{b_{1j}}}{2d_j} \tag{49}$$

Or, $T_{M3} > T_{M2}$:

$$T_{S2,3} = t_{s,d_i} + \frac{h + L_i}{v_{b_{ni}}} - \frac{v_{b_{ni}}}{2a_i} + \frac{v_{b_{ni}}^2}{2a_i^2} + \frac{B_{sj} - h + e}{v_{b_{1j}}} + \frac{v_{b_1}^2}{2d_j^2} - \frac{v_{b_{1j}}}{2d_j} + \frac{v_{b_2}^2 - v_{b_{1j}}^2}{2d_j^2} + \frac{B_1}{v_{b_{1j}}} - \frac{v_{b_2}^2 - v_{b_{1j}}^2}{2d_j v_{b_{1j}}}$$

$$= t_{s,d_i} + \frac{h + L_i}{v_{b_{ni}}} - \frac{v_{b_{ni}}}{2a_i} + \frac{v_{b_{ni}}^2}{2a_i^2} + \frac{B_1 + B_{sj} - h + e}{v_{b_{1j}}} + \frac{v_{b_2}^2}{2d_j^2} - \frac{v_{b_2}^2}{2d_j v_{b_{1j}}}$$

$$\rightarrow t_{s,d_i} + \frac{h + L_i}{v_{b_{ni}}} - \frac{v_{b_{ni}}}{2a_i} + \sqrt{\frac{v_{b_{ni}}^2}{2a_i^2}} + \frac{B_1 + B_{sj} - h + e}{v_{b_{1j}}} + \sqrt{\frac{v_{b_2}^2}{2d_j^2}} - \frac{v_{b_2}^2}{2d_j v_{b_{1j}}} \tag{50}$$

32

State $\boxed{3}$:

$$T_{S2,3} = T_{M1} + T_{M2} \text{ (for train } j)$$

$$= td_j + \frac{h + L_j}{v_{b_{nj}}} - \frac{v_{b_{nj}}}{2a_j} + \frac{v_{b_{nj}}^2}{2a_j^2} + \frac{\hat{B}_{nj}}{v_{b_{nj}}}$$

$$= td_j + \frac{\hat{B}_{nj} + h + L_j}{v_{b_{nj}}} - \frac{v_{b_{nj}}}{2a_j} + \frac{v_{b_{nj}}^2}{2a_j^2}$$

$$\rightarrow td_j + \frac{\hat{B}_{nj} + h + L_j}{v_{b_{nj}}} - \frac{v_{b_{nj}}}{2a_j} + \sqrt{\frac{v_{b_{nj}}^2}{2a_j^2}} \tag{51}$$

4.2.4   Quality Check of Formulas Type II

For the formulas developed for the Type II station, the same sanity checks are performed, to validate the formulas.

4.2.4.1   Formula Quality Check 1

For a test sample, take a Type II station and trains i and j with the following parameters:

Parameter		Side notes
B_{1j}	$500m$	
$\hat{B}_{1j}$	$650m$	
B_{sj}	$800m$	
B_{ni}	$450m$	
$\hat{B}_{ni}$	$600m$	
$v_{b_{ni}}$ for $\hat{B}_{ni}$	$40km/h$	Equals $11.11m/s$
$v_{b_{ni}}$ for B_{ni}	$80km/h$	Equals $22.22m/s$
$v_{b_{1j}}$ for $\hat{B}_{1j}$	$40km/h$	Equals $11.11m/s$
$v_{b_{1j}}$ for B_{1j}	$80km/h$	Equals $22.22m/s$
v_{b_2}	$100km/h$	Equals $27.77m/s$
t_{s,d_i}	$54s$	
h	$10m$	
L_i	$410m$	
a_i	$1.1m/s^2$	
d_j	$1.3m/s^2$	
e	$250m$	$= 9 \cdot v_{b_2}$

Table 5: Test Sample Type II

$T_{S2,2}$ and $T_{S2,3}$ are calculated according to Equations 37 or 38 and 49 or 50.

Starting with T_{M2} and T_{M3} for $T_{S2,2}$, $T_{M2} = 20.25$ and $T_{M3} = 49.93$. concluding that $T_{M3} \geq T_{M2}$ and thus $T_{S2,2}$ can be calculated by Equation 38. This gives $T_{S2,2} = 217.61$ which is 3 minutes and 38 seconds.

Furthermore, for $T_{S2,3}$ calculations show that T_{M2} is equal to 54.01 and $T_{M3} = 26.76$. Hence, $T_{M2} \geq T_{M3}$ and $T_{S2,3}$ can be calculated by Equation 49, which gives 198.24. this is equal to 3 minutes and 18 seconds.

Note that these results are not to be compared with the 3 minutes and 47 seconds calculated earlier, because now the layout of the station is very different. What can be done is to also calculate

33

case (1) for the Type II station:

$$T_{M2} = \frac{B_{ni}}{v_{b_{ni}}} = \frac{450}{22.22} = 20.25$$

$$T_{M3} = \sqrt{\frac{v_{b_2}^2 - v_{b_{1j}}^2}{2d_j^2}} + \frac{B_{1j}}{v_{b_{1j}}} - \frac{v_{b_2}^2 - v_{b_{1j}}^2}{2d_j v_{b_{1j}}} = \sqrt{\frac{(27.77)^2 - (22.22)^2}{2 \cdot (1.3)^2}} + \frac{500}{22.22} - \frac{(27.77)^2 - (22.22)^2}{2 \cdot 1.3 \cdot 22.22} = 26.76$$

It can be concluded that $T_{M3} > T_{M2}$, hence $T_{S2,1}$ can be calculated by using Equation 24.

$$T_{S2,1} = t_{s,d_i} + \frac{h + L_i}{v_{b_{ni}}} - \frac{v_{b_{ni}}}{2a_i} + \sqrt{\frac{v_{b_{ni}}^2}{2a_i^2}} + \frac{e + B_{sj} - h + B_{1j}}{v_{b_{1j}}} - \frac{v_{b_2}^2}{2d_j v_{b_{1j}}} + \sqrt{\frac{v_{b_2}^2}{2d_j^2}}$$

$$= 54 + \frac{10 + 410}{22.22} - \frac{22.22}{2 \cdot 1.1} + \sqrt{\frac{(22.22)^2}{2 \cdot (1.1)^2}} + \frac{250 + 800 - 10 + 500}{22.22} - \frac{(27.77)^2}{2 \cdot 1.3 \cdot 22.22} + \sqrt{\frac{(27.77)^2}{2 \cdot (1.3)^2}}$$

$$= 148.14 = 2 \text{ min and } 28 \text{ sec.}$$

Now that cases (1) to (3) are calculated, case (1) can be compared with Max{case (2); case (3)}. Hence, it can be observed that when two trains go straight on the main rail, the headway according to the formulas defined before is 2 minutes and 28 seconds. Whereas when a train takes the side platform, the headway is 3 minutes and 38 seconds in the test sample station. This is expected because when a train takes the side platform, it has to travel at a lower speed limit due to going at an angle through the switch, together with the fact that the angle of the switch makes the train travel more meters, causing a slightly higher headway. This was also found by Abril et al., 2008, Francesco Rotoli and Ricci, 2016 and Yung-Cheng Lai and Jong, 2010; that longer blocks give higher headway and thus lower capacity.

However, in practice, Type II stations are very practical when it comes to Sprinter trains and Intercity trains sharing the rails. At such a station, it can be scheduled that the Intercity train can pass by the Sprinter train and this Intercity train will then not have to stop at every Type I station where the Sprinter is scheduled to stop. Type II stations also increase robustness (Abril et al., 2008, Ryus et al., 2013).

4.2.4.2 Formula Quality Check 2

Again, the effect of changing a parameter is evaluated.

- $t_{s,d_i}, h, L_i, a_i, d_j, v_{b_2}, \hat{B}_{1j}, B_{1j}, B_{sj}, \hat{B}_{ni}, e$ and B_{ni} will have the same reasoning as for Type I stations.

- $v_{b_{1j}}$ for b_{1j} and $v_{b_{1j}}$ for $\hat{b}_{1j}$ ↓: if the speed limit in block b_{1j} or $\hat{b}_{1j}$ decreases this means train j will need more time to travel the same distance. This means the headway will increase.

- $v_{b_{ni}}$ for b_{ni} and $v_{b_{ni}}$ for $\hat{b}_{ni}$ ↓: when the speed limit in block b_{ni} or $\hat{b}_{ni}$ decreases, train i needs less time to accelerate, but more time to travel over the same distance. If $v_{b_{ni}}$ decreases, as in this case, T_{M3} will still be bigger than T_{M2} and nothing changes. If this was not already the case, it could cause T_{M3} to become bigger than T_{M2} and this causes the headway to increase.

Parameters	(217.60) $T_{S2,2}$ (%)	(198.24) $T_{S2,3}$ (%)	Expectations
$t_{s,d_i} \uparrow 5\%$	220.30 ($\uparrow$ 1.24)	200.95 ($\uparrow$ 1.36)	$T_{S2} \uparrow$
$h \uparrow 5\%$	217.58 ($\downarrow$ 0.01)	198.27 ($\uparrow$ 0.01)	—— $T_{S2,2} \downarrow, T_{S2,3} \uparrow$
$L_i \uparrow 5\%$	218.52 ($\uparrow$ 0.42)	200.09 ($\uparrow$ 0.93)	$T_{S2} \uparrow$
$a_i \downarrow 5\%$	217.82 ($\uparrow$ 0.10)	198.36 ($\uparrow$ 0.06)	—— $T_{S2} \uparrow$
$d_j \downarrow 5\%$	216.99 ($\downarrow$ 0.28)	198.43 ($\uparrow$ 0.10)	$T_{S2,2} \downarrow, T_{S2,3} \uparrow$
$v_{b_2} \downarrow 5\%$	219.45 ($\uparrow$ 0.85)	198.24 ($=$ 0.00)	—— $T_{S2} \uparrow$
$v_{b_{1j}}$ for $\hat{b}_{1j} \downarrow 5\%$	224.20 ($\uparrow$ 3.03)	-	$T_{S2} \uparrow$
$v_{b_{1j}}$ for $b_{1j} \downarrow 5\%$	-	200.53 ($\uparrow$ 1.15)	—— $T_{S2} \uparrow$
$v_{b_{ni}}$ for $\hat{b}_{ni} \downarrow 5\%$	-	202.97 ($\uparrow$ 2.39)	$T_{S2} \uparrow$
$v_{b_{ni}}$ for $b_{ni} \downarrow 5\%$	218.39 ($\uparrow$ 0.36)	-	—— $T_{S2} \uparrow$
$\hat{B}_{1j} \downarrow 5\%$	214.68 ($\downarrow$ 1.34)	-	$T_{S2} \downarrow$
$B_{1j} \downarrow 5\%$	-	198.24 ($=$ 0.00)	—— $T_{S2} \downarrow$
$B_{sj} \downarrow 5\%$	214.00 ($\downarrow$ 1.65)	196.45 ($\downarrow$ 0.91)	$T_{S2} \downarrow$
$\hat{B}_{ni} \downarrow 5\%$	-	195.55 ($\downarrow$ 1.36)	—— $T_{S2} \downarrow$
$B_{ni} \downarrow 5\%$	217.60 ($=$ 0.00)	-	$T_{S2} \downarrow$
$e \downarrow 5\%$	216.48 ($\downarrow$ 0.52)	197.68 ($\downarrow$ 0.28)	—— $T_{S2} \downarrow$

Table 6: Formula Quality Check 2 T_{S2}

Table 6 is to be read like Table 4 and shows the effect of in-/decreasing the different parameters.

4.2.4.3 Comparison with Yung-Cheng Lai and Jong, 2010

In this section the formulas of $T_{S2,c}$ are compared to $T_{s,A2}$ by Yung-Cheng Lai and Jong, 2010. It was previously established that Yung-Cheng Lai and Jong, 2010 take $T_{M3} \geq T_{M2}$ as a given. Also, note that the formula for $T_{M3} \geq T_{M2}$ for $T_{S2,3}$ and $T_{S2,3}$ are very similar except for taking another measure for B_1, as the one takes the side rail and the other takes the main rail. $T_{s,A2}$ of Yung-Cheng Lai and Jong, 2010 is compared with the formula of this work.

$$T_{S2,c} = t_{s,d_i} + \frac{h + L_i}{v_{b_{ni}}} - \frac{v_{b_{ni}}^2}{2a_i} + \sqrt{\frac{v_{b_{ni}}^2}{2a_i^2}} + \frac{B_1 + B_{sj} - h + e}{v_{b_{1j}}} + \sqrt{\frac{v_{b_2}^2}{2d_j^2} - \frac{v_{b_2}^2}{2d_j v_{b_{1j}}}}, \qquad (52)$$

$$T_{s,A2} = \frac{v_j}{K_b b_j(G_i)} - \sqrt{\frac{2s_e}{K_a a_i(G_0)}} + \frac{v_y^2}{2K_a b_j(G_i)v_j} + \frac{B_2 + B_1 + B_s - s_x}{v_j} + t_o + t_r + t_{dj}$$

$$= \frac{v_j}{K_b d_j} - \sqrt{\frac{2s_e}{K_a a_i}} + \frac{v_y^2}{2K_a d_j v_j} + \frac{B_2 + B_1 + B_s - h}{v_j} + t_{s,d_j}$$

When comparing the two formulas, there are again a few terms that match, but there are also terms that are different. Set $v_j = v_{b_{1j}}$, $K_a = -1$ and $v_y = v_{b_2}$ to get the terms of $\frac{B_{1j} + B_s - h}{v_{b_{1j}}}$ and $\frac{v_{b_2}^2}{2d_j v_{b_{1j}}}$ appear in both formulas. It can also be seen that the terms $\frac{B_2}{v_{b_{1j}}}$ and $\frac{e}{v_{b_{1j}}}$ are related, as e is part of B_2. This research chose to work with the sight distance rather than the entire block length of block b_2, to allow for more efficiency.

Note however, that the formulas take into account a different dwell time, and all other terms are different too. Again it is unclear where the differences come from, due to a lack of full information. Hence, we can only assume these difference are caused by differences in rules and/or assumptions for the different countries.

5 Case Study

As a case study the Dutch rail network between Alkmaar and Zaandam is chosen to analyse bottlenecks in performance and to study potential measures which can be taken to increase the capacity. This part of the network was chosen as it is a diverse part of the railway network, but not too complex to interpret the results and the train schedule easily. It is, though, complex enough to be representative of many other parts of the Dutch railway network.

First the data is introduced, then the data is used to identify bottlenecks. For the critical station, potential measures are studied which can be taken to increase railway capacity. The data is then edited, to see if the bottleneck is solved. All results are validated through XANDRA, the simulation tool used by Arcadis.

5.1 Data

To be able to follow the case study fully, we start by looking at what this part of the railway network looks like.

5.1.1 Railway

Figure 9 shows a map of how the chosen railway part looks; this figure is a screenshot of the real railway network map, where there are 7 stations between the station of Alkmaar and the station of Zaandam and where two branches show other ways the train can travel to but these are not relevant for this case study. An additional drawing made as a simple yet detailed overview of the railway is shown in the Appendix in Figure 12. In this overview, it can be seen that stations Kz and Zzs are very close to each other. This is important to keep in mind as changes in either one of the station's layout could also influence the other station significantly.

Figure 9: Alkmaar ⟷ Zaandam

Next, an overview is shown in Table 7 of the type of stations; for a better overview, two pictures are added to clarify what a Type I (a) and a Type II (b) station looks like for the specific case study, see Figure 10. Note that is is similar to Figure 1.

StationName	Abbreviation	StationType
Alkmaar	Amr	o.o.s.
Heiloo	Hlo	Type 1
Castricum	Cas	Type 1
Uitgeest	Utg	Type 2 (incl. blue)
Krommenie-Assendelft	Kma	Type 1
Wormerveer	Wm	Type 2 (excl. blue)
Zaandijk Zaanse Schans	Zzs	Type 1
Koog aan de Zaan	Kz	Type 1
Zaandam	Zd	o.o.s.

[o.o.s. = Out of Scope]

Table 7: Station Types along Alkmaar → Zaandam

(a) Type 1.   (b) Type 2.
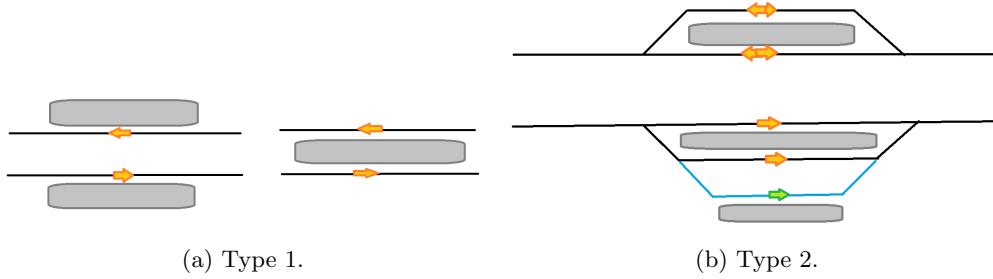
Figure 10: Station Types

Furthermore, for the capacity analysis the train schedule of the trains that travel in the direction of Alkmaar → Zaandam is used. Table 8 shows more details on the stations along the studied part of the railway network. The table shows the block lengths around the stations' platforms. This information is to be filled into the formulas described in Section 4.

StationName	B_2	B_1	B_s	B_n
Heiloo	915 m	750 m	469 m	1741 m
Castricum	687 m	604 m	762 m	1402 m
Uitgeest (1)	1244 m	994 m	331 m	1344 m
Uitgeest (3)	1244 m	885 m	209 m	1466 m
Uitgeest (4)	1244 m	1021 m	290 m	1358 m
Uitgeest (5)	1244 m	1011 m	300 m	1358 m
Krommenie-Assendelft	976 m	1173 m	400 m	886 m
Wormerveer (3)	886 m	620 m	879 m	456 m
Wormerveer (4)	886 m	620 m	879 m	456 m
Zaandijk Zaanse Schans	456 m	707 m	1154 m	1133 m
Koog aan de Zaan	707 m	1154 m	1133 m	429 m

Table 8: Station Details

A few details need to be noted about the information shown in Table 8:

- It is unexpected that both the main rail and the side platform have the same lengths of blocks at Wormerveer. Because for a Type II station, the side rail is usually longer than the main rail.

- The stations Zaandijk Zaanse Schans and Koog aan de Zaan are situated very close together, causing Zzs's b_1 to be the same block as Kz's b_2 as well as Zzs $b_s \longleftrightarrow$ Kz b_1 and Zzs $b_n \longleftrightarrow$ Kz b_s.

- For Uitgeest, platform 2 is not taken into account as this one is solely dedicated to the other direction, which is not past of the current case study.

- For Uitgeest all b_2's are the same as all trains arrive from one rail and then split into the various platforms.

- The stations Krommenie-Assendelft and Wormerveer are also situated quite close to each other, causing Kma's b_n to be the same block as Wm's b_2.

In addition Table 9 shows the speed limits in the blocks around each station:

Station	Hlo	Cas	Utg 1	Utg 3	Utg 4	Utg 5	Kma	Wm 3	Wm 4	Zzs	Kz
v_{b_2} (km/h)	130	130	x	130	80	80	130	80	80	80	100
v_{b_1} (km/h)	65	60	x	50	40	80	65	80	80	50	130
v_{b_n} (km/h)	130	130	40	115	70	70	100	80	40	130	130

Table 9: Speed Limits

To come to the data presented in Tables 7-9, the OS-files and the OBE-files (by ProRail, n.d.) were studied and as the case where all trains come to a stop is considered, the speed limit in b_1 is divided by 2, when there is no switch the train needs to decelerate for. Also, when the speed limit is unclear or changes halfway in the block, the average is taken to complete the data.

Platform 1 of the Uitgeest station, does not need information on v_{b_1} and v_{b_2} as trains arrive from the other direction there and stay at the platform for a while and before leaving the platform again in the direction considered, it is therefore only important to have information for movement 1 and 2 for this platform.

5.1.2 Train Schedule

Secondly, the current schedule for the chosen part and direction of the railway network is looked at in more detail. Table 10 shows the passenger trains that use this part of the railway network every hour. It also shows at what stations the trainseries stops and what the train type is.

Series	Origin/Destination	Type	Route*
3000 H	Nijmegen → Den Helder	IC VIRM 10	Zd → Cas → Hlo → Amr
3000 T	Den Helder → Nijmegen	IC VIRM 10	Amr → Hlo → Cas → Zd
3400 H	Haarlem → Alkmaar	IC SLT 6	Cas → Hlo → Amr
3400 T	Alkmaar → Haarlem	IC SLT 6	Amr → Hlo → Cas
4000 H	Rotterdam Centraal → Uitgeest	SPR SLT 10	Zd → Kz → Zzs → Wm → Kma → Utg
4000 T	Uitgeest → Rotterdam Centraal	SPR SLT 10	Utg → Kma → Wm → Zzs → Kz → Zd
4800 H	Hoorn → Amsterdam Centraal	SPR SNG 9	Amr → Hlo → Cas → Utg
4800 T	Amsterdam Centraal → Hoorn	SPR SNG 9	Utg → Cas → Hlo → Amr
7400 H	Driebergen-Zeist → Uitgeest	SPR SLT 10	Zd → Kz → Zzs → Wm → Kma → Utg
7400 T	Uitgeest → Driebergen-Zeist	SPR SLT 10	Utg → Kma → WM → Zzs → Kz → Zd
800 H	Maastricht → Alkmaar	IC VIRM 10	Zd → Cas → Amr
800 T	Alkmaar → Maastricht	IC VIRM 10	Amr → Cas → Zd

[*within the scope of this Case Study]

Table 10: First Overview of Trains on Alkmaar ⟷ Zaandam

Since the direction of Alkmaar → Zaandam is studied, all trains going in the other direction are deleted from Table 10 which results in Table 11 when the dwell-times of all trains at all stations within the case study and the platforms for Uitgeest and Wormerveer are added for more detail.

Series	Type	Route, incl. dwell-times, within the scope of this Case study
3000 T	IC VIRM 10	Amr → Hlo (54) → Cas (54) → Utg (0, Pl.3) → Kma (0) → Wm (0, Pl.3) → Zzs (0) → Kz (0) → Zd
3400 T	IC SLT 6	Amr → Hlo (42) → Cas (42) → Utg (60, Pl.5)
4000 T	SPR SLT 10	Utg (60, Pl.1) → Kma (42) → Wm (42, Pl.3) → Zzs (42) → Kz (42) → Zd
4800 H	SPR SNG 9	Amr → Hlo (42) → Cas (42) → Utg (42, Pl.4)
7400 T	SPR SLT 10	Utg (60, Pl.1) → Kma (42) → Wm (252, Pl.4) → Zzs (42) → Kz (42) → Zd
800 T	IC VIRM 10	Amr → Hlo (0) → Cas (54) → Utg (0, Pl.3) → Kma (0) → Wm (0, Pl.3) → Zzs (0) → Kz (0) → Zd

Table 11: Second Overview of Trains on Alkmaar ⟷ Zaandam

As shown in Tables 10 and 11 there are different kinds of trains traveling along the Dutch railway network. It is therefore important to include some of the main properties of these trains; these details are shown in Table 12. The information was collected through various references (Treinen-Web, 2023, Everything Explained Website, n.d.), and The missing details where filled in by looking at the default details in XANDRA (see Section 5.4).

Type	Length	Acceleration	Deceleration	# seats
IC VIRM 10	271m	$0.96\ m/s^2$	$1.00\ m/s^2$	980
SPR SNG 9	179m	$1.10\ m/s^2$	$1.30\ m/s^2$	500
SPR SLT 10	170m	$0.96\ m/s^2$	$1.20\ m/s^2$	558
IC SLT 6	101m	$0.96\ m/s^2$	$1.20\ m/s^2$	342

Table 12: Train Details

Lastly, Table 13 shows the real-life current times between two arriving trains at each station. This is important to compare with the formula results later on.

	Time between two consecutive trains	
Station	Min	Max
Heiloo	5 min. and 18 sec.	10 min. and 30 sec.
Castricum	5 min. and 6 sec.	9 min. and 42 sec.
Uitgeest	4 min. and 18 sec.	9 min.
Krommenie-Assendelft	3 min. and 24 sec.	11 min. and 42 sec.
Wormerveer	2 min. and 18 sec.	13 min. and 18 sec.
Zaandijk Zaanse Schans	3 min. and 54 sec.	10 min. and 54 sec.
Koog aan de Zaan	3 min. and 18 sec.	10 min. and 54 sec.

Table 13: Time Between Two Consecutive Trains, Case Study

5.2 Results

Now that all the necessary data has been discussed, we can use the formulas on the Dutch railway network to analyze the parameters used in the formulas and identify bottlenecks in this case study.

5.2.1 Formulas

Firstly, the formulas presented in Section 4 are applied to the data described above. Where $h = 10m$ since the OVS-files (ProRail, 2023) state that h needs to be at least 10 meters. All trains travel as scheduled in the existing timetable and the maximum headway for each station is saved. Results are shown below:

Station	Headway according to Formulas
Heiloo	164.31 sec. = 2 min. and 45 sec.
Castricum	175.88 sec. = 2 min. and 56 sec.
Uitgeest	174.92 sec. = 2 min. and 55 sec.
Krommenie-Assendelft	157.80 sec. = 2 min. and 38 sec.
Wormerveer	166.59 sec. = 2 min. and 47 sec.
Zaandijk Zaanse Schans	206.75 sec. = 3 min. and 27 sec.
Koog aan de Zaan	201.06 sec. = 3 min. and 21 sec.

Table 14: Results on Minimal Headway per Station for the Case Study

It can be concluded that the headway of station Zaandijk Zaanse Schans is determining the overall headway. The real-life time that is left between two consecutive trains on the specific part of the network in the current timetable is shown in Table 13 and this table shows that the results from the formulas fit in with the real-life minimal headway, except for Wormerveer. This is because at Wormerveer the occurrence described before occurs, where the Sprinter train (7400_T) arrives first but then waits on the side platform to let an Intercity train (800_T) pass; as this occurrence was not implemented in the formulas, this could explain the difference

The data presented in Section 5.1.2 shows that 8 trains per hour pass through stations Hlo, Cas, Kma, Wm, Zzs and Kz and 12 trains per hour pass through or at least use station Utg. When the results in Table 14 are studied, it can be observed that currently the maximal headway is 3 minutes and 27 seconds. In order to have at least 3 minutes and 27 seconds between each train at every station, this will result approximately in the current timetable. It is therefore important to find some potential solutions to reduce the headway in order to increase capacity. To do so, we need to take a look at the minimal headway and how to improve the bottlenecks there. This will be done in the next subsection.

5.3   Parameter Analysis

Station Zaandijk Zaanse Schans resulted in the highest necessary headway. As this station is a Type I station, Table 4 is studied, which suggests that in-/decreasing the parameters as below helps to decrease the headway for Zzs:

Parameter	t_{d_i}	h	L_i	a_i	d_j	v_{b_2}	v_{b_1}	v_{b_n}	B_1	B_s	B_n	e
$\uparrow/\downarrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\downarrow$	x	$\downarrow$

Table 15: Parameter Effects on Headway Based on Table 4 (educated guess)

Next, it is known that parameters a_i and d_j are tough to change as interchanging trains or buying new trains is needed to have better characteristics. The possibility of changing these two parameters is therefore not considered. All other parameters can be changed more easily by moving a signal or changing a sign along the railway (this is for $h, v_{b_2}, v_{b_1}, v_{b_n}, B_1, B_s, B_n$ and e), or by adding/removing another train part to the train (this is for L_i). The change of these parameters is discussed in a little more detail:

- t_{d_i}: The dwell time of a train can be changed in theory, but it is important to consider that passengers need sufficient time to enter or leave the train. It can, therefore, be lowered only to a certain level. The dwell time could also be increased, but when the dwell time is too long, passengers might get impatient and less satisfied with the overall travel experience by train. NS has a minimal dwell time of 42 seconds and no maximal dwell time (NS.nl, 2021).

- h: According to the design rules of ProRail, h has to be equal to at least 10 meters, but it can increase depending on the length of the train and the length of the platform (ProRail, 2018b). Hence there is only the possibility of increasing this parameter.

- L_i: According to another design rule by ProRail (ProRail, 2018b), L_i can be at most 400 meters and it is essential to note that the smaller the train, the fewer passengers can be transported at once. So, having a smaller train length will not help increase the capacity of the railway from this point of view.

- v_{b_2}: TreinenWeb, 2023 and Everything Explained Website, n.d. state that the Dutch trains have a maximal speed of $140km/h$. Next to that, note that in practice, it is very important to take into account switches, the gradient, and angles of the rail to set an appropriate and safe speed limit.

- v_{b_1}: The trains have a maximum speed of $140km/h$. Next to that, it does not make sense to accelerate before braking for a stop; the constraint of $v_{b_1} \leq v_{b_2}$ is therefore added. Of course, similar to v_{b_2}, v_{b_1} also depends on the layout of the rails.

- v_{b_n}: The assumption of accelerating in $h + L_i$ still needs to hold, so $v_{b_n} \leq \sqrt{2a_i(h + L_i)}$. Furthermore, the same constraints as with the other speed limits are still active: $v_{b_n} \leq 140km/h$ and rail layout is of importance in practice.

- B_1: B_1 needs to be at least long enough for the train to fit into the block. Else, multiple assumptions on the formulas do not hold anymore. Also, note that when any block is very long, it is also occupied for very long, this does not seem efficient for improving capacity.

- B_s: Depending on the layout, this block length could be less than the train length due to the switch being in a different position on a Type II station. But since Zzs is a Type I station, It is still assumed that this block needs to be bigger or equal to L_i.

- B_n: Similar to B_1, B_n needs not be too long but at least long enough for the train to fit in the block.

- e: Because of safety reasons, e cannot decrease. Increasing e is safe but not desirable.

Considering these established constraints on changing the parameters, Figures 13 to 21 (in the Appendix) show the range of change per parameter and its effect on the headway using the formulas defined in Section 4 and keeping all other parameters as described in Section 5.1. The findings presented in Table 16 are concluded.

Parameter	Headway lowest at ...	Headway
t_{d_i}	100%	206.75 sec.
L_i	5%	202.28 sec.
v_{b_2}	175%	186.66 sec.
v_{b_1}	155%	157.70 sec.
v_{b_n}	80%	206.44 sec.
B_1	55%	185.79 sec.
B_s	15%	136.13 sec.
B_n	100%	206.75 sec.
e	100%	206.75 sec.

Table 16: Minimal Headway Possibly Found, Considering Constraints

Table 16 shows that decreasing B_s by 85% gives the biggest improvement in headway. Secondly, increasing v_{b_1} might hold the key to solving the high headway of station Zzs, which is in line with the finding of Abril et al., 2008 and Francesco Rotoli and Ricci, 2016 of increasing speed limits decreases headway. Note that we can only make these changes if the timetable can still operate safely and all assumptions taken before still hold. Hence decreasing B_s and increasing v_{b_1} is interesting but taking into account all assumptions taken before. A small optimization model is used because it is obvious that both parameters have effects on each other, since trains drive at speed v_{b_1} in b_s.

Min Headway
s.t.
$\Delta y_{\{v_{b_1}, 0\}} \leq (B_s - h)$
$\Delta y_{\{v_{b_2}, v_{b_1}\}} \leq B_1$
$B_s \leq$ Train Length
$v_{b_1} \leq v_{b_2}$ and a rounded value of speed

Min T_{S1}
s.t.
$\frac{v_{b_1}^2}{2d_j} \leq (B_s - h)$
$\frac{v_{b_2}^2 - v_{b_1}^2}{2d_j} \leq B_1$
$B_s \leq L_i$
$v_{b_1} \in \{50km/h, 55km/h, 60km/h, 65km/h, 70km/h, 75km/h, 80km/h\}$

Min T_{S1} (See Section 4)
s.t.
$\frac{v_{b_1}^2}{2*1.2} \leq (B_s - 10)$
$\frac{(22.22)^2 - v_{b_1}^2}{2*1.2} \leq 707$
$B_s \leq 169.9$
$v_{b_1} \in \{13.88, 15.28, 16.67, 18.06, 19.44, 20.83, 22.22\}$

Solving the minimization problem gives an optimal headway of 117.26 seconds with $B_s \downarrow 80\% = 231m$ and $v_{b_1} \uparrow 50.1\% = 75km/h$. The data from this station can be edited and to calculate the new results as in Table 14; this gives Table 17 and leaves station Cas as the new bottleneck (very closely with Utg) and a minimum headway of 2 minutes and 56 seconds. Hence, editing station Zzs resulted in a decrease in minimum headway of 14.93%.

When looking at theoretical capacity, the theoretical capacity was $\frac{3600}{206.75} = 17.41$ trains per hour and is now improved to $\frac{3600}{175.88} = 20.47$ trains per hour, which is an increase of 17.58% in theoretical rail capacity.

Station	Headway according to Formulas
Heiloo	164.31 sec. = 2 min. and 45 sec.
Castricum	175.88 sec. = 2 min. and 56 sec.
Uitgeest	174.92 sec. = 2 min. and 55 sec.
Krommenie-Assendelft	157.80 sec. = 2 min. and 38 sec.
Wormerveer	166.59 sec. = 2 min. and 47 sec.
Zaandijk Zaanse Schans	117.26 sec. = 1 min. and 57 sec.
Koog aan de Zaan	149.94 sec. = 2 min. and 30 sec.

Table 17: Results on Minimal Headway per Station After Changes in Parameters for Zzs

5.4 Validation

In an attempt to validate the findings above, Arcadis offered for me to use their long-used simulation tool of the railway network called XANDRA.

Arcadis utilizes the simulation model XANDRA to evaluate solutions for train problems on the railway. The model describes the movement of trains and the related processes, like energy consumption and passing locations. The model includes track layout, rolling stock, timetables, signaling and management (Colleagues, 2020). XANDRA was written in C Sharp (C#).

Checking the results of this thesis was not easy as it is necessary to manually add each timetable point to the program and it only shows the results but doesn't do any optimization or suggestion. Below, an explanation of what exactly was done is discussed. In addition, it is discussed how this

shows the results described in this thesis can be validated.

5.4.1 Headway at 207 seconds is Conflict-Free

To start, the real-life case from the case study is entered into the program. All signs and signals are exactly like the OBE- and OS-files of ProRail (ProRail, n.d.) show and the used layout is also as shown on the design papers. The real-life schedule of all trains going twice an hour at the exact times as in the timetables provided by ProRail are entered as well. When the simulation tool was run on this data, we got Distance Graphs for all trainseries; see Figures 22a, 23a, 24a, 25a, 26a and 27a in the appendix. Checking the actual headway between the real-life case shows that the minimal headway is around 3 minutes and 30 seconds, which is very close to the minimal headway this thesis found (3 minutes and 27 seconds) using the formulas for the real-life case. Hence, this shows that there is little room for extra trains. For the rest of this validation through XANDRA, the real-life case without adaptations is taken as the case where headway is 207 seconds.

5.4.2 Zaandijk Zaanse Schans is Bottleneck Station

Figures 22a, 23a, 24a, 25a, 26a and 27a show the cruising speed of the trains on traveling from Alkmaar to Zaandam or on part of this route. In Section 5.1, it was discussed when a train is supposed to make a stop according to schedule. In the Figures, a stop means cruising speed is equal to zero. If a train breaks or stops in the middle of two stations or at a station where no stop was scheduled, the train experiences disruption due to another train and conflict-free planning has failed. To show an example, we let every train depart every 15 minutes instead of every 30 minutes. This makes minimal headway a lot smaller. Also, any constraints on travel times were removed to make the trains move as freely as possible.

In Figures 22b, 23b, 24b, 25b, 26b and 27b the results are shown, Figures 22b, 23b, 24b, 26b and 27b show some clear disruptions. Trainseries 4800_H does not seem to be constrained by any trains and even travels at a faster speed now that the constraints on travel times no longer need to hold. A closer look at Figures 22b, 24b, 26b and 27b shows that almost all trains experience a delay around station Zzs. This observation aligns with the findings that station Zzs is the current bottleneck station.

5.4.3 Adaptations to Zzs Make for Less Conflicts

After adapting the layout to make B_s of Zzs 231m and to increase the speed limit in b1. The same case where the frequency of trains is once every 15 minutes is looked at once more and it is checked whether the conflicts have become less. Figures 22c, 23c, 24c, 25c, 26c and 27c show the results gotten. It is fair to mention that not all trains from the same trainseries show the same graph, but to show the results, a graph that shows the most common graph for the trainseries was carefully selected.

When taking a look at Figures 22c, 23c, 24c, 25c, 26c and 27c it can be observed that no train is unnecessarily making a stop at station Zzs and also the braking is less or not present in the graph around this station. What is now visible in Figures 22c, 23b, 23c, 27b and 27c is the conflicts around station Cas, which is also very much in line with the findings of this station being the new bottleneck station when station Zzs had some adaptations.

Even though XANDRA does not give exact headway times or creates train schedules for the minimal headway, looking at the output graphs does give insights that were also found through the formulas. These findings can, therefore, be used to prove that this thesis' findings are valid and that the formulas work. There were, however, a lot of limitations on this validation and it is therefore not clear to what extent the formulas relate to reality. We can only conclude that this validation shows similar results.

6 Conclusion

This research used an analytical approach to create an useful model to match how the Dutch railway system works. The analytical model is then checked, used to identify bottlenecks for a small case study and then validated through the case study using a simulation tool. The research was led by four sub research questions to come to a clear conclusion to the main research question: "*What are bottlenecks in the Dutch railway network and how can they be found?*". In order to conclude on the research question all four sub research questions are first discussed shortly.

6.1 Sub Research Questions

The first sub research question that is important to address is "*What model is simple yet effective to calculate capacity on the Dutch railway network?*". From the literature research (Section 2), it became clear that using an analytical approach would be most desirable since it allows for parameter choices and it allows for complexity to be added or left out as fits the objectives of the research (Abril et al., 2008). Through assumptions and including more and more parameters, this model can be brought closer to reality to show more realistic results.

For this thesis, an analytical model with 12 parameters was developed and multiple assumptions for simplification. The model takes into account the train mix, speed limits and block lengths, which sums up the second sub research question: "*What parameters are important to railway capacity and thus need to be included in the model?*".

Trainmix is considered by various parameters: train lengths, train acceleration, train deceleration and train routes/timetables. The case study considers four different trains in the train mix. In addition, the system with signals is included in the model by using block lengths and conflict-free planning as the guidelines. Speed limits are also included, looking at switches, stations, signs, and signals.

Thirdly, the sub research question of "*What assumptions need to be taken, to keep the model simple yet somewhat realistic?*" is discussed in Section 3.4.1 and during the formulation of the analytical model. In short, the assumptions are a perfect world, with no gradient of the rail and no unexpected delays. All trains are planned conflict-free, and operation and reaction time are not taken into consideration. It is also assumed that the rail was well designed and that it fits with the train time schedules.

Lastly, the fourth sub research question is ground to Section 5 where an applied example of the analytical model defined was shown. The formulas fit Type I and Type II stations and give a good insight into what stations are causing the headway to be high. Furthermore, the model can then be used to identify the impact of the parameters on the bottleneck station to conclude a possible solution for solving the bottleneck. This process was shown and documented.

6.2 Research Question

The research question can now be answered as well. As shown, bottlenecks are stations along the Dutch railway network that limit the minimal headway. Adaptations can solve these bottlenecks to the specific bottlenecks. Data on the station layout and the timetables is needed to find out what stations are bottlenecks. To create a clear overview, the steps taken during this thesis to identify bottlenecks for the Dutch railway network using the defined model are shown below:

Step 1: Collect data on station layouts, train characteristics and train schedules.

Step 2: For each two consecutive trains calculate the headway.

Step 3: Find the minimal headway for each station.

Step 4: Find the overall minimal headway and identify the bottleneck station.

Step 5: Analyse the parameters for the bottleneck station to find what parameters are best to change for better results.

Step 6: Change the found parameters to an optimal amount.

Step 7: Edit the data on the station layout.

Step 8: Recalculate the overall minimal headway through steps (2) to (4).

Step 9: Steps (5) to (8) can now be repeated to solve for more bottlenecks.

6.3 Future Use

For Arcadis, the model developed is useful in the future to help find potential bottlenecks on parts of the Dutch Railway network. The model can also help point out important parameters that need more focus when capacity is desired to increase.

As the model is analytical and holds under the assumptions made in Section 3.4.1, the numerical outcomes should not be used officially but more as indicators for where to focus. Next to that, the model gives insight into the parameters' effects on capacity.

7   Limitations and Future Research

This thesis defines a simple analytical model to calculate the headway for the Dutch railway network. However, a six-month internship is not long enough to cover all important and interesting fields that come with this topic of study. Therefore, possible research topics that expand upon the research done during this thesis internship will be addressed shortly.

7.1   The Current Output

To start, all headway presented as the results in Table 14 are close to 3 minutes. As stated before, this is hopeful as ProRail took 3 minutes as the general headway to calculate capacity with up until 2020. But this is also a factor that could be explored more. Adding reaction time, operation time, or gradient into the model will only raise the current headway. As ProRail has stated before, they are aware that headway is less than 3 minutes in most stations, it shows that the current model needs some adaptations to meet these new developments of ProRail. It could be a very interesting topic for future research to extend the current model and make it more realistic with a result of lower headway.

7.2   ERTMS

Currently, a lot of research is done on the possible use of ERTMS (see Section 2) on the European and, therefore, also the Dutch railway network. This new way of safety measures creates a dynamic block around the train that is adjusted to the characteristics of the specific train; the block will move along with the train as the train travels. This way, blocks can be as small as the characteristics of the train allow and do not need to be a general length such that all trains can fit in it. This new development in possible safety measures on the railway also significantly influences the current model. If ERTMS should be considered, the model would need changes, and this is an interesting field to look into. Perhaps some comparisons could be made between the current system in place and the upcoming ERTMS ((Mikulcic and Mlinaric, 2021), (Abril et al., 2008)).

7.3   Parameter Correlation

As described in Section 5, B_s and v_{b_1} can have an impact on each other, since trains travel at speed v_{b_1} in B_s as well. Like this example, many more parameters might have some correlation with each other. Their effects on each other and their effects on the minimal headway are of importance. This would be a valuable addition to the current research, but it was not included due to time limitations.

7.4   Robustness

Another field of study to research along with the current model is the robustness of the railway. Is it desirable to have more trains on the railway to meet an increase in demand? or is it more desirable to make trains longer but have fewer trains so that there is more robustness and, thus, customer satisfaction? What is the impact on headway when changing the station type? And what is the impact on the robustness of the change of the station type? It is best for the robustness to have all type II or bigger stations, but is this desirable? All these questions would make an extension to this thesis project and could be of great interest to the field of rail transport.

7.5 Randomness

Furthermore, an extension to the current model would be adding some random noise to the model. A perfect world was assumed, but this is rarely the case in practice. Stochastic factors play an important role in the realistic headway on the railway, and thus, it would be of interest to the field of study to add in randomness and get more realistic results.

However, adding randomness to the model brings a lot of complexity. All random factors have their own distribution, and data is needed to draw conclusions on these distributions. This data is usually collected through simulation, which adds a whole other analysis method to the current topic.

7.6 Cost-Benefit Analysis

Lastly, stations commonly have a specific layout due to constraints on costs and materials. An addition to the current research done in this thesis internship would be to add a cost-and-benefit analysis. In this thesis, it was found that it is desirable for station Zzs to change the length of block b_s. But what are the costs of this change in layout? And what are the benefits? And is it worth making such a change according to a cost-benefit analysis? Perhaps changing the station type is more beneficial to the railway capacity, but is it worth it compared to the costs? The answer to these questions will give a more economical but critical overview of station layout options and their consequences in terms of costs.

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9 Appendix

9.1 Changing Block Lengths
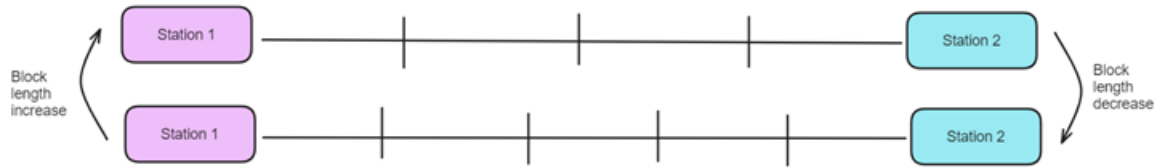
Figure 11: Simple Explanation In-/Decreasing Block Lengths

9.2 Overview Case Study
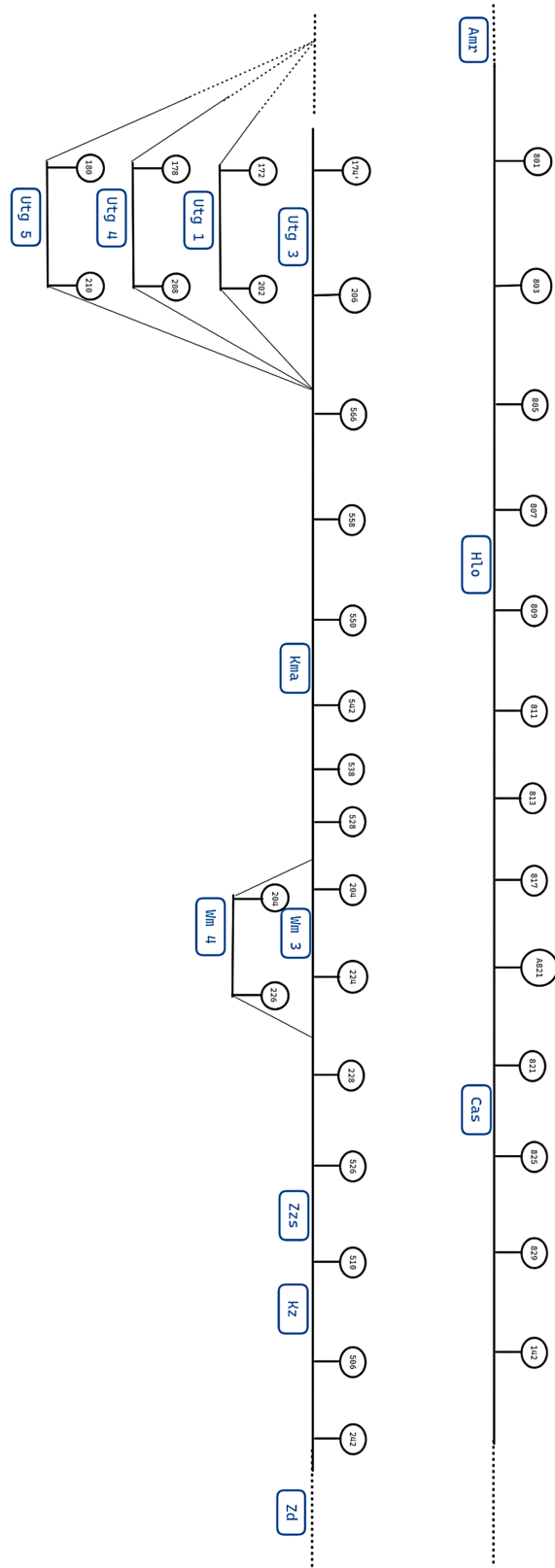
Figure 12: Simplified Overview Signs/Blocks and Stations

9.3   Parameter Analysis
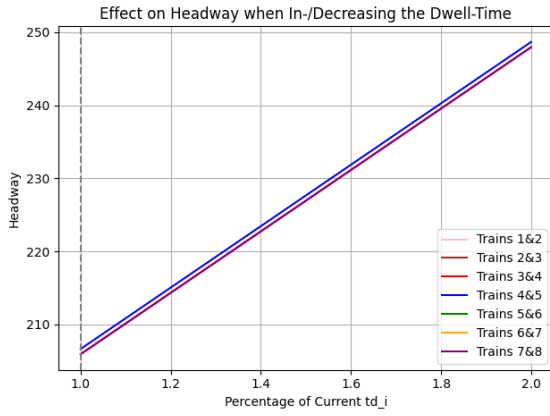
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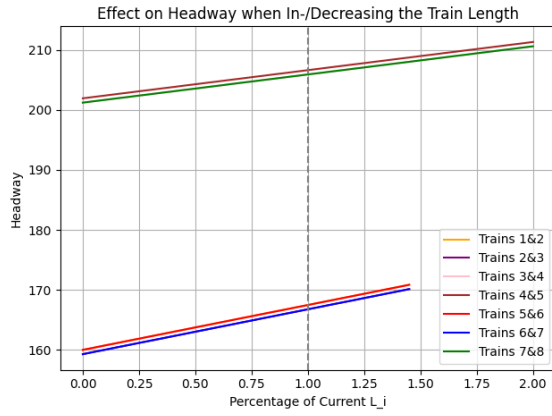
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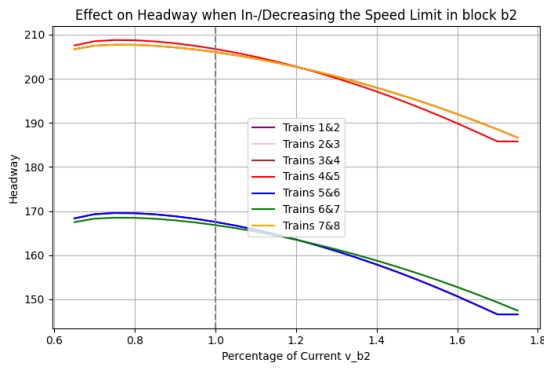
Figure 15
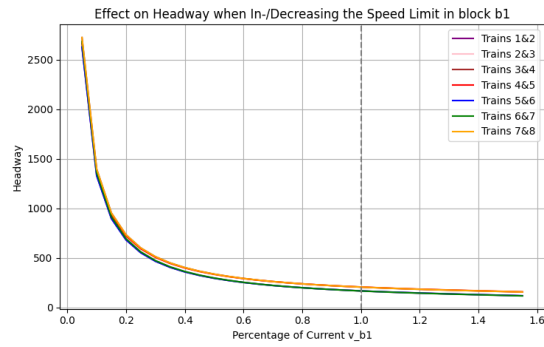
Figure 16
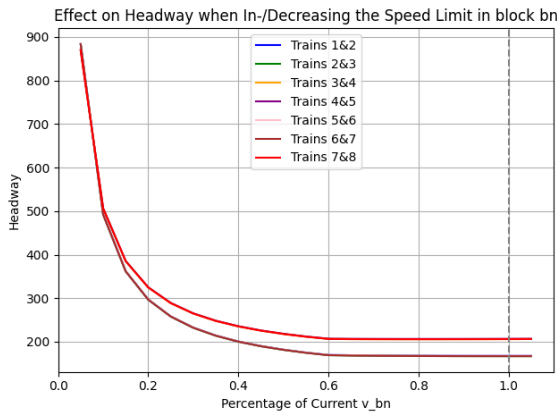
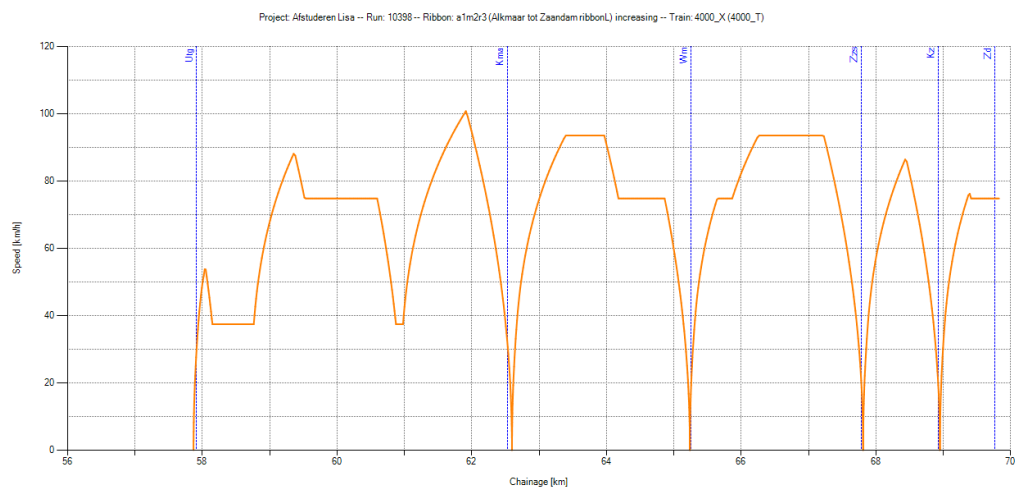Figure 17
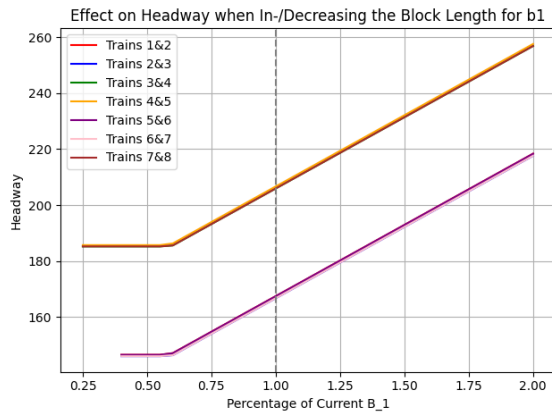
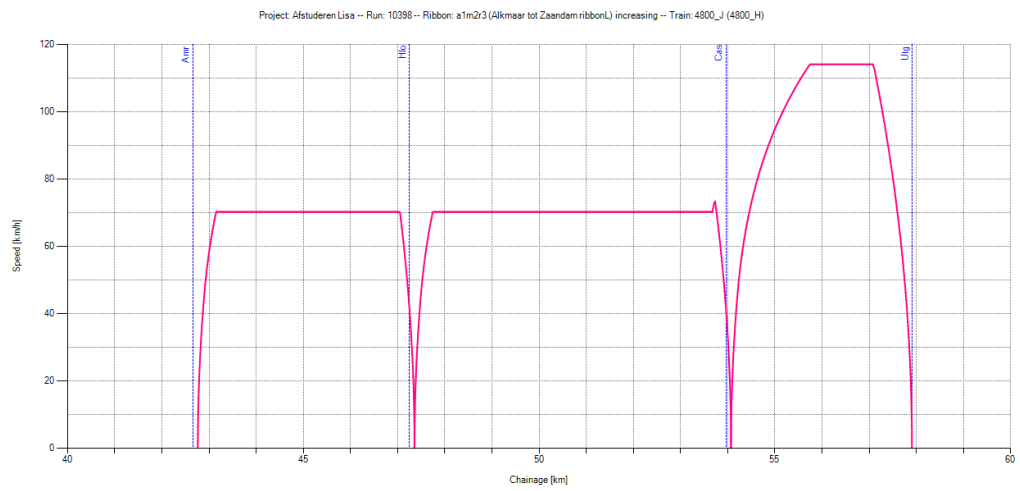
Figure 18
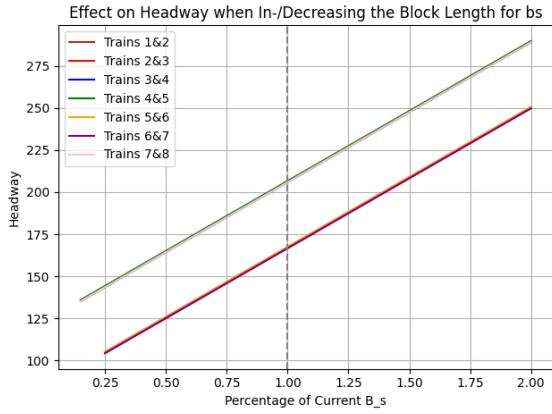
Figure 19
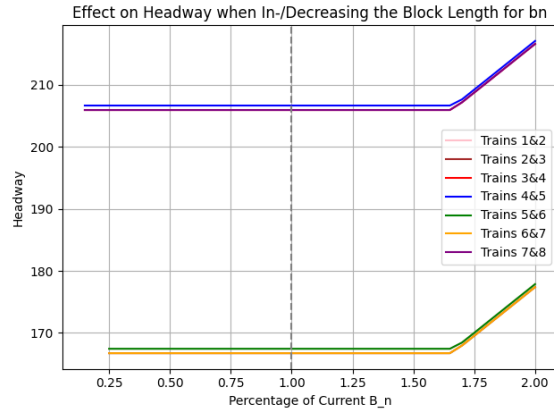
Figure 20
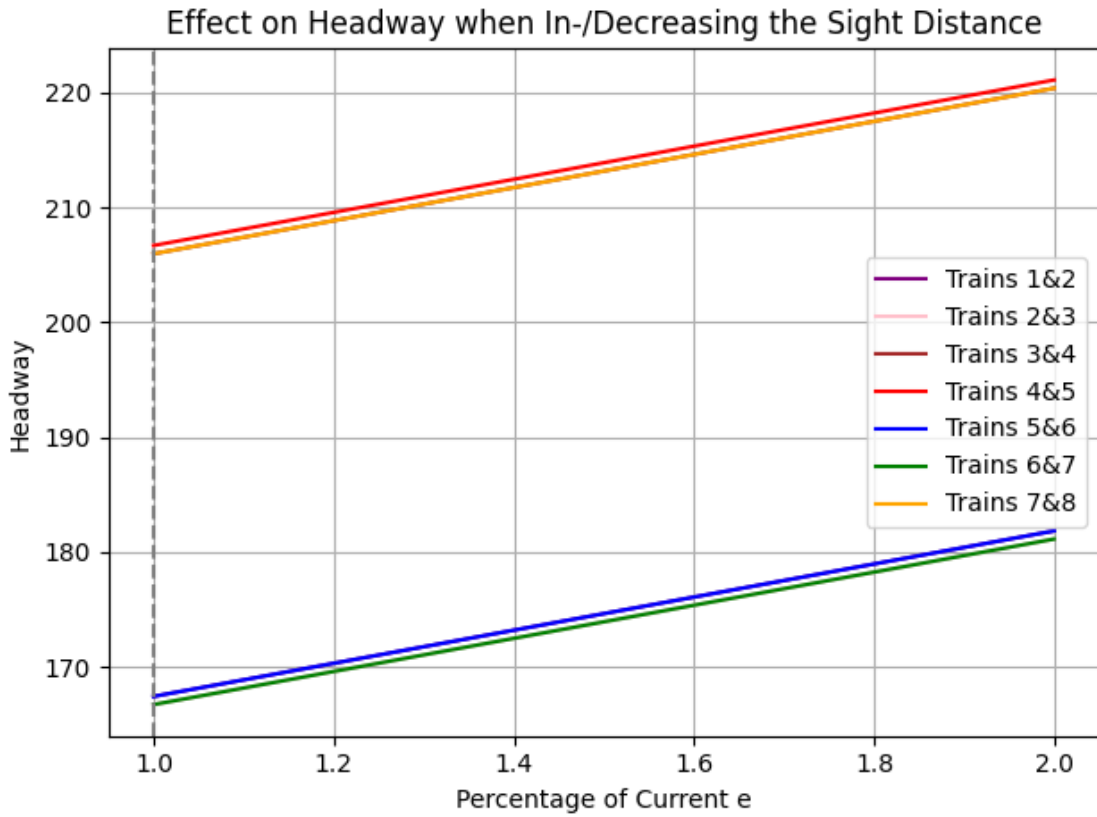
Figure 21

9.4 Xandra Output Trainseries
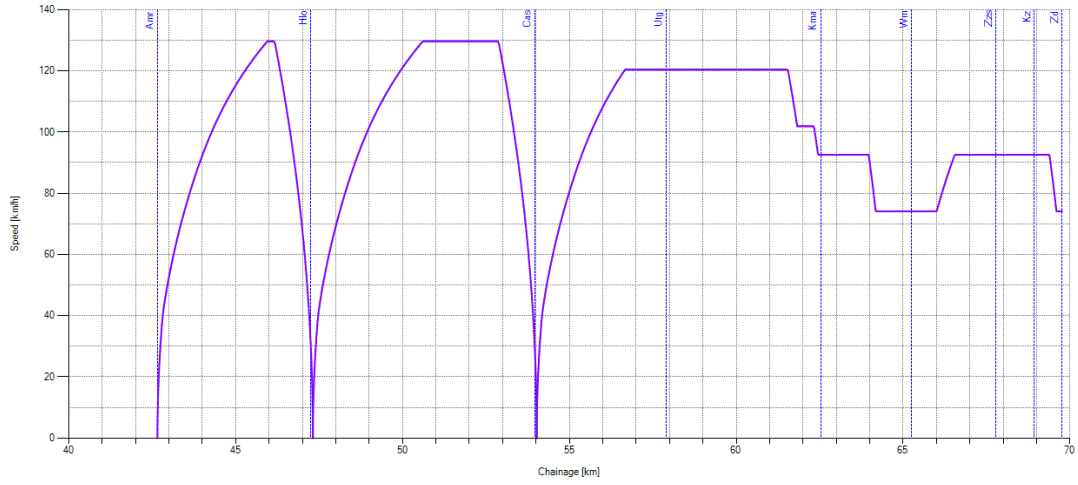
(a) Headway = 207 seconds
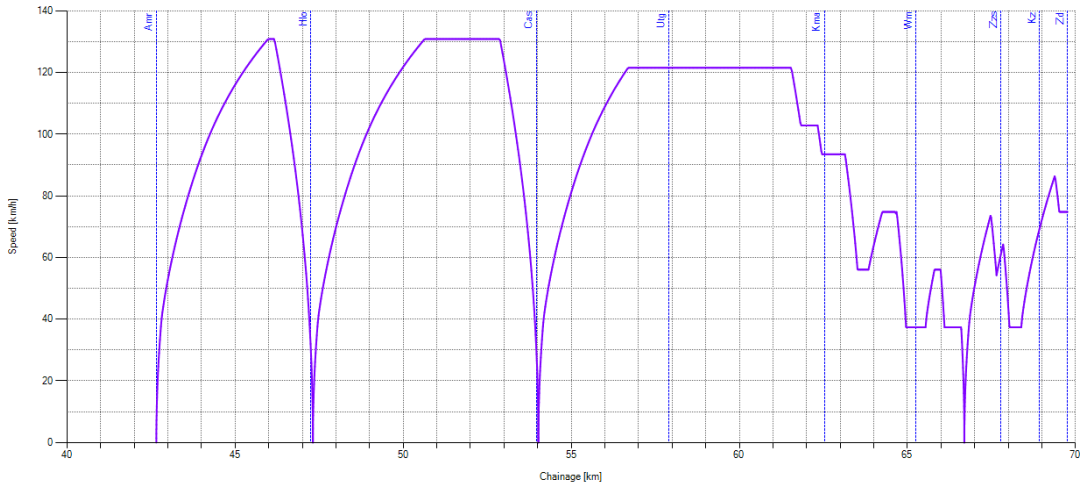
(b) Train Departure each 15 minutes
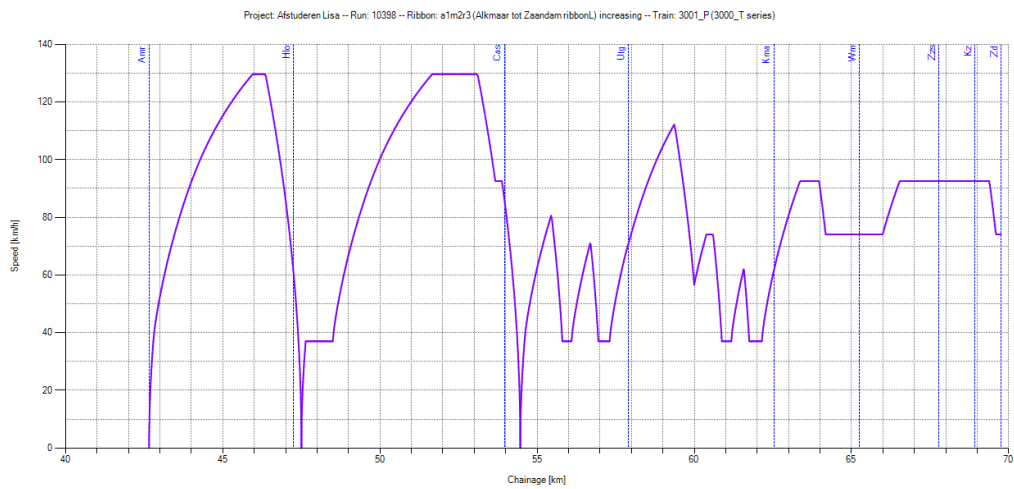
(c) After adaptations to Zzs

Figure 22: Output Xandra Trainseries 3000_T
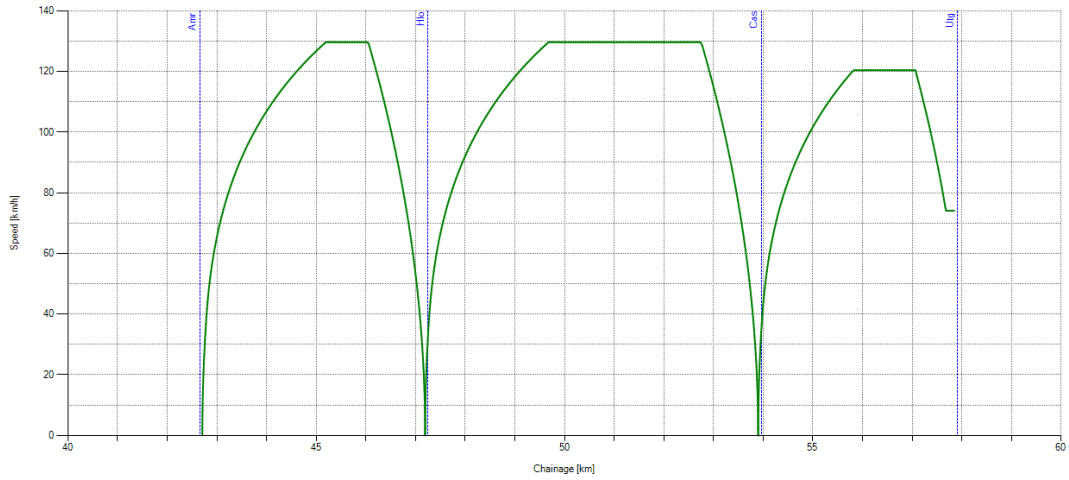
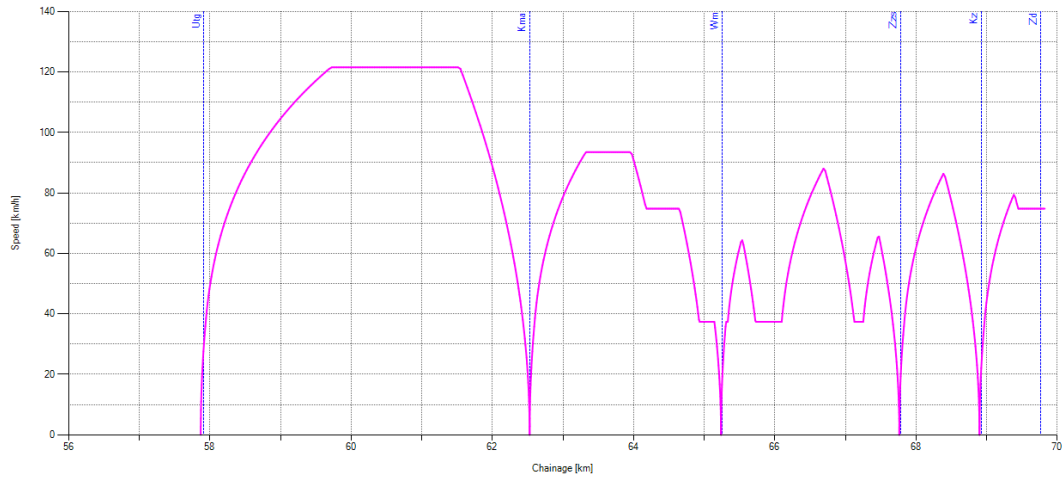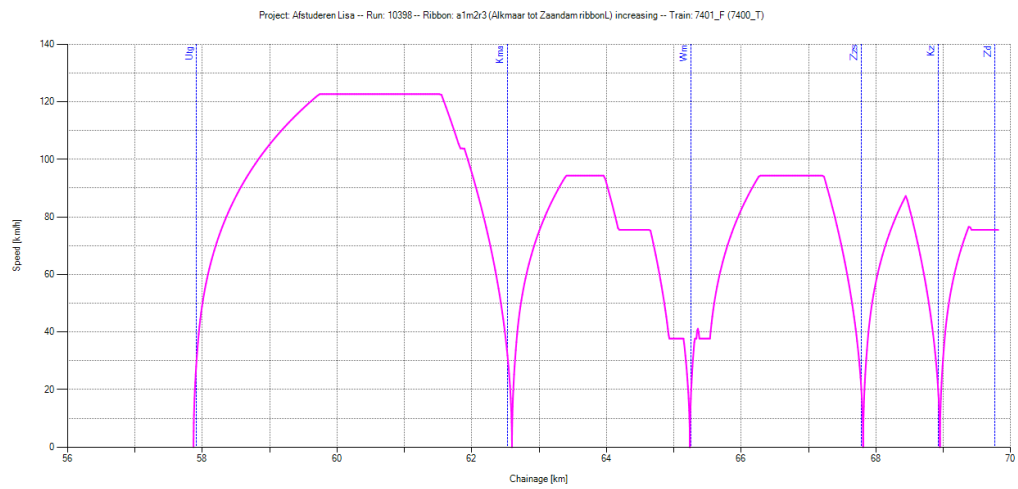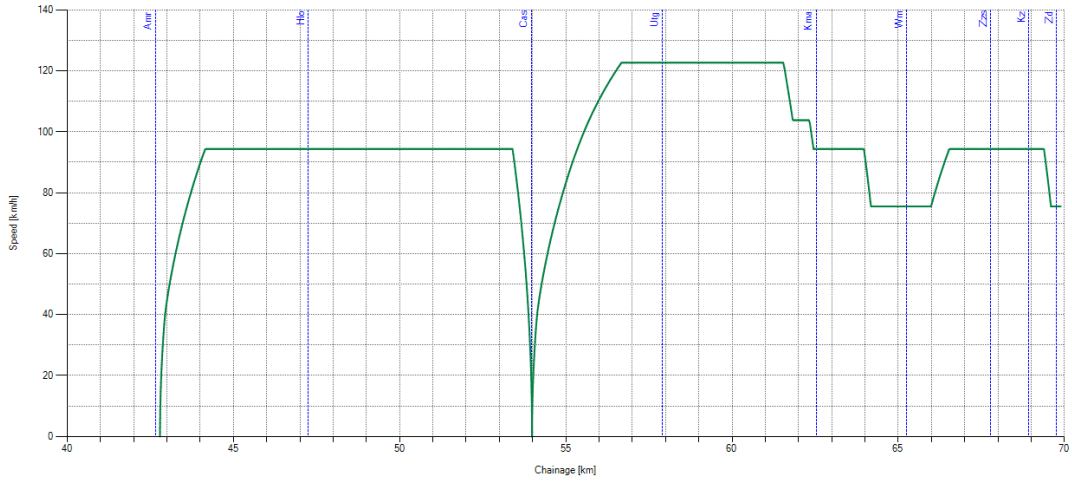(a) Headway = 207 seconds
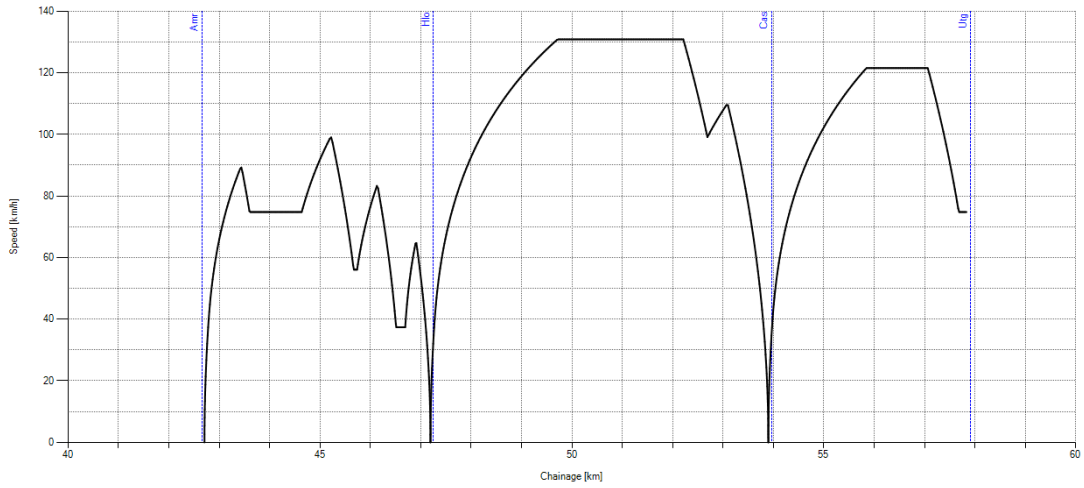
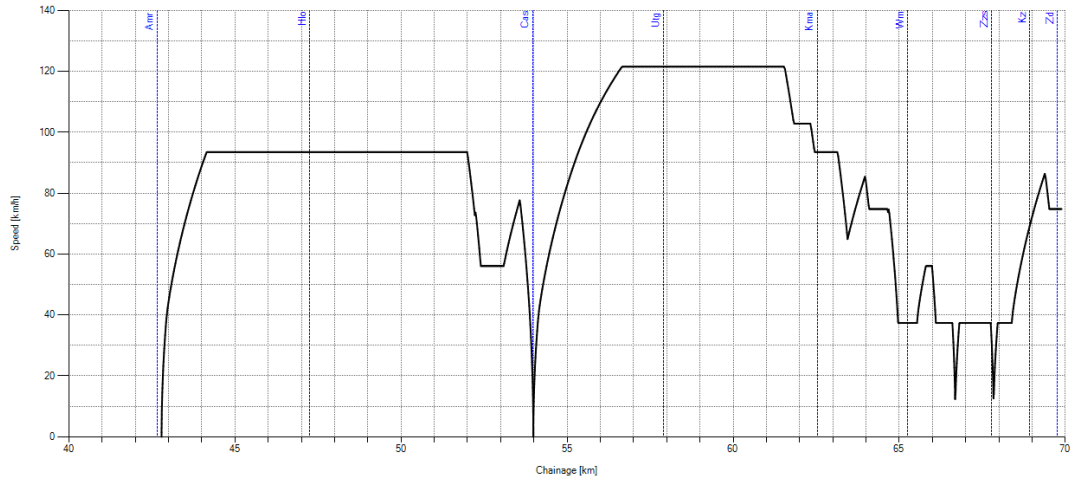(b) Train Departure each 15 minutes
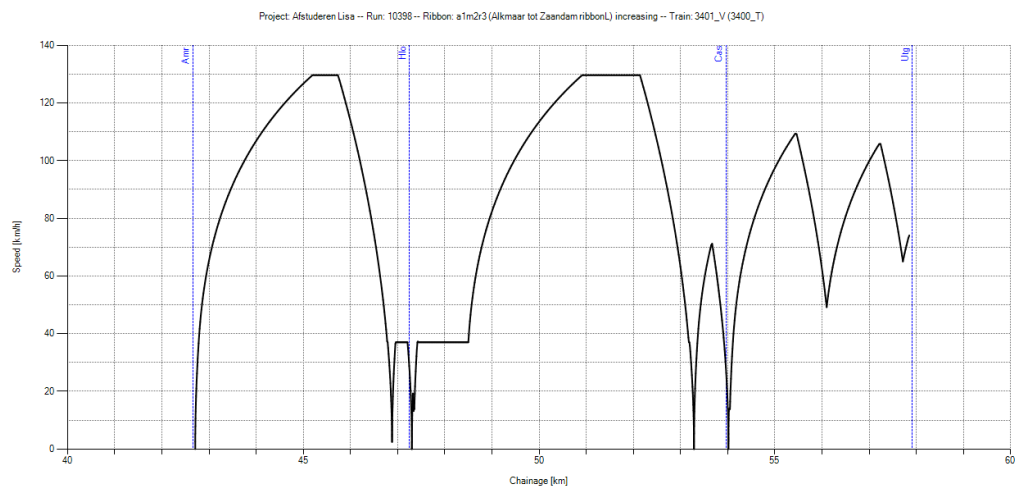
(c) After adaptations to Zzs

Figure 23: Output Xandra Trainseries 3400_T
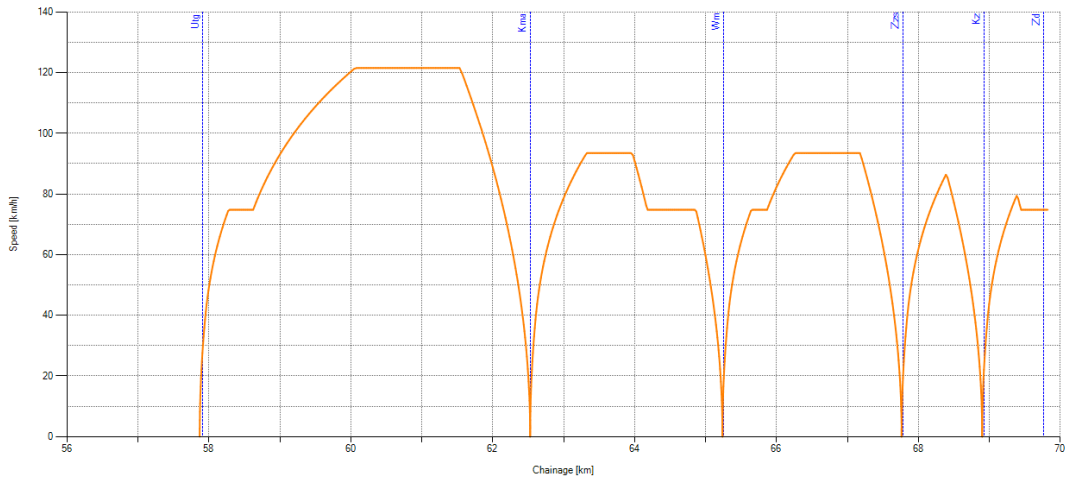
(a) Headway = 207 seconds
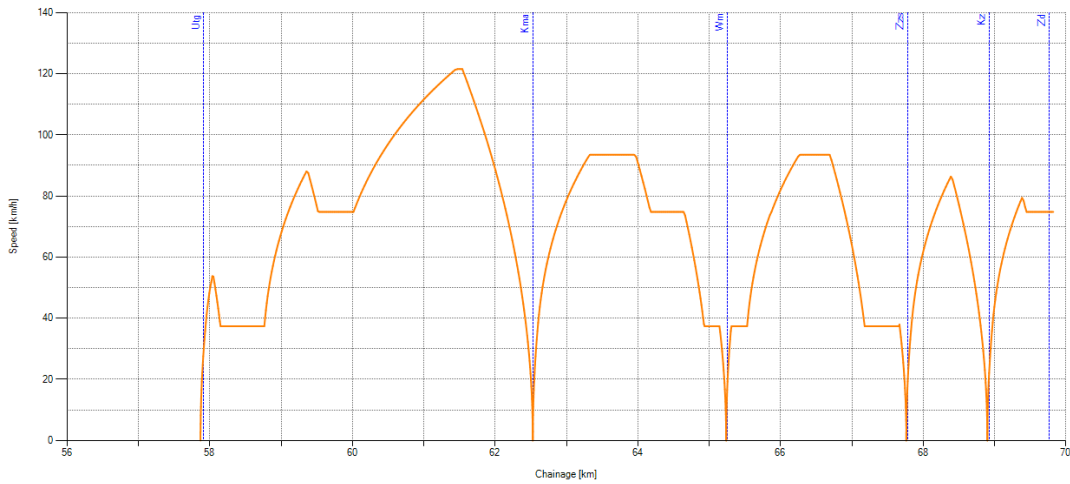
(b) Train Departure each 15 minutes

(c) After adaptations to Zzs

Figure 24: Output Xandra Trainseries 4000_T
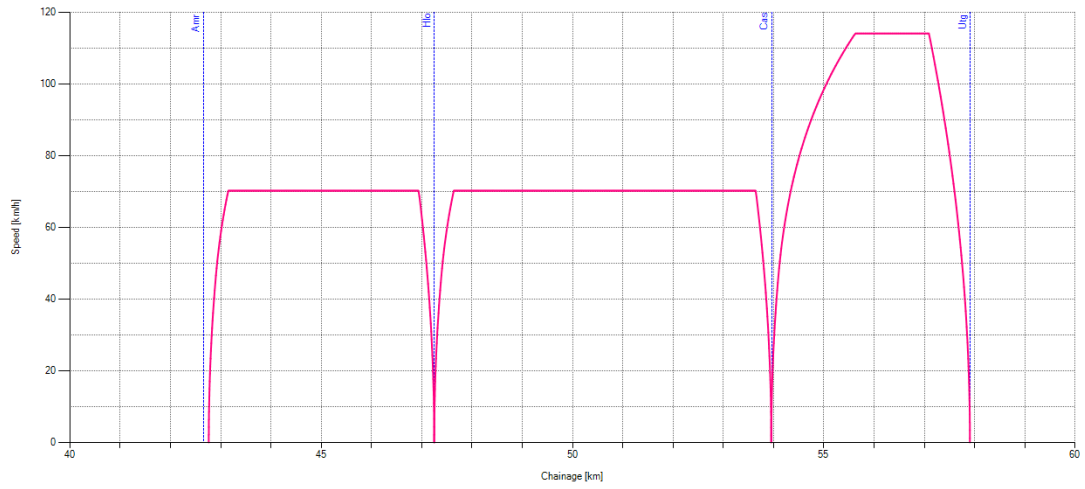
(a) Headway = 207 seconds
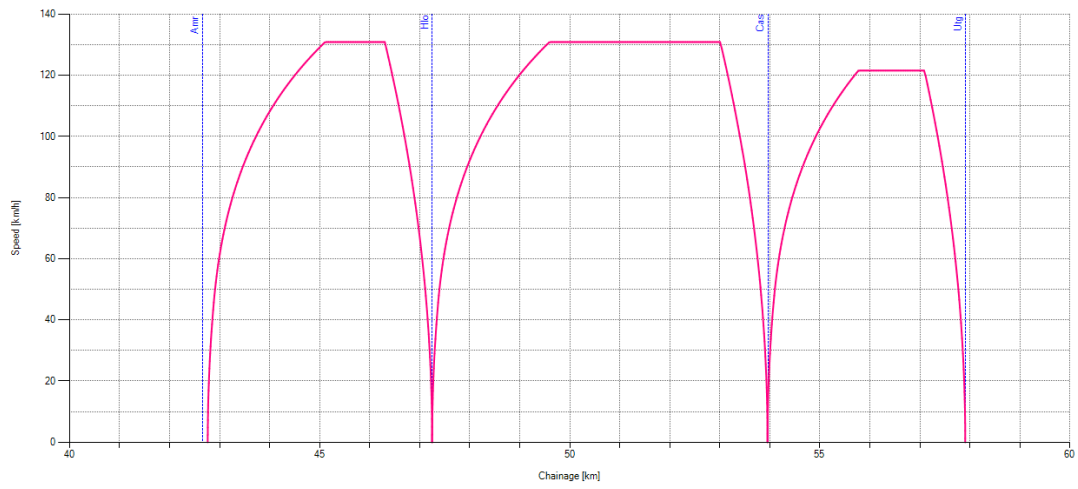
(b) Train Departure each 15 minutes

(c) After adaptations to Zzs

Figure 25: Output Xandra Trainseries 4800_H
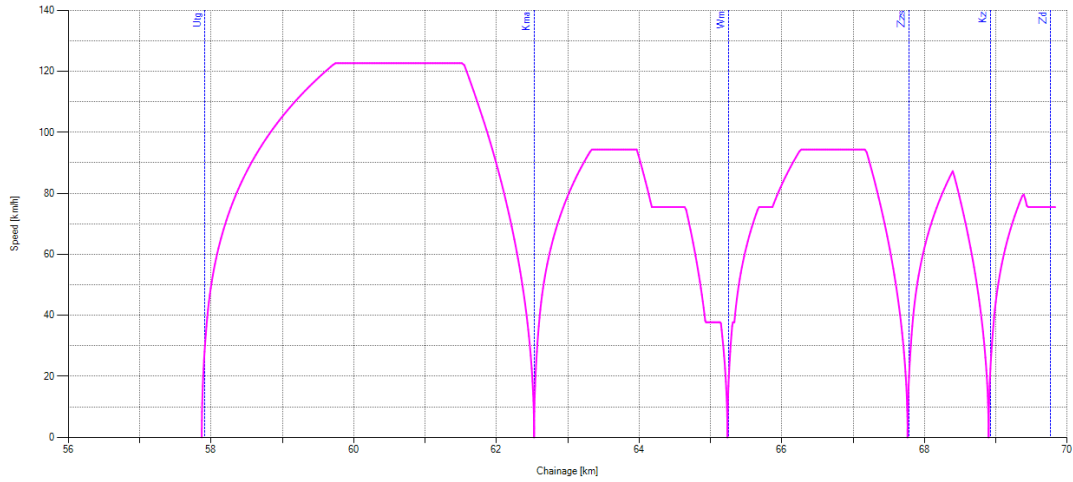
(a) Headway = 207 seconds

(b) Train Departure each 15 minutes
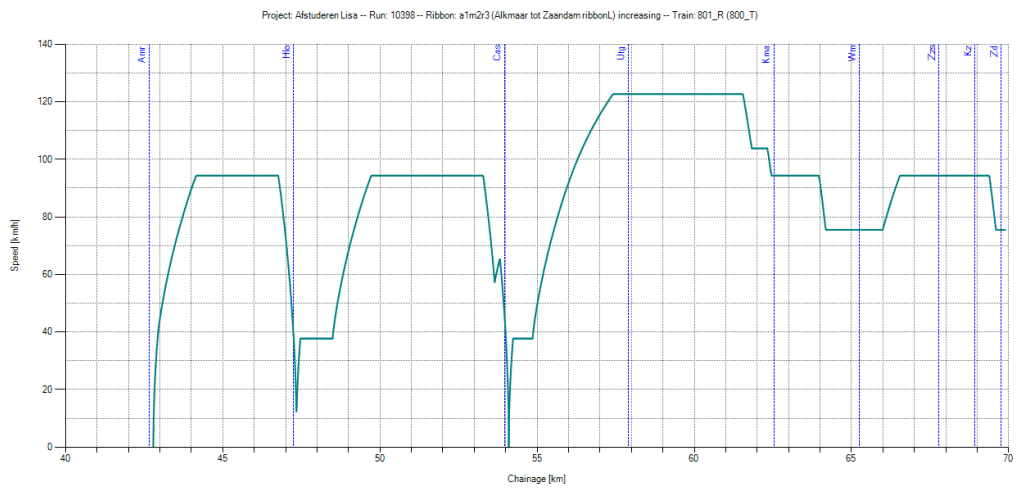

(c) After adaptations to Zzs

Figure 26: Output Xandra Trainseries 7400_T

(a) Headway = 207 seconds

(b) Train Departure each 15 minutes

(c) After adaptations to Zzs

Figure 27: Output Xandra Trainseries 800_T