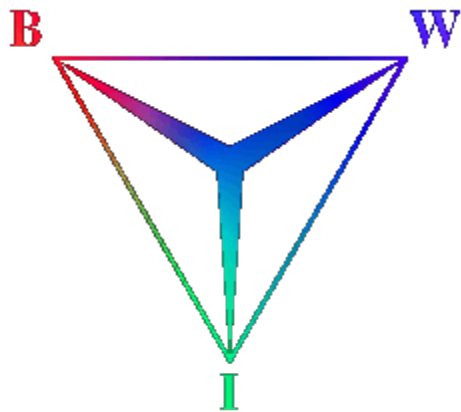


Uncertainty Analysis in Economic Evaluations

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Summary

Large companies like Shell often deal with big projects. The decision to invest is based on the evaluation of the project profitability. But how certain is the calculated profitability? What if the costs overrun during implementation of the project? What if the reservoir performance is less than estimated? And what if the project completion is delayed?

My focus will be on how to make people more aware of the risks and uncertainties in economic evaluations and to show the influence of these uncertainties on the economic indicators.

Economic evaluations in the oil industry are carried out with cash flow models. Traditionally, these evaluations are carried out with the estimated (most likely) set of parameters. Usually some parameters, such as project costs or reserves, are varied manually as ‘sensitivities’ to show the potential impact on profitability.

In this report, it is proposed to treat the uncertainties by defining stochastic parameters with carefully specified supports based on inputs from discipline experts. In this manner a better insight is gained in the distribution of the project profitability. Some of the key uncertainties in oil and gas investments have been investigated in detail.

Crystal Ball is one of the software packages used to quantify the impact of uncertainties through Monte Carlo Simulation¹. To use Crystal Ball for this purpose the following steps are required:

1. Build a discounted cash flow model of the project in a spreadsheet,
2. Identify the main uncertainties,
3. Define a realistic statistical distribution for these uncertainties that represent the full range of uncertainty (positive and negative),
4. Generate the distribution of the profitability indicators.

The distribution of the profitability indicators will then show the estimated likelihood that the project will meet the required profitability criteria.

Specific attention has been paid to the modelling of the following key uncertainties in typical oil and gas projects:

1. The oil price,
2. The investments (capex) and operating expenses (opex) for the project,
3. The number of wells and associated capex to recover the reserves,
4. The oil and gas reserves and production profiles,
5. The production start-up date,
6. The inflation rate.

¹ Forecasting & Risks analysis for spreadsheet users, 1988-1996

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Chapter 1 Introduction

This chapter will give a general description of the research of my thesis prepared at Shell. The research and the approach to quantify the risks and uncertainties will be the next step and finally the structure of the report will be described.

1.1 Research

For over 30 years Shell has used scenarios, rather than forecasts, to explore ways in which the future may behave. An important benefit of this approach with scenarios is that it makes us realise that the future is uncertain, that it may evolve in a number of different ways, and that decisions must be made within this context.

The idea of scenario thinking is the search for flexibility in our decisions. When just one path is adopted with no alternatives, any deviation between forecast and reality can mean a big loss. If the costs are higher in a certain year or the production is lower than estimated, things do not eventuate in the way we expected and the project may fail. Recognizing and managing possible negative scenarios should avoid this situation. It will allow us to think in advance about possible alternative scenarios and decisions to manage a negative impact on the project.

Thinking in terms of scenarios will help us to take better decisions (e.g. about field development concepts) that are robust against a range of scenarios.

For any oil and gas development project the decision to implement the project needs a clear view of the project's profitability and of the presented results. Cash flow models are often used to evaluate the profitability of a project.

Economists often enter values given by the domain specialists in the cash flow models and ignore the influence of the uncertainties that are hidden in the assumed values. The profitability of the whole project is mis-represented when the uncertainties in the values are ignored.

To examine the economic feasibility of the project there is a need to understand the elements of the project's evaluation. In the example project the key uncertainties of the cash flow model will be defined. The statistical approach will define these factors as stochastic variables that have statistical distributions. With the use of simulations and statistical distributions, the economic evaluations will say more about the risks and uncertainties of the project.

I have focussed on the uncertainties in the economic evaluations for an oil field. The economic evaluation is carried out with a discounted cash flow.

The contract terms are fixed once negotiations have been completed. Other elements contain a lot of uncertainty even after contract signature. This is especially the case for projects, where many uncertainties exist at present. The dependency between some elements needs to be considered, like the increasing costs when more wells need to be

drilled to recover the reserves. These risky elements are estimated as future values with ranges. The estimated values could have any values within the ranges.

1.2 Approach

With the knowledge of statistics and simulation by Crystal Ball the key uncertainties will be defined by stochastic variables, each having its own distribution. These distributions could have high variances depending on the extent of the uncertainty.

The approach will be outlined corresponding to the following steps:

- Getting into the Discounted Cash Flows and understanding its elements,
- Examining where the uncertainties are and what the key risk factors are,
- Defining stochastic variables, distributions and generating scenarios,
- Doing the analysis by varying of the parameters manually and by simulation,
- Analysing the results and making recommendations.

A clear overview can be achieved in this way about how big the risks are and how robust the model or the contract is. With such an overview people will be more aware of the uncertainties in the estimates and will know more about the risks they are facing.

1.3 Report structure

This thesis discusses the economic evaluations of an oil and gas field development. The economic evaluation is generated with a cash flow model. The project elements and the different scenarios are described. The design of the statistical model follows. With the help of simulations different analyses are done, after which the observations and results are reviewed.

Firstly, Chapter two discusses the several types of oil contracts that could be considered. Chapter three gives an introduction to the uncertainty analysis in cash flow models. The statistical work of the thesis starts in Chapter four, where the contract elements are explained and where some of them are treated as stochastic variables. The observations from the simulations and results are outlined in Chapter five. Based on the results, the conclusions and some recommendations are made in Chapter six.

For readers who are interested a basic knowledge of statistics will be outlined in Appendices I and II, where the main statistical definitions and probability distributions used in this thesis, are explained.

I hope you will enjoy reading this thesis.

Chapter 2 Types of Field Development Contracts

Governments and companies negotiate their interest in oil and gas fields in one of the two basic systems: concessionary and contractual². The fundamental difference between them comes from different attitudes towards the ownership of mineral resources.

- The concessionary systems allow private ownership of mineral resources, like in the United States. In this case the government transfers title of the minerals to a company if they are produced. The company is then subject to payment of royalties and taxes.
- Under the contractual systems the government retains ownership of minerals. Oil companies have the right to receive a share of the production or revenues from the sale of oil and gas. This is outlined in the so-called Production Sharing Contract (PSC), like in Indonesia, and in the Service Contract.

Host governments are usually represented by either a national oil company or an oil ministry or both. The term contractor is used to denote the international oil company operating the oil or gas field. The contractor then funds the required activities and is eventually reimbursed out of a dedicated share of the production plus a share of the remaining oil as a reward. In practical use the term contractor cuts across the boundaries between the PSCs and the concessionary systems.

The contractual systems are divided into two types of contracts: the production sharing contract and the service contract.

- Under the PSC the contractor receives compensation in kind (crude)
- The contractor receives compensation in cash (share of profits) under the service contract, which is also divided into a pure service contract and a risk service contract. The difference between the service contracts depends upon whether the fee is based upon a flat fee (pure), or a profit share (risk).

In pure service contracts the contractor carries out exploration and/or development work on behalf of the host country for a fee. All risk is borne by the state. This is sometimes the case in the Middle East where the state often has capital but seeks expertise and/or technology.

On the one hand many service agreements are identical to PSCs in all but the method of payment. The differences between the service agreements and the PSCs are modest. In practice PSCs effectively cover the whole contractual branch. In many cases a mix of contracts can be used, like in the Philippines, where the government alternates to his contractual arrangement as either a service contract or a PSC.

² International Petroleum Fiscal Systems and Production Sharing Contracts, 1994

On the other hand the two main systems, concessionary and contractual, generally are quite similar from a financial point of view. In many cases we see a PSC with royalties as well.

In Iran there is a special type of contract that has been defined and used; the Buy-Back contracts. Oil companies receive a share of the profit from the government after the start of production the minerals. This Buy-Back contract is a short-term contract since the company/contractor transfers the facilities to the National Oil Company and gets its share within a few years. The transfer of the facilities ends the contract immediately.

In our case of production of oil and gas in a field a long-term project has been assumed where the terms and calculations are based upon a PSC agreement. The reason for using this contract type is that PSCs work well in many countries.

The PSC agreement has been modelled as a Discounted Cash Flow, describing the project costs and revenues over forty years.

This and more will be explained in the next chapter.

Chapter 3 Discounted Cash Flows and Project Profitability

The proposed venture of implementing the project is basically seen as a ‘black box’, which initially absorbs shareholders’ investment funds and later generates money. Inside the black box the investments are turned into steel, concrete, facilities, etc.; oil sales revenues are received; and operating costs, royalties and taxes are paid. The forecast of the annual amounts of money generated is called the **cash flow** of the venture.

A company’s ability to add value is determined by its ability to generate future positive cash flows. Increasing value can be measured by Discounted Cash Flows (DCF). The DCF technique is used to determine the *Net Present Value (NPV)*³.

The Net Present Value (NPV) is a function of the project results in dollars, the discount rate and the time period. The Net Present Value must therefore be quoted with the discount rate and the reference date. The reference date is the date to which future amounts have been valued. It is the date to which the Present Value is related. So the NPV is the sum over the years of the project of its discounted cash flow. This represents the value of the project to the investor.

The profitability indicators result from “discounting” the cash flow. In this process the cash flow elements of later years are reduced by discount factors reflecting the time value of money.

In the cash flow calculations of the oil field project three different discount rates have been used; 0%, 10%, and 15%. According to the different discount rates three different series of calculations have been evaluated, each with the associated discount rate.

The model first calculates the *gross revenue* of a project from which royalties, costs and taxes should be subtracted.

The gross revenues are simply the outcome of the production of oil (in barrels) and gas (in standard cubic feet) times the oil and gas prices. After subtracting the royalties (not assumed in this case) the *net revenues* remain.

Before sharing of production, the contractor is allowed to *recover costs* out of revenues. Most PSCs will place a limit on cost recovery. In this case the cost recovery is limited to 40%. Revenues remaining after cost recovery are referred to as *profit oil* or profit gas, for which the contractor’s share of profit oil is assumed to be 20%. This contractor’s share of profit oil may be subject to taxation.

In cash flow terminology the calculations are made in two different types of money:

³ Shell Learning, 2001

- MOD (Money of the Day) is the amount of dollars handed over on the day when a purchase or payment is made. This amount increases with inflation.
- RT (Real terms) is an imaginary currency on the reference date, which never devalues with inflation.

The cash flow model takes into effect the influence of inflation on costs and revenues. The NPV is expressed in RT money to avoid that an “inflated” figure is shown that obscures the real profitability. Therefore, in this thesis only the RT values of the calculated elements will be shown.

The limitation of cost recovery, the percentage of profit oil, as well as the participation of other parties, belong to the terms of the contract. These terms have been fixed in the model since it is assumed that the contract is about to be signed. All the other elements like the (technical) costs, the future oil prices, the reserves, and inflation rate remain uncertain after contract signature.

It is the petroleum economist’s job to advise on the economic attractiveness of the opportunities, taking into account the many uncertainties regarding reservoir behaviour, development costs, future oil prices, and relationships with governments.

The accuracy of the information used for generating the cash flow varies considerably. In order to appreciate the effect of possible variations, a set of uncertainties will be defined, evaluated, and analysed. Typical examples are changes in:

- Oil prices,
- Oil reserves,
- Production behaviour,
- Capital expenditure,
- Operating expenses,
- Time of production start-up.

These uncertainties are contained in the project elements that are evaluated and analysed. The four main elements are:

- Oil price
- Costs
- Production Profiles
- Inflation

The next chapter, Chapter 4, will discuss the elements more in detail.

Chapter 4 Key Uncertainties

Study of sensitivities as mentioned in the previous chapter, reveals the project's vulnerability to parameter variations. Measures can then be taken to reduce its probability of occurrence. If, for instance, the economics are very sensitive to the reserves estimates, a suitably placed appraisal well could limit the risk. If the main risk is the capex increaser, special attention needs to be paid to estimating the costs of facilities or drilling and planning the project.

As said, there are uncertainties in the estimates of the technical and commercial elements. Technicians and economists use their expertise and experience to predict the future values. The challenge in this assignment is to refine these estimates while recognising the uncertainties.

There is no perfect way to predict how the **oil price** will behave during the life of the project. A high oil price results in higher revenues and a low one can lead to loss. It is very important to take a look at both possibilities and to avoid the unbalanced cases like an estimated oil price of \$50 or \$15 per barrel for all the years. The uncertainty of the oil prices needs to be considered with regard to the project profitability.

The reserves, and the number of wells required to produce these reserves, depend on many subsurface parameters, which cannot be measured accurately at reasonable costs. The (technical) **costs** depend on the number of wells drilled. When more wells have to be drilled, the costs will increase, which means that the net present value of the project would reduce.

The estimated number of wells and the associated costs are full of risk and the uncertainties should be taken into account.

Elements that are not less important are the **production profiles** and the **reserves**. The production profiles depend on the reserves and on the reservoir conditions. Do we know how much oil there is in the ground? Or how much reserves can be produced by each well?

When the reserves per well are overestimated, more wells have to be drilled to keep the estimated production profiles at the same level, which means higher costs. Since the reservoir, and the reserves per well, are hard to estimate it is normal to face uncertainties about the production profiles.

Also the **start-up date** of production has an impact on the project economics. Any delay will lead to delayed revenues and to negative cash flows in the delayed years. So are we sure that there is no delay?

The last element, the **inflation rate**, should not be forgotten. The uncertainty of the inflation rate is high as well. In periods of high inflation "recovered costs" (in Money of the Day) have less value than the actual costs when expressed in Real Terms. If oil price and investment costs inflate equally the effect on profitability can be small. But what if local costs escalate while oil prices stay constant?

In short the uncertainties in the element's values have a great impact on the model results and on the project's profitability in particular. Decision makers need to know how sure the results are and how to deal with deviations from the *base case*, which is the economical analysis based on the "most likely" assumptions.

Most assumptions in an economic evaluation are uncertain. The assumptions made in constructing the project cash flow come from the best available assessment of the technical and the economical parameters. But the reality will be different, and the impact of a realistic range of possible deviations from the base case should always be tested. Varying the parameters manually is a way to test the impact on the project results of possible deviations from the base case. This process is known as sensitivity analysis, which will be further explained in Chapter four.

The impact of the uncertainties in the economic evaluation of the oil and gas field development project will be analysed. To do the analysis we do need to discuss the selected uncertain parameters in more detail to create a better understanding of the analysis.

The parameters are divided into technical and economical parameters:

- The oil price depends on the density of the oil and thus on the quality of the oil produced in the project field. Different oil leads to different oil prices. Low oil density means light oil with a better quality. The cost to refine the oil then is low. This oil will be sold with low prices and vice versa. Since technical experts make the measurements of the oil quality, the parameter oil price is considered as a technical parameter.
- The costs estimated for the life duration of the project are made by the cost estimators who are technical engineers. Therefore the costs parameters are technical.
- The production profiles and the reserves are estimated by the domain experts, who are engineers from the required technical background. The production profiles and the reserves and of course the related start-up date are technical parameters too.
- The inflation rate does not depend on any technical parameter. If the project has higher or lower costs, produces more or less or even if the oil quality is different, it will not influence the inflation rate. This parameter is seen as a commercial/economical parameter.

All the parameters and their statistical description will be discussed in the next sections after which the analysis can be done.

Technical Variables

4.1 Oil Prices

The history of oil prices (Figure 1) has seen big fluctuations starting from the year 1974. The reason for the increase in 1974 was the powerful effect of the OPEC organisation on the oil prices for the first years after the setting up of OPEC in 1960.

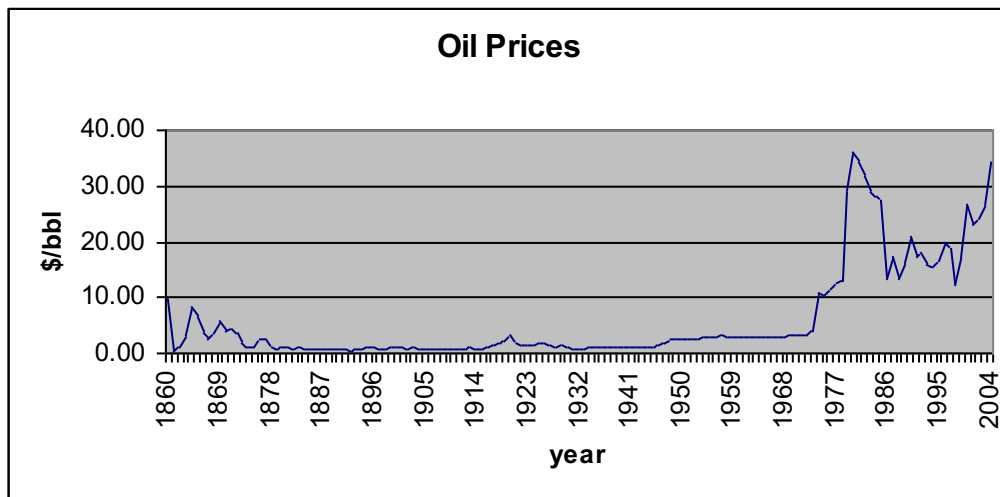


Figure 1 History of oil prices

In present years the oil prices have been high. Several factors have led to keep the oil prices high. The main reason is the continuous high demand for oil, especially from Asia. Moreover doubts about Saudi Arabia's spare capacity, further supply disruption in Nigeria, geopolitical uncertainties in the Middle East, and general worries about reserves have put an additional risk premium on the oil price. These factors are expected to continue supporting the oil prices to be above their fundamental level, which is seen by many analysts between \$20-\$30 per barrel.

The history of oil prices above shows how the oil prices have gone through big changes in the period between 1860 and 2004. The fluctuations in oil prices affect the economic growth. There are many ways in which the prices of oil do effect the economic growth. Some ways are:

- Higher oil prices lead to an increase of the general price level and thus inflation. With strong monetary policies the reaction to higher inflation will raise the interest rates. This has a dampening effect on the investments and consumption.
- The change in inflation affects also the value of the bonds and equities, which affects the exchange rates.
- Higher oil prices raise the cost of production of goods, putting pressure on profit margins/sales and thus raising prices of goods.

So changes in the oil price affect the economy. Historic fluctuations of the oil prices can be used as basis to predict future fluctuations. Fluctuations of oil prices will influence the economical results of any oil and gas project that is going to be executed in the future, since the expected project's revenues depend very strongly on the oil prices.

The history of the oil price behaviour points to the real need to be careful with the economic calculations and to take into account the different ways the future oil prices may behave. Through analysis of the history and paying attention to the forecast of the oil prices we may define the risks and uncertainties in several scenarios. In these scenarios we should consider the effect that the oil prices may have on the project profitability and the decision-making.

4.1.1 Analysis of historical oil prices

There are many methods of forecasting future data. One easy to understand method is the method of *exponential smoothing*⁴. The Exponential Smoothing method uses a weighted average of past and current values, adjusting weights to current values to account for the effects of fluctuations in the data. Using an alpha term (between 0 and 1), one can adjust the sensitivity of the smoothing effects. Exponential Smoothing is often used on large-scale statistical forecasting problems, because it is both robust and easy to apply.

Exponential Smoothing is a popular scheme to produce a smoothed time series. Whereas in other methods for forecasting methods, like Single Moving Averages, the past observations are weighted equally, Exponential Smoothing assigns exponentially decreasing weights as the observation get older.

In other words: recent observations are given relatively more weight in forecasts than the older observations. In the case of Moving Averages, the weights assigned to the observations are the same and are equal to $1/N$, where N is seen as the number of the observations. In Exponential Smoothing, however, there are one or more smoothing parameters to be determined (or estimated) and these choices determine the weights assigned to the observations.

In this thesis, based on historical data the future oil price will be estimated with Exponential Smoothing, since it is very easy in concept, easy to understand, and very powerful because of its weighting process.

The basic model for exponential smoothing is:

$S(0) = Y(0)$, initial value

$S(t+1) = \alpha Y(t) + (1-\alpha) S(t)$, where

$S(t+1)$ = estimated oil price value at period $t+1$

$S(t)$ = estimated oil price value in period t

⁴ Engineering Statistics Handbook

$Y(t)$ = actual oil price value at period t
 α (Alpha) = the smoothing constant

If alpha is set to 1, the forecast for the next period is based entirely on the actual value from the last period. If alpha is set to 0, the actual value from the last period is completely ignored. Since neither of these cases is close to reality to estimate the future data properly, the alpha should have a value somewhere between 0 and 1.

The objective is to determine smoothed oil prices and the alpha-smoothing constant while minimizing the error, which is in the base case seen as the differences between the actual oil prices and the estimated ones.

As an example, we have chosen an arbitrary α , say 0.5, and applied the exponential smoothing function on the data. Figure 2 illustrates the smoothed oil prices with the chosen alpha.

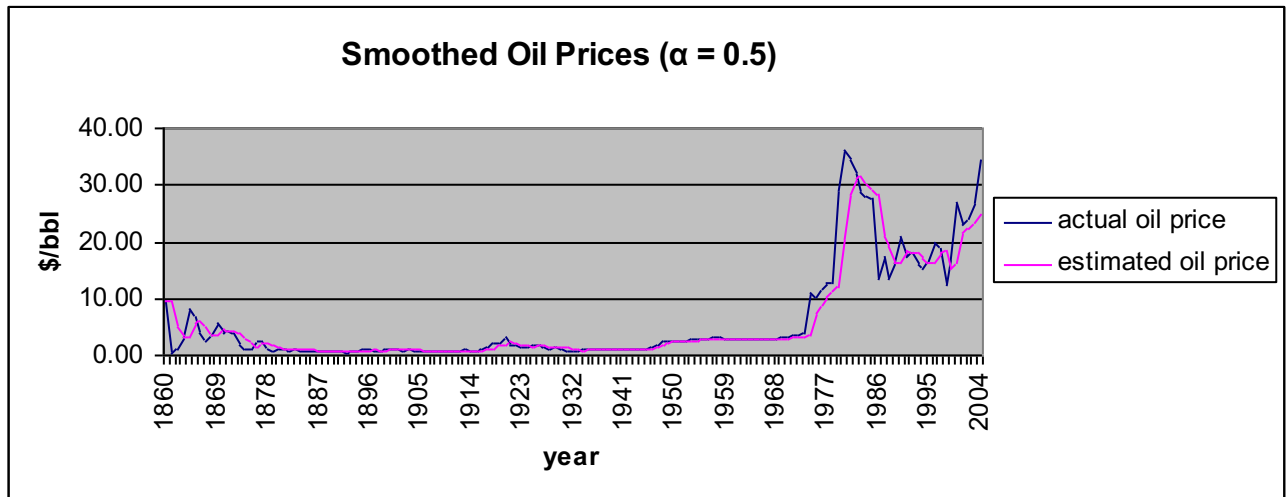


Figure 2 Smoothed oil prices (alpha = 0.5)

Selecting an optional value for α may affect the smoothing of the data. There are several ways to choose the best value for alpha to predict with, like the mean square error method, the mean absolute error method, the percentage error and so on. Gardner (1985) discusses various theoretical and empirical arguments for selecting an appropriate smoothing parameter α .

The most widely used method to choose the alpha is to minimize the mean squared error (MSE). The parameter α that minimizes the squared differences between the actual and the estimated oil prices at time period t seems to be the best α to predict with.

There is an option in Excel⁵, called the Solver that minimizes the MSE and optimises α . For this historic oil data the Solver has chosen $\alpha = 1$ as the best α . This means that

⁵ Microsoft Excel 2000 Bible, Gold Edition

the estimation is based on the actual oil price from the last period only, ignoring the past totally.

However, if we would use $\alpha = 1$ to forecast future oil prices, the future oil price would stay constant forever. This is not the pattern that has been observed historically, and does not represent a reasonable scenario of the future oil prices.

The observations have always to be considered when the estimations have to be done. But we should notice that in general more attention is paid to the last few years than to the older observations.

Therefore it seems reasonable to choose a value for α , that generates a more realistic pattern for the oil prices. This is surely possible since exponential smoothing and its parameter α are very flexible and the weighting and smoothing of the data is up to the user.

The idea is to define the positive dependency or the so-called *positive correlation* between the annual oil prices into the exponential smoothing method. This correlation is the parameter alpha, which we are looking for. Generally the correlation is best represented by the range $[-1, 1]$. When the correlation coefficient is closer to -1 , we speak about a negative correlation, whilst a coefficient value near to $+1$, means a positive correlation.

The history of the oil prices suggests a degree of the dependency between the annual oil prices. When the oil price of an arbitrary year raises, the oil price of the year after will increase too but by a smaller amount. Since the raise of the oil price in the following year is as nearly high as the oil price increase in that arbitrary year, we then speak about a high positive correlation.

Considering this high positive correlation, after some trials an alpha of 0.7 has been selected. This gives a reasonable weight to the last years as well to the earlier years. The value 0.7 describes the moderated/high positive dependency between the annual oil prices.

The new smoothed oil prices are plotted in Figure 3.

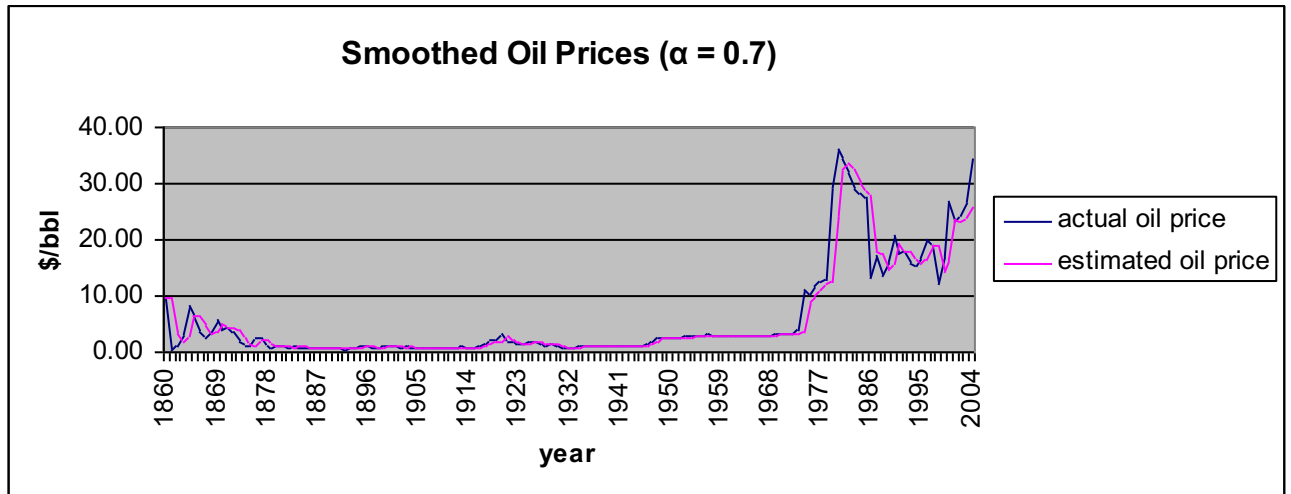


Figure 3 Smoothed oil prices (alpha = 0.7)

Figure 3 shows that the smoothed oil prices are close to the original observations. This shows the reasonable choice of the alpha value of 0.7, which can be used to generate the future oil prices. This will be described in the next section.

4.1.2 Generating future oil price scenarios

Based on historical oil prices, several scenarios of the future oil prices can be generated.

If the forecast of the oil price for the next year can be directly expressed from previous forecasts and previous observations, then the model for forecasting should be:

$$\hat{Y}(t+1) = S(t+1), \text{ where}$$

$\hat{Y}(t+1)$ = forecasted oil price value at period t+1

$S(t+1)$ = estimated oil price value at period t+1

The forecasted oil price in period t+n, $\hat{Y}(t+n)$, would have after some future calculations the same value for the oil prices as the estimated price at initial period t+1, $S(t+1)$, since at the moment observations cannot be made for the future $Y(t+1)$. The $S(t+1)$ then becomes the same as $S(t)$ (this follows from the equations in Section 4.1.1). Since the estimated value at period t+1 is the same as the forecasted value at period t+1 (this follows from the equation above), every forecasted oil price will be the same as $S(t)$. This means a fixed oil price for the next 40 years when the project will be implemented!

As mentioned before, history shows that oil prices fluctuate much more on a year-to-year basis. A fixed oil price does not represent a realistic scenario for the future. Therefore a degree of uncertainty has been introduced by adding a random variation

to the predicted oil price. The random variation has been expressed as the so-called *white noise*⁶.

The white noise consists of a sequence of mutually independent random variables... $Z(-1), Z(0), Z(1), \dots$, each with *mean*, μ , zero and a finite variance σ^2 . Because of the independence of its *random variables* white noise itself is not that interesting as a model for time series (annual oil prices). However, it is often used as a building block to construct series of which variables are dependent. Such series are more interesting for practical purposes since with one of them as a model the future can to a certain extent be predicted from the past.

The objective now is to define the white noise distribution and its *variance* σ^2 . A *Normal distribution* has been selected to represent the range of the oil prices fluctuations. This distribution is symmetric around the *mean*. So the oil price values will be equally distributed around the estimated *mean* and thus higher or lower than expected.

There are several ways to estimate the variance. The one we have chosen is to take the variance over periods of x years and repeat until the last years are reached.

A look at the historic data could help to decide how many years should be taken to determine the variance. The last 145 years of oil production can be divided into 15 periods of 10 years, or 16 periods of 9 years etc. However, to determine the variance ranges of years should be considered that include the jump from low to high price periods. The variance has been taken over periods of 10 years until the last year is reached. The several calculations lead to a *standard deviation* (the square root of the variance), σ , of around 5, which seems to be quite representative for uncertainties over the expected oil prices when the *mean* in recent years is about \$30 per barrel. This standard deviation allows the future oil price to have realistic minimum and maximum values around the mean oil price.

Exponential smoothing constructed with the defined white noise will generate future oil price scenarios. In mathematical terms:

$$\hat{Y}(t+1) = S(t+1) + Z(t+1), \text{ where}$$

$\hat{Y}(t+1)$ = forecasted oil price in period t+1

$S(t+1)$ = estimated oil price at period t+1

$Z(t+1)$ = white noise process with $\mu = 0$ and $\sigma = 5.2$

The scenarios have been generated with the simulation tool Crystal Ball. During a run of 1000 trials, each trial chooses a random value for the white noise. So every trial gives a new oil price value. Figure 4 and Figure 5 show two scenarios of the thousands of runs.

⁶ Statistical Models, 1997

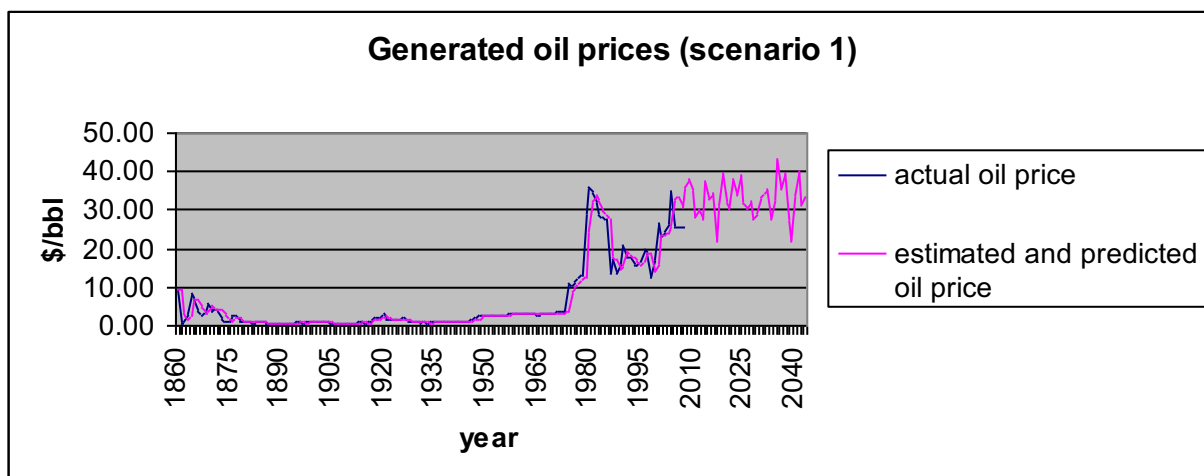


Figure 4 The first future oil prices scenario

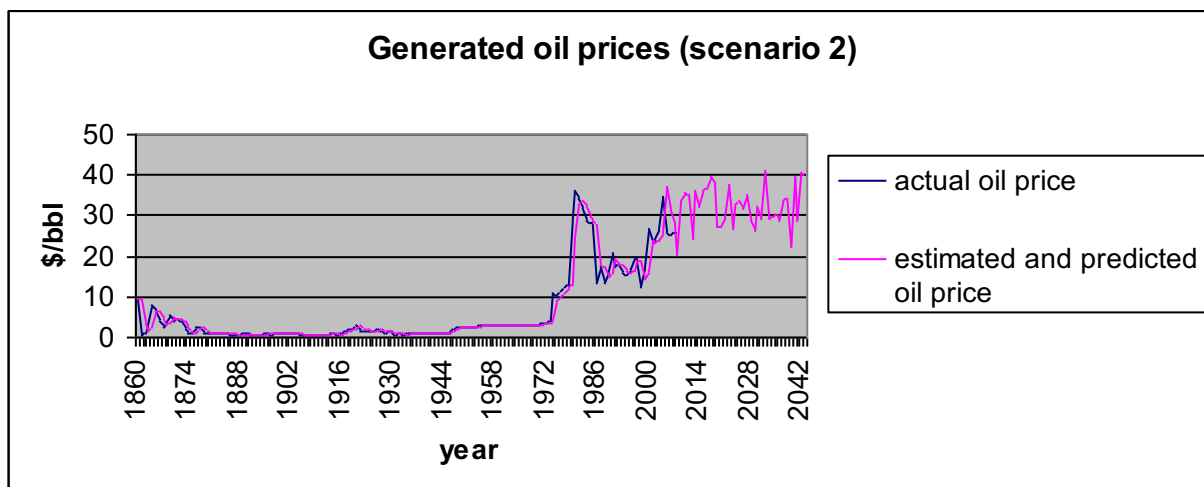


Figure 5 The second future oil price scenario

The generated oil prices represent a realistic behaviour of the oil prices with the normal life cycle. Usually the increase in oil price leads to more investments, more wells drilled and thus higher oil production. High production of oil and stable demand will lead in short term to lower prices again. This effect can be recognised in the generated scenarios.

The generated stochastic oil prices will be used in our model and every time a simulation is run, the scenarios will be generated. In this way the uncertainty in the oil price is introduced in the project's evaluation.

4.2 Costs

Project costs represent how much is going to be spent during the construction and the implementation phase of the project. The project results depend very strongly on the magnitude of the costs. If the costs are higher than estimated, the project's profit will be less than expected. In some cases higher costs can lead to a big loss, especially when the profit is low or in case the project's revenue is very dependent on the amount of the costs.

Therefore decision-making should take into account the risks and the uncertainties of the costs since we would never know exactly what the costs would be, e.g., we would never know if the facilities cost more or if more wells are going to be drilled. A range of possible costs values should be defined and analysed.

Firstly, the cost elements will be discussed to develop an understanding of their importance. The chosen distributions and parameters will follow.

Costs are split up into two categories:

- **Opex** (operation cost) which is divided into fixed and variable costs,
- **Capex** (capital cost) which is split up into exploration and production costs.

The Opex and the Capex again are divided into several components.

Opex

| | |
|-----------------------|-----------|
| Pre_operation_costs | MM\$RT |
| Opex_fixed | MM\$RT |
| Opex_variable | \$/boe RT |
| Transportation_Tariff | \$/bbl RT |

Exploration Capex

| | |
|----------------|--------|
| Exploration_RT | MM\$RT |
| Appraisal_RT | MM\$RT |

Production CAPEX

| | |
|---------------|--------|
| Facilities_RT | MM\$RT |
| Drilling_RT | MM\$RT |

The units in the table are defined as:

MM\$RT = million dollar in real terms
\$/boe RT = dollar per barrels of oil equivalent
\$/bbl RT = dollar per barrel in real terms

The expected cost elements of the project are estimated like the other parameters in our model. Since we do not exactly know how many wells should be drilled during

the implementation, significant uncertainty may exist about the precision and the certainty of the estimations.

If the reservoir behaviour is overestimated and production should go up, more wells cannot be avoided. We also do not know how much it will cost in the pre-operation phase and how much the facilities exactly will cost.

The deterministic estimation of the elements is not useful in this case and a range of capex *overrun* and *underruns* should be added. There is an overrun when the real value of the costs is higher than the estimated number. And when the real outcome of the costs is lower than the estimated value there is an underrun.

We have assumed a factor in an overrun of about 40% and an underrun of 25% in the cost estimations of the capex. This means that the capital cost estimations may be 40% higher than expected and 25% lower than expected. These factors are dependent on the stage of the project and can be higher or lower.

The capex is more important than the opex since the latter is a fraction of the capex. A typical profile for the opex has been plotted in Figure 6.

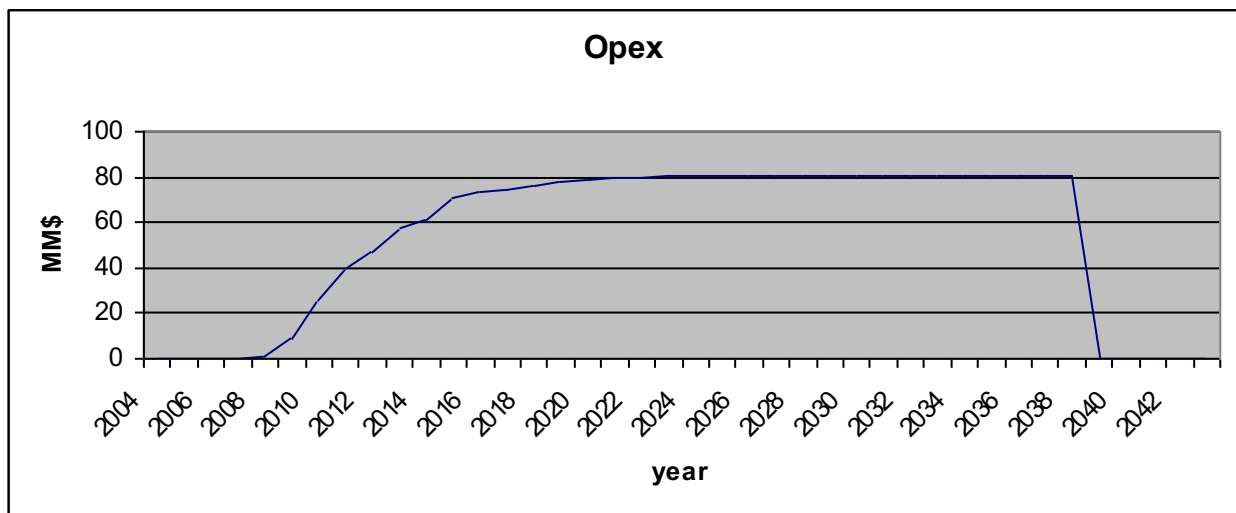


Figure 6 The opex profile in the oil and gas field development project

Pre-operation costs are incurred before starting with production whereas the fixed and variable costs start with production.

Capex and especially the production capex are the main costs elements. The total capex profile is plotted in Figure 7.

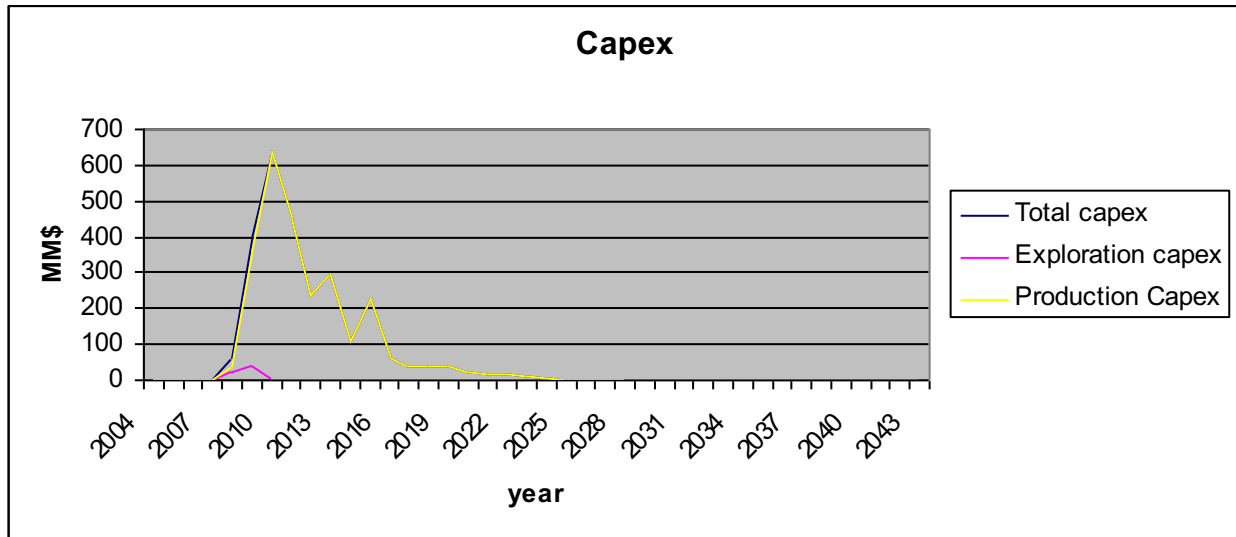


Figure 7 The capex profile in the field development project

The real capital costs can be much higher than estimated. In onshore fields the drilling costs are the largest costs element. It costs approximately \$5 million dollar to drill one well. The drilling costs can be incurred during the life cycle of the project since at any point in time the need to drill more wells may occur.

Therefore the drilling costs represent an important technical parameter. The production capex profile is plotted in Figure 8.

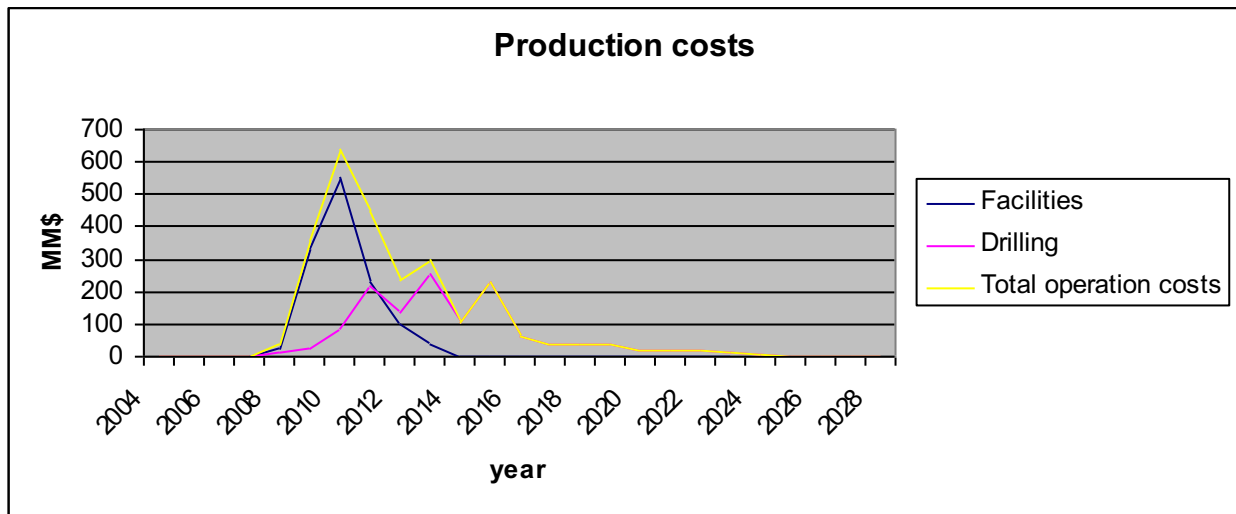


Figure 8 The production capex profile

Drilling costs depend on the number of wells that should be drilled. The number of wells needed depends on how good the estimates are about the production profiles and the reservoir (see Chapter 5.3). There are big uncertainties about the wells that should be drilled to reach the estimated production profiles. The forecasts for the production in the end of the field life are very uncertain due to information about how much oil there exactly is in the reservoir and how the reservoir will behave in the long term.

4.2.1 Defining the distributions

The annual costs will be defined as stochastic variables. Each cost element will have a defined series of time variables. Every year the different cost elements will be assigned to the cost variables defined with their own distribution. Depending on how the future cost elements may behave, the distributions and the parameters will be assigned.

Exploration capex

- Exploration and appraisal costs have been given a range between 140% and 75%. This corresponds to 40% more than the expected value and 25% less. A *Triangular Distribution* will represent the distribution of future exploration costs.

Production capex

- The costs of facilities may behave the same as the exploration capex. These costs are also assigned to a range between 140% and 75%.
- Future drilling costs behave according to a *Normal Distribution* with a *variance* of 20% from the expected mean. Thus the number of wells that is going to be drilled during the execution of the project can vary by 20% from the expected number of wells, which is seen as 100%.

Opex

- Operation costs are estimated as a percentage of the capital costs. A *normal distribution* represents the risks and uncertainties of the opex with a *variance* of +/-10%.

4.2.2 Generating future costs scenarios

All the defined cost elements have been used as assumptions in our Crystal Ball model. Using Crystal Ball, the simulations have been run. Several scenarios of the future costs can be generated. The focus will be on the costs of facilities and drilling since these costs are the largest and therefore their influence may affect the project results significantly.

Some of the scenarios generated by Crystal Ball within the defined ranges are plotted below. The first plot (Figure 9) shows the different ways the costs of facilities may behave. The second plot (Figure 10) shows the possible future behaviour of the drilling costs.

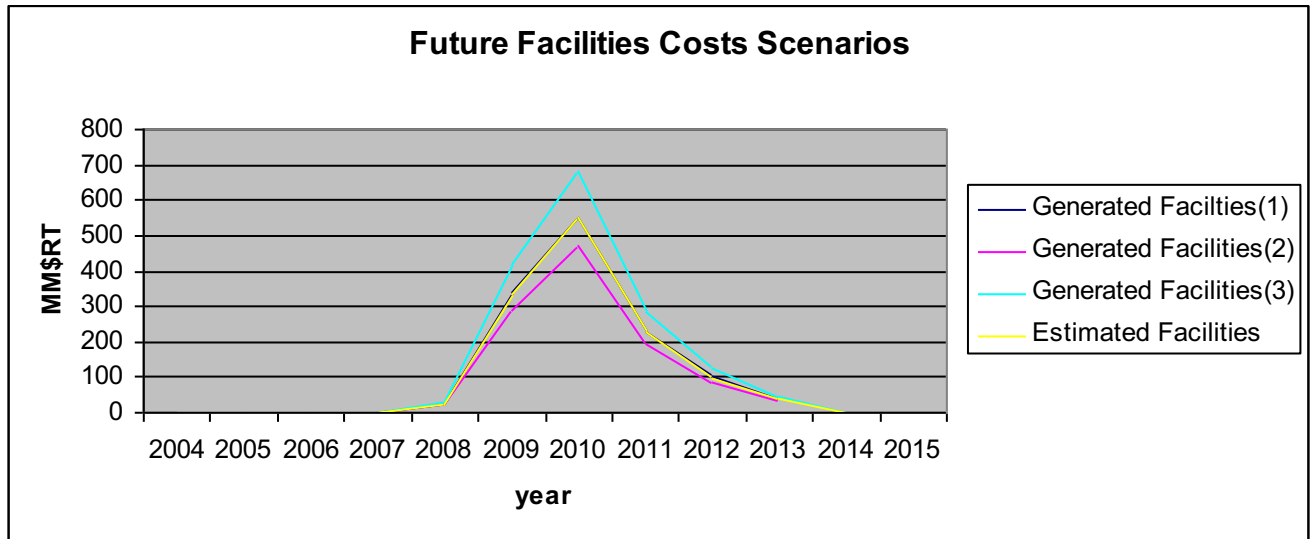


Figure 9 Three different future facility costs scenarios

The three generated scenarios show three different possible outcomes of the costs of facilities. The second scenario generated almost the same cost estimations as the original estimated facilities costs. Nevertheless, the future costs might be a lot higher (3rd scenario) or much lower (2nd scenario), whereas in the low cost case the project profitability is higher and a certain profit is then booked. There are no facility costs after 2013, which is three years after the production starts.

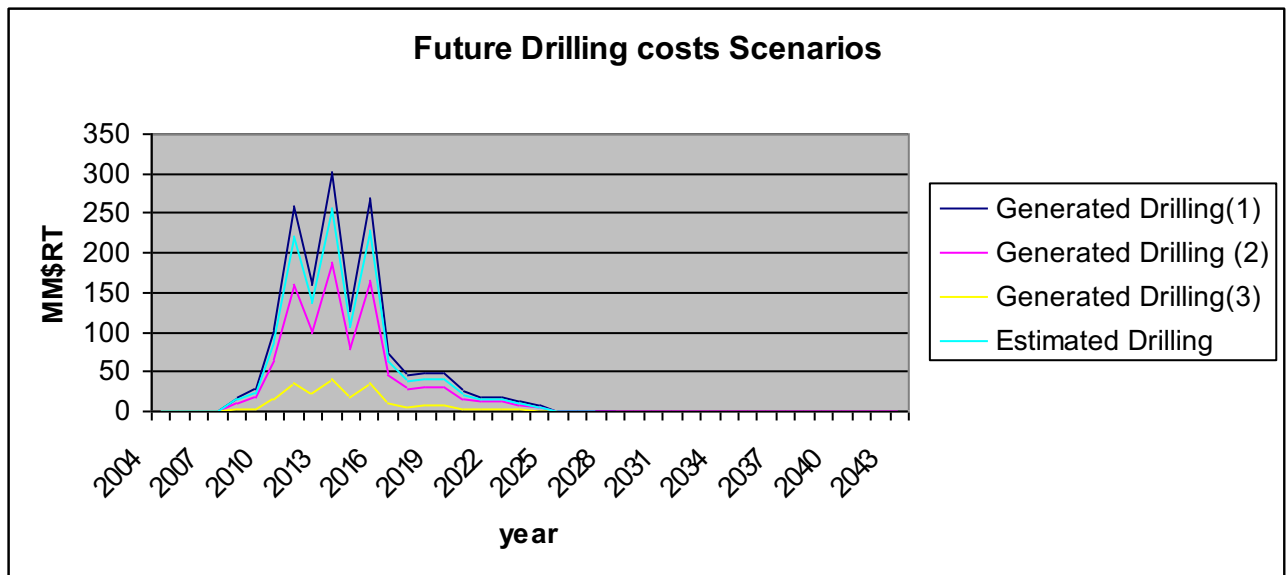


Figure 10 Three different scenarios for future drilling costs

The main costs are composed of drilling costs. Cumulative drilling costs are very high and are incurred over a long period. The costs of drilling start two years before first production and are incurred till 2028, over the course of production.

The different costs scenarios show the different ways the future costs may behave. Lower costs are profitable, but at the same time higher costs have negative consequences on the project results. Since we do not know how much the costs exactly are, we should be aware of the uncertainty in the cost estimates. Therefore ranges of costs value possibilities have been defined. The ranges generate all the possible scenarios to be taken into account in the decision-making.

4.3 Production Profiles and Reserves

The forecasting of production profiles is an iterative process, in which information gained from appraisal wells and from actual production is continuously used by the reservoir engineers to update their previous view. The confidence level of the forecast will therefore be fairly low in the early stage of the venture and will be higher after the field has been producing for a few years.

Key factors influencing the production profile include the amount of oil (or gas) in place, the drive mechanism, the fluid properties, and the initial rate of wells.

The production of oil and gas is assumed to start in 2010. After a build up period of 4 years the plateau of 250 thousands barrels per day will be reached. The plateau lasts four years, from 2014 till 2017. After the plateau the field goes into decline. The production declines gradually and finally stops in 2039. In this thesis the focus is on the production of oil, since the gas has to be transferred to the government for free and does not impact the economic evaluations.

The production profile of oil is estimated over the duration of this field. The estimation for the base case is plotted in Figure 11.

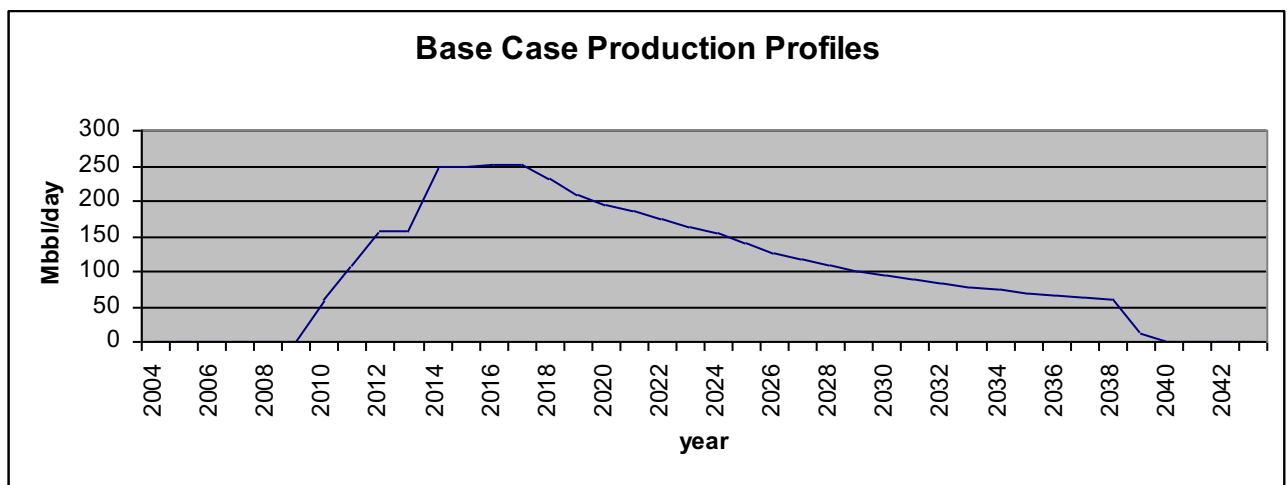


Figure 11 The base case production profiles of the oil and gas field development project

Figure 11 shows the three phases in the production profile:

- Build up period:
Newly drilled producers are brought successively on stream.
- Plateau period:
Production is more or less stable. The last producing wells are brought in; the older ones begin their decline.

- Decline period:
In this final period all producers show declining production and may eventually “water out” or “gas out”.

The start up date for production can be highly uncertain. It depends on many factors: any delay with the installation or the drilling can postpone the production start date/year. When the delay happens and the start of the production is postponed, the estimated production profile for the second year of production will be postponed as well.

It has been assumed that a delayed start-up will also effect the build up in the next years. Between the first two years of production there is a total dependence of 100%.

Figure 11 shows that four years after the start of production the plateau of a level of approximately 250.000 barrels per year will be achieved. The plateau will stay stable for four years after which the production will start to decline and finally stops in 2039 (past the project end date of 2031).

There is a positive correlation⁷ among the annual production profiles. The history of many oil and gas field developments shows a certain dependency among the annual production profiles. When the production is higher than expected in a certain year, the next year is likely to have more production as well. This is the so-called positive correlation between the years. The correlation does not mean a total dependency of 100% of course. The correlation must be less than 1. At the same time the correlation clearly is higher than 0.5. Therefore the domain experts assume a correlation rate of 0.7, which is not too high but strong enough to emphasize the correlation.

The produced reserves are directly related to the production profile. Subtracting the produced reserves from the initial reserves gives the remaining reserves. The total amount of oil and gas is fixed in the field. The challenge is to get the most out of it. How much oil and gas is there? How much are we going to produce? Is there any delay? Do we drill enough wells? And so on ...

In the first years of plateau production, the estimated values are assumed to be more certain than the build-up period. Further into the future, estimates of the production profiles become more uncertain. The uncertainties increase with the length of the forecast period.

The estimated production profiles are generated from the expected capacity of the reservoirs. If these expectations are higher or lower than the actual capacity, the project’s evaluation will be different. During the development and production of the field it becomes clearer what the capacities are and how much of the reserves will remain. Then one can get a better view about the possibilities to produce more or for longer years with the same wells. It also becomes clearer whether the production performance gets better if we drill more wells.

⁷ Inleiding in de Waarschijnlijkheidsrekening, 1998

The plateau may also behave differently from the estimated profile. The estimations show some certainties over the profiles of the plateau but the start of decline is not certain. The plateau may last longer which will lead to higher project profitability, and vice versa.

Drilling more wells in the future may lead to higher production in the period of decline. In the same way, it is unknown how fast the decline will be.

So risks and uncertainties over the reservoir and the drilling of wells lead to uncertainties over the production profiles of oil.

All the mentioned risks and the dependencies of the production profiles on the reserves are taken into account in the process of choosing the distributions for the production profiles.

4.3.1 Defining the distributions

The annual production profiles will be defined as stochastic variables. The estimated profiles are then the expected values. Based on the explained risks and uncertainties in the estimations these expected values will vary within a range, with a maximum and a minimum value.

Domain experts tell us that during the implementation of the project, the expected profiles may be higher or lower than expected. There is no reason to use a normal distribution since the deviation from the expected profiles shows no symmetry. Experts assume a *triangular distribution*, which represent uncertainties as best as possible. The defined ranges of minima and maxima of the production profile distributions will then allow the estimations to include the uncertainties.

Every single year can vary between different ranges concerning the different sensitivities and uncertainties in the production profiles. The ranges wherein the production profiles could vary are defined as follows:

Build up

- The first year of production may have a range between 0% and 120%. This corresponds to 20% more than the expected most likely value and no production (delay).
- In the second year, production can also be increased by 20% more than expected but the estimated value may decrease to 50% and not to 0% as for the start year. When production starts in the first year it will be unlikely to stop producing at once in the second year.
- The third and fourth years, the years before reaching the plateau, show less sensitivity since production already has started. The production profiles may be up to 30% less and 20% more than estimated.

Plateau

- The first year of reaching the plateau is quite stable. The estimation may vary between 25% sooner and 10% later.
- The uncertainty over production increases further into the future as mentioned before. The range may vary between 30% less and also 10% more.
- The likelihood of producing less than expected is higher. Therefore the production profile may be 35% less than expected but unlikely to be more than 10 % higher.
- There is greater risk in the last year of plateau. The question is whether the decline already started or if the plateau production lasts longer. The minimum is assumed to be 40% less than expected, while production could increase to 110% like the other plateau years.

Decline

- The first 12 years of decline show the same behaviour. The estimation of the production profiles is very difficult. It is hard to predict the rate of decline. Generally, the estimate can fluctuate within a range of 30% more and 30% less production.
- The further the project progresses, the bigger the uncertainty of the estimates are. The years from 2030 to 2038, the last years before the end of production show large uncertainties with a range of 40% more and 40% less than expected. Again the production profiles depends on the decline rate, the size of the reservoir, and on the number of wells that will be drilled.
- The last years of the project life are difficult to estimate and the challenge is to keep on producing. The values estimated could therefore be 50% higher and 50% lower than the expected production profiles.

4.3.2 Generating future production profiles scenarios

This classification of the annual production profiles helps to generate several scenarios. These scenarios represent the way the production can behave during the implementation of the project. Each year may behave differently from the other, moving between the associated ranges that are pre-defined. Simulations runs sample 1000 times and generate several scenarios of the production profiles (Figure 12). This is very useful when one would like to see the way the production profiles may look like keeping in mind that different outcomes could have a big impact on the project revenues.

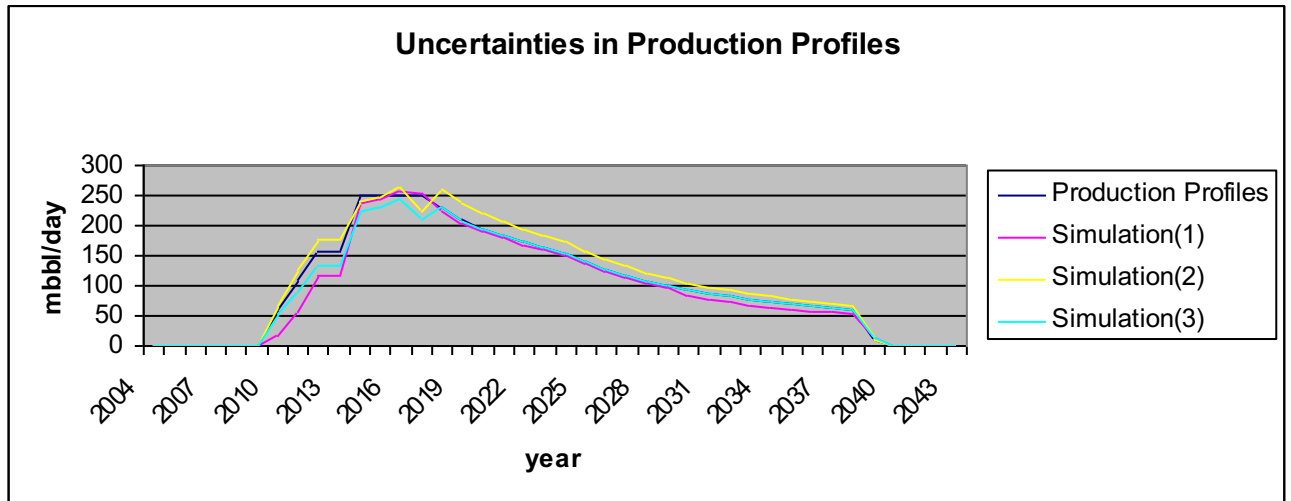


Figure 12 Three generated future production profiles

Figure 12 shows the differences between the deterministic method of calculating the production profiles and the simulated method that consists of different scenarios. It shows, for example, that the production in the first years may be less than expected, or that, plateau production is not necessarily constant. Decline can set in later than estimated. The decline can also be managed with more production every year, which means higher profitability.

The scenarios shown above are a few of the thousand that can be generated in every run. All the different scenarios will be included in the calculations to generate the project's results.

Economical Variables

4.4 Inflation Rate

Inflation is a general rise in prices across the economy. This is distinct from a rise in the price of a particular good or service. Individual prices can rise and fall all the time in a market economy, due to consumer demand. For example, when the price of a particular model of car increases because of high demand, this is not seen as inflation. Inflation occurs when most prices rise by some degree across the whole economy.

At the same time any individual price change could cause the measured rate of inflations to change, particularly if it is large or if the item has a significant weight in the *price index*. A price index is made up of the prices of hundred of goods and services that consumers buy. Prices are sampled across the country every month; in supermarkets, petrol stations, travel agents, insurance companies, and many other places. All these prices are combined together to produce an overall index of prices. The goods and services included in the index are chosen and weighted on the basis of the spending patterns of households.

The oil price has a significant weight in the price index. Therefore a large rise in the oil price, which is quite imaginable, might affect the overall rate of inflation. But unless this price continues to rise, the annual inflation rate would eventually fall back again.

If petrol prices had been 50 cents a litre in February 2004 and increased to \$1 dollar a litre a year later while no other price changed, the annual rate of consumer price index (CPI) would increase. If petrol prices remained unchanged after that, the annual rate of inflation would then fall back by February 2006. That is because the annual rate of inflation in February 2006 measures the change in prices between February 2005 and February 2006, during which time the price in our example has stayed the same at \$1 a litre. So, although the price of petrol remains at the higher level, annual inflation is not higher after a year or more.

The *Inflation rate* is a measure of the average change in prices across the economy over a specified period, most commonly the annual rate of inflation, which is also used in our model. If, say, the annual inflation rate for this year were 2%, then prices overall would be 2% higher than last year. So a typical basket for goods and services costing \$100 last year would cost \$102 this year.

If the value of the euro or dollar falls against other currencies the price of imported goods might rise. But only if the exchange rate keeps falling, this influence on the inflation rate will continue.

So the inflation rate may fluctuate when the prices of goods rise and fall or if the exchange rate changes over time.

In most operating companies, cash flows are constructed in local currency since that is the currency in which taxes are calculated and paid. Payments or receipts in other currencies must be converted to local currency. The final cash flow, however, is presented in RT US dollars. All cash flow elements should first be calculated explicitly in MOD of the currencies in which they are paid, and then converted to MOD local currency using the historic/forecast exchange rate. The resulting MOD local-currency cash flow is then translated into an explicit MOD US dollars cash flow, which in turn will be deflated to RT dollars using a US deflator.

The cash flow model we consider has been established in MOD. The calculations are made in both the local currency and the operating units functional currency. These calculations are then converted to MOD US Dollars using the appropriate exchange rate.

Exchange rates can fluctuate significantly over time. It will be adequate to use an average long term, constant MOD exchange rate, which is determined by considering historical rates, forward market rates, etc. Hence, constant MOD exchange rates imply that local inflation rates are equal to 2% inflation.

However such rates for a model are expected to deviate significantly. The impact of possible deviations should be shown as sensitivities, since the fluctuations can have significant impact on the project's revenues.

In an oil and gas development, movements in the local exchange rate are likely to have an impact on the economic outcome of the project, especially through prices of materials and equipment during the constructing phase or through operating costs and revenues in local currencies during the operating phase.

Exchange rate movements may on the one hand influence the project's cash flow directly. When the local currency falls, costs will become higher since we must pay more for the same imported goods. In this particular case the risks and uncertainties over the costs are addressed by defining several possible values and ranges for the cost parameters (see Section 4.2.1).

On the other hand the exchange rate movements may also influence the economic results indirectly. Fluctuations in operating costs and the oil prices, for example, will lead to fluctuations in the inflation rate. By contrast, fluctuations in costs but with stable oil prices will not necessarily lead to inflation movements.

The economic results of our project have been converted from MOD to RT assuming a constant deflator of an inflation rate of 2% per year. Since the deflator is constant, the fluctuations in the exchange rate might have no effect on the project's results. But in contrast to the other economic indicators the oil prices are calculated in MOD only. If the oil prices were calculated in RT there would be no need to use an inflation rate since the uncertainties over fluctuations in the inflation already would have been taken into account via the other parameters that depend on the inflation rate.

But since the oil prices are in MOD and significant volatility in exchange rates and inflation is expected, the impact on the project economics should be evaluated for a realistic range of possible inflation rates.

High inflation tends to be more variable and uncertain. Many of the costs of inflation are associated with its uncertainty. That is why price stability is very important because high and volatile inflation creates additional future uncertainties. Since there is no price stability in at the moment, such risks and uncertainties should be considered.

The cost estimation of the oil and gas field development depends on the local inflation and currency on the one hand and on the other hand on the operation unit's inflation and currency as explained before.

Considering the possible influence of foreign exchange and inflations rates on the costs of any project, it is worthwhile to show the inflation rate behaviour of other important countries like the United States and the European Union (see Figures 13, 14 and 15).

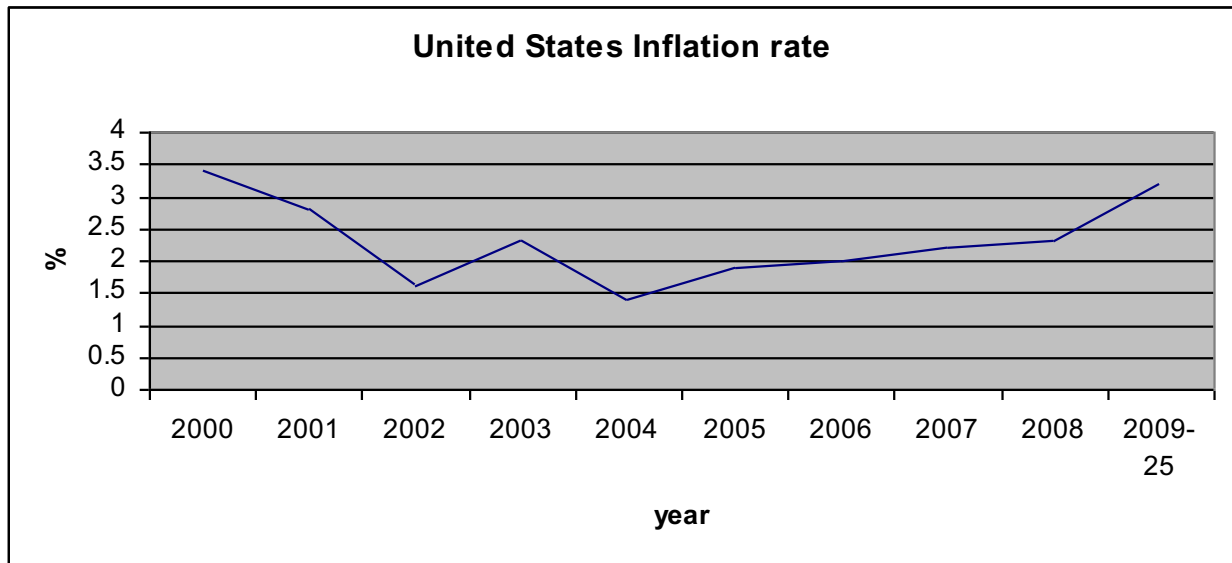


Figure 13 The course of the inflation rates in the US

We can observe big uncertainties in the United States' inflation rate behaviour. Figures 14 and 15 show more stability in the European economy.

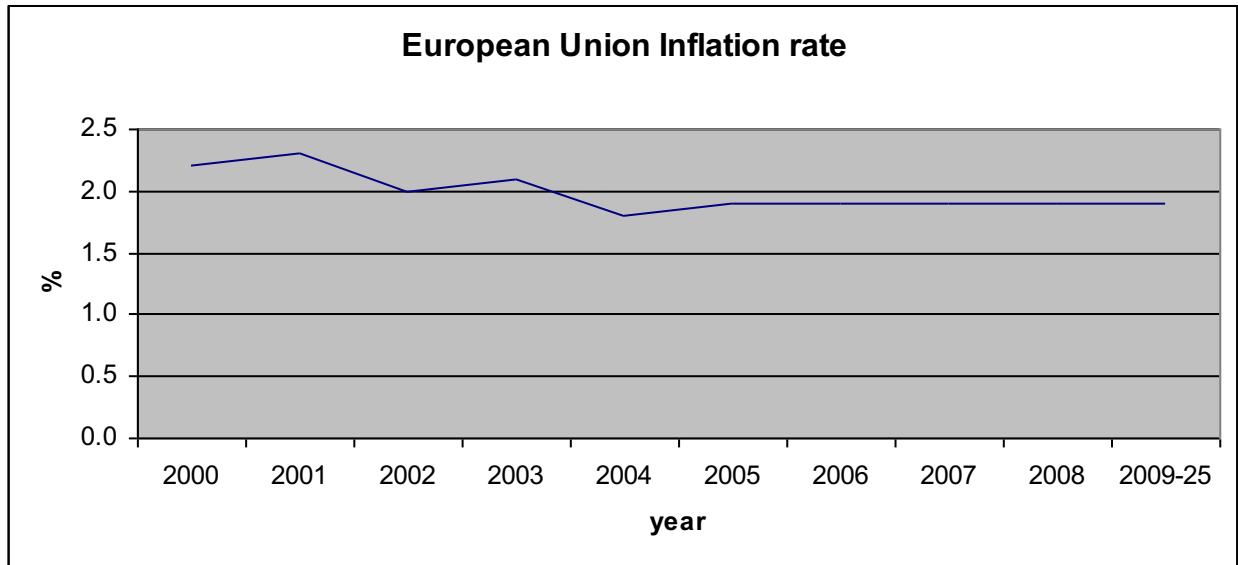


Figure 14 The course of the inflation rates in the EU

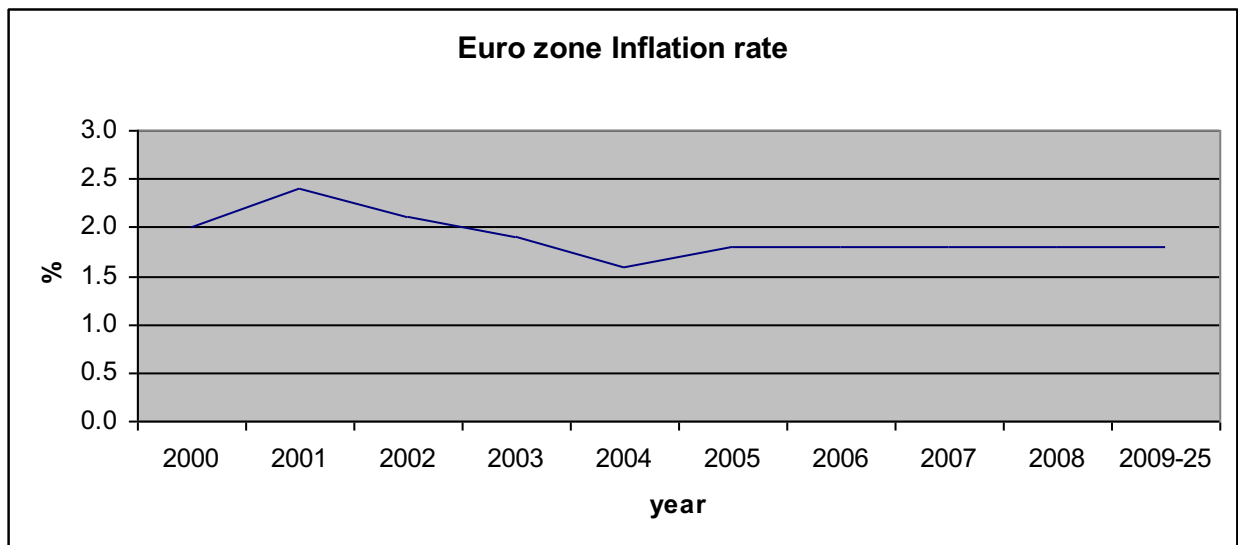


Figure 15 The course of the inflation rates in the Euro zone

4.4.1 Defining the distributions

Considering the different behaviour of the inflation rates and the uncertainties about the mix of currencies used, it seems appropriate to define a range of possibilities for the inflation rate in our model.

Based on the chosen inflation rate of 2% as the most likely rate, we should define a range of a maximum and a minimum value. On the one hand the local inflation rate could be 7%. On the other hand the European inflation rate is expected to be around 2% and the US inflation rate around 3%. Since the inflation rate used for the project is

a mix of the local and the operation unit's rates, it is reasonable to assume a maximum range of 3.5%.

The figures above show that the inflation rates could have a minimal value of 1.5%. This percentage is also used in our project since it represents the lowest minimum inflation rate a project may have.

The inflation rate is expected to vary between 1,5% and 3,5%, most likely 2%. We choose the *triangular distribution* because this distribution describes a situation where one can estimate the minimum, maximum, and most likely values to occur.

4.4.2 Generating future inflation rates scenarios

Like what we did before to generate future scenarios, simulations runs sample 1000 times and generate several scenarios of the inflation rate. One of the many possible scenarios is shown in Figure 16.

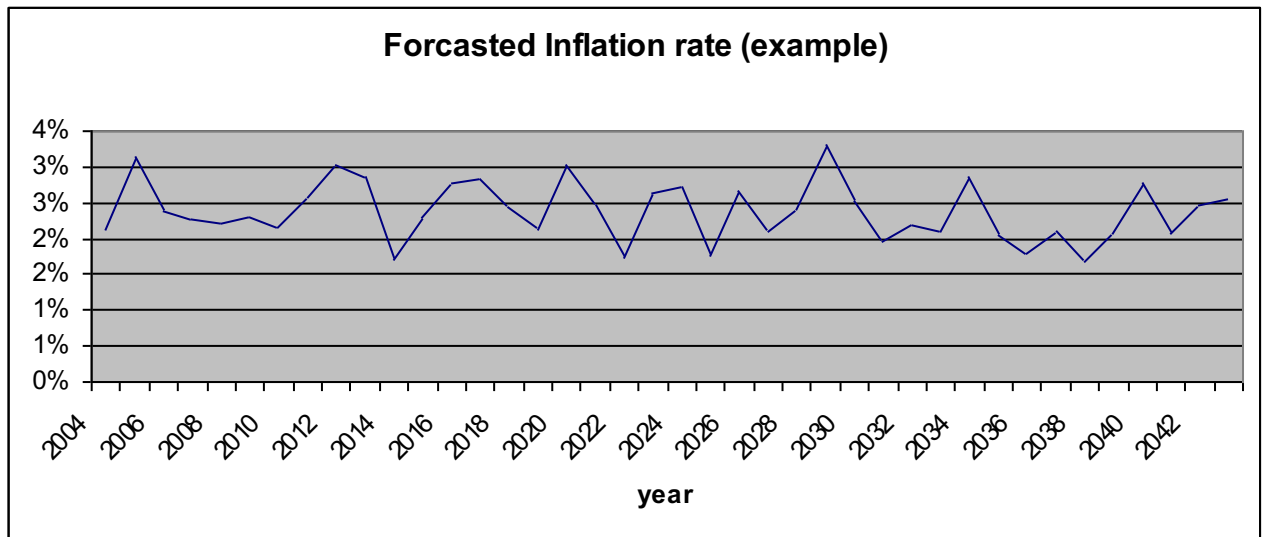


Figure 16 Generated future inflation rate scenario

The behaviour of the future inflation rate resembles to reality closer, since the normal life cycle of rising and falling down again is represented. The assumed distribution allows inflation rates between the defined ranges in a realistic way. The fluctuations show the almost expected stability, which is not so strong as in the operation unit's countries.

Chapter 5 Uncertainty Analysis and Results

There is a need to analyse the uncertainties to have a better view of likely project profitability. In the cash flow model the key uncertainties have been selected: the oil price, the costs, the production profiles and the inflation rate.

We have estimated min/max ranges and distributions for these variables.

A sensitivity analysis has been done to determine which variables have a significant impact on the project results and which variables can be ignored.

There are several manners in which the risks and uncertainties can be analysed for their impact on economic evaluations. One choice is to analyse the parameters separately or all together. The analysis can be done manually or by simulation. The different methods illustrate the effect of each parameter and also the differences of the analysis between the different approaches. The effects on the final project results will be analysed. The NPV of the project at 10% discount rate is assumed to be the most important result that is considered in the decision-making.

The sensitivities of a project evaluation are generally evaluated by varying one parameter at a time. We may test the effect of a higher or lower capex, various oil price levels, a higher or lower inflation rate, and so on.

In this chapter the different approaches will be discussed. Some of these are already commonly applied and others will be introduced.

I Manual approach

- A** The first well-known approach is to keep all the elements deterministic, and to analyse the elements separately by manual variation of the elements by a fixed percentage, say +/- 10%.
- B** The second way is to assign realistic ranges of minimum and maximum values for each parameter separately. The impact on the economic results of varying each parameter to the maximum and minimum expected value is evaluated.

II Simulation approach

- A** The first way is to define all the elements stochastically, but to analyse the impact of variations in each element separately with simulation.
- B** The last way is to analyse the impact of variations in all the elements together.

Each of these methods gives insights at different levels:

- Method IB is best used to understand and manage individual risks (e.g., capex or production),
- Method IIB is most suited for presentation to senior management, since it will show the overall likelihood of failing to meet minimum profitability.

5.1 *Manual variation*

5.1.1 Fixed variation

The uncertainties in economic evaluations can be analysed by a manual variation of the parameters individually. A growth of, for example, 10% of the parameter value may give an indication of the importance of the parameters and their influence on the project revenues.

In this evaluation, the parameters have each separately been given an increase of 10% as an example, to show the impact of the increases on the project results:

- Oil prices get a 10% increase in the nominal and in the estimated oil price for the first year (2004).
- Production profiles have been split up in the build up, the plateau and the decline phases. These phases have been divided into sub-phases, which include the years when production may behave the same (Chapter 4.3).
- The costs categories also have been divided into several components. Again an increase of 10% has been applied.
- The inflation rate has been given a 10% increase.

The manual variation has been applied to the estimates. Every time the effect on the NPV has been calculated. The calculations are illustrated in the table below (Table 1).

Base case NPV = 321

| | Original value | New value | New NPV | Change in NPV (%) |
|------------------------------------|----------------------------------|-----------|------------|-------------------|
| Oil Price | Nominal: 34.73 | 38.203 | 392 | 22% |
| | Estimated: 25.9 | 28.49 | | |
| Production Profiles Factors | All production-Years 110% | | 384 | 20% |
| DelayAtOnstream_FirstYear | 100% | 110% | 325 | 1.2 |
| DelayAtOnstream_SecondYear | 100% | 110% | 327 | 1.9 |
| BeforePlateau_2Years | 100% | 110% | 333 | 3.7 |
| Plateau_2014 | 100% | 110% | 328 | 2.2 |
| Plateau_2015 | 100% | 110% | 326 | 1.6 |
| Plateau_2016 | 100% | 110% | 326 | 1.6 |
| Plateau_2017 | 100% | 110% | 325 | 1.2 |
| AfterPlateau_till2029 | 100% | 110% | 342 | 6.5 |
| Years2030_2038 | 100% | 110% | 322 | 0.3 |
| LastYears | 100% | 110% | 321 | 0.0 |
| Costs Factors | All costs 110% | | 293 | -9% |
| Exploration_RT_factor | 100% | 110% | 321 | 0.0 |
| Appraisal_RT_factor | 100% | 110% | 320 | -0.3 |
| Facilities_RT_factor | 100% | 110% | 306 | -4.7 |
| Drilling_RT_factor | 100% | 110% | 313 | -2.5 |
| Pre_operation_costs_factor | 100% | 110% | 321 | 0.0 |
| Opex fixed factor | 100% | 110% | 318 | -0.9 |
| Infl Rate | 2% | 2.2% | 309 | -4% |

Table 1 The impact on the NPV by a fixed variation of +10% of individual factors

The table shows the impact of the different elements on the project NPV:

- Higher oil prices or higher production both lead to higher project profitability in contrast to a higher inflation or higher costs, which lead to lower NPV.
- A 10% change in the oil price has the biggest influence on the NPV with an increase of 22%.
- A higher oil production in all years leads to an increase of the NPV as well with almost 20%, but a bit less than the effect of higher oil prices.
- The impact of a 1-year delay in the production start-up is difficult to show through this sensitivity analysis by varying production by +10%.
- An increase in the total costs causes a nearly 9% decrease in the NPV.

5.1.2 Variation within realistic ranges

In this section the expected range for each parameter will be used to analyse the effect on the NPV. The calculation of the project NPV at the minimum and maximum values will say more about the risks in the estimates, since there is a quite range of possible values of which we must be aware of.

Each parameter will be assigned its minimum and its maximum values. In this way a better analysis can be made about the impact of the parameter ranges on the project results.

- In recent years, the oil price typically has varied between around \$25 and \$35, with average oil price of \$32 per barrel. The minimum and maximum value for the oil price have been between \$15 and \$50 with the same defined mean of \$32. The choice of the different ranges is further explained in Section 5.2.1.
- The annual production profiles already have their defined ranges (see Section 4.3.1). Each annual production profile has its range. The impact of the different ranges will be analysed. Especially, the impact years before plateau and the first 10 years after plateau will be analysed, since the fixed variation (see Section 5.1.1) pointed out their importance. Also the impact of the minimal and maximal oil production in the first year of production is considered, since we know the high probability with which a delay in production may occur. Of course the impact of a minimum and maximum oil production during the whole life of the project is analysed as well.
- The cost elements have their defined ranges as explained in Section 4.2.1. The different ranges are applied to the cost values. The costs of facilities and drilling are the most important ones. Therefore the impact of these costs on the project NPV are separately analysed. Also the impact of a minimum and a maximum for the total project costs is analysed. The choice of the ranges is explained in Section 5.2.1.
- The inflation rate has been given a minimum of 1.5% and a maximum of 3.5% (see Section 4.4.1). So it is quite possible to have an inflation rate of 2.2% (10% increase in Section 5.1.1) since the maximum can go up to 3.5%. A fixed variation is not always the best method to do the analysis.

The base case NPV = \$321 million has been retained. The minimum and maximum values have been implemented for every element separately. Within each element some categories have been assigned to their minimum and maximum values to see what the special effect is of these categories within the elements on the project profitability.

To illustrate which elements and which sub-elements are influencing the NPV, the calculations are shown in Table 2.

Base case NPV = 321

| | Mean | Min value | Change in NPV | Max value | Change in NPV |
|---------------------------------------|------|-----------|---------------|-----------|---------------|
| Oil Price | 32 | 27 | -34% | 37 | 32% |
| | | 14 | -187% | 51 | 124% |
| Production Profiles (in total) | | | -78% | | 40% |
| delayAtOnstream_FirstYear | 100% | 0% | -33% | 120% | 13% |
| delayAtOnstream_SecondYear | 100% | 50% | | 120% | |
| BeforePlateau_2Years | 100% | 70% | -12% | 120% | 7% |
| Plateau_2014 | 100% | 75% | | 110% | |
| Plateau_2015 | 100% | 70% | | 110% | |
| Plateau_2016 | 100% | 65% | | 110% | |
| Plateau_2017 | 100% | 60% | | 110% | |
| AfterPlateau_till2029 | 100% | 70% | -21% | 130% | 21% |
| Years2030_2038 | 100% | 60% | | 140% | |
| LastYears | 100% | 50% | | 150% | |
| Costs (in total) | | | 27% | | -43% |
| Exploration_RT_factor | 100% | 75% | | 140% | |
| Appraisal_RT_factor | 100% | 75% | | 140% | |
| Facilities_RT_factor | 100% | 75% | 11% | 140% | -19% |
| Drilling_RT_factor | 100% | 43% | 12% | 157% | -15% |
| Pre_operation_costs_factor | 100% | 60% | | 140% | |
| Opex_fixed_factor | 100% | 60% | | 140% | |
| Infl Rate | 2% | 1.5% | 10% | 3.5% | -28% |

Table 2 The impact of the variation within realistic ranges on the project NPV

The table above shows that:

- With the assigned oil price ranges, the project NPV is most affected. The NPV then reaches its highest level with the maximum prices and its lowest level with the minimum oil prices.

It is more realistic to consider a minimum oil price of 27 \$/bbl rather than the extreme of 14 \$/bbl. Since we input a minimum constant oil price for a period of forty years, it is more reasonable to assume an oil price of 27 \$/bbl.

Also, a constant maximum oil price of 37 \$/bbl is more reasonable than a price of around 50 \$/bbl, even if the oil price is 60 \$/bbl at the moment.

- Even with modest oil price variations between 27 and 37 \$/bbl, the oil price has a large impact on project NPV.
- A delay to start of production has a big impact of –33% on the NPV. It is assumed that a delay by 1 year will reduce the production in the second year by 50%.
- Because of limited capacity of the designed facilities, in combination with reservoir uncertainties, it is more likely to produce less than estimated and the NPV is expected to be often lower rather than higher. We should avoid lower production by drilling more wells or collect more data about the reservoir.

From this analysis, it is suggested that the greatest risks to the project profitability are (ranked by impact):

1. Lower oil price (-34%),
2. Delay to production start (-33%),
3. Higher inflation impacting the oil price (-28%),
4. Lower recovery in later years (-21%),
5. Higher facility costs (-19%),
6. Higher drilling costs (-15%).

The largest upside to profitability are (ranked in order of impact):

1. Higher oil prices (+32%),
2. Higher recovery in later years (+21%),
3. A 20% higher production in first year (+13%),
4. Reduction in drilling costs/wells (+12%),
5. Reduction in facility costs (+11%),
6. Lower inflation impacting oil prices (+10%).

If production (and reserves) are higher or lower in all years then this would have an even greater impact than the variation in oil prices (-78%, +40%).

If costs for drilling and facilities vary simultaneously, then this would also have a great impact (+27%, -43%).

Until now the manual variation has been used and the results have been analysed. Variation within the realistic ranges gives more information about the importance and the sensitivity of the parameters than the variation with a fixed percentage. The next section will discuss the simulation approach to do the analysis on the sensitivities of the project.

5.2 *Simulation*

In this chapter the simulation tool Crystal Ball has been used to run the simulations. Crystal Ball extends the analytical capability of the spreadsheet model and provides the information needed to make more accurate, efficient and confident decisions.

As spreadsheet users we know that spreadsheets have two major limitations by manually variation:

- Only one spreadsheet cell can be changed at a time (like the 10% variation in Section 5.1.1). As a result, exploring the entire range of possible outcomes is next to impossible. The aggregated risk that may impact the project results cannot be realistically determined.
- “What-if” analysis always results in single point estimates, which do not indicate the likelihood of achieving any particular outcome. While single-point estimates tell what is possible, they will not tell what is probable.

The use of probability distributions and Monte Carlo simulation overcomes both limitations:

- A range of possible values for each uncertain cell in the spreadsheet can be described. Everything known about the parameters is expressed simultaneously.
- After a Monte Carlo Simulation, results can be displayed in a forecast chart that shows the entire range of possible outcomes and the likelihood of achieving each of them. In effect, this moves us beyond what-if analysis by providing a statistical picture of the range of possibilities inherent in the assumed parameters.

Firstly, simulations are run with the parameters separately. The results will be analysed. And secondly, all the parameters will be varied together while the simulation is run.

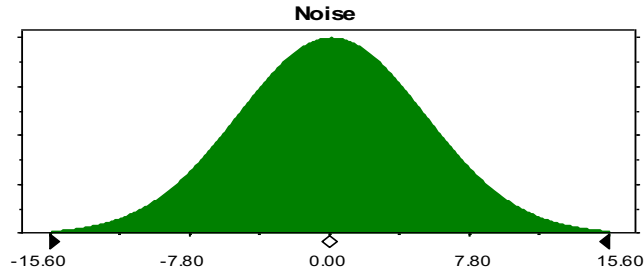
5.2.1 **Simulation of parameters separately**

When multiple random elements are involved in a simulation the focus will be on the aggregated influence of the parameters on the project results but not on the exact influence of each individual element.

One alternative is to keep the elements stochastic and to analyse each element separately with Crystal Ball. In that way we will know which element has the most influence on the project’s profitability and which element is more important and which one is less important.

With the use of statistics and the defined distributions for each element the analysis can be done with each element separately. Also the ranges of the minimum and maximum values are described that have been used in the previous section.

- The parameter oil price has been described by a *Normal Distribution* with: *mean* = 32 dollars per barrel and *standard deviation* = 5.2.



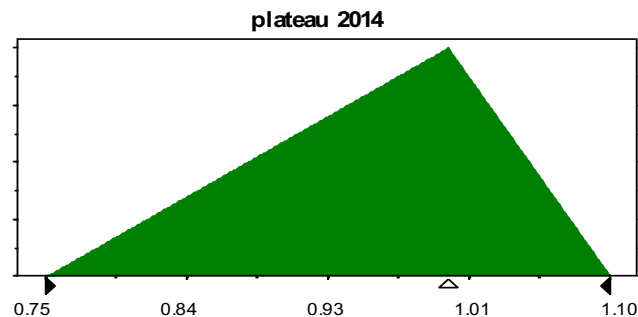
Based on the 10%-90% percentile of the assigned distribution the minimum and maximum values the oil price have been chosen. The oil price may vary +/- 6.66 from the expected oil price.

If we implement the minimum value for the oil price in the first two years of the project (2004 and 2005) the generated future oil price will have a minimum of 27 \$/bbl.

If we input the mentioned minimum value until the start year of production (2010), the generated future oil price will have a minimum of 14 \$/bbl.

Similarly, the maximum value of 6.66 has been implemented to get a maximum future oil price of 37 \$/bbl otherwise 51\$/bbl.

- The different phases within the production profiles have been assigned with the *Triangular Distribution*. The maximum and minimum values in the different stages differ from each other. Based on that, every stage has its own range, for example, for a plateau year (see Section 4.3.1) the probability the production may occur may look like the distribution below.



Of course we also may consider the case when the production is less than expected because of an overestimated reservoir or failed well drilling. In the

most extreme case the production profiles will have their minimum values during the whole life of the project. Similarly, production could be higher than estimated and could have its maximum level during the entire production life of the project.

- The costs elements have been divided in categories. Some have been assigned a *Triangular Distribution* and some have a *Normal Distribution*.

The costs of exploration, appraisal, and facilities behave according to a *Triangular Distribution*. Therefore the defined minimum and maximum values can be used during the analysis.

Drilling costs and opex have a *Normal Distribution*. Based on the 10%-90%-percentile the minimum and maximum values are chosen to do the analysis.

- The inflation rate has a *Triangular Distribution* with a minimum of 1.5% and a maximum of 3.5% as defined before.

When a simulation is run, Crystal Ball uses the Monte Carlo method to generate random numbers for the variables that conform to real-life possibilities. Each set of random numbers effectively simulates a single “what-if” scenario for the cash flow model. As the simulations runs, the model is recalculated for each scenario and the results are then dynamically displayed in an easy-to-understand display chart.

The final forecast chart of the project profitability that is expressed in the NPV reflects the uncertainty of the variables on the models results.

The generated distributions of the NPV represent the probability to obtain a certain profit. Simulations of 10.000 samples of each individual parameter have been run. During each run the generated distribution has been analysed.

Running the simulation with the assumed oil price distribution generates the distribution of the NPV as plotted in Figure 19.

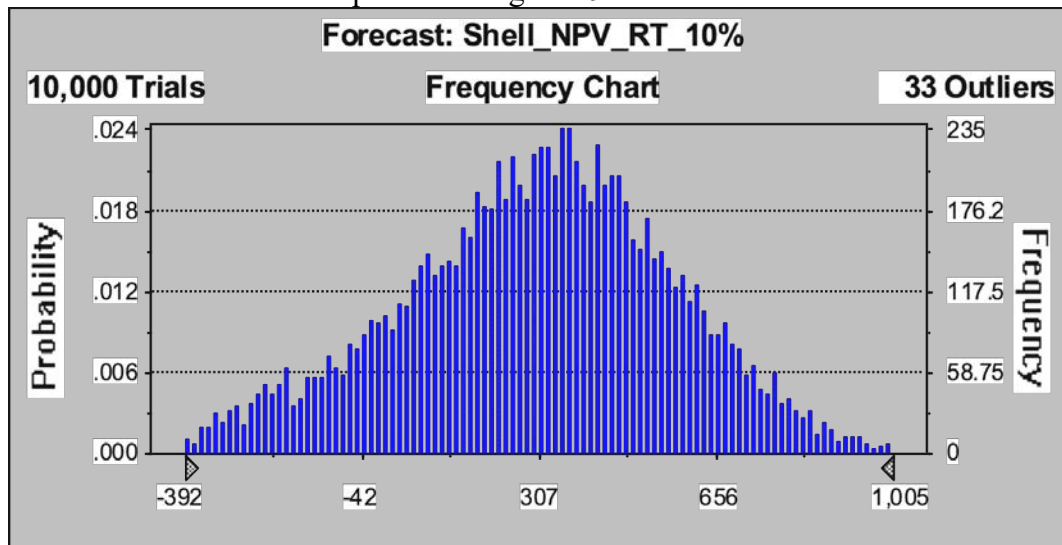


Figure 19 The NPV distribution after running the simulation with the oil price

The generated distribution above indicates the wide range of possible project outcomes. Some outcomes deviate from the base case (NPV = 321) significantly.

We can examine the probability that the implementation of the project will give a positive result. Moving the left side of the distribution to zero makes it possible to get the required information as shown in Figure 20. This indicates that the probability that the NPV will be positive is approximately 86%.

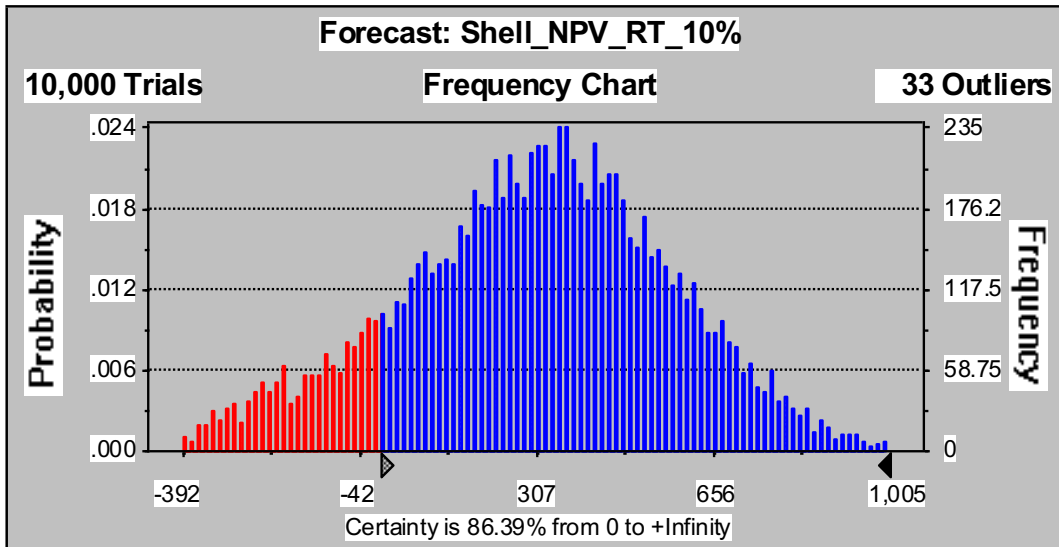


Figure 20 The probability to achieve a positive NPV given the uncertain oil prices

This analysis can now be applied to each random parameter individually. The analysis is summarised and presented in Table 3.

Base case (NPV = 321)

| | P(NPV >= 0) | P(NPV >= 321) | P(NPV < 0) |
|---------------------|-------------|---------------|------------|
| Particularly | | | |
| Oil prices | 86.3% | 49.5% | 13.6% |
| Production Profiles | 99.5% | 25.8% | 0.5% |
| Costs | 99.2% | 41.6% | 0.8% |
| Inflation | 99.9% | 24.1% | 0.1% |

Table 3 The analysis results of each parameter individually

The analysis shows that with the assumed distributions for the random parameters:

- The oil price presents the highest risk as illustrated by the 13.6% probability of an NPV < 0.
- The oil price distribution is more or less symmetrical around the assumed base case of 32 barrels per day. Hence, the likelihood that the NPV will exceed 321 million is close to 50% (49.5%).

- Running the simulation with the production profiles only shows the low probability (25.8%) to improve the project results above the base case NPV 321 million dollars. The production profiles have “more downsides than upsides”.
- The assumed cost distribution is relatively symmetrical around the base case. This is illustrated by the likelihood that the NPV will exceed \$321 mm being close to 50% (41.6%). There is a 41.6% probability of profitability exceeding \$321 mm if costs come in lower than estimated, and vice versa.
- The probability that the variations in production profiles, costs or inflation by itself will lead to a loss is quite small (0.5%, 0.8%, and 0.1%). Low oil prices are more likely (13.6%) to lead to a loss.
- The inflation rate is the least important parameter. Although it is likely to have a negative impact on the expected NPV.

Higher risks lead to wider ranges and so to higher maximum values and lower minimum values. This leads to a higher likelihood of making a large loss or large profit.

It is important to have an idea about how accurate the estimates are and how confident we are about the forecasts. The spread of the project NPV can be tested. The distribution of the different outcomes with the base case as the mean can be analysed. When the distribution of the project results has a significant deviation around the base case, we can draw conclusions about the estimates and forecasts and should be aware of the uncertainty of the elements' values.

To test the spread there are several techniques that one can use. A simple and useful technique is to calculate the *coefficient of variation* for each simulation. The coefficient of variation is the fraction of the *standard deviation* of the *mean*. A small coefficient, e.g., indicates the estimations. A large coefficient indicates that there is a large uncertainty in the estimates and forecasts. Perhaps a large coefficient then indicates that more accurate estimates can be made.

Each parameter has been analysed individually. A simulation is run with each parameter and the mean and the standard deviation of the NPV distribution has been used to calculate the required coefficient of variation as shown in Table 4.

Base case NPV = 321

| | Mean | St. dev. | St. dev. /mean |
|---------------------|------|----------|----------------|
| Particularly | | | |
| Oil prices | 307 | 269 | 0.88 |
| Production Profiles | 285 | 53 | 0.19 |
| Costs | 309 | 42 | 0.14 |
| Inflation | 301 | 25 | 0.08 |

Table 4 The analysis of the coefficient of variation for each parameter

The table above shows that:

- The oil prices contain the highest uncertainty. The fraction is closest to 1, which indicates the high variation of every single ‘what-if’ scenario during the calculations. The random samples of higher and lower oil prices lead to a large spread of the project profitability. The profitability is distributed around the mean in the negative as well as the positive side. Again the oil price presents the highest risk.
- The production profiles show the highest coefficient of variation compared to the estimated costs and inflation. Based on the analysis shown in Table 3, which indicated that the production profiles have more downsides than upsides, we can conclude that the base case production profiles are overestimated. The estimated base case production profiles should be estimated lower since the mean NPV (\$285 mm) related to the production profiles is less than the base case NPV (\$321 mm).
- The variation of the costs shows a closer mean (\$309 mm) compared to the base case. The deviation from the expected mean is symmetrically distributed around the mean as shown in Table 3. Still the expected mean is less than the base case. This indicates that the costs tend to be higher than expected and would lead to lower profitability.
- The fraction of the deviation is the lowest for the inflation rate. Again the inflation rate is the least important factor in the simulation. It has a negative impact (NPV = \$301 mm) on the project NPV.

This analysis indicates that the base case assumptions are not always considering a true 50/50 estimate by the experts that have provided the estimates. Shell uses the term 50/50 for the median. It refers to the middle value of a distribution, i.e., the value for which there is a 50% probability on a value higher than or equal to the median and a 50% probability on a value lower than or equal to the median. For an average number of items the median is calculated as the average of the middle two data points.

A dialogue between the economist and the technical experts about the uncertainty ranges will help to refine the base case estimates, as to make an adjustment to the estimates so that the expected profitability of the project is properly presented.

After doing the analysis by simulation of each parameter separately, the next section will present the analysis of simulation of all parameters together.

5.2.2 Simulation of all parameters together

Now all the elements will be simulated together. This is useful to see how the risks and uncertainties in the economic parameters influence the project profitability in an aggregated manner.

The generated distribution has been plotted in Figure 21, where the probability to have a positive project result has been analysed.

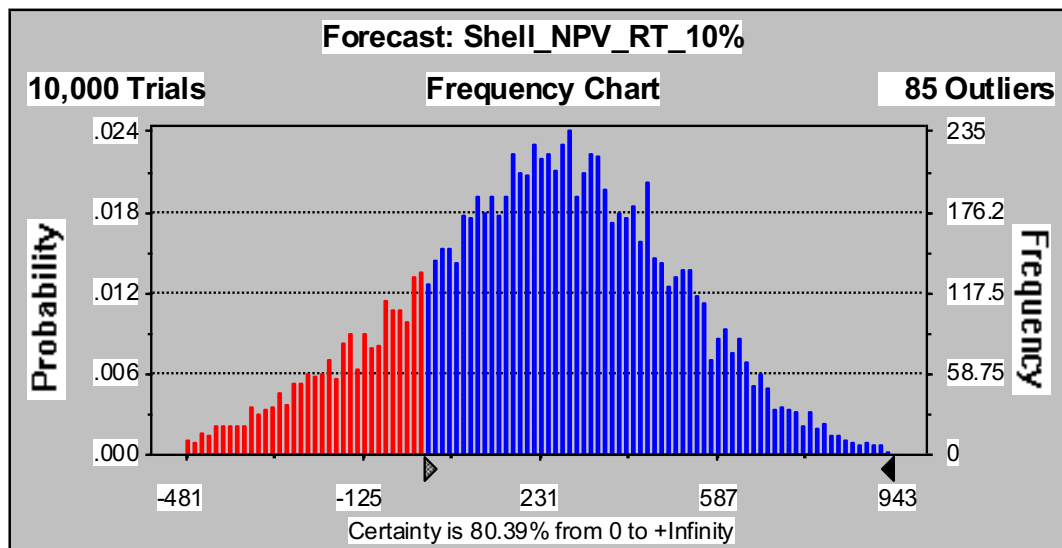


Figure 21 The probability to get positive project NPV

The figure shows that the probability that the NPV is positive is about 80%. The probability of a higher NPV than expected might be interesting as well. Figure 22 shows that the probability that the NPV will exceed the base case of \$321 mm is 38%.

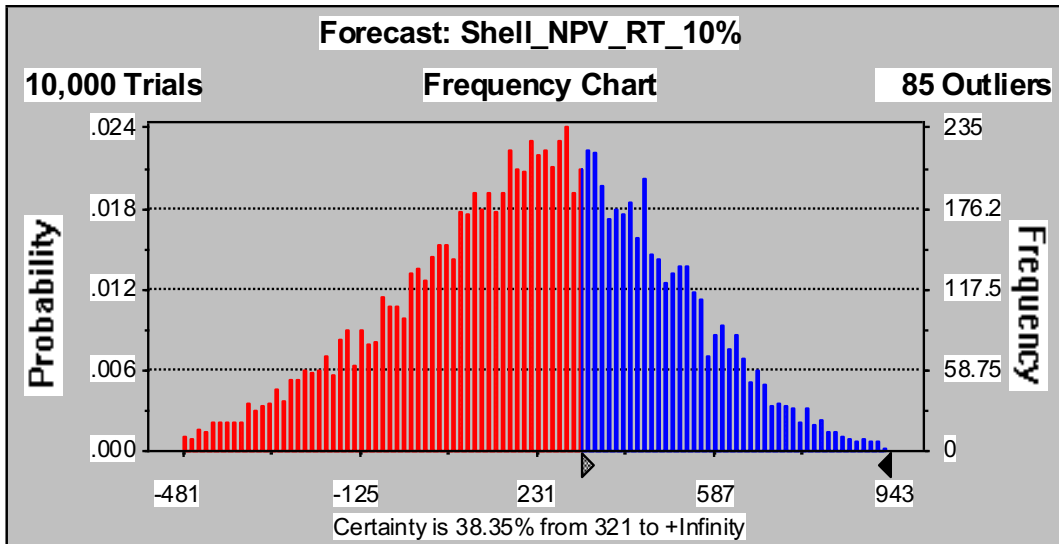


Figure 22 The probability to exceed the base case

The analysis is summarised in Table 5.

| | P(NPV >= 0) | P(NPV >= 321) | P(NPV < 0) |
|------------|-------------|---------------|------------|
| All | | | |
| | 80% | 38% | 20% |

Table 5 The analysis results of running all the parameters together

The table shows an 80% probability of achieving positive project NPV if implemented. But it also means that there is almost a 20% probability to have a significant loss (i.e., NPV<0)! The profitability that the project will earn less than the base case \$321 mm is 100% - 38% = 62%.

Therefore decision makers should be aware of the risks and uncertainties and the effect on the economic evaluations.

As before, the impact of the estimates will be represented with the calculated *mean* and *standard deviation*. Table 6 summarises the results.

| | Mean | St. dev. | St. dev. /mean |
|------------|------|----------|----------------|
| All | | | |
| | 231 | 274 | 1.19 |

Table 6 The analysis of the coefficient of variation for all parameters together

The high result of the fraction should not be a surprise, since in the cash flow analysis large positive random numbers (production profiles*oil price) and large negative random numbers (costs) are subtracted. The resulting difference (profit) is relatively small but has the large aggregated uncertainty range of all the large random parameters. It is more likely to have a project NPV (\$231 mm) less than expected. The aggregated distribution shows the impact of the uncertainties in the estimates.

The high deviation stresses the influence of the economic indicators on the project profitability.

After doing the analysis and presenting the results, the conclusions of the thesis are made in the next chapter. Based on the analysis and the results, some recommendations will be presented.

Chapter 6 Conclusions and Recommendations

In this thesis the uncertainties in the project estimates have been described. The key uncertainties have been selected. These uncertainties have been assigned as statistical variables with ranges that have been advised by domain experts.

In the last chapter, Chapter five, sensitivity analysis has been used with manual variation. The risky variables have been assigned distributions with associated parameters and simulations have been run to provide a better analysis of the project profitability.

This chapter describes the conclusions based on the analysis after which some recommendations are presented.

6.1 Conclusions

The use of Monte Carlo simulation is a relatively untested method to examine the project profitability under uncertainties.

The analysis done by simulation shows that:

1. Generally, defining the uncertainties as stochastic variables and running simulations gives a better insight in the project results. The information provided tells more about the likelihood to earn profit or to face loss.
2. In the analysed project, the profitability is likely to be less than originally expected because:
 - The uncertainty range of the production profile typically has more downsides (project delays, subsurface set backs) than upsides (restricted due to facility constraints).
 - The costs are more likely to be higher than expected, which is shown in the defined distributions. The costs lead to lower project profitability.
 - The inflation rate is more likely to be higher than expected. A high rate leads to lower Real Term oil prices and lower profitability.
3. The correlations between the oil prices ($\alpha = 0.7$) and the correlation between the production in individual years create a degree of dependence between the values in individual years. This reflects the typical periodic life-cycle for oil projects and oil prices where a scarcity of oil and high prices lead to increases in investment in projects executed over several years and thus a slow return to lower oil prices, and vice versa.

The dependence in production from year to year reflects that the reason for higher or lower production in any particular year probably still exists (but to a lesser extent) in the next year.

Without the correlation factor, “high” and “low” years would follow each other randomly and thus cancelling each others impact and thus under-representing the true risk or upside.

4. Based on the presented results and conclusions about the impact of the parameters, the NPV base case of \$321 mm should be reduced to reflect the likelihood of higher costs and inflation and lower production. Even the fluctuation in the oil price will have a lower impact on the results when the estimated base case is more realistic.

6.2 Recommendations

The analysis has been based on the selected key uncertainties and the advised distributions. Accurate estimates lead to more certain project results. Therefore the awareness of the uncertainties should lead:

1. To ensure that estimates for key parameters such as capex, start-up date, production profiles, and reserves are not given as a single number but with carefully considered min/max ranges representing the degree of confidence in these estimates.
2. To ensure that the base case estimates represent a true 50/50 estimate.
3. To consider the aggregated impact of uncertainties in project schedule, well/reservoir performance, reserves, and constraints such as facility capacity, to arrive at an uncertainty range for the production profiles and revenues.
4. To consider that capex ranges also can reflect the likelihood that more or less wells are required than expected.
5. To consider that the probability distribution of the project profitability is included in investment proposals.
6. To ensure in particular that economic models of petroleum production contracts with “asymmetrical” risks and profits (more downsides than upsides through capping mechanisms) take into account the expected range of oil prices, costs and production.
7. To analyse the actual profitability of projects against that originally predicted, and thus to get a better understanding of the risks and uncertainties.

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Appendices

I Statistical Definitions

Random Variable:

The outcome of an experiment need not be a number, for example, the outcome when a coin is tossed can be 'heads' or 'tails'. However, we often want to represent outcomes as numbers. A random variable is a function that associates a unique numerical value with every outcome of an experiment. The value of the random variable will vary from trial to trial as the experiment is repeated.

Mean:

The mean, notation \bar{x} , of a set x_i of n random variables is the average data point value within a data set. To calculate the mean, add all of the individual data points then divide that figure by the total number of data points.

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i,$$

Variance:

The variance, notation σ^2 or s_N^2 , of a set x_i of n values is a measure of how spread out a distribution is. It is computed, as the average squared deviation of each number from its mean. Variance is the deviation from what was expected. The variance of a random variable is a non-negative number which gives an idea of how widely spread the values of the random variable are likely to be; the larger the variance, the more scattered the observations are on average.

$$s_N^2 \equiv \frac{1}{N} \sum_{i=1}^N (x_i - \bar{x})^2,$$

Stating the variance gives an impression of how closely concentrated around the mean the distribution is; it is a measure of the 'spread' of a distribution about its average value.

The larger the variance is, the further that individual values of the random variable (observations) tend to be from the mean, on average. The smaller the variance, the closer the individual values of the random variable (observations) tend to be to the mean, on average.

Standard Deviation:

The standard deviation is a statistic used to measure the variation in a distribution. Sample standard deviation, notation s_N or σ , is equal to the square root of the variance.

$$s_N = \sqrt{\frac{1}{N} \sum_{i=1}^N (x_i - \bar{x})^2}.$$

Mode:

The mode is the data value, which occurs with the highest frequency.

Correlation:

Correlation is a technique for investigating the relationship between two quantitative, continuous variables. In correlation the emphasis is on the degree to which a linear model may describe the relationship between two variables.

The correlation coefficient may take on any value between plus and minus one.

$$-1.00 \leq r \leq +1.00$$

The sign of the correlation coefficient (+, -) defines the direction of the relationship, either positive or negative. A positive correlation coefficient means that as the value of one variable increases, the value of the other variable increases; as one decreases the other decreases. A negative correlation coefficient indicates that as one variable increases, the other decreases, and vice versa.

Taking the absolute value of the correlation coefficient measures the strength of the relationship. A correlation coefficient of $r = 0.50$ indicates a stronger degree of linear relationship than one of $r = 0.40$. Likewise a correlation coefficient of $r = -0.50$ shows a greater degree of relationship than one of $r = 0.40$. Thus a correlation coefficient of zero ($r = 0.0$) indicates the absence of a linear relationship and correlation coefficients of $r = +1.0$ and $r = -1.0$ indicate a perfect linear relationship.

II Probability Distributions

Normal Distribution



A normal distribution in a variate X with mean and variance σ^2 is a statistical distribution with probability function

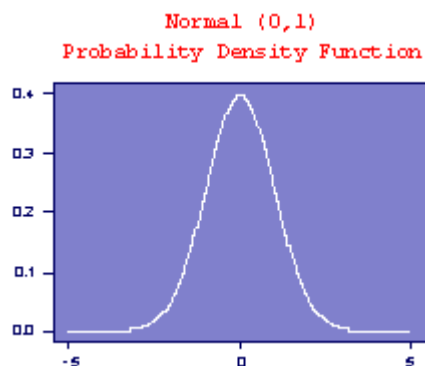
$$P(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/(2\sigma^2)}$$

on the domain $x \in (-\infty, \infty)$.

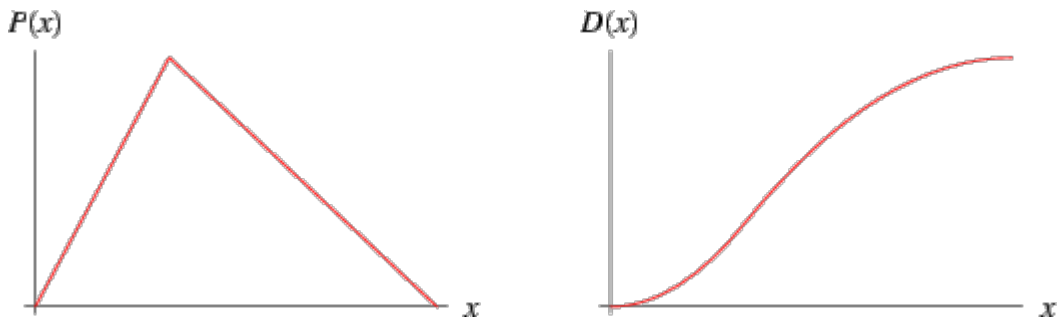
A variate is a generalization of the concept of a random variable that is defined without reference to a particular type of probabilistic experiment. It is defined as the set of all random variables that obey a given probabilistic law.

It is common practice to denote a variate with a capital letter (most commonly X). The set of all values that X can take is then called the range. The probability that a variate X assumes the value x is denoted $P(X = x)$.

The so-called “standard normal distribution” is given by taking $\mu = 0$ and $\sigma^2 = 1$ in a general normal distribution



Triangular Distribution



The triangular distribution is typically used as a rough model for the time required performing some task when no real-world data are available. A triangular distribution takes on values in the finite interval [minimum, maximum] (minimum ≥ 0.0 , mode $>$ minimum, and maximum $>$ mode), with values near the mode being most likely to occur. Subjective estimates of the three parameters are obtained from domain experts. The mean of a triangular distribution is only equal to the mode when the distribution is symmetric