RISK MANAGEMENT AT INSURANCE COMPANIES
PROFIT SHARING PRODUCTS

By
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Abstract

This thesis gives a comprehensive analysis of typical profit sharing products sold in the Dutch life insurance industry. The dynamics and parameters that influence the value of the product are revealed using replicating portfolios consisting of swaptions. An alternative model, which explicitly considers sensitivity towards the government curve and the euro swap curve, is introduced for this. This model provides additional insights, as not modeling exposure to credit risk can have severe consequences in the valuation of these products and consequently also in the construction of risk mitigating strategies. Therefore, this study considers several hedge strategies that try to capture this exposure by including Credit Default Swap (CDS) contracts. Results show that these strategies perform well but, because the payoff structure of these contracts remains linear, do not completely capture the optional element in profit sharing products for extreme movements in the credit spread. Not considering exposure to credit spread results in a hedge that only performs well when the government curve and the swap curve move in equal direction simultaneously, but severely under performs when this is not the case.

Keywords: Profit sharing, embedded options, life insurance, replicating portfolio, guaranteed returns, hedge strategies, BPV, credit risk, credit default swap.

Preface

This thesis is written accompanying an internship that is an integral part of the Business Mathematics & Informatics Master program at VU University Amsterdam. The purpose of this internship is to perform research on a practical problem individually during six months. The problem and methods used should display all elements of the program, i.e., practical relevance to the industry, mathematical modeling and computer science. This thesis is written at Cardano Risk Management, a company that specializes in risk management using derivative overlay structures.

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Special thanks go out to my two supervisors at Cardano, Vincent van Antwerpen and Niels Vermeijden, for their continuous guidance and support during the internship. Finally, I would like to thank Cardano as a whole for providing the internship and therefore the opportunity to graduate, and all colleagues there for contributing to an environment and atmosphere that helped me considerably.

The subject of this thesis, and therefore also theory and terminology used, is finance related. Because the program Business Mathematics & Informatics does not require comprehensive knowledge of all this terminology, but everybody from this program should be able to understand this thesis, a short description of the most important terms is given in the appendix. These terms are formatted *italic* when they are first introduced.

Jos van Gulick
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Contents

Introduction

Insurers play a vital role in today's society. They enable individual persons to hedge the risk of ending up in a situation costing more than they can afford. The question however remains to whom the insurance companies can turn to hedge their own risks, how can insurance companies insure themselves for situations they cannot afford?

Regulation for insurance companies has left a great deal of the responsibility at the companies, maybe because everyone assumed that "the experts of insuring" would surely insure themselves properly. When Equitable Life, an over 200 years old insurance company with around 1.5 million policy holders, nearly collapsed in 2000, this assumption was proven wrong. It seemed that a lot of insurance policies were sold while the insurer did not fully understand the value of the promise it had made to the policyholder. This caused a shift in the regulatory requirements and in the risk management practices within insurance companies as a whole. Though most insurance companies currently still value their liabilities using a fixed *discount rate*, they have been very busy with preparations for *Solvency II*, which is expected to go into force in the near future. This framework requires extensive market based valuation practices. The thesis will therefore focus on market based risk management strategies for insurance contracts with profit sharing elements. These contracts are among the most sold insurance policies and are similar to the policies that caused the problems described above. The contracts promise to pay the policy holder a minimum guaranteed return over the life of the contract. In addition they allow the policyholder to share in profits when interest rates are high. The description immediately reveals the optional character of the contract because the policyholder essentially receives a guaranteed return and a *call option* on the return of a given portfolio. While the concept is easy to grasp this product can adopt a rather complicated form due to this optional element.
The type of contract in its general form can be found in many countries, but very different specifications of the product exist at every insurance company and within every country. The focus here will be on the Dutch "Overrente polis", which allows for profit sharing when the return on a given investment portfolio, that is based on fixed income assets, exceeds a predetermined threshold. The product is among the most important in terms of market size within the Dutch life insurance industry. The details for this specific product can still differ per insurance company but the most important parameters, requirements and specifications are the same.

The type of product of which the yield depends on a given reference portfolio became popular in the

eighties[1] when rising interest rates led to a significant flow of capital into financial markets. This, in turn, resulted in increased competition between financial institutions, forcing also life insurance companies to sell products with a higher yield, making them more sensitive to interest rate changes. Equitable life was not the first insurer to get into trouble, already in the late eighties some life insurance companies got insolvent with many more to follow. The main reason was that these products had always been considered very safe because the low guaranteed rate represented an option with a *strike* very far *out of the money*. When interest rates fell sharply at the start of the nineties, the first problems however arose quickly.
These circumstances sparked an amount of academic research, focusing mainly on *unit- and equity linked products* at first. Because these products give a return that is directly linked to the return on a given reference portfolio, they are generally easier to understand and to value than most profit sharing products including a guaranteed return.
Around the year 2000, the risks from the optional character in profit sharing products became apparent as several companies had to file for bankruptcy as a direct consequence of it. This caused an emergence of academic literature and regulatory reforms. Among the first to address the valuation of the optional character were Briys and de Varenne (1994),Hipp (1996),Miltersen and Persson (2000) and Grosen and Jorgensen (2000). Research in the following years contributed to the ideas from these authors by including mortality and surrender options, but also by addressing issues more specific to insurance policies sold in different countries. This has led to a substantial amount of literature on the fair valuation of contracts that are sold in several countries, including the Netherlands. The first to address the problem of guaranteed returns offered by Dutch insurance companies was Donselaar (1999). He showed that the demand for these products quickly rose during the nineties when they started to be used as pension plans as well. But also that most insurers probably did not charge enough for the products they sold and that they were likely to use investment strategies that did not match with their liabilities, exposing them to risks. Bouwknegt and Pelsser (2001) used the optional character of a simple profit sharing product and came up with a fair valuation based on a replicating portfolio. Later, Plat and Pelsser (2008) found an analytical expression for the fair value of a profit sharing product that can be considered as a type of "Overrente polis".

It is clear from the above that quite some research has been done on the fair valuation of a wide range of insurance policies with embedded options during the last two decades. However, little attention has gone to the risks involved with these contracts and possible hedge strategies. Some basic elements have been discussed but they are mainly theoretical, as they are often a result of the replicating portfolios used in the valuation.

[1]The first form of a with profit sharing product was sold already in 1806 according to Sibbett (1996).

Because the profit sharing of the "Overrente polis" is determined by a complex yield that is based on Dutch government bonds, the swap rate is often taken as an approximation to simplify calculations. This can lead to problems in modeling these products and in the construction of risk mitigating strategies. An important contribution to existing literature lies in the reevaluation of this assumption. For this an alternative model is introduced that quantifies these risks, provides additional insights and allows for the construction of strategies and testing environments that should be more effective. The aim of this work is to create understanding in the value and dynamics of typical profit sharing products. Questions that will be answered are: *a*) How can the fair value of the product be computed?; *b*) What factors influence the value of the profit sharing?; and *c*) What approach can be taken to mitigate the risks that these factors introduce?

In summary the results of this thesis show that a replicating portfolio of swaptions and a zero coupon bond can estimate the fair value of these contracts consistently. The value of these swaptions should however be computed by explicitly considering sensitivity towards a government curve and a discount curve.

Neglecting sensitivity towards country credit risk can result in significant modeling errors and the construction of weak performing hedge strategies, as the credit risk premium can cause strong fluctuations in the value of these products.

The long maturity of these contracts, liquidity issues and laws prohibiting short selling of government debt, impose limitations in the construction of effective hedge strategies that incorporate exposure to credit risk. The use of CDS contracts in this however seems to be an effective alternative. Furthermore, the results in this paper strongly suggest that the fair value of these contracts is significantly higher than the price for which they can be bought.

The remainder of this thesis will have the following structure. Chapter 1 will describe the profit sharing product in general. First the position of the profit sharing product within the Dutch pension and insurance system will be drawn. In the subsequent section the most popular type of profit sharing product sold in the Netherlands is discussed and an example will be provided. Next, it will be shown how a replicating portfolio can be constructed which is consistent with precise contract specifications. With this, the value and the sensitivity of these products at a given moment in time can be estimated. At the end of the first chapter two products with alternative structures that are often encountered will be considered and the results from the earlier sections will be applied.

In chapter 2 the results of the first chapter will be used to determine the risks that these contracts introduce to the books of the insurance company. Several risks will be analyzed for different scenarios and methods that can help mitigating these risks will be discussed. A model that considers exposure to credit risk will be introduced and evaluated. Both the dynamics of the guarantee and the profit sharing will be analyzed.

Several risk mitigating strategies will be discussed in more detail in chapter 3, where special attention is given to the practical implementability of the suggested hedging strategies, as the findings of this thesis are meant to provide methods that work and can readily be applied in the daily operations of risk managers. This chapter will describe products and strategies that can be used. The effectiveness of these strategies will also be assessed for several scenarios. Special attention here will be given to the use of CDS contracts in hedging the exposure to country credit risk.

The last chapter concludes.

Chapter 1

Profit Sharing Products

There is profit sharing in a number of products, organizations and industries. The type of profit sharing discussed here stems from a product which is a form of defined benefit pension plan offered by insurance companies, together with *unit-linked* products. The profit sharing products mainly differ from unit-linked products in that the return is not directly linked to some reference portfolio which can be chosen by the policyholder to match his risk appetite. Instead the return on profit sharing products is typically based on a predefined, fictive, investment policy. Though unit-linked products generally also offer some kind of minimum rate of return guarantee (MRRG), by promising a fixed amount at expiration of the contract, it is lower than in a typical profit sharing contract and the policyholder bears a much larger part of the risks.

The motivation behind profit sharing products is twofold. First, it should provide a stable return to the policyholder, with low risk through the minimum return guarantee while still being competitive with other financial assets through the profit sharing. Secondly, the smoothing of returns by the investment policy, that on the one hand provides stable returns for the policyholder, should also ensure less volatile market values of the liabilities.

This chapter will begin by shortly discussing the function and position of the profit sharing product in society. In section 1.2 a precise specification of a Dutch profit sharing contract, the "Overrente polis", will be given. Section 1.3 will discuss the so called u-yield, a rate that is used to determine the bulk of Dutch profit sharing products. In section 1.5 an efficient and consistent way to value these products by the use of a replicating portfolio will be given. Using this valuation some important properties and sensitivities will be addressed. Finally the results will be applied to two contracts of known form in section 1.6.

1.1 The position of profit sharing products within the Dutch pension system

The Dutch pension system is based on three pillars:

1. A state pension that every citizen receives after the age of 65. It is linked to the statutory mimimum wage and provides a minimum income to prevent real poverty. This pillar is based on a pay as you go framework, meaning that the people currently having a job provide for the retirees, and is a result of two acts; 1) The general Old Age Pensions Act [*Algemene Ouderdomswet* (AOW)], that came into force in 1957 and 2) the National Survivor Benefits Act [*Algemene Nabestaandenwet(ANW)]*.

2. A supplementary pension build up during the working life of a citizen. This pillar is, as it is in other countries, still a crucial one for the citizen to enjoy a decent pension after retiring. It consists of collective pension schemes that are administered by either a pension fund or an insurance company. Because membership of a pension fund is mandatory for many sectors and professions, today about 94% of the employees belong to a pension fund, of which there were 514 (end 2010). There are three different types of pension funds:
   - Corporate pension funds (for one single company or corporation).
   - Industry-wide pension funds (for all employees of a whole sector).
   - Pension funds for independent professionals.[1]

   These pension funds are non-profit and strictly separated from the companies. Therefore financial trouble for the company will not directly effect the pension plans of the employees. The pension plans are financed by capital funding. Meaning that they are paid for by contributions and returns on investments made by the funds. Today the managed capital of all Dutch pension funds amounts to over €746 billion, exceeding the dutch GDP by about 26%.

3. The third pillar consists of individual pension products. It is used by employees not participating in a collective pension scheme or people that prefer a more assuring pension plan in addition to the second pillar. Utilizing savings for the purpose of a pension one can often take advantage of tax benefits.

[1] For example pilots, dentist and doctors all have separately managed pension plans.

Profit sharing products can be placed in either the second or the third pillar when a person buys such a product at a insurance company through his or her employer or individually. In a less obvious way they are also used indirectly by pensions funds and insurance companies through reassurance. A companies pension fund with defined benefit pension contracts might want to transfer some of the risk by use of a profit sharing product sold by an insurance company.

1.2 Product specification

In the contract of a profit sharing product the insurer usually promises to pay a fixed amount to the policyholder in the future. This amount is then either discounted using a fixed interest rate, called the technical rate or the guaranteed rate r^g, and paid as a lump sum at inception of the contract or paid by regular premiums over the life of the contract.
Every year the reserve of the account the policyholder has at the insurer grows by the guaranteed rate but the policyholder also has an option, representing the profit share, on the return of some investment portfolio the insurer manages according to a predetermined policy. Often the profit sharing return r^{PS} is also subject to a certain participation level α and/or fee δ subtracted before the sharing. The profit sharing return can then be defined as

$$r^{PS} = \max \alpha \left[r^{fi} - (r^g + \delta), 0 \right], \tag{1.1}$$

with r^{fi} the return on the investment portfolio, which is often defined by a fictive investment policy, hence fi. This portfolio should not be confused with how the insurance company actually manages its assets and is only used for the determination of the profit sharing rate. Because r^{fi} is the only parameter that varies each period in equation 1.1, this is the most important element in the valuation of the profit sharing part of this product and it ensures the stable and smoothed returns discussed earlier. The investment policy can differ substantially between insurance companies but is mostly based on fictitious investments in the u-rate. This is a weighted average yield on a number of bonds issued by the Dutch state and will be discussed in more detail later. The use of the u-rate is the first step in smoothing returns as it is a weighted average of a number of underlying yields. To stabilize the profit sharing further, a weighted average return on the investments made against different u-rates in the fictive portfolio is used, setting r^{fi} equal to the return on a portfolio of fixed income products.

For a product that pays out the profit sharing component each year and a fictive portfolio that invests in M year bonds the above can be summarized by the following steps.

At the start of the year the premium, the coupons from previous investments and possible redemptions are received. This amount is then invested in bonds yielding the then prevailing u-rate with a fixed maturity of M years and a fixed turnover structure. At the end of the year the weighted return on all investments made up to then can be computed by considering all coupons. Based on this return the amount of profit sharing can be determined.

Essentially it is a simple idea and this is why insurers promote it as being perfectly transparent. This structure will however cause some difficulties in determining efficient hedging strategies and consistent valuation.

Based on the above a contract that pays out the profit sharing every year is defined by:

1. CF_i The cash flow to be paid to the policyholder in year i, the horizon.
2. r^g The minimum guaranteed rate of return.
3. M Maturity of assets in the reinvestment strategy.
4. T A turnover structure of the investments.
5. δ The fee subtracted from the profit sharing.
6. α The participation level of the policyholder in the profit sharing.

To give some more insight in how the profit sharing evolves an example will be given in section 1.4.

1.3 U-yield

The profit sharing rates of European life insurance contracts are most often based on the return of a reference portfolio containing fictitious fixed income assets. In the Netherlands it is common practice to invest in fixed income products that have a rate that is based on bonds issued by the Dutch state. Three yields are used in the Dutch life insurance industry for this purpose; the s-yield, the t-yield and the u-yield. All yields are computed in a similar complicated way where the s-yield computes a yield that is corrected for inflation compared to the t-yield and u-yield. The main difference between the latter two is that the t-yield considers bonds with a longer maturity, i.e., it includes maturities from 7 years on (except perpetual bonds). All yields are published by "Verbond van Verzekeraars", a Dutch association of insurance companies. The u-yield is by far the most popular yield, hence this will be the one discussed here.

Definition of the u-yield.
The u-yield is determined every 15^{th} *of the month as the average yield on 6 past "part" yields, computed at the same time as a weighted*[2] *average of medians of yields over different maturity segments of all bonds issued by the Dutch state that have a principal of at least €255 mln and a remaining maturity between 2 and 15 years.*[3]

The above definition makes clear that this rate has been setup in a way that makes any computation of a valuation or projection of the profit sharing at the least very inefficient, if not impossible. Because of this a proxy is often used for computations requiring the u-rate. From historical data the 7-year swap rate has been known to be a good proxy (see figure 1.1).

Earlier literature does not elaborate on possible reasons why this is a good proxy and start calculations based on the 7 year swaprate right away. One thing that is obvious from the figure is that the 7 year swap rate is more volatile than the u-rate, what makes sense since the latter is an averaged value. This mismatch will cause differences in the valuation of the embedded option.

For now the 7 year swap rate will be taken as basis for projections of the u-rate as this is the best indicator at hand. Later, in section 2.2.2, the risks implied by making this assumption will be discussed.

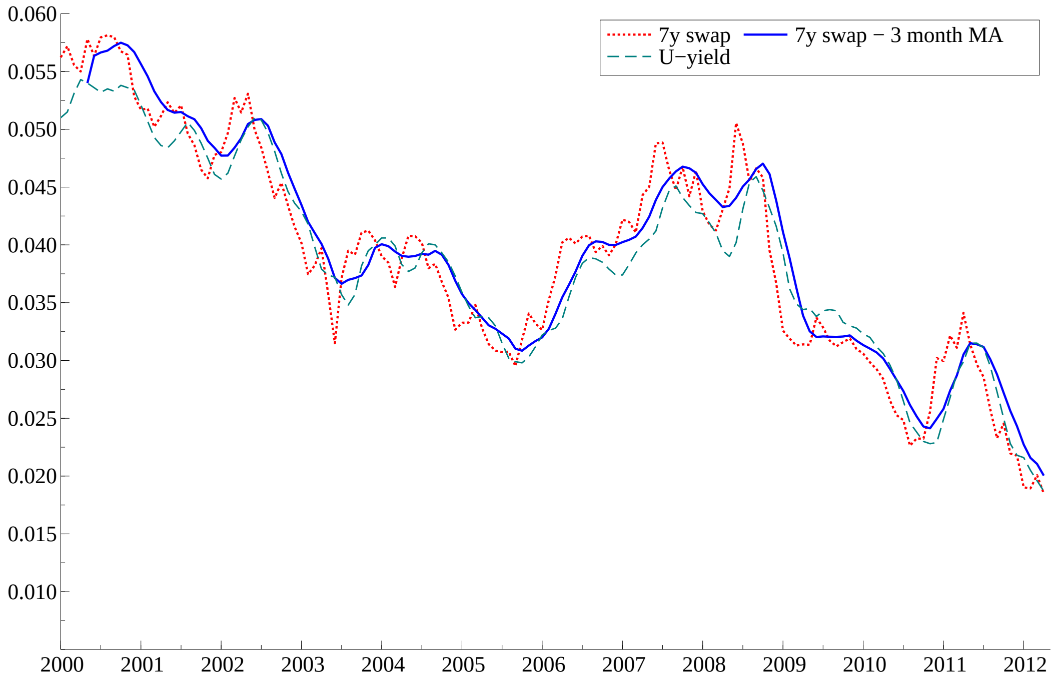
It should also be clear that, though the 7 year swap rate is a good proxy for the u-yield, this does not mean it is a good proxy for the profit sharing rate as this rate crucially depends on the investment policy and timing of the reference portfolio.

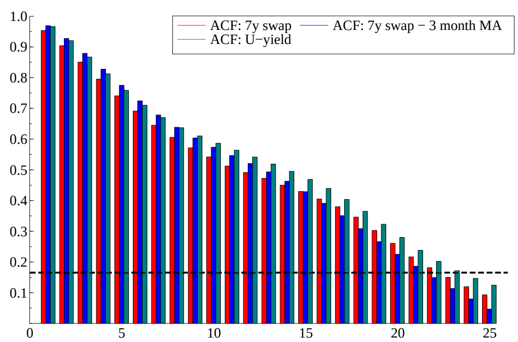
1.4 An example

To give some more insight into how the profit sharing is determined, how the investment portfolio and reserve are related, and how they evolve over time a simple example is worked out in this section.

Consider a contract that pays €100 mln in 30 years with a guaranteed interest rate of 1.0%, a reinvestment strategy in coupon bonds with a maturity of 7 years and a yield equal to the u-yield prevailing in that period, no fee, 100% participation and is paid by one lump sum at inception of the

[2]This weighing scheme is not based on the principals but on fixed percentages that depend on the maturity.
[3]For a more precise definition of the u-yield, but also the t- and f-yield see: `http://www.verzekeraars.nl/UserFiles/File/cijfers/Definitie%20rendementen%20s%20t%20u.pdf`





Variable	min	mean	max	σ
7y swap	0.0185	0.0392	0.0581	0.00968
7y swap - 3 month MA	0.0200	0.0394	0.0575	0.00902
U-yield	0.0187	0.0386	0.0543	0.00864

Figure 1.1. *This figure shows the 7 year swap rate as obtained from Bloomberg, its 3 month moving average, computed with 6 half month datapoints, and the u-yield over the period January 2000 to April 2012. In the bottom left the autocorrelations are displayed. The table in the bottom right displays some summary statistics of these series.*

contract. The return of this contract is computed in the following way:

$t = 0$ The amount in the reserve and in the fictive investment portfolio managed by the insurer is equal and computed using a discount factor based on the horizon of the cash flow and the technical rate r^g

$$R_0 = I_0 = df_i CF_{30} = (1+r^g)^{-30} CF_{30} = €\,74{,}192{,}292,$$

with R_0 the reserve and I_0 the amount in the investment portfolio at time $t = 0$. I_0 is then invested completely in a bond with a yield u_0, the prevailing u-yield at $t = 0$, and has a maturity of 7 years. At the end of the first period the return on the fictive investment portfolio equals the coupon from the bond. If the u-yield at inception was $u_0 = 2.5\%$, the profit sharing can be computed from equation 1.1 as

$$r_0^{PS} = \max\left[\alpha(r_0^{fi} - (r^g + \delta), 0\right] = \max[2.5\% - 1.0\%, 0] = 1.5\%.$$

This means that there is a profit sharing at the end of the period of

$$PS_0 = 1.5\% \cdot R_0 = 1.5\% \cdot €\,74{,}192{,}292 = €\,1{,}112{,}884.$$

This amount would be paid out to the policyholder and only $1\% < u_0$, equal to the increase of the reserve, will be available for investment in the fictive portfolio. If u_0 would have been lower then $r^g = 1\%$, the insurer would have to honor the contract and let the reserve R_0 increase by the guaranteed rate of 1% to $R_1 = R_0(1 + r^g)$, while the investment portfolio only grew by $u_0 < r^g$. Only u_0 would then be available for investment I_1 at the beginning of the next period, ensuring a lower likelihood for future profit sharing if u-rates will rise to compensate the insurer because u_0 will have a weight in r_i^{fi} for 7 years. As a consequence the amount available for investment at the beginning of a period is always based on $\min(r_{t-1}^{fi}, r^g)$.

$t = 1$ Continuing with the example, $I_1 = \min(u_0, r^g)I_0 = 1\% \cdot I_0 = €\,741{,}923$ is invested in 7 year coupon bonds yielding u_1. At the end of the second period the total return on the fictive investment portfolio is equal to the coupons from the investments at the beginning of period 0 and the beginning of period 1, divided by the amount in the investment portfolio at the beginning of the second period. If $u_1 = 0.5\% (< r^g)$ this results in

$$r_1^{fi} = \frac{u_0 I_0 + u_1 I_1}{I_{\Sigma,1}} = \frac{1,854,807 + 3,710}{74,192,292 + 741,923} = 2.48\% \qquad \leftrightarrow \qquad \begin{aligned} PS_1 = 1.48\% \cdot R_1 = 1,109,175 \\ I_2 = 1\% \cdot R_1 = 749,342 \end{aligned}$$

with $I_{\Sigma,1}$ the total investments at the start of period 1. This immediately makes clear in what way the weighted structure in which r_t^{fi} is computed influences the profit sharing rate r_t^{PS} each period. Although the u-rate dropped by 2% there is still profit sharing as a result of the large weight I_0 has. This can be seen very clearly in figure 1.2(a)[4] in which the cubes represent the u-yield, the crosses the weighted u-yield (r_t^{fi}), the stacked bar the investment portfolio that visualizes the investment layers each year and the light red bar representing the reserve. The amount invested at inception determines the profit sharing rate almost completely for the coming 7 years and it is only when the investment is rolled over that the weighted u-yield r_t^{fi} really changes.

[4]This figure uses different u-yields than in the example.

One way to prevent this large dependence on one layer is to pay regular premiums during the life of the contract. This way the amount invested every period is similar and the evolution of r_t^{fi} will be more gradual. The sum of all premiums to be paid every year together with the interest received on them should be equal to €100 mln at the end of the contract. The computation to find the appropriate premium will not be done here but is described in the tool description which can be found in the Appendix A.1.1.2. The results in figure 1.2(b) confirm the above.

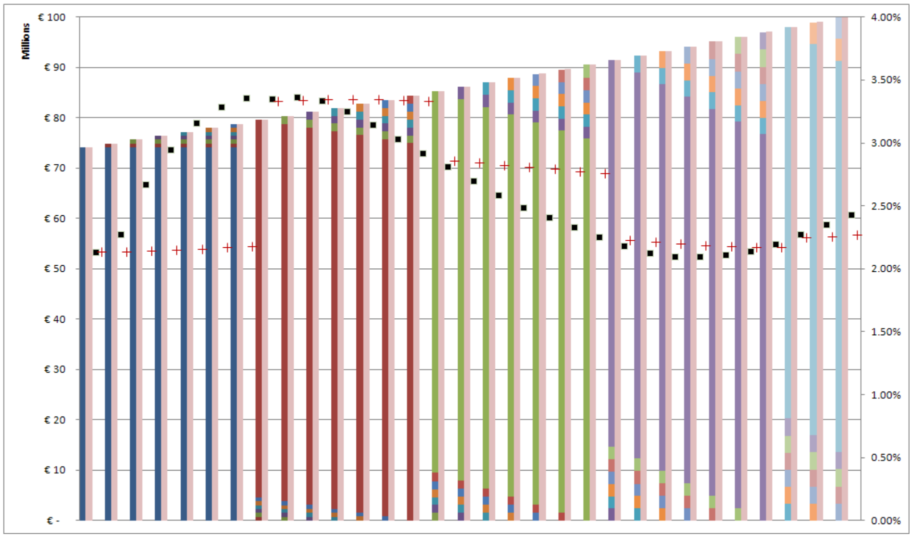
The figure shows the smoothing effect discussed earlier as the rate r^{fi}, that determines the profit sharing, can vary significantly from the u-yield in a period. This means that the periods in which profit sharing is to be paid out do not have to coincide with the periods in which the u-yield exceeds the guaranteed rate and the number of profit sharings do not have to match the number of times the u-yield exceeds the guaranteed rate.

....

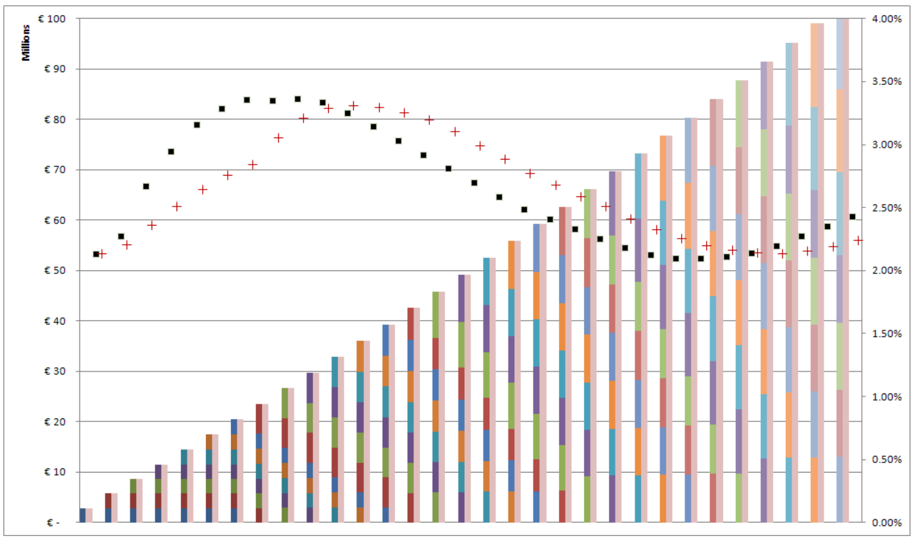
$\boldsymbol{t = 30}$ After the length of the contract, here 30 years, the guaranteed amount is paid to the policyholder. The amount in the fictive investment portfolio can however differ substantially from the one in the reserve. This is not the case in this example as the u-rate is above the guaranteed rate in every period but one can see already here that, when the insurance company sold contracts with a guaranteed rate just above 2%, significant losses can present themselves. It is emphasized once more that the both the amounts stated for the investment portfolio and the reserve have little to do with what the insurance company actually does with the premium that is received from the policyholder. This is traditionally invested in a combination of shares, real estate and bonds.

1.5 Valuation of product sharing products

A substantial amount of research has been done on the fair valuation of contracts that contain a form of embedded option. Not surprisingly this started shortly after the pioneering work of Black and Scholes (1973) on option pricing, beginning with Boyle and Schwartz (1997) on guarantees in equity linked products. As mentioned in the introduction the research related closely to the products discussed in this thesis emerged at the beginning of this century. First with more general approaches and continuing through the first decade to treatment of products sold specifically in the country of the authors origin. In the Netherlands Plat (2005) discussed the valuation of a contract in which the profit sharing rate is based on a portfolio consisting of fixed income products



(a) One lump sum payment at inception.



(b) Regular premiums.

Figure 1.2. *This figure shows how the reserve (light red bar), the fictive investment portfolio (stacked bar) and the weighted u-yield (cross) evolve over time as a consequence of a given RTS and set of u-yields (cubes) for a contract that promises €100 mln in 30 years based on an annual guaranteed return of 1%.*

and modeled the profit sharing as an Asian option to find an analytical approximation. In the work of Plat and Pelsser (2008) the earlier work of Plat is combined with results of Schrager and Pelsser (2006) on the valuation of swaptions in affine term structure models[5] in which the swap rate is modeled as an affine function of factors as well. Their results apply almost directly to the products

[5]These are arbitrage free models in which bond yields are a linear function of some state vector and in which cross and auto correlations can be modeled.

discussed in this thesis, though they will not be used here in the sense that a precise analytical formula is used for valuation. The structure used for the replication of the profit sharing part will however be similar.
The value of the profit sharing will also be approximated by creating a replicating portfolio and using the no-arbitrage argument that if the replicating portfolio has exactly the same cash flows as the profit sharing their value should be the same. Otherwise a riskless profit can be made by selling the one and buying the other.

The amount of profit sharing at the end of every period i is a function of the variable return on the fictive investment portfolio, the reserve at the beginning of the period, which is predetermined if the profit sharing is to be paid out, a constant guaranteed rate, a fee and a participation level:

$$PS_i(r_i^{fi}, R_i) = R_i \max\left[r_i^{fi} - (r^g + \delta), 0\right],$$

with R_i being the reserve at the beginning of period i, r^g the guaranteed rate and δ the fee, assuming that the participation α is equal to 1.

r_i^{fi}, the return of the investment portfolio at the end of period i, is a weighted function of the investments done in previous periods. For most insurance policies sold in the Netherlands r_i^{fi} would then be

$$r_i^{fi} = \sum_{q=(i-M+1)^+}^{i} \frac{u_q I_q}{I_{\Sigma,i}} \qquad I_{\Sigma,i} = \sum_{q=(i-M+1)^+}^{i} I_q, \tag{1.2}$$

with M the maturity of the bonds, u_q the u-yield at the beginning of period q, I_q the amount invested in period q, often called a layer and $I_{\Sigma,i}$ the sum of all investments at the beginning of period i. I_q, the amount that can be invested every period, depends on the turnover structure of the investments through the payments every period, the historical u-yields which determine the coupons and the premiums.

If r_i^{fi} would be an interest rate quoted in the market, this cash flow could be replicated by use of a strip of European *swaptions* on the interest rate r_i^{fi} of which one expires every period and has a strike $r^g + \delta$, a *notional* R_i and lasts one period. Using the distribution of r_i^{fi} the value can then be computed by use of standard option theory.

The fact that r_i^{fi} is a return on investments driven by a specific investment policy, that these investments have a yield that is itself a complex weighted average of yields and that the amount in the reserve can depend on profit sharing if the profit share is reinvested each year, complicates things.

First consider the simplified case in which the profit share is paid out every period and the turnover structure of the reinvestments specifies that the principal is paid back in full at maturity.
The element that is least straightforward but most crucial in modeling the replicating portfolio, as to match the cash flows of the profit share as good a possible, is determining the notionals of the swaptions. Intuivitively is it clear that these notionals should depend on the amount in the reserve, because this is the amount over which the profit sharing rate is due at the end of every period, and on the reinvestments at the beginning of the period, because this determines the weighting.
In the case the u-yield exceeds the guaranteed rate every period the weighing does not influence the profit sharing as to it is paid out or not because there will be profit sharing every period. The notionals of the underlying swaptions are then known for every period as the amount in the reserve R_i grows every year by a fixed rate r^g and this increase is equal to the reinvestment in the fictive portfolio next period . The notionals for the swaptions every period are in this case therefore equal to the amount available for investment in the fictive portfolio (return - profit sharing). The sum of all cashflows from the underlying *swaps* (swaptions are sure to end *in the money*) will now exactly match the profit share because the notionals perfectly replicate the weighing scheme or reinvestments.

However, in general the amount in the investment portfolio and the amount in the reserve will not be equal. If the return on the fictitious investments in a period is below the guaranteed rate, $r_i^{fi} < r^g$, the reserve will grow faster than the fictive investment portfolio. If $\delta > 0$ and $0 < r_i^{fi} - r^g < \delta$ the reserve will just grow by r^g but the investments will grow by $r_i^{fi} > r^g$. Therefore the investments made every period by the investment portfolio, though they match the weighing part perfectly, can not be used as notional for the swaptions.
The solution is that the notionals should be based on the reserve, as this is the amount that determines the profit sharing together with r^{fi}. To incorporate the weighting effect correctly the premiums paid should be invested using the same policy as the fictive portfolio but based on the guaranteed rate r^g instead of the u-yields.

Under the assumption that profit share is paid out every period and the turnover structure is just that the principal is paid back in full at maturity M, the swaption notional N_i for period i is determined by the recursive formula

$$N_i = N_{i-M} + r^g \sum_{q=(i-M)^+}^{i-1} N_q = N_{i-M} + r^g R_{i-1}.$$

Assuming also that there is a good proxy for the u-yield, which seems to be a reasonable assumption considering the analysis in the last subsection (see also 1.1), the value of the profit sharing element at the start of period t is then equal to the strip of swaptions

$$V_t^{PS} = \sum_{i=t}^{n} V_{t,i}^{swaption}(N_i, \sigma_{u_i}, r^g + \delta, M), \qquad (1.3)$$

with σ_{u_q} the *implied volatility* of the u-yield proxy, $r^g + \delta$ the strike, i the exercise date, M the length of the underlying swap and n the horizon of the contract.[6]

For two interest rate term structures the example of section 1.4 is worked out for the first eleven years. Table 1.1 shows the amount of profit sharing PS and the cash flows of the replicating portfolio together with general information on the evolution of the reserve, investment portfolio and the 7 year swap rate, taken as proxy for the u-rate. Figure 1.3(a) and 1.3(b) show how the notionals influence the cash flows from the replicating portfolio for these examples.

			Flat 2%					
	R_i	$I_{\Sigma,i}$	I_i	N_i	r^i_{swap}	r^{fi}	PS_i	CF_i^{rs}
2012	€ 74,192,292	€ 74,192,292	€ 74,192,292	€ 74,192,292	2.00%	2.00%	€ 741,923	€ 741,923
2013	€ 74,934,215	€ 74,934,215	€ 741,923	€ 741,923	2.00%	2.00%	€ 749,342	€ 749,342
2014	€ 75,683,557	€ 75,683,557	€ 749,342	€ 749,342	2.00%	2.00%	€ 756,836	€ 756,836
2015	€ 76,440,393	€ 76,440,393	€ 756,836	€ 756,836	2.00%	2.00%	€ 764,404	€ 764,404
2016	€ 77,204,797	€ 77,204,797	€ 764,404	€ 764,404	2.00%	2.00%	€ 772,048	€ 772,048
2017	€ 77,976,845	€ 77,976,845	€ 772,048	€ 772,048	2.00%	2.00%	€ 779,768	€ 779,768
2018	€ 78,756,613	€ 78,756,613	€ 779,768	€ 779,768	2.00%	2.00%	€ 787,566	€ 787,566
2019	€ 79,544,179	€ 79,544,179	€ 74,979,858	€ 74,979,858	2.00%	2.00%	€ 795,442	€ 795,442
2020	€ 80,339,621	€ 80,339,621	€ 1,537,365	€ 1,537,365	2.00%	2.00%	€ 803,396	€ 803,396
2021	€ 81,143,017	€ 81,143,017	€ 1,552,738	€ 1,552,738	2.00%	2.00%	€ 811,430	€ 811,430
2022	€ 81,954,447	€ 81,954,447	€ 1,568,266	€ 1,568,266	2.00%	2.00%	€ 819,544	€ 819,544
			Example section 1.4					
	R_i	$I_{\Sigma,i}$	I_i	N_i	r^i_{swap}	r^{fi}	PS_i	CF_i^{rs}
2012	€ 74,192,292	€ 74,192,292	€ 74,192,292	€ 74,192,292	2.50%	2.50%	€ 1,112,884	€ 1,112,884
2013	€ 74,934,215	€ 74,934,215	€ 741,923	€ 741,923	0.50%	2.48%	€ 1,109,175	€ 1,112,884
2014	€ 75,683,557	€ 75,683,557	€ 749,342	€ 749,342	1.00%	2.47%	€ 1,109,175	€ 1,112,884
2015	€ 76,440,393	€ 76,440,393	€ 756,836	€ 756,836	1.50%	2.46%	€ 1,112,959	€ 1,116,669
2016	€ 77,204,797	€ 77,204,797	€ 764,404	€ 764,404	2.00%	2.45%	€ 1,120,603	€ 1,124,313
2017	€ 77,976,845	€ 77,976,845	€ 772,048	€ 772,048	0.50%	2.43%	€ 1,116,743	€ 1,124,313
2018	€ 78,756,613	€ 78,756,613	€ 779,768	€ 779,768	0.50%	2.41%	€ 1,112,844	€ 1,124,313
2019	€ 79,544,179	€ 79,544,179	€ 74,979,858	€ 74,979,858	0.50%	0.53%	€ -	€ 11,428
2020	€ 80,339,621	€ 79,964,681	€ 1,162,425	€ 1,537,365	1.00%	0.53%	€ -	€ 11,428
2021	€ 81,143,017	€ 80,385,679	€ 1,177,759	€ 1,552,738	2.00%	0.52%	€ -	€ 26,955
2022	€ 81,954,447	€ 80,807,603	€ 1,201,314	€ 1,568,266	2.50%	0.52%	€ -	€ 50,479

Table 1.1. *This table shows how the cashflows from the replicating portfolio CF_i^{rs} evolve compared to the profit sharing PS_i for a product that is specified a guaranteed amount of € 100 mln in 30 years based on a technical rate of 1% in 2 scenarios. r^i_{swap} is the swap rate used for the swaptions, in this case the euro 7 year swap rate.*

From table 1.1 it can be seen that the replicating portfolio exactly matches the profit sharing in case

[6]See equation 2.2 for the definition of $V_q^{swaption}$.

the weighted u-rate, or investment portfolio return r_i^{fi}, is always above r^g, which is the case if the interest rate term structure is flat at 2%.[7]

The more realistic scenario from the example reveals an important property of the replication method. The replicating portfolio pays out more often than there is profit sharing. This is also seen in figure 1.3(b) and it is a consequence of the swaptions because the replicating portfolio in period i pays out the excess of u_i to r^g for the coming 7 years, based on just one u_i, while the product pays out the excess of $r_i^{fi} - r^g$ once, based on the weighted average of multiple u_i in r_i^{fi}.[8] It will therefore never be the case that the amount of profit sharings over the life of the product exceeds the amount of cash flows coming from the replicating portfolio. It can neither be the case that the amount of profit sharing in a period exceeds the cash flows from the replicating portfolio, this shows the dominance of this replication method but also that the it is not exact.

The value of the profit sharing in this product for the realistic term structure from figure 1.2(a) in section 1.4 is then computed using equation 1.3 and amounts to €31,513,290. This immediately shows how significant the contribution of the profit sharing element is to the value of the product.

In case the turnover structure is different, one has to consider declining investments in all layers, resulting in lower coupons, and reinvestments depending also on payments from investments in multiple periods. This will be discussed in the next subsection.

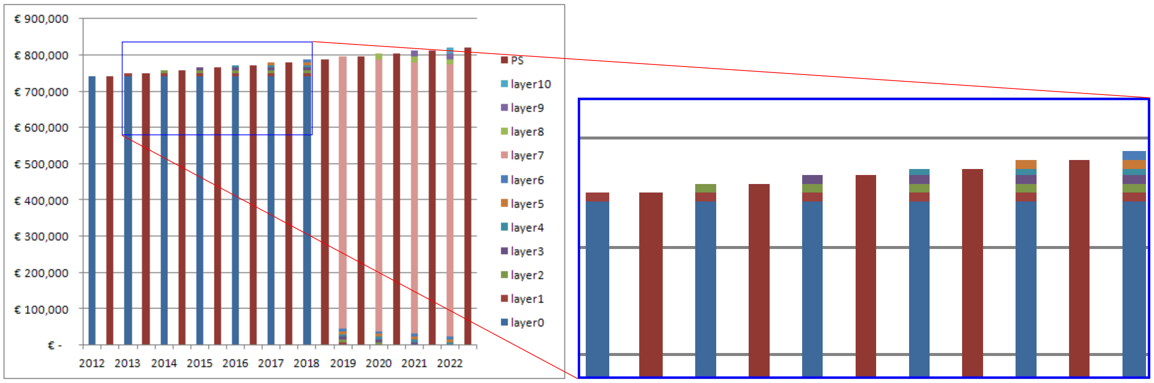
If the profit share is not paid out every period the reserve will not increase by a fixed amount of r^g but will also depend on the profit sharing. This means that R_i is not predetermined anymore but will depend on the stochastic u-yields as well and this essentially means that the insurer promises additional future profit sharing on already uncertain profit sharing. This further complicates the value of the embedded options.

1.6 Some standard profit sharing products

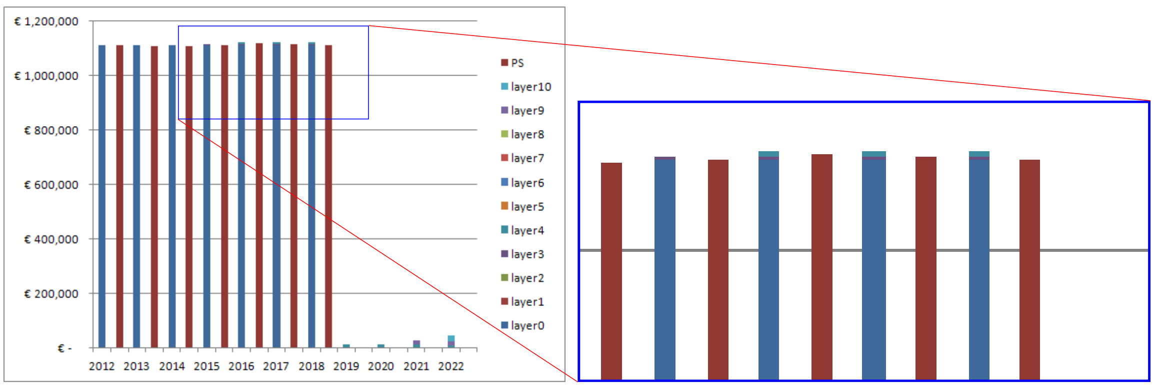
As mentioned in the last section the fictive investment portfolio is led by a certain policy using assets with a specific turnover structure. In fact most of the insurance companies have their own investment policies and all use a different turnover structure for their fictive assets. By doing this the insurance company aims to reduce the variance of the weighted u-rate r_i^{fi} or let r_i^{fi} to be less dependent on

[7]The same would be the case if the u-yield would always be below the guaranteed rate as there will be no profit sharing and all swaptions will end out of the money.

[8]Actually the 7 cash flows from the swap are discounted and the trade is settled when the option expires, but this has no further consequences.



(a) Flat 2%



(b) Example of section 1.4

Figure 1.3. *These figures show how the underlying swaps in the replicating portfolio contribute to the cash flow from the replicating portfolio for two scenarios. One that should be similar to the profit sharing, for an interest rate term structure that is flat at 2%, and for the example of section 1.4*

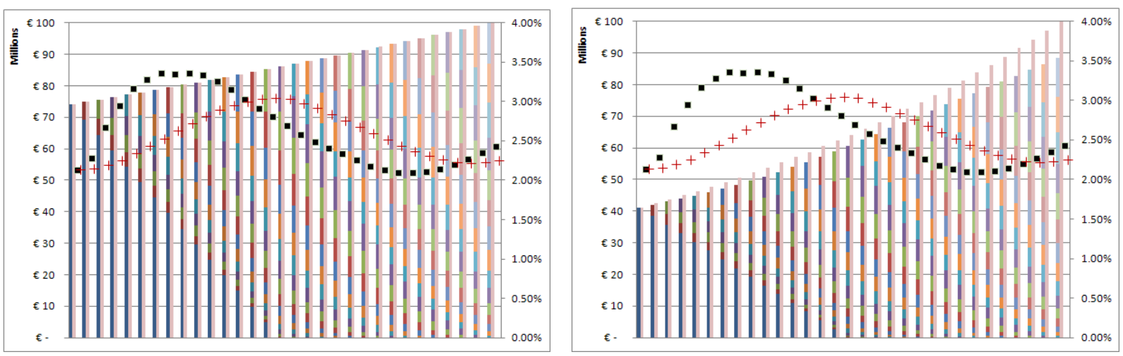
u-yields further back in time. In this section two turnover structures will be treated that are known to represent a significant amount of the profit sharing products sold in the Netherlands.

1.6.1 Company A

Company A sells a profit sharing product similar to the type described above with the only difference that the fictive investment portfolio invests in bonds with a 15 year maturity and a turnover structure specifying payments of $\frac{1}{15}^{th}$ of the principal at the end of each period. This means that the weight of the u-yield u_q of period q, in r_i^{fi} (equation 1.2) declines every period by $\frac{1}{15}^{th}$. Figure 1.5(a) shows how the investments, the u-yield, r^{fi} and the reserve evolve for this product for the same scenario as figures 1.2(a) and 1.2(b).

The figure implies that the replicating strategy from the previous section adds too much weight to the investment layers further away. A replicating strategy in line with the last section requires the use of swaptions of which the underlying swap notionals decline by $\frac{1}{15}^{th}$ every period. Although it might be possible to find an analytical valuation of this, it is not within the scope of this thesis. Another way of modeling this would be to use fifteen swaptions for every investment layer: one on a 15 year swap with $\frac{1}{15}$ of the investment layer as notional, one on a 14 year swap with $\frac{2}{15}$ of the investment layer as notional, etc. Because this is a bit cumbersome and again not in line with the purpose of this thesis a solution could be to work with underlying swap maturities equal to the weighted average maturity M_q of an investment layer, the use of an average notional that results in swaptions paying too little in beginning periods, too much during the last periods but are good on average, or a combination of the two. The trick is then to replace the fifteen swaptions that are optimally required by significantly less. The trade off in selecting the optimal replicating strategy will be in the extent to which the M year swap rate is still a good proxy for the u-rate and the similarity of the swaption payoffs to the profit sharing that is determined by the turnover structure, both in time and in size.

Using the weighted average maturity of an investment layer essentially entails trying to mimic



(a) Guaranteed rate $r^g = 1\%$ **(b)** Guaranteed rate $r^g = 3\%$

Figure 1.4. *This figure shows how the reserve (light red bars), the fictive investment portfolio (colored bars) and the weighted u-yield (red crosses) evolve over time as a consequence of a given RTS and set of u-yields (black cubes) for a contract that promises 100 mln in 30 years, based on a guaranteed rate of a)1% and b)3%. The contract further specifies that the underlying investment portfolio used bond with maturity of 15 years of which $\frac{1}{15}$ is paid back every year.*

15 swaptions with 15 different swap maturities and notionals, by one swaption with the average swap maturity and a notional equal to the entire investment layer. In this case the weighted average maturity M_w equals

$$M_w = \sum_{q=0}^{15-1} (15-q)\frac{q}{15} = 8$$

The expiration dates of the swaptions still coincide with the investments layers as before. Theoretically this would then cause cash flows that will be too large from the second year until the 8^{th} year and cash flows that will be too low (0) from the 9^{th} until the 15^{th} year.
A strategy that would create cash flows over the entire period of the 15 years in which the investment layer influences the profit sharing, but still uses only one swaption per layer, could be the use of an average notional in 15 year swaptions. The average investment over the 15 years for investment I_i is

$$N_i = \frac{1}{15}\sum_{q=0}^{15-1}(1-\frac{q}{15})I_i^g = 0.533I_i^g.$$

The superscript in I_i^g here means that these are again the investments based on the guaranteed rate, as explained in the last section and, more elaborately, in appendix A.1.1.2.
This strategy would theoretically cause cash flows being too low during the first years, exactly right in the middle and to large during the last years.
A disadvantage that is also encountered using a weighted maturity, is that the 8 year and the 15 year swap rates might not match the u-yield very closely anymore. To adjust for the mismatch between the u-yield and the 8 and 15 year swap rate an adjustment can however be made to the strike of the swaptions that should be quite reliable. This would amount in adjustments of respectively 10 basis points(0.10%) and 50 basis point (0.5%) upwards, following the average difference from historical time series over the last 12 years.

Besides a motivation to use 8 year swaptions is that the 8 year swap rate can be expected to match the u-rate more closely then the 15 year swap rate, and that the notional might result in cash flows more close to the profit sharing during the first 8 years compared to the use of an average notional, the disadvantage remains that the cash flows following the underlying swaps will not coincide with the weight of the investment layers. This is because the underlying swaps last 8 years and the layers 15 years. Another question that remains is whether the notionals of the 8 year swaptions should b based on the 15 year investment policy, in line with the fictive investment portfolio, or on an 8 year investment policy.

The use of 15 year swaptions with an average notional will circumvent the problem of not taking into account layers which do influence the profit sharing but might result in differences in cash flows that are too large. This will be both due to the use of an average notional and due to the fact that 15 year swaptions will cause 15 cash flows, when the swap rate is higher than the strike, whereas profit sharing causes a maximum of one cash flow, based on the weighted rates over the last 15 years in r^{fi}.

To prevent too much divergence in the cash flow pattern a hybrid of the two, that tries to capture the fifteen swaptions ideally required for an investment layer in two swaptions, will also be considered. One hybrid will be based on two swaptions that both have a maturity of 7 years to minimize the difference between the swap rate and the u-yield. One will have a notional of the average investment during the first 7 years with a start date equal to the start of the period of the investment, and one with a notional equal to the average remaining investment during the subsequent 7 years. Note that this strategy will cause at most fourteen cash flows, seven based on the swap rate in the period the investment is done, seven based on the swap rate 7 years later. This strategy will cause a u-rate in period i to have an influence based on the investments in period i and investments of 7 periods ago. This way it will ensure a less diverge cash flow pattern while on average the whole notional of every layer is still considered, but giving up some of the right weighing because a u-rate in year i is only partially weighted by an investment of year i.
Another approach would be to try mimicking the weighing more precise. This would mean that the swap rates match the u-rate less and it causes more diverge cash flows. A way to do this with two swaptions would be to use one swaption with a maturity of 15 years and a notional equal to the average amount of the investment that lasts more than 7 years, and one swaption with a maturity of 7 years and a notional equal to the average amount of the investment the first 7 years minus the portion covered by the 15 year swap. Both starting when the investment is made. If the swap rate in period i would then be higher then the strike this would cause two cash flows during the first 7 years and one cash flow during the last 8. In summary there are then 5 replicating strategies that try to mimic the profit sharing by a maximum of 2 swaptions per investment layer:

1. Use of one swaption per investment layer using a weighted average swap maturity of 8 years, notionals based on the investment policy of the fictive portfolio and a strike of $r^g + \delta + 10bps$.
2. Use of one swaption per investment layer with a weighted average swap maturity of 8 years, notionals based on a investment policy following a turnover structure specifying notionals are paid back in full after 8 years and a strike of $r^g + \delta + 10bps$.
3. Use of one swaption per investment layer with a swap maturity of 15 years, a weighted average notional based on the investment policy of the fictive portfolio and a strike of $r^g + \delta + 50bps$.
4. Use of two swaptions per investment layer with a swap maturity of 7 years, one with a weighted notional based on the first 7 years, the other one with maturity of 7 years and a weighted notional based on the subsequent 7 years. Both following the investment policy of the fictive portfolio and a strike of $r^g + \delta$.
5. Use of two swaptions per investment layer, one with a swap maturity of 15 years, a weighted notional based on investments lasting over 7 years (30% I_q) of the notional and a strike of

$r^g + \delta + 50bps$, the other one with maturity of 7 years, a notional that sets the combined notional of the two swaptions to the weighted average layer over the first 7 years ((80% - 30%)I_q = 50%I_q) and a strike of $r^g + \delta$. Both following the investment policy of the fictive portfolio.

To evaluate which of the strategies will be likely to be most effective two scenarios will be considered. The first will be a scenario for which the product is originally designed with a u-yield that is always above the guaranteed rate. This is the scenario in figure 1.4(a) in which the guaranteed rate is again 1%. The second scenario will be one in which there is profit sharing in some years and non in other years, for this the same interest rate term structure is considered as in the first scenario but with a guaranteed rate of 3%, this is displayed in 1.4(b).[9] Figures 1.5(a) and 1.5(b) show the

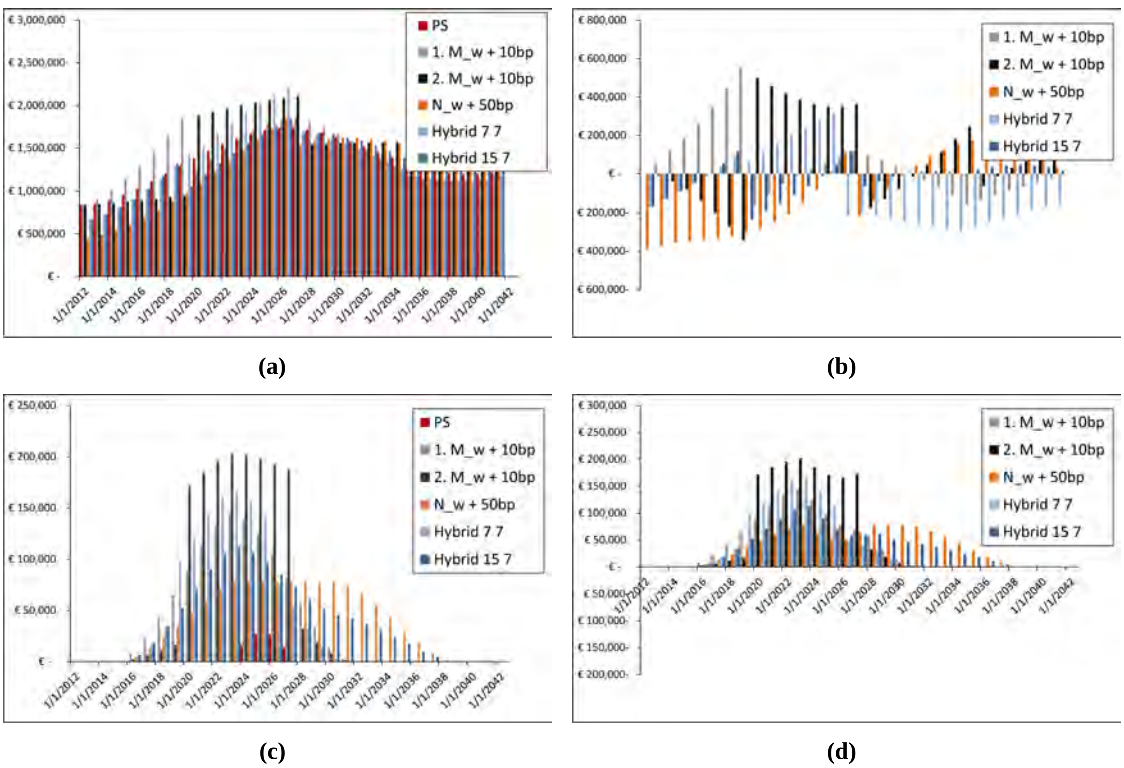


Figure 1.5. *This figure shows the result of the 5 replicating strategies discussed in this subsection (1.6.1) for a profit sharing product that pays €100 mln in 30 years based on a guaranteed rate of 1% [fig a,b] or 3% [fig c,d]. Figures b and d show the difference with the profit sharing cash flows.*

results of the five strategies for the first scenario, figures 1.5(c) and 1.5(d) for the second. It should be mentioned that the 15 and 8 year swap rates are assumed to be exactly the same amount of basis

[9]The third scenario that could be considered would be one in which the u-yield is always below the guaranteed rate but this would be matched perfectly by all strategies because there will be no profit sharing and all swaptions will end out of the money.

points above the 7 year swap rate as the upward adjustment of the strikes and the the 7 year swap rate is a good proxy for the u-rate. This means that the focus here is purely on the replication of the cash flows from profit sharing by use of a few swaptions when the underlying decreases by a fixed turnover pattern. From this figure it is clear that most strategies are not really satisfactory during the first years. The first two strategies make use of one swaption for every layer based on a weighted average maturity of 8 years but converge more to the profit sharing in later years. The main advantage of this strategy is that the cash flow pattern will match more closely to the amount of profit sharings than the strategies that use 15 year swaptions. Also the cash flows match the profit sharing better in the first years because no average notional is used. This result is most pronounced in figure 1.6(a), comparing the orange bar (strategy 3) with the black or grey bar (strategies 1 and 2). However, after a couple of years it is clear that cash flows start to diverge significantly compared to the profit sharing in scenario 1 and exceed the profit sharings by far in scenario 2. Comparing the two with each other suggest it is important to base the swaption notionals on the re-investments done in the fictive portfolio and not based on a simplified turnover structure that matches the maturity of the swaptions. The black bar, representing strategy 2, does not move as gradual as one would like, though some smoothing seems to occur at the end.
The third strategy, that uses an averaged notional and 15 year swaptions, results in significantly less then the profit sharing in the first years of scenario 1 and shows a very diverge cash flow pattern of twenty three swap payoffs compared to four profit sharing in scenario 2, where they are also at least twice as large as the profit sharing.
The use of a hybrid strategy seems a good compromise. The use of two 7 year swaptions is attractive because it is simpler and the 7 year swap rate proxies the u-rate best, a worrying observation is however that the differences with the profit sharings start to become more significant in periods further in the future, whereas the other strategies show a greater amount of convergence there. Though the use of lower maturities results in a number of cash flows from the replicating strategies that is more close to the number of profit sharings in times like scenario 2, this advantage is offset by the the extent in which the cash flows are larger than the profit sharings. This strategy also seems to be expensive, probably due to the use of swaptions having a longer time to expiration, increasing the time value in the options.
The second hybrid strategy seems to be a good remedy for the disadvantages of the third strategy that were just pointed out. Also the disadvantage of non-convergence of the first hybrid model does not occur, this suggests that the last strategy might be the best way to model the replicating portfolio by just two swaptions per layer.

Table 1.2 gives some summary statistics of all replicating strategies. The statistics seem to confirm with the results from the above analysis as the second hybrid strategy (strategy 5) shows the

	Strategy									
	1.		2.		3.		4.		5.	
					Scenario 1 : $r^g = 1\%$					
$\bar{\Delta}$	€ 42,244	(0.06%)	€ 81,369	(0.09%)	€ 88,148-	(-0.12%)	€ 86,323-	(-0.09%)	€ 22,569-	(-0.03%)
min	€ 162,995-	(-0.20%)	€ 342,880-	(-0.43%)	€ 391,727-	(-0.53%)	€ 304,261-	(-0.33%)	€ 234,918-	(-0.30%)
max	€ 552,987	(0.70%)	€ 500,461	(0.62%)	€ 177,337	(0.19%)	€ 316,803	(0.38%)	€ 115,021	(0.15%)
σ	€ 168,441		€ 226,237		€ 201,241		€ 185,928		€ 84,203	
value	€ 32,644,645		€ 30,485,836		€ 25,006,648		€ 31,482,620		€ 28,738,478	
					Scenario 2 : $r^g = 3\%$					
	1.		2.	3.	4.		5.			
$\bar{\Delta}$	€ 35,237	(0.06%)	€ 49,854	(0.09%)	€ 35,214	(0.06%)	€ 36,646	(0.07%)	€ 35,237	(0.06%)
min	€ -	(0.00%)	€ -	(0.00%)	€ -	(0.00%)	€ -	(0.00%)	€ -	(0.00%)
max	€ 144,430	(0.25%)	€ 202,318	(0.35%)	€ 78,361	(0.13%)	€ 165,761	(0.29%)	€ 111,574	(0.20%)
σ	€ 47,280		€ 79,321		€ 30,630		€ 56,419		€ 34,641	
value	€ 11,230,454		€ 10,530,109		€ 8,021,951		€ 11,694,748		€ 9,654,201	

Table 1.2. *This table shows some summary statistics about the strategies proposed in this section to replicate the profit sharing from a contract that pays €100 mln in 30 years in two scenarios. The fictitious investments portfolio is driven by a turnover structure that specifies principal payments of $\frac{1}{15}$ of the notional from bonds invested in that have a maturity of 15 years. $\bar{\Delta}$ represents the average difference between the profit sharing and the cash flows from the replicating strategy.*

smallest average difference between the profit sharings and swap cash flows, has the smallest global difference ($\max - \min$) and the lowest standard deviation in scenario 1. Surprisingly, strategy 3 seems to outperform strategy 5 for all measures in scenario 2 by a small amount. The percentages behind the measures represent the mismatch in proportion to the amount in the reserve, this suggest that all strategies perform reasonably well. Considering the percentages one might give the most weight to the "normal" scenario 1, and then the second hybrid model would be classified as best with an average difference of 0.3% of the reserve between the swap cash flows and the profit sharing over the life of the product, while showing also a significantly lower standard deviation than the other strategies.

An important conclusion that can be made is that, when choosing a replicating strategy, the "weighing" factor is more important than the "timing" factor. What is meant with this is that giving u-rates the best weight as possible over the life of the investment layer is preferred over trying to match the amount of profit sharings by choosing lower maturities. Furthermore it seems best to base the notionals on the fictive investment policy, not adjusting for the swap maturity chosen in the replicating strategy.

1.6.2 Company B

Company B sells a profit sharing product using a fictive investment portfolio that invests in bonds with a 10 year maturity and a turnover structure specifying payments of $\frac{1}{5}^{th}$ of the principal during the last 5 years. This represents a kind of compromise between a full redemption at maturity and the gradual even turnover structure seen in the last subsection for Company A. In principle this could be replicated by 5 swaptions for every period: a 10 year swaption with a notional of $\frac{1}{5}^{th}$ of the investment layer, a 9 year swaption with a notional of $\frac{2}{5}^{th}$ the investment layer, etc., and ending with a 5 year swaption having a notional equal to the entire amount of the investment layer. Here this portfolio will be replicated using again a maximum of two swaptions. It can be expected that the use of a weighted average maturity will result in better results than in the last subsection because repayments of an investment layer begin only after 5 years and last only 5 years. Results from the last section however suggest that the weighing should be replicated as good as possible. This would result in using a hybrid strategy consisting of 1 swaption with a notional equal to the 40% of the investment and a maturity of 5 years and 1 with a maturity of 10 years and a notional equal to the average investment layer during the last 5 years (60%). The strategy that rivaled the hybrid one in the last section, using a weighted average notional over the full maturity, will also be considered.

In summary the following strategies will be considered:

1. Use of one swaption per investment layer with a weighted average swap maturity of

$$M_w = \frac{1}{5}\big(10+9+8+7+6\big) = 8,$$

   a notional equal to the full amount of the investment layer and a strike of $r^g + \delta + 10bps$.
2. Use of one swaption per investment layer using a notional based on the weighted average investment of

$$N_i = \frac{1}{10} M_w I_i^g = 0.8 I_i^g,$$

   a swap maturity equal to the maturity of the investments of the fictive portfolio and a strike of $r^g + \delta + 20bps$.
3. Use of two swaptions per investment layer, one with maturity of 10 years, a weighted notional based on investments lasting over 5 years (60% of the layer) and a strike of $r^g + \delta + 20bps$, the other one with maturity of 5 years, a notional equal to 40% of the full amount of the investment layer of the period and a strike of $r^g + \delta - 15bps$.

All notionals are based on the investment policy of the fictive portfolio.

The figures below give the results for these replicating strategies in the same way as the last subsection. Again two scenarios are considered, one where all u-rates during the life of the contract are above the guaranteed rate of 1%, causing profit sharings every year, and one scenario where there is profit sharing in some years and none in others due to a higher guaranteed rate of 3%. From figures

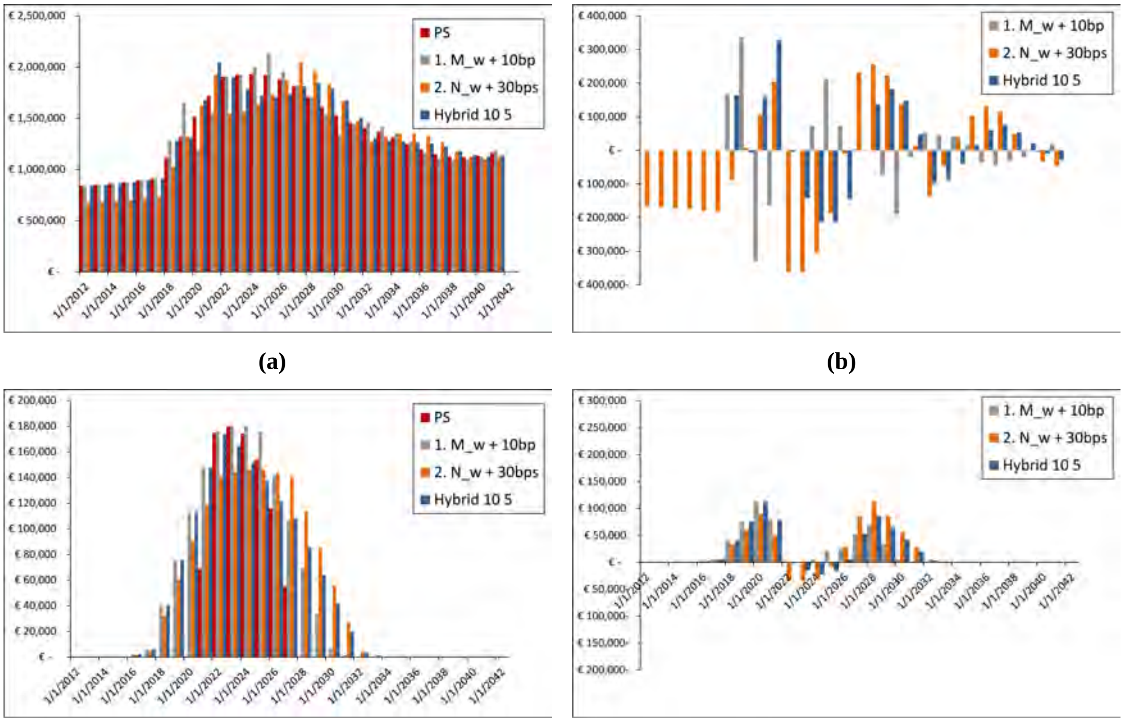


Figure 1.6. *This figure shows the result of the 3 replicating strategies discussed in this subsection (1.6.2) for a profit sharing product that pays €100 mln in 30 years based on a guaranteed rate of 1% [fig a,b] or 3% [fig c,d]. Figures b and d show the difference with the profit sharing cash flows.*

1.6(a) and 1.6(b), considering scenario 1, the second replicating strategy, using one swaption with a weighted notional and a underling swap maturity of 10 years, seems to perform less well than the other two. This underperformance is not so clear in the second scenario. Though the hybrid model seems to outperform the first strategy, that is based on just one swaption and a weighted average swap maturity, it is not really significant. Table 1.3 suggest that the first strategy is however most effective with an average difference between the cash flows from the strategy and the profit sharing of 0.01% of the reserve and a slightly lower standard deviation. This outperformance is relatively low and, as such, might be insignificant. The fact however remains that, when choosing between two replicating strategies that perform equally well, one that uses only one swaption per layer is preferred.

Results from the past two sections then give slightly mixed signals as to what approach is generally the best. It is clear that it is optimal to base the notionals on the swaptions on the same investment policy (maturity and turnover) as the fictive portfolio, not adjusting for the maturity of the swaption that is used to replicate the profit sharing. However, where the first example (Company A) clearly shows a preference for a strategy that uses swaptions with the same swap maturity as the maturity

	Strategy 1.		Strategy 2.		Strategy 3.	
			Scenario 1 : $r^g = 1\%$			
$\bar{\Delta}$	€ 4,128	(0.01%)	€ 32,494-	(-0.05%)	€ 12,936	(0.02%)
min	€ 329,228-	(-0.41%)	€ 361,616-	(-0.44%)	€ 214,647-	(-0.25%)
max	€ 337,422	(0.42%)	€ 255,808	(0.29%)	€ 327,864	(0.40%)
σ	€ 113,308		€ 168,919		€ 114,870	
value	€ 31,420,578		€ 28,075,902		€ 30,502,333	
			Scenario 2 : $r^g = 3\%$			
$\bar{\Delta}$	€ 17,223	(0.03%)	€ 17,223	(0.03%)	€ 17,223	(0.03%)
min	€ -	(0.00%)	€ 35,382-	(-0.06%)	€ 22,246-	(-0.04%)
max	€ 112,705	(0.22%)	€ 113,220	(0.17%)	€ 112,705	(0.22%)
σ	€ 30,019		€ 37,175		€ 33,900	
value	€ 10,862,386		€ 9,907,173		€ 10,711,014	

Table 1.3. *This table shows some summary statistics about the strategies proposed in this section to replicate the profit sharing from a contract that pays €100 mln in 30 years in two scenarios. The fictitious investments portfolio is driven by a turnover structure that specifies principal payments of $\frac{1}{5}$ of the notional during the last 5 years from bonds invested in that have a maturity of 10 years. $\bar{\Delta}$ represents the average difference between the profit sharing and the cash flows from the replicating strategy.*

of the assets in the fictive investment portfolio, the second example (Company B) does not display a real preference. This suggests that there is not one optimal approach to choose the best replicating strategy and this should be tested on a case by case basis.
Both tables also show the value of the replicating portfolio at $t = 0$. This shows that the use of an average notional results in the lowest valuation. Interesting is also that the value of the profit sharing from Company B is slightly above the value of the profit sharing from Company A. This displays that a fictive investment policy using a turnover structure that equalizes investment layers quickly, reduces the value of the profit sharing while the expected amount of profit sharing remains very similar.

1.7 Summary

In this chapter the profit sharing product was discussed elaborately. The main purpose of the chapter was to create insight in the purpose, determinants and dynamics of the product, with special attention for the profit sharing element. In this, the importance of the u-yield and the fictive investment strategy was pointed out.
A general method to value the profit sharing by use of a replicating portfolio of swaptions was given

and the performance of this valuation was evaluated for several scenarios. Results showed that the method produces robust results for a basic profit sharing product. For products specified by a more complicated investment strategy, a replicating portfolio could in theory still produce accurate results. Because this does however not have any practical value, here strategies to replicate the products by use of a simple portfolio were constructed. In doing this one has to find the optimal trade off between trying to replicate the number of profit sharings and trying to replicate the right weight that investments have. What this trade off should be depends specifically on the turnover structure of the fictitious investments and has to be determined on a case by case basis.

Chapter 2

Risks

Chapter 1 discussed the history of the profit sharing product, gave a detailed description on how such a product behaves as a consequence of combinations of market circumstances and contract specifications, and what approach can be taken to value it. This chapter will go into details on the risks that these products bring to the books of the insurance company that sold them. The most important risk is interest rate risk, this will be described in the first section. A slightly different way of replication than used in section 1.5 will be used here and the hazardous situation in which some insurance companies now find themselves due to these products will be addressed. It will also become clear that the optional character of the product is much more pronounced in scenarios in which the u-yield is close to the guaranteed rate. Furthermore, an important assumption made in the valuation, namely that the 7 year swap rate will proxy the u-rate, will be reconsidered and the consequences of some scenarios in which it might not be a good proxy will be evaluated. For this, a first some theory will be presented that motivates the use of two curves for the valuation of swaps and swaptions. A model based on this theory will then be proposed and evaluated. The results will be applied to analyze the evolution in the value of the guarantee and the profit sharing using historical data for several European countries. The chapter concludes with a summary.

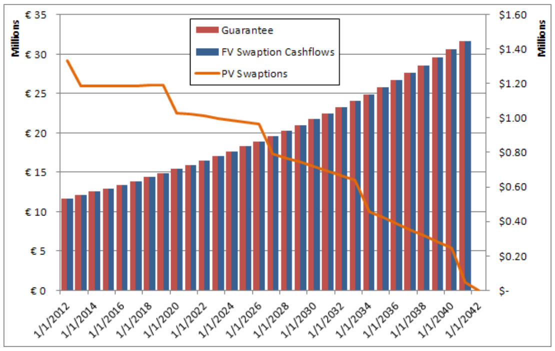
2.1 Interest rate risk

By far the most influential variable to the value of a profit sharing product is the interest rate. To be more precise, the difference between the guaranteed rate r^g and the u-rates. An insurance company can get in to serious trouble if the u-rates drop below the guaranteed rate. This scenario was not really discussed in chapter 1 because this is not a scenario for which this product was designed. The insurance company will be obligated to live up to the contractual agreement of paying the policyholder interest equal to r^g in this case, while the fictive investment portfolio only generates r^{fi}, that will be lower than r^g if the u-rates stay low over a large enough time span. For a scenario in which all u-rates are below the guaranteed rate ($u_i < r^g$), the value of the profit sharing will be very close to zero.[1] The swaptions specified in the replicating portfolio from section 1.5 will all end out of the money and the value of the contract will be equal to a zero coupon bond paying the guaranteed amount at maturity. Though this valuation is still correct it is not very informative for the interest rate risk anymore. Another approach to the replication of this product would be to use swaptions for the downside ($r^{fi} < r^g$), modeling the guarantee explicitly, instead of the upside ($r^{fi} > r^g$), that models the profit sharing. In this way it will become clear how expensive these products currently are for the insurance companies, as interest rates are at an all time low. Essentially exactly the same scenario repeats itself now as in the late nineties. Back then insurance companies sold the product with guaranteed rates of around 8%, because interest rates were above 10% and the implicit guaranteed rates were considered harmless. Most insurance companies that experienced the drop in interest rates to 4-6% just lowered the guaranteed rate for the policies to around 3% and are therefore likely to face similar difficulties due to this guarantee as before.

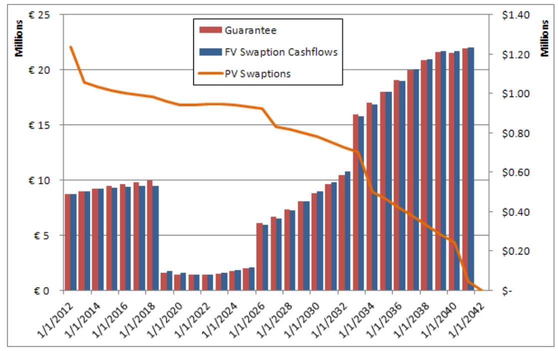
In summary two methods to replicate the cash flows from a contract that promises to pay €100 mln in 30 years based a guaranteed rate of 3% and a fictive investment portfolio that invests in 7 year u-rate bonds with a turnover structure specifying full principal payments at maturity of the bond, are:

1. Modeling the profit sharing explicitly with the results of section 1.5 and a zero coupon bond.
   i) 30 payer swaptions with expiration dates just before the start of every year, an underlying swap maturity of seven years, a strike equal to r^g and a notional determined by the the investment strategy of the fictive portfolio based on the guaranteed rate (see section 1.5 and appendix A.1.1.2).
   ii) A zero coupon bond paying €100 mln in 30 years.
2. Modeling the guarantee explicitly using receiver swaptions and bonds yielding u_i.
   i) 30 receiver swaptions with expiration dates just before the start of every year, an underlying swap maturity of seven years, a strike equal to r^g and a notional determined by the the investment strategy of the fictive portfolio based on the guaranteed rate.
   ii) Investments in bonds equal to the ones specified by the investment policy of the fictive portfolio.

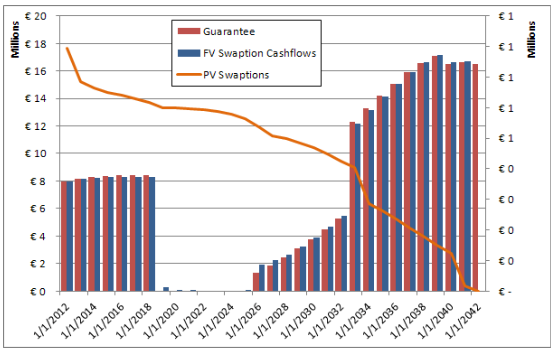
[1] There will be some time value for the optional profit sharing.



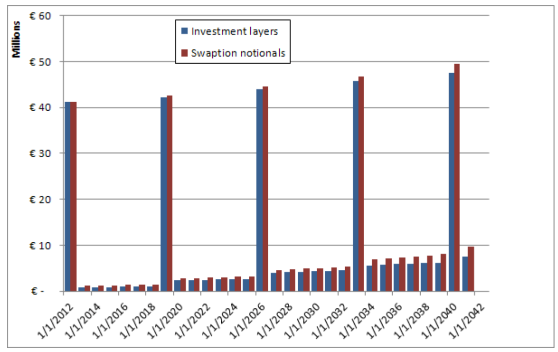
(a) r^g: 3.5% , RTS: Flat 2%



(b) r^g: 3.5% , RTS: Chapter 1 (27-12-2011)



(c) r^g: 3.0% , RTS: Chapter 1 (27-12-2011)



(d) r^g: 3.5% , RTS: Chapter 1 (27-12-2011)

Figure 2.1. *This figure shows the results of replicating the value of the guarantee, that promises €100 mln in 30 years, through time using a strip of receiver swaptions (left axis) and (on the right axis) the cash flows from the underlying swaps in comparison with the payments due to the guarantee. Figure **d** aims to show the difference between the swaption notionals and the investment layers for a scenario in which the interest rate is always below the guaranteed rate.*

The assumption is made that it is possible to invest in a bond yielding the then prevailing u-rate in the second replication method. Theoretically this would be possible by investing in a portfolio of bonds issued by the Dutch state every year using the same relative composition as is used in determining the u-rate, in practice this might be harder. This is however not important for the purpose of this exposition.

From figure 2.1(a) it seems that the replication is perfect for a scenario in which the u-rates are always below the guaranteed rate. This would be in line with the performance of the profit share replications of chapter 1 for scenarios in which the u-rates are always above the guaranteed rate. Figure 2.1(b) however shows that this reasoning is not correct. In a scenario in which the u-rates are always below the guaranteed rate but do vary, the cash flows from the replicating portfolio do not, though they are very close, exactly match the losses due to the guarantees. The reason why the replication is perfect for the profit sharing in case the u-rates are always above the guaranteed rate, is that the value of the fictive investment portfolio is exactly equal to the reserve at all times because the profit share is paid out. This has as result that all investment layers in the different u-rates, of which the coupons set the portfolio return on which the profit sharing is based, will match the swaption notionals exactly. For the replication of the guarantee this does however not hold because the fictive investment portfolio will start to diverge from the reserve when the u-rates are below the guaranteed rate, meaning that the investments layers will in general not be equal to the swaption notionals. Figure 2.1(d) shows this. The mismatch in all scenarios is however very small, meaning that this replication method can assign a value to the guarantee in which one can be confident. The scenario of figure 2.1(c) is probably the most representative with a guaranteed rate of 3%, here the value of the guarantee is close to €18 mln and in this scenario all cash flows resulting from the guarantee eventually amount to over €13 mln, taking into account profit sharing in 7 periods and accrued interests. The most important message coming forth from this section is then that the value of this guarantee is very significant and that it can be expected that a lot of insurance companies are currently facing big losses due to the selling of these embedded options. This is also confirmed by a paragraph in a recent semiannual financial stability report from the Dutch central bank, see DNB (2012), in which they explicitly mention "the pressure on capital buffers due to the issuance of high guarantees in the past...".

2.2 Valuation and Swap-Government spread

All computations in this thesis and in most of the earlier research have been based on the assumption that the swap rate is a good proxy for the u-rate, or at least the "part" u-yield (see section 1.3). This assumption makes life a lot easier with respect to valuation and risk management practices, but it entails a substantial risk. If the u-rate does not proxy the 7 year swap rate, this will result in a faulty valuation or an ineffective hedge. For example, using the replication strategy of the last chapter, the swaptions pay out if the swap rate is above the guaranteed rate but the profit sharing pays out when the weighted u-rate is above the guaranteed rate. This means that if such a scenario presents itself, an insurance company will sell products on which it is likely to make a loss or buys a hedge that is ineffective. [2]

What is needed to value this product using the same replicating strategy as in chapter 1, while not making this assumption, is a way to value swaptions based on two interest rate curves; one as a reference rate and one for discounting the cash flows. To see why this makes sense and how this would be done, some swap and swaption theory will be discussed.

[2]It should be emphasized that this risk is imposed due to the use of the 7 year swap rate as a proxy for the u-rate and not due to the use of replicating portfolio for valuation.

2.2.1 Swaps, Swaprates and Swaptions

A swaption gives the right, but not the obligation, to enter a swap at a predetermined fixed rate. An interest rate swap represents the exchange of a fixed interest for a floating interest during a specified period of time on an agreed notional. The value of this (payer) swap is then best defined by the difference between a floating rate bond and a fixed rate bond:

$$V_t^{swap} = B_{fl} - B_{fix} = N \sum_{q=1}^{n} df_{t,q} \left(r_{t,q}^{fw} - r^{fix} \right), \tag{2.1}$$

with V_t the value of the swap at time t, N the notional, $df_{t,q}$ the discount factor at time t for a cash flow in period q and $r_{t,q}^{fw}$ the forward rate in period q at time t. This formula can be used to compute the swap rate r_t^{swap} at time t . The swap rate is the rate that sets the value of this contract equal to zero using forward rates and zero rates, this is usually done at inception of a swap contract to compute $r^{fix} = r_t^{swap}$. Later, the floating rates (and swap rate that sets the value of the contract to zero at a different point in time) will vary from the rates at time t and the swap will take on a value different from zero. In this case the value will be greater than zero if the expected floating rates are higher than the fixed rate during the period of the swap.

The same curve is often used in computing the discount factors $df_{t,q}$ and in computing the forward rates $r_{t,q}^{fw}$. The interest rate curve used to compute the discount factors should be a "risk free" curve as swap contracts allow for several measures that minimize risk of payments not being made. The forward rates include a small credit risk premium[3], mostly representing the creditworthiness of banks providing Libor[4]. It is then convenient to use the same curve and add the premium to this for the computation of the forward rates.

To determine the value of a swaption at t only the swap rate is used. Because this rate sets the value of a similar contract to zero at different points in time, the difference between the contractual fixed rate and the swap rate at time t can capture the value. The value of the swaption is then computed as the discounted difference (if positive) between the expected swap rate at expiration and the fixed rate that was agreed upon for the amount of interest exchanges n. This expectation is computed in a risk neutral setting, hence the subscript Q, in the same way as Black and Scholes (1973) introduced

[3]It is important to realize that this credit risk premium is different from the premium that the contract holder would pay if he issued a bond.

[4]London interbank offered rate.

it for the pricing of options:

$$V_{t,T}^{swaption} = df_{t,T} E_Q[V_T] = N \cdot df_{t,T} E_Q\left[\sum_{q=T+1}^{n} df_{T,q}\left(r_T^{swap} - r^{fix}\right)^{+}\right]. \tag{2.2}$$

Taking the swap curve as the risk free curve, and hence using only one curve to compute the discount factors and the swap rate, explains why it is so convenient to have a fictive portfolio using investments of 7 years and a 7 year swap rate that proxies the u-rate.

2.2.2 Modeling government interest rate based swaptions

One could motivate the use of a risk free rate for discounting the cash flows from a profit sharing product as insurance companies are under strict supervision of regulatory authorities and, if an insurance company would get into trouble, the Dutch state might protect deputized policyholders. Moreover, regulation requires valuation of expected cash flows using the swap curve. It is however less straightforward why the use of the swap rate (possibly plus some fixed risk premium) will equal the government bond yields that eventually determine the profit sharing.

The yield on a bond can be decomposed into a number of factors that need to compensate the investor for the risks he takes. The most important compensations are for the devaluation of future cash flows, i.e., expected inflation, and for the expectation of the event that the counterparty defaults and the loan is not paid back. Furthermore there is also a compensation asked for the expected change in these expectations, i.e., if the expectations are very volatile the investors ask an additional premium. This suggests that the Dutch government yields are fundamentally only different from the swap rates in the expected credit risk and the volatility of this expectation, because the factors regarding inflation should be the same within a monetary union. If, for example, the credit rating of the Dutch government would be downgraded, the yields would go up but the swap rates are expected to stay the same. Such a scenario would force an insurance company to buy bonds of lower quality to be able achieve a return that is sufficient for paying the policyholders their promised profit sharing. If the event of a downgrade would represent a probability that is very small, the spread between the swap rate and the government yield will be insignificant. Assuming that the 7 year swap rate is a robust proxy of the u-rate is thus equal to assuming that this spread is always constant. Figure 1.1 showed that this spread has been quite constant over the last years, even during some very turbulent times. Figure 2.2 shows how these curves and the spread between them have changed during the financial crises. It can be observed that, though the spread on the 7 year point has not changed much, it has not remained very constant over the entire curve.

Also, recent developments in the Eurozone have brought enough uncertainty to reassess the

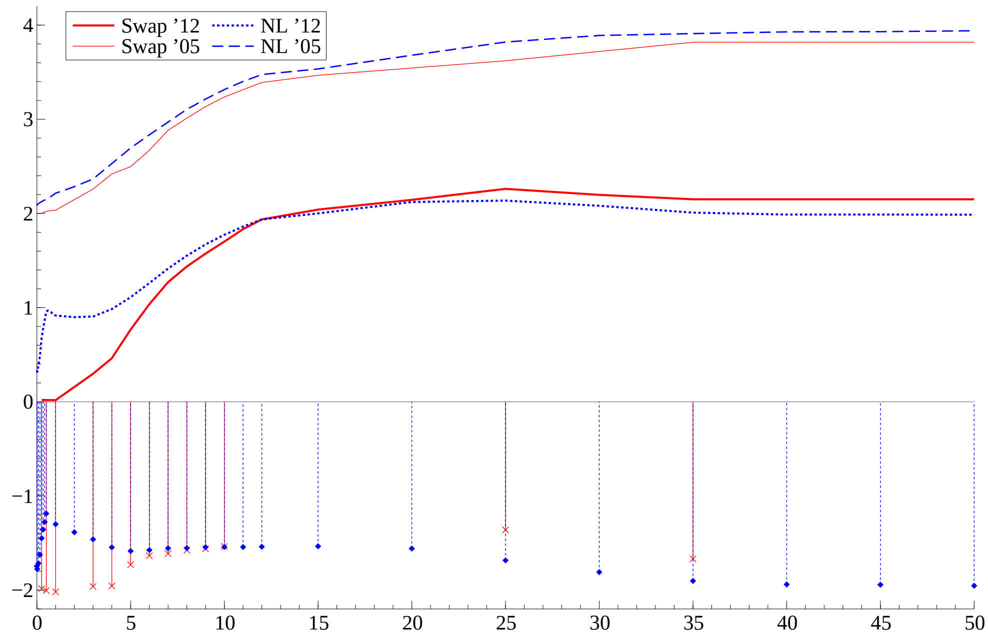


Figure 2.2. *This figure shows the Dutch government yield curve, the Libor swap curve and the spread between them on 24/5/2012 and 24/5/2005. The y-axis shows the yield for different tenors (x-axis).*

meaning of "risk free". Where most governments were considered risk free 5 years ago almost none are considered this today. Figure 2.3 shows that the spread between the swap yields and the government yields were close to zero up till late 2007, but that it diverged significantly in subsequent years. This suggests that, although the spread between the Dutch government bond yields may have remained quite constant, research on a scenario in which this is not the case would not be entirely hypothetical.

The question then is how to value the profit sharing in such a scenario. The amount of profit sharing will rise as yields will go up, but the valuation of the profit sharing will not rise because only the swap curve is used in this and the swap rate does not move together with the government curve in such a scenario. This makes clear that two curves are needed to assess the value of the profit sharing in such a scenario. The last subsection made clear that, though it is common practice to value swap and swaptions using one interest rate curve, it is not necessary. The curve that is used to compute the forward rates to assess the value of the floating rate leg can be separated from the curve that is used for computing the discount factors

$$V_t^{swaption} = N \cdot df_{t,T}^{swapcurve} E_Q \left[\sum_{q=T+1}^{n} df_{T,q}^{swapcurve} \left(r_T^{gvtcurve} - r^{fix} \right)^+ \right]. \tag{2.3}$$

Theoretically, the government curve should always be above the risk free curve. Because the forward rates computed from the government curve are therefore expected to be above the ones computed from the swap curve, formula 2.3 should value the swaptions higher than formula 2.2, that uses only the swap curve. With respect to a profit sharing product the results are not this straightforward. Theoretically the value of the profit sharing element should also be higher and the value of the guarantee should be lower if the government curve, that determines the amount of profit sharing eventually, lies above the swap curve. The replication strategy that was used to value the profit sharing and the guarantee should first be reassessed in this new setting.
Up till now the assumption was made that the 7 year swap rate is a good proxy for the u-rate and this was motivated mainly by the historical time series in figure 1.1. Though the main motivation of this exercise is that the government yield provides a more representative proxy for the u-rate, it is not straightforward which point on this curve is optimal. As the u-rate is an average of past "part" u-yields, that are a weighted average of all issued bonds by the state, the optimal point should reflect the weight of the different maturities that the government bonds have. Furthermore, the spread of this point should also reflect the spreads that the other maturities have. In this work the assumption is made that the spread between the government yield and the swap rate is about equal over the entire curve. Based on figure 2.2 this assumption does not seem too unreasonable, especially in

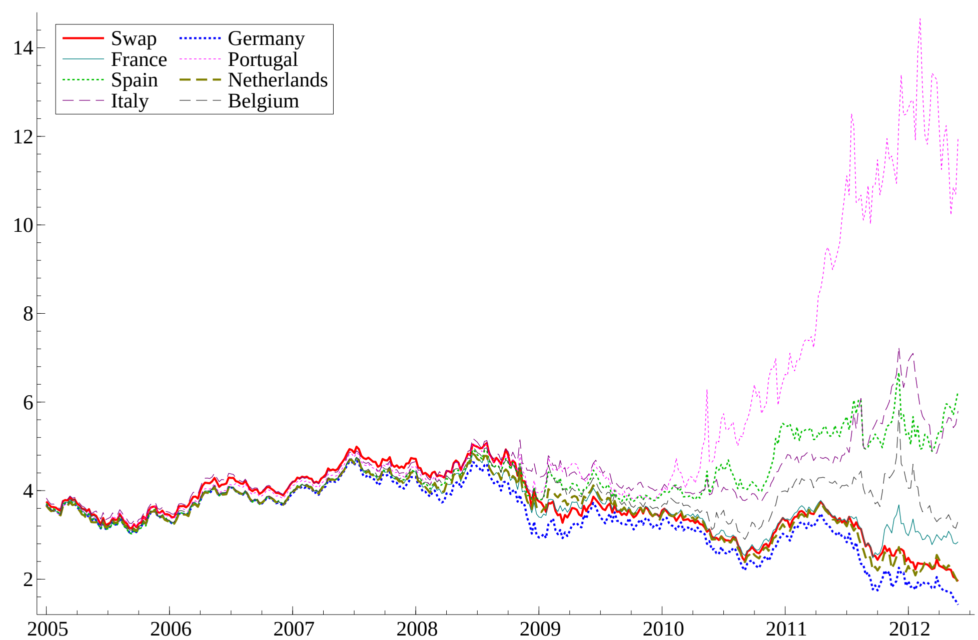


Figure 2.3. *This figure shows the 10 year swap rate and the yield on 10 year government bonds over the period 7/1/2005 to 21/5/2011 using weekly data obtained from Bloomberg.*

Identifier	Coupon	Maturity	Curr	Rating	Principal (mln)	Duration	YearsTM	Weight
ED3904759	3.75 %	7-15-2014	EUR	AAA	15,325	2.0	2.1	
EH8926899	2.75 %	1-15-2015	EUR	AAA	15,489	2.5	2.6	
EI9395480	0.75 %	4-15-2015	EUR	AAA	9,674	2.8	2.9	
ED9903094	3.25 %	7-15-2015	EUR	AAA	15,110	2.9	3.1	10%
EF5517158	4 %	7-15-2016	EUR	AAA	13,311	3.7	4.1	
EI7197110	2.5 %	1-15-2017	EUR	AAA	15,186	4.3	4.6	
EG6293930	4.5 %	7-15-2017	EUR	AAA	14,655	4.5	5.1	
EH1795374	4 %	7-15-2018	EUR	AAA	15,081	5.4	6.1	
EH6998361	4 %	7-15-2019	EUR	AAA	14,056	6.1	7.1	65%
EI1321401	3.5 %	7-15-2020	EUR	AAA	15,070	7.0	8.1	
EI5371022	3.25 %	7-15-2021	EUR	AAA	15,494	7.8	9.1	
EJ0062152	2.25 %	7-15-2022	EUR	AAA	8,899	9.0	10.1	
EF2774059	3.75 %	1-15-2023	EUR	AAA	9,870	8.9	10.6	
GG7145913	7.5 %	1-15-2023	EUR	AAA	4,200	8.0	10.6	25%
GG7295270	5.5 %	1-15-2028	EUR	AAA	12,144	11.4	15.6	
ED9083541	4 %	1-15-2037	EUR	AAA	13,038	16.9	24.6	
EI2397178	3.75 %	1-15-2042	EUR	AAA	12,126	19.6	29.6	
						weighted duration		**6.8**

Table 2.1. *This table shows all outstanding Dutch government bonds on 31/05/2012, that have a maturity between 2 and 15 years and a minimal principal amount of €225 mln. The duration is computed as the modified duration. The weighted duration is computed using the weights specified in the definition of u-yields and the median of durations belonging to maturity ranges that are also in the definition*

2005. The current yield curves suggest that this assumption does not hold for maturities lower than 6 years.

This means that the curve point that matches the weighted average *duration* of the bonds issued by the Dutch government is most representative for the u-yield. Table 2.1 shows the current outstanding bonds that satisfy the required conditions for taking part in the computation of the the u-rate. The weighted average *modified duration* is currently 6.8.[5]

The result suggest that it is best to use the 7 year point, this also motivates why the 7 year swap rate has been the best proxy and is in line with the reasoning at the beginning of this section that effectively the only difference between the two yields is the credit risk premium.

Modeling the profit sharing and guarantee can then be done using the same replication method as before with the only difference that the swaptions are valued based on two curves. Because the 7 year point is chosen on the curve the cash flows from the use of 7 year swaptions again match the cash flows from the fictitious investments that determine the profit sharing.

[5] This duration is weighted using the percentages specified in the definition of the u-yield (outer left column of the table) and not by the notional amounts.

2.2.3 Model evaluation

To evaluate the performance of the model, results from the new replication method will be analyzed for both the profit sharing element and the guarantee. In both cases the value is computed for scenarios of varying government spreads and guaranteed rates. By using two curves in the valuation, the value of the portfolio of swaptions is also sensitive to changes in these two curves. The value and the sensitivity, measured as the change in value of the swaption portfolio as a consequence of a parallel shift upwards of 1 basis point in the curve, is displayed in tables 2.2 and 2.3 (respectively the profit sharing and the guarantee). The percentages to the right of all values computed using the new model represent the percentual change in value relative to the original (old) model. Because the original model is only sensitive towards one curve, these changes concerning the sensitivities are not entirely comparable. Therefore the difference between the sum of the sensitivities in the new model and the one of the old model is also computed and expressed as percentage fourthly in all scenarios. Summing these values can be motivated by the fact that a change in the swap curve is often followed by the same change in government curves. This is also in line with the reasoning that the government curve from a Eurozone member country is equal to a risk free curve plus some credit spread.

NB.

It should be emphasized that in the computation of the value of all swaptions, so also those based on government curves, only the volatilities of the swap curves are used. These are implied volatilities extracted from swaption quotes in the market and, as there are no government interest rate based options traded, are not available for government rates. The reader should be aware that this can however have a significant effect on the value of the profit sharing and guarantee. Although volatilities can be expected to rise when spreads increase and this effect is not expected to be negligible, in this paper only the direct effect of the swap government spread is evaluated.

The model seems to value the profit sharing correctly. If the spread between the swap curve and the government curve is zero, the value is exactly similar for the two models. Also the sum of the two sensitivities equals the one from the original model. If the spread is exactly one basis point, the value change by the amount that is displayed for the sensitivity towards the forward curve (Δ_{fw}). Furthermore, the direction of the value is as expected, because it increases when the spread increases and decreases when the guaranteed rate decreases. One can also observe that the sensitivity towards the swap curve is always negative and the sensitivity is always positive towards the forward curve. This is a logical consequence of the fact that the present value of all swaptions decrease when the discount rate increases and the "in the moneyness" of these options increase when spreads increase.

		Old	Spread									
r_g		0.00	0		0.0001		0.01		0.02		0.05	
0.01	Value	€ 33.16	€ 33.16	0%	€ 33.35	1%	€ 51.96	57%	€ 71.14	115%	€ 129.89	292%
	Δ_{sw}	€ 0.13	€ - 0.06	-143%	€ - 0.06	-143%	€ - 0.09	-166%	€ - 0.12	-190%	€ - 0.21	-262%
	Δ_{fw}	€ 0.00	€ 0.19	43%	€ 0.19	43%	€ 0.19	47%	€ 0.19	49%	€ 0.20	53%
				0%		0%		-19%		-41%		-109%
0.02	Value	€ 18.10	€ 18.10	0%	€ 18.24	1%	€ 32.87	82%	€ 48.41	167%	€ 97.26	437%
	Δ_{sw}	€ 0.11	€ - 0.04	-134%	€ - 0.04	-135%	€ - 0.06	-157%	€ - 0.09	-181%	€ - 0.17	-258%
	Δ_{fw}	€ 0.00	€ 0.14	34%	€ 0.14	35%	€ 0.15	44%	€ 0.16	49%	€ 0.17	57%
				0%		0%		-13%		-32%		-101%
0.03	Value	€ 11.80	€ 11.80	0%	€ 11.88	1%	€ 20.87	77%	€ 33.28	182%	€ 73.70	524%
	Δ_{sw}	€ 0.05	€ - 0.03	-148%	€ - 0.03	-148%	€ - 0.04	-182%	€ - 0.07	-221%	€ - 0.14	-350%
	Δ_{fw}	€ 0.00	€ 0.08	48%	€ 0.08	48%	€ 0.12	120%	€ 0.13	135%	€ 0.14	157%
				0%		0%		38%		14%		-92%
0.04	Value	€ 8.34	€ 8.34	0%	€ 8.40	1%	€ 15.15	82%	€ 23.31	179%	€ 56.70	580%
	Δ_{sw}	€ 0.04	€ - 0.02	-148%	€ - 0.02	-148%	€ - 0.03	-183%	€ - 0.05	-225%	€ - 0.11	-369%
	Δ_{fw}	€ 0.00	€ 0.06	48%	€ 0.06	48%	€ 0.07	78%	€ 0.10	145%	€ 0.12	182%
				0%		0%		-5%		21%		-87%
0.05	Value	€ 6.20	€ 6.20	0%	€ 6.25	1%	€ 11.61	87%	€ 18.02	191%	€ 44.41	617%
	Δ_{sw}	€ 0.03	€ - 0.02	-148%	€ - 0.02	-149%	€ - 0.03	-187%	€ - 0.04	-231%	€ - 0.09	-391%
	Δ_{fw}	€ 0.00	€ 0.05	48%	€ 0.05	49%	€ 0.06	84%	€ 0.07	111%	€ 0.10	206%
				0%		0%		-2%		-20%		-85%

Table 2.2. *This table shows the value of the profit sharing element from a contract that guarantees €100 mln in 30 years based on 5 guaranteed rates (seen in the outer left column) for the valuation method using one curve (old) and for the valuation method using two curves. The different curves are based on the Libor swap curve of 27/12/2011 and bumped with the spread over the entire curve.* Δ_{sw} *and* Δ_{fw} *respectively show the sensitivity towards the discount curve and the forward curve for a basis point shift upwards of the curve. The percentages represent the differences with the old model.*

Though this is quite straightforward it is a nice property of this model as it gives additional insight. Something that is also clear from this table is that the value of these embedded options are undervalued considerably using the original model when spreads increase. For a fixed guaranteed rate the misvaluation seems approximately linear. For fixed spread and varying guaranteed rates this relation does not seem linear. Furthermore, the error in valuations increases as guaranteed rates increase together with increasing spreads, i.e., the error multiplies by 5 if the spread increases from 1 to 5 percent when the guaranteed rate is 1 percent but the error multiplies by six when the guaranteed rate is 5 percent for the same increase in the spread. Another interesting observation is that the sensitivity towards the discount curve is quite constant for a fixed spread and varying guaranteed rates but changes quickly for varying spreads. The sensitivity towards the discount curve also becomes relatively more important as spreads increase, this is because the value of the options increase and a change in the discount curve therefore has a larger impact.

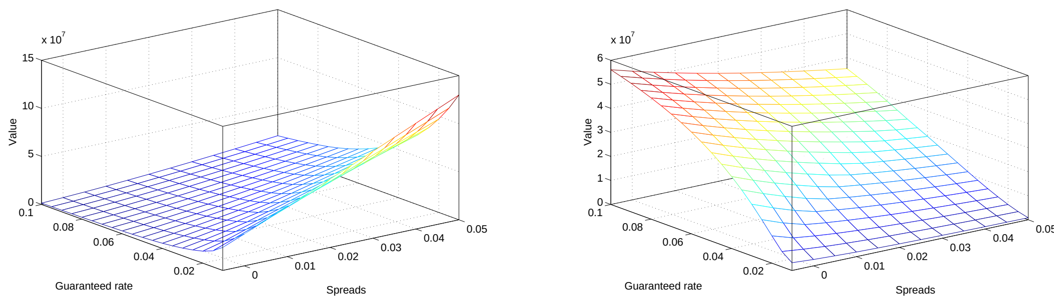
The sensitivity towards the forward curve remains quite constant for fixed guaranteed rates, while it changes significantly for varying guaranteed rates and fixed spreads. This is because the forward rates do not change much by parallel shifts but the sensitivities decrease quickly when the swaptions

		Old	Spread									
r_g		0.00	0		0.0001		0.01		0.02		0.05	
0.01	Value	€ 2.66	€ 2.66	0%	€ 2.65	0%	€ 1.94	-27%	€ 1.50	-44%	€ 0.84	-68%
	Δ_{sw}	€ - 0.02	€ - 0.01	-60%	€ - 0.01	-60%	€ 0.00	-70%	€ 0.00	-76%	€ 0.00	-86%
	Δ_{fw}	€ 0.00	€ - 0.01	-40%	€ - 0.01	-41%	€ - 0.01	-65%	€ 0.00	-78%	€ 0.00	-91%
				0%		-1%		-35%		-54%		-78%
0.02	Value	€ 8.61	€ 8.61	0%	€ 8.59	0%	€ 6.66	-23%	€ 5.38	-37%	€ 3.32	-61%
	Δ_{sw}	€ - 0.04	€ - 0.02	-55%	€ - 0.02	-55%	€ - 0.02	-65%	€ - 0.01	-71%	€ - 0.01	-81%
	Δ_{fw}	€ 0.00	€ - 0.02	-45%	€ - 0.02	-45%	€ - 0.02	-65%	€ - 0.01	-76%	€ 0.00	-90%
				0%		0%		-30%		-47%		-71%
0.03	Value	€ 18.05	€ 18.05	0%	€ 17.99	0%	€ 12.64	-30%	€ 10.50	-42%	€ 6.81	-62%
	Δ_{sw}	€ - 0.10	€ - 0.04	-64%	€ - 0.04	-64%	€ - 0.03	-71%	€ - 0.02	-75%	€ - 0.02	-83%
	Δ_{fw}	€ 0.00	€ - 0.06	-36%	€ - 0.06	-36%	€ - 0.03	-74%	€ - 0.02	-82%	€ - 0.01	-92%
				0%		0%		-45%		-57%		-75%
0.04	Value	€ 26.50	€ 26.50	0%	€ 26.44	0%	€ 20.64	-22%	€ 16.07	-39%	€ 10.87	-59%
	Δ_{sw}	€ - 0.12	€ - 0.05	-56%	€ - 0.05	-56%	€ - 0.04	-63%	€ - 0.04	-68%	€ - 0.03	-78%
	Δ_{fw}	€ 0.00	€ - 0.07	-44%	€ - 0.06	-45%	€ - 0.05	-55%	€ - 0.03	-78%	€ - 0.01	-90%
				0%		0%		-17%		-46%		-67%
0.05	Value	€ 33.48	€ 33.48	0%	€ 33.41	0%	€ 27.69	-17%	€ 22.86	-32%	€ 15.16	-55%
	Δ_{sw}	€ - 0.13	€ - 0.07	-49%	€ - 0.07	-49%	€ - 0.06	-56%	€ - 0.05	-62%	€ - 0.04	-72%
	Δ_{fw}	€ 0.00	€ - 0.06	-51%	€ - 0.06	-51%	€ - 0.05	-60%	€ - 0.04	-66%	€ - 0.02	-88%
				0%		0%		-16%		-27%		-60%

Table 2.3. *This table shows the value of the guarantee element from a contract that guarantees €100 mln in 30 years based on 5 guaranteed rates (seen in the outer left column) for the valuation method using one curve (old) and for the valuation method using two curves. The different curves are based on the Libor swap curve of 27/12/2011 and bumped with the spread over the entire curve. Δ_{sw} and Δ_{fw} respectively show the sensitivity towards the discount curve and the forward curve for a 1 basis point shift upwards of the curve. The percentages represent the differences with the old model.*

are further out "of the money". Fore completeness the same table is shown for the guarantee. The model again values correctly and the sensitivity agrees with the values shown for a spread increase of one basis point. Essentially the same goes as for the profit sharing but then the other way around. When the swaptions, representing the guarantee, move further *out of the money* (spreads increase), the old model does not capture this and therefore overvalues the guarantee. The errors are not as extreme as for the profit sharing element because, as spreads increase, the value of the guarantee decreases. Another interesting observation is that the sensitivity towards both curves is negative. If one assumes that a decrease in the discount curve (the "risk free" curve) is followed by a decrease in the government curve, the value of the guarantee increases through both curves, whereas these effects partially offset each other in the case of profit sharing.

Though the relations can be observed from the tables, figure 2.4 might show them clearer. From these graphs it is clear that an increasing guaranteed rate quickly decreases the value of the profit sharing, as the embedded option moves out of the money, but increases the value of the guarantee. This is a consequence of the fact that the profit sharing is modeled by payer swaptions and the guarantee is modeled by receiver swaptions. If one moves out of the money, the other has to move into the money. Interesting is that, if both the guaranteed rate and the spread move up by the same



(a) Profit Sharing

(b) Guarantee

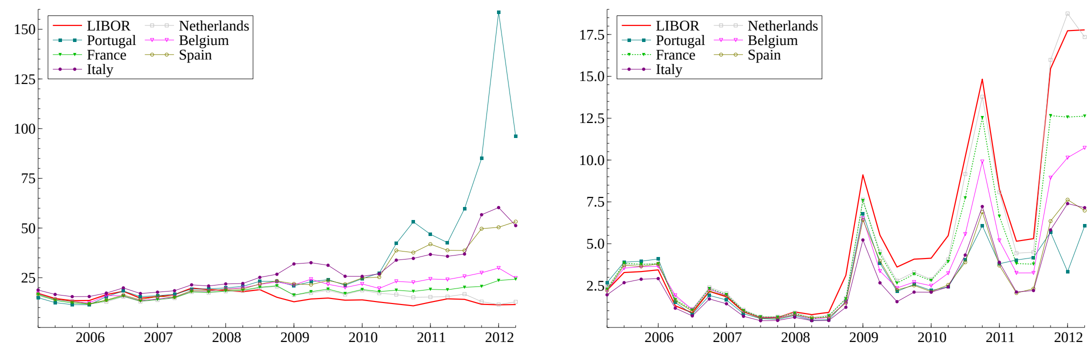
Figure 2.4. *This figure shows a 3d plot of the value of a contract that guarantees €100 mln in 30 years following a fictive investment strategy in 7 year government bonds with varying guaranteed rates and spreads, based on the swap curve of 27/12/2012.*

amount, the value does not remain equal. This is a consequence of the volatility having a larger impact when interest rates are higher.

2.2.4 A practical implementation

Using this new model, the impact of valuing the profit sharing and guarantee using the government curve can be assessed in a more practical setting. Considering figure 2.3, two interesting scenarios can be considered. One is the current situation for the Netherlands and Germany, in which the government curve is actually below the "risk free" swap curve. As will become clear later, this is not so much because the Dutch government is incredibly risk free, but should be seen in the perspective of relatively risk free compared to the peripheral countries, in times that investors are looking for a safe haven. One would then expect the guarantee to increase in value and the profit sharing to decrease. The second scenario is the one in which the spread diverges and the government curve shifts up relative to the swap curve. This is what happened to, for example, Portugal and to a lesser extent to Italy and Spain, as a result of increasing budget deficits and credit downgrades. In this scenario the value of the profit sharing is expected to rise and the value of the guarantee is expected to fall.
Figure 2.5(a) and 2.5(b) show the value of both elements from a profit sharing contract, that guarantees an amount of €100 mln after 30 years based on a guaranteed return of 3 % a year, over time for the scenarios in figure 2.3.

Figure 2.5(a) and 2.5(b) give insight into the sensitivity of the value of the profit sharing element and the guarantee to the swap-government spread. The expectations seem to be true for the profit sharing

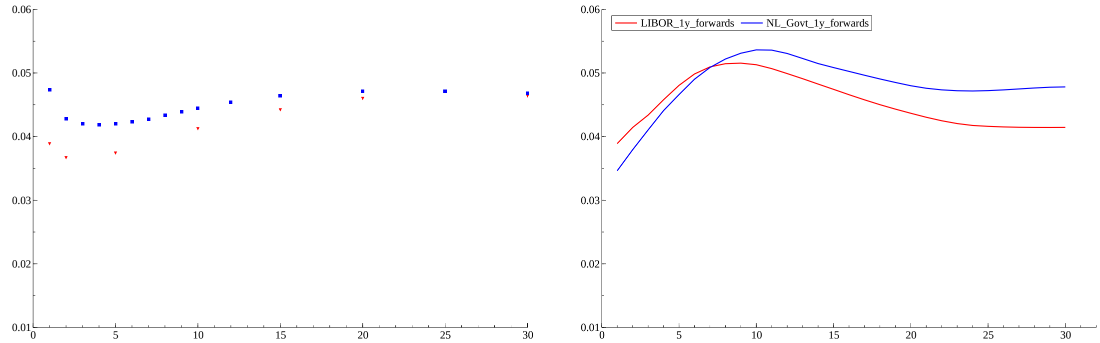


(a) Profit Sharing **(b)** Guarantee

Figure 2.5. *This figure shows a time series of the value of the profit sharing element (a) and the value of the guarantee (b) in a contract that guarantees €100 mln in 30 years with a guaranteed annual return of 3 % and uses a fictive investment strategy in 7 year government bonds. These values are computed using historical government and swap curves that were constructed using data from bloomberg over the spanned period.*

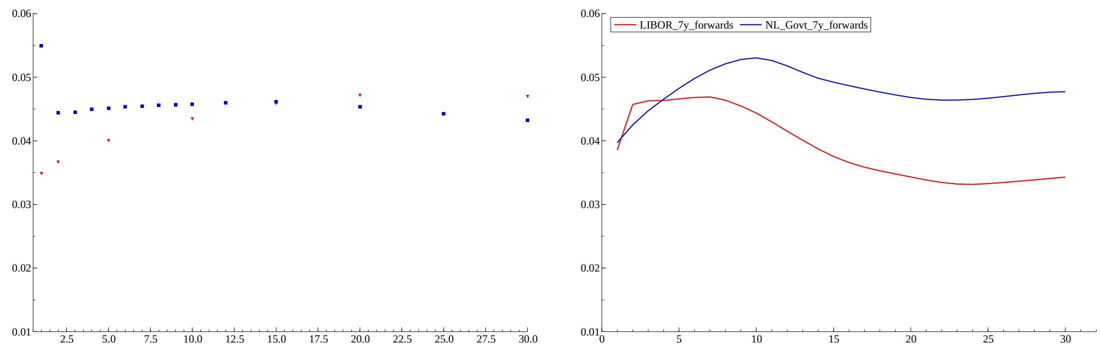
element; if the spread increases in figure 2.3 the element of the profit sharing increases. Something interesting however occurs when the spread becomes negative (Germany and the Netherlands). What was expected is that the value of the profit sharing would be lower using a valuation based also on the government curve. This is however not the case, the value of the profit sharing element computed for the Dutch interest rate scenario has been above the swap curve valuation over the entire period. This seems strange because the Dutch interest rates are below the swap rate over the entire period in figure 2.3. Moreover, when the spread increases (negatively) the values converge instead of further diverge. Looking at figure 2.5(b), the same goes for the guarantee. This suggest that the relation between the interest rates, the profit sharing and the guarantee is not as straightforward as was just assumed.

The difference between the profit sharing element in a valuation using the Dutch government curve is largest compared to a valuation based on the swap curve in the third quarter of 2008(+ €6.71 mln). This was at the height of financial crises just after Lehman Brother had collapsed. The profit sharing gives the policyholder the right to share in profits in every year during the life of the contract. This means that all options that are used in the valuation are sensitive to different parts of the interest rate curve. Furthermore, the valuation depends to a large degree on the expected forward rates in different periods, meaning that the shape of a curve can have considerable influence on the value of a swaption. Figures 2.6(a) and 2.6(b) show the scenario on 31-03-2008, just before the period in which the largest difference occurs, and display that it is perfectly possible for the Libor curve to be above the government curve entirely while a large part of the 1 year Libor forward rates are below the government 1 year forward rates. This is caused by the convexity of the government curve being much stronger. Figure 2.7(a) below shows the situation at 30-09-2008. Very different curves are observed here as the government curve is concave and the swap curve is almost flat, with



(a) **(b)**

Figure 2.6. *This figure shows the Libor swap curve and the Dutch government curve (a.) and the 1 year forward rates (b.) of these curves on 31-03-2008.*



(a) **(b)**

Figure 2.7. *This figure shows the Libor swap curve and the Dutch government curve (a.) and the 7 year forward rates (b.) of these curves on 30-09-2008.*

a relatively high yield on short term financing due to lack of confidence between banks. Here the spread between the government curve and the Libor swap curve is also mostly negative, especially in the first part of the curve. Figure 2.7(b) then explains why, while the swap curve is above the government curve for the greater part, the profit sharing is still worth significantly more when it is valued using the appropriate government curve. The 7 year government forward rates are almost all above the 7 year Libor swap rates.

Returning to figures 2.5(a) and 2.5(b), that show how the profit sharing value and the guarantee value evolve over time, it is clear that the consideration of the swap government spread in the valuation can have a substantial impact. Looking at the profit sharing for an insurance company in Portugal that would have sold these type of contracts, it could cause the company to go bankrupt quite quickly if it had to use market value accounting methods. It would also not take long to reach the point where it has insufficient reserves to pay out the profit sharing if it was invested in, for example, stocks or a portfolio of bonds containing mainly credit from northern countries. In this case the reserves would decrease the liabilities would quickly rise. This highlights an important risk for insurance companies in for example France, Spain and Belgium, countries of which the credit ratings are under fire. Furthermore, it reveals that the contract specifications of this type of product, i.e., linking the profit sharing directly to the yield of a countries debt, results in wrong incentives. When the government of a country in which the insurance companies is located will receive a credit downgrade, the company will have to buy government debt of decreasing quality to achieve adequate return needed for profit sharing payments. This can then lead to the same kind of voodoo economics as one currently sees in peripheral countries that have a distressed banking system: governments are lending money at increasing interest rates and the distressed banks are buying it. This results in increasing government debt (and rates) again and decreases the healthiness of bank balance sheets. Linking profit sharing products to one government can then lead to exactly the same vicious circle in the life insurance industry. Meaning that it does not only introduce risk to the insurance company but to the countries real economy as well.

Not only insurance companies which are located in countries of which credit ratings are uncertain and interest rates rising are at risk. Figure 2.5(b) shows clearly that the value of the guarantee is very sensitive to interest rate changes. While this value was relatively constant up till 2008, it has become very volatile during the last years, as a result of the uncertainty in the Eurozone. With interest rates that keep decreasing for the "safe havens" of Europe, the guarantee value keeps increasing. The volatility that is displayed in figure 2.5(b) for the Libor curve and Dutch government curve is however remarkable. While the volatility of the interest rates of the different countries are certainly not the same, the values of the guarantee show considerable similarities and seem sensitive to the same factors. Also the expectation of a decrease in the value of the guarantee when government curves move up and spreads increase do not seem correct. The guarantee, though most valuable based on the swap curve and the Dutch government curve, moves up in in all scenarios. As the use of two curves in the valuation of swaptions results in a sensitivity towards those two curves, this suggests that the valuation of the guarantee shows substantial sensitivity towards the discounting curve, because this is their common denominator. To asses this, the change in the value of the profit sharing and the guarantee as a result of a basis point change in the curve[6] is computed over time for two countries. This is displayed figure 2.8 below for the Netherlands (increasingly negative spread towards the swap curve) and for Spain (increasingly positive spread). The figures show that both elements are more sensitive to the forward curve and that this curve has the most influence in every scenario. In all cases, the sensitivity towards the discount curve is negative and increases when the value of the profit sharing or guarantee increases. It can then be seen that in the profit sharing case the sensitivity towards both the the discount curve and the forward curve decreases when the spread

[6]The curve, either the discount curve or the government curve, makes a parallel shift of 1 basis point upwards.

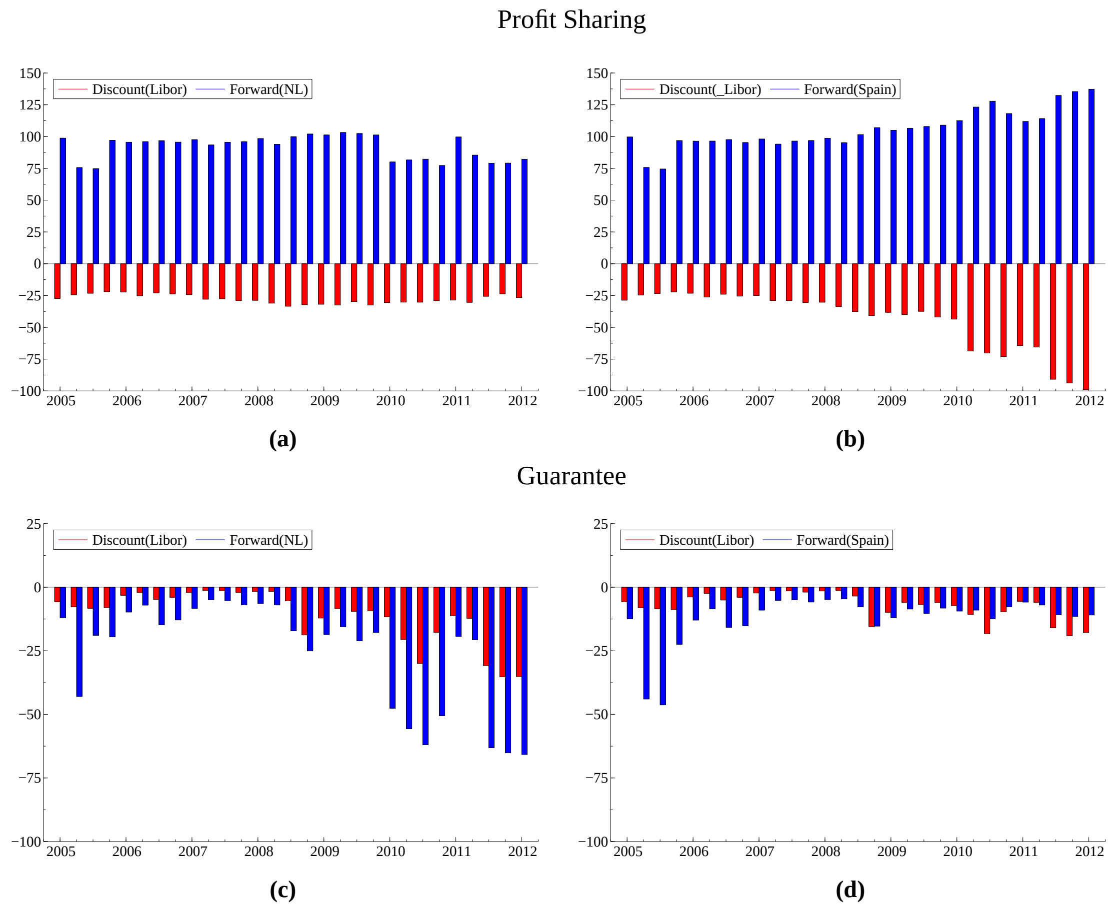
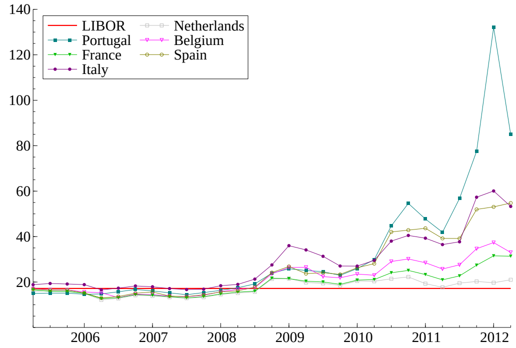


Figure 2.8. *This figure shows the sensitivity to the discount curve and the forward (government) curve for the profit sharing (top) and the guarantee (bottom) over time. On the left the Netherlands, on the right Spain. Values are in €millions.*

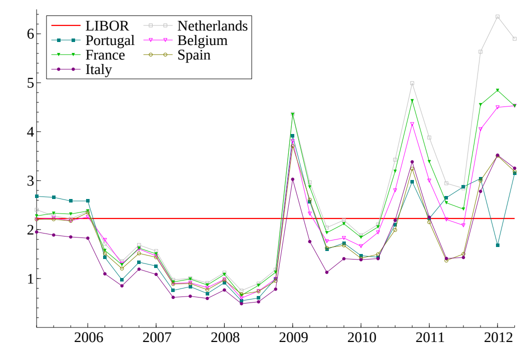
is negative and they both increase when the spread is positive. This is a nice property because the sensitivities have an opposite sign in the profit sharing case, causing them to partially offset each others effects if interest rates rise (assuming that a shift up in the swap curve is followed by the same shift in the government curve). This is not the case for the guarantee, as both sensitivities have the same sign, meaning that a drop in interest rates cause the value to increase through the forward curve and trough the discount curve.

A disturbing fact is that the sensitivities increase simultaneously with the value of the profit sharing and the guarantee. This is a consequence of the swaptions being more in the money. The same reasoning can be used to explain the high volatility in the sensitivities seen for the Netherlands in the guarantee case, as swaption values are most volatile when they are at the money.

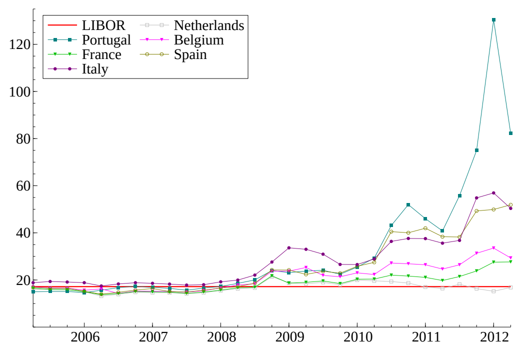
These figures, while they show that there is indeed substantial sensitivity towards the discount curve, do not really explain why the value of the guarantee increases over time for scenarios in figure



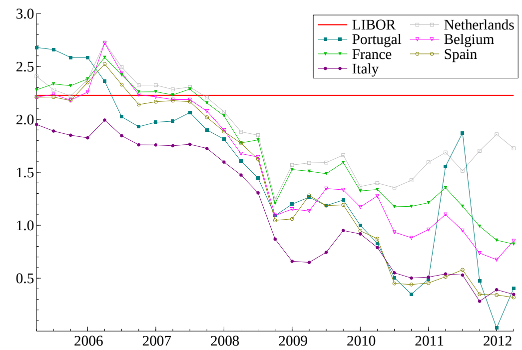
(a)



(b)



(c)



(d)

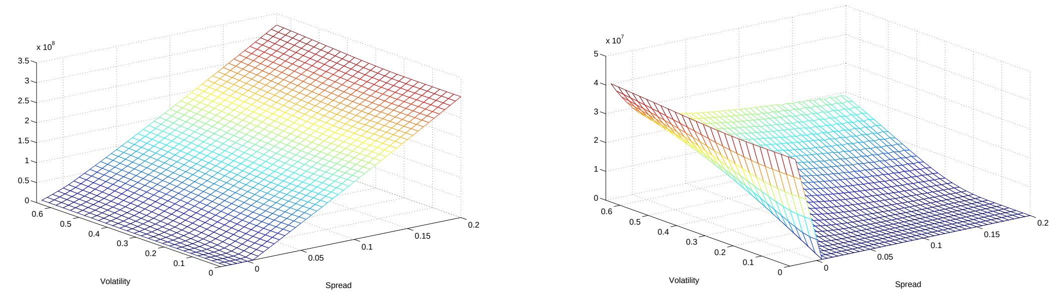
Figure 2.9. *This figure shows a time series of the value of the profit sharing element (left) and the value of the guarantee (right) for a contract that guarantees €100 mln in 30 years based on a guaranteed return of 3 % and uses a fictive investment strategy in 7 year government bonds. In the top two graphs the discount curve is kept constant and only the spreads and volatilities change. In the bottom two graphs only the spreads change and also the volatility is kept constant (Q1 2005). Values are in €millions.*

2.5(b), in which spreads increase. Because this can be expected to be caused by decreasing swap (discount) rates, the same graphs are plotted again for these government curve scenarios, but using a constant Libor swap curve (Q1 2005) as discount curve. The aim is to exclude the effect of decreasing swap rates and capture better the effect of the government spreads. To achieve this, the government spreads are computed for all quarters and for a number of points on the curve. These spreads are then added to the same curve points of the Libor swap curve of Q1 2005 to construct the new government curves at that quarter. For completeness this is also done for the profit sharing part.

These figures suggest that the the guarantee is much more sensitive to the discount curve, as all values have approximately halved compared to figure 2.5(a), what seems also reasonable when one looks at the relative bar heights in figure 2.8. While figure 2.9(b) does show that the discount curve is important for the value, it does not seem to explain the increase in the value of the guarantee over a period in which spreads increase. Figure 2.9(b) does suggest that from the fourth quarter in 2009 something changes in the way interest rate swaptions were valued as everything shifts up. If the decreasing discount rates do not explain this, the only thing that can is the volatility which is used in pricing these swaptions or, as we saw earlier, the shape of the curves. Lastly then, figure 2.9(c) and 2.9(d) show the values when only the spreads vary over time. The graphs takes on a completely different shape what makes clear that the volatility has a substantial influence on the value of the guarantee. The effect of the spreads is far more clear from this graph, as one can see that the value of the guarantee decreases for the countries of which the spreads increased since the Eurozones sovereign crises started and the value increases since 2010 for the Netherlands (negative spread towards the swap curve). The drop in value during the burst of the housing bubble (Q3 2007 - Q3 2008) is explained by government, showing an increased negative spread.[7] As from the start of the financial crises (fall of Lehman in Q3 2008) the spreads of the peripheral countries in Europe increased slightly but the government curve of the "safe" countries stayed below the swap curve, resulting in respectively a decrease and increase in the value of the guarantee in these countries. During Q3 of 2009 these spreads slightly converged again until the sovereign debt crises in Europe started that made them diverge again. The directions in figure 2.9(d) clearly show that increasing spreads generally decrease the value of the guarantee and increasing negative spreads increase the value. Exactly opposite of the profit sharing case. The values of the guarantee for Portugal however seem to disagree with this during Q1 and Q2 of 2011, showing a significant increase while spreads also increased. This is due to a shift in the curve, as the interest rates of low maturities increased quickly due to a lot of uncertainty over shorter horizons while longer term yields reacted less. This resulted in lower forward rates further away, increasing the value of the swaptions. The hops that are observed in the profit sharing series of Portugal in figure 2.9(c), where the value suddenly increases and decreases, are also explained by the spreads. Looking at those periods in figure 2.3 one sees that all interest rates move parallel to the swap curve but this correlation is broken for the Portuguese interest rates.

Figure 2.9 showed that both the spread and the volatility have an influence on the value of the guarantee and on the value of the profit sharing. The shape of the curve however still influences the values as well and this distorts a transparent observation of the relative effect that the volatility and spread have. To overcome this, figure 2.10 shows the relation clearer, because all curves are kept flat and only the spreads and volatility vary. As expected, both the value of the guarantee and the value of the profit sharing increases when volatility rises. The values react oppositely to expanding spreads; profit sharing increases, guarantee decreases. An interesting observation, that agrees with the big differences between figures 2.9(b) and 2.9(d), is that the value of the guarantee is relatively

[7] Until the financial crises all government yields were below the Libor swap curve, as governments were considered entirely risk free.



(a) Profit Sharing **(b)** Guarantee

Figure 2.10. *This figure shows a 3d plot of the value of a contract that guarantees €100 mln in 30 years with a guaranteed annual return of 3 % and a fictive investment strategy in 7 year government bonds. All curves are kept flat and only the volatility and spread varies.*

Cost of contract (initial premium paid)	€47,674,269
Market value 30y zero coupon bond of €100 mln	€46,780,031
Market value Profit Sharing	€16,118,359
Profit/Loss	€15,224,121-

Table 2.4. *This table shows the market value a profit sharing contract pays €100 mln based on a guaranteed rate of 2.5% and the value for which this product can be bought on 30/03/2012. It makes clear that the contract is sold under its market value.*

much more sensitive to differences in volatility than the value of the profit sharing.

The value of a contract that promises €100 mln in 30 years based on guaranteed rate of 2.5%[8] can be computed using the current Dutch government and swap curve by summing the value of replicating portfolio of the profit sharing with the value of a zero coupon bond that pays €100 mln in 30 years. As mentioned, the idea of this product is that the insurance company guarantees a below market guaranteed rate, but promises profit sharing in return for this. The difference between the market value of the zero coupon bond and the amount that the policy holder has to pay for this contract should then cover for the profit sharing option. From table 2.4 it is however clear that the profit sharing can probably not be financed from this difference and that insurance companies are likely to sell these products for well below the market value.

[8]This is currently available at several insurance companies.

2.3 Summary

In summary this chapter has shown that there are two ways in which these profit sharing products can be modeled. Though both should result in the same value, one explicitly models the profit sharing and one explicitly models the guarantee. The value of both elements was subsequently analyzed for scenarios of varying swap-government spreads, using a valuation method that considers both the government rates and the swap rates. This showed that there is indeed significant exposure towards both curves and that the value of the profit sharing is extremely sensitive to swap government spreads. This can lead to perverse situations in which insurance companies are almost obligated to invest in lower quality government bonds in order to be able to pay out the profit sharing.

The value of the guarantee is low compared to the profit sharing but is relatively more sensitive towards the discount curve and to fluctuations in volatility. Furthermore, it has become clear that the value of both elements are very sensitive to the shape of the curve, which essentially determines the value of the swaptions.

Although guarantee values are generally lower than profit sharing values, the risks involved with the guarantee might actually be higher. Profit sharing should in essence not be a problem because, if yields go up, the insurance companies can invest in the bonds. Except for the (possibly substantial) risk of a counterparty defaulting, the company can then share the profits with the policyholder.[9] The guarantee might however be more dangerous, as this means that there are no bonds of the desired credit rating that yield a return high enough to cover the guarantee. This means that following the investment policy that specifies how the profit sharing is determined is not an option and investments in theoretically riskier assets, yielding more, have to be made.

[9] Book value would then coincide, using market based accounting rules there would however still be a gap between assets and liabilities.

Chapter 3

Hedging strategies

Traditionally risk management in the life insurance industry has always relied on long term expectations about longevity, inflation, investment returns and regulatory frameworks. This resulted in the common view on this industry being one of the most stable and reliable. With the introduction of an increased amount of complex products and a regulatory framework relying more on market based valuation techniques, this era belongs to the past. Especially now, as markets are more volatile than ever.

Combining the long term nature of the insurance industry with market consisting valuation and increasingly complex products creates a challenging environment for solvency and risk management. This chapter will therefore discuss several basic strategies that can be applied in mitigating the risks that profit sharing products introduce. The main question that will be answered is how insurers can hedge the risks on their balance sheet which policyholders seek protection from. This essentially entails hedging the two components that underlie the existence of these products: 1) a guaranteed return somewhere in the future and, 2) profit from high interest rates.

The first section will introduce the framework that will be used to evaluate all hedging strategies and will discuss some related literature as well. Section 3.2 will discuss various products and strategies that can be used to mitigate risks. The proposed strategies will then be evaluated in section 3.3 to asses the effectiveness in different scenarios. Finally, in section 3.4 the performance of the strategies are assessed based on a backtest using historical data on European countries interest rates. The chapter will again conclude with a short summary of the main results.

3.1 Framework

Though the main emphasis has definitely been on the fair valuation of the embedded options that are present in the products that life insurers sell, literature on risk management practices for them is also available. Originally risk management in this industry followed the passive actuaries approach of holding enough risk free assets to meet the liabilities with high probability. This has however become hard due to the nature of the products which are currently available. Another approach is to use results of finance to compute the fair value of these contracts and match the liabilities by rebalancing positions in different basic asset classes. But most of the literature that is specifically on hedging life insurance products use risk minimizing strategies. For this, the results of Föllmer and Sondermann (1986) are often applied, whom proved the existence of a unique risk minimizing hedging strategy for any square-integrable contingent claim with a fixed maturity. This approach aims to minimize total discounted profits and losses. Coleman et al. (2003) discuss piece wise linear risk minimizing strategies based on local risk minimization (match final value and minimize intermediate cash flows individually) as an alternative, avoiding computationally difficult dynamic stochastic programming problems encountered in minimizing over all periods. They find that this method has a high probability of outperforming the quadratic programming model under the assumption of a complete market. Subsequent research adds by introducing mortality risk (Moller (2001)) or considering more realistic price, market and interest rate modeling. A useful introduction and summary of the performance of several strategies, e.g. delta hedging, static hedging, risk minimizing strategies and the modeling of mortality and financial risk is given by Nteukam et al. (2011).

However, the amount of literature on hedging strategies is still limited and remains very theoretical, especially for the type profit sharing products discussed here. The work done by Pelsser (2008) is probably most related to the approach that will be taken in this thesis. He discussed hedging the minimum guarantee in British Guaranteed Annuity Options using a static portfolio of receiver swaptions and found this to be effective based on historical data. An important difference between all previous literature and this work is that all products are "directly" unit-linked. This means that the hedging strategies that are discussed focus all on mitigating the risks introduced by the guarantee given to the policyholder. The risks introduced by the other element, being the profit sharing, are not really considered because this can be mitigated quite simply by investing in the asset to which it is linked, assuming that this is possible.

It has been made clear that the profit sharing products in the Netherlands are in general not directly linked to an asset in which one can invest because the u-rate is not traded on financial markets. It was revealed in the last chapter that an important factor to the value of these products is the credit spread between the Libor swap rate and the government yields, and that neglecting this feature can amount in substantial errors in the valuation of it. As a consequence the consideration of this spread in determining risk mitigating strategies is essential to achieve required effectiveness. Therefore, in contrast to earlier research, the downside risk will be quickly dealt with by the use of a zero coupon bond that returns the guaranteed amount. The focus will then mainly be on what strategies can be used to hedge the profit sharing and the swap-government spread that was discussed in chapter 2.

3.2 Instruments & Strategies

The profit sharing product used here will be the one that is considered throughout this thesis; it promises €100 mln in 30 years based on some guaranteed rate and uses a fictive investment policy that is characterized by investing in 7 year bonds with coupons equal to the then prevailing u-rate. Figure 2.8 in chapter 2 showed that the value of the profit sharing in this product is sensitive to two interest rate curves. The sensitivity is measured in BPV (basis point value) which is one of two, the other one being duration, measures which are most often used in assessing sensitivity to interest rate changes. BPV measures exactly what is in the name, the change in value when the interest rate goes up by one basis point (0.01%). Duration is a mathematical expression introduced by Macaulay (1910) that captures the weighted average term to maturity of a fixed income product by considering all cash flows associated with the product. The modified duration is a linear approximation for the percentage change in value of a fixed income product when the yield changes by one percent and is closely related to the duration and BPV. Throughout this chapter the measure that will be used to asses sensitivity will be BPV.
The interest rate sensitivity of the value of the guarantee, modeled by a zero coupon bond paying €100 mln, can be visualized in a similar way as the profit sharing in figure 2.8. What is not displayed in figure 2.8, but is essential in determining hedging strategies, is that the sensitivity varies between different part of the interest rate curve. This means that every bar in figure 2.8 can be split up in a sensitivity to the parts (yields computed from traded bonds with different maturities) that are used to build the curve. These parts are often called buckets, because sensitivity to a range of yields is usually hedged by only a few instruments, the sensitivities are therefore "bucketed". When this is done for a contract with guaranteed rate of 2.5% it reveals that the 7y and 30y point influence the value most. This is due to the zero coupon bond that is only sensitive to this point (negatively) but also due to the profit sharing as the value is sensitive (positively) to the government curve.

From this point on the hedge for the guaranteed amount of €100 mln will not be discussed further and the focus will be solely on the profit sharing part.

3.2.1 Delta hedging

Using the sensitivities that were just discussed one can compose an offsetting position with sensitivities that have exactly the opposite sign using the products that were also available to construct the curve. This would result in a portfolio of cash, swaps and zero coupon government bonds that should equal the value of profit sharing if there is a change of one basis point in the curve. The swap hedge should be a position in receiver swaps[1] that increases in value when swap rates decrease, as the value of the product increases when discount rates drop and vice versa. This hedge would then also require a short position in government bonds because profit sharing increases when yields increase and a short position increases when this happens as well. This might sound counter intuitive as the amount of profit sharing increases when government bond yields rise (u-rates increase). In the first place the aim should however not be to replicate the cash flows exactly but the market value itself. If government bond yields rise, the hedge value increases and the holder of the hedge can finance the risen promised profit sharing from this. This actually reveals the difference in contract specification between a profit sharing contract with a guarantee and a unit-linked contract. In a unit-linked product that would have promised the return on a portfolio of government bonds, the insurer could hedge this by investing in this portfolio and the risks will be for the policyholder. For the profit sharing product, the situation is exactly opposite as, when the insurer promises the return on the government bonds and the government defaults, the insurer still has to pay the guarantee but has no underlying asset to pay it with. Hence the insurer bears the risk. This means that for a unit-linked portfolio the value of the investments, when done according to the specification of the product, always exactly matches the liabilities value. But for a profit sharing product with a guarantee, the market value of the product will generally differ from the specified investments of the fictitious investments. The easiest way to see this is considering a downgrade of the government. The yields will rise, making the investments worth less (this would be a bad scenario for a policyholder with a unit-linked product), but the u-rates rise, increasing the value of the profit sharing (so a positive scenario for a policyholder with a profit sharing product).
It is clear that the aim of a hedge consisting of swaps and zero coupon bonds is to mimic the optional character of the profit sharing by using a portfolio of linear products. This is in fact similar to applying a delta hedging strategy. The selling of profit sharing is in a way equal to selling put options on government bonds as the value of these put options increase when yields go up. To delta

[1]It can well be that the portfolio consists of both payer and receiver swaps, this statement aims at the net position.

hedge a put option one has to sell the underlying as the options go "in-the-money".[2]

3.2.2 Delta hedging and CDS

Though in theory delta hedging can be applied to hedge any derivative, the real world has different rules that sadly make this strategy difficult or impossible to implement. The long maturities of the insurance products make a delta hedging strategy difficult to apply and another practical limitation is that delta hedging requires the selling of bonds. The strategy would then require the selling of sovereign debt, this increases yields and therefore results a vicious circle.[3] Furthermore delta hedging requires continuous rebalancing, imposing additional costs.

In chapter 2 the importance of modeling the swap-government spread was emphasized. It was discussed here that the Dutch government yield can theoretically be split up in a credit spread part and a risk free (Libor) part. This is in line with the explanation given above that the selling of profit sharing based on u-yield results in exposure to default risk. It would mean that the short position in government bonds can be seen as a short position in Libor and exposure to a product that increases in value when the credit spread rises. This exposure can be found in CDS (Credit Default Swap) contracts, which, when having a long position, protects the investor.

3.2.2.1 Credit Default Swap

In a credit default swap two parties enter into an agreement that ends at at stated maturity or when a "credit event" occurs. In this agreement one party (the insurance buyer) pays a fixed fee at the end of every period until maturity of the contract and receives protection on some underlying in return from the other other party (the insurance seller). Hence, this contract transfers the credit risk from one party to another. The relevant underlying in this setting is a government bond but CDS contracts are available for a wide range of fixed income products. The "credit events" that trigger payment are specified in the contract. The most important ones, defined by the ISDA[4], are :

- Failure to pay payments when they are due.

[2]In line with section 2.1 one could also delta hedge the product with receiver swaps and a long position in government bonds, not using a zero coupon bond. This is however not discussed here as this does not model the profit sharing explicitly.

[3]There is in fact a draft law that poses limitations on the selling of sovereign debt products in some situations, see Dutch State Treasury Agency (2011)

[4]International Swaps and Derivatives Association

- Restructuring.
- Bankruptcy.

The value of such a contract is determined by the probability that the reference entity defaults in any given period during the life of the contract and the expected recovery rate as percentage of the face value of the underlying. Using the (risk neutral) default probabilities the CDS spread can be computed, which is the fee to be paid to set the value of the contract to zero at initiation

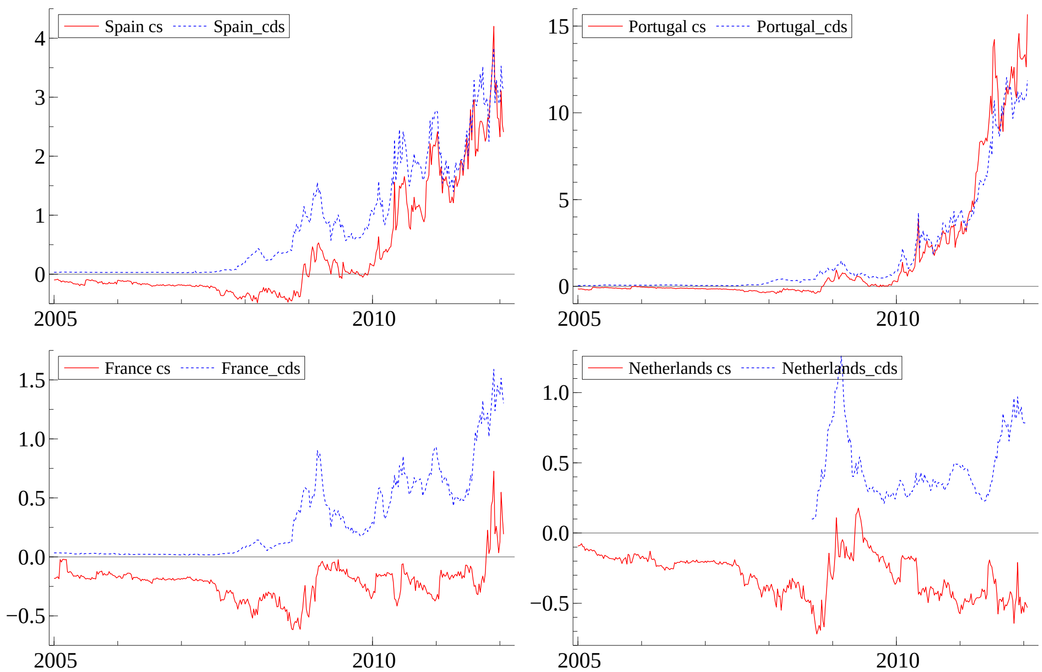
$$S_T \sum_{i=1}^{T} DF_i \cdot PnD_i \cdot \Delta_i + S_T \sum_{i=1}^{T} DF_i (PnD_{i-1} - PnD_i) \frac{\Delta_i}{2} = (1-R) \sum_{i=1}^{T} DF_i (PnD_{i-1} - PnD_i). \tag{3.1}$$

Here S_T is the CDS spread in basis points that sets the value of a T maturity CDS to zero, PnD_i is the probability of no default up till period i , Δ_i is the accrual period between period T_{i-1} to T_i and $(1-R)$ is the percentage which is not recovered. This formula then sets the present value of the expected fee payments, considering that they are paid, equal to the value of the claim in case a default occurs between any of the periods $(PnD_{i-1} - PnD_i)$. To compute the current value of the contract the default probabilities do not have to be computed every time because they are implied by CDS spreads quoted in the market. The value of the contract for the insured on a later time is then determined by

$$V(t,T) = \big(S_T(t) - S_T(0)\big) RPV01(T), \tag{3.2}$$

where $S_T(t)$ is the current CDS spread, $S_T(0)$ the contractual spread and $RPV01(T)$ is the 'risky present value" of a basis point given the notional and the assumed recovery rate (the present value of the premium payments, taking into consideration the default probability).

It is clear that a CDS contract should be able to hedge credit risk. The question however remains to what extent it mimics the credit spread and what approach can be taken to achieve the optimal sensitivities. Duffie (1999) showed that if the credit spread is not equal to the CDS spread it would lead to arbitrage opportunities. Subsequent research has however revealed that these arbitrage opportunities are not easily exploited and that it is common for the spreads in CDS contracts on sovereign debt to be above the credit spread. This can be observed in figure 3.1 as well, which shows the 5 year CDS spreads and credit spreads for several sovereigns. Quite some research has also focused on reasons why the two are not equal, why these arbitrage opportunities are so hard to exploit. One of those reasons is that CDS contracts on European entities are mostly dollar denominated as insurance takers want to avoid vulnerability to severe depreciation in case of a credit event. This exchange rate risk is priced into the contract. Secondly, the volumes available are not



(a) Credit Spreads vs CDS spreads

	CS			CDS			Difference				
	Average	Max	Min	Average	Max	Min	Average	Min	Max	σ	ρ
Spain	0.38	4.20	0.48-	0.83	3.84	0.03	0.46	0.38-	1.15	0.95	**0.95**
Netherlands	0.29-	0.18	0.72-	0.47	1.26	0.10	0.81	0.28	1.60	0.16	**0.07**
Portugal	1.65	15.68	0.42-	1.78	12.04	0.04	0.13	4.03-	1.47	3.61	**0.98**
France	0.21-	0.73	0.62-	0.31	1.59	0.02	0.52	0.05	1.50	0.16	**0.38**

(b) Summary statistics

Figure 3.1. *This figure shows (a) plots of the credit spread (computed as swap spread) at the 5 year point and the CDS spread on 5 year euro denoted CDS contracts of four countries, using weekly data over the period 1/1/2005 to 1/2/2012 obtained from bloomberg. (b) Shows some summary statistics. The difference is computed as CDS spread - credit spread, ρ represents the correlation between the two. All figures are in hundreds of basis points (percentages).*

yet comparable to the amount of sovereign debt outstanding, leading to liquidity premia. This is in sharp contrast with other sovereign derivative markets. Another reason could be that counterparty risk plays a greater role in the sovereign CDS market, i.e. one could question the value of such a contract when the financial sector (selling the contract) itself is supported by the government. Fontana and Scheicher (2010) give an analysis of the Euro area sovereign CDS market and study the difference between credit spread and CDS spread explicitly. They find that indeed most sovereign CDS spreads are above the credit spread and use several econometric models and some theory to explain this difference. They report to have found evidence for failure of arbitrage due to market

frictions and structural changes that limit traders to arbitrage differences away. Furthermore, they do observe strong correlation between the two and conclude that price discovery shifts between the derivative market and the bond market, depending in which the most informed investors are. This could have implications for the hedging strategy as, when price discovery takes place in the derivative market, the buying of CDS contracts leads to increased credit spread. This would then lead to the same vicious circle as hedging by selling government bonds does. Something similar would happen if bond traders would link a countries creditworthiness to the health of its financial sector. If insurance companies would hold CDS contracts on sovereign debt that other financial institutions have sold, this would lead to an increasing credit spread, resulting in a vicious circle once more. Dieckmann and Plank (2010) and Ejsing and Lemke (2010) both study this subject and do find some evidence on relations between the financial health of banks and sovereign CDS spreads. Finally "flight to safety" can pose problems as this drives down government yields leading to possibly negative credit spreads while CDS spreads do not show this effect, increasing differences between the two (this is illustrated for the Netherlands in figure 3.1(a)).
Though considering all these effects is important in designing hedge strategies, this is outside the scope of this thesis. In the evaluation of the strategies the assumption is made that the CDS spread is perfectly correlated with the credit spread. From the table in figure 3.1 one can see that the correlations are very high in the times it matters (Portugal and Spain, but also during last year for France), so the CDS spread definitely seems to capture jumps in creditworthiness and therefore the assumption seems reasonable.

A hedge portfolio that consists of cash, swaps and a CDS contract should then create the right exposures. In this portfolio the short position in government bonds from the last strategy is modeled as a position in Libor (net payer swaps) and credit spread (long protection). The value of the CDS contract is sensitive to both changes in the Libor swap curve and changes in the government curve. If the Libor rates drop, the present value of future cash flows will be higher but, more importantly, the credit spread increases if the government rates remain equal. The CDS contract chosen is ideally linked to the credit spread that influences the profit sharing most, hence in this case seven year CDS contracts will be considered.[5] The notional amount insured is determined by the sensitivity of the profit sharing to the government curve in the same way that the notional amounts of the government bonds are determined in the previous strategy. This strategy then requires an additional swap hedge to hedge unwanted exposure to Libor swap rates introduced by the CDS. Assuming that the CDS spread is a deterministic function of the credit spread this is visualized below by the use of a first order Taylor expansion around $PS(r^f, u)$, where r^f is the Libor rate and , $u = r^f + cs$ the government

[5] One could also consider a hedge that uses multiple CDS contracts, this might result in better results for non-parallel curve shifts. This is not done here because its expected to give only minor improvements in immunity while being less cost efficient.

rate as a function of r^f and the credit spread cs. Using $PS(r^f,u) \leftrightarrow PS(r^f,cs)$ then gives

$$\mathrm{d}PS(r^f,cs) = \frac{\partial PS}{\partial r^f}\mathrm{d}r^f + \frac{\partial PS}{\partial cs}\mathrm{d}cs, \text{ with} \qquad \mathrm{d}CDS(r^f,cs) = \frac{\partial CDS}{\partial r^f}\mathrm{d}r^f + \frac{\partial CDS}{\partial cs}\mathrm{d}cs,$$

setting $\frac{\delta CDS}{\delta cs} = \frac{\delta P}{\delta cs}$ then results in

$$\mathrm{d}PS = \mathrm{d}CDS + \left(\frac{\partial PS}{\partial r^f} - \frac{\partial CDS}{\partial r^f}\right)\mathrm{d}r^f.$$

This means that the sensitivity towards the government curve is modeled as sensitivity towards the credit spread. For scenarios in which the swap curve varies there is an offsetting hedge determined by $\frac{\partial CDS}{\partial r^f}$. The notional for the CDS contract is determined by $\frac{\partial PS}{\partial cs}$ and $\frac{\partial CDS}{\partial cs}$. The Libor hedge remains the same as in the previous hedge because $\frac{\partial PS}{\partial r^f}$ does not change.

3.2.3 Static Swaption and CDS hedge

In expectation the two hedges above will give similar results when applied instantaneously. They however both use instruments with a linear payoff to offset changes in value of a non-linear product. A straightforward way to build a hedge portfolio of which it can be expected to have better results is to use the replicating portfolio that is used to value the profit sharing. Because there are only swaptions traded based on the Libor swap curve the remainder should again be covered by the use of a CDS contract. This would then result in a hedge portfolio of 30 payer swaptions, of which the notionals are determined by the investment strategy of the fictive portfolio based on the guaranteed rate.[6] The strike of the swaptions depends on the guaranteed rate r^g and the spread between Libor and the u-rate[7]. The notional used for the CDS contract is again determined by $\frac{\partial PS}{\delta cs}$ and $\frac{\partial CDS}{\partial cs}$. Lastly, this portfolio consists of a cash amount equal to the difference between the profit sharing value and the swaption portfolio.
It should be clear that the swaption hedge does not represent a hedge against changes in the Libor curve in a similar way as the strategies above do but also includes a part of the sensitivity towards the government curve.

The strategy described above will result in a hedge that would partially capture the optional character of the product if the government curve is modeled as a risk free Libor rate plus credit spread.

[6]See section 1.5 for more details.
[7]In this portfolio the strike is set equal to $r^g - cs$. One can also choose to set the strike of the swaptions equal to the guaranteed rater^g and buy the CDS at a spread of 0 (value > 0). This seems to result in slightly better performance.

3.2.4 Linear and non-linear hedge portfolios ignoring swap-government spread

For completeness, also the hedge portfolio that would be constructed when not considering varying swap government spreads are evaluated. Under this assumption there is only sensitivity towards the Libor curve because the u-rate is modeled as $u = r^f + cs$, in which the spread cs is fixed. Under this assumption again a linear and a non-linear hedge can be constructed using respectively swaps or swaptions. The linear hedge would consist of cash and swaps, in which the notionals are determined by $\frac{\partial P(r^f)}{\partial r^f}$. The non-linear hedge is just the replicating portfolio of Libor swaptions that is also used in the previous hedge plus the difference between this value and the profit sharing value in cash. The CDS is not used to provide protection for varying swap government spreads, therefore it is expected that this hedge will display a similar performance when the Libor and government curve move in the same direction simultaneously.

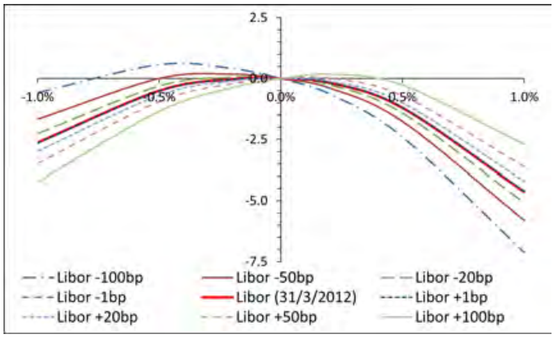
3.3 Performance evaluation

To asses the performance of these strategies, first the difference between the value of the hedge portfolio and the value of the profit sharing are considered for instantaneous parallel shifts in both the Libor swap curve and the government curve.[8]. Subsequently, also the performance over a larger time span will be evaluated.

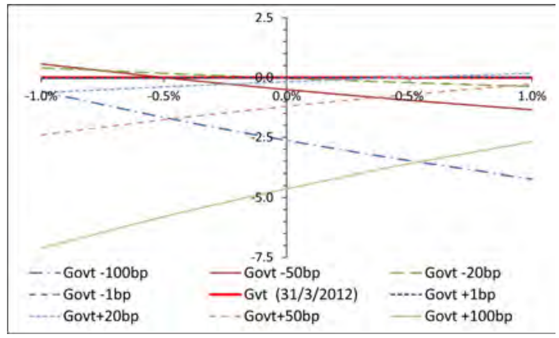
3.3.1 Instantaneous performance

For this the Libor swap curve of 31/3/2012 will be used as standard reference and the government curve is modeled by adding 20bp (basis points) as credit spread. In this scenario the value of the profit sharing is approximately € 15.8 mln. The shifts, in both swap and government curve, that are applied to this reference scenario are -100bp, -50bp, -20bp, -1bp , 0bp, 1bp, 20bp, 50bp and 100bp resulting in a total of 81 scenarios. This means that the maximum credit spread will be 220bp in the most extreme scenario.

[8]Constructing a hedge in this way should however result in a similar performance for non parallel shifts



(a) Mismatch (Fixed Libor)



(b) Mismatch (Fixed Government)

Figure 3.2. *This figure shows the differences in the value of the profit sharing and the value of a hedge portfolio consisting of swaps and government bonds for fixed libor rates and instantaneous changes in credit spreads (a) and for fixed government curves and instantaneous changes in the libor curve (b). The values on the y-axis are in €millions, the product under consideration guarantees €100 mln in 30 years using a guaranteed rate of 2.5%. At (0,0) the value of the profit sharing is approximately €15.8 mln.*

3.3.1.1 Delta Hedging

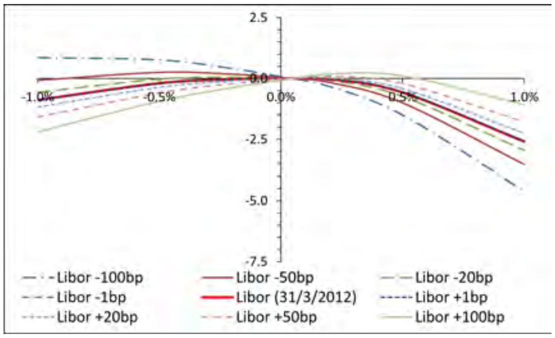
The delta hedging strategy, consisting of Libor swaps and government bonds, is expected to perform very well for small changes in both directions. This is confirmed by figure 3.2 in which the mismatch between the hedge portfolio and the profit sharing (computed as *Hedge - Profit Sharing* is shown. The bold red line represents the reference scenario. In graph 3.2(a) the Libor (discount) curve is fixed and on the horizontal axis varying shifts (forward curves) are considered. The 220bp credit spread scenarios are in the lower left (€-4.2 mln) and lower right (€-7.1 mln), when the profit sharing values are respectively €1.6 mln and €27.1 mln. As expected, the mismatch increases steepest for rising credit spreads combined with decreasing discount rates. This explains the slight skew to the right.

Figure 3.2(b) shows exactly the same but from another angle. Here the government curves are fixed and varying shifts in the Libor curve are considered. This visualizes that swap hedge performs very well with a maximum mismatch of €0.26 mln within the 20bp range.

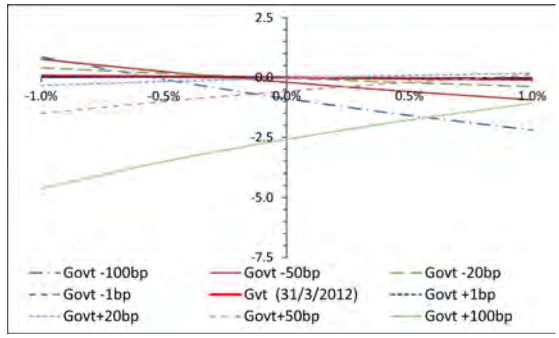
In summary figures 3.2(a) and 3.2(b) display that the mismatch for small changes in both ways is approximately linear and confirm that this hedge should be effective when rebalanced frequently.

3.3.1.2 Delta Hedging and CDS

Though the delta hedging portfolio that makes use of government bonds has good results, discussion earlier has suggested that the implementation is probably hard due to liquidity and regulatory



(a) Mismatch (Fixed Libor)



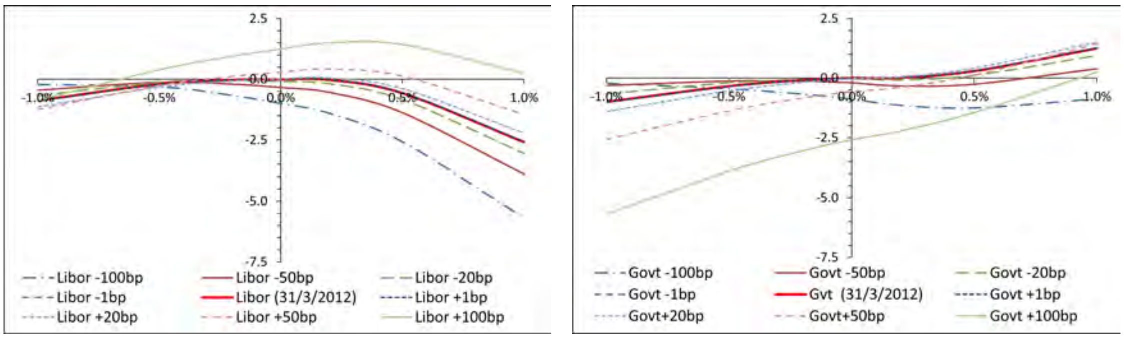
(b) Mismatch (Fixed Government)

Figure 3.3. *This figure shows the differences in the value of the profit sharing and the value of a hedge portfolio consisting of swaps and a CDS for fixed libor rates and instantaneous changes in credit spreads (a) and for fixed government curves and instantaneous changes in the libor curve (b). The values on the y-axis are in €millions, the product under consideration guarantees €100 mln in 30 years using a guaranteed rate of 2.5%. At (0,0) the value of the profit sharing is approximately €15.8 mln.*

reasons. An alternative is a delta hedging strategy that makes use of the credit spread to hedge for the swap government spread. Figure 3.3 displays the results of this strategy. The lines display a very similar pattern as in the previous strategy, though this is partly due to the assumptions that were made. Under these assumptions the use of a CDS contract even seems to outperform the strategy of using government bonds when considering instantaneous changes. The curvature in figure 3.3(a) is less steep and the Libor swap hedge is more effective for shifts in the government curve. From both figure 3.2 and figure 3.3 it is clear that the optional character is stronger for varying government curves than for shifts in the Libor curve. In both figure 3.3(a) and figure 3.3(b), a move over the horizontal axis also implies a move in credit spread. In figure 3.3(b), a move to the left (swap curve lower) results in an increase of the credit spread but the value of the profit sharing only increases slightly due to lower discount rates because the government curve remains equal. This (linear) increase in value is almost completely off set by the CDS hedge. In figure 3.4(a) a move to the right from the reference point increases the credit spread as well. But this time the government curve is affected. This also influences the value of the profit sharing positively but in a non linear way because the forward curve used to compute the value of the portfolio of swaptions has changed. The linear CDS only partly captures this increase, resulting in a less effective hedge.

3.3.1.3 Static Swaption hedge and CDS

This strategy is expected to outperform the linear hedges as it should be able to partly capture the non-linearity in the profit sharing that was just discussed. The results, displayed in figure 3.4, however show that the performance is very similar to the linear hedge that uses swaps and a CDS



(a) Mismatch (Fixed Libor) **(b)** Mismatch (Fixed Government)

Figure 3.4. *This figure shows the differences in the value of the profit sharing and the value of a hedge portfolio consisting of swaptions and a CDS for fixed libor rates and instantaneous changes in credit spreads (a) and for fixed government curves and instantaneous changes in the libor curve (b). The values on the y-axis are in €millions, the product under consideration guarantees €100 mln in 30 years using a guaranteed rate of 2.5%. At (0,0) the value of the profit sharing is approximately €15.8 mln.*

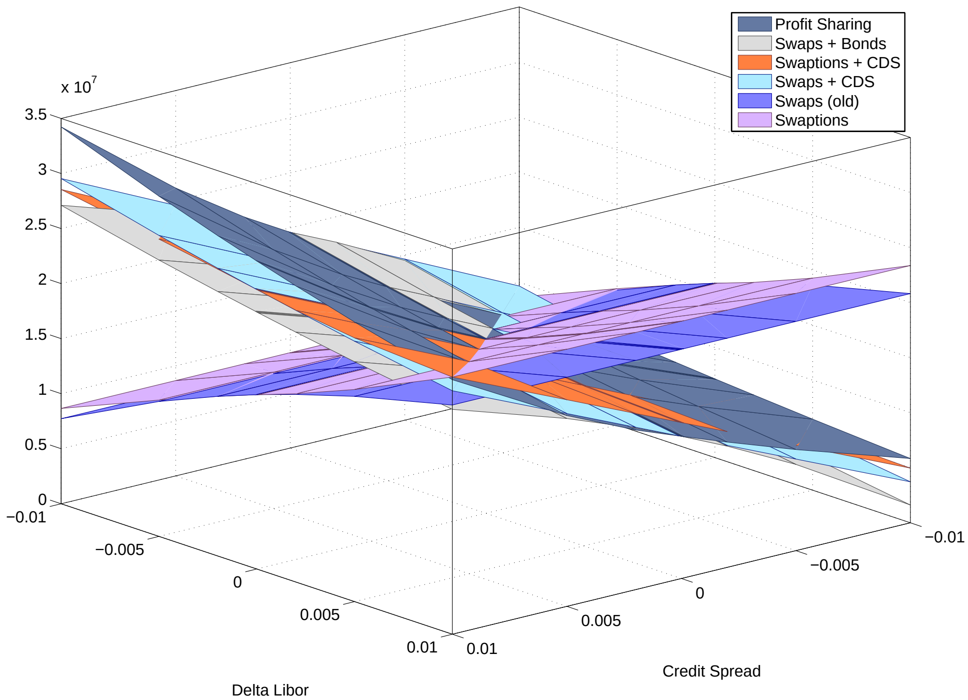
for the reference scenario but worse for others. Under the assumption that the government curve is modeled as swap plus credit spread, the portfolio of swaptions tracks the profit sharing very well for simultaneously decreasing or increasing swap and government curves (constant spread), the CDS should then capture increases in the spread. The lines in 3.4(b) are not as linear as before because changes in the discount curve do effect the swaptions in a non-linear way while they do not affect the value of the profit sharing much (more sensitive to forward curve). This also explains that the lines in figure 3.4(a) show larger distances from the reference rate.

3.3.1.4 Comparison

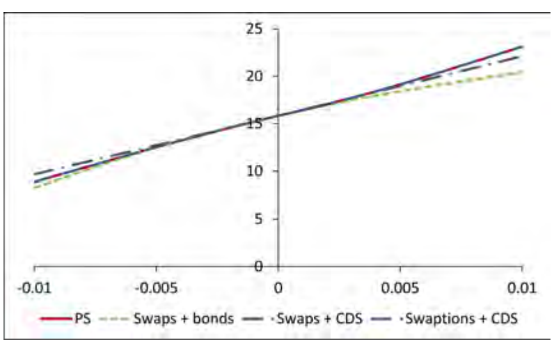
To compare all strategies a 3d plot is shown in figure 3.5(a). This figure also includes the two hedge portfolios (linear and non-linear) as they would be constructed under the assumption that the spread between the Libor rate and the government rate is constant (section 3.2.4). The non-linear (swaption) portfolio is exactly similar to the portfolio of swaptions that is used in the swaptions plus CDS hedge. The linear (swap) hedge is constructed differently because, where the hedge of section 3.2.1 only used swaps to hedge for the discounting effect, in this setting there is not differentiated between sensitivity towards the forward (government) curve and the discount (Libor) curve. Therefore the BPV's towards both curves are modeled to be captured by only one. This means that the swap hedge here mimics the sensitivity of the portfolio of swaptions.

Figure 3.5(a) indeed shows that the portfolio of swaptions and the just described portfolio of swaps (swaps (old)) lie close together. As expected, they also mimic the value of the profit sharing well when the Libor curve and the government curve move in the same direction simultaneously and the credit spread remains equal. This is shown explicitly in figures 3.5(b) and 3.5(c). However, when spreads do vary, these hedge portfolios display sensitivities that have the wrong sign, meaning that the value of the hedge moves in the opposite way of the profit sharing value. Finance theory states that such a scenario should not present itself as a government curve should always be on the same level or above the risk free curve. Today it is however clear that in times of crises this is exactly what happens: investors move away from riskier countries (increasing yields) and look for safe havens, resulting in very low (sometimes negative) yields for these countries, while central banks lower borrowing (lowering also Libor) rates to stimulate the economy. This stresses once more the importance of acknowledging the fact that the value of these profit sharing products is determined by two curves and that this should be used in designing and implementing effective hedge strategies for profit sharing.

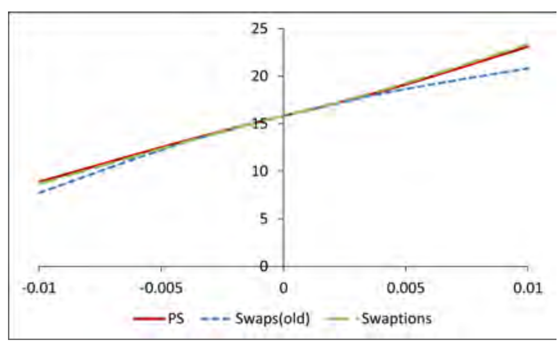
Figure 3.5 furthermore visualizes what has already been analyzed in the previous paragraphs and confirms also by the numerics in the table that, when considering instantaneous changes, a linear hedge consisting of swaps and a CDS seems to perform best.



(a) 3d plot



(b) Diagonal view



(c) Diagonal view

	Swaps + Bonds		Swaps + CDS		Swaptions + CDS		Swaptions		Swaps (1 curve)	
Mismatch										
Max (-)	€7.11-	-45%	€4.61-	-29%	€5.32-	-34%	€25.28-	-160%	€26.49-	-167%
Max (+)	€0.57	4%	€0.86	5%	€0.72	5%	€16.82	106%	€14.98	95%
Average	€1.02-	-6%	€0.48-	-3%	€0.55-	-3%	€0.65-	-4%	€1.05-	-7%
σ	€0.09	1%	€0.06	0%	€0.07	0%	€0.49	3%	€0.53	3%

(d) Summary statistics

Figure 3.5. *This figures gives a comparison between five hedge strategies using (a) a 3d plot and (b,c) a diagonal view from front to back, and (d) a table of summary statistics. The values are in €millions and in percentages of the initial value of the profit sharing (0,0). The product under consideration guarantees €100 mln in 30 years using a guaranteed rate of 2.5%.*

3.3.2 Performance over time

In the previous subsection only the performance of the strategies over a very small time increment was evaluated. This simplified the analysis because the time aspect of every element could then be discarded. As delta hedging can not be implemented in a continuous way (transaction cost, liquidity, administrative reasons, etc.) a more practical approach has to be found. This would entail rebalancing only a few times a year or the use of a static swaption portfolio as hedge. The latter does not require additional transactions after buying all swaptions at initiation. Results of the previous section showed that a CDS contract should be able hedge the spread between the government and the swaprate, but this contract requires to pay a periodic fee (CDS spread) to the insurance seller and will attain a value of zero at maturity if no credit event occurs. Therefore, the use of a CDS will require rolling over the CDS periodically or maybe buying new ones with the notional based on the reserve and the sensitivity towards the credit spread. The cost of doing this should be financed by the premium that is paid for the contract. An immediate problem that then arises is that the CDS contract used in the performance appraisal for instantaneous changes had an underlying notional of €145 mln. This was determined by matching the BPV's of the CDS contract and the profit sharing contract. Because the spread at initiation is 0.2%, this means that an annual fee of approximately €290,000 has to be paid. Though, ceteris paribus, the BPV of the profit sharing to the credit spread will lower over time because the period over which profit sharing has to be paid decreases, the difference between the value of the profit sharing and the swaption portfolio (around €1 mln) is not enough to finance the CDS. The BPV has a strong positive correlation with the maturity of the contract.This means that, the higher the maturity, the lower the notional has to be to achieve the right BPV. The maturity of CDS contract in the previous section was set at 7 years, because this made the most sense regarding the influence of the 7 year point on the u-rate. This however seems impossible so because of this, together with the fact that the profit sharing product has also shown a strong sensitivity toward the 30 year point and the fact that 30y CDS contracts are about the ones with the longest maturity traded, the CDS contracts used in this section will have a maturity of 30 years.[9] This will require a notional of approximately €46 mln to achieve the same BPV. After 30 years this would amount to $30 \cdot 0.2\% \cdot 46mln = 2.8mln$. The €1 mln in excess cash will be worth €2.13 mln. So still a bit less but as just mentioned, the BPV of the profit sharing to the credit spread will decrease over time, allowing for smaller notionals in the CDS, resulting in lower fees. Meaning that this strategy should be possible.

The approach that will be followed here is to monitor the hedge on a monthly basis and readjust when a given threshold is breached. This threshold is defined as a mismatch between the BPV's

[9]It is assumed here that the term structure of the CDS spread is flat.

of the hedge and the profit sharing. Using this approach the loss due to time of the instruments will be captured automatically as it will decrease the BPV of the hedge. The threshold is set at a mismatch of .05% of the value of the profit sharing. The performance of three hedge portfolios will be analyzed in a total of 4 scenarios. All scenarios will start from the reference point in the last section and are defined by different shocks to both the swap and the government curve. The scenarios all evolve over a six month time span. The three hedge portfolios considered are the static swaption portfolio described in section 3.2.4, the hedge that combines this with a CDS, described in section 3.2.3, and the completely linear hedge of swaps and a CDS contract, described in section 3.2.2.

3.3.2.1 Scenario A

This scenario is relatively mild, the swap curve and the government curve move in the same direction. At the end of 6 months the swap curve has made a parallel shift upwards of 65 basis points and the government curve shifted 70 basis points up. This results in a spread increase of 5 basis points (total of 25bp). The results of the 3 strategies are given in table 3.1 below. One could expect the standard swaption portfolio to perform best as not much happens with the credit spread, resulting mainly in extra costs when using a CDS. These costs (about €42,000) however are off set by the small increase in the spread, making the swaption portfolio perform worse then the swaption hedge that additionally uses a CDS. Remarkable is that, while the CDS spread deviates by a maximum of 5 basis point, two CDS trades are needed to adjust the mismatch in credit spread sensitivity. This is due to the fact that the liabilities rise by over €5 mln, forcing the notional amount in the CDS to increase from approximately €46 mln to €64 mln.
The hedge consisting of swaps and a CDS has the lowest performance. This is due to the linearity of the swaps, resulting in a profit from this hedge of only €3.7 mln while the swaptions increase by €4.1 mln. The strong increase in the mismatch during the last two months of all strategies is mainly due to the mismatch in the swap BPV, for which the hedge is not periodically adjusted.

3.3.2.2 Scenario B

Where in the last scenario both curves move up over six months, in this scenario both move down. The swap curve shifts down by 50 basis points, while the government curve only shifts down by 30 basis points. The results are shown in table 3.2 and display that the credit spread varies quite a bit

	Scenario A			Profit sharing			HEDGE 1 - Swaptions			Mismatch		
Date	$\Sigma\Delta r^f$	$\Sigma\Delta cs$	cds	Value	$BPVr^f$	$BPVcs$	Value	$BPVr^f$	$BPVcs$	Value	$BPVcs$	$BPVr^f$
2-4-2012	0	0	20	15,877,028	33,103-	95,616	15,877,028	65,179	-	-	95,616-	2,666
30-4-2012	15	20	25	17,295,258	35,606-	95,599	16,860,562	62,941	-	434,696-	95,599-	2,948
31-5-2012	30	30	20	17,737,005	36,070-	95,709	17,817,093	60,671	-	80,088	95,709-	1,032
28-6-2012	40	35	15	17,896,709	36,067-	102,749	18,445,573	59,220	-	548,864	102,749-	7,462-
31-7-2012	35	35	20	18,108,059	36,417-	104,249	18,178,222	60,462	-	70,163	104,249-	7,369-
30-8-2012	45	50	25	19,545,943	38,470-	125,084	18,810,019	59,055	-	735,924-	125,084-	27,559-
28-9-2012	65	70	25	21,278,579	40,358-	123,454	19,994,476	55,864	-	1,284,103-	123,454-	27,232-
									σ	567,167	11,978	12,314

	HEDGE 2 - Swaptions + CDS			Mismatch			HEDGE 3 - Swaps + CDS			Mismatch		
Date	Value	$BPVr^f$	$BPVcs$	Value	$BPVcs$	$BPVr^f$	Value	$BPVr^f$	$BPVcs$	Value	$BPVcs$	$BPVr^f$
2-4-2012	15,877,028	30,437-	95,616	-	-	2,666	15,877,028	33,088-	95,616	-	-	15
30-4-2012	17,321,594	31,415-	93,735	26,336	1,864-	4,191	17,273,020	34,443-	93,735	22,238-	1,864-	1,163
31-5-2012	17,801,800	31,230-	91,901	64,795	3,808-	4,839	17,691,882	34,487-	91,901	45,123-	3,808-	1,583
28-6-2012	17,969,126	30,888-	90,699	72,417	12,050-	5,179	17,815,076	34,312-	90,699	81,633-	12,050-	1,755
31-7-2012	18,211,834	43,635-	103,421	103,775	828-	7,218-	18,062,664	47,647-	103,421	45,395-	828-	11,230-
30-8-2012	19,340,342	44,358-	102,085	205,601-	22,999-	5,888-	19,132,232	48,561-	102,085	413,711-	22,999-	10,092-
28-9-2012	20,518,940	66,021-	121,884	759,639-	1,570-	25,662-	20,212,786	69,962-	121,884	1,065,793-	1,570-	29,603-
			σ	285,518	7,835	10,363			σ	362,147	7,835	10,714

Table 3.1. *This table shows the the performance of the hedge strategies for scenario A, defined in the upper left part of the table. $\Sigma\Delta r^f$ and $\Sigma\Delta cs$ respectively show the cumulative shift in the swap and government curve.*

in this scenario, with a maximum of 45 basis points. Remarkable here is that the hedges using a CDS do not require an adjustment, where the last scenario did while the CDS spread moved by a maximum of 5 basis points there. This shows how important the exact value of the profit sharing (liability) is relative to the spread change in designing and adjusting the hedge.

In this scenario the static portfolio of swaptions plus a (dynamic) CDS and the linear hedge portfolio consisting of only swaps and a CDS both perform very good. The BPV match to the swap rate seems to be slightly better for hedge 2. Hedge 1, consisting of only swaptions performs a lot worse, resulting in a mismatch of over €2 million. This is mainly due to the increase in the credit spread of 20 basis points, as the BPV to the government curve shows a relatively large mismatch.

3.3.2.3 Scenario C

The third scenario represents a situation in which the country is seen as a safe haven, pushing yields below the swap curve, resulting in a negative credit spread. This creates an interesting scenario since a CDS spread cannot be negative. As government yields drop the value of the profit sharing lowers. The swaptions in hedge 1 and 2 also lower in value but to a lesser extent, as the shift down is smaller. The CDS contract has dropped in value by approximately €1.9 million but does not require any adjustment as the BPV mismatch remains low.

From 31/7/2012 the credit spread is negative. One could question the need of a CDS contract in this case as it would only impose costs to the hedge if the original contract is held on to. The method of using BPV mismatch to trigger rebalancing however ensures consistent hedging automatically.

	Scenario B			Profit sharing			HEDGE 1 - Swaptions			Mismatch		
Date	$\Sigma\Delta r^f$	$\Sigma\Delta cs$	cds	Value	$BPVr^f$	$BPVcs$	Value	$BPVr^f$	$BPVcs$	Value	$BPVcs$	$BPVr^f$
2-4-2012	0	0	20	15,877,028	33,103-	95,616	15,877,028	65,179	-	-	95,616-	2,666
30-4-2012	20-	-20	20	14,607,822	30,692-	96,679	14,556,614	68,580	-	51,208-	96,679-	2,593
31-5-2012	25-	-30	15	13,806,521	28,982-	96,107	14,226,742	69,571	-	420,221	96,107-	2,445
28-6-2012	40-	-35	25	13,764,351	29,020-	98,338	13,176,691	71,777	-	587,660-	98,338-	2,458
31-7-2012	25-	-20	25	14,798,359	30,781-	98,177	14,250,228	70,067	-	548,131-	98,177-	2,671
30-8-2012	45-	-20	45	15,441,821	32,319-	102,468	12,832,467	72,812	-	2,609,354-	102,468-	2,663
28-9-2012	50-	-30	40	14,585,023	30,513-	101,776	12,472,981	73,544	-	2,112,042-	101,776-	2,281
									σ	1,055,028	2,506	138

	HEDGE 2 - Swaptions + CDS			Mismatch			HEDGE 3 - Swaps + CDS			Mismatch		
Date	Value	$BPVr^f$	$BPVcs$	Value	$BPVcs$	$BPVr^f$	Value	$BPVr^f$	$BPVcs$	Value	$BPVcs$	$BPVr^f$
2-4-2012	15,877,028	30,437-	95,616	-	-	2,666	15,877,028	33,088-	95,616	-	-	15
30-4-2012	14,548,972	29,659-	98,239	58,850-	1,560	1,033	14,604,306	32,332-	98,239	3,516-	1,560	1,639-
31-5-2012	13,716,888	28,670-	98,913	89,633-	2,805	312	13,787,532	31,577-	98,913	18,989-	2,805	2,596-
28-6-2012	13,658,527	29,871-	100,956	105,824-	2,617	851-	13,778,544	32,333-	100,956	14,193	2,617	3,313-
31-7-2012	14,714,151	29,509-	98,904	84,208-	727	1,272	14,794,660	33,191-	98,904	3,699-	727	2,409-
30-8-2012	15,335,573	32,339-	101,655	106,248-	814-	19-	15,489,532	35,212-	101,655	47,711	814-	2,892-
28-9-2012	14,474,172	31,636-	102,355	110,851-	579	1,123-	14,655,293	34,437-	102,355	70,270	579	3,924-
			σ	36,376	1,236	1,215			σ	29,756	1,236	1,186

Table 3.2. *This table shows the the performance of the hedge strategies for scenario B, defined in the upper left part of the table. $\Sigma\Delta r^f$ and $\Sigma\Delta cs$ respectively show the cumulative shift in the swap and government curve.*

This is because, when the credit spread lowers, the delta to the credit spread (BPV_{cs}) lowers as well. In turn, this will then trigger a roll over of the CDS contract when the decrease in credit spreads is large enough.

The linear hedge (3) and the swaption hedge with CDS again show a very similar performance. This seems because the swaptions capture the sensitivity of the profit sharing to the swap rate better then the swaps. The swaption hedge underperforms since the drop in credit spread does not lead to a loss on a CDS hedge as it does in the hedge 2.

Interesting here is that the swap hedge, though closest to the value of the profit sharing, displays the highest mismatch in sensitivity to the swap curve after six months. This is again because the linear hedge is more sensitive than the embedded options in the profit sharing, as they are out of the money (7y swap rate is 0.0172 at 28/9/2012).

3.3.2.4 Scenario D

The last scenario is the most extreme and represents a situation in which the countries finances deteriorate quickly. This scenario resembles what has happened to several European countries while eventually, partly because of this, also the Libor rates dropped. In this scenario the Libor rates first rise due to the lack of confidence between banks, after six months the swap curve has however shifted down by 20 basis point. The government curve shifts up by a total of 425 basis points, increasing the credit spread to 445 bp.

The value of the profit sharing increases due to both a sharp increase in the government yields

	Scenario C			Profit sharing			HEDGE 1 - Swaptions			Mismatch		
Date	$\Sigma\Delta r^f$	$\Sigma\Delta cs$	cds	Value	$BPVr^f$	$BPVcs$	Value	$BPVr^f$	$BPVcs$	Value	$BPVcs$	$BPVr^f$
2-4-2012	0	0	20	15,877,028	33,103-	95,616	15,877,028	65,179	-	-	95,616-	2,666
30-4-2012	5-	-10	15	15,101,551	31,464-	95,304	15,568,158	66,263	-	466,607	95,304-	2,422
31-5-2012	15-	-25	10	13,993,348	29,210-	95,012	14,915,331	68,107	-	921,983	95,012-	2,306
28-6-2012	20-	-35	5	13,198,918	27,538-	94,344	14,586,311	69,090	-	1,387,393	94,344-	2,284
31-7-2012	10-	-30	-	13,402,016	27,679-	93,476	15,285,508	67,888	-	1,883,492	93,476-	2,091
30-8-2012	15-	-40	-	12,617,533	26,037-	92,733	14,961,172	68,901	-	2,343,639	92,733-	2,205
28-9-2012	20-	-55	-	11,372,132	23,459-	90,767	14,627,464	69,856	-	3,255,332	90,767-	2,549
									σ	1,041,725	1,590	185

	HEDGE 2 - Swaptions + CDS			Mismatch			HEDGE 3 - Swaps + CDS			Mismatch		
Date	Value	$BPVr^f$	$BPVcs$	Value	$BPVcs$	$BPVr^f$	Value	$BPVr^f$	$BPVcs$	Value	$BPVcs$	$BPVr^f$
2-4-2012	15,877,028	30,437-	95,616	-	-	2,666	15,877,028	33,088-	95,616	-	-	15
30-4-2012	15,079,162	29,362-	96,271	22,389-	966	2,102	15,091,574	32,363-	96,271	9,977-	966	899-
31-5-2012	13,924,230	28,156-	97,581	69,118-	2,569	1,054	13,965,843	31,396-	97,581	27,505-	2,569	2,185-
28-6-2012	13,089,792	27,152-	98,238	109,126-	3,895	386	13,152,513	30,612-	98,238	46,405-	3,895	3,075-
31-7-2012	13,316,584	26,419-	96,916	85,432-	3,440	1,260	13,343,402	30,573-	96,916	58,614-	3,440	2,894-
30-8-2012	12,971,339	26,043-	97,578	353,806	4,846	6-	13,019,158	30,483-	97,578	401,625	4,846	4,447-
28-9-2012	12,616,765	25,720-	98,238	1,244,633	7,471	2,261-	12,693,132	30,370-	98,238	1,321,000	7,471	6,911-
			σ	455,557	2,300	1,494			σ	471,488	2,300	2,122

Table 3.3. *This table shows the the performance of the hedge strategies for scenario C, defined in the upper left part of the table.* $\Sigma\Delta r^f$ *and* $\Sigma\Delta cs$ *respectively show the cumulative shift in the swap and government curve.*

and the lower discount rates, resulting in a profit sharing value of € 76 mln. In this scenario, the government curve and the swap curve move in opposite direction, this means that a swaption hedge has decreased in value. This is confirmed by the first hedge consisting only of swaptions, that is no where close to the value of the profit sharing (€ 61.5 mln mismatch). This is in line with figure 3.5. Both hedges that use a CDS contract perform much better but are still short close to € 7 mln. While this means that the hedge captures about 90% of the increase in value, it displays that a linear CDS hedge misses much of the convexity in the profit sharing for large yield changes. In this scenario only two adjustment are made, one at 31/5/2012 and one at 31/7/2012. These adjustment result in an increase in the notional amount in the CDS from € 46 mln to € 74 mln. The costs increase as well because spreads rise, to a total of approximately € 56,000 a month. This is again off set easily by the increase in value over the previous periods of the CDS hedge to € 65 mln. More frequent adjustments should theoretically improve the result but would imply higher costs as well.

A significant part of the mismatch in all hedges is also due to the fact that no adjustment are made to match the BPV to the swap curve, doing this would increase the performance by about € 1 mln.

3.3.2.5 Comparison

Both hedge strategies that use a CDS show a very similar performance. This is because in all scenarios the optional character in the swaptions, that is suppose to mimic the optional element in the risk free part of the profit sharing, is not often far in or out of the money, meaning that the portfolio of swaps still suffices. Scenario A is the only scenario in which hedge 2 really outperforms

	Scenario D			Profit sharing			HEDGE 1 - Swaptions			Mismatch		
Date	$\Sigma\Delta r^f$	$\Sigma\Delta cs$	cds	Value	$BPVr^f$	$BPVcs$	Value	$BPVr^f$	$BPVcs$	Value	$BPVcs$	$BPVr^f$
2-4-2012	0	0	20	15,877,028	33,103-	95,616	15,877,028	65,179	-	-	95,616-	2,666
30-4-2012	25	25	20	17,414,654	35,664-	94,902	17,481,342	61,209	-	66,688	94,902-	1,971
31-5-2012	25	75	70	23,483,380	46,038-	130,951	17,511,631	61,544	-	5,971,749-	130,951-	23,368-
28-6-2012	25	175	170	37,028,467	68,017-	138,398	17,537,555	61,844	-	19,490,912-	138,398-	8,537-
31-7-2012	-	425	445	76,360,052	134,909-	154,133	15,957,607	66,335	-	60,402,445-	154,133-	47,111
30-8-2012	30-	375	425	72,589,343	130,857-	161,032	13,912,773	70,983	-	58,676,570-	161,032-	40,808
28-9-2012	20-	405	445	76,178,243	135,536-	159,454	14,628,349	69,856	-	61,549,894-	159,454-	45,938
									σ	27,338,085	26,221	26,731

	HEDGE 2 - Swaptions + CDS			Mismatch			HEDGE 3 - Swaps + CDS			Mismatch		
Date	Value	$BPVr^f$	$BPVcs$	Value	$BPVcs$	$BPVr^f$	Value	$BPVr^f$	$BPVcs$	Value	$BPVcs$	$BPVr^f$
2-4-2012	15,877,028	30,437-	95,616	-	-	2,666	15,877,028	33,088-	95,616	-	-	15
30-4-2012	17,473,699	31,295-	92,504	59,045	2,398-	4,369	17,395,428	34,192-	92,504	19,226-	2,398-	1,472
31-5-2012	22,121,745	37,054-	92,508	1,361,635-	38,443-	8,984	22,029,562	40,404-	92,508	1,453,818-	38,443-	5,635
28-6-2012	35,208,808	86,343-	130,942	1,819,659-	7,456-	18,326-	35,109,376	90,093-	130,942	1,919,091-	7,456-	22,076-
31-7-2012	71,261,670	136,918-	135,360	5,098,382-	18,772-	2,008-	71,260,072	141,221-	135,360	5,099,980-	18,772-	6,312-
30-8-2012	65,778,264	85,162-	160,520	6,811,079-	511-	45,695	65,903,845	89,006-	160,520	6,685,498-	511-	41,851
28-9-2012	69,462,409	88,491-	158,347	6,715,834-	1,107-	47,045	69,552,159	93,141-	158,347	6,626,084-	1,107-	42,395
			σ	2,805,360	13,197	22,775			σ	2,738,432	13,197	22,499

Table 3.4. *This table shows the the performance of the hedge strategies for scenario D, defined in the upper left part of the table. $\Sigma\Delta r^f$ and $\Sigma\Delta cs$ respectively show the cumulative shift in the swap and government curve.*

hedge 3, confirming this statement.

The effectiveness of a strategy that uses a CDS contract is never below one that does not, where the reverse does not hold. A strategy that does not have exposure specifically to the credit spread, can be very ineffective when both the government and the swap curve move in opposite directions. A direct consequence of the fact that the optional character of hedge 2 does not really outweigh the use of swaps in hedge 3, is that a large part of the optionality in profit sharing can come forth from the exposure to credit spread. Therefore, no hedge performs extremely well for severe movements in the credit spread, though the ones that use a CDS contract perform much better than the one that does not use as CDS. To improve result further, the use of products that have a non-linear exposure to credit spread could be considered. For example spread options or credit default swaptions might be useful, but this is left for further research.

It seems that the CDS hedge can be financed from the value that the insurance company should receive from selling profit sharing. This is best shown by the second strategy, that uses swaptions and a CDS. If everything remains as it is at initiation of the contract the swaption portfolio will give the cash flows needed for the profit sharing. The cost of the CDS are €90,000 a year from initiation, using a 30y CDS.[10] If all else remains equal the contract can be rolled over at the end of the year, keeping the BPV to the credit spread equal (drops by only €2,100 over the year). The BPV to the credit spread will decrease in value in the same way that the profit sharing value decreases. This can be projected in the same way as was done for the guarantee in figure 2.1. This shows that the decrease depends on the investment strategy (and the resulting swaptions) but also that a linear approximation should do. Meaning that on average the BPV to the credit spread that has to be off set by the CDS, is half the amount at initiation. This would result in an average

[10]Using the right maturity here has shown to be important.

notional of €23 mln, equaling total cost of €1.4 mln over 30 years. The cost of the CDS is thus less than the amount received as premium, what seems plausible as the CDS is not a perfect match. One should then be able to buy the optional character with what is left.

If the situation does change, and profit sharing increases, this is due to the fact that the government yields have increased. If the credit spreads have remained equal, the cash flows from the swaptions should still match the ones needed for profit sharing, as the swap rates have increased. If the swap curve has remained the same, the increase is due to a larger credit spread. This value is captured by the CDS hedge. Rolling over the CDS will now result in larger costs due to the increased CDS spread. These cost can be financed by the increase in the hedge portfolios value.

3.4 A practical implementation

In this section the performance of the hedge strategies is analyzed using historical data on the CDS spread and government yields of Spain over the period 31/3/2005 - 1/4/2012. In the computation of these results the assumption is not made that the CDS spread is equal to the credit spread. 5 year CDS spreads are used to compute the values of 30 year contracts. This is done because quotes on 30 year Spanish CDS contracts are limited as a result of the fact that Spain generally issues debt with low maturities. Normally the term structure is slightly upward sloping, so this might result in costs that are estimated a bit to low. The Libor hedges, both by swaps and by swaptions, are again static. The CDS contract is rolled over based on similar criteria as discussed in the last section, only the monitoring is done quarterly. Finally, because the government curve is modeled from actual historical data, the shifts do not have to be parallel as they were in the last section. All results are displayed in figure 3.6.

The figure immediately reveals that the two portfolios that do not consider exposure to the government curve explicitly (*Swaps(Old) and Swaptions*) perform significantly worse than the two strategies that do. The hedge that uses swaptions instead of swaps together with the CDS contracts, performs very well up till 30/6/2010, while over this period the CDS spread has risen 240 basis points. The spikes that follow are due to large jumps in the CDS spread, not exactly coinciding with the jumps in credit spread (see also figure 3.1). This shows an important property of CDS spreads, as they can be very volatile compared to bond spreads.

The portfolio that uses swaps does not seem to track the profit sharing as well but does result in the best performance at the end of the period. This mostly due to the fact that the CDS spreads moves stronger than the bond spreads do over this period. But as well because of the fact that the swap

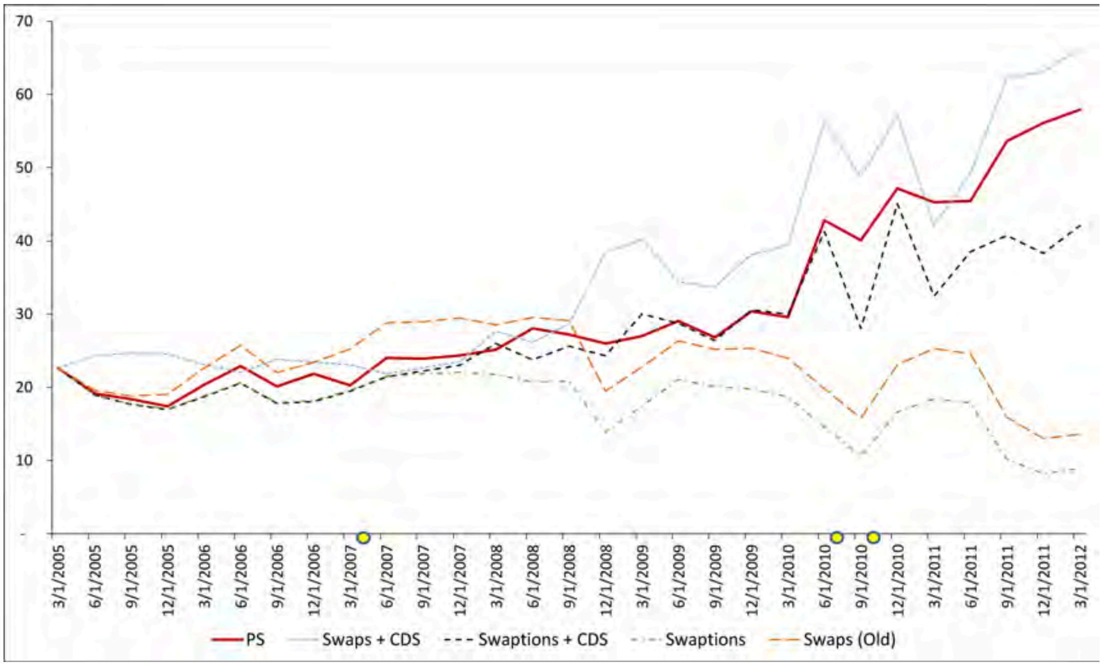


Figure 3.6. *This figure shows the value (in €mln) of a profit sharing contract that was settled on 1/3/2005 and promises €100 mln in 30 years based on a guaranteed rate of 2.5%. The value of 4 hedge strategies are displayed as well. The bullets represent the moments in which the CDS contract is rolled over.*

hedge returns close to €5.5 mln over the period and does therefore capture the increase as a result of lower discount rates reasonably well. The swaption hedge should capture this effect as well but still loses €12 mln. This shows again how different the approaches are. The swaptions are modeled to capture part of the positive sensitivity towards the government rate whereas the swaps are only used to offset the discounting effect. Therefore the swaptions only work when the CDS spread proxies the difference between the government curve and the swap curve accurately, and then still misses the optional part in the profit sharing due to credit risk exposure.

The CDS hedge is rolled over three times, displayed by the bullets in 3.6. The cost of this hedge is around €2.5 mln over this period, of which €2.4 is incurred over the last two years. The profits from this hedge are however close to €35 mln.

The mismatch in BPV to the swap curve is again large between the hedge portfolios and the profit sharing (missing around 45,000), rebalancing this as well would result in better performance.

Concluding, this scenario confirms with previous results and shows that the strategy that uses swaptions is slightly preferred over the one that uses swaps. It is however confirmed again as well that performance drops when movements are more extreme. This is partly due to the fact that the optional character is not captured by CDS contracts. This would imply that better results can be achieved by the use of CDS swaptions, or other non-linear products. But the effect is also because the CDS spread is more volatile and does not always approximate the bond spread well.

3.5 Summary

This chapter started with a short overview on what has been done in the field of risk management at insurance companies. Several hedge strategies were discussed, following in a way the approach of Coleman et al. (2003), by local optimization (match BPV's) and the use of a zero coupon bond to equal the final value.

In the construction of the hedge strategies special attention has gone to the exposure to credit spread, of which the importance was discussed in the previous chapter. For this, the use of a CDS contract was considered elaborately, including several effects that could influence the effectiveness of the hedge but were not analyzed here.

In the evaluation of the strategies, the hedges that use a CDS have shown to be a significant improvement to the ones that do not. The one that uses swaptions and a CDS should match the profit sharing better, but this is only visible when swap rates increase substantially. A large part of the value of the profit sharing can result from the non-linear exposure towards the credit spread. If movements are not to extreme, the CDS contract seems to be able to capture the effect reasonably well.

In chapter 2 it was already suggested that the value of the profit sharing is probably to large to finance from the difference between the premium received by the insurer and the cost of the guarantee. Because the cost of the contract should equal the price in the market in the end and the CDS hedge proposed here does not replicate the value of the profit sharing exactly, results imply that this mismatch pointed out in chapter 2 might even be larger.

Conclusion

This work started off with a comprehensive discussion of a profit sharing product that is typically sold by Dutch insurance companies. The motivation for the existence of this product is twofold; 1) it provides a level of security to the policyholder through a guaranteed return and keeps upside potential when interest rates are high 2) for the insurance company its value lies in the stability of the profit sharing and in that it allows the insurance company to market a product that is competitive. The product differs from many profit sharing products in other countries with respect to the underlying factor that determines the return. This is because profit sharing is based on a fixed income investment portfolio that invests in fictitious bonds. The yield of these bonds, called the u-yield, is determined by the yields on Dutch government bonds via a complicated weighting scheme.

In chapter 1, methods were discussed to construct replicating portfolios that are consistent with the value of the product. These portfolios were build of swaptions in combination with a zero coupon bond. Also a hands on approach was provided to quickly estimate the value of such a product when the investment policy gets more complicated. This came down to finding an optimal trade off between matching the weight of investments and the timing of profit sharing. These methods should prove useful in preparations for Solvency II, which requires market based accounting and will force insurance companies to estimate the value of the embedded options in this product accurately.

Due to the complicated structure of the u-yield and to simplify computations, most calculations involving these products are currently based on the seven year swap rate. This is because these rates have shown to behave very similar historically. In the second chapter it is however reasoned that these rates are fundamentally different and that it can be dangerous to assume a constant spread between the two. The implications of doing this were assessed using an alternative model to value the replication portfolios. This model estimates the value of the profit sharing product while acknowledging sensitivity to both the risk free rate and the countries credit spread. Results show that not considering the credit spread can lead to substantial errors in valuation and consequently also in the design of risk mitigating strategies. Furthermore, the model can provide additional insight in the dynamics of the profit sharing products sensitivities, which is important in the construction and performance appraisal of hedge strategies.

This chapter has also shown that the value of the guarantee is likely to add significant pressure on the capital buffers of many insurance companies today. It was observed that the shape of the government curve is of large influence to the value of the profit sharing product through the forward rates used to value the swaptions. This confirms once more the importance of explicitly considering two curves in the valuation of these products. It was shown that the credit spread has a strong positive effect on the value of the profit sharing.

The last chapter has pointed out important differences between unit-linked contracts and profit sharing contracts. The most relevant difference is that credit risk in profit sharing contracts is to be held by the insurance company, where it is held by the policyholder in unit-linked contracts. The exposure to credit risk should be taken into consideration when designing risk mitigating strategies. The importance of doing this, and the implications of not doing this, were analyzed using various hedge strategies in scenarios with varying discount curves and credit spreads. Both the instantaneous performance and the effectiveness over a larger time span were assessed. The hedge strategies considered are constructed by matching the sensitivity to several parts of the applicable curve in basis point value (BPV). Both non-linear hedges consisting of swaptions and linear hedges consisting of swaps were evaluated. The effectiveness of including CDS contracts in hedging credit risk was assessed as well. Results show that the performance of the strategies that use swaps instead of swaptions are very similar. Only when swap rates increase significantly a portfolio of swaptions is preferred. The use of a CDS contract in mitigating exposure to credit risk seems very effective when a roll over strategy based on BPV mismatch is applied. Only when the credit spread increases dramatically the linear payoff from the CDS contract does not completely capture the optional character that the spread displays in the profit sharing. Ideally, multiple CDS contracts should be used to exactly match exposure to credit risk on different parts of the curve. The most important parts are determined by the fictive investment policy and contract maturity. CDS contracts with long maturities can be used to minimize costs, as these allow for a lower fee. This method seems to result in an effective hedge, despite the mismatch in exposure. Hedge strategies constructed while not considering exposure to credit risk perform well for scenarios in which the government and swap curve move in an equal direction simultaneously. However, they severely under perform when this is not the case.

Results of chapter 1 and 2 already suggested that the profit sharing could not be financed from the difference between the amount that the insurers receive as a premium from the policy holder and the cost of the guarantee in the market. The fact that the best performing hedge strategies in chapter 3 are still not perfect, again raise the question what the fair value of such a contract should be. The results all point in the direction that these contracts are being sold below their market value. This might however still be equal to the fair value of this contract to the consumer, as he or she might not expect to receive payments with a 100% probability.

Although this work has discussed many aspects of the profit sharing product in great detail, it still allows for enough improvements and contributions to be made. The model can be extended by developing a way to incorporate the volatility matrix of the forward curve[11] in the valuation of the replicating portfolio, instead of using the volatility of the discount curve. Furthermore significant contributions could be added by considering more instruments to hedge the credit risk in these products. But also by assessing effectiveness in a more complete way, i.e., considering the side effects that were discussed in paragraph 3.2.2.1. Interesting would also be to consider different measures in constructing a hedge portfolio. Here, this was done by minimizing the mismatch in BPV's, meaning that risk is minimized locally. One could also use linear or dynamic programming to minimize risk over a larger quadrant, i.e., constructing the hedge based on minimization of the mismatch over multiple curve shifts. Here, also non-parallel curve shifts could be considered.

Finally, in this thesis it was assumed that the contractual promise made to the policyholder is very strict. Often however, there is a clause in such a contract that states the insurance company can keep itself from paying the profit sharing under certain circumstances. This is not considered here because it is outside the cope of this thesis and the likelihood of an insurer doing this is estimated to be small. It would significantly damage the credibility of the insurance company and the trust of the consumer in the product. The clause however still represents some kind of counter option that would be interesting to consider in the valuation.

In summary, this work has discussed the profit sharing product elaborately and provided the reader with insights into the factors that influence the value and into the risks that these products introduce. Important here was the exposure to country credit risk, that was assessed using a model that considers sensitivity towards two interest rate curves. Results showed that the use of a portfolio consisting of swaptions and a CDS contract can be expected to provide a robust hedge for the profit sharing element. Furthermore, the discussion of the risks and the hedge strategies, the insights provided by the model and the suggestions given above should provide a fruitful ground for further research. A useful recommendation to insurers could be to modify the contract slightly, by promising profit sharing based on the swap rate instead of the u-rate, or to model exposure to credit risk explicitly. Neglecting this exposure in risk management can lead to severe errors in the valuation and in the design of an effective hedge.

[11] The applicable government curve.

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Appendix A

A.1 Function descriptions

A.1.1 Profit sharing function

This appendix provides information about the tool that is used in this thesis to simulate expected cash flows originating from profit sharing products. Because the theory behind these products is discussed in chapter 1 the focus here will be more on tool specific subjects as input, output and calculations that are not trivial.

The main purpose of the tool is to make projections about profit sharing based on some interest rate scenario and product specifications and to construct a replicating portfolio that mimics the profit sharing payments that are expected. Before computations are discussed first a note on the installation will be given and all the input parameters that are used need to be defined.

Installation instructions.

The tool can be accessed by installing the Excel Add-In in which the code of the function resides. The best way to do this is to put the Add-In in the Microsoft AddIns folder with the default address: "C:/Documents and Settings/<user>/Application Data/Microsoft/AddIns". Then install the AddIn in Excel by going to:

File → Options → Add-Ins → Go.., and selecting "Winstdeling Functie".

A.1.1.1 Input and parameters

The tool can be used as a function in Excel after the Add-In has been installed under the function name "Overrente". This function requires 8 parameters and has 5 additional optional parameters:

Curve. This is the most important parameter, it represents the interest rate term structure on which all calculations are based and needs to be passed as a Curve object as it is computed by the CardanoLib.

Template. This provides information about what output has to be printed. Possible values, that mostly have a trivial meaning but will also be explained in the text below, are: *"All, FV Premium, FV Reserve, FV Investments, FV PL, 7ySwaprate,* U_{Rate}*,* U_w*, FV Profit Share, FV Swaption Cashflows, FV Swaptions, PV Swaptions, DF, Notionals, NRPD"*

Expected cash flows. This parameter needs to be entered as a n by 2 array, where n is the number of cash flows. In the first column the date of the cash flows, in the second the amount that has to be paid to policyholders. Or it can be entered as a n by 1 array with only the cash flows. In this case the $periodlength$ has to be entered as optional parameter as well.

Guaranteed rate. This is the rate that is guaranteed to the policyholders annually no matter what the return on investments has been.

Fee. This parameter represents the fee the insurer can charge by subtracting a percentage from the profit share. If there is no profit sharing in a year no fee is paid, if there is, first the fee is paid and the remainder goes to the policyholder.

Maturity. This parameter represents the maturity of the investments that are made in the investment portfolio.

Type. This specifies what should be done with possible profit sharing. Values can be either 1) "Payout", meaning that the amount is paid out to the policyholder or 2) "Reinvest", meaning that the amount is added to the reserve and invested.

Distribute premiums. This parameter specifies whether all expected cash flows are presented as one lump sum payment at inception or that premiums are equalized for all cash flows and distributed to according periods.

Turnover. This parameter is optional. By default it is set to 1, meaning that the notionals of the investments are paid back in full at maturity. If another payment structure is preferred it should be entered as a m by 1 array, where m is the maturity of the investments that are made in the fictive portfolio. The elements in the array should sum to 1.

Model. This is an optional parameter that gives the user the ability to specify what should be modeled; either "Profit Sharing" or "Guarantee".

DiscountCurve. This parameter is optional. If the user wants to value the product using two curves, the discount curve should be entered here. The forward curve should be entered as first variable.

Period length. This is an optional parameter that should be entered if the expected cash flows are entered as an n by 1 array. Values should be in the tenor "im" or "iy", for respectively months or years. i must be an integer.

FixedVol. This is an optional parameter that gives the user the ability to use a fixed value for the volatility instead of using the entire volatility matrix from the function library that belongs to the start date of the curve.

The function can then be used by of selecting a range, based on the preferred output (template), and following the standard procedure of entering array formulas in Excel with

"=Overrente(Curve, Template, ExpectedCashFlows, r^g, Fee, Maturity, Type, DistributePremiums)"

A.1.1.2 Calculations

The calculations start with the evaluation of the expected cash flows to the policy holders. Depending on the parameter *Distribute premiums*$\{False, True\}$ the cash flows are discounted to:

1. False : One big lump sum payment at origin:

$$P_j = \begin{cases} \sum_{i=0}^{n} df_i CF_i & \text{if } j = 0, \\ 0 & \text{if } j \neq 0, \end{cases}$$

   with df_i the discount factor of period i at inception based on the guaranteed rate r^g that is provided to the function and CF_i the cash flow belonging to this date.

2. True : All cash flows are distributed to fixed premiums in a way that the sum of all premiums, including the interest, exactly matches the expected cash flow at the period the cash flow has to be paid to the policyholders. The sum of all the premiums in a period contributing to the cash flows expected from that period onward then represents the total amount received by the insurer.

To distribute CF_j , the cash flows belonging to date j, equally to all periods up to date j consider that

$$CF_j = P_{qj} \sum_{q=0}^{j} \exp[(j-q) f_{q,j}],$$

with P_{qj} the premium in period q belonging to the cash flow of period j and $f_{q,j}$ the forward rate for period $[q, j]$, computed based on the guaranteed rate r^g that is entered. Because the premiums should be equal each year P_{qj} is a constant.[1]

$$P_{qj} = CF_j \left(\sum_{q=0}^{j} \exp\left[(j-q) f_{q,j}\right] \right) = CF_j \left(\sum_{q=0}^{j} \frac{df_q}{df_j} \right)^{-1}. \tag{A.1}$$

The total payment received by the insurer in a period is than equal to the sum of all premiums belonging to the cash flows at the end of this and future periods:

$$P_j = \sum_{q=j}^{n} P_{jq},$$

with n the number of expected cash flows. When the premiums received by the insurer in each period are determined they are invested in coupon paying bonds with a maturity equal to what has been entered in the function and a coupon equal to the then prevailing u-rate. This u-rates are computed as the 7-year swap rate for the period, extracted from the (forward) *Curve* that is entered as interest rate term structure.

The second optional parameter is the turnover rate. This rate applies to the bonds just mentioned. If the parameter is not entered this turnover rate is set to 1, meaning that the principal is paid back in full at maturity of the bond. If this is not the case it has consequences for the investment scheme and replicating portfolio. This will be discussed at the end of this section; first the standard case of turnover 1 at maturity of the investment is treated.

It is best to distinguish clearly the reserve that is build up by the policyholder, the fictive investment portfolio managed by the insurer according to some investment policy that determines the profit share and the replicating portfolio.

The fictive investment portfolio.

This investment portfolio has as single purpose to determine the profit sharing rate at the end of every period and should not be confused with how the insurance company actually manages its

[1]Notice that continuously compounding rates are used in these computations.

assets. This fictive portfolio is usually defined by a simple investment policy that invests in one type of fixed income asset with a fixed maturity of M years and a certain turnover structure for the principal payments. The policy is described in more details below.

For each period i the premiums and the payments and coupons of the previous investments are received. Depending on the *Type* also the profit share over the last period is added to this sum and the total amount is then invested in bonds (the fixed income asset) yielding the then prevailing u-rate u_i.

At the end of each period a weighted u-rate can be determined by computing the return on the fictive investment portfolio r_i^{fi}. This weighted u-rate then determines the profit share over the last period i :

$$r_i^{fi} = \sum_{q=(i-M+1)^+}^{i} \frac{r_q^u I_q}{I_{\Sigma,i}}, \qquad I_{\Sigma,i} = \sum_{q=(i-M+1)^+}^{i} I_q,$$

$$(i-M+1)^+ = \max(i-M+1,0),$$

with M the maturity, I_q the amount invested in period q, often called a layer, and $I_{\Sigma,i}$ the sum of all investments in previous layers. The profit sharing PS_i is then

$$PS_i = R_i \max\left[r_i^{fi} - (r^g + \delta), 0\right],$$

where R_i is the amount in the reserve at the beginning of period i and δ is the fee entered in the function. The amount that has to be paid to the policyholders at the end of the period is then the profit sharing and the cash flow based on the contracts entered as array in the function. The amount of the cash flows has to be subtracted from the investment portfolio by reducing the notionals invested against the u-rates by a factor $\frac{1-CF_i}{I_{\Sigma,i}}$, resulting in coupons and payments reduced by the same factor.[2]

The reserve.

This portfolio tracks what rights the policy holders have build up and what, at the end of the contracts, has to be paid out. In the case the profit share is paid out every period this is the sum of the premiums and the accrued interest. In the case that the profit share is reinvested two things need to be considered:

- The extra interest over the profit share.
- Future profit share over the amount received as profit share now. This means the policyholder will receive options on profit sharing in future dates, complicating the product.

[2]This suggest assets are liquidated equally over all layers in the fictive investment portfolio to cover the liabilities.

The reserve is linked to the investment portfolio through the initial amount in the investment portfolio that equals the premiums paid at inception by the policy holders. If PS_i is positive and the profit share will be paid out to the policyholder, the investment portfolio will decrease by the same amount. If the profit share is reinvested the reserve R_i increases with PS_i.
If the return on the fictive portfolio r_i^{fi} is lower than the guaranteed rate there will be no profit share. Because the reserve enjoys a minimum return equal to the guaranteed rate the amounts in both portfolios will diverge. The loss that is made by the insurer is tracked by a loss account that has Template value *FV PL*. The interest on the loss account is determined at the end of each period and equals the swap rate with appropriate maturity extracted from the discount object entered into the function.
In the case $0 \leq r_i^{fi} - r^g \leq \delta$ the amount in the investment portfolio increases with a rate exceeding the reserves by $r_i^{fi} - r^g$.

The replicating portfolio.

This is the portfolio that replicates the profit sharing part of the product, not the entire portfolio.[3]

The profit sharing part of the product resembles the position in a strip of payer swaptions by the policyholder on the weighted u-rate rate r_i^{fi} as this cannot be modeled by one swaption because the option of a positive profit share is each period and the notional amount changes each period. As mentioned in the text it is not possible, or to expensive, to buy swaptions on the u-rate, let alone a weighted u-rate, and therefore the swaptions are based on the 7-year swap rate assuming that this is a good proxy for the u-rate (see section 1.3). The use of a weighted u-rate causes it to look more like an Asian option on a 1 year swap, but then not using just an average but a weighing scheme proportional to the investments made up to q periods back by the investment portfolio, where q is equal to the maturity of the investments. A strip of M year swaptions is used to model the weighing, with M the maturity of underlying investments.
The way the swaption portfolio is the build up is by buying a swaption on the M year swap rate with an expiration date that equals the beginning of each period, a maturity equal to the maturity of the investments, a notional proportional to the amount that is invested that period and a strike equal to $r^g + \delta$.[4] This means that at the end of every period there will be possible payoffs of a number of swaps and these payoffs will have a weighing to the total payoff equal to the weights that the u-rates have in the computation of the fictive investment portfolio return r_i^{fi}. Because the sum of the investments in the fictive portfolio does not have to match the reserve and the profit sharing is paid out over the reserve the notionals are not exactly equal to the investments of the fictive portfolio.

[3]If this is required the calculation can be done easily by summing with the values of zero coupon bonds with similar maturity and principal as the cash flows resulting from the contracts.
[4]If the maturity differs from 7 years a adjustment should be made to the strike. This is explained in section 1.6.

The correct way to model this is to use notionals following the guaranteed rate, but that are based on the investment policy of the fictive portfolio.[5] This way the correct size and weighing is combined.[6] One thing that should be mentioned is that, by modeling it in this way, there might be multiple payoffs from the replicating portfolio every period but there is only paid one possible profit sharing, based on r_i^{fi}. In other words, it will never be the case that there is no payoff from the replicating portfolio when there is a profit sharing. This reflects that this replication is quite dominant and not exact.

The above can be shown in a similar recursive way as before:

At the beginning of period i the swaption needed has a notional of

$$N_i = \sum_{q=(i-M+1)^+}^{i} r_q^g I_q^g + I_{i-M}^g + \mathbf{1}_{\{Type\}} PS_i,$$

with $\mathbf{1}_{\{Type\}}$ an indicator function returning 1 if Type is "Reinvest" and the superscript g means that the investments are based on the guaranteed rate r^g instead of the u-yield u_i.

The swaption needed at inception then has the properties:

1.	Notional	N_i
2.	Expiration date	$Date_i$
3.	Strike	r^g
4.	Maturity	Maturity of investments.
5.	Volatility	Implied volatility of 7 year swap rate.
6.	Tenors	1 year.

At the end of each period one swaption expires and a maximum of M cash flows are received from the swaps underlying previously expired swaptions that were in the money. The swap payoffs, showed by *FV Swaption Cashflows*, are

$$CF_{swaps,i} = \sum_{q=(i-M+1)^+}^{i} N_q \max\left[u_q - (r^g + \delta), 0\right].$$

[5] In case the profit share is reinvested this amount should be added.
[6] A more extensive explanation is given in section 1.5.

These are the cash flows that should replicate the profit share at year *i*:

$$R_i \max\left[r_i^{fi} - (r^g + \delta), 0\right] \approx \sum_{q=(i-M+1)^+}^{i} N_q \max\left[u_q - (r^g + \delta), 0\right].$$

The replicating portfolio is linked to the reserve by the use of the guaranteed rate r^g in determining the notionals and simultaneously to the investment portfolio through the use of the same turnover structure and maturity of the fictive investments.

From the above it should be clear that the required swaptions are first determined based on the guaranteed rate r^g and the policy of the investment portfolio. Based on the *Curve* object the present value of the swaptions at each period can be calculated by use of the expected forward curve in a period. This is dislayed by *PV Swaptions* of which the value at the beginning of period *i* is:

$$PV_i(Swaptions) = \sum_{q=i}^{n} Swaption_q\big(FCurve_i, Notional_q, \Sigma_u, r^g + \delta, q, M\big)],$$

with $FCurve_q$ the forward curve in period *i* and *M* the maturity of the swap that equals the maturity of the investments.

If a specific turnover structure is used in the investment portfolio this has two important consequences for the investment policy and replicating portfolio.

The consequences for the investments policy are that at each period the payments of all previous investments up to the maturity of the investments have to be taken into account. The principals of the underlying investments that are made against the different u-rates then decline which in turn make that the coupons will vary over time.
This turnover structure has consequences for the replicating portfolio as well. The notionals change in the first place because of the amount available for investments in a period includes more payments. The declining value of the principles also decline and therefore the weight the different u-rates have in determining r_i^{fi} will change, making it less sensitive to investments further back. To be able to exactly match this, one would need a swaption with a declining notional. Another approach would be, though swap rates make it slightly different, to use a number of swaptions per investment layer. For example, if the principal of a bond with a maturity of 5 years is paid back in 5 periods by equal amounts ($\frac{1}{5}$), this can be modeled by using 5 swaptions: one with a notional of $\frac{1}{5}$ of the investment and a maturity of 5 years, one with a notional of $\frac{1}{5}$ of the investment and a maturity of 4 years, etc..[7] Because for the tool a general approach is needed, the problem is tackled

[7] See section 1.6 for more information on replicating strategies for different turnover structures.

by changing the maturity of the underlying swap to be the weighted maturity of the investments based on the turnover structure.

$$M_w = \sum_{i=0}^{M-1} (M-i)\tau_{\Sigma,i}, \qquad \tau_{\Sigma,i} = \sum_{q=1}^{i+1} \tau_q,$$

where τ_q is the turnover in the q^{th} year of the investments.

Section 1.6 shows that this is however not always the best way to replicate the profit sharing and that this should be tested on a case by case basis, the code in the tool can be adjusted relatively easily to speed up this process.

A.1.2 Hedge evaluation function

This section describes the function that quickly analyzes the value and the BPV of a hedge strategy. It is part of the AddIn that also contains the "Profit sharing function".

A.1.2.1 Input and parameters

The tool can be used as a function in Excel under the name "EvaluateHedge". It can be used to evaluate hedge portfolios consisting of swaps, swaptions or CDS contracts.[8] The function requires 3 parameters and has one additional optional parameter:

Date. This is the date on which the portfolio is bought.

Instruments. This parameter is entered as a range. The size of the range depends on the type of instruments:

Swap	swaption, *endtenor, rate, notional*
Swaption	swap, *starttenor, endtenor, strike, p/r, notional*
CDS	CDS, *starttenor, endtenor, CDS spread, notional*

Curve. The curve is used to value the instruments. It is entered as a curve object computed by the CardanoLib.

Evaluation Arguments. This is an optional argument. Here the CDS spread and default probabilities can be entered.

[8]It can be extended to include other instruments in a simple way.

A.1.2.2 Calculations

The swaps and swaptions are valued by use of the Cardano Function Library. The value of the CDS in computed as was described in section 3.2.2.1. The BPV is calculated by the difference in value when the curve shifts up by one basis point. This shift is parallel and does therefore not compute sensitivities to all different parts of the curve.

The output is given in a 1 by 2 array. In the first cell the value of the hedge portfolio, in the second cell its BPV for a parallel shift.

A.2 Financial products

This section contains a brief overview of the financial products that are used in this thesis. Only a very short description of the purpose of the product and its valuation will be given. For a more detailed treatment see Hull (2008), sections 13.8, 7.7 and 28.3 respectively.

A.2.1 Option

In finance an option gives the holder the right, but not the obligation, to buy or sell something. This something is called the underlying and can be anything from a stock to a container of pork bellies. A call option is defined as the option to buy the underlying on a future date for a predetermined price (the strike price), the right to sell a put option. The term European specifies that the option can only be exercised at this future date (the expiration date). If it would be the case that the option can be exercised at any time up to the expiration date it would be an American option. Under some assumptions Black and Scholes (1973) found an analytic expression for the value of an European call option, from which the value of the put option can be deduced.

$$C = S\mathcal{N}(d_1) - K\exp^{-rT}\mathcal{N}(d_2), \tag{A.2}$$

with,

$$d_1 = \frac{\log S/K + (r+\sigma^2)T}{\sigma\sqrt{T}}, \qquad d_2 = d_1 - \sigma\sqrt{T}.$$

In this formula S represents the value of the underlying, K the strike price, r the risk free interest rate, σ the standard deviation of the underlying, T time to the expiration date, $\mathscr{N}$ the cumulative standard normal distribution function.

A.2.2 Swap

A swap represents an exchange of cash flows between two entities. The most well known, and also the one used in this thesis, is the interest rate swap. In this swap one entity agrees to pay a floating interest rate (Libor[9] + spread) to receive a fixed rate. At inception the value of a swap is usually zero because the swap rate sets the value of the fixed leg exactly equal to the value of the floating leg. At a later point in time the swap will have a value different from zero as Libor will vary over time. The easiest way to value a swap is to see it as the difference between two bonds:

$$S = B_{fix} - B_{fl},$$

where B_{fix} is a fixed bond with coupons set by the swap rate, B_{fl} a floating rate bond paying Libor + spread and both bonds having the same notional and maturity.

A.2.3 Swaption

A swaption is a combination of the two financial products described above. The underlying in the option, S in equation A.2, is simply the value of the swap. The expiration date of the swaption, in the way it is used in this thesis, is the start date of the swap.

A.3 Terminology

At of the money If the value of the underlying is currently equal to the strike price of an option the option is said to be at the money.

Discount rate The value of €1 today (the present value) is greater than €1 next year. The discount rate is used to translate future values to the present values and it is closely related to the

[9] London inter bank offered rate

discount factor.

Duration Duration is a mathematical expression introduced by Macaulay (1910) that captures the weighted average term to maturity of a fixed income product by considering all cash flows associated with the product.

Implied volatility The implied volatility of an option contract is the volatility that, based on a pricing model, yields a value for the option equal to the current market price of that option.

In the money If the value of the underlying is currently above (below) the strike price of an option the option is said to be in the money for a call (put) option.

Modified duration The modified duration is a linear approximation for the percentage change in value of a fixed income product when the yield changes by one unit and is closely related to the duration and BPV.

Notional A financial contract is always based on some underlying asset. For fixed income product, e.g. bonds and swaps, this is just some amount of cash over which the interest is paid or yield is computed. This amount is called the notional amount, or, mainly for bonds, the principal amount.

Option In finance an option is a derivative contract, it derives its value from some underlying asset. The variety in options is enormous but in general there are two types of options: 1) a **call** option and 2) a **put** option. They respectively give the right, but not the obligation, to buy and sell the underlying for a predetermined price (strike price) during (American option) or at (European option) expiration of the option.

Out of the money If the value of the underlying is currently above (below) the strike price of an option the option is said to be out of the money for a put (call) option.

Solvency II Solvency II reviews the capital adequacy and risk management standards for the European insurance industry and will replace Solvency I. The main difference with Solvency I will be the extensive use of market based valuation practices.

Strike This is the predetermined price used in the valuation of an option. (See option).

Swap A swap represents an exchange of cash flows between two entities. The most well known, and also the one used in this thesis, is the interest rate swap. In this swap one entity agrees to pay a floating interest rate (Libor + spread) to receive a fixed rate. At inception the value of a swap is usually zero because the swap rate sets the value of the fixed leg exactly equal to the value of the floating leg. At a later point in time the swap will have a value different from zero as Libor will vary over time.

Swap rate The swap rate is the fixed rate that sets a swap contracts market value at initiation to zero. (See also swap)

Swaption This product gives the holder the the right, but not the obligation, to buy a underlying swap for a predetermined price. The underlying swaps are interest rate swaps. There are two kinds of swaptions; 1) a **payer** swaption and 2) **receiver** swaption. Respectively for an underlying swap that pays a fixed rate and one that receives a fixed rate over the notional.

Unit- and equity linked products These are two type of insurance plans of which the value changes based on an existing underlying reference portfolio of assets. **Unit-linked** contracts generally allow investments in a combination of both equity and and debt instruments. **Equity-linked** products only allow investments in equities. It is often easier to withdraw funds from equity-linked saving plans and the cost charges are lower. Most equity-linked contracts do not have an insurance element in them, whereas unit-linked contracts usually do.