# Valuation of Residential Mortgage-Backed Securities 

- A Monte Carlo Simulation approach -

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## Table of Contents

1 Introduction ..... 1
2 Prepayment ..... 4
2.1 Twelve-Year Prepaid Life ..... 5
2.2 Conditional Prepayment Rate (CPR) ..... 5
2.3 FHA Experience ..... 5
2.4 PSA Model ..... 6
2.5 <<confidential>> ..... 6
3 Cash Flow Structure of an MBS ..... 7
3.1 Cash Flows without Prepayment ..... 7
3.2 Cash Flows with Prepayment ..... 8
4 Implementation ..... 10
4.1 Monte Carlo Simulation ..... 10
4.1.1 Step 1: Simulate short-term interest rate and refinancing rate paths ..... 10
4.1.2 Step 2: Project the cash flow on each interest rate path ..... 12
4.1.3 Step 3: Determine the present value of the cash flows on each interest rate path. ..... 13
4.1.4 Step 4: Compute the theoretical value of the MBS ..... 13
4.2 One-Factor Hull White Model ..... 14
4.2.1 Zero Curve Generation ..... 15
4.2.2 <<confidential>> ..... 16
4.3 Other implementations ..... 16
5 Simulation Results ..... 18
5.1 Results Comparing with the Market ..... 18
5.2 Model Sensitivity ..... 19
5.2.1 Number of Simulations ..... 19
5.2.2 Prepayment Sensitivities ..... 19
6 Conclusion ..... 23
Reference List ..... 24
Appendix A: Derivation of Monthly Mortgage Payment ..... 25
Appendix B: Bloomberg Prints ..... 26
Appendix C: User Manual ..... 32
Appendix D: Example Chart ..... 33
Appendix E: Cash Flow Structure ..... 34
Appendix F: Correlated Rates ..... 35
Appendix G: Hull-White Calibration Model ..... 36
Appendix H: Source Code ..... 38

## 1 Introduction

Financial investors who have bought structured products such as Mortgage-Backed Securities (hereinafter MBS) and Collateralized Debt Obligations (hereinafter CDO) had issues with valuation of these products. This was because of the underlying structure which gives rise to complexity. Since those products take a large place in financial markets, one of the reasons for the emergence of the crisis is that those products have not always been low risk as assumed. The Residential Mortgage-Backed Securities (hereinafter RMBS) in which residential mortgage loans are being passed through to investors in the form of packages, have been assigned the best possible credit ratings by rating agencies. Those ratings have given the impression to the investors that the structure of the underlying mortgage loans brings along almost no credit risk. As a result, they have invested without hesitation of credit risk and the accompanying consequences. When early 2007 the first problems began to stand out and each time piled up, the RMBS seemed not to be low risk as assumed. The matter was, in particular, the speed of how the problems piled up and spread over the whole market within a short course of time. Due to the complexity of the products and lack of adequate information, no one could exactly tell when the problems were started and where they came from. It is expected that the next few years these valuation problems will continue to occur because the context is still complicated from a technical perspective.

As mentioned above, mortgages are used to produce MBS ${ }^{1}$. They are gathered in a pool and this pool of mortgages are sliced in small pieces and finally sold to the investors as packages. We would like to give some brief introduction about mortgages before we go into the MBS. Fabozzi gives the definition of a mortgage in his book the Handbook of Fixed Income Securities as follows: "A mortgage loan is a loan secured by the collateral of some specified real estate property, which obliges the borrower to make predetermined series of payments." A mortgage is a contract between the lender (mortgagee) and the borrower (mortgagor) in which the mortgagee has the right of foreclosure on the loan in case the mortgagor defaults.

Types of real estate properties that can be mortgaged are represented as follows:

[^0]

Figure 1-1: Types of real estate properties

Residential properties include houses, cooperatives and apartments while the non-residential properties include commercial and farm properties etc. We will restrict us to the residential properties since we are interested in valuation of RMBS.

There are several types of mortgage loans that are applicable to residential properties. The most common type is level payment, in other words fixed-rate mortgages. Other types are graduated-payment mortgages, growing-equity mortgages, adjustable-rate mortgages, fixed-rate tiered-payment mortgages, balloon mortgages, two-step mortgages and last but not least fixed/adjustable-rate mortgage hybrids. We will restrict us to the most common type of fixed-rate mortgages whose principal are amortizing to zero until the maturity.

In fixed-rate mortgages the borrower has the obligation to pay a predetermined equal amount on a monthly basis. A monthly amount consists of interest payment and repayment of a portion of the outstanding mortgage balance. Below you find the formula that calculates the monthly mortgage payment ${ }^{2}$ [See Fabozzi-1995]:
$M P=M B_{0} \times \frac{\left[i(1+i)^{n}\right]}{\left[(1+i)^{n}-1\right]}$
where
MP is the monthly mortgage payment (\$)
$\mathrm{MB}_{0}$ is the original mortgage balance (\$)
$i$ is the simple monthly interest rate (annual interest rate/12)
$n$ is the maturity in months

[^1]An important issue for the investors of the RMBS is the prepayment risk. Each homeowner has the option to prepay the whole or a part of the outstanding mortgage balance regardless of time to maturity while there is no penalty imposed. This results in uncertainty for the investor because he/she never knows whether the mortgage will be prepaid 1 year or 30 years after the agreement. There are different motives why a mortgagor prepays. Mortgagors prepay the entire mortgage if they sell their home, for example, for personal reasons. It is also common practice that the mortgagors refinance a part or the whole if the current mortgage rates fall by a sufficient amount below the contract rate. Refinancing costs should be taken here into consideration. Another possibility is that the mortgager defaults and as a result the collateral is repossessed and sold.

If we assume a pool consisting of thousands of mortgages of which each one bears prepayment risk and as a result each one has different cash flow structures, we can understand how complicated the valuation of such an instrument is. Prepayment models arise here in order to reflect the reality as much as possible. We will discuss this topic entirely later in the upcoming chapters.

The purpose of this thesis is to provide a quantitative approach of how the MBS may be valued. After giving an introduction to the topic, we will go further with the challenging issues such as generic prepayment models, changes in cash flow structures as a result of prepayments and Monte Carlo implementation. We will finalize with simulation results and of course the conclusion.

## 2 Prepayment

As mentioned earlier, there are different motives why a mortgagor prepays. In other words, a variety of economic, demographic and geographic factors influence a mortgagor's prepayment decision. The most common factors are interest rates (refinancing incentive), burnout, seasoning, seasonality, heterogeneity and overall economy3.

Prepayment is directly related with interest rates. As interest rates fall below the mortgage coupon rate, mortgagors will have the incentive to prepay their loan. Historical research shows that not all mortgagors prepay even if it is favourable to do so. Burnout phenomenon arises here.

Heterogeneity of individual mortgagors is closely related to the burnout phenomenon. Burnout of a mortgage pool means that the mortgagors will remain in the pool if the interest rates decline below the mortgage coupon rate while others will prepay and leave the pool through refinancing. Those who leave the pool are regarded as being in an interest rate sensitive layer while sitters are regarded as insensitive. In other words, the higher the fraction of the pool that has already prepaid, the less likely are those remaining in the pool to prepay at any interest rate level. This burnout effect causes heterogeneity in the pool.

Seasoning refers to the aging of mortgage loans and can be described as the increasing prepayment incentive of borrowers who are willing to prepay their mortgages if they have not prepaid yet. The Public Securities Association model (also known as "the PSA aging ramp") assumes that prepayment rates increase linearly in the first thirty months of the contract before levelling off.

Seasonality measures the correlation between prepayment rates and the corresponding month of the year. As one might expect, the seasonal increases do occur in the spring and gradually reach a peak in the late summer and decline in the fall and the winter, with the school year calendar and the weather as the driving forces behind the seasonal cycle.

A variety of models has been developed to forecast the prepayment behaviour and to represent the prepayment rates and as a result, the timing and the amounts of the cash flows. Prepayment rates tend to fluctuate with interest rates, coupon and age of the underlying mortgage as well as with non-economic factors such as burnout and seasoning. The next section describes different types of prepayment models.

[^2]
### 2.1 Twelve-Year Prepaid Life

This approach has ever been broadly used in the mortgage industry. Twelve-year life assumes that there will not be any prepayment during the first twelve year of the mortgage life and then suddenly the mortgage will fully prepay at the twelfth year. This approach seems not to give reliable results since prepayment itself tends to fluctuate with interest rates and other factors.

### 2.2 Conditional Prepayment Rate (CPR)

CPR for a given period is the percentage of mortgages outstanding at the beginning of the period that terminates during that period. The CPR is usually expressed on an annualized basis, whereas the term single monthly mortality (SMM) refers to monthly prepayment rates [The Handbook of Fixed Income Securities, 1995 - Fabozzi]. More specifically, SMM indicates, for any given month, the fraction of mortgages principal that had not prepaid by the beginning of the month but prepaid during the month. The relationship between the CPR and SMM can be formulated as:

$$
C P R=1-(1-S M M)^{12}
$$

An approximation to the above formula is:
$C P R=12 \times S M M$
In the early years of the mortgage pool, prepayment occurs rarely in comparison to the rest of the period. Taking a constant CPR will give rise to misleading results since the prepayment in those years will be overestimated. In order to handle this shortcoming, Public Securities Association (PSA) model has been developed. This model combines Federal Housing Administration (FHA) experience with the CPR method.

### 2.3 FHA Experience

FHA is one of the federal agencies who provides insurance to mortgages of the qualified borrowers. FHA builds yearly a survivor table. This table consists of a row of thirty numbers that each represents the annual survivorship rates of FHA-insured mortgages. The prepayment rates are derived from this table. Although FHA experience is not often being used nowadays, this has been a widely used prepayment model. This model is based on historical prepaid mortgages and therefore is unable to project the future prepayments.

### 2.4 PSA ModeI

The PSA model has been developed by combining FHA experience with the CPR method. This method implies that the prepayment is zero at the initial month and increases each month with increments of $0.2 \%$ up to 30 months and then levels off till the end. Thus there is a linear increase from $0 \%$ to $6 \%$ until the thirtieth month and then it remains constant. This is called $100 \%$ PSA. Multiples of PSA are available employing different coefficients to the slope. For $150 \%$ PSA the CPR will be $0.3 \%$ in month one and increase with the increments of $0.3 \%$. See the figure below for an illustration:


Figure 2-1

## 2.5 <<confidential>>

## 3 Cash Flow Structure of an MBS

Since we have worked out the prepayment phenomenon, one might expect to discuss the cash flow structure with and without the prepayment effect.

### 3.1 Cash Flows without Prepayment

We assume that the mortgage pool consists of fixed rate loans without prepayment of any mortgagor. Then the equal monthly installments are:
$M P=M B_{0} \times \frac{\left[i(1+i)^{n}\right]}{\left[(1+i)^{n}-1\right]}$
where
MP is the monthly mortgage payment (\$)
$\mathrm{MB}_{0}$ is the original mortgage balance (\$)
i is the simple monthly interest rate (annual interest rate/12)
n is the maturity in months.

The remaining principal balance at month t will then be:
$M B_{t}=M B_{0} \times \frac{(1+i)^{N}-(1+i)^{t}}{(1+i)^{N}-1}$
where
$N$ is the maturity of the mortgage in months.

The initially scheduled interest payment at month t is:
$I_{t}=M B_{t-1} \times i$

The total cash flow at month $t$ is the change of principal balance from $t-1$ to $t$ and the interest rate excluding the service fee:

$$
C F_{t}=\left(M B_{t-1}-M B_{t}\right)+\left(\frac{C}{C+S}\right) \times I_{t}
$$

where
C is the coupon rate of the MBS and
$S$ is the servicing fee ${ }^{4}$.

### 3.2 Cash Flows with Prepayment

Cash flows with a prepayment option differ from those without prepayment which could be considered as a risk-free bond. We assume that there are in total $K$ mortgagers in the pool and each one owns the same amount of mortgage loan. This implies that we could switch the concept of prepayment ratio measured in terms of money into the ratio measured in terms of number of remaining mortgagors. We also assume that there is no partial prepayment. In case of partial prepayment, each mortgagor will tend to prepay in different amounts caused by different factors mentioned earlier. This will add another dimension of complexity to the actual problem. We should then not only be interested in number of prepaid mortgagors but also the amount of each prepayment. However, if prepayment occurs, the mortgage will be fully prepaid.
Denote the random variable $L_{t}=$ the number of mortgagors who prepay up to t .
Then the actual principal balance for month $t$ is expressed in terms of $L_{t}$ and $K$ is

$$
\overline{M B}_{t}=\frac{M B_{t}}{K} \times\left(K-L_{t}\right)=M B_{t} \times\left(1-\frac{L_{t}}{K}\right),
$$

and the actual interest at month t is
$\bar{I}_{t}=\overline{M B}_{t-1} \times i=M B_{t-1} \times\left(1-\frac{L_{t-1}}{K}\right) \times i=I_{t} \times\left(1-\frac{L_{t-1}}{K}\right)$
The total cash flow at month $t$ is the change of the actual principal balance from $t-1$ to $t$ and the interest rate excluding the service fee:

$$
\overline{C F}_{t}=\left(\overline{M B}_{t-1}-\overline{M B}_{t}\right)+\left(\frac{C}{C+S}\right) \times \bar{I}_{t}
$$

If we work the equation out:

$$
\overline{C F}_{t}=\left[M B_{t-1} \times\left(1-\frac{L_{t-1}}{K}\right)-M B_{t} \times\left(1-\frac{L_{t}}{K}\right)\right]+\left(\frac{C}{C+S}\right) \times I_{t} \times\left(1-\frac{L_{t-1}}{K}\right)
$$

[^3]$$
=a_{t} \times\left(1-\frac{L_{t}}{K}\right)+b_{t} \times\left(1-\frac{L_{t-1}}{K}\right)
$$
where
$$
a_{t}=-M B_{t}
$$
$$
b_{t}=M B_{t-1}+\left(\frac{C}{C+S}\right) \times I_{t}
$$

## 4 Implementation

We have so far tried to explain what a MBS is from the theoretical perspective. Since we are interested in the valuation of MBS using Monte Carlo simulation, we will explain further the needs to develop a simulation model. The simulation model is developed in <<confidential>>. See Appendix H for the source code.

Monte Carlo methods are used in finance to value complex instruments and portfolios by simulating the components that include uncertainty. We have chosen a 1 -factor Hull-White model as short-rate model since it is widely used by practitioners and generates reliable results. In Hull-White, the theoretical rate $r_{t}$ has an uncertain movement according to the term structure of interest rates at each point $t$ in time. Next to the short rate $r_{t}$ in the Hull-White model, the refinancing rate P in the <<confidential>> is uncertain and will be simulated. We will firstly focus on the simulation steps and secondly Hull-White formulae implemented in our simulation model and then explain our implementation further.

### 4.1 Monte Carlo Simulation

Monte Carlo simulation involves a series of interrelated steps. Using the approach stated in the book "Mortgage-Backed Securities: Products, Structuring and Analytical Techniques" of Fabozzi, we define these steps in the methodology as follows:

Step 1: Simulate short-term interest rate and refinancing rate paths.
Step 2: Project the cash flow on each interest rate path.
Step 3: Determine the present value of the cash flows on each interest rate path.
Step 4: Compute the theoretical value of the MBS.
We will further work the implementation out according to the simulation steps mentioned above.

### 4.1.1 Step 1: Simulate short-term interest rate and refinancing rate paths

An interest rate path is simulated for interest rates over the life of the mortgage security. If, for example, the MBS has a remaining life of 25 years, then interest rates will be simulated for 300 months ${ }^{5}$. The number of months on an interest rate path is called weighted average maturity

[^4](WAM) ${ }^{6}$, which defines the average remaining time of the underlying mortgage pool. The simulation requires the generation of multiple interest rate paths.


Figure 4-1: Generation of 20 interest rate paths over 360 months (simulated zero curves)
1-factor Hull-White model generates a series of random interest rate paths. The generated rates on each simulated path are used to calculate the present value of the future cash flows on that specific path. The term structure of interest rates is the theoretical spot rate (zero-coupon) curve for the market on the valuation date. The simulations should be calibrated so that the average simulated price of a zero-coupon bond equals today's actual price.

The random paths of interest rates will be generated from the arbitrage-free Hull-White model, using values obtained from the derivatives markets. By arbitrage-free it is meant that the model replicates today's term structure of interest rates, an input of the model, and that for all future dates there is no possible arbitrage within the model. We use, for example, USD-LIBOR ${ }^{7}$ rates (together with the par swap rates for the long-term) for the euro currency. Those rates should first be bootstrapped in order to get the zero curve, which serves as input to our HullWhite model. We have used a third-party model (<<confidential>>) to bootstrap the interest rates. In absence of the <<confidential>>, the user must input ${ }^{8}$ an alternative (bootstrapped)

[^5]discount factor (df) curve to the <<confidential>>. In this case, our model will convert the df curve to the zero curve.

While the short-term interest rate paths are eventually used to discount the MBS cash flows, they are also used to generate the prepayment path, and thus the cash flows, for each interest rate path. What determines the prepayment path is the refinancing rate available at each point in time, relative to the coupon rate of the MBS. The refinancing rate represents the opportunity cost the mortgagor is facing each month. If the refinancing rates are low relative to the mortgagor's original coupon rate, the mortgagor will have incentive to refinance and vice versa. Our model makes use of the simulated refinancing rates only in the <<confidential>>. The initial refinancing rate will be the average known refinancing rate in the mortgage market as of the valuation date and be simulated using the Hull-White model as well.

The short-term interest rates and the refinancing rates are highly correlated to each other. To handle with this, we have given the user the option to choose for a correlation factor as input when <<confidential>> will be chosen as prepayment model. In the upcoming chapters the sensitivity of the correlation factor will be analyzed.

### 4.1.2 Step 2: Project the cash flow on each interest rate path

The cash flow for any given month on any given interest rate path is equal to the scheduled principal for the mortgage pool, the net interest and prepayments. Calculation of the scheduled principal is straightforward, given the projected mortgage balance in the prior month. The prepayment models used determine the unscheduled principal (i.e., prepayments) to be assumed for that month. Thus the prepayment model employed heavily affects the valuation of the MBS. To determine the monthly unscheduled prepayments, we have given the user the option to choose several prepayment models mentioned in Chapter 2: Prepayments. Those are CPR, PSA model and <<confidential>>. Note that in case of CPR and PSA model, the amount of prepayment will remain unchanged over all simulated paths. These figures are coming from the derivatives market and represent the last known prepayment performance of the underlying mortgage pool. Thus different CPR and PSA percentages are assigned to each specific MBS contract for each valuation date. However, we are mostly interested in <<confidential>> since this model generates a different prepayment rate (CPR) on each month at each simulation step. A comparison of those 3 prepayment approaches is shown in the following figure:
<<confidential>>
Figure 4-2: A comparison of three prepayment models that are used in our simulation

### 4.1.3 Step 3: Determine the present value of the cash flows on each interest rate path

Given the cash flows on an interest rate path, the path's present value can be calculated. The discount rate for determining the present value is the simulated zero rate for each month on the interest rate path. The zero rate on a path can be determined from the simulated future monthly rates. The relationship that holds between the simulated zero rate for month $T$ on path $n$ and the simulated future 1-month rates is:
$z_{T}(n)=\left\{\left[1+f_{1}(n)\right] *\left[1+f_{2}(n)\right] * \ldots *\left[1+f_{T}(n)\right]\right\}^{1 / T}-1$
where
$z_{T}(n)=$ the simulated zero rate for month $T$ on path $n$
$f_{j}(n)=$ the simulated future 1-month rate for month $j$ on path $n$
Consequently, the interest rate path for the simulated future 1-month rates can be converted to the interest rate path for the simulated monthly zero rates. Therefore, the present value of the cash flows for month $T$ on interest rate path $n$ discounted at the simulated zero rate for month $T$ is:
$P V\left[C_{T}(n)\right]=\frac{C_{T}(n)}{\left[1+z_{T}(n)\right]^{T}}$
where
$P V\left[C_{T}(n)\right]=$ the present value of the cash flow for month $T$ on path $n$
$C_{T}(n)=$ the cash flow for month $T$ on path $n$
$z_{T}(n)=$ the zero rate for month $T$ on path $n$
The present value for path $n$ is the sum of the present value of the cash flows for each month on path $n$. Thus,
$P V[\operatorname{Path}(n)]=P V\left[C_{1}(n)\right]+P V\left[C_{2}(n)\right]+\cdots+P V\left[C_{360}(n)\right]$
where $P V[\operatorname{Path}(n)]$ is the present value of interest rate path $n$.

### 4.1.4 Step 4: Compute the theoretical value of the MBS

The theoretical value of the MBS can be determined by taking the average present value of all the simulated paths. Thus the theoretical value is:

Theoretical value $=\frac{P V[P a t h(1)]+P V[P a t h(2)]+\cdots+P V[P a t h(N)]}{N}$
where $N$ is the number of simulations.

### 4.2 One-Factor Hull White Model

The Hull-White model is a single-factor, no-arbitrage yield curve model in which the short-term rate of interest is the random factor or state variable. By no-arbitrage, it is meant that the model parameters are consistent with the bond prices implied in the zero coupon yield curve.

The model assumes that the short-term rate is normally distributed and subject to mean reversion. The mean reversion parameter ensures consistency with the empirical observation that long rates are less volatile than short rates. In the special case where the mean reversion parameter is set equal to zero, the Hull-White model reduces to the earlier Ho and Lee model. The use of the normal distribution affords a good deal of analytic tractability, thereby resulting in very fast computation times relative to competing no-arbitrage yield curve models. A disadvantage is that the model allows negative short rates.

The stochastic differential equation describing the form of the Hull-White interest rate model implemented in our model is:
$d r_{t}=\left[\theta(t)-a r_{t}\right] d t+\sigma d z$
where
$d r_{t}$ is the change in the short-term interest rate over a small interval;
$r_{t}$ is the short-term interest rate ${ }^{9}$ a time t ;
$\theta(t)$ is a function of time determining the average direction in which $r$ moves, chosen such that movements in $r$ are consistent with today's zero coupon yield curve;
$a$ is the mean reversion rate, governing the relationship between short and long rate volatilities; $d t$ is a small change in time;
$\sigma$ is the annual standard deviation of the short rate;
$d z$ is the derivative of the Wiener process $z_{t}$ (a drawing from a standard normal stochastic process).

In practice, the Hull-White model is calibrated by choosing the mean reversion rate and short rate standard deviation in such a way so that they are consistent with option prices observed in the marketplace. Empirical values for the parameter a (mean reversion rate) are in the order of $0 \%$ to $10 \%$, while the parameter sigma tends to be between $1 \%$ and $3 \%$. It should be noted that the sigma of the Hull-White is totally different than the sigma used in BlackScholes model. While one refers to the short rate standard deviation, the other refers to the stock price standard deviation respectively. We have used a third-party model (<<confidential>>)

[^6]to calibrate the mean reversion rate and sigma. This model firstly uses as input the volatility cube and the term structure of interest rates as of the valuation date and then generates hypothetical swaptions related to the input. The second and the final step in this model is the calibration of the Hull-White parameters using the Levenberg Marquardt error minimization method. The generated hypothetical swaptions from the first step are used in this final step as input. The output of this <<confidential>> is the calibrated a and sigma values which serve as input ${ }^{10}$ to the Hull-White model. See Appendix G for the volatility cube and the <<confidential>> screenshots.

The Hull-White model has a great deal of analytic tractability. The parameter $\theta(t)$ is chosen to be
$\theta(t)=F_{t}(0, t)+a F(0, t)+\frac{\sigma^{2}}{2 a}\left(1-e^{-2 a t}\right)$
The price at time $t$ of a discount bond maturing at time $T$ is given by
$P(t, T)=A(t, T) * e^{-B(t, T) r_{t}}$
where
$B(t, T)=\frac{1}{a}\left[1-e^{-a(T-t)}\right]$
and
$\log A(t, T)=\log \frac{P(0, T)}{P(0, t)}+B(t, T) * F(0, t)-\frac{\sigma^{2}}{4 a}\left(1-e^{-2 a t}\right) * B(t, T)^{2}$

### 4.2.1 Zero Curve Generation

In our simulation model, after generating the short rates using Equations 4-5 and 4-6, the bond price formulae in Equations 4-7 to 4-9 are used to determine the simulated forward rates applicable between $t$ and $T$. A forward rate $f$ applicable between $t$ and $T$ is given by:
$f(t, T)=-\frac{\partial}{\partial T} \log P(t, T)$
<<confidential>>

[^7]

Figure 4-3: Theoretical short rates follow the term structure of interest rates


Figure 4-4: Discount factor curves resulting from short rates

### 4.2.2 <<confidential>>

### 4.3 Other implementations

Next to the topics above, a minor adjustment has been implemented such as determining the payment dates (accrual method).

There are two options available in the model for payment dates. The user may choose for "No Adjustment" where the payments will be passed through to the investors based on given actual delay. For example, if actual delay is 10 and the accrual method is chosen for "No Adjustment", the investor will be paid on each $11^{\text {th }}$ of the month until the maturity. If chosen for "Modified Following Business Day" with the same actual delay, the model checks whether the $11^{\text {th }}$ ever comes to a holiday or weekend. If so, the following business day will be applied. This adjustment has a slight effect on the valuation since the cash flows are discounted at the payment dates and the discount factors of each payment day are different.

## 5 Simulation Results

In this chapter simulations with different input parameters will be performed and results will be analyzed. We will first introduce a couple of generic ${ }^{11}$ MBS whose terms are obtained from the Bloomberg ${ }^{12}$. We will value these contracts in our developed model (See Appendix C for user manual) and compare the valuations to the market values ${ }^{13}$. Secondly, we will measure the sensitivity of some input parameters by shifting them up and down.

### 5.1 Results Comparing with the Market

MBS of whose characteristics (see Appendix B: description, prepayment and historical price screenshots, respectively) are given as follows:

| Type | Pool | Effective Date | Maturity Date | Pass Through <br> Rate | W AC | Original <br> W AM | CPR \% | PSA \% |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| GNMA | 658017 | $1-7-2006$ | $15-7-2036$ | $5.50 \%$ | $6 \%$ | 357 | $14.8 \%$ | $355 \%$ |
| FNMA | 783390 | $1-8-2004$ | $1-9-2034$ | $6.50 \%$ | $7.15 \%$ | 359 | $24.0 \%$ | $501 \%$ |
| GNMA II | 2231 | $1-6-1996$ | $20-6-2026$ | $7 \%$ | $7.75 \%$ | 347 | $24.8 \%$ | $448 \%$ |

Table 5-1: The characteristics of MBS
We have valued the MBS given in Table 5-1 as of 31-12-2009. As seen from the table, different types of MBS with different maturities and coupon rates are chosen. The corresponding CPR and PSA rates are determined by generic Bloomberg models. The results and the motivation of the input parameters are given as follows:

```
<<confidential>>
```

Table 5-2: The simulation results vs. Bloomberg prices
At first glance we see that the model average is very close to the market quote for each MBS type. The seperate valuations differ slightly from the market quotes as well. This gives a good indication that the model is not unacceptable. The used assumptions are given as follows:

```
<<confidential>>
```

[^8]The same assumptions have been used for further analysis except the ones whose sensitivity is analyzed. FNMA MBS is picked from Table 5-1 for the sensitivity analysis in the next chapter.

### 5.2 Model Sensitivity

### 5.2.1 Number of Simulations

The FNMA MBS whose characteristics are given in Table 5-1 is simulated using PSA model and different number of simulations. The figure below represents the results:


Figure 5-1: Convergence as the number of simulation steps increases
We see that the simulations converge to an acceptable level at approximately 1000 steps. Number of simulations above this number will affect the fair value merely. Note that this analysis has been done at the calibrated Hull-White interest rate volatility of $1.78 \%$. One may need to use more steps in case sigma gets higher. The higher the volatility, the more steps are needed for convergence.

### 5.2.2 Prepayment Sensitivities

The given CPR and PSA rates in Table 5-1 are derived from the in-house models of Bloomberg and therefore remain predictions of the future prepayments. There exist enough endogenous and exogenous factors that affect the prepayment behavior of the underlying homeowners. Changing these rates has enormous effect on the valuation as well as cash flow structure.

After the simulations are run, a chart (see Appendix D for an enlarged example) is generated where the averages of the net interest, the scheduled principal and the prepayments are represented for the remaining life of the MBS. These three values summed up give the cash flows ${ }^{14}$. The figure below represents the cash flow structures in different scenarios together with the fair values:


CF Structure under 150\% PSA


CF Structure under 300\% PSA


CF Structure under 100\% PSA


Figure 5-2: Cash flow structure under different PSA rates
In the top left chart, the amount prepaid each month is very high and therefore exposed to risk that the cash flows will amortize soon and the investor will not be able to profit much longer from the interest. The less the prepayment rates, the more the cash flows are distributed over time and as a result, the more the value of the MBS gets.

Figure 5-3 represents how the simulated cash flows based on the <<confidential>> are distributed over the course of time. Note that the maturity of this MBS is adjusted to 360 months for illustration purposes:

[^9]

The seasonality effect of the prepayment behavior is clear to see from the dark blue line where the rates follow sine curve. The seasoning effect is also clear to see in the first 2.5 years where the prepayment rates increase gradually until the pool becomes seasoned.

Figure 5-3: Cash flows under <<confidential>>

Another aspect that effects the prepayment under <<confidential>> is the correlation coefficient. In our valuations the correlation factor is set to 1 since the interest rates and the refinancing rates are highly correlated. The effect of the correlation on the fair value is given in the below table:

| Correlation | Fair value |
| ---: | ---: |
| 1 | 107.03 |
| 0.75 | 108.26 |
| 0.5 | 109.35 |
| 0 | 111.45 |
| -0.3 | 112.78 |
| -1 | 115.68 |

Table 5-3: In Appendix F the corresponding short rates are represented.
The fair value gets higher as the correlation decreases (or inversely correlation increases). To analyze this further, we have compared the simulated prepayment rates of $\rho=1$ to those of $\rho=-1$. The differences in \% seem to be very small but they have enormous impact on the fair value. However, we have converted the differences to the basis points ( $1 \mathrm{bp}=0.01 \%$ ). The prepayment rate $r_{\mathrm{i}, \rho=-1}$ for month $i$ is deducted from $r_{\mathrm{i}, \mathrm{p}=1}$ and represented in Figure 5-4 in basis points:


Figure 5-4: The difference in simulated prepayment rates given in basis points In the early months, where the discount factors of the prepaid amounts are high, the $r_{\rho=-1}$ are higher as well. This results in higher fair values. The reason to this could be explained by the fact that under the lower (or inverse) correlations, the homeowners will have the incentive to prepay even if the available market rates get higher than the contractual rates.

## 6 Conclusion

<<confidential>>
Simulation models are used very often for valuation purposes in the financial world but the trickiest part is the input parameters that are driving the model. We have seen how prepayments affect the results significantly. Thus one should be sure of the input parameters used in the model, especially the Hull-White parameters. Simulation results without calibrated parameters will be meaningless.

Since the model performs properly, this may be taken to further developments such as incorporating the Waterfall structure, where the pools are securitized in credit rating tranches and the investors are getting paid in order of the credit quality. Another addition would be developing RMBS for the Dutch market, where the payments to the investors passed through in floating rate coupons. Next to this, some improvements could be performed such as reestimating <<confidential>> parameters and extending Hull-White to two-factor model.

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## Appendix A: Derivation of Monthly Mortgage Payment

Amount owed at month 0: $M B_{0}$
Amount owed at month 1: $(1+i) M B_{0}-M P$
Amount owed at month 2: $(1+i)\left((1+i) M B_{0}-M P\right)-M P=(1+i)^{2} M B_{0}-(1+(1+i)) M P$
Amount owed at month 3: $(1+i)\left((1+i)\left((1+i) M B_{0}-M P\right)-M P\right)-M P=(1+i)^{3} M B_{0}-(1+(1+i)$
$\left.+(1+i)^{2}\right) M P$
...

Amount owed at month $N:(1+i)^{N} M B_{0}-\left(1+(1+i)+(1+i)^{2}+(1+i)^{3}+\ldots+(1+i)^{N-1}\right) M P$

The polynomial $p_{N}(x)=1+x+x^{2}+\ldots+x^{N-1}$ with $x=(1+i)$ has a simple closed-form expression obtained from observing that $x p_{N}(x)-p_{N}(x)=x^{N}-1$ because all but the first and last terms in this difference cancel each other out. Therefore, solving for $p_{N}(x)$ yields the much simpler closedform expression which can be formulated as:
$p_{n}(x)=1+x+x^{2}+\ldots+x^{N-1}=\frac{x^{N}-1}{x-1}$
Applying this fact to the amount owed at Nth month:
Amount owed at month $N$
$=(1+i)^{N} M B_{0}-p_{N}(1+i) M P$
$=(1+i)^{N} M B_{0}-\frac{(1+i)^{N}-1}{1+i-1} M P$
Since the amount owed at month $N$ must be zero because the mortgagor agrees to fully pay off, the monthly mortgage payment MP can be obtained by:

$$
M P=M B_{0} \frac{i(1+i)^{N}}{(1+i)^{N}-1}
$$

(Source: Wikipedia - http://en.wikipedia.org/wiki/Mortgage_Calculator)

## Appendix B: Bloomberg Prints

## Ginnie Mae Pool 658017



<HELP> for explanation, 〈MENU〉 for similar functions.

## Mtge HP

FTIM/NY/CLOSE/MID/YIELD


Finnie Mae Pool 783390




Ginnie Mae II Pool 2231


| DES |  |  | Mtge | DES |
| :--- | :--- | :--- | :--- | :--- |
|  |  |  |  | Security Description |
| Agency | G2 | Issue Date | $06 / 01 / 96$ | Originator |
| Pool | 2231 | Mty Date | $06 / 20 / 26$ | Multiple Issuer |
| CUSIP | $36202 C P Q 0$ |  |  |  |
| Type | (SF) Level pay |  |  |  |
| Traits | $30 / 360$ |  |  |  |
| Generic | G2SF 7 1996 |  |  |  |

Information As Of Feb10
1 Month CPR History

| Date | Pool |  |
| :---: | ---: | ---: |
| $02 / 2010$ | 1.1 |  |
| $01 / 2010$ | 1.1 | 4.6 |
| $12 / 2009$ | 21.9 | 5.3 |
| $11 / 2009$ | 4.9 | 18.6 |
| $10 / 2009$ | 26.2 | 14.2 |
| $09 / 2009$ | 1.6 | 21.1 |
| $08 / 2009$ | 15.8 | 10.4 |
| $07 / 2009$ | 1.8 | 15.0 |
| $06 / 2009$ | 12.6 | 8.5 |
| $05 / 2009$ | 30.8 | 17.6 |
| $04 / 2009$ | 52.3 | 16.3 |
| $03 / 2009$ | 16.2 | 18.9 |



1) Summary / 2) Generic
2) Prepay 4) Geo/LOY




Appendix C: User Manual <<confidential>>

## Appendix D: Example Chart



## Appendix E: Cash Flow Structure

Represented figures below are the averages of all the simulated paths

| Date | Month | Outstanding Balance | SMM | Mortgage Payment | Net Interest | Gross Interest | Scheduled principal | Prepayments | Total Principal | Cash <br> Flow | PV Cash Flow |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 16-1-2010 | 1 | 1,000,000 | 0.03\% | 6,754 | 5,417 | 5,958 | 796 | 250 | 1,046 | 6,463 | 6,462 |
| 16-2-2010 | 2 | 998,954 | 0.05\% | 6,752 | 5,411 | 5,952 | 800 | 500 | 1,301 | 6,712 | 6,709 |
| 16-3-2010 | 3 | 997,653 | 0.08\% | 6,749 | 5,404 | 5,944 | 805 | 751 | 1,555 | 6,959 | 6,954 |
| 16-4-2010 | 4 | 996,098 | 0.10\% | 6,744 | 5,396 | 5,935 | 809 | 1,001 | 1,810 | 7,205 | 7,196 |
| 16-5-2010 | 5 | 994,288 | 0.13\% | 6,737 | 5,386 | 5,924 | 813 | 1,250 | 2,063 | 7,449 | 7,435 |
| 16-6-2010 | 6 | 992,225 | 0.15\% | 6,729 | 5,375 | 5,912 | 817 | 1,500 | 2,316 | 7,691 | 7,671 |
| 16-7-2010 | 7 | 989,909 | 0.18\% | 6,718 | 5,362 | 5,898 | 820 | 1,748 | 2,568 | 7,930 | 7,902 |
| 16-8-2010 | 8 | 987,341 | 0.20\% | 6,707 | 5,348 | 5,883 | 824 | 1,995 | 2,819 | 8,167 | 8,130 |
| 16-9-2010 | 9 | 984,522 | 0.23\% | 6,693 | 5,333 | 5,866 | 827 | 2,241 | 3,068 | 8,401 | 8,352 |
| 16-10-2010 | 10 | 981,454 | 0.25\% | 6,678 | 5,316 | 5,848 | 830 | 2,486 | 3,316 | 8,632 | 8,570 |
| 16-11-2010 | 11 | 978,138 | 0.28\% | 6,661 | 5,298 | 5,828 | 833 | 2,729 | 3,562 | 8,860 | 8,783 |
| 16-12-2010 | 12 | 974,576 | 0.31\% | 6,642 | 5,279 | 5,807 | 835 | 2,971 | 3,806 | 9,085 | 8,993 |
| 16-1-2011 | 13 | 970,770 | 0.33\% | 6,622 | 5,258 | 5,784 | 838 | 3,210 | 4,048 | 9,306 | 9,196 |
| 16-2-2011 | 14 | 966,722 | 0.36\% | 6,600 | 5,236 | 5,760 | 840 | 3,447 | 4,287 | 9,524 | 9,397 |
| 16-3-2011 | 15 | 962,435 | 0.38\% | 6,577 | 5,213 | 5,735 | 842 | 3,683 | 4,525 | 9,738 | 9,595 |
| 16-4-2011 | 16 | 957,910 | 0.41\% | 6,551 | 5,189 | 5,708 | 844 | 3,915 | 4,759 | 9,948 | 9,786 |
| 16-5-2011 | 17 | 953,151 | 0.44\% | 6,525 | 5,163 | 5,679 | 845 | 4,145 | 4,990 | 10,153 | 9,973 |
| . | . | . | - | . | . | . | . | . | . | . |  |
| - | - | - | $\cdot$ | - | - | - | . |  | . |  |  |
| 16-2-2029 | 230 | 113,797 | 0.78\% | 1,254 | 616 | 678 | 576 | 886 | 1,462 | 2,079 | 739 |
| 16-3-2029 | 231 | 112,335 | 0.78\% | 1,244 | 608 | 669 | 575 | 875 | 1,450 | 2,058 | 728 |
| 16-4-2029 | 232 | 110,885 | 0.78\% | 1,234 | 601 | 661 | 574 | 864 | 1,437 | 2,038 | 718 |
| 16-5-2029 | 233 | 109,448 | 0.78\% | 1,225 | 593 | 652 | 572 | 852 | 1,425 | 2,018 | 707 |
| 16-6-2029 | 234 | 108,023 | 0.78\% | 1,215 | 585 | 644 | 571 | 841 | 1,413 | 1,998 | 696 |
| 16-7-2029 | 235 | 106,611 | 0.78\% | 1,206 | 577 | 635 | 570 | 830 | 1,400 | 1,978 | 686 |
| 16-8-2029 | 236 | 105,210 | 0.78\% | 1,196 | 570 | 627 | 569 | 819 | 1,388 | 1,958 | 676 |
| 16-9-2029 | 237 | 103,822 | 0.78\% | 1,187 | 562 | 619 | 568 | 808 | 1,376 | 1,939 | 666 |
| 16-10-2029 | 238 | 102,445 | 0.78\% | 1,177 | 555 | 610 | 567 | 798 | 1,365 | 1,919 | 656 |
| 16-11-2029 | 239 | 101,081 | 0.78\% | 1,168 | 548 | 602 | 566 | 787 | 1,353 | 1,900 | 646 |
| 16-12-2029 | 240 | 99,728 | 0.78\% | 1,159 | 540 | 594 | 565 | 776 | 1,341 | 1,881 | 636 |
| 16-1-2030 | 241 | 98,387 | 0.78\% | 1,150 | 533 | 586 | 564 | 766 | 1,330 | 1,862 | 627 |
| 16-2-2030 | 242 | 97,057 | 0.78\% | 1,141 | 526 | 578 | 563 | 755 | 1,318 | 1,844 | 618 |
| 16-3-2030 | 243 | 95,739 | 0.78\% | 1,132 | 519 | 570 | 562 | 745 | 1,307 | 1,825 | 608 |
| 16-4-2030 | 244 | 94,433 | 0.78\% | 1,123 | 512 | 563 | 561 | 735 | 1,295 | 1,807 | 599 |
| 16-5-2030 | 245 | 93,137 | 0.78\% | 1,114 | 504 | 555 | 559 | 725 | 1,284 | 1,789 | 590 |
| 16-6-2030 | 246 | 91,853 | 0.78\% | 1,106 | 498 | 547 | 558 | 715 | 1,273 | 1,771 | 582 |
| 16-7-2030 | 247 | 90,580 | 0.78\% | 1,097 | 491 | 540 | 557 | 705 | 1,262 | 1,753 | 573 |
| 16-8-2030 | 248 | 89,318 | 0.78\% | 1,088 | 484 | 532 | 556 | 695 | 1,251 | 1,735 | 564 |
| 16-9-2030 | 249 | 88,067 | 0.78\% | 1,080 | 477 | 525 | 555 | 685 | 1,240 | 1,717 | 556 |
| 16-10-2030 | 250 | 86,827 | 0.78\% | 1,071 | 470 | 517 | 554 | 675 | 1,229 | 1,700 | 547 |
| 16-11-2030 | 251 | 85,597 | 0.78\% | 1,063 | 464 | 510 | 553 | 666 | 1,219 | 1,682 | 539 |
| 16-12-2030 | 252 | 84,378 | 0.78\% | 1,055 | 457 | 503 | 552 | 656 | 1,208 | 1,665 | 531 |
| 16-1-2031 | 253 | 83,170 | 0.78\% | 1,046 | 451 | 496 | 551 | 647 | 1,198 | 1,648 | 523 |
| 16-2-2031 | 254 | 81,972 | 0.78\% | 1,038 | 444 | 488 | 550 | 637 | 1,187 | 1,631 | 515 |
| 16-3-2031 | 255 | 80,785 | 0.78\% | 1,030 | 438 | 481 | 549 | 628 | 1,177 | 1,615 | 507 |
| . | . | . | . | . | - | . | . | . | . | . |  |
| . | - | - | - | . | - | - |  |  | . |  |  |
| 16-5-2038 | 341 | 9,853 | 0.78\% | 524 | 53 | 59 | 465 | 73 | 539 | 592 | 122 |
| 16-6-2038 | 342 | 9,314 | 0.78\% | 520 | 50 | 55 | 464 | 69 | 534 | 584 | 120 |
| 16-7-2038 | 343 | 8,780 | 0.78\% | 516 | 48 | 52 | 464 | 65 | 529 | 576 | 117 |
| 16-8-2038 | 344 | 8,252 | 0.78\% | 512 | 45 | 49 | 463 | 61 | 524 | 568 | 115 |
| 16-9-2038 | 345 | 7,728 | 0.78\% | 508 | 42 | 46 | 462 | 57 | 519 | 561 | 113 |
| 16-10-2038 | 346 | 7,209 | 0.78\% | 504 | 39 | 43 | 461 | 53 | 514 | 553 | 111 |
| 16-11-2038 | 347 | 6,696 | 0.78\% | 500 | 36 | 40 | 460 | 49 | 509 | 545 | 109 |
| 16-12-2038 | 348 | 6,187 | 0.78\% | 496 | 34 | 37 | 459 | 45 | 504 | 537 | 107 |
| 16-1-2039 | 349 | 5,683 | 0.78\% | 492 | 31 | 34 | 458 | 41 | 499 | 530 | 105 |
| 16-2-2039 | 350 | 5,184 | 0.78\% | 488 | 28 | 31 | 457 | 37 | 494 | 522 | 103 |
| 16-3-2039 | 351 | 4,689 | 0.78\% | 484 | 25 | 28 | 456 | 33 | 490 | 515 | 101 |
| 16-4-2039 | 352 | 4,200 | 0.78\% | 481 | 23 | 25 | 456 | 29 | 485 | 508 | 99 |
| 16-5-2039 | 353 | 3,715 | 0.78\% | 477 | 20 | 22 | 455 | 26 | 480 | 500 | 97 |
| 16-6-2039 | 354 | 3,234 | 0.78\% | 473 | 18 | 19 | 454 | 22 | 476 | 493 | 95 |
| 16-7-2039 | 355 | 2,759 | 0.78\% | 469 | 15 | 16 | 453 | 18 | 471 | 486 | 93 |
| 16-8-2039 | 356 | 2,288 | 0.78\% | 466 | 12 | 14 | 452 | 14 | 467 | 479 | 92 |
| 16-9-2039 | 357 | 1,821 | 0.78\% | 462 | 10 | 11 | 451 | 11 | 462 | 472 | 90 |
| 16-10-2039 | 358 | 1,359 | 0.78\% | 459 | 7 | 8 | 450 | 7 | 458 | 465 | 88 |
| 16-11-2039 | 359 | 902 | 0.78\% | 455 | 5 | 5 | 450 | 4 | 453 | 458 | 86 |
| 16-12-2039 | 360 | 449 | 0.78\% | 451 | 2 | 3 | 449 | (0) | 449 | 451 | 85 |

## Appendix F: Correlated Rates



## Appendix G: Hull-White Calibration Model

Volatility Cube for USD as of 31-12-2009 as taken from Blomberg


Volatility Cube visualized for USD as of 31-12-2009


Levenberg Marquardt method is used to calibrate the Hull-White parameters <<confidential>>

## Appendix H: Source Code

<<confidential>>


[^0]:    ${ }^{1}$ Since RMBS is a part of MBS and we restrict us only to the residential variant, for sake of simplicity, we call the residential mortgage-backed securities as MBS through the whole paper.

[^1]:    ${ }^{2}$ See Appendix A for a derivation of this formula

[^2]:    ${ }^{3} \mathrm{~A}$ behaviour of regional or national economy as a whole

[^3]:    ${ }^{4}$ A percentage of each mortgage payment made by a borrower to a mortgage servicer as compensation for keeping a record of payments, collecting and making escrow payments, passing principal and interest payments along to the note holder, etc. Servicing fees generally range from $0.25-0.5 \%$ of the remaining principal balance of the mortgage each month (source: www.investopedia.com)

[^4]:    ${ }^{5}$ In our examples and figures in Chapter 4, we will assume a recently issued MBS which has a remaining life of 30 years (i.e., 360 months).

[^5]:    ${ }^{6}$ The weighted average of the time until all maturities on mortgages in a mortgage-backed security (MBS). The higher the weighted average to maturity, the longer the mortgages in the security have until maturity.
    ${ }^{7}$ The London Interbank Offered is a daily reference rate based on the interest rates at which banks borrow unsecured funds from other banks in the London wholesale money market (or interbank market).
    ${ }^{8}$ The user should input the discount factor curve in <<confidential>>

[^6]:    ${ }^{9}$ Initial short rate $r_{0}$ is equal to the initial zero rate $z_{0}$ which is an input.

[^7]:    ${ }^{10} a$ and $\sigma$ should be input in the <<confidential>>

[^8]:    ${ }^{11}$ The valuation will be based on generic pool basis rather than MBS notes (contracts) because the MBS contracts are assigned to those pools, where the notional amounts depend on each request of investors.
    ${ }^{12}$ A major global provider of 24-hour financial news and information including real-time and historic price data, financials data, trading news and analyst coverage, as well as general news and sports. Its services, which span their own platform, television, radio and magazines, offer professionals analytic tools
    ${ }^{13}$ Please note that the MBS values of Bloomberg are represented in American format (i.e., 10502. This is equal to $105+2 / 32=105.06$ ).

[^9]:    ${ }^{14} \mathrm{~A}$ complete overview of the simulated cash flow structure is represented in Appendix E

