Fuel usage estimation and optimization in temperature-controlled vehicle routing

Thesis MSc Business Analytics at ORTEC

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23 August 2021

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Acknowledgements

This research has been performed as graduation project for the Master's program Business Analytics at the Vrije Universiteit Amsterdam and as an internship at ORTEC. ORTEC is the world's leading supplier of mathematical optimization software and advanced analytics, with around 1,000 employees and offices in 13 countries around the globe. They leverage data and mathematics to create value for business and society at large.

One of the projects they are involved in is the SensLog research project in cooperation with the Vrije Universiteit Amsterdam and industry specialists. The goal of this research is to leverage data generated by innovative sensors in order to develop new models, algorithms and strategies in logistics. Part of this research specifically focuses on cold-chain vehicle routing in cooperation with Mandersloot, a company that performs (mainly temperature regulated) transport of products across the Benelux and Central Europe. Their trucks have been equipped with sensors taking measurements in and around the vehicle, generating data that forms an important basis for this research as well as for this graduation project.

I would like to thank everyone who has supported this project. Alessandro Zocca acted as university supervisor from the Business Analytics program, Rosario Paradiso has been the second university supervisor from the SensLog research group and Joaquim Gromicho was my supervisor from ORTEC, the company at which I have been able to perform this internship project. I also want to thank Henri Groothuis from Mandersloot and all colleagues at ORTEC who have helped me obtaining and understanding the data and problem as well as introducing me in the company as a whole in these mainly digital times.

Management Summary

Temperature-controlled vehicle routing problems are a specific kind of vehicle routing problem concerning trucks with temperature regulatable loading spaces, that are used for the transport temperature sensitive products. On board of such a truck is a refrigerator unit that can cool or heat the loading space to reach a desired temperature. At the same time, this temperature is also influenced by the temperature outside the truck, especially while the truck's door is opened for loading purposes. This means that a vehicle does not only consumes energy for traveling, but also for the continuous temperature regulation by the refrigerator unit.

Companies that offer temperature-controlled transportation, usually have a set of required pickups and deliveries that should all be visited within a specified time window. Each of the products that are picked up, have specific temperature requirements for transportation. The goal is to create feasible routes covering all visits as efficiently as possible. This are usually the routes that minimize the total required fuel consumption, and therefore the resulting costs and emissions. Traditionally, such a vehicle routing problem can be approached by finding the routes with the lowest total distance. But for temperature-regulated trucks, this are not necessarily also the routes that require the least fuel for the truck and the refrigerator unit. So might it be the case that taking slightly longer routes can prevent a truck from a long loading stop while at the same time keeping the rest of the load frozen and consuming a lot of fuel. This makes this problem different from a regular vehicle routing problem, requiring different approaches.

The first issue that arises when aiming to solve such a problem is that it is not trivial to compute an expected fuel usage, given a certain route planning. That is why the first part of this research will use historical data, containing sensor measurements collected in and around a truck during real performed routes. Based on this historical data, there will be searched for ways to estimate fuel consumption based on features of the route that can be known or estimated in advance. The second part of this research will use these estimations while searching for routes that minimize this total estimated fuel consumption. A MILP is formulated and solved that explicitly minimizes fuel usage instead of minimizing the total route distance. Furthermore, it is also investigated what the effect is of allowing a dynamic target temperature that can change during the route versus a fixed setpoint per vehicle.

The results show that this is a very promising approach, that is able to decrease the estimated amount of consumed fuel compared to using a distance objective. Especially when also allowing a dynamic target temperature. These gains are achieved by selecting different routes, that prefer certain combinations of products with matching temperature requirements as well as optimizing at which moments possible waiting times are included. In this way, there is more fuel saved for temperature regulation than is required for the extra traveling distances, lowering the total consumption. This proof of concept shows that extending the model, could make it a very usable approach in practice to save fuel in cold-chain logistics.

1 Introduction

Within the domain of logistics and transportation, the optimization of all tasks and processes involved can be of great value. Planning and performing all operations efficiently can greatly reduce the time and resources required. Generally, this does not only lead to a decrease in operational costs, but can also lower the total consumption of energy and materials, and therefore the emission and waste generated. It could even create additional profit and an improved market position due to better customer service and experience. A lot of research in this area has already been done and is used in practice, but as a result of ever increasing computational power the possibilities and effectiveness for solving bigger and more complex problems, and thereby the interest of both research as well as industry, keeps growing.

The problem that will be discussed in this paper is based on a realistic scenario concerning companies in the transport and logistics sector that possess vehicles with temperature regulatable loading spaces. Such a company usually has a list of pickup and delivery requests that need to be served within a certain time window. Each pair of requests involves the transportation of a certain amount of goods from one location to another, while maintaining a product dependent temperature inside the loading space. The company has to assign each pair of requests to one of its vehicles and also has to decide in which order each vehicle visits those assigned requests. The aim is to do this such that routes are created for each of the vehicles that respect all temperature and time window requirements while preventing unnecessary high transportation costs. Two different cases will be considered regarding the temperature regulation inside the loading spaces. In the first case, each vehicle has a fixed setpoint temperature to which the loading space will be cooled or heated. This is based on the currently most common situation where a specialized technician is required to make changes to the settings of the refrigerator. In the second case, the setpoint temperature of a vehicle can be changed dynamically at any point during a route. This is based on the more advanced situation where either a dynamic temperature regime can be set up at the start of a route, or the target temperature can be changed manually by the driver based on the instructions from his board computer.

Traditionally, this can be seen as a Capacitated Vehicle Routing Problem (CVRP) with additional constraints for pickup and delivery, for the time windows of each visit and for the allowed temperature ranges for each product type. It is assumed that there is a fixed number of available vehicles with only variable costs for their fuel consumption. This can then be solved as a Mixed Integer Linear Programming (MILP) model with the objective to minimize the total distance of all the resulting routes. These shortest routes are then assumed to require the lowest amount of fuel for both traveling and temperature regulation.

However, the assumption that the shortest routes also lead to the lowest fuel consumption might not always be correct. There could be many other factors that influence the fuel consumption that are not taken into account when using this approach. For example, the fuel required to maintain a certain temperature inside the vehicle might depend on the type and amount of loaded products. Some products are picked-up pre-frozen, while others might be stored at room temperature and do neither require, nor mind to be transported in a cold environment. Also, while (un)loading a truck, the door of the loading space will need to be open. This means that the longer the service time, the more the temperature inside the loading space is affected by the outside temperature. After the door is closed, the refrigerator will need to cool or heat the loading space to match the target temperature again. Since this could lead to peaks in the fuel used by the refrigerator unit, it would make sense to take this into account as well. While finding the best routes, there might be chosen for routes with a slightly higher total distance if this for instance leads to better load combinations or avoid long (un)loading stops when there is a big difference between the setpoint and outside temperatures. So could it be that transporting certain electronics together with frozen meat at a low temperature is allowed, but consumes more fuel than splitting those tasks over two different trucks. There are many such non-trivial factors that possibly influence the total amount of fuel that is used by a vehicle for traveling and temperature regulation. That is why the first goal of this research will be to come up with a way to make an accurate and reliable estimation of the total fuel that will be consumed by a vehicle when executing a certain planned route of pickups and deliveries. This will be done based on historical data that has been collected by sensors that have been placed on board of temperature regulated trucks that are currently in operation at Mandersloot. These sensors do not only measure the fuel levels in the tank, but also take a large number of other measurements in and around the truck. The learned estimation methods will then be used to achieve the second goal of this research. Namely, comparing the fuel usage estimations of solutions that result from finding routes by minimizing the total travel distance with those resulting from minimizing on the total fuel usage explicitly. This will be done for both the situation where the temperature setpoint is fixed, as well as for the situation where this setpoint is dynamic along a route. If can be shown that it is possible to decrease the total fuel usage by explicitly optimizing over this, this would be of great value to the industry of cold chain logistics. In this way, it would be possible to gain from an economical perspective by reducing the total costs as well as gaining from an sustainability perspective by reducing the total emissions.

A number of studies have been looking into similar cold-chain vehicle routing problems before. Aung and Chang (2014) looked into clustering methods in order to make combinations of different products that could be transported at the same target temperature. The objective was to minimize the quality decay of each product, by making sure that the target temperatures were close enough to a product's optimal required temperature. Hsiao et al. (2017) formulated a MILP with an objective function that minimized both travel costs, refrigeration costs as well as product quality decay translated into costs based on the expected residual shelf life after delivery. Refrigeration costs were estimated by a fixed amount per loaded product and per degrees of temperature difference between each loaded product and the current target temperature, without taking external causes for temperature fluctuations into account. An evolutionary algorithm was presented to solve the problem instances heuristically. Stellingwerf, Kanellopoulos, et al. (2018) incorporated an objective function where fuel usage for traveling is computed as a function of certain characteristics of the route and vehicle load including traffic, loaded weight and temperature regulation. Fuel usage for temperature regulation is computed based on the energy required to counter thermal processes due to difference with the outside temperature. The used values are based on a theoretic situation with a single fixed target and outside temperature. It was shown that including the fuel usage for temperature regulation could lead to different solutions and routes compared to solving the default load dependent problem. Stellingwerf, Groeneveld, et al. (2021) extended this by deriving more accurate quality decay estimations using real supermarket data. It was concluded that quality decay could be reduced by having shorter service times, shorter travel times and better cooling units. The required energy from temperature regulation also depended on the isolation quality of a truck as well as on the outside temperature. Only a fixed outside temperature and a single product type with a fixed target temperature were considered.

2 The Data

This research has been based on historical transportation data of temperature controlled transport across the Benelux and Central Europe performed by the company Mandersloot. They granted access to the planned schedules and realized routes in May 2020 and January until March 2021 as have been created by the ORTEC Routing and Dispatching (ORD) application, as well as the sensor measurements collected from the vehicles while performing these tasks. This data has been enriched with publicly available historical weather data from the National Centers for Environmental Information (NCEI) within the National Oceanic and Atmospheric Administration Agency (NOAA). All datasets have been used to learn how to estimate the fuel usage for a planned route, but also to create reality-based problem instances to solve. For the creation of instances and visualizations of the results has also been made use of ORTEC's Vehicle Routing API, in order to construct the fastest truck routes over the road and compute the route's distance and expected (time dependent) travel time.

2.1 ORTEC Routing and Dispatching data

ORTEC keeps a record of the historical routing and dispatching data as has been scheduled by their planning application and has been realized afterwards. This information is stored in a relational SQL database with many different tables and columns, which are not all relevant for the current research. A query was used to extract a list of all the actions planned and performed in the time periods for which sensor data is available. This list contains multiple sequences of subsequent actions that together form a so called shift. At each shift, a container is first coupled to a truck, followed by a number of pickup, travel and delivery tasks, before the container is finally uncoupled from the truck. For each of those actions, the following information has been extracted:

- Type of action (couple/pickup/delivery/travel/decouple).
- Plate of the vehicle that performed the action.
- Planned start and ending time of the action.
- Realized start and ending time of the action.
- For pickup & delivery actions the earliest and latest allowed starting times.
- For pickup & delivery actions the load amount, product type and allowed temperature range.
- Total load amount and product types present in the container during the action.
- Geographic location of the action (for travel actions both a start and ending location).
- For travel actions the expected distance and duration of the route.

Figure 1: Two example shifts from the routing and dispatching data

2.2 Sensor data

The sensors on board of the containers do regular measurements in and around the container while the container is in use. It tracks the temperature at different positions inside the container, but also performs measurements at the tires and coupling point. Furthermore, this data contains information about the operating state of the refrigerator unit as well as information from the on board GPS device. It includes continuous measurements per vehicle on the following data:

- The measured temperature at multiple locations in the container, evaporator and air vents.
- The refrigerator unit's mode, status and temperature setpoint.
- The remaining fuel level in the refrigerator unit's tank.
- The tires' temperature, pressure and rotation speed.
- Whether the container is coupled to a truck.
- Whether the container door is open.
- The geographic location, altitude, vehicle speed and time from the GPS device

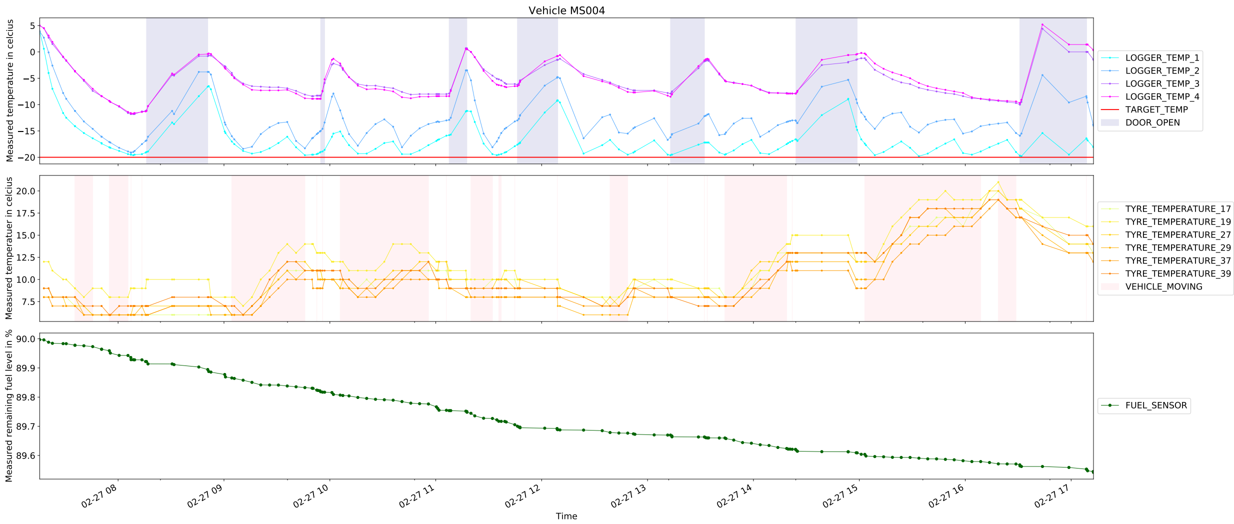


Figure 2: Example of sensor data while the refrigerator is mainly cooling

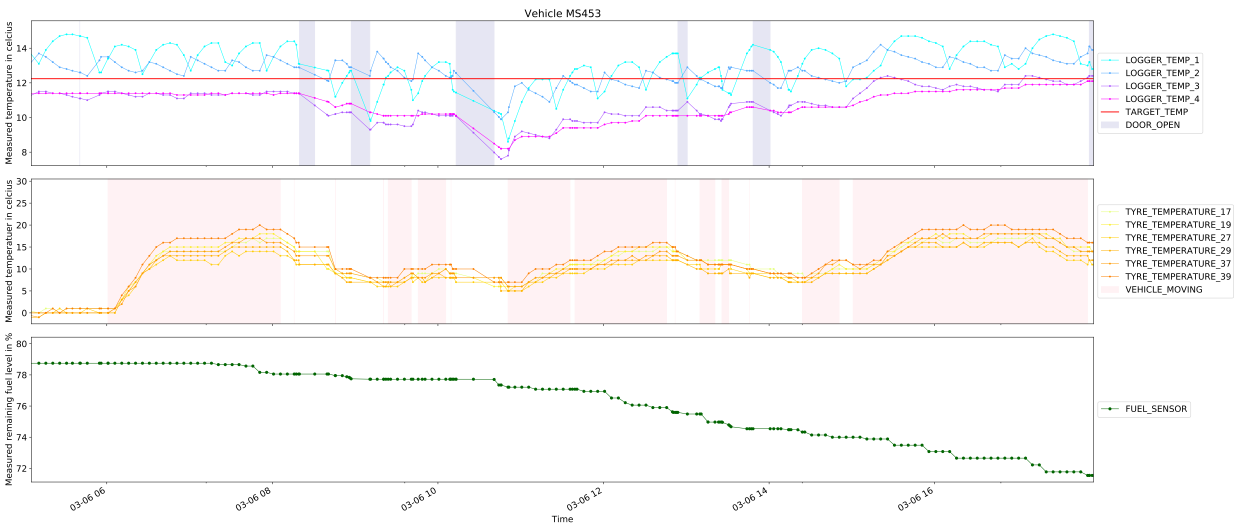


Figure 3: Example of sensor data while the refrigerator is mainly heating

2.3 Weather data

The NOAA's National Centers for Environmental Information provides public access to environmental data around the world. Among their datasets is the Global Surface Summary of the Day (GSOD). This dataset contains daily weather measurement averages from over 9,000 weather stations since 1929. From this dataset, the following information has been extracted for the relevant stations and time periods:

- The mean, maximum and minimum temperature over a day.
- The mean and maximum wind speed over a day.
- The total amount of precipitation and snow over a day
- The mean air pressure over a day
- The mean visibility over a day
- The occurrence of any snow, rain, hail, thunder or extreme wind over a day

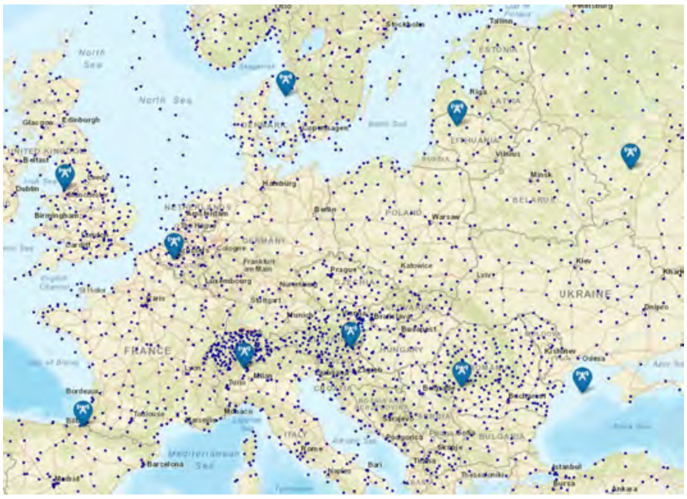


Figure 4: Locations of weather stations across central Europe

2.4 Routing API

ORTEC's Routing API is part of ORTEC's cloud service solutions that includes the functionality of constructing the fastest truck routes over the road between any two given coordinates. The API is able to not only return the generated routes and their distances, but also includes the expected travel time both in normal circumstances as well as based on the expected congestion during a specific time of the week. This service is used for enriching the historical data, for creating instances as well as for making route visualizations.

3 Methods

3.1 Modeling assumptions

In order to turn reality into a solvable MILP model without any non-linearity, a number of modeling assumptions and simplifications have to be made. The model will be formulated as a Multi Vehicle Routing Problem with Time Windows. Each of the requests is represented by a node of a fully connected graph. Each of the vertices between two nodes represents the shortest truck route between the locations of the two requests. The goal is now to find the best route for each vehicle such that every request is exactly visited once while all additional constraints are satisfied.

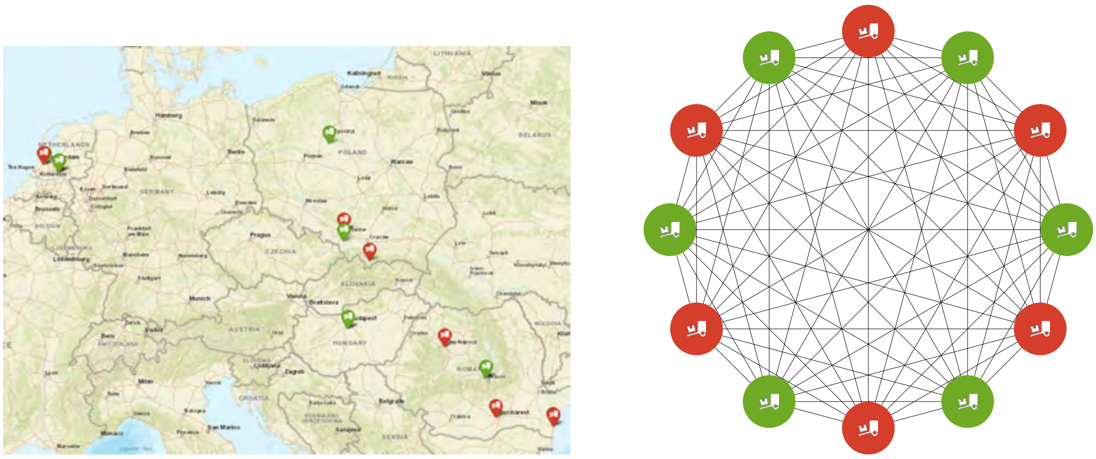


Figure 5: An example of a real routing problem instance and it's graph representation.

3.1.1 Instance

A problem instance should specify a fixed number of vehicles, nodes and product types. Each of the vehicles should have some maximum loading capacity as well as a specific starting node that specifies where the vehicle is located at time zero. Each node should specify the amount and type of product picked up or delivered, the service time required at this node, the earliest and latest allowed starting time and the average outside temperature during this time window. Each product type should specify the minimum and maximum temperatures at which it should be transported. Finally, both a matrix with the distance as well as a matrix with the expected travel times between each combination of nodes should be included. The exact locations of a node are not required to solve an instance, but could be used for visualization purposes.

Node	Start Time (min)		Load (lm)		Service Time (min)	Outside Temp (℃)
	Earliest	Latest	Type	Size		
1	0	9567.4	Plants	0.62	120	4.06
2	10055.7	108007.5	Food	-0.40	130	1.18
3	0	71340.8	Food	1.40	120	-2.87
4	2455.6	12615.5	Flowers	-4.40	206	10.11
5	788.4	17050.5	Plants	-0.62	3262	4.89
...	...	...	...	...	...	...

Vehicle	Capacity (lm)	Start Node
1	12.8	1
2	12.8	14
3	14.8	17

Distance (km)	1	2	3	4	...
1	0	346.2	346.2	836.0	...
2	345.3	0	0	979.7	...
3	345.3	0	0	979.7	...
4	758.9	452.4	450.0	0	...
...	...	...	...	...	...

Product	Min Temp (℃)	Max Temp (℃)
Fruit	2	10
Meat	-18	2
Plants	4	14
Flowers	6	16
Electronics	-18	18

Duration (min)	1	2	3	4	...
1	0	298.2	298.2	651.5	...
2	294.2	0	0	424.7	...
3	294.2	0	0	424.7	...
4	647.3	422.5	422.5	409.4	...
...	...	...	...	...	...

Table 1: Example Instance

3.1.2 Solution

The solution to such an instance should specify a route for each of the instance's vehicles. Each route contains a sequence of one or more nodes, beginning at the vehicle's starting node, as well as the times at which the vehicle is planned to enter and leave each of those nodes. Furthermore, a solution should specify either a fixed target temperature for each vehicle, or a dynamic target temperature at every part of each vehicle's route.

Figure 6: Routes of two example solutions with respectively one and two vehicles

3.1.3 Temperature over time

While at a node, a vehicle is (un)loading at a fixed location and it's door will be continuously open. This means that the inside temperature (t) is gradually changing towards the outside temperature (w) during service. This loss of cold or heat is assumed to happen at fixed rates (L^+ and L^-) that will be derived from the historical data. From the moment that the inside temperature equals the outside temperature onwards, the inside temperature will not change any further. Given a known starting temperature t_0, this means that the temperature at time τ during service is defined as:

$$t(\tau) = \begin{cases} \max\{t_0 - L^- \cdot \tau, w\} & \text{if } t_0 \geq w \\ \min\{t_0 + L^+ \cdot \tau, w\} & \text{if } t_0 < w \end{cases}$$

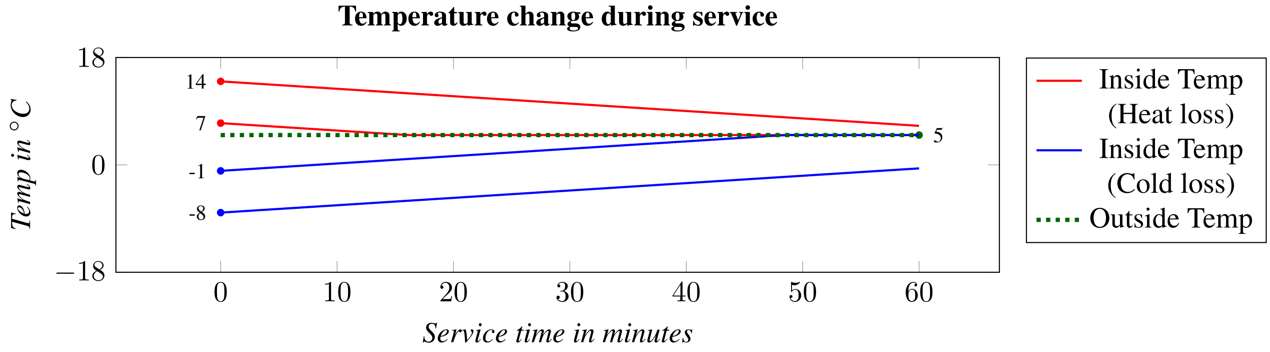


Figure 7: Examples of temperature change during service for four different starting temperature

While at a connection, a vehicle is traveling from one location to another with it's door closed. This means that the inside temperature is gradually heated or cooled towards the target temperature (s). This heating and cooling also happens at fixed rates (C^+ and C^-) that will also be derived from the historical data. From the moment that the inside temperature equals the target temperature, the inside temperature will not change any further. Given a known starting temperature t_0, this means that the temperature at time τ during travel is defined as:

$$t(\tau) = \begin{cases} \max\{t_0 - C^- \cdot \tau, s\} & \text{if } t_0 \geq s \\ \min\{t_0 + C^+ \cdot \tau, s\} & \text{if } t_0 < s \end{cases}$$

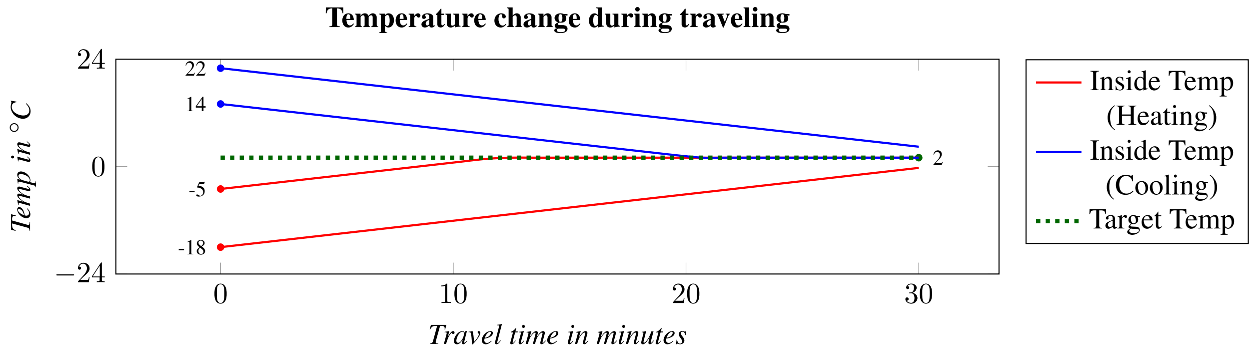


Figure 8: Examples of temperature change during traveling for four different starting temperature

Vehicles are allowed to stay longer at a node than the service time requires, either by waiting with the door open or closed. Waiting time with the door opened is represented by an extended time spent in a node, while waiting with a closed door is represented by an extended time spent at an arc.

3.1.4 Fuel usage

A solution can either be scored by the total travel distance or by the total estimated fuel usage. This estimation is done by a linear function that will be learned from the historical data. This function will map the information that can be derived from the decision variables, such as the time, load, inside, outside and target temperatures at each node and arc into a fuel consumption estimation. The linearity of this function is very important for solving purposes of the MILP. More details on how this function will be learned from the historical data follows in Section 3.3.

3.2 Data processing

In order to turn the available datasets into usable training data for fuel estimation and solvable problem instances, a number of processing steps are required. The data from the different sources has to be preprocessed and matched into a coherent dataset first. In order to learn estimating fuel usage, this data should then be resampled into records that match complete historical service or travel segments that matches the situation as modeled by a single node or arc. Furthermore, reality-based instances should be generated from the entire set of available historical requests.

3.2.1 Data preprocessing

The sensor data per container reports measurements between each few seconds up to thirty minutes while the refrigerator is switched on. To prepare this data for further usage, it requires some parsing, cleaning and transformations. So was the categorical feature containing information on the refrigerator state one-hot encoded, to allow for summarizing over time intervals later. Some of the continuous measurements contain unexpected fluctuations that are expectedly caused by faulty measurements. Take measurements such as the fuel level, for which it is known that it should only gradually decrease over time and should just increase with a big step towards a full tank level when it is refueled. Any small increases at other moments in time have to be caused by disturbances of the sensors or fuel surface, such as due to bumpy or sloped roads. Since both those disturbances as well as refueling are not of interest for determining the fuel consumption, this data is converted by taking the difference over each time step with a lowerbound of zero:

$$\mathit{fuel\_consumption}_{[t]} = \max\left\{\mathit{fuel\_level}_{[t]} - \mathit{fuel\_level}_{[t-1]}, 0\right\}$$

In order to enrich this sensor data with the weather measurements from the GSOD dataset, it is required to find the closest weather stations for each of the sensor measurements. For each sensor measurement the average weather measurements of the nearest stations, using Euclidean distance, on the corresponding day can then be added. This datasets do only contain a daily average, minimum and maximum temperature and not the exact temperature at each time of the day. The temperature level over a day, given these average minimum and maximum value can however be modeled by a sinusoidal function. This sinusoid should have a period of 24 hours, that is shifted such that the minimum temperature occurs at time t_{min} which is known to be dependent on the month of the year (Johnson and Fitzpatrick, 1977). The amplitude and mean should then be chosen such that the min, max and mean temperatures are fulfilled. In order to achieve this for any $w_{\text{min}} \leq w_{\text{mean}} \leq w_{\text{max}}$, the amplitude is a sinusoidal function in itself that shifts the average over the entire 24 hours up or down. This means that the temperature at time t during the day could be estimated by:

$$A(t) = \underbrace{(w_{\text{max}} + w_{\text{min}} - 2 \cdot w_{\text{mean}})}_{\text{difference between the amplitudes above and below average}} \cdot \sin\left(\frac{\pi}{12}(t - t_{\text{min}} - 6)\right) + \overbrace{\frac{1}{2}\left(w_{\text{max}} - w_{\text{min}}\right)}^{\text{average amplitude}}$$

$$w'(t) = A(t) \cdot \sin\left(\frac{\pi}{12}(t - t_{\text{min}} - 6)\right) + w_{\text{mean}} - \frac{1}{2}\overbrace{\left(w_{\text{min}} + w_{\text{max}} - 2 \cdot w_{\text{mean}}\right)}$$

$$w(t) = \min\left\{ \max\left\{ w'(t), w_{\text{min}} \right\}, w_{\text{max}} \right\}$$

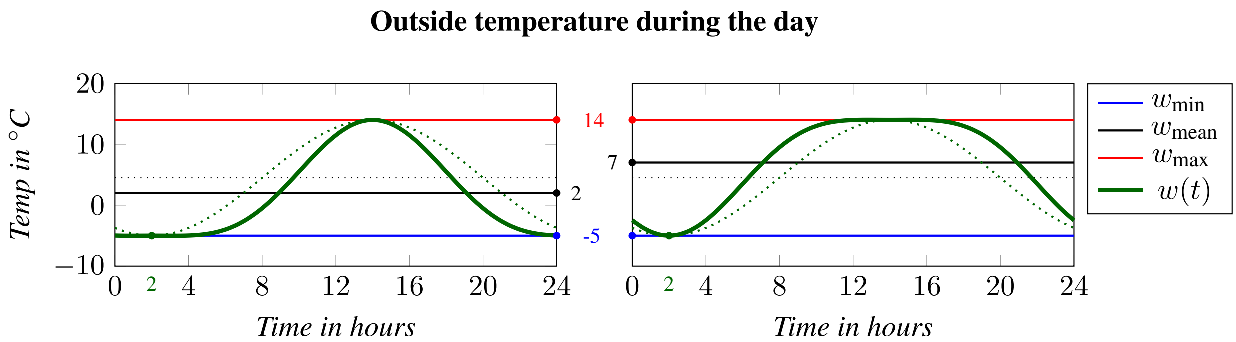


Figure 9: Examples of temperature fluctuation modeling over a day

3.2.2 Create training data

In order to create training data, the ORD data is chopped into segments representing either an entire service or an entire travel for which the refrigerator had been active and expanded with the sensor and weather measurements of the same vehicle between the segment's start and finish time. Each segment is then turned into a single record of features, by summarizing each of the measurements by taking the mean, min, max, total increase, total decrease and total difference over the time window. Since the measurements only include the fuel used at the refrigerator's engine, an estimation of the fuel used at the trucks main engine of 0.133L per minute (Hsiao et al., 2017) is added for each of the travel segments. All these records are then randomly split into a train and a test dataset, that will be used to find average temperature change rates as well as for learning and validating the function to estimate fuel usage.

3.2.3 Create instances

In order to generate solvable reality based instances, it is required to turn the continuous data into finite time problems existing out of $|N|$ requests to be served by $|K|$ vehicles starting at a time $\tau = 0$. An instance is generated by randomly taking $\frac{1}{2}|N|$ pairs of pickup and delivery requests from the complete set of requests available in the historical ORD data. Similarly, a subset of $|K|$ different vehicles are randomly drawn from all known vehicles. All requests' time windows are aligned by subtracting the original route's actual starting time. Finally, the MILP as will be defined in Section 3.4 is used without any objective function other than minimizing the amount of selected arcs, to quickly check if any feasible solutions exists for the given time windows and distances. Only if that is proven to be the case, the created instance is maintained.

Figure 10: Example subset of pickup and delivery pairs

3.3 Fuel estimation

In order to estimate the total fuel consumption of a vehicle during each service and travel segment, two linear functions will be fitted on the training dataset using ElasticNet regression (Zou and Hastie, 2005). The goal is to find a set of weights $\vec{\sigma}$ that, taking a dot product with an equal sized array of features, yields the best fuel consumption estimations. ElasticNet is a linear regression model that combines the strength of Ridge regularization (Hoerl and Kennard, 1988) and Lasso regularization (Tibshirani, 1996). Both regularization methods extend a Ordinary Least Squares loss function with a penalty term to prevent overfitting on the training data. Ridge regression uses the continuous squared magnitude $\alpha_2||\vec{\sigma}||_2^2$ to penalize large weights. Lasso regression uses $\alpha_1||\vec{\sigma}||_1$ that apart from penalizing large weights, also acts as feature selection method by shrinking all less important weights to zero. All features are chosen such that they should not lead to any fuel consumption when all equal to zero themselves, meaning that the intercept is fixed to zero.

The features that will be considered for fuel estimation during service are apart from the total service duration also the mean, total increase, total decrease and total difference during service of the load, the inside, outside and target temperatures as well as the differences between these different temperatures. For fuel estimation during travel, additionally also the total travel distance and expected travel time are considered. There might be other factors available in the historical data that also influence the fuel usage and could improve the predictions. However, those cannot be expressed in advance in terms of the decision variables of the MILP of Section 3.4 and are therefore not regarded.

3.4 Mixed Integer Linear Programming Model

The following parameters, variables, objective function and constraints together form a Mixed Integer Linear Programming model that gives a formal definition of the problem. Given a set of instance specific parameters, the Gurobi Optimizer (Gurobi Optimization, 2021) is used to find the optimal solution.

3.4.1 Parameters

The total set of parameters used in the model exists out of parameters that describe a specific problem instance, parameters that have been derived from the historical data and are used regardless of the specific problem, as well as auxiliary big-M parameters that are derived from the instance specific parameters and act as theoretical upper bounds on certain decision variables.

Instance specific parameters

o: Dummy depot node
N: Set of request nodes
V: Set of dummy depot and request nodes $N \cup \{o\}$
S_i: Service time at node $i \in N$
Θ_i^-: Earliest starting time at node $i \in N$
Θ_i^+: Latest starting time at node $i \in N$
Δ^{+p}_i: Amount of product $p \in P$ picked up at node $i \in N$
Δ^{-p}_i: Amount of product $p \in P$ delivered at node $i \in N$
$Z_{(i,j)}$: Travel time at the arc from node $i \in N$ to node $j \in N$
$D_{(i,j)}$: Travel distance at the arc from node $i \in N$ to node $j \in N$
W_i: Outside temperature at node $i \in N$

K: Set of vehicles
Q^k: Load capacity of vehicle $k \in K$
$n_0^k \in N$: Starting node of vehicle $k \in K$

P: Set of product types
T_p^-: Minimum allowed temperature setpoint for product of type $p \in P$
T_p^+: Maximum allowed temperature setpoint for product of type $p \in P$

Learned global parameters

C^+: Heating rate while traveling in degrees/second
C^-: Cooling rate while traveling in degrees/second
L^+: Cold loss during service in degrees/second
L^-: Heat loss during service in degrees/second

$\vec{\sigma}_{\text{service}}$: Weights for the linear function mapping decision variables to fuel usage estimate during service
$\vec{\sigma}_{\text{travel}}$: Weights for the linear function mapping decision variables to fuel usage estimate during travel

Big-M constants

Latest possible finish time of a segment

$$M_\tau: \quad \max_{i \in N} \left[\Theta_i^+ + S_i + \max_{j \in N} Z_{(i,j)} \right]$$

Longest possible distance of one travel segment

$$M_D: \quad \max_{i \in N} \max_{j \in N} D_{(i,j)}$$

Highest possible total load size at any moment

$$M_\ell: \quad \max_{k \in K} Q^k$$

Highest outside temperature at any moment

$$M_W: \quad \max_{i \in N} W_i$$

Highest possible difference between outside and target temperatures at any moment

$$M_t: \quad \max \left\{ \max_{i \in N} W_i, \quad \max_{p \in P} T_p^+ \right\} - \min \left\{ \min_{i \in N} W_i, \quad \min_{p \in P} T_p^- \right\}$$

Highest possible temperature difference cooled and heated within a single segment

$$M_{C^-}: \quad C^- \cdot \max_{i \in N} \max_{j \in N} Z_{(i,j)}$$

$$M_{C^+}: \quad C^+ \cdot \max_{i \in N} \max_{j \in N} Z_{(i,j)}$$

Highest possible cold and heat temperature loss within a single segment

$$M_{L^+}: \quad L^+ \cdot \max_{i \in N} S_i$$

$$M_{L^-}: \quad L^- \cdot \max_{i \in N} S_i$$

Highest possible fuel usage during one service segment

$$M_f: \quad \vec{\sigma}_{\text{service}} \cdot \begin{bmatrix} M_t & \dots & M_\ell & \dots & M_\tau \end{bmatrix}^\intercal$$

Highest possible fuel usage during one travel segment

$$M_g: \quad \vec{\sigma}_{\text{travel}} \cdot \begin{bmatrix} M_D & M_\tau & M_t & \dots & M_\ell & \dots & M_\tau \end{bmatrix}^\intercal$$

3.4.2 Decision Variables

The following decision variables are used to describe a solution and track the time, load and temperatures at each segment for fuel estimation.

Routing variables

$x_i^k \in \mathbb{B}$	Node $i \in N$ is visited by vehicle $k \in K$
$y_{(i,j)}^k \in \mathbb{B}$	Arc from $i \in V$ to $j \in V$ is visited by vehicle $k \in K$

Time tracking variables

$\tau_i^k \in \mathbb{R}^+$	Time at which vehicle $k \in K$ enters node $i \in N$
$\hat{\tau}_i^k \in \mathbb{R}^+$	Time at which vehicle $k \in K$ leaves node $i \in N$

Load tracking variables

$\ell_i^{k,p} \in \mathbb{R}^+$ Cumulative load of type $p \in P$ when vehicle $k \in K$ enters node $i \in N$

$\hat{\ell}_i^{k,p} \in \mathbb{R}^+$ Cumulative load of type $p \in P$ when vehicle $k \in K$ leaves node $i \in N$

$z_i^{k,p} \in \mathbb{B}$ Cumulative load contains type $p \in P$ when vehicle $k \in K$ enters node $i \in N$

$\hat{z}_i^{k,p} \in \mathbb{B}$ Cumulative load contains type $p \in P$ when vehicle $k \in K$ leaves node $i \in N$

Temperature tracking variables

$t_i^k \in \mathbb{R}$ Inside temperature when vehicle $k \in K$ enters node $i \in N$

$\hat{t}_i^k \in \mathbb{R}$ Inside temperature when vehicle $k \in K$ leaves node $i \in N$

Temperature setpoint variables

In case of a dynamic setpoint per vehicle

$s_i^k \in \mathbb{R}$ Temperature setpoint when vehicle $k \in K$ enters node $i \in N$

$\hat{s}_i^k \in \mathbb{R}$ Temperature setpoint when vehicle $k \in K$ leaves node $i \in N$

*In case of a fixed setpoint per vehicle**

$s^k \in \mathbb{R}$ Temperature setpoint for vehicle $k \in K$

* (*The rest of the model is defined for the dynamic setpoint case, which can be translated to the fixed setpoint case by replacing each variable s_i^k and $\hat{s}_i^k$ by the variable s^k*)

Outside temperature difference tracking variables

${\delta^{\oplus}}_i^k \in \mathbb{R}^+$ Positive temperature difference with the outside temperature when vehicle $k \in K$ enters node $i \in N$

${\delta^{\ominus}}_i^k \in \mathbb{R}^+$ Negative temperature difference with the outside temperature when vehicle $k \in K$ enters node $i \in N$

${\delta^{\odot}}_i^k \in \mathbb{B}$ Outside temperature is higher than the inside temperature when vehicle $k \in K$ enters node $i \in N$

${\hat{\delta}^{\oplus}}{}_i^k \in \mathbb{R}^+$ Positive temperature difference with the outside temperature when vehicle $k \in K$ leaves node $i \in N$

${\hat{\delta}^{\ominus}}{}_i^k \in \mathbb{R}^+$ Negative temperature difference with the outside temperature when vehicle $k \in K$ leaves node $i \in N$

${\hat{\delta}^{\odot}}{}_i^k \in \mathbb{B}$ Outside temperature is higher than the inside temperature when vehicle $k \in K$ leaves node $i \in N$

Setpoint temperature difference tracking variables

$\gamma^{\oplus}{}_{i}^{k} \in \mathbb{R}^+$	Positive temperature difference with the setpoint temperature when vehicle $k \in K$ enters node $i \in N$
$\gamma^{\ominus}{}_{i}^{k} \in \mathbb{R}^+$	Negative temperature difference with the setpoint temperature when vehicle $k \in K$ enters node $i \in N$
$\gamma^{\odot}{}_{i}^{k} \in \mathbb{B}$	Setpoint temperature is higher than the inside temperature when vehicle $k \in K$ enters node $i \in N$
$\hat{\gamma}^{\oplus}{}_{i}^{k} \in \mathbb{R}^+$	Positive temperature difference with the setpoint temperature when vehicle $k \in K$ leaves node $i \in N$
$\hat{\gamma}^{\ominus}{}_{i}^{k} \in \mathbb{R}^+$	Negative temperature difference with the setpoint temperature when vehicle $k \in K$ leaves node $i \in N$
$\hat{\gamma}^{\odot}{}_{i}^{k} \in \mathbb{B}$	Setpoint temperature is higher than the inside temperature when vehicle $k \in K$ leaves node $i \in N$

Temperature change during service tracking variables

$\lambda^{\oplus}{}_{i}^{k} \in \mathbb{R}^+$	Part of temperature difference $\delta^{\oplus}{}_{i}^{k}$ that is not lost during service
$\left[\lambda^{\oplus}_{\odot}\right]_{i}^{k} \in \mathbb{B}$	There is some remaining coolness $\lambda^{\oplus}{}_{i}^{k}$ after service
$\lambda^{\ominus}{}_{i}^{k} \in \mathbb{R}^+$	Part of temperature difference $\delta^{\ominus}{}_{i}^{k}$ that is not lost during service
$\left[\lambda^{\ominus}_{\odot}\right]_{i}^{k} \in \mathbb{B}$	There is some remaining heat $\lambda^{\ominus}{}_{i}^{k}$ after service

Temperature change during traveling tracking variables

$\mu^{\oplus}{}_{i}^{k} \in \mathbb{R}^+$	Part of temperature difference $\gamma^{\oplus}{}_{i}^{k}$ that could not be cooled during traveling
$\left[\mu^{\oplus}_{\odot}\right]_{i}^{k} \in \mathbb{B}$	Not the full temperature difference $\gamma^{\oplus}{}_{i}^{k}$ could be cooled during traveling
$\mu^{\ominus}{}_{i}^{k} \in \mathbb{R}^+$	Part of temperature difference $\gamma^{\ominus}{}_{i}^{k}$ that could not be heated during traveling
$\left[\mu^{\ominus}_{\odot}\right]_{i}^{k} \in \mathbb{B}$	Not the full temperature difference $\gamma^{\ominus}{}_{i}^{k}$ could be cooled during service

Fuel estimation variables

$f_{i}^{k} \in \mathbb{R}^+$	Fuel usage by vehicle $k \in K$ at node $i \in N$
$g_{(i,j)}^{k} \in \mathbb{R}^+$	Fuel usage by vehicle $k \in K$ at the arc from $i \in N$ to $j \in N$

Additional node tracking variables

$\left[t_{\Delta}^{\oplus}\right]_{i}^{k} \in \mathbb{R}^{+}$	Inside temperature increase during service by vehicle $k \in K$ at node $i \in N$
$\left[t_{\Delta}^{\ominus}\right]_{i}^{k} \in \mathbb{R}^{+}$	Inside temperature decrease during service by vehicle $k \in K$ at node $i \in N$
$\left[t_{\Delta}^{\odot}\right]_{i}^{k} \in \mathbb{B}$	Inside temperature did increase during service by vehicle $k \in K$ at node $i \in N$
$\left[s_{\Delta}^{\oplus}\right]_{i}^{k} \in \mathbb{R}^{+}$	Setpoint temperature increase during service by vehicle $k \in K$ at node $i \in N$
$\left[s_{\Delta}^{\ominus}\right]_{i}^{k} \in \mathbb{R}^{+}$	Setpoint temperature decrease during service by vehicle $k \in K$ at node $i \in N$
$\left[s_{\Delta}^{\odot}\right]_{i}^{k} \in \mathbb{B}$	Setpoint temperature did increase during service by vehicle $k \in K$ at node $i \in N$
$\left[\delta_{\Delta}^{\oplus}\right]_{i}^{k} \in \mathbb{R}^{+}$	Outside temperature difference increase during service by vehicle $k \in K$ at node $i \in N$
$\left[\delta_{\Delta}^{\ominus}\right]_{i}^{k} \in \mathbb{R}^{+}$	Outside temperature difference decrease during service by vehicle $k \in K$ at node $i \in N$
$\left[\delta_{\Delta}^{\odot}\right]_{i}^{k} \in \mathbb{B}$	Outside temperature difference did increase during service by vehicle $k \in K$ at node $i \in N$
$\left[\gamma_{\Delta}^{\oplus}\right]_{i}^{k} \in \mathbb{R}^{+}$	Setpoint temperature difference increase during service by vehicle $k \in K$ at node $i \in N$
$\left[\gamma_{\Delta}^{\ominus}\right]_{i}^{k} \in \mathbb{R}^{+}$	Setpoint temperature difference decrease during service by vehicle $k \in K$ at node $i \in N$
$\left[\gamma_{\Delta}^{\odot}\right]_{i}^{k} \in \mathbb{B}$	Setpoint temperature difference did increase during service by vehicle $k \in K$ at node $i \in N$
$\left[\ell_{\Delta}^{\oplus}\right]_{i}^{k} \in \mathbb{R}^{+}$	Load increase during service by vehicle $k \in K$ at node $i \in N$
$\left[\ell_{\Delta}^{\ominus}\right]_{i}^{k} \in \mathbb{R}^{+}$	Load decrease during service by vehicle $k \in K$ at node $i \in N$
$\left[\ell_{\Delta}^{\odot}\right]_{i}^{k} \in \mathbb{B}$	Load did increase during service by vehicle $k \in K$ at node $i \in N$

Additional arc tracking variables

$\left[t_{\Delta}^{\oplus}\right]_{(i,j)}^{k} \in \mathbb{R}^{+}$ Inside temperature increase during travel by vehicle $k \in K$ from node $i \in N$ to $j \in N$

$\left[t_{\Delta}^{\ominus}\right]_{(i,j)}^{k} \in \mathbb{R}^{+}$ Inside temperature decrease during travel by vehicle $k \in K$ from node $i \in N$ to $j \in N$

$\left[t_{\Delta}^{\odot}\right]_{(i,j)}^{k} \in \mathbb{B}$ Inside temperature did increase during travel by vehicle $k \in K$ from node $i \in N$ to $j \in N$

$\left[W_{\Delta}^{\oplus}\right]_{(i,j)}^{k} \in \mathbb{R}^{+}$ Outside temperature increase during travel by vehicle $k \in K$ from node $i \in N$ to $j \in N$

$\left[W_{\Delta}^{\ominus}\right]_{(i,j)}^{k} \in \mathbb{R}^{+}$ Outside temperature decrease during travel by vehicle $k \in K$ from node $i \in N$ to $j \in N$

$\left[W_{\Delta}^{\odot}\right]_{(i,j)}^{k} \in \mathbb{B}$ Outside temperature did increase during travel by vehicle $k \in K$ from node $i \in N$ to $j \in N$

$\left[s_{\Delta}^{\oplus}\right]_{(i,j)}^{k} \in \mathbb{R}^{+}$ Setpoint temperature increase during travel by vehicle $k \in K$ from node $i \in N$ to $j \in N$

$\left[s_{\Delta}^{\ominus}\right]_{(i,j)}^{k} \in \mathbb{R}^{+}$ Setpoint temperature decrease during travel by vehicle $k \in K$ from node $i \in N$ to $j \in N$

$\left[s_{\Delta}^{\odot}\right]_{(i,j)}^{k} \in \mathbb{B}$ Setpoint temperature did increase during travel by vehicle $k \in K$ from node $i \in N$ to $j \in N$

$\left[\delta_{\Delta}^{\oplus}\right]_{(i,j)}^{k} \in \mathbb{R}^{+}$ Outside temperature difference increase during travel by vehicle $k \in K$ from node $i \in N$ to $j \in N$

$\left[\delta_{\Delta}^{\ominus}\right]_{(i,j)}^{k} \in \mathbb{R}^{+}$ Outside temperature difference decrease during travel by vehicle $k \in K$ from node $i \in N$ to $j \in N$

$\left[\delta_{\Delta}^{\odot}\right]_{(i,j)}^{k} \in \mathbb{B}$ Outside temperature difference did increase during travel by vehicle $k \in K$ from node $i \in N$ to $j \in N$

$\left[\gamma_{\Delta}^{\oplus}\right]_{(i,j)}^{k} \in \mathbb{R}^{+}$ Setpoint temperature difference increase during travel by vehicle $k \in K$ from node $i \in N$ to $j \in N$

$\left[\gamma_{\Delta}^{\ominus}\right]_{(i,j)}^{k} \in \mathbb{R}^{+}$ Setpoint temperature difference decrease during travel by vehicle $k \in K$ from node $i \in N$ to $j \in N$

$\left[\gamma_{\Delta}^{\odot}\right]_{(i,j)}^{k} \in \mathbb{B}$ Setpoint temperature difference did increase during travel by vehicle $k \in K$ from node $i \in N$ to $j \in N$

3.4.3 Objective function

Two different objective functions will be considered for this model, namely:

Minimize the travel distance of each vehicle

$$\min \sum_{k \in K} \left(y^k_{(i,j)} \cdot D_{(i,j)} \right) \tag{1a}$$

Minimize the fuel usage of each vehicle at each node and at each arc

$$\min \sum_{k \in K} \sum_{i \in N} \left(f^k_i + \sum_{j \in N} g^k_{(i,j)} \right) \tag{1b}$$

3.4.4 Constraints

The following constraints are used to describe all conditions that each solution should fulfill as well as to define the values of all decision variables.

Routing constraints

Each vehicle should begin with visiting its starting node

$$y^k_{(o,n^k_0)} = 1 \qquad \forall k \in K \tag{2}$$

Each vehicle should leave the dummy depot exactly once

$$\sum_{j \in N} y^k_{(o,j)} = 1 \qquad \forall k \in K \tag{3}$$

Each node must be entered by any vehicle exactly once

$$\sum_{k \in K} \sum_{i \in V} y^k_{(i,j)} = 1 \qquad \forall j \in N \tag{4}$$

Each vehicle that enters a node should leave the node afterwards

$$\sum_{i \in V} y^k_{(i,j)} = \sum_{i \in V} y^k_{(j,i)} \qquad \forall k \in K, \forall j \in V \tag{5}$$

Each node is visited by a vehicle if any of the incoming arcs is used

$$x^k_j \geq y^k_{(i,j)} \qquad \forall k \in K, \forall i \in V, \forall j \in N \tag{6}$$

Time constraints

Track the time spent at each node

$$\hat{\tau}^k_i \geq \tau^k_i + S_i - \left(1 - x^k_i\right) \cdot M_\tau \qquad \forall k \in K, \forall i \in N \tag{7}$$

Track the time spent at each arc

$$t^k_j \geq \hat{t}^k_i + Z_{(i,j)} - \left(1 - y^k_{(i,j)}\right) \cdot M_\tau \qquad \forall k \in K, \forall i \in N, \forall j \in N \tag{8}$$

Force that no request starts before its time window opens

$$\tau^k_i \geq \Theta^-_i - \left(1 - x^k_i\right) \cdot M_\tau \qquad \forall k \in K, \forall i \in N \tag{9}$$

Force that no request starts after its time window closes

$$\tau^k_i \leq \Theta^+_i + \left(1 - x^k_i\right) \cdot M_\tau \qquad \forall k \in K, \forall i \in N \tag{10}$$

Load constraints

Initialize the starting load to zero

$$\ell_j^{k,p} \leq \left(1 - y_{(0,j)}^k\right) \cdot M_\ell \qquad \forall k \in K, \forall p \in P, \forall j \in N \tag{11}$$

Track the load change at each node

$$\begin{aligned}\hat{\ell}_i^{k,p} \geq \ell_i^{k,p} + \Delta^{+p}_i - \Delta^{-p}_i \\ - \left(1 - x_i^k\right) \cdot M_\ell\end{aligned} \qquad \forall k \in K, \forall p \in P, \forall i \in N \tag{12}$$

$$\begin{aligned}\hat{\ell}_i^{k,p} \leq \ell_i^{k,p} + \Delta^{+p}_i - \Delta^{-p}_i \\ + \left(1 - x_i^k\right) \cdot M_\ell\end{aligned} \qquad \forall k \in K, \forall p \in P, \forall i \in N \tag{13}$$

Track the load change at each arc

$$\begin{aligned}\ell_j^{k,p} \geq \hat{\ell}_i^{k,p} \\ - \left(1 - y_{(i,j)}^k\right) \cdot M_\ell\end{aligned} \qquad \forall k \in K, \forall p \in P, \forall i \in N, \forall j \in N \tag{14}$$

$$\begin{aligned}\ell_j^{k,p} \leq \hat{\ell}_i^{k,p} \\ + \left(1 - y_{(i,j)}^k\right) \cdot M_\ell\end{aligned} \qquad \forall k \in K, \forall p \in P, \forall i \in N, \forall j \in N \tag{15}$$

Track the loaded products at each node

$$\ell_i^{k,p} \leq z_i^{k,p} \cdot M_\ell \qquad \forall k \in K, \forall p \in P, \forall i \in N \tag{16}$$

Track the loaded products at each arc

$$\hat{\ell}_i^{k,p} \leq \hat{z}_i^{k,p} \cdot M_\ell \qquad \forall k \in K, \forall p \in P, \forall i \in N \tag{17}$$

Force that no vehicle can have more load than its capacity

$$\sum_{p \in P} \hat{\ell}_i^{k,p} \leq x_i^k \cdot Q^k \qquad \forall k \in K, \forall i \in N \tag{18}$$

Force that the delivery at each node can be served with the loaded products

$$\begin{aligned}\ell_i^{k,p} \geq \Delta^{-p}_i \\ - x_i^k \cdot M_\ell\end{aligned} \qquad \forall k \in K, \forall p \in P, \forall i \in N \tag{19}$$

Force that the setpoint always respects all loaded product's requirements when entering a node

$$\begin{aligned}s_i^k \geq T_p^- \\ - M_t \cdot (1 - \ell_i^{k,p})\end{aligned} \qquad \forall k \in K, \forall p \in P, \forall i \in N \tag{20}$$

$$\begin{aligned}s_i^k \leq T_p^+ \\ + M_t \cdot (1 - \ell_i^{k,p})\end{aligned} \qquad \forall k \in K, \forall p \in P, \forall i \in N \tag{21}$$

Force that the setpoint always respects all loaded product's requirements when leaving a node

$$\begin{aligned}\hat{s}_i^k \geq T_p^- \\ - M_t \cdot (1 - \ell_i^{k,p})\end{aligned} \qquad \forall k \in K, \forall p \in P, \forall i \in N \tag{22}$$

$$\begin{aligned}\hat{s}_i^k \leq T_p^+ \\ + M_t \cdot (1 - \ell_i^{k,p})\end{aligned} \qquad \forall k \in K, \forall p \in P, \forall i \in N \tag{23}$$

Temperature initialization constraints

Initialize the inside temperature equal to the outside temperature at the starting node

$$t_j^k \geq W_j - \left(1 - y_{(0,j)}^k\right) \cdot M_W \qquad \forall k \in K, \forall j \in N \tag{24}$$

$$t_j^k \leq W_j + \left(1 - y_{(0,j)}^k\right) \cdot M_W \qquad \forall k \in K, \forall j \in N \tag{25}$$

Outside temperature difference tracking constraints

Calculate the difference between the inside and outside temperature when entering each node

$$\delta^{\oplus}{}_i^k - \delta^{\ominus}{}_i^k = W_i - t_i^k \qquad \forall k \in K, \forall i \in N \tag{26}$$

Calculate if the outside temperature is higher than the inside temperature when entering each node

$$\delta^{\oplus}{}_i^k \leq \delta^{\odot}{}_i^k \cdot M_t \qquad \forall k \in K, \forall i \in N \tag{27}$$

$$\delta^{\ominus}{}_i^k \geq (1 - \delta^{\odot}{}_i^k) \cdot M_t \qquad \forall k \in K, \forall i \in N \tag{28}$$

Calculate the difference between the inside and outside temperature when leaving each node

$$\hat{\delta}^{\oplus}{}_i^k - \hat{\delta}^{\ominus}{}_i^k = W_i - \hat{t}_i^k \qquad \forall k \in K, \forall i \in N \tag{29}$$

Calculate if the outside temperature is higher than the inside temperature when leaving each node

$$\hat{\delta}^{\oplus}{}_i^k \leq \hat{\delta}^{\odot}{}_i^k \cdot M_t \qquad \forall k \in K, \forall i \in N \tag{30}$$

$$\hat{\delta}^{\ominus}{}_i^k \geq (1 - \hat{\delta}^{\odot}{}_i^k) \cdot M_t \qquad \forall k \in K, \forall i \in N \tag{31}$$

Setpoint temperature difference tracking constraints

Calculate the difference between the inside and target temperature when entering each node

$$\gamma^{\oplus}{}_i^k - \gamma^{\ominus}{}_i^k = W_i - t_i^k \qquad \forall k \in K, \forall i \in N \tag{32}$$

Calculate if the setpoint temperature is higher than the inside temperature when entering each node

$$\gamma^{\oplus}{}_i^k \leq \gamma^{\odot}{}_i^k \cdot M_t \qquad \forall k \in K, \forall i \in N \tag{33}$$

$$\gamma^{\ominus}{}_i^k \geq (1 - \gamma^{\odot}{}_i^k) \cdot M_t \qquad \forall k \in K, \forall i \in N \tag{34}$$

Calculate the difference between the inside and target temperature when leaving each node

$$\hat{\gamma}^{\oplus}{}_i^k - \hat{\gamma}^{\ominus}{}_i^k = W_i - \hat{t}_i^k \qquad \forall k \in K, \forall i \in N \tag{35}$$

Calculate if the setpoint temperature is higher than the inside temperature when leaving each node

$$\hat{\gamma}^{\oplus}{}_i^k \leq \hat{\gamma}^{\odot}{}_i^k \cdot M_t \qquad \forall k \in K, \forall i \in N \tag{36}$$

$$\hat{\gamma}^{\ominus}{}_i^k \geq (1 - \hat{\gamma}^{\odot}{}_i^k) \cdot M_t \qquad \forall k \in K, \forall i \in N \tag{37}$$

Temperature change during service tracking constraints

Calculate which part of the coolness is left after service

$$\begin{aligned}\lambda^{\oplus}{}^{k}_{i} \geq \delta^{\oplus}{}^{k}_{i} - L^{+} \cdot \left(\hat{\tau}^{k}_{i} - \tau^{k}_{i}\right) \\ - \left(1 - x^{k}_{i}\right) \cdot M_{L^{+}}\end{aligned} \qquad \forall k \in K, \forall i \in N \tag{38}$$

Calculate if there is some coolness left after service

$$\begin{aligned}\lambda^{\oplus}{}^{k}_{i} \leq \delta^{\oplus}{}^{k}_{i} - L^{+} \cdot \left(\hat{\tau}^{k}_{i} - \tau^{k}_{i}\right) \\ + \left[\lambda^{\oplus}_{\odot}\right]^{k}_{i} \cdot M_{L^{+}} \\ + \left(1 - x^{k}_{i}\right) \cdot M_{L^{+}}\end{aligned} \qquad \forall k \in K, \forall i \in N \tag{39}$$

$$\begin{aligned}\lambda^{\oplus}{}^{k}_{i} \leq \left(1 - \left[\lambda^{\oplus}_{\odot}\right]^{k}_{i}\right) \cdot M_{L^{+}} \\ + \left(1 - x^{k}_{i}\right) \cdot M_{L^{+}}\end{aligned} \qquad \forall k \in K, \forall i \in N \tag{40}$$

Calculate which part of the heat is left after service

$$\begin{aligned}\lambda^{\ominus}{}^{k}_{i} \geq \delta^{\ominus}{}^{k}_{i} - L^{-} \cdot \left(\hat{\tau}^{k}_{i} - \tau^{k}_{i}\right) \\ - \left(1 - x^{k}_{i}\right) \cdot M_{L^{-}}\end{aligned} \qquad \forall k \in K, \forall i \in N \tag{41}$$

Calculate if there is some heat left after service

$$\begin{aligned}\lambda^{\ominus}{}^{k}_{i} \leq \delta^{\ominus}{}^{k}_{i} - L^{-} \cdot \left(\hat{\tau}^{k}_{i} - \tau^{k}_{i}\right) \\ + \left[\lambda^{\ominus}_{\odot}\right]^{k}_{i} \cdot M_{L^{-}} \\ + \left(1 - x^{k}_{i}\right) \cdot M_{L^{-}}\end{aligned} \qquad \forall k \in K, \forall i \in N \tag{42}$$

$$\begin{aligned}\lambda^{\ominus}{}^{k}_{i} \leq \left(1 - \left[\lambda^{\ominus}_{\odot}\right]^{k}_{i}\right) \cdot M_{L^{-}} \\ + \left(1 - x^{k}_{i}\right) \cdot M_{L^{-}}\end{aligned} \qquad \forall k \in K, \forall i \in N \tag{43}$$

Temperature change during traveling tracking constraints

Calculate how much could not be cooled during traveling

$$\mu^{\oplus k}_{i} \geq \gamma^{\ominus k}_{i} - C^{-} \cdot \left(\tau^{k}_{j} - \hat{\tau}^{k}_{i}\right) - \left(1 - y^{k}_{(i,j)}\right) \cdot M_{C^{-}} \qquad \forall k \in K, \forall i \in N, \forall j \in N \tag{44}$$

Calculate if the temperature could not be fully cooled during traveling

$$\mu^{\oplus k}_{i} \leq \gamma^{\ominus k}_{i} - C^{-} \cdot \left(\tau^{k}_{j} - \hat{\tau}^{k}_{i}\right) + \left[\mu^{\oplus}_{\odot}\right]^{k}_{i} \cdot M_{C^{-}} + \left(1 - y^{k}_{(i,j)}\right) \cdot M_{C^{-}} \qquad \forall k \in K, \forall i \in N, \forall j \in N \tag{45}$$

$$\mu^{\oplus k}_{i} \leq \left(1 - \left[\mu^{\oplus}_{\odot}\right]^{k}_{i}\right) \cdot M_{C^{-}} + \left(1 - y^{k}_{(i,j)}\right) \cdot M_{C^{-}} \qquad \forall k \in K, \forall i \in N, \forall j \in N \tag{46}$$

Calculate how much could not be heated during traveling

$$\mu^{\ominus k}_{i} \geq \gamma^{\oplus k}_{i} - C^{+} \cdot \left(\tau^{k}_{j} - \hat{\tau}^{k}_{i}\right) - \left(1 - y^{k}_{(i,j)}\right) \cdot M_{C^{+}} \qquad \forall k \in K, \forall i \in N, \forall j \in N \tag{47}$$

Calculate if the temperature could not be fully heated during traveling

$$\mu^{\ominus k}_{i} \leq \gamma^{\oplus k}_{i} - C^{+} \cdot \left(\tau^{k}_{j} - \hat{\tau}^{k}_{i}\right) + \left[\mu^{\ominus}_{\odot}\right]^{k}_{i} \cdot M_{C^{+}} + \left(1 - y^{k}_{(i,j)}\right) \cdot M_{C^{+}} \qquad \forall k \in K, \forall i \in N, \forall j \in N \tag{48}$$

$$\mu^{\ominus k}_{i} \leq \left(1 - \left[\mu^{\ominus}_{\odot}\right]^{k}_{i}\right) \cdot M_{C^{+}} + \left(1 - y^{k}_{(i,j)}\right) \cdot M_{C^{+}} \qquad \forall k \in K, \forall i \in N, \forall j \in N \tag{49}$$

Additional node tracking constraints

Temperature change during service

$$\left[t_{\Delta}^{\oplus}\right]_i^k - \left[t_{\Delta}^{\ominus}\right]_i^k = \hat{t}_i^k - t_i^k \qquad \forall k \in K, \forall i \in N \tag{50}$$

Temperature did increase during service

$$\left[t_{\Delta}^{\oplus}\right]_i^k \leq \left[t_{\Delta}^{\odot}\right]_i^k \cdot M_t \qquad \forall k \in K, \forall i \in N \tag{51}$$

$$\left[t_{\Delta}^{\ominus}\right]_i^k \leq \left(1 - \left[t_{\Delta}^{\odot}\right]_i^k\right) \cdot M_t \qquad \forall k \in K, \forall i \in N \tag{52}$$

Setpoint temperature change after service

$$\left[s_{\Delta}^{\oplus}\right]_i^k - \left[s_{\Delta}^{\ominus}\right]_i^k = \hat{s}_i^k - s_i^k \qquad \forall k \in K, \forall i \in N \tag{53}$$

Setpoint temperature did increase after service

$$\left[s_{\Delta}^{\oplus}\right]_i^k \leq \left[s_{\Delta}^{\odot}\right]_i^k \cdot M_t \qquad \forall k \in K, \forall i \in N \tag{54}$$

$$\left[s_{\Delta}^{\ominus}\right]_i^k \leq \left(1 - \left[s_{\Delta}^{\odot}\right]_i^k\right) \cdot M_t \qquad \forall k \in K, \forall i \in N \tag{55}$$

Outside temperature difference change during service

$$\left[\delta_{\Delta}^{\oplus}\right]_i^k - \left[\delta_{\Delta}^{\ominus}\right]_i^k = \hat{\delta}_i^k - \delta_i^k \qquad \forall k \in K, \forall i \in N \tag{56}$$

Outside temperature difference did increase during service

$$\left[\delta_{\Delta}^{\oplus}\right]_i^k \leq \left[\delta_{\Delta}^{\odot}\right]_i^k \cdot M_t \qquad \forall k \in K, \forall i \in N \tag{57}$$

$$\left[\delta_{\Delta}^{\ominus}\right]_i^k \leq \left(1 - \left[\delta_{\Delta}^{\odot}\right]_i^k\right) \cdot M_t \qquad \forall k \in K, \forall i \in N \tag{58}$$

Setpoint temperature difference change during service

$$\left[\gamma_{\Delta}^{\oplus}\right]_i^k - \left[\gamma_{\Delta}^{\ominus}\right]_i^k = \hat{\gamma}_i^k - \gamma_i^k \qquad \forall k \in K, \forall i \in N \tag{59}$$

Setpoint temperature difference did increase during service

$$\left[s_{\Delta}^{\oplus}\right]_i^k \leq \left[\gamma_{\Delta}^{\odot}\right]_i^k \cdot M_t \qquad \forall k \in K, \forall i \in N \tag{60}$$

$$\left[\gamma_{\Delta}^{\ominus}\right]_i^k \leq \left(1 - \left[\gamma_{\Delta}^{\odot}\right]_i^k\right) \cdot M_t \qquad \forall k \in K, \forall i \in N \tag{61}$$

Load change after service

$$\left[\ell_{\Delta}^{\oplus}\right]_i^k - \left[\ell_{\Delta}^{\ominus}\right]_i^k = \hat{\ell}_i^k - \ell_i^k \qquad \forall k \in K, \forall i \in N \tag{62}$$

Load did increase after service

$$\left[\ell_{\Delta}^{\oplus}\right]_i^k \leq \left[\ell_{\Delta}^{\odot}\right]_i^k \cdot M_t \qquad \forall k \in K, \forall i \in N \tag{63}$$

$$\left[\ell_{\Delta}^{\ominus}\right]_i^k \leq \left(1 - \left[\ell_{\Delta}^{\odot}\right]_i^k\right) \cdot M_t \qquad \forall k \in K, \forall i \in N \tag{64}$$

Additional arc tracking constraints

Temperature change during travel

$$\left[t_{\Delta}^{\oplus}\right]_{(i,j)}^{k} - \left[t_{\Delta}^{\ominus}\right]_{(i,j)}^{k} = t_j^k - \hat{t}_i^k \qquad \forall k \in K, \forall i \in N, \forall j \in N \tag{65}$$

Temperature did increase during travel

$$\left[t_{\Delta}^{\oplus}\right]_{(i,j)}^{k} \leq \left[t_{\Delta}^{\odot}\right]_{(i,j)}^{k} \cdot M_t \qquad \forall k \in K, \forall i \in N, \forall j \in N \tag{66}$$

$$\left[t_{\Delta}^{\ominus}\right]_{(i,j)}^{k} \leq \left(1 - \left[t_{\Delta}^{\odot}\right]_{(i,j)}^{k}\right) \cdot M_t \qquad \forall k \in K, \forall i \in N, \forall j \in N \tag{67}$$

Outside temperature change during travel

$$\left[W_{\Delta}^{\oplus}\right]_{(i,j)}^{k} - \left[W_{\Delta}^{\ominus}\right]_{(i,j)}^{k} = W_j - W_i \qquad \forall k \in K, \forall i \in N, \forall j \in N \tag{68}$$

Outside temperature did increase during travel

$$\left[W_{\Delta}^{\oplus}\right]_{(i,j)}^{k} \leq \left[W_{\Delta}^{\odot}\right]_{(i,j)}^{k} \cdot M_t \qquad \forall k \in K, \forall i \in N, \forall j \in N \tag{69}$$

$$\left[W_{\Delta}^{\ominus}\right]_{(i,j)}^{k} \leq \left(1 - \left[W_{\Delta}^{\odot}\right]_{(i,j)}^{k}\right) \cdot M_t \qquad \forall k \in K, \forall i \in N, \forall j \in N \tag{70}$$

Setpoint temperature change after travel

$$\left[s_{\Delta}^{\oplus}\right]_{(i,j)}^{k} - \left[s_{\Delta}^{\ominus}\right]_{(i,j)}^{k} = s_j^k - \hat{s}_i^k \qquad \forall k \in K, \forall i \in N, \forall j \in N \tag{71}$$

Setpoint temperature did increase after travel

$$\left[s_{\Delta}^{\oplus}\right]_{(i,j)}^{k} \leq \left[s_{\Delta}^{\odot}\right]_{(i,j)}^{k} \cdot M_t \qquad \forall k \in K, \forall i \in N, \forall j \in N \tag{72}$$

$$\left[s_{\Delta}^{\ominus}\right]_{(i,j)}^{k} \leq \left(1 - \left[s_{\Delta}^{\odot}\right]_{(i,j)}^{k}\right) \cdot M_t \qquad \forall k \in K, \forall i \in N, \forall j \in N \tag{73}$$

Outside temperature difference change during travel

$$\left[\delta_{\Delta}^{\oplus}\right]_{(i,j)}^{k} - \left[\delta_{\Delta}^{\ominus}\right]_{(i,j)}^{k} = \delta_j^k - \hat{\delta}_i^k \qquad \forall k \in K, \forall i \in N, \forall j \in N \tag{74}$$

Outside temperature difference did increase during travel

$$\left[\delta_{\Delta}^{\oplus}\right]_{(i,j)}^{k} \leq \left[\delta_{\Delta}^{\odot}\right]_{(i,j)}^{k} \cdot M_t \qquad \forall k \in K, \forall i \in N, \forall j \in N \tag{75}$$

$$\left[\delta_{\Delta}^{\ominus}\right]_{(i,j)}^{k} \leq \left(1 - \left[\delta_{\Delta}^{\odot}\right]_{(i,j)}^{k}\right) \cdot M_t \qquad \forall k \in K, \forall i \in N, \forall j \in N \tag{76}$$

Setpoint temperature difference change during travel

$$\left[\gamma_{\Delta}^{\oplus}\right]_{(i,j)}^{k} - \left[\gamma_{\Delta}^{\ominus}\right]_{(i,j)}^{k} = \gamma_j^k - \hat{\gamma}_i^k \qquad \forall k \in K, \forall i \in N, \forall j \in N \tag{77}$$

Setpoint temperature difference did increase during travel

$$\left[\gamma_{\Delta}^{\oplus}\right]_{(i,j)}^{k} \leq \left[\gamma_{\Delta}^{\odot}\right]_{(i,j)}^{k} \cdot M_t \qquad \forall k \in K, \forall i \in N, \forall j \in N \tag{78}$$

$$\left[\gamma_{\Delta}^{\ominus}\right]_{(i,j)}^{k} \leq \left(1 - \left[\gamma_{\Delta}^{\odot}\right]_{(i,j)}^{k}\right) \cdot M_t \qquad \forall k \in K, \forall i \in N, \forall j \in N \tag{79}$$

Fuel estimation constraints

Linear function for estimating fuel usage during service

$$
f_i^k \geq \vec{\sigma}_{\text{service}} \cdot \left[\begin{array}{c} \\ \frac{1}{2}\left(t_i^k + \hat{t}_i^k\right) \\ \left[t_\Delta^\oplus\right]_i^k - \left[t_\Delta^\ominus\right]_i^k \\ \left[t_\Delta^\oplus\right]_i^k \\ \left[t_\Delta^\ominus\right]_i^k \\ \\ W_i \\ 0 \\ 0 \\ 0 \\ \\ \frac{1}{2}\left(s_i^k + \hat{s}_i^k\right) \\ \left[s_\Delta^\oplus\right]_i^k - \left[s_\Delta^\ominus\right]_i^k \\ \left[s_\Delta^\oplus\right]_i^k \\ \left[s_\Delta^\ominus\right]_i^k \\ \\ \frac{1}{2}\left(\delta_i^k + \hat{\delta}_i^k\right) \\ \left[\delta_\Delta^\oplus\right]_i^k - \left[\delta_\Delta^\ominus\right]_i^k \\ \left[\delta_\Delta^\oplus\right]_i^k \\ \left[\delta_\Delta^\ominus\right]_i^k \\ \\ \frac{1}{2}\left(\gamma_i^k + \hat{\gamma}_i^k\right) \\ \left[\gamma_\Delta^\oplus\right]_i^k - \left[\gamma_\Delta^\ominus\right]_i^k \\ \left[\gamma_\Delta^\oplus\right]_i^k \\ \left[\gamma_\Delta^\ominus\right]_i^k \\ \\ \frac{1}{2}\sum_{p \in P}\left(\ell_i^{k,p} + \hat{\ell}_i^{k,p}\right) \\ \left[\ell_\Delta^\oplus\right]_i^k - \left[\ell_\Delta^\ominus\right]_i^k \\ \left[\ell_\Delta^\oplus\right]_i^k \\ \left[\ell_\Delta^\ominus\right]_i^k \\ \\ \hat{\tau}_i^k - \tau_i^k \\ \end{array}\right] \begin{array}{l} \textbf{Inside temperature} \\ \quad \textit{Average} \\ \quad \textit{Difference} \\ \quad \textit{Increase} \\ \quad \textit{Decrease} \\ \textbf{Outside temperature} \\ \quad \textit{Average} \\ \quad \textit{Difference} \\ \quad \textit{Increase} \\ \quad \textit{Decrease} \\ \textbf{Target temperature} \\ \quad \textit{Average} \\ \quad \textit{Difference} \\ \quad \textit{Increase} \\ \quad \textit{Decrease} \\ \textbf{Outside temperature } \Delta \\ \quad \textit{Average} \\ \quad \textit{Difference} \\ \quad \textit{Increase} \\ \quad \textit{Decrease} \\ \textbf{Target temperature } \Delta \\ \quad \textit{Average} \\ \quad \textit{Difference} \\ \quad \textit{Increase} \\ \quad \textit{Decrease} \\ \textbf{Total load} \\ \quad \textit{Average} \\ \quad \textit{Difference} \\ \quad \textit{Increase} \\ \quad \textit{Decrease} \\ \textbf{Duration} \\ \quad \textit{Total} \\ \end{array} - \left(1 - x_i^k\right) \cdot M_f \quad \forall k \in K, \forall i \in N \tag{80}
$$

Linear function for estimating fuel usage during travel

$$g^k_{(i,j)} \geq \vec{\sigma}_{\text{travel}} \cdot \left[\begin{array}{c|l}
 & \textbf{Route} \\
D_{(i,j)} & \textit{Distance} \\
Z_{(i,j)} & \textit{Duration} \\
 & \textbf{Inside temperature} \\
\frac{1}{2}\left(\hat{t}^k_i + t^k_j\right) & \textit{Average} \\
\left[t^{\oplus}_{\Delta}\right]^k_{(i,j)} - \left[t^{\ominus}_{\Delta}\right]^k_{(i,j)} & \textit{Difference} \\
\left[t^{\oplus}_{\Delta}\right]^k_{(i,j)} & \textit{Increase} \\
\left[t^{\ominus}_{\Delta}\right]^k_{(i,j)} & \textit{Decrease} \\
 & \textbf{Outside temperature} \\
\frac{1}{2}\left(W_i + W_j\right) & \textit{Average} \\
\left[W^{\oplus}_{\Delta}\right]^k_{(i,j)} - \left[W^{\ominus}_{\Delta}\right]^k_{(i,j)} & \textit{Difference} \\
\left[W^{\oplus}_{\Delta}\right]^k_{(i,j)} & \textit{Increase} \\
\left[W^{\ominus}_{\Delta}\right]^k_{(i,j)} & \textit{Decrease} \\
 & \textbf{Target temperature} \\
\frac{1}{2}\left(\hat{s}^k_i + s^k_j\right) & \textit{Average} \\
\left[s^{\oplus}_{\Delta}\right]^k_{(i,j)} - \left[s^{\ominus}_{\Delta}\right]^k_{(i,j)} & \textit{Difference} \\
\left[s^{\oplus}_{\Delta}\right]^k_{(i,j)} & \textit{Increase} \\
\left[s^{\ominus}_{\Delta}\right]^k_{(i,j)} & \textit{Decrease} \\
 & \textbf{Outside temperature } \Delta \\
\frac{1}{2}\left(\hat{\delta}^k_i + \delta^k_j\right) & \textit{Average} \\
\left[\delta^{\oplus}_{\Delta}\right]^k_{(i,j)} - \left[\delta^{\ominus}_{\Delta}\right]^k_{(i,j)} & \textit{Difference} \\
\left[\delta^{\oplus}_{\Delta}\right]^k_{(i,j)} & \textit{Increase} \\
\left[\delta^{\ominus}_{\Delta}\right]^k_{(i,j)} & \textit{Decrease} \\
 & \textbf{Target temperature } \Delta \\
\frac{1}{2}\left(\hat{\gamma}^k_i + \gamma^k_j\right) & \textit{Average} \\
\left[\gamma^{\oplus}_{\Delta}\right]^k_{(i,j)} - \left[\gamma^{\ominus}_{\Delta}\right]^k_{(i,j)} & \textit{Difference} \\
\left[\gamma^{\oplus}_{\Delta}\right]^k_{(i,j)} & \textit{Increase} \\
\left[\gamma^{\ominus}_{\Delta}\right]^k_{(i,j)} & \textit{Decrease} \\
 & \textbf{Duration} \\
\tau^k_j - \hat{\tau}^k_i & \textit{Total}
\end{array}\right] - \left(1 - y^k_{(i,j)}\right) \cdot M_g \qquad \forall k \in K, \forall i \in N, \forall j \in N \tag{81}$$

4 Results

4.1 Temperature change rates

The temperature fluctuation over time has been analyzed by looking at the service and travel segments obtained from the historical data. Figure 11 shows for both kind of segments the distribution of the temperature change rates for two different cases. Although the values show correlation with certain weather condition features, a fixed rate per case should be taken to comply with the model assumptions and perserve linearity in the MILP. That is why the average temperature change rates per second have been determined, resulting in the following values:

$$L^+ = 0.00207\ ^\circ C/\text{second} \qquad C^+ = 0.00984\ ^\circ C/\text{second}$$
$$L^- = 0.00206\ ^\circ C/\text{second} \qquad C^- = 0.00975\ ^\circ C/\text{second}$$

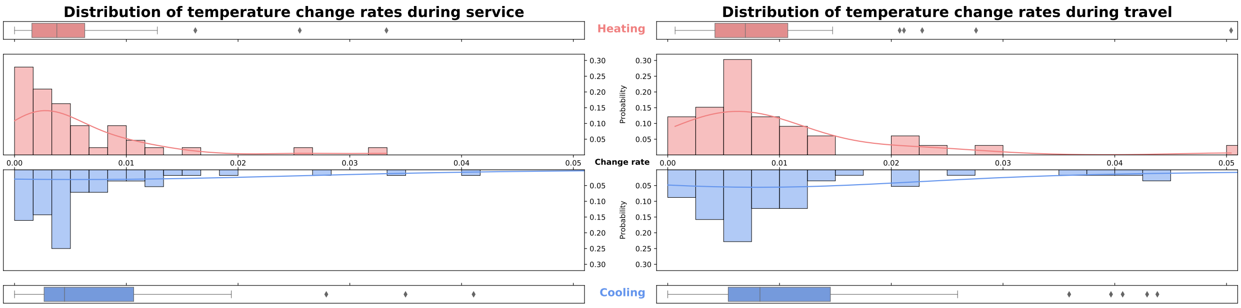


Figure 11: Distribution of the temperature change rates per case over all training records

4.2 Fuel usage

The same historical data segments have been used to train the fuel estimation functions. Figure 12 shows the distribution of the fuel usage during a single service and a single travel segment in the training dataset. The consumed fuel during service shows a high correlation with the outside temperature and the amount of (un)loaded products, which is also an indirect measure for the time the door has to be opened. During travel segments, the fuel consumption is mainly correlated with the route distances as well as the size of the gap between the starting and the target temperature. The exact weights for each of the features have been learned using ElasticNet regression on the training data and are validated on a different set of historical segments. All resulting (nonzero) weights that led to the best estimations, can be found in Table 2.

Weights for fuel estimation during service

Feature		Weight
Inside temperature	Average	-0.05240
	Difference	0.41678
	Increase	0.73342
Outside temperature	Difference	1.05491
	Decrease	0.13503
Target temperature	Difference	0.67446
	Increase	0.06926
	Decrease	-0.32350
Outside temperature Δ	Decrease	0.17867
Target temperature Δ	Average	0.03402
	Decrease	-0.30712
Total load	Average	0.22448
	Increase	0.52944
	Decrease	-0.07351
Duration	Total	-0.00001

Weights for fuel estimation during travel

Feature		Weight
Route	Distance	0.00026
	Duration	-0.00190
Inside temperature	Increase	0.11279
	Decrease	-0.08096
Target temperature Δ	Increase	-0.23766
Duration	Total	0.00006

Table 2: Fuel estimation weights

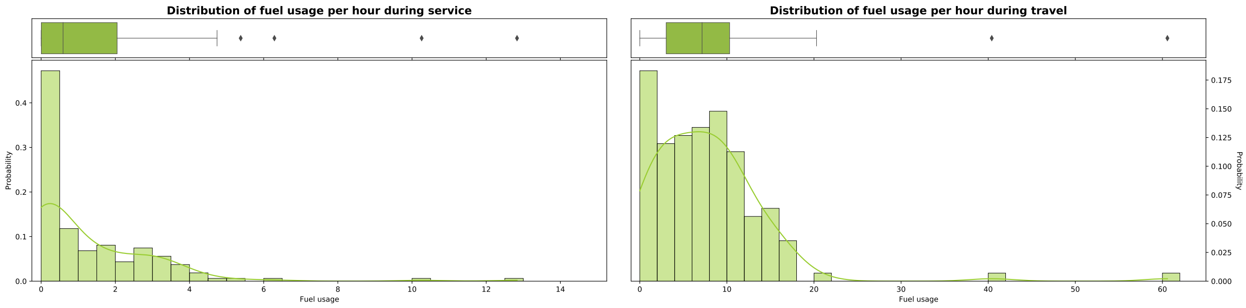


Figure 12: Distribution of fuel usage over all training records

4.2.1 Fuel usage during service

The best results for the fuel usage estimation during service have been obtained using regularization parameters $\alpha_1 = \alpha_2 = 1.5$. Figure 13 shows the distribution of the errors for the training segments as well as a comparison between all measured and predicted values. The mean absolute error was 7.81L and is depicted in red. The remaining errors were mainly correlated with the occurrence of refrigerator error states that are not regarded in the current MILP.

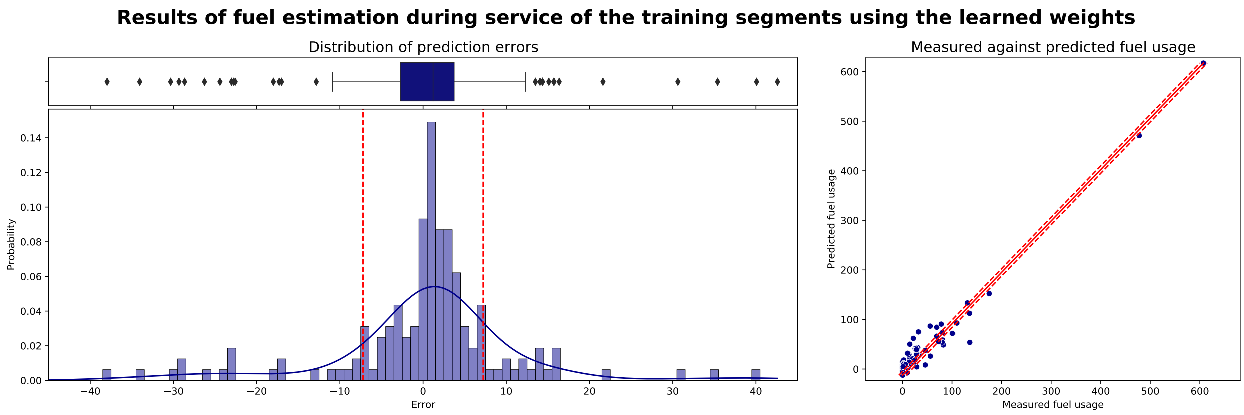


Figure 13: Prediction results for fuel usage during service on the training segments

For the segments in the test set, the mean absolute error was with 8.67L slightly higher. Figure 14 shows that the biggest errors are made in the segments where relatively high amounts of fuel were used. For those segments, the predicted values are usually still higher than average, but the exact values can deviate somewhat from the truth.

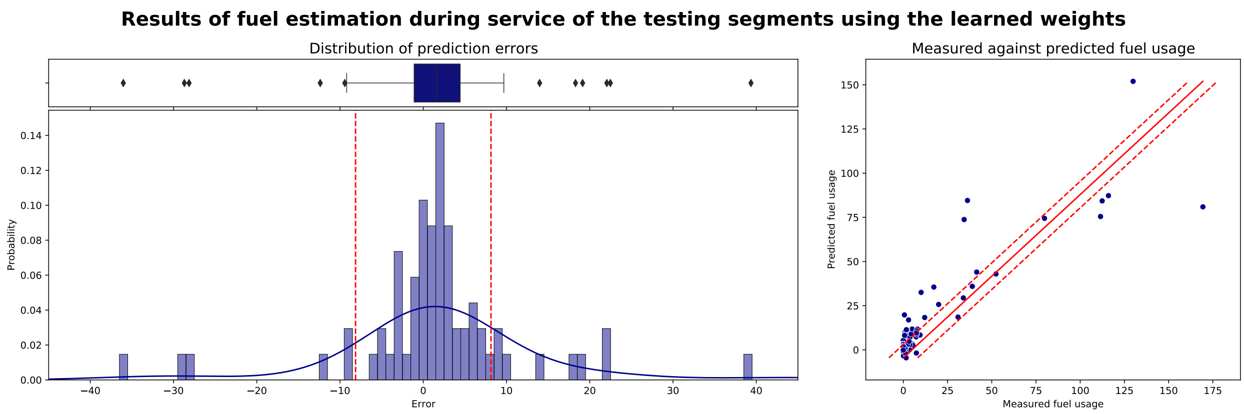


Figure 14: Distribution of fuel usage over all training records

4.2.2 Fuel usage during travel

For the travel segments, the best performing regularization parameters are $\alpha_1 = \alpha_2 = 20$. The distribution of the errors within the training set can be found in Figure 15, where the mean absolute error is equal to 7.74L. The remaining errors were mainly correlated with rare weather events such as fog or snow, which are not incorporated in the MILP.

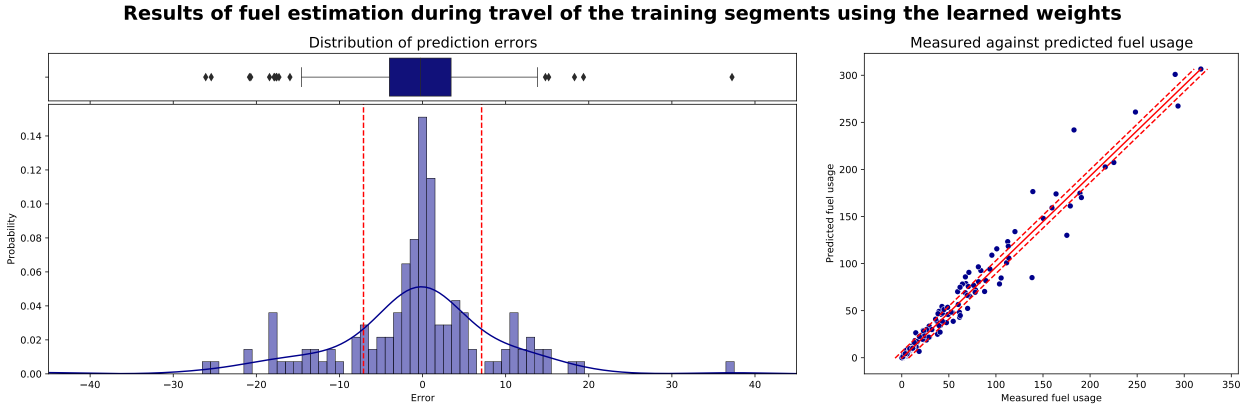


Figure 15: Distribution of fuel usage over all training records

The mean absolute error was with 12.09L a bit higher for the test segments. Again, it stands out that the higher errors are made in those segments were the fuel usage is high. In those cases, the fuel usage can be slightly overestimated.

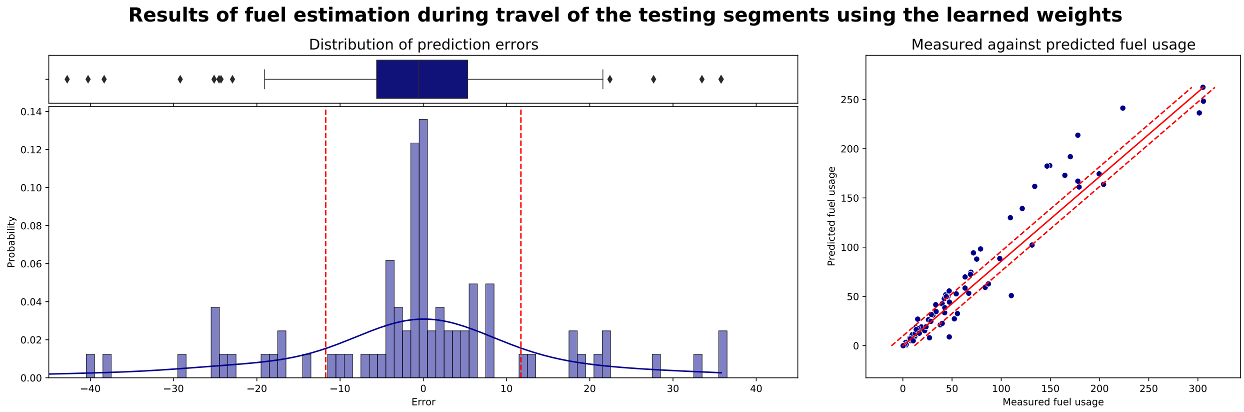


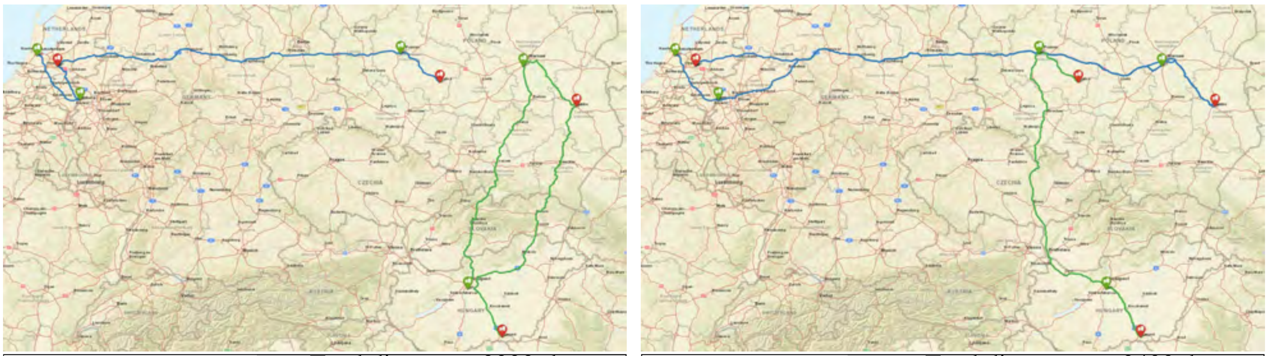
Figure 16: Distribution of fuel usage over all training records

4.3 Minimizing fuel consumption

As explained in Section 3.2.3, it is possible to create a large number of unique problem instances by sampling from the historical data. Each of these instances can be formulated as a Mixed Integer Linear Program and solved by the Gurobi solver in order to find the best solution. There are four different strategies for which each instance will be solved and the resulting estimated fuel consumptions as well as other route details will be compared. The MILP can either have the objective to find the routes with the smallest total distance (distance objective) or the objective to find the routes with the lowest estimated fuel consumption explicitly (fuel objective). Both are regarded in the situation where the target temperature is fixed over the entire route of a truck (fixed setpoint) as well as in the situation where the target temperature can dynamically change over time (dynamic setpoint).

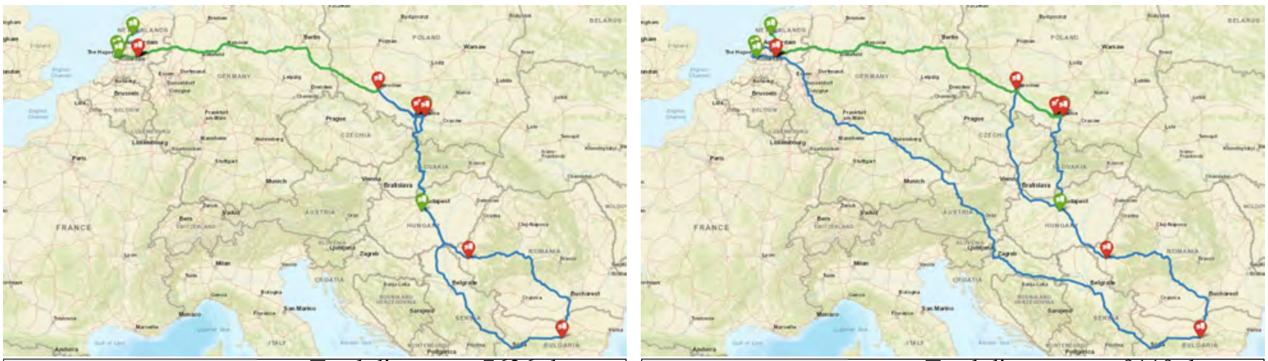
4.3.1 Example solutions

For three example instances, the resulting solutions for both the distance objective as well as the fuel objective are shown on a map in Figures 17, 18 and 19. When visually comparing the two solutions for each of these examples, it can be seen that the solutions found by optimizing over the total route length tend to prefer different routes than those obtained using the fuel objective. The latter solutions have a decreased estimated fuel usage at the expense of an increased route length and total duration, leading to very different routes.



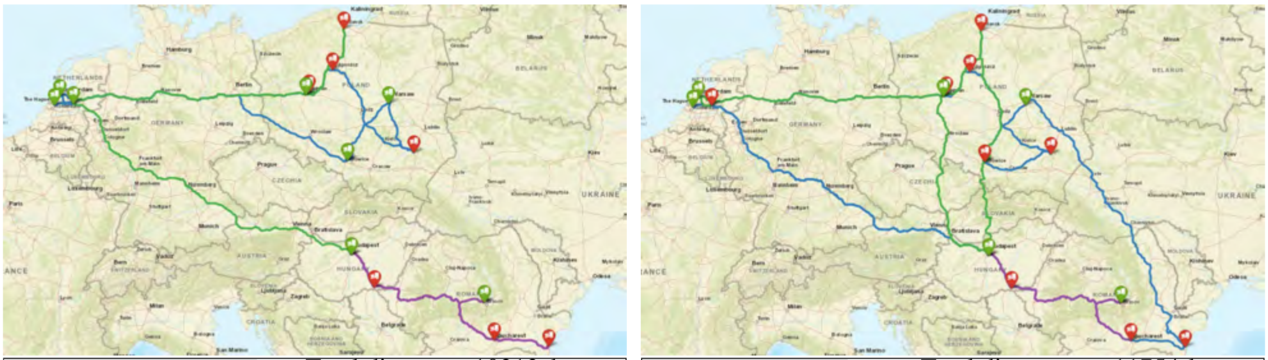
Distance objective:	Total distance:	2923	km	**Fuel objective:**	Total distance:	3400	km
	Total duration:	45	hours		Total duration:	52	hours
	Total fuel:	2554	L		Total fuel:	2310	L

Figure 17: Routes of two example solutions for an instance with two vehicles and ten requests



Distance objective:	Total distance:	7626	km	**Fuel objective:**	Total distance:	8119	km
	Total duration:	111	hours		Total duration:	129	hours
	Total fuel:	4961	L		Total fuel:	4750	L

Figure 18: Routes of two example solutions for an instance with two vehicles and twenty requests



Distance objective:	Total distance: 10313 km Total duration: 146 hours Total fuel: 3165 L	**Fuel objective:**	Total distance: 11754 km Total duration: 166 hours Total fuel: 2885 L

Figure 19: Routes of two example solutions for an instance with three vehicles and twenty requests

4.3.2 Solution characteristics

A total of 200 unique instances with ten up to twenty requests and one up to three vehicles have been created and solved. Those reality based instances do all have varying characteristics regarding the strictness of the service time windows, the distances between the different requests and the present product types. The average percentage of estimated fuel usage that is saved when comparing between the four different strategies can be found in Table 3. The results show that both optimizing over fuel usage as well as using a dynamic setpoint can lead to a total average fuel usage reduction of up to 13% compared to the other strategies.

			Fuel usage reduction by using			
			Distance objective		**Fuel objective**	
			Setpoint		Setpoint	
Instead of using			Fixed	Dynamic	Fixed	Dynamic
Distance objective	Setpoint	Fixed		6.27%	8.82%	13.12%
		Dynamic			0.13%	6.10%
Fuel objective	Setpoint	Fixed				4.61%
		Dynamic				

Table 3: Fuel estimation reduction between the different strategies

Apart from the difference in resulting fuel usage, there are a number of other differences to be found when comparing the solutions between the different strategies. Figure 20 shows the relative distributions of a number of other solution characteristics.

Solving problem instances by using the explicit fuel objective instead of the distance objective, does usually lead to an increased time required for the solver to find the optimal solution. However, the fuel consumption for the solutions is lower. This decrease of used fuel seems to be gained both during travel as well as during service. Apart from longer route distances and durations, a number of other main differences stand out when comparing to the solutions from using the distance objective. Firstly, the average load is usually higher meaning that the solution tend to have fuller trucks during service and travel. Secondly, the fuel objective does also lead to higher service times by idling trucks when the doors are open instead of waiting with the doors closed. This is probably done to make use of cold outside temperatures when applicable, while waiting for another request's time window to open, without having to refrigerate using fuel in the mean time. Finally, it can be observed that the average inside temperatures during service are slightly higher for the fuel objective overall.

When comparing the effect of allowing a dynamic setpoint instead of a fixed setpoint, it stands out that fuel can be saved both during service and travel as well. The number of different product types that are loaded simultaneously in a truck is lower. This means that the target temperature does now only have to be set as cold as the currently loaded products require, instead of having to take the products that are present at any point during the entire route into account. On average, this leads to higher target temperatures that are only temporarily lowered when strictly necessary, which can save a lot of fuel.

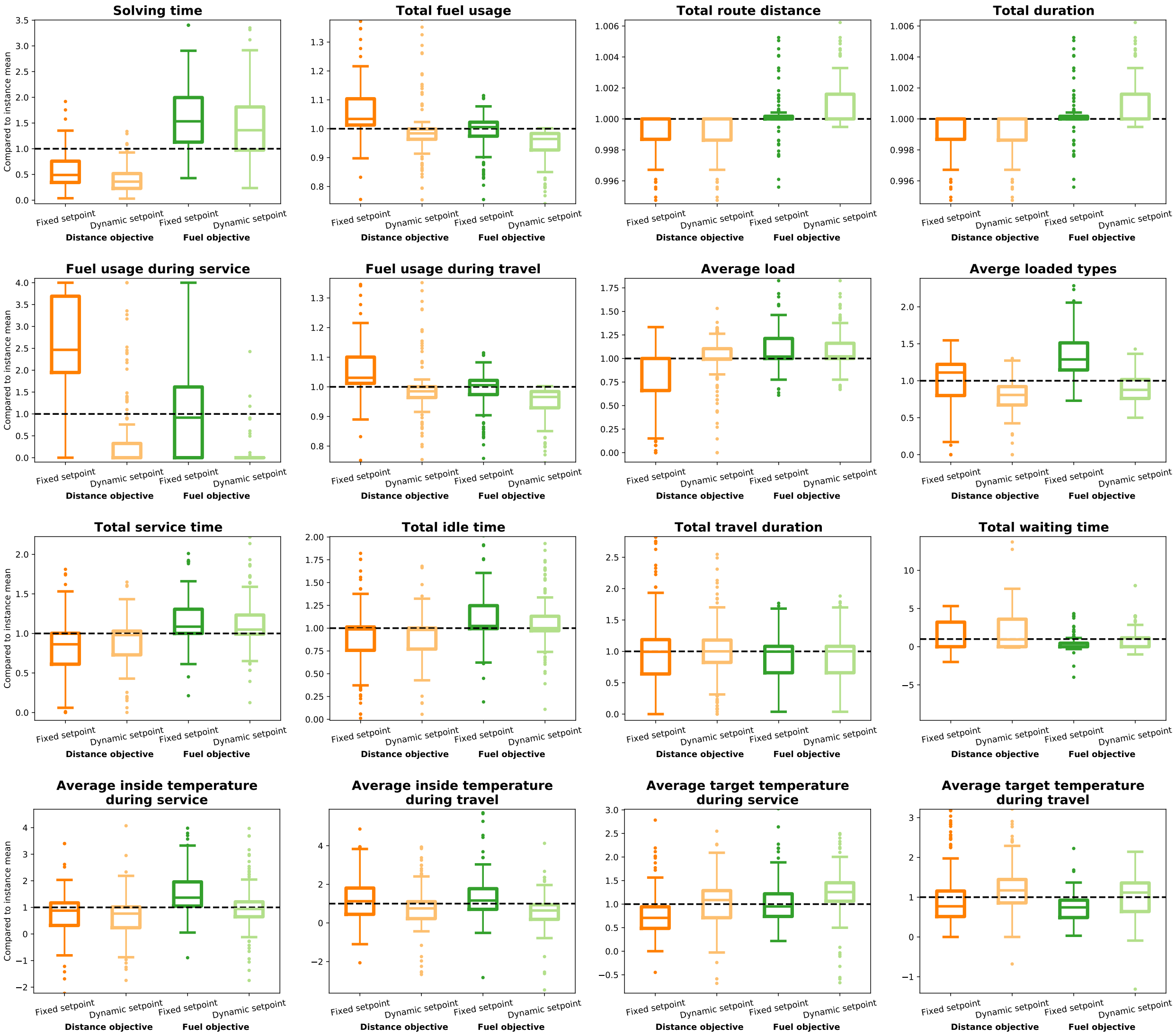


Figure 20: Relative distributions of all solution characteristics between the different strategies

4.3.3 Product combinations

The results did show that using a dynamic setpoint led to an overall decrease of simultaneously loaded product types. But the exact product types that occur together do differ as well when comparing between the two regarded objective functions. Figure 21 shows how frequently certain product combinations occurred over all solved instances. When explicitly optimizing over the fuel usage, it stands out that combinations like chicken and fruit or electronics and flowers occur relatively often, while electronics are less often combined with food, fruit or vegetables. These difference have to do with the allowed temperature ranges for the different products, as well as the corresponding service locations and durations that might differ between the varying product types.

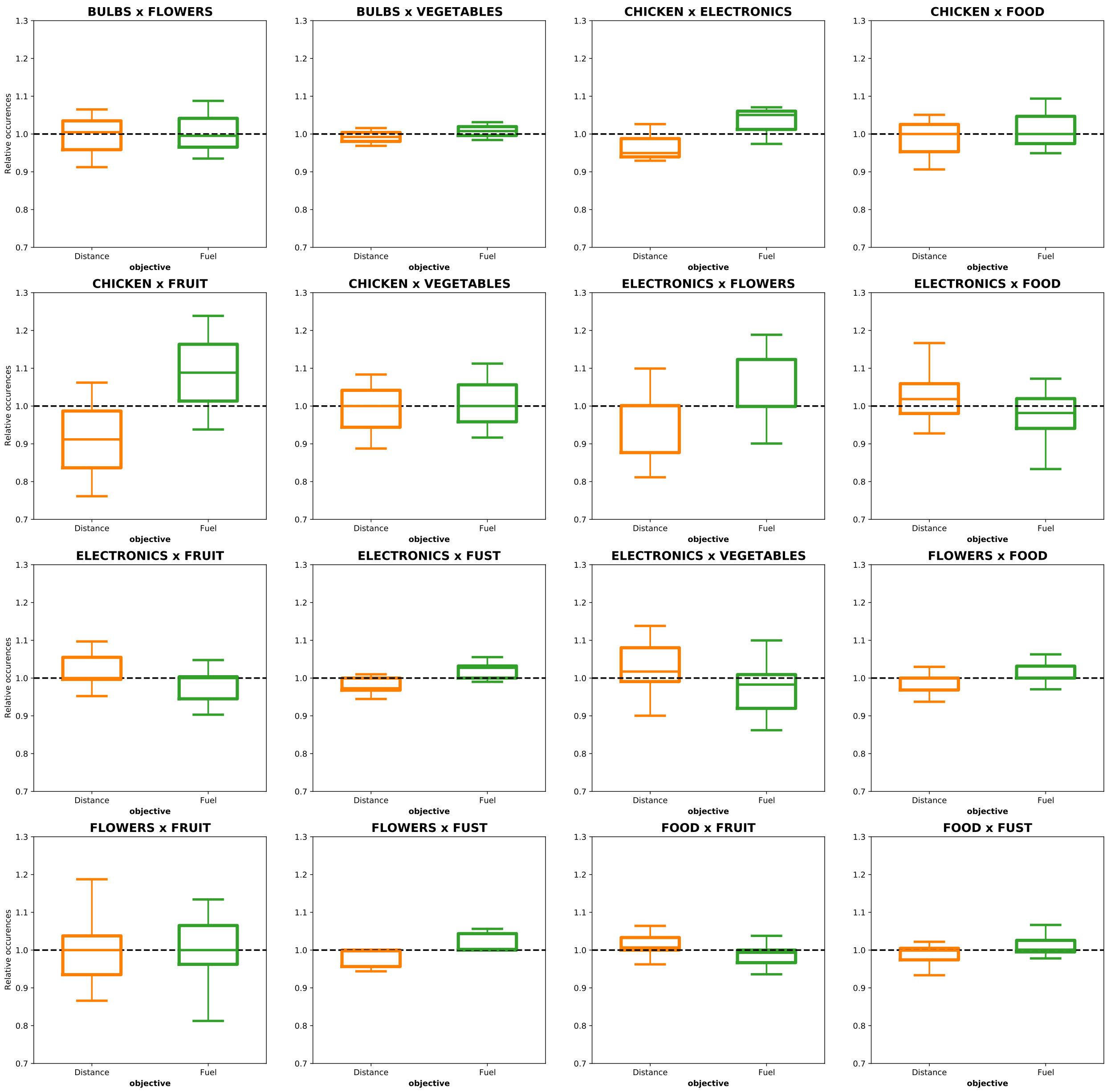


Figure 21: Relative frequency that a product combination occurs in the solutions per objective

4.3.4 Scalability

The amount of requests considered in a single problem instance were relatively small compared to a realistic situation. The complexity of the regarded problem and MILP formulation causes that the solving times that are required to find an optimal solution quickly increase for instances with more nodes, which makes it difficult to solve bigger instances as well. Figure 22 shows an example of the progress of the Gurobi optimizer solving a single problem instance. What can be observed for this example is representable for all of the solved instances. It shows that the solver is usually able to quickly find the optimal solution, but requires a far longer time to increase the lower bound and proof the optimality of this solution.

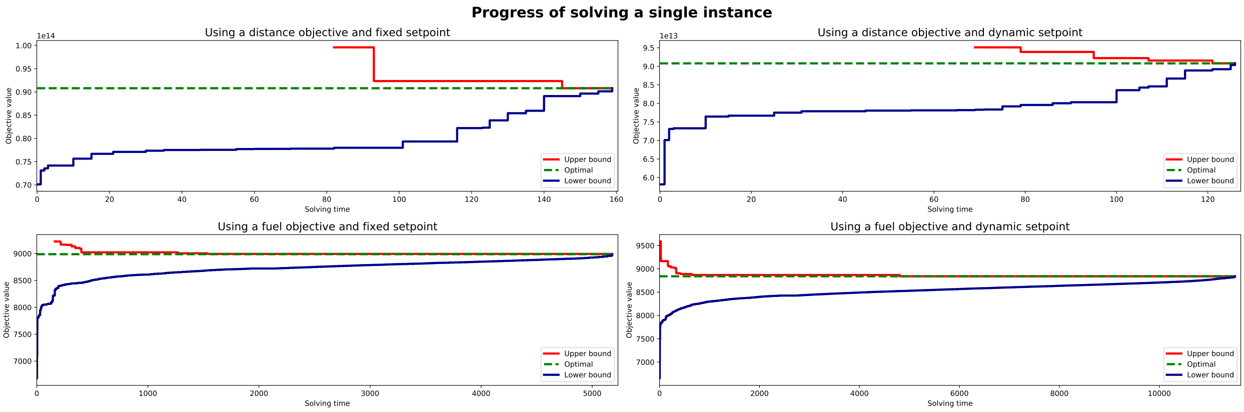


Figure 22: Progress of upper and lower bounds over time while solving an example instance

5 Conclusions

The aim of this research was to estimate and optimize the fuel usage in temperature-controlled vehicle routing. The first goal was to make reliable estimations of the total fuel consumption required for executing a planned route of pickups and deliveries. The second goal was to minimize this estimated fuel consumption by explicitly optimizing over it instead of letting it be a result of optimizing over route distances. Additionally, the effect of allowing the temperature setpoint to change dynamically during a route was investigated.

It has been shown that a linear function can be fitted to the historical data that is able to map variables describing a certain route planning to an estimated fuel usage. The performance of the learned function on a validation dataset showed that it was possible to distinguish segments with higher or lower fuel consumption and make predictions with a mean absolute error of at most 12L. The errors that were made in the predictions, were mainly correlated with events that are not included in the model and are difficult to account for in advance, such as failures in the refrigerator unit. Secondly, it has been shown that both optimizing over fuel usage explicitly as well as allowing a dynamic setpoint can lead to a clear reduction of the fuel consumption. When combined, the two strategies led to a fuel usage reduction of up to 13% over the reality based instances, compared to the fixed objective with fixed setpoint. These fuel savings can involve very different routes to be constructed, that come at the expense of an increased total route distance and duration. The gains in fuel usage are achieved by combining more and different types of load and making better use of the moments at which the vehicle is waiting.

These results are very promising and show that explicitly taking fuel usage into account when optimizing routes in cold-chain logistics, could lead to different routes and solutions with reduced fuel consumption. However, a lot of further research steps can be done in order to obtain more reliable results and increase the practical usability of this approach. The current research has been done using historical data from a limited time period on which the fuel consumption and temperature behavior are based. The quality and reliability of the predictions can probably be improved when more historical segments would be available for testing. When more data is available, the effect of possible measurement errors or exceptional situations will decrease, leading to an increase of the overall performance. This additional historical data segments should preferably be as equally spread over a year as possible, to make sure all possible seasons and weather conditions are represented.

Furthermore, the MILP formulation has been based on a number of assumptions that could be further improved. In order to preserve linearity and limit the complexity of the model, not all possibly relevant factors have been included. So are the temperature change rates probably not constant over time and for all possible environmental conditions. Similarly, weather conditions and travel times will usually be time dependent. Furthermore, a lot of practical issues have not been considered, such as possible different refrigerator modes to choose from, or vehicles with more than one compartment each having a separate setpoint. Finally, the model has also not been made robust for coping with possible travel delays or refrigerator failures. Another issue that should be tackled in order to further improve the usability of this approach, is related to the maximum number of requests for which a single instance can still be solved. In order to achieve this with the currently available computational power, it would probably be required to make use of problem specific heuristics or other non-exact solving methods. The results have namely shown that obtaining a good solution can usually be done a lot quicker than proving optimality.

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