Hedging Interest Rate Risks in Dutch Pension Funds and Life Insurance Companies

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Abstract

The peculiar structure of Pension Funds and Life Insurance Companies with substantial duration gap and dominance of interest rate risk call for a shift of focus to Liability Driven Investment (LDI) instead of the conventional asset-driven strategic asset allocations. This paper analyzes different approaches available to LDI to assess which strategy works best in different interest rate scenarios and over different time horizons.

The analysis was carried out using Asset and Liability (ALM) Model comprising a Vector-Auto Regressive Model for economic scenario generation. The results show that all three strategies : usage of Swaps, swaptions and dynamic swap/swaption switching add value to the ALM with respect of funding status and contribution levels, although their efficacy varies with the time horizon. Moreover, the difference of hedging effects under different strategies becomes less distinct over time.

Key Words : Liability Driven Investment (LDI), Asset & Liability Management (ALM), Interest Rate Risk Hedging, Pension Funds, Life Insurance Companies

JEL Classification: G11

Ⅰ Introduction

Pension funds and life insurance companies with conventional asset and liability structures have high exposures mainly to two types of risks: equity risk and interest rate risk. Traditional asset portfolio risks are dominated by equity risks, which are considered to be the curse of declines of funding status. However, closer study reveals the funding status problem is more relevant to interest rate risks rather than equity risks. Liability values are highly sensitive to interest rates, and responsible for higher volatility levels of funding status. Some studies provide empirical evidences that interest rate risks are the key for funding status (see Ross 2007): two golden periods for pension funds are 1978-1981 and 1993-2000, during the first period funding status raised by an interest rate increase from 7.5% to 15%; the latter rise was triggered by increased excess returns on equities. However, during 1984-1992, when equity markets were also good, decreased interest rates cancelled out the benefit from excess equity returns and brought negative net effects to funding status. More recently, from 2000 to 2004, pension funds experienced their disastrous time because of continuously low interest rates. As a result of funding status' plummeting, higher contributions are required from active pension or life insurance plan participants, defined benefits to beneficiaries may also be cut.

These situations are preventable by restructuring asset portfolios to hedge the interest rate risks which cause liability value changes. Typical pension funds and life insurances have liabilities with 10-17 years' duration. Interest rate changes trigger liability values' fluctuations; whereas, in normal assets portfolios, fixed income allocations usually range from 30 percent to 60 percent of the total asset values, with much shorter durations than liability durations. When interest rates go down, pension funds or life insurance will suffer from mismatched duration gaps and drastically experience funding status declines. We can shift, or at least partly shift the focus from asset-driven allocations to liability-driven investments (LDI).

There are two groups of LDI strategies, which can immunize portfolio from interest rate risks (see Moody, 2006, and Schweitzer, 2006), first is bond asset restructuring and second is the usage of alternative investments. The bond strategies structure bond assets to match the duration with liabilities. These strategies, however, have some disadvantages. First, pension funds and life insurance companies have existing asset allocations, shifting from current situations to liability-matched target may have tremendous structural adjustments and transaction cost; second, long term bonds, which are required in structuring new asset portfolios, may not have sufficiently liquid market; third, to fully match duration gaps between asset portfolios and liabilities, vast majority of assets needs to be reallocated to bond products, this asks for enormous switches from equities and sacrifice of the excess return from equity.

Interest rate derivatives are alternatives which can help immunizing portfolios. Compared to long term bonds, long-duration interest rate derivatives may have more liquid market, don't require 100% funding, and don't require large changes in existing asset portfolios. Some interest rate derivatives, for instance, Roller Coaster Swaps, are designed to have different underlying notional in order that for each tenor, the interest rate sensitivity is zero. The nature of these swaps can help structuring ideal LDI strategies. Notice that no strategy can fully hedge interest rate risk since liability structure changes over time, and there are always credit risks from counterparties.

In this paper, I concentrate on swaps, swaptions and swap/swaption dynamic LDI strategies. Engel, Kat, Kocken 2005 investigated these three strategies to handle interest rate risk problems for pension funds, focused on strategies' impacts on funding status. They draw the conclusions that the decision whether to choose swaps or swaptions is highly interest rate environment dependent, swaps can hedge most of the interest rate risks except when interest rates are lower than historical means; swaptions are preferred in low interest rate environments since they protect the upside potential. Dynamic swap/swaption strategies are recommended which buy fixed receiver swaptions in low interest rate environments and switch to swaps when interest rates go up. Their results are robust with assumed interest rates historical mean reversion levels. The results in my research paper are derived in both initial historical average interest rates and initial low interest rates, since current interest rates are lower than historical mean levels. The aims of my paper are firstly to assess swap, swaption and swap/swaption strategies' performance by measuring funding status, secondly to investigate how they influence contribution levels, thirdly to analyze their consistencies on a 25-year time horizon. All the investigations are conducted on both low initial interest rate environment and normal (close to historical average) interest rate environment.

The answers for the first research question: in an increasing interest rate environment, strategy's impact on funding status, on 5 years and 10 years horizons, are in coincidence with Engel, Kat, Kocken (2005), that is, dynamic swaption/swap's risk reduction performances is the best, followed by swaption strategies, then swap strategies. On a 25 years' horizon, swaptions bring lower expected funding ratios than the other two; and for risk reductions, swaptions' performances are highly structure dependent. In a smooth interest rate environment, three strategies perform similarly in short term; their long term performances are in coincide with those in increasing interest rate environment.

The answers to the second question are structure dependent. On a 5 years' horizon, pension funds and life insurances with the dynamic swap/swaption strategy require least annual contributions, followed by the swap strategy, then the swaption strategy. On 10 years and 25 years' horizons, the swap/swaption strategy keeps ahead, while the swaption strategy involved pension funds and life insurance require less contributions than the swap strategy involved cases.

To the third question, performance of all the three strategies is converging to non-derivatives pension funds and life insurance companies' performance over time. That is, hedging effects become less distinct over time.

In chapter Ⅱ the methodology and model are demonstrated in details. All the assumptions of plan policies and general profiles are made, with description of the model which generates economic scenarios and evaluate long term ongoing pension funds and life insurances' performance. Scenario generation model is a vector auto-regression model used to create main economic environments in the future, since pension funds and life insurances don't exist in an isolated world. Several articles discussed the scenario generation; Hoevenaars, Molenaar, and Steenkamp (2003) introduced a vector auto-regression model with OLS-estimated parameters; Boender, Dert, Heemskerk and Hoek also use a vector auto-regression model with Yule-Walker-estimated parameters. The scenario generation model in this paper is a similar multi-variants model with stepwise-estimated parameters. I developed it during internship project. This long term ongoing asset and liability model developed based on the assumptions and generated economic environments, funding status, contribution levels and values of liabilities and asset portfolios with various structures are calculated and evaluated. In chapter Ⅲ results from models are presented and explained.

Ⅱ Methodology

Measuring Criteria

Two main measurement criteria which are used to evaluate funds' performance are expected funding ratios and probabilities of being underfunded. A VAR model (which is demonstrated in next section) is developed to simulate 2000 future economic scenarios; each scenario contains a quarterly evolution path of a group of economic factors, including yield curve, implied volatility, inflation and equity premium, on a 25 years horizon. At the beginning of the year, we hold a certain structure of asset portfolio and a certain liability portfolio; at the end of the year, according to VAR model, all the assets and liabilities will be reevaluated. In this manner, we can get 2000 pairs of new asset and liability values; and 2000 expected funding ratios are calculated by (Value of Assets/Value of Liabilities); the probability of being underfunded, is then decided by:

*Prob. Of being underfunded = 100% * (Amount of funding ratios which is smaller than 1)/2000*

Here when we mention the probability of being underfunded, it refers to the underfunding risk on a one year horizon. As Dutch regulation requires, if the funding ratio is less than 105% (or 100%, still in discussing, I use 105% since it is more likely to be preferred by regulators) at the end of the year, cash has to be collected to make funding ratio bounce to 105%. So anyway the starting funding ratio is higher than 105%, the probability of being underfunded, in this case, is a one year horizon risk probability.

The reason why to choose the expected funding ratio and the probability of being underfunding as main measurement criterions, is motivated by the Dutch pension funds regulations: FTK (Financieel Toetsingskader), it require a solvency test that pension fund have 97.5% probability that funding ratio higher than 100% at the end of the year. So in this research the expected funding ratio and the one year probability of being underfunded, become the main measurement criteria.

Besides these two main criteria, I have chosen contribution levels to be another criterion. The contributions are collected from plan participants at the beginning of each year; if the funding ratio is then high, say, higher than 120%, then the amount of new contributions is equal to the amount of new liabilities; otherwise, if funding ratio is lower, more contributions are required. From my point of view, contribution level is an important factor to evaluate funds' health; if a fund's lower risk and higher expected funding ratio come from a higher contribution level, then this fund is not healthy.

Inflation indexations, which will be mentioned in the next section, is linearly decided by funding ratios, so here we do not specially set it as a separate criterion.

Derivatives Strategy Assessment

To assess whether a derivatives strategy is a good one, I have compared the performance of the non-derivatives-involved fund to the derivatives-involved fund (with the similar structure, only adding derivatives. Say, if non-derivatives fund has 70% bonds and 30% equities, derivatives-involved fund first exclude derivative premium, then invest 70% to bonds and 30% to equities). Three criteria discussed in last section are used to compare performance. If a derivative strategy helps to reduce underfunded risks while maintaining similar expected funding ratios with non-derivatives, then this derivatives strategy is considered to be a good one.

Model

An Asset and Liability Management (ALM) model has been developed. This model contains two parts: a vector auto-regression (VAR) part used to generate economic environments and, a part describing how assets and liabilities evolve. From this model funding status and contribution levels for different structured portfolios are calculated.

VAR model

First a VAR model containing selected variables is developed. The VAR idea was demonstrated in literatures, for example, Enders (2003) and Hamilton (1995), it is an econometric model in which each variable is explained by its own lag and lags of all the other variables. In this model, all the variables are interdependent.

Variable selection criteria are set as whether the variable plays main roles in Assets & Liability profile. In my model four factors are included in the VAR model, yield curve (interest rates related), implied volatility (VIX), inflation and equity premium. These four factors build the main external economic environment for further analysis. Among these four factors, yield curves can not be modeled by a single variable. Instead of using real market rate as variable, we use the Nelson-Sigel model to construct yield curves, described by Nelson Siegel (1987), and further developed by Diebold and Li (2004):

$$y_t(\tau) = \beta_{1t} + \beta_{2t}\left(\frac{1-e^{-\lambda_t \tau}}{\lambda_t \tau}\right) + \beta_{3t}\left(\frac{1-e^{-\lambda_t \tau}}{\lambda_t \tau} - e^{-\lambda_t \tau}\right) + \varepsilon$$

λ_t governs the exponential decay rate; as stated in the paper Diebold and Li (2004), λ_t can be fixed in such a value that maximizes the loading on β_{3t}, I fixed it as 0.0598 (0.0607 in Diebold and Li (2004)). β_{1t}, β_{2t} and β_{3t} are three latent dynamic factors in Nelson-Siegel model. The loading on β_{1t} is constant as 1, it is a long-term factor. The loading on β_{2t} is a function that starts at 1 but decays monotonically and quickly to 0, it is a short-term factor. The loading on β_{3t} is a function which starts at 0, increases, and then decays to 0, it is a medium-term factor. All the three β_{1t}, β_{2t} and β_{3t} are OLS regressed from historical data.

The estimated empirical results are shown in Appendix 1.

This VAR model is used for generating 2000 scenarios in the next 25 years. Two groups of initial variable values are set, first the current values, where interest rates are lower than historical mean second historical averages. By this we could learn how derivative strategies work in different economic environments.

Appendix 2 tells more details about VAR model and its empirical results.

Data

All the data are US based.

Data for real variables and estimated values for structural variables are collected for VAR model estimation. Time series of data are selected and adjusted in a quarterly manner during 1983 quarter one to 2006 quarter three. Equity return is quarterly return while Inflation is annualized. Quarterly data seems to be good compromise between annual and monthly data. I have 95 quarterly observations available while only 24 yearly observations, which could hardly support the coefficient estimation. Monthly data are comparatively noisy and not appropriate for capturing long-term dynamics (Hoevenaars, Molenaar and Steenkamp, 2003).

The data are collected from 1983 mainly because only since then the interest rates have been controlled reasonably by the appointment of the 12th president of the US Federal Reserve, Paul Volcker. When making forecast about future it is believed that the current policy will keep on working in long term thus we collect data from 1983.

Asset and Liability Evolution Model

Contributions collected at the beginning of each year fund asset portfolio. The amount of contributions are decided by funding status, new liabilities and pension funds and life insurances' policies. Defined benefits, which will be distributed to plan participants after their retirement, keep increasing with participants' working ages until they retire. These increases, together with the amount of defined benefits for new-entering participants, form the new liabilities part. Once the amount of new liabilities has been decided, according to funds' policies, the contribution levels are decided by funding status. In this model, I made the policy assumption that if the funding ratio is higher than 120%, then the amount of new contributions are equal to that of the new liabilities; if the funding ratio is lower than 105%, then the amount of new contributions are equal to 1.5 times of the new liabilities; if between, then linearly related; if below 105%, then the amount to contributions is required to feed the asset value back to 105% of the liability value. Meanwhile, the liabilities (new-entering participants' parts exclusive) need to be inflation indexed. I made the assumptions that if funding ratios are equal to or above 120%, the liabilities have full indexation of the inflation; if below 105%, no indexation; if between, linear indexation.

During the year I assume that only asset portfolio evolves. I made the assumptions that assets are only allocated to bond portfolios and equity portfolios; when after adding derivatives, derivatives are also in the asset portfolios. Bonds portfolios are structured with duration of 7 years, which is representative in funds, and are renewable at the start of each year; equity portfolios are assumed to be a fully-diversified portfolio with the equity market return. In this model, bond portfolios and equity portfolios of 100%/0%, 80%/20%, 60%/40%, 40%/60%, 20%/80, 0%/100% are included.

Derivatives Strategies

Swaps and swaptions are included in asset portfolios in such a way that fully bridge the duration gap between fixed income assets and liabilities. Look at the formula below:

*Value Change of Assets = - Duration of Bonds * Yield change * Bond Value*

*- Duration of Swap * Yield change * Swap Notional*

*Value Change of Liabilities = - Duration of Liabilities * Yield change * Liabilities Value*

Swap and swaptions are structured to make *Value Change of Assets* roughly equals to *Value Change of Liabilities* arising out of a change in interest rate. Then if interest rates change, yield curves change, then asset and liability value change in the same direction with a similar amount. I have chosen 10 year quarterly swap, and have calculated the notional using the formula shown above.

The operations are entering into swap agreements or buying swaptions at the beginning of the year and at the end of the year evaluating the market value of swaps and swaptions in assets portfolios. I made the assumptions that swaps and swaptions are renewable without transaction costs; all the swaps and swaptions positions are closed at the end of this year and, new positions are established at the start of next year. The swap and swaption markets are assumed to be sufficiently liquid.

When adding derivatives, relevant derivatives are firstly priced. Swaptions are priced by Black formula, as described in Black, Fisher 1976. All the swaptions are assumed to be at the money swaptions. When deciding the assets allocations, first derivatives are bought and the rests amount is allocated to bond and equity portfolio. In individual swap and swaption strategies, the amount of notional are decided in such a way that the interest rate risks are fully hedged, that is, the duration gaps between asset portfolio and liability portfolio are covered. When using dynamic swaption and swap strategies, however, I made a simple assumption to invest in swaptions when interest rates are **below 5% and switch to swap if it is above 5%;** the reason here why I choose, 5%, is that according to the my VAR model, the historical mean of 10 year yield curve is roughly 5% (I adjusted the historical mean of 30 years yield to 5.5%, the observed mean is 7.44%, but main academic schools do not expect it can bounce to so high). This assumption is too simple, when considering what a low interest rate environment to use swaption is, it is hard to make a clear definition. I am thinking an alternative, which recognize the interest rate as low rate when it has kept decreasing for a certain period without bounce.

Then asset values after one year can be calculated with the VAR model, which provides simulated yield curve and equity premium results.

At end of the year, defined benefits are subtracted from both asset portfolios and liability portfolios and the funding status are decided. Then next year starts with new contribution collections and inflation indexations like demonstrated before.

Ⅲ Results

In this chapter I show results for 5 years, 10 years and 25 years horizons, at each time point comparisons are made among non-derivatives portfolios and three derivatives-added cases, in two interest rate environments: ⅰ, initial interest rate levels are close to historical mean; ⅱ, initial interest rate levels are lower than historical mean

Short Term Results

First look at the funding status on 5 years' horizon. Left figure is funding status results from environment (i) while right is from (ii).

The x-axis is the probability of being underfunded and y-axis is the expected funding ratio. Red pots refer to the non-derivatives cases with (100% bonds, 0% Equity), (80% bonds, 20% Equity), (60% bonds, 40% Equity), (40% bonds, 60% Equity), (20% bonds, 80% Equity), and (0% bonds, 100% Equity) assets allocations. Yellow pots refer to the swap-included cases with the same corresponding allocation. Blue pots are swaption cases and greens are dynamic swap and swaption cases.

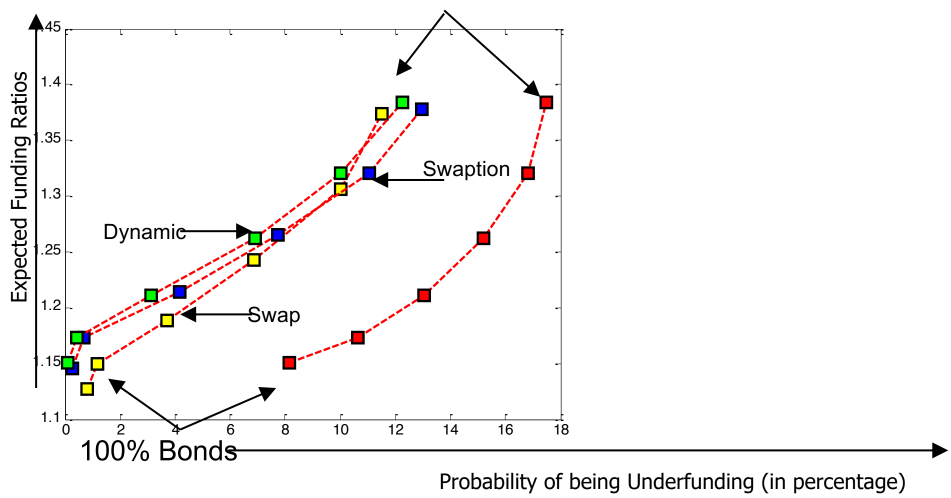


Figure 3.1.a: Funding status after 5 years (Smooth Interest Rate Environment)

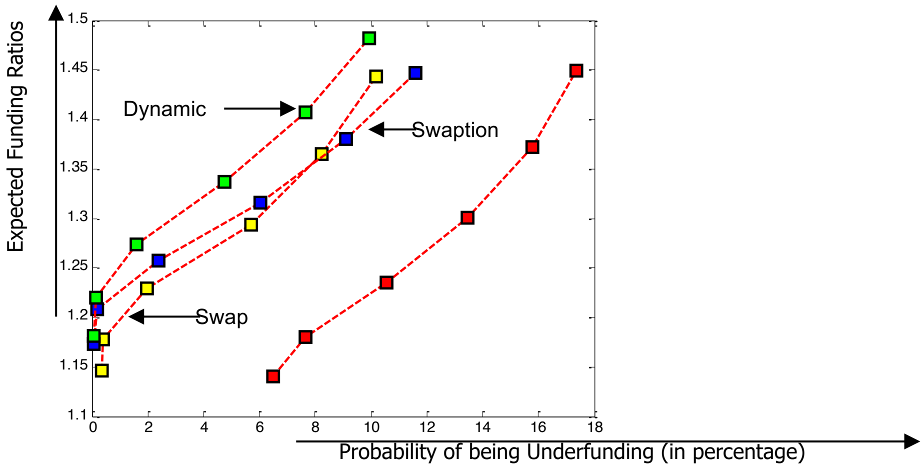


Figure 3.1.b: Funding status after 5 years (Increase Interest Rate Environment)

In smooth interest rate environment, three strategies have similar risk reduction performance, while in increasing environment, they have different performance: the Dynamic strategy is the least risk reduction strategy; the swaption strategy works better than the swap strategy especially with more bonds in the portfolio; with the increasing equities, however, more swaption premiums are paid since the duration gap becomes larger, then this strategy loses its advantages in risk reduction. In both environments the dynamic swaption and swap strategy performs best among the three.

The contribution level is another key factor implying funds' health.

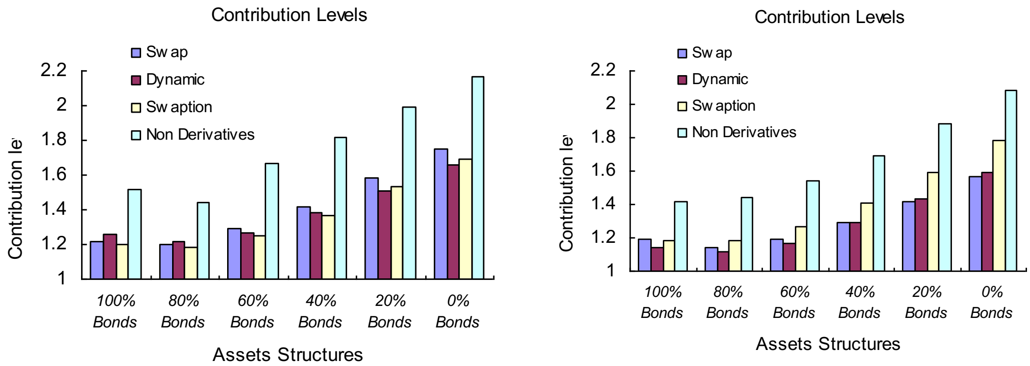


Figure 3.2: Contribution levels at the beginning of 5th year

From the figure, all three strategies bring lower contribution levels as compared to non-derivatives cases. In the smooth interest rate cases, as shown in the left graph of figure 3.2, swaption strategies perform better than swap strategies with all asset structures; when there are few equity assets the dynamic swaption and swap strategy performs worst among three strategies, while with increasing share of equity, it gradually surpasses other two and becomes the best strategy.

In the increasing interest rate environment as shown in the left graph of figure 3.2, however, dynamic swaption and swap gradually loses its leading performance when amount of equity increases. Almost for all the asset structures the swap strategy precedes the swaption strategy. That's because in an increasing environment, swaptions usually expire without being exercised. In a short horizon although swaptions keep upside potentials, they can't compensate for the premiums paid.

Medium Term Results

Second look at the funding status on 10 years' horizon. Left figure is funding status results from environment i while right is from ii.

Red pots: non-derivatives; Yellow pots: swap-included;

Blue pots: swaption; Green pots: dynamic swap and swaption.

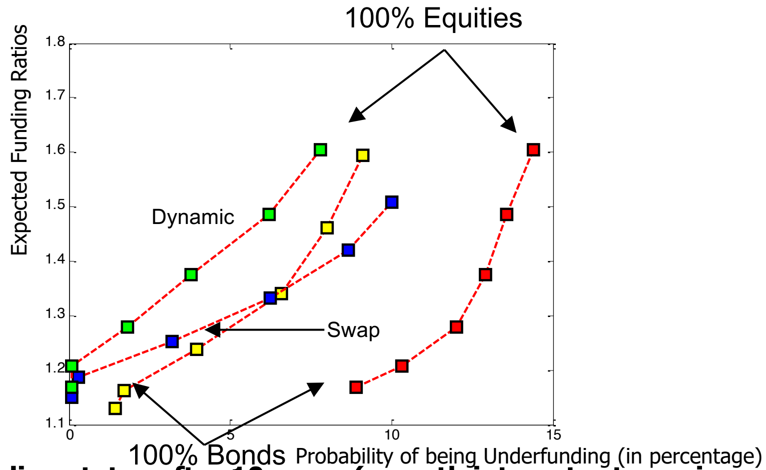


Figure 3.3.a: Funding status after 10 years (smooth interest rate environment)

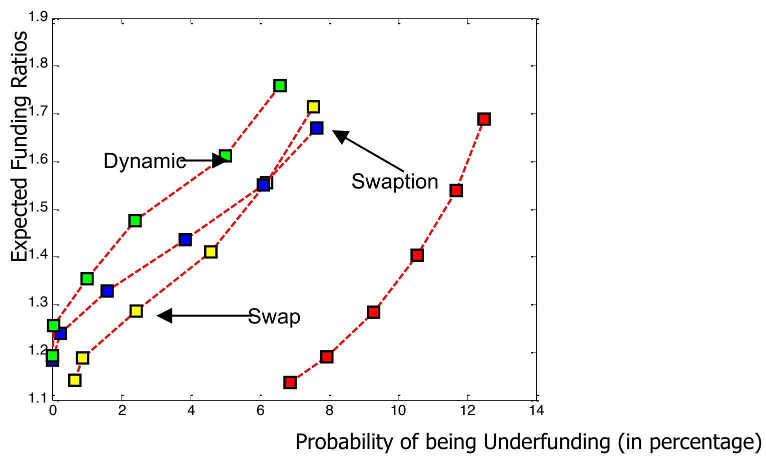
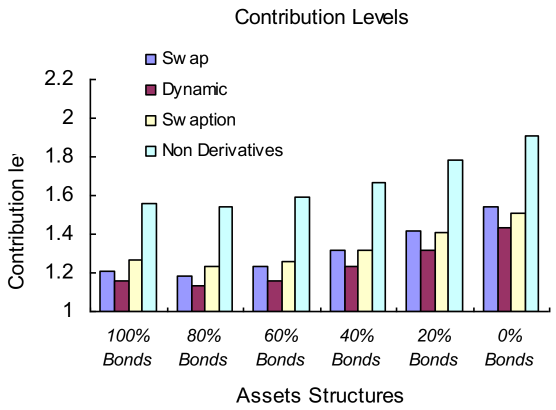


Figure 3.3.b: Funding status after 10 years (Increasing interest rate environment)

In smooth interest rate environment on 10 years' horizon, three strategies don't have similar risk reduction performance as was on short term. Three strategies become having similar performance in both interest rate environments. The dynamic swaption and swap strategy performs best among the three.

The comparison between swap and swaption strategies' risk reduction performance is asset-structure dependent. With more bond assets than equity assets, swaptions reduce more risks; while when more equities are included, swaptions fail to win.

The contribution levels:



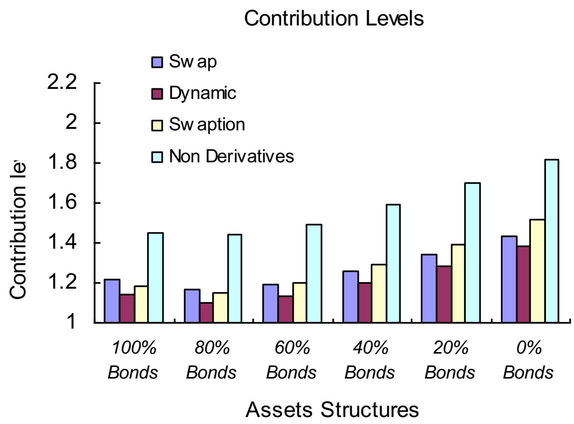


Figure 3.4: Contribution levels at the beginning of 10th year

Again from the figure, it is found that all three strategies bring lower contribution levels as compared to non-derivative cases. And in the figure, dynamic swaption and swap strategy shows overwhelming advantages in risk reductions.

In the left smooth interest rate cases, the swap strategy performs better than swap strategies when there are few equity assets; while with increase in the share of equity, it is gradually surpassed by the swaption strategy. The reason can be that with more equity, more swaptions are used and more upside potential is gained.

In the increasing interest rate environment as depicted in the right panel of figure 3.4, however, the resluts are in contrast. Swaptions win in less equity asset cases and fail when more equities are added in. Possible explanations, again, few swaptions are exercised; interest rates' increasing potential doesn't compensate swaption premiums.

Long Term Results

Finally look at the funding status on 25 years' horizon. Again left figure is funding status results from environment i while right is from ii.

Red pots: non-derivatives; Yellow pots: swap-included;

Blue pots: swaption; Green pots: dynamic swap and swaption.

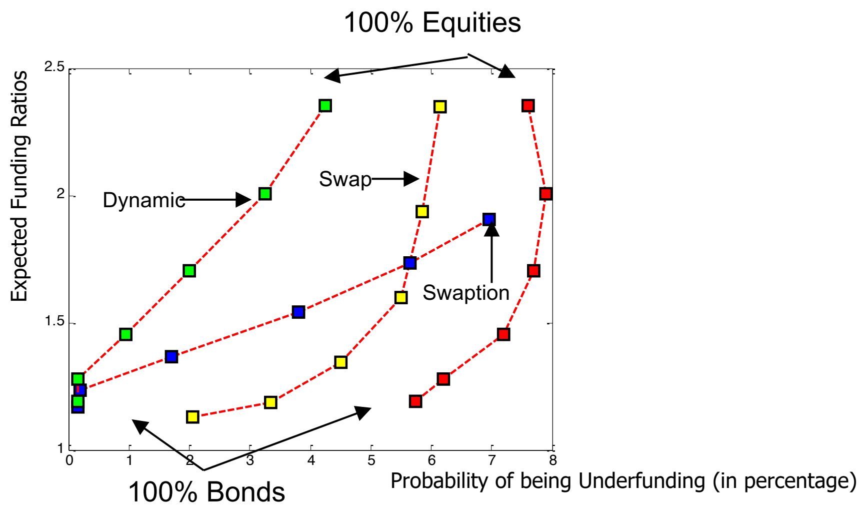


Figure 3.5.a: Funding status after 25 years (smooth interest rate environment)

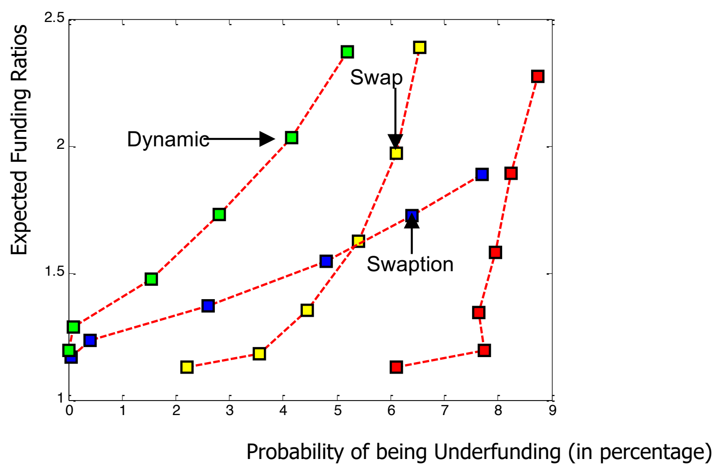


Figure 3.5.b: Funding status after 25 years (increasing interest rate environment)

On 25 years' horizon, three strategies work similarly in both environments. The dynamic swaption and swap strategy performs best among the three.

The comparison between swap and swaption strategies' risk reduction performance is asset-structure dependent. Swaptions reduce more risks in less equity cases; when equities are more than bonds, swaptions fail to win. The contribution levels in long term are different from short term and medium term in the sense that contribution levels of these strategy-involved funds converge to non-derivatives cases.

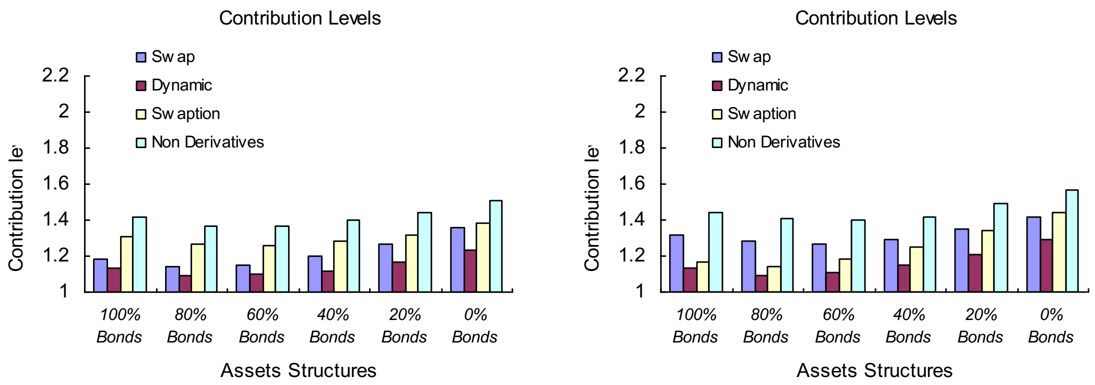


Figure 3.6: Contribution levels at the beginning of 25th year

As is found in medium term, all three strategies bring lower contribution levels as compared to non-derivatives cases, however, only to a smaller extent. Dynamic swaption and swap strategy succeed again.

Ⅳ Conclusions

In this paper the importance of Liability-Driven Investment (LDI) is discussed and solutions with derivatives are evaluated. It is proven that swaps and swaptions add value to the Asset and Liability Management (ALM) with respect of funding status and contribution levels. This paper agrees with Engel, Kat, Kocken (2005) that swaps and swaptions add value to ALM, and what strategy to use is highly dependent on the interest rate environment. Moreover, this paper investigates their long term performance and find that in long term, swaptions and swaps, and with structured dynamic strategies, have similar performance in both smooth and increasing interest rate environments. Furthermore, this paper analyzes another important factor which influences pension funds and life insurance companies' health, contribution levels. Adding derivatives help to reduce contributions, which greatly favors participants.

Further research can be done in the area of credit risks of relevant derivatives.Further research can be in the area of credit risks of relevant derivatives and also in the structure of dynamic swaption and swap strategies. The arbitrary 5% (roughly 10 years' yield) boundary in this paper is too simple so alternative value could be set; the point is deciding a level below which interest rate is considered to be low so swaptions are preferred than swaps. As I mentioned, a solution could be that have a look at the previous interest rate record, if it has been decreased for a certain period, say, 12 months, then we buy swaptions, otherwise we enter swap agreement.

One disadvantage about the model is that Nelson Siegel term structured yield curves is not in coincidence with the Black model's assumption: the forward rate is lognormal distributed. The reason I choose Nelson Siegel model to forecast yield curve is because its empirical estimation results can well fit observed data (See Appendix 1), and for funds' management, yield curve is crucial factor. However, the Nelson Siegel term structure is in conflict with Black model's lognormal distribution assumption. A possible solution is to use other yield curve model which is in coincidence with Black model's assumptions, while in that case, the accuracy of yield curve estimations could also be sacrificed.

Appendix

Appendix 1: Empirical Results on Nelson Siegel Model

To learn how well Nelson Siegel model's estimation fit to historical data, I made the following figures to compare real historical data and the NS fitted results by three types: 12 month yield, 60 month yield and 120 month yield, represent short term, medium term and long term respectively.

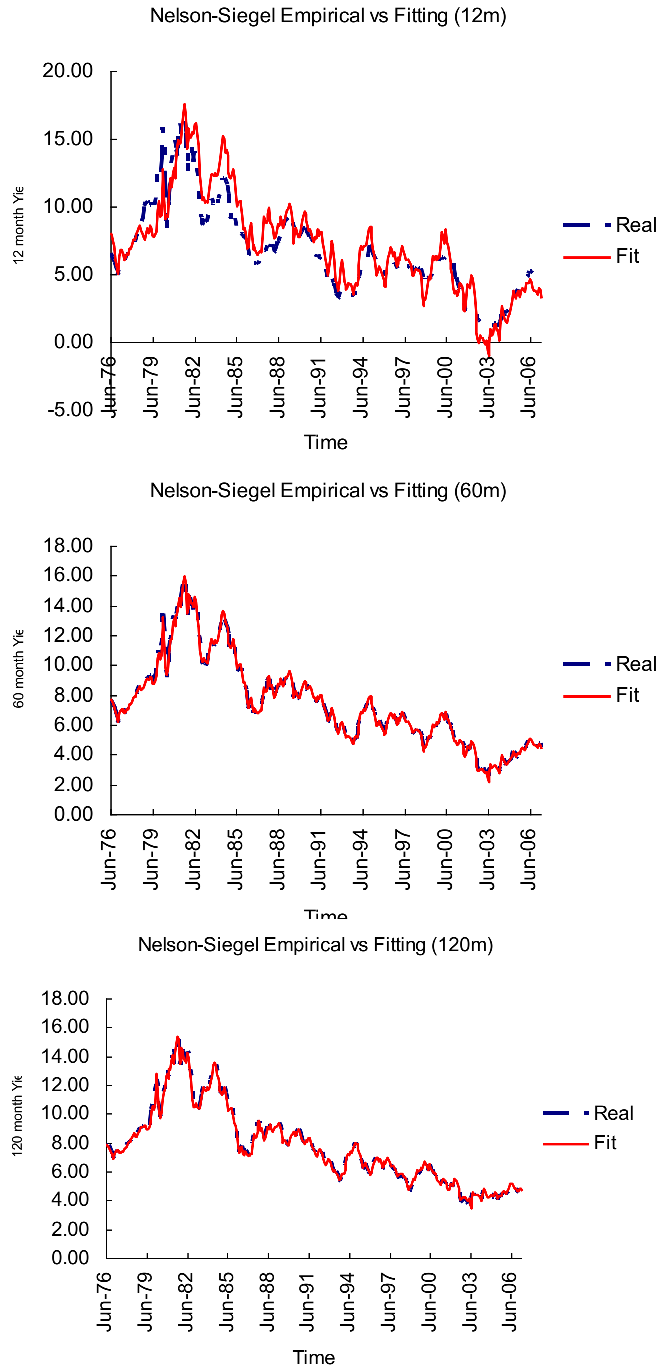


Figure A.1: Nelson-Siegel 's Empirical Result

From Figure A.1 we can see that Nelson-Siegel estimation works better in Longer term than short term, for instance 12m. Basically, we could drawn the conclusion that Nelson-Siegel can well explain yield curve.

Appendix 2:

VAR model

Notations in the model vectors are:

S_{tn} = [R_{tn}, E_{tn}, I_{tn}, V_{tn}]; t=1,…,4*25 (25 years horizon, quarterly) ; n=1,…2000 (2000 scenarios).

With:

R_{tn} = Yield curve factors in <t,n>
E_{tn} = Returns of asset classes in <t,n>
I_{tn} = Price inflation in <t,n>
V_{tn} = Implied Volatility in <t,n>

In each future time node a number of scenarios are generated by VAR model, consisting of yield curve, implied volatility, inflation and equity return in a future time node t.

Explain how VAR and NS linked:

R_{tn} here refers to yield curve structural factors in <t,n>; these yield curve structural factors, are β_{1t} , β_{2t} and β_{3t} in the Nelson Siegel model. After get these three estimated β_{1t} , β_{2t} and β_{3t} factors by VAR model, loadings (explained in Chapter 2) on β_{1t} , β_{2t} and β_{3t} will be added and structure yield curves.

Parameter Estimation

We use the VAR model to construct scenarios of the future environment. In a VAR model, the value of each year's object in a vector depends on linear combination of object values from previous time point in a multidimensional manner:

$$(Y_t - \mu) \sim N(\sum_{p=1}^{P} \Phi_p * (Y_{t-p} - \mu), C).$$

Here μ refers to vector of average value for the six factors during the given time horizon; Φ_p refers to a matrix of coefficients for independent variable $(Y_{t-p} - \mu)$; *p* is the order of lags; *C* is a matrix of noise.

When building the VAR model, Returns of asset classes, Price inflation are observed variables. Yield Curve is characterized by three variables β_{1t} , β_{2t} and β_{3t} , which capture most of information of the yield curves.

When estimate Φ_p, we need to decide whether the statistical insignificant coefficients should be included. So in order to decide which model, including or excluding insignificant coefficients, to be adopted as our VAR model, and also in order to learn VAR model's empirical estimation results, we check the fitting graphs of these 6 factors, all the data here are $(Y_{t-p} - \mu)$:

Observed $(Y_{t-p} - \mu)$, Estimated $(Y_{t-p} - \mu)$ with all coefficients and estimated $(Y_{t-p} - \mu)$ with significant coefficients.

Blue lines: Observed data; red lines: Fitting line based on the model with all the coefficients including the insignificant ones; Yellow line: Fitting lines without insignificant coefficients.

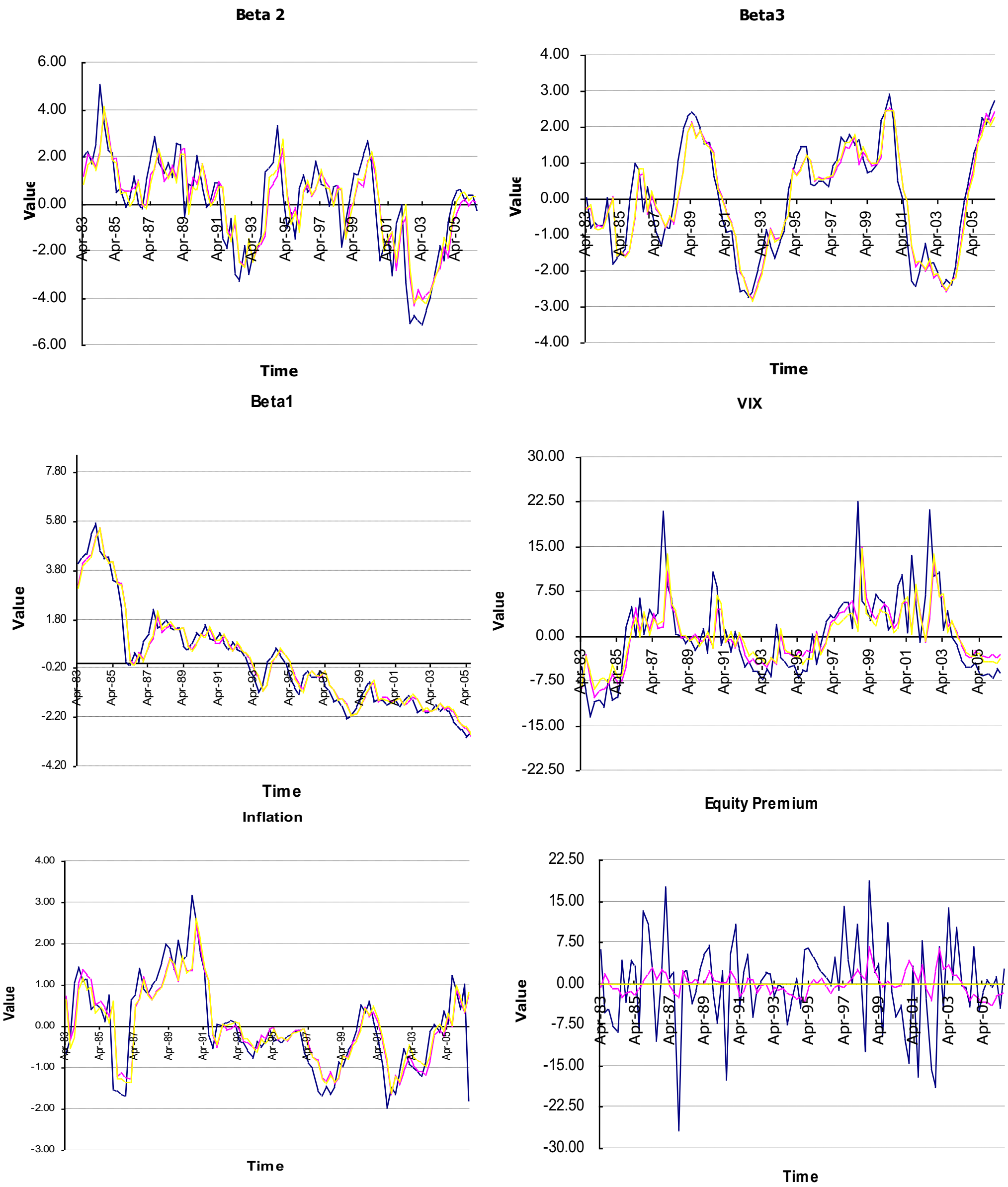


Figure A.2: Fitting lines with or without insignificant coefficients

As we can see from the figures, for the first five plots, with or without insignificant coefficients in the estimated model would well fit the historical data. That is, including or excluding insignificant coefficients would cause slight difference when creating scenarios. However, in last plot we could find obvious differences between these two fitting lines. As we can see in the plot, the maximum difference was up to roughly 7 percent in some quarter 1998, concerning that it is quarterly equity premium, the difference between annualized equity premiums is roughly 28 percent. When making forecasting of equity premium for the next years, we expected a comparatively stable mean value around historical mean 1.71 percent per quarter. Since it is weak-linked with previous performance, adopting model with all the coefficients would harm this stability, as it is shown in the last plot; it would add positive or negative drifts to cause the expected equity premium to be too volatile. Given the results we got above, we decide to continue with the estimated model with all the coefficients for Yield curve, Inflation, VIX; while for equity premium, set each coefficient equals to zero.

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