Capacity models for investigating the patient flow between the Intensive Care and Medium Care

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Preface

This paper is written for an obligatory course in the Master program of Business Mathematics and Informatics of the VU University in Amsterdam. The Master's in Business Mathematics and Informatics (BMI) is a multidisciplinary program, aimed at improving business processes by applying a combination of methods based on mathematics, computer science and business management. The aim of this paper is to reflect on the research performed to find answers for a problem definition. The research is based on computer generated data.
In this paper the relation of the patient-flow between the Intensive Care and the Medium Care will be investigated. This will be based on three models for the capacity of the Intensive Care, the Medium Care and an additional joint ward. Different kinds of objectives will be investigated using simulation programs for these models.

I would like to thank René Bekker for his support, advices and critical insights during the process and writing of this paper.

Abstract:

In this paper we investigate the influence of the patient-flow between the Intensive Care (IC) and the Medium Care (MC). The best model that represents a hospital is that internal patients remain at the IC if they find the MC and the JW full. For investigating this care chain we therefore created three models, consisting of an IC, MC and an additional joint ward (both patient types have access to this ward if their own ward does not have enough available beds left. In the first model the patients from the IC to the MC are blocked when they find the MC and the joint ward fully occupied. In the second model those patients remain at the IC when they find the MC and joint ward both fully occupied. The third model consists of an independent IC and MC without the patient-flow in between. This model is easy to use because it has an analytic solution for calculating blocking probabilities, which is called the product-form. Using a simulation we compared what the differences of the blocking probabilities between those models are and we found out what the influence of MC patients is on the IC. We found that the blocking probabilities of all models approximately show the same result especially when we look at the realistic bed allocations. For all different models the minimum number of beds per ward which give a blocking probability lower than 5 % is similar, namely for the IC 28 beds are needed to reach this and for the MC 11 beds. Therefore we can see the wards as independent. Thus when making the calculations for the blocking percentages we might use the product-form for calculating blocking probabilities of each ward. Another interesting conclusion is that if the MC does have enough capacity for handling external MC patients, the influence of the internal MC patients on the IC is negligible.

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1. Introduction

Over the past years hospital managers have been stimulated to make the organization structure of the hospital more efficient by reducing the number of beds and increase the occupancy rates to improve operational efficiency. This strategy is questionable because generally the management does not consider the total care chain from admission to discharge, but mainly focuses on the performance of individual units. This has often lead to less patient access without any significant reduction in costs. [4]

Another reason why managers must make their organization structure more efficient is because of the competition between hospitals and moreover, the increased focus on the service to the patient. To distinguish themselves they can keep costs as low as possible and, to reach a high service level, they can reduce waiting times. This way they make sure that the quality of the service health care is optimal for improving patient satisfaction.

With the current budget cuts, one of the reasons why managers want to make the structure of hospitals more efficient is to reduce costs. Therefore we have to look at the cost drivers to know in which care chain the structure needs to be adapted. One of the main cost drivers for hospitals is the Intensive Care (IC). The costs of the IC can become very large in comparison with other wards. For example the costs for an IC bed are twice as high as for a bed on the Medium Care (MC). In VUmc, when patients leave the IC, 21,5 percent of the IC patients are admitted to the MC, otherwise they are transferred to another ward such as the Normal Care (NC). To keep the costs for the IC as low as possible the IC should be utilized as effectively as possible, meaning that blockings from the MC should be avoided. So if managers of hospitals want to gain profit by reducing the costs for IC, it is useful to examine this care chain more carefully. Another reason for examining this is that the IC needs to be accessible for patients who need acute and intensive attention.

Based on economies of scale it is preferable to have large clinical wards. For large, for example merged wards, it is possible to achieve a higher occupancy rate while refused admissions are lower. In most queuing systems economies of scale occur. From queuing theory, in particular the Erlang B model, we know that large systems may benefit from economies of scale, implying efficiency gains and potential cost reductions [7]. Larger service systems can operate at higher occupancy rates than smaller ones while attaining the same percentage of blocking or delay. This means that the best thing a hospital can do is to merge clinical wards as much as possible. But this is not always beneficial or desirable for the costs because then the staff needs to be multi-skilled and the planning and control as well as the organization of the different kind of patients becomes more difficult, which will lead to higher costs.

As mentioned above, patients who leave the IC are admitted to the MC, the NC or another nursing unit. I also take into account a joint ward; a joint ward can be seen as a merged ward for IC and MC patients and contains flex-beds that can be used by IC patients as well as by MC patients. This will be the case if there are no available beds left anymore on the particular ward. A fixed number of beds are guaranteed for the IC and MC, although this number is usually less than for the case in which the hospital has fully separate wards. The joint ward allows the hospital management to protect the high valued patients or patients with a small average length of stay. Multi-skilled nurses, who are more expensive than specialized nurses, are only required for the joint ward, whereas specialized nurses can be used on the IC and the MC.
I created three different models A, B and C (explained in Section 2) to examine the already above mentioned patient-flow between the IC and MC, care chain in more detail. These models are based on three factors.
First of all, we want to avoid that those patients who leave the IC and go to the MC encounter a full MC. Second, it is beneficial to combine the economics of scale of merging wards. Third, we want to use the benefits of single wards. Therefore the basis of the models consists of the IC, the MC and a joint ward.

The goal of this paper is to investigate the influence of the patient-flow between the IC and the MC based on two objectives:

1. Do the blocking probabilities for the models show similar results?
2. What is the influence of the MC on the IC?

Note that I also take into account that the hospital is using a joint ward for patients who find the IC or the MC full when admitting.

In order to obtain the answers for the mentioned objectives, I investigated the patient-flow from the IC to the MC. To investigate this flow I created different kind of simulation models based on the models A, B and C. These models were used to calculate the performance measures defined in Section 2.

First the three models are presented in Section 2. The objectives are listed in Section 3. Section 4 consists of model assumptions and parameters of the arriving and outgoing patients on the IC, MC and the patient-flow between the IC and the MC. The performance measures of the simulation models, formulas for the blocking probabilities and a description of the simulation models are given in Section 5. The results are presented in Section 6 and the conclusion can be found in Section 7.

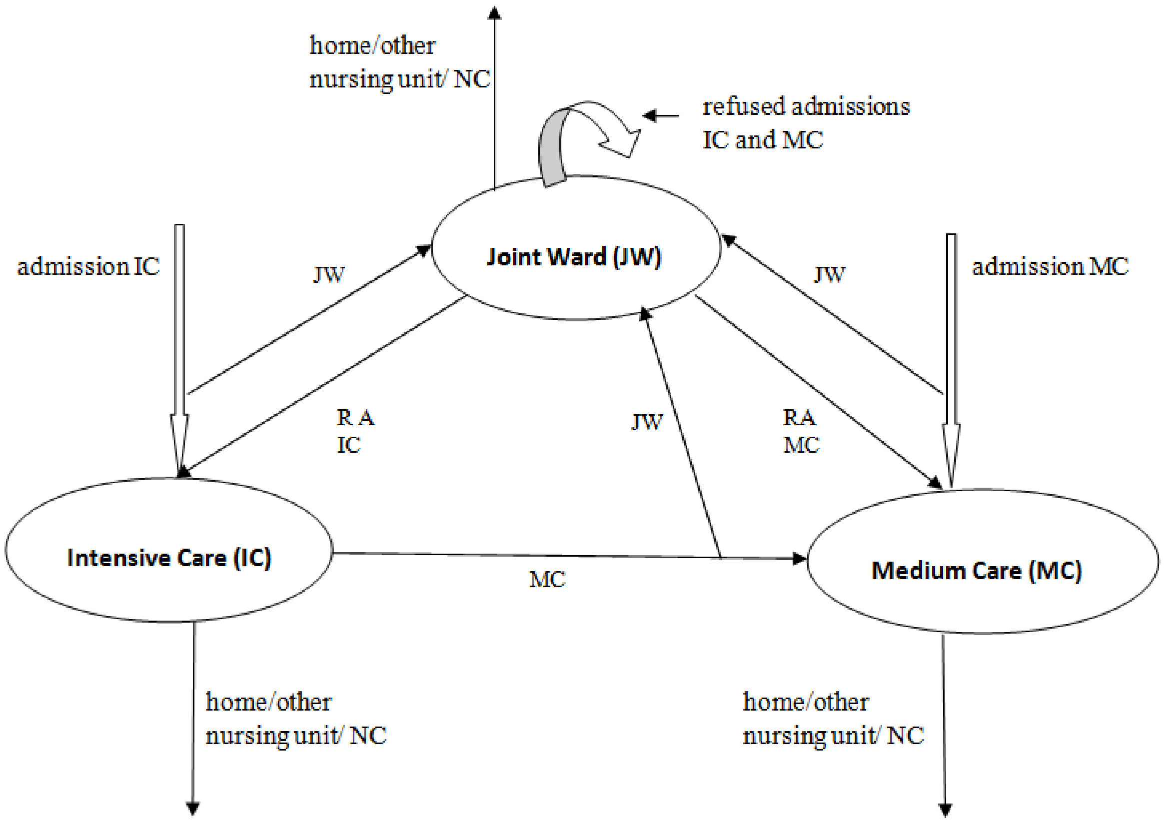
2. The three different kind of patient flow models

This section contains the definition of the three structural models and formulas to calculate the blocking probabilities for each model. These models describe the different patient routings in a qualitative manner and define the relations between different hospital units. I decided to identify three different kinds of patient flows. Model A is a model with internal refusals for the patient-flow from the IC to the MC. Model B is comparable with model A with the only difference that if a leaving IC patient who is transferred to the MC finds the MC as well as the joint ward full then the patient stays on the IC. Model C is a model with a joint ward and independent flows, without a patient-flow from the IC to the MC.

Models A and C can be compared to find out what the influence is of the patient flow from the IC to the MC. Model B can be compared with model A and model C to find out what the impact is on the IC if patients stay there when the joint ward and the MC are both occupied.

All models contain a joint ward. The name for this bed allocation policy is called the earmarking policy. Each patient type has a guaranteed number of beds. For this policy, the earmarked beds (available beds on the IC and MC) should always be used first before turning to the joint ward. Moreover, in the model we assume that as soon as an earmarked bed becomes available, a patient is transferred from the joint ward to an earmarked bed. In practice, this patient transfer will not occur immediately, but this generally happens in case the joint ward is fully occupied. Therefore, the modeling assumption of immediate patient transfers seems a realistic simplification [2]. Models A and B have an internal patient-flow from the IC to the MC. In model C the MC can be seen as an independent ward.

Model A: With internal refusals for patients going from IC to MC



Model A describes the overall patient flow where patients admit the system at the IC or MC. If an entering IC patient finds the ward full then the patient is sent to the joint ward (JW). If the JW is also full then the patient gets blocked and is sent home, to another nursing unit or to the NC. The same holds for an entering MC patient.

Patients lying on the IC can be transferred to the MC after their stay on the IC, or the patients leave the IC by going to another nursing unit such as the NC. If a patient coming from the IC finds both the MC and the joint ward full then the patient gets blocked (refused admission) and is transferred to another hospital or to another nursing unit. Patients lying on the MC can leave this particular ward by going home or to another nursing unit like the NC.

It is assumed in this model that, if an IC or a MC patient leaves respectively the IC or the MC then an IC or a MC patient lying on the joint ward is immediately transferred back to their belonging ward. This is called a re-admission (RA). IC and MC patients who enter the joint ward and stay there leave the joint ward by going home or to another nursing unit including the NC.

This model is used to investigate the influence of the patient-flow between the IC and the MC with internal refusals. This means that if the IC patient who is transferred to the MC gets blocked then there is no possibility that this patient can return to the IC again. Therefore, the patient is transferred to another nursing unit or another hospital.

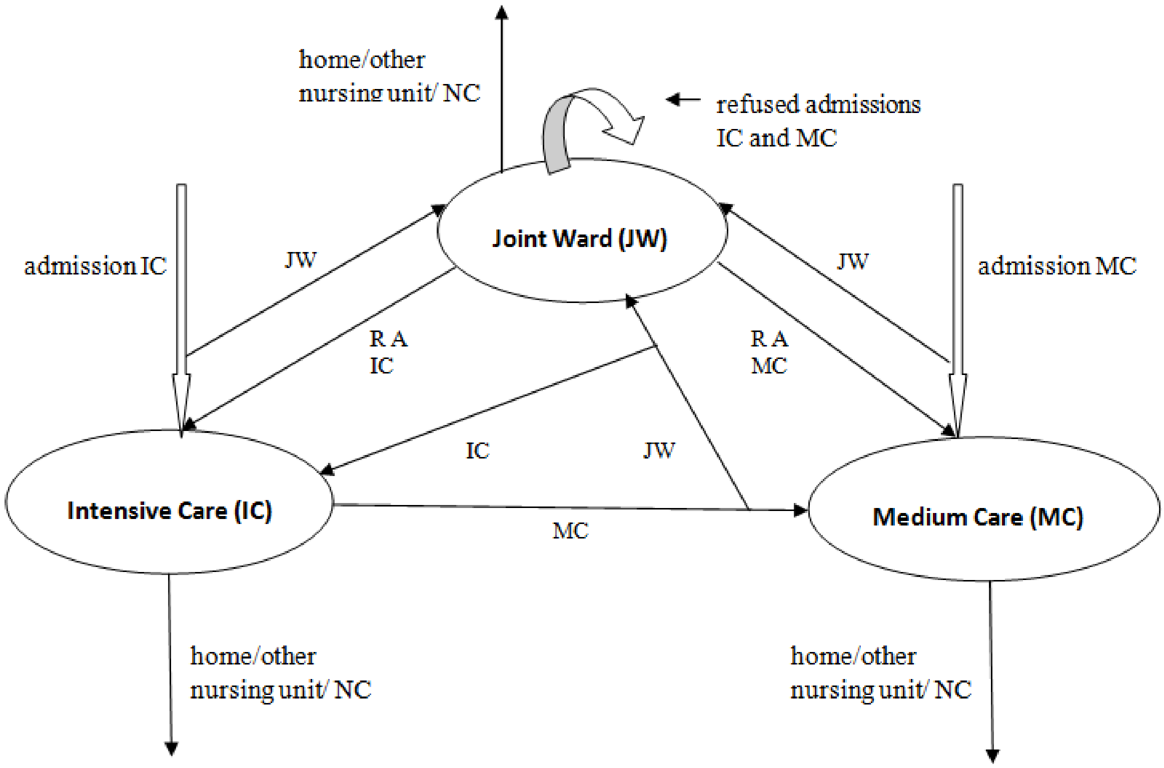
The internal refusals avoid that the IC can be fully occupied with MC patients and also serve as a comparison to model C, for which closed-form expressions are available.

The performance measures of this model are

- The blocking probability for the IC
- The blocking probability for the MC

The blocking probabilities are measured using a simulation model.

Model B: Without internal refusals for patients going from IC to MC



Model B describes the patient-flow where patients enter the system at the IC or MC. If an entering patient from the IC finds the ward full then the patient is sent to the joint ward. If the joint ward is full then the patient gets blocked and is sent to another hospital or to another nursing unit such as the NC. The same holds for an entering MC patient.
Patients leaving the IC can be transferred to the MC after their stay on the IC or these patients leave the IC by going to another nursing unit like the NC. If the patients coming from the IC find both the MC and the joint ward full, then instead of sending the patient to another hospital or another nursing unit the patient remains at the intensive care, thus the patient will not get blocked. Patients lying on the MC can leave this particular ward by going home or to another nursing unit including the NC.

Just like model A, it is assumed that if IC or MC patients leave respectively the IC or the MC then an IC or MC patient lying on the joint ward is transferred back to their belonging ward right away. This is called a re-admission (RA). If IC and MC patients who entered the joint ward are not transferred back to their ward they just leave the joint ward by going home, to another nursing unit or to the NC.

This model is used to investigate the influence of the patient-flow between the IC and the MC without internal refusals. This means that if the IC patient who goes to the MC after their stay on the IC finds both the MC and the joint ward full, then there is a possibility that this patient can return to the IC. Therefore the patient can never be transferred to another nursing unit or another hospital because the patient originally came from the IC.

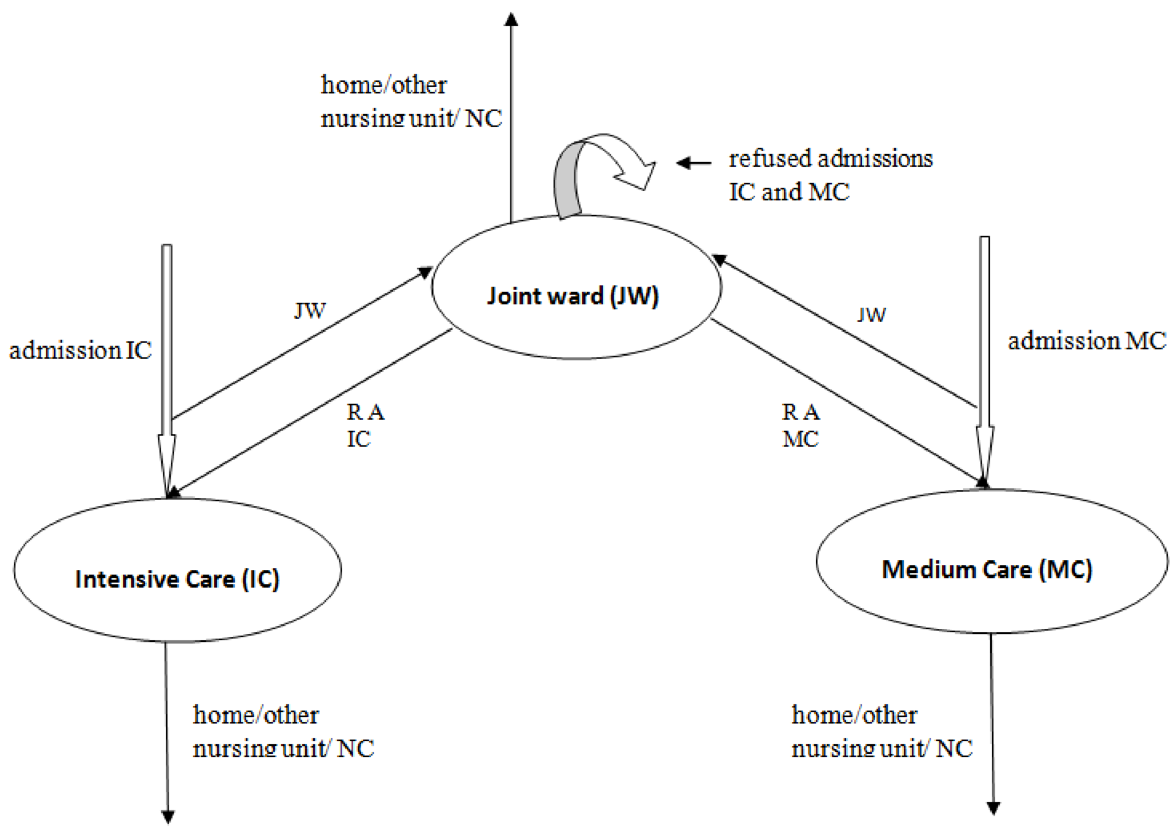
This model seems to reflect current practice, although there will be tighter bounds on the number of MC patients at the IC. Hence, a combination of model A and model B might be most appropriate. It, however, depends on many (external) factors whether patients are refused or not making any model an abstraction of reality.

The performance measures of this model are:

- The blocking probability of the IC
- The blocking probability of the MC
- The blocking probability of the MC without counting the MC patients who remain on the IC as blocked MC patients.
- Number of patients who find the MC and the JW full after their stay on IC
- Time MC patients spend on the IC.

The blocking probabilities are measured with a simulation model.
The blocking probability for the MC can be defined in two different ways. Namely, with and without counting the patients who remain at the IC as blocked patients.

Model C: Model with only independent admission flows



Model C describes the patient-flow where patients enter the system at the IC or MC. If an entering patient from the IC finds the ward full then the patient is sent to the joint ward. If the joint ward is full then the patient gets blocked and is sent to another hospital or another nursing unit like the NC. The same holds for an entering MC patient. This model does not contain a patient-flow between the IC and the MC. The IC and MC can be seen as independent wards. The MC has only external admissions.

Patients lying on IC leave the IC by going to another hospital or another nursing unit such as the NC. Patients lying on the MC can also leave this particular ward by going home, to another nursing unit including the NC. IC and MC patients who entered the joint ward because of finding their own ward full leave the ward by going to another hospital or to another nursing unit.

When the number of patients exceeds the specified number, then the patient can be admitted to a joint ward. An arriving patient of type IC or MC is admitted in case there is a bed available among the allocated (earmarked) beds on the IC or MC, or on the joint ward, and refused otherwise. This model does not have a patient-flow from the IC to the MC.

This model is used for the purpose to investigate the difference with and without a patient-flow between the IC and the MC.

The performance measures of this model are:

- The blocking probability of the IC
- The blocking probability of the MC

Because this model does not have a patient-flow between the IC and the MC and the IC and the MC can be seen as independent wards, we can use the product-form formula for calculating the blocking probabilities for the IC and the MC (see Section 5.4.1) [2].

3. Objectives and scenarios

In this section the objectives of this paper are described. The purpose of the first objective is the comparisons of the blocking probabilities between the three different models. This is interesting because for model C there already exists a product-form (see section 5.4.1) solution for calculating the blocking probabilities [2]. This objective is important for investigating whether the product-form formula for calculating the blocking probabilities gives an appropriate approximation for the models A and B which both have an internal patient flow between the IC and the MC.

As mentioned, the two main objectives are:

Objective 1:

- Do the blocking probabilities for models A, B and C show similar results?
  - Is it possible to use the product-form formula for model A?
  - How many beds are needed on each ward to achieve 5 % blocking probability for each patient type?
  - What is the optimal allocation combination of beds on the MC and IC in the absence of a joint ward? This gives IC* and MC*.
  - What is the optimal allocation combination of beds on the MC and IC while having a joint ward?
  - What might be the costs for a bed on the joint ward?

Objective 2:

- What is in model B the influence of the MC on the IC and what are the consequences for the admission of IC patients due to internal obstacles?

In the absence of a joint ward:

- What is the average time that an MC patient lies on the IC?
- How many IC beds are on average occupied by MC patients?
- What is the percentage of IC beds that are taken by MC patients?

With a joint ward:

- What is the average time that an MC patient lies on the IC?
- How many IC beds are on average occupied by MC patients?
- What is the percentage of IC beds that are taken by MC patients?

3.1 Clarifying objectives:

Objective 1 will be investigated considering the blocking probabilities of the IC and the MC for different kinds of scenarios. The scenarios are fully described below.

For Scenario 1 the number of beds on the joint ward is 0. Calculate the blocking probabilities for different bed allocations.

1. IC and MC are independent of each other but now the total admission rate of the MC consists of the direct admissions of the MC plus the admission of patients coming from the IC. The total number of available beds per ward on the IC and the MC varies from 0 to 40. [use model C]
2. Consider the internal patient flow and use models A, B and C. The total number of available beds per ward on the IC varies from 0 to 40.
3. The number of beds on the IC is fixed and the number of beds on the MC varies from 0 to 16. Repeat this for different fixed number of beds on the IC. [use models A,B,C]
4. Total number of beds on the MC and the IC stays 29. This is a *tight* scenario. The number of beds on the MC varies and therefore the number of beds on the IC varies as well. MC$\in \{0,1,2,\ldots,15,16\}$ [use models A, B and C]. Note that model B has different performance measures as model A and C.
5. Total number of beds on the MC and the IC stays 33. This is an *average* scenario. The number of beds on the MC varies and therefore the number of beds on the IC varies as well. MC$\in \{0,1,2,\ldots,15,16\}$ [use models A, B, C]. Note that model B has different performance measures as models A and C.
6. Total number of beds on the MC and the IC stays 37. This is a *wide* scenario. The number of beds on the MC varies and therefore the number of beds on the IC varies as well. MC$\in \{0,1,2,\ldots,15,16\}$ [use models A, B, C]. Note that model B has different performance measures as model A and C.
7. Compare models A and C to see if the product-form solution can be used.
8. Highlight for each model the blocking probabilities that are less than 5 %.
9. Find the optimal allocation of beds for the IC and MC for each scenario. This gives an IC* and MC* for each scenario.

Scenario 2: Take into account the joint ward (JW). Calculate all performance measures for each model for different bed allocations.

10. Total number of beds is MC*+IC*. Change the beds on the JW form 0 to10 with steps of 1 by decreasing the number of beds on the MC and the IC. [Use models A, B and C]. Note that model B has different performance measures.
    After each step the number of beds on the IC and the MC becomes:

    $$IC = IC^* - \left\lceil \frac{Number\ of\ beds\ on\ the\ JW}{2} \right\rceil$$

    $$MC = MC^* - \left\lfloor \frac{Number\ of\ beds\ on\ the\ JW}{2} \right\rfloor$$

    Where ⌈ ⌉ means rounding up and ⌊ ⌋ means rounding down.
    Repeat this for each scenario; *average, wide* and *tight*.

11. Use the optimal allocation formula (see section 5.5) and obtain results with taking different costs of a bed on the JW.

Objective 2 will be investigated considering all performance measures of the IC and the MC for different kinds of scenarios.

Scenario 1: the number of beds on the joint ward is 0.

12. Total number of beds on the MC and the IC stays 29. The number of beds on the MC varies and therefore the number of beds on the IC varies as well. MC$\in$ $\{0,1,2,\ldots,15,16\}$ [Use model B]. This is a tight scenario.
13. Total number of beds on the MC and the IC stays 33. The number of beds on the MC varies and therefore the number of beds on the IC varies as well. MC$\in$ $\{0,1,2,\ldots,15,16\}$ [use model B]. This is an average scenario.
14. Total number of beds on the MC and the IC stays 37. The number of beds on the MC varies and therefore the number of beds on the IC varies as well. MC$\in$ $\{0,1,2,\ldots,15,16\}$ [use model B]. This is an average scenario.

Scenario 2: Take into account the joint ward (JW). Calculate all performance measures for each model for different bed allocations.

15. Total number of beds is MC*+IC*. Change the number of beds on the JW form 0 to10 with steps of 1 by decreasing the number of beds on the MC and the IC. [Use Model B]. Note that model B has different performance measures.
    After each step the number of beds on the IC and the MC becomes:

    $$IC = IC^* - \left\lceil \frac{Number\ of\ beds\ on\ the\ JW}{2} \right\rceil$$

$$MC = MC^* - \left\lfloor \frac{Number\ of\ beds\ on\ the\ JW}{2} \right\rfloor$$

Where ⌈ ⌉ means rounding up and ⌊ ⌋ means rounding down.
Repeat this for each scenario; *average, wide* and *tight*.

16. Now we take for the start allocation the number of beds on the IC and the MC at which each ward does have a maximum blocking probability of 5 percent.
The rest of the process is the same as in 16.

4. Model assumptions and parameters.

4.1 Arrivals

We assume that patients in a hospital arrive according to a Poisson process; this is often a good approximation [3]. A Poisson process is a stochastic process which is related to the Poisson distribution. It is a counting process N (t) that counts the number of related events (for example arrivals at a bank, earthquakes or calls at a call center) during a specified time period t. To show what the properties are for a Poisson process we also need to define
$N(s, t) = N(t)-N(s)$ as the number of arrivals in (s, t]. Then, the counting process N(t) on [0,) is called a (homogeneous) Poisson process with rate λ and interarrival times $X_1, X_2, \ldots$ between events if the two definitions below hold:
- N(s, t) has a Poisson distribution with expectation $\lambda(t - s)$ for all $0 \leq s < t$;
- N(s, t) and N(s', t') are stochastically independent for all 0 s $\leq$ t $\leq$ s' < t'.
If N_1 and N_2 are two independent Poisson processes with rates λ_1 and λ_2, then $N = N_1 + N_2$ is also a Poisson process with rate $\lambda_1 + \lambda_2$ [5].

Based on historical data of the VUmc we assume the following:

- The total number of annual arrivals for the IC fluctuates around 1500 arrivals. Therefore the average number of patients arriving per day is 4,11. The external arrival process at the IC is modeled as a Poisson process with intensity $\lambda_{IC} = 4{,}11$.

- The total number of external annual arrivals for the MC (without the internal patient flow from the IC) fluctuates around 550 arrivals, that have an average of 1,51 arriving patients per day. The external arrival process at the MC is modeled as a Poisson process with intensity $\lambda_{MC} = 1{,}51$.

- The probability that an admitting IC patient who does not get blocked will go to the MC after his treatment is 0,215. The rest of the 0,785 patients will leave the IC and go elsewhere.

- If the number of beds on the IC is large enough to cover all IC patients, so that the blocking probability of the IC is close to zero, an additional 1500* 0,215= 322,5 patients are admitted the MC. That gives an average of 0,88 arriving patients per day.

Model C does not have an internal patient flow. For investigating the influence of the patient flow between the IC and the MC, model C must be compared with models A and B. For model C the MC is considered as an independent entity, so both external and internal arrivals should be taken into account, yielding a load comparable to models B and C. So for model C the $\lambda_{MC\ model\ C} = \lambda_{MC\ external} + \lambda_{ICMC}$.= 1,51+0,88 = 2,39 patients each day.

The λ_{IC} for the IC is the same for all models.

4.2 Length of stay:

The number of days in hospital for a patient is described by the term length of stay (LOS). LOS is defined as the time of discharge minus time of admission. Similarly, the average length of stay is denoted as ALOS. We assume that the LOS is exponentially distributed.

Based on historical data we use the following parameters:

- The average length of stay on the IC is 5,4 days.
- The average length of stay on the MC for external patients is 1,51 days.
- The average length of stay on the MC for patients coming from the IC is 5,3 days.

Model C does not have an internal patient flow. For investigating the influence of the patient flow between the IC and the MC, model C must be compared with models A and B. This comparison must be valid. Therefore the length of stay must be adapted. Moreover the MC consists of two different kinds of admission types. These admissions types do not have the same length of stay.

So for model C the average length of stay on the MC is:

$\lambda_{ICMC} / \lambda_{MC} * LOS_{ICMC} + \lambda_{Mc\ external} / \lambda_{MC} * LOS_{MC\ external:}$

(0,88 / 2,39) * 5,3 + (1,51 / 2,39) * 1,5 = 2,903

The length of stay for the IC is the same for all the models.

4.3 Total available beds:

Currently the available number of beds for the IC in VUmc is around 28 beds and the available number of beds for the MC is around 9 beds. In our model the total available number of beds per ward is pre-determined.

5. Performance Analysis

In this section I first explain what kind of simulation is used and what a simulation program is in general. Second, the basic assumptions and characteristics of the two different simulation programs I created are described. After that a more detailed description is given for models A and B about what the characteristics of each simulation program are and how the performance measures are calculated. Model C does have its own decision support system. This is explained in section 5.4. At the end of this section the formula for calculating the optimal bed allocation is explained.

5.1 Simulation:

The simulation program I created is an example of long term simulation. Another expression that can be used for long term simulation is the term steady sate simulation. This term presents very well the purpose of long term simulation. The aim of the program is to analyze the behavior of the system after that the start- up is finished and the system has statistical reached equilibrium. So we are interested in the average behavior of the system when the system runs for a long period. In our case the hospital is busy day and night and a hospital does not close at the end of the day. Therefore long term simulation is a good model for the simulation of the hospital. The long term performance measures for models A,B and C are the blocking probabilities for the IC as well as for the MC. However, model B has additional performance measures, namely the blocking probabilities of the MC without counting the patients, who remain at the IC, as blocked MC patients, the number of patients who find the MC and the joint ward full, and therefore remain at the IC, and the average time that a patient who actually belongs on the MC uses an IC bed. These performance measures are calculated with using a simulation program.

The technique that is used for these models is discrete-event simulation. We assume Xt $\subseteq N_0^m$. This means that Xt, which denotes the state of the system at time t, is an element from N_0^m, where N denotes all integers in the range from 0 to m (state space).
In models A, B and C, Xt is a vector: Xt = (x_1, x_2), where x_1 is the number of patients on the IC and x_2 denotes the number of occupied beds on the MC Every trajectory is a piece-wise constant function, where the process makes discrete jumps. For this reason such a process is called a discrete-event system. Discrete-event simulation is about generating trajectories of discrete-event systems. This is done by starting at time 0 and then constructing a trajectory by sampling one by one events in the system. Every event happens on a certain time step and forces the state to change. All discrete events are stored in an event list. [5]

The basic functionality of any simulation program is the fact that it can generate pseudo-

random numbers. Pseudo-random numbers are not really random (they are generated by some deterministic algorithm), but a sequence of pseudo-random numbers resembles, for most practical purposes, sufficiently well to real random numbers. Pseudo-random numbers are usually integers between 0 and some very large number (say N). From that we can construct realizations of other probability distributions. For example, by dividing by N we get numbers that are uniformly distributed on [0, 1]. Any random variable X with a known inverse distribution function F^{-1} can now be sampled as follows: if U is uniformly distributed on [0, 1], then $F^{-1}(U)$ has the same distribution as X:
$P(F^{-1}(U) \leq x) = P(U \leq F(x)) = F(x) = P(X \leq x)$. The interarrival times and length of stay of patients in a hospital are exponentially distributed. Therefore to generate pseudo-random numbers we need the inverse of the exponential distribution for a given random variable X. The code below gives an idea how the arrivals and the length of stay for patients on the IC are generated.

```
float draw_expArrivalIc() {                    // random sample from exponential distribution
        float dummy;
        dummy = rand()+1;                      // random from 1 to 1
        return -(1/(lambdaIc))*log((float)dummy/(RAND_MAX+2)); // Inverse Trafo Method
};      [6].
```

To understand how we end up with the performance measures first some assumptions and characteristics about the simulation model must be explained.
The runtime of the simulation is 5 years with a start-up period of half a year.
The start-up period achieves that a steady state is simulated, due to neglecting the impact of the initial state.
Each model does have its own simulation program. The similarities between these models is that before you can run the simulation you have to provide the program with the number of beds for each ward. This number will not change during the simulations. Another similarity is that they all have an event list and that the simulation starts by first initializing the first two events which consists of an admission to the IC and an admission to the MC. Both of these events have the parameters: arrival time, type of event and an event number. The events are placed in the event list. The arrival time is determined by a sample that is drawn from the exponential distribution with rate λ. As long as the event list has events the simulation program is continued. For each type of patient different calculations are done in different methods. Next to an event list we need separate lists for the IC, MC and the joint ward so that we can check what types of patients each ward consists of at any given moment.

There are six different kinds of events: admitting IC patient, admitting MC patient, admitting MC patient after a stay on the IC, departure of IC patient, departure of MC patient, departure of a joint ward patient.

5.2 Simulation model A:

An arriving IC patient has a chance of 0.215% that the patient is transferred to the MC after his stay on the IC. This is determined with a random generator. After this determination the first thing that has to be done is to check if the IC is full, if not then the patient is transferred to the IC. If the IC is full then the patient goes to the joint ward. If the joint ward is full as well then the patient gets blocked. For each patient that stays a sample is drawn from the exponential distribution to determine how long the patient will stay, then a departure event is created with parameters departure time, type of event and event number.

An arriving MC patient goes to the MC. If the MC is full then the patient will be transferred to the joint ward if still having available beds and otherwise the patient is blocked. For each patient that stays a sample is drawn from the exponential distribution to determine how long the patient will stay. Then a departure event is created with parameters departure time, type of event and event number.

MC patients coming from the IC first enter the MC if the MC has available beds and otherwise they are transferred to the joint ward. If all beds on the joint ward are occupied as well then the patient is blocked. For each patient that stays a sample is drawn from the exponential distribution to determine how long the patient stays, then a departure event is created with parameters departure time, type of event and event number.

An admitting joint ward patient is a patient that actually belongs to the IC or the MC.

When a new IC or MC patient arrives at the ward, either it gets blocked or not, a new arrival is created by drawing a sample from the exponential distribution.

If an IC or MC patient departs then one bed on respectively the IC or the MC becomes available again. Then first a check is done to see whether the joint ward does have a patient that belongs on that particular ward. If that is the case then the patient is transferred back to their belonging ward.

A departure of a patient on the joint ward simply results into one extra available bed on the joint ward.

The performance measures for this model are the blocking probabilities for the IC and the MC

The blocking probability for the IC is measured by counting the arriving IC patients who get blocked (so who find both the IC and the joint ward full) divided by the number of total arrived IC patients.

There are two different kinds of MC patient admissions. The patient can enter the MC immediately or the MC patient first could have had a stay on the IC. Therefore the blocking probability of the MC consists of the immediately entering MC patients who find the MC and the joint ward full plus the patients coming from the IC who find the MC and the joint ward full, divided by the total entering MC patients (patients coming directly or coming via the IC).

5.3 Simulation model B:

Model B does have the same characteristics as model A, with one important difference. When an MC patient is admitted that comes from the IC which finds the joint ward completely occupied then the patient is transferred back to the IC instead of getting blocked. The IC must have an available bed because the admitting patient on the MC comes originally from the IC.

Next to the check if the joint ward does have an MC patient when an MC patient departs from the MC, first the IC is checked to see whether the IC has patients who belong to the MC.

The performance measures for this model are

- The blocking probability of the IC
- The blocking probability of the MC
- The blocking probability of only the external MC.
- Number of patients who find the MC and the JW full after their stay on IC
- Time MC patients spend on the IC

When we know these performance measures, the percentage of the number of beds of the IC that is on average occupied by MC patients can now be calculated. This is done using the cost equation Little's Law[5];

$L=\lambda w$.

The λ denotes the number of patients who remain at the IC per day and w denotes the average time those patients spend on the IC per day. Now L, the number of IC beds that are taken by MC patients, can be calculated. The percentage of IC beds that are taken by MC patients is calculated by: (L / number of beds on the IC) * 100.

The blocking probability for the IC is measured by counting the arrived IC patients who get blocked (who find the IC and the joint ward full) divided by the total number of arrivals for the IC.

There are two different kinds of MC patient admissions, external and internal. Therefore the blocking probability of the MC consists of the immediately entering MC patients who find the MC and the joint ward full plus the patients coming from the IC who find the MC and the joint ward full, divided by the total entering MC patients (patients coming directly or coming from the IC).

To see what the influence of the MC patients is on the IC we also measure the blocking probability of the MC only by counting the immediate entering MC patients who get blocked divided by the total number of external patients on the MC.

5.4 Decision support system for model C:

Model C does not have an internal patient-flow from the IC to the MC. Therefore IC patients only leave the IC by going to another nursing unit or to the NC. As mentioned in Section 2 the blocking probabilities of the IC and MC when using model C can be calculated with the so called product-form solution [2]. Instead of calculating the blocking probabilities of the IC and the MC with a simulation program a decision support system with the implementation of the product form is used. This formula of the product form is derived below:

5.4.1 The calculation of blocking probabilities for model C

The loss fraction or the blocking probability can easily be derived for model C. In this model we have two types of patients namely IC and MC patients. Denote the number of type IC($_1$) patients and MC($_2$) patients at some time by x_j , j = 1,2 and let x = (x_1, x_2) be the corresponding vector. We assume that M_1 beds are reserved for IC patients and M_2 beds are reserved for MC patients, with $\sum_J \mathrm{Mj} \leq$ N,where N denotes the total available beds.
In case all beds for type j are occupied there is a ward of overflow that is shared by all patient types. The size of this joint ward is $M_{joint} = N - \sum_{j=1}^{2} Mj$Let Mi, i = 1, 2 and N be fixed and denote by x(t) the vector of the number of patients at time t. The stochastic process $\{x(t), t \geq 0\}$ then clearly is a Markov process with state space

$$S = \{x \in Z_1^2: x_1 < M_1 + (Mjoint - (x_2 - M_2)^+) \\ : x_2 < M_2 + (Mjoint - (x_1 - M_1)^+)\}$$

The transition rates q(x, x′) are given by $q(x, x') = \begin{cases} \lambda_i, & x' = x + e_i, \\ x_i \;{}_i, & x' = x - e_i. \end{cases}$ $x + e_i \in S$, Let $\pi(x)$ denotes the stationary distribution of x(t). The stationary distribution has a product form: $\pi(x) = G^{-1}\prod_{j=1}^{2}\frac{\rho_j^{x_j}}{x_j!}$ with $G = \sum_{x\in S}\prod_{j=1}^{2}\frac{\rho_j^{x_j}}{x_j!}$ as the normalizing constant.

ρ denotes the load of the ward that is: $\rho=\lambda$*average length of stay

This result can be derived by verifying that $\pi(x)$ satisfies the balance equations:

$\pi(x)\lambda_i = \pi(x + e_i)(x_i + 1)$ $_i$, $x, x + e_i \in S$. [2]

To obtain the fraction of refused admissions (blocking probabilities), define all the sets

$Sj = \{x \in S : x_1 = M_1 + Mjoint - (Mjoint - (x_2\text{-}M_2)^+)$

$x_2 = M_2 + Mjoint - (Mjoint - (x_1\text{-}M_1)^+)$

to get all the blocked states for the IC and the MC.

Using PASTA, it follows directly that blocking probabilities for the IC (w_1) are obtained using the formula: $w_1 = \sum_{x\in S1}\pi(x)$, and for the MC($w_2$) with the formula $w_2 = \sum_{x\in S2}\pi(x)$

Finally, we note that the product-form distribution is insensitive to the length of stay distribution. Hence, we only require the average length of stay to determine the performance of the earmarking policy with independent admission flows.[2] So this formula can also be used when the length of stay is not exponentially distributed as in this paper.

In brief, we give the number of beds of the IC, MC and the joint ward. Then the accepted states for each ward are obtained and for each accepted state it is checked whether the next patient gets blocked or not. Finally find the blocking probability for each ward by summing up the stationary distributions of all those blocked states.

The above calculations are implemented in a decision support system which determines for each ward the blocking probability. This decision support system has the following characteristics:

Input variables:

J= # type patients=2 (IC and MC)

M = total number of beds

M_i = number of beds for ward i (in this case IC and MC) number of beds for the joint ward is calculated by Mjoint= $M\text{-}M_{IC} - M_{MC}$.

λ_i = expected arrival rate for patient of type i. $i\in \{IC, MC\}$

β_i = expected service time (length of stay) for patient of type i. $i\in \{IC, MC\}$

Output variables:

b_{i} = blocking probability for patients of type i. $i \in \{IC, MC\}$

5.5 Optimal bed allocation:

For each type of patient, in this case the patients of the IC and the MC, we assign an α and the costs for a bed. α is the importance rate for a specific ward.
For the calculation of the optimal bed allocation we use the formula presented below.

$$\min \sum_{i=1}^{2} \alpha_i * w_i + \sum_{i=1}^{2} C_i * M_i + C_{joint} * M_{joint}$$

α_i = importance factor for patient of type i
w_i= blocking probability for patient of type i
C_i =cost of a bed for patient of type i
C_{joint} = costs of a bed for a patient on the "joint ward"
M_i = total number of beds for type i
Mjoint= number of beds on the joint ward
w_i= Blocking probability for patient of type i

Before we can use this formula, we need to assign all the parameters (alpha, costs and the number of beds) a relative value between 0 and 1. This is achieved by first adding all alphas, and calculating for each ward the new alpha by dividing alpha with the sum of the alphas. This way the fraction of the alphas is calculated. The same thing is done for the costs and the number of beds. So now all the parameters are between 0 and 1 and we can combine them together to use the formula.

In this paper we assume that the α_{IC} is twice as high as α_{MC} so that the cost for a bed on the IC is twice as high as a bed on the MC as well. To find the optimal bed allocation first, we obtain for each bed allocation the result of the formula. The bed allocation with the lowest value represents the optimal bed allocation.

6. Results

In this section I present the results of the objectives, which are described earlier in Section 3.

6.1 Objective 1: “Do the blocking probabilities for the models show similar results?”

6.1.1 Scenario 1: without a joint ward

In figure 1 the blocking probabilities of the independent IC and MC are presented without using a joint ward.

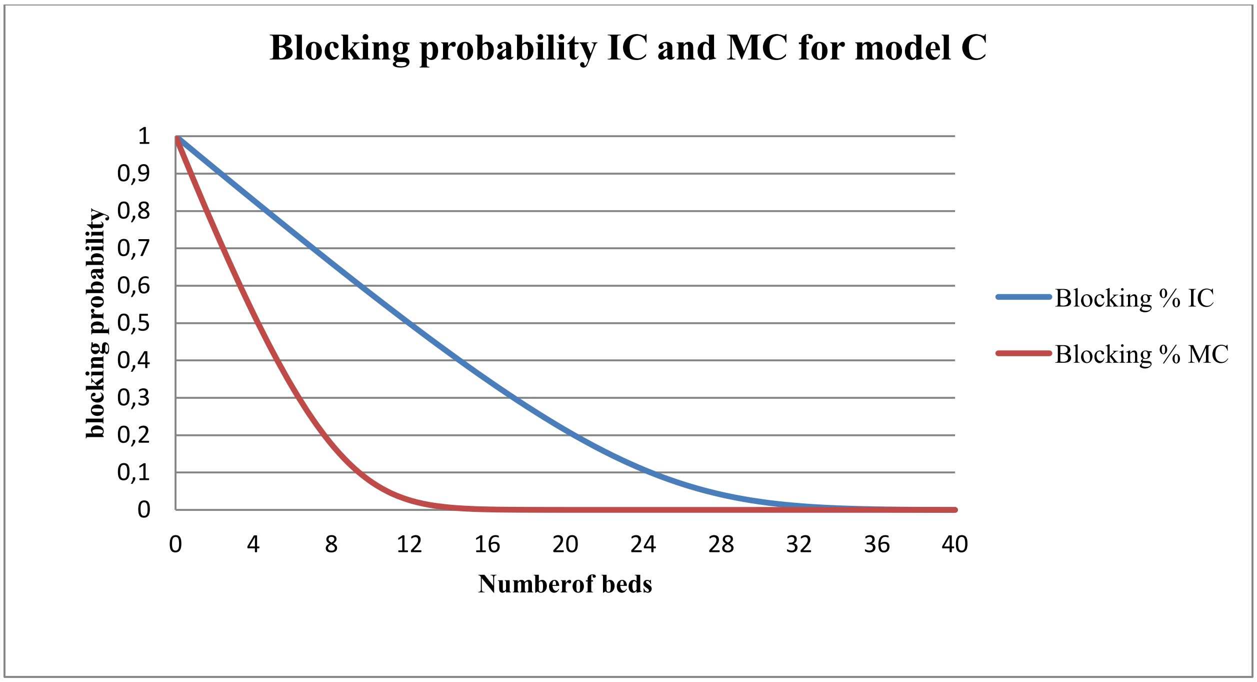


Figure 1 Blocking probability for independent IC and MC for model C

Treating the IC and MC as independent wards this situation can be considered as two independent wards with their own load (see Sections 4.1 and 4.2). We see that approximately 11 beds for the MC and 28 beds for the IC are needed to achieve a blocking probability of approximately 5 %.

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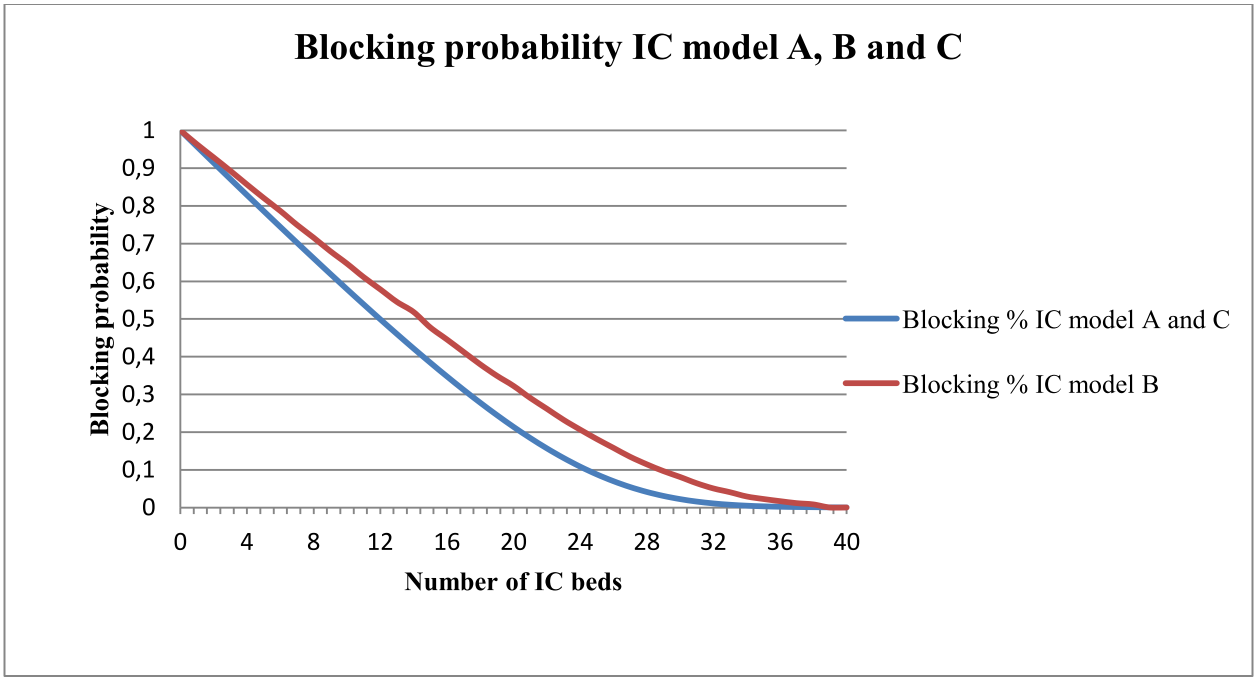

Figure 2 Blocking probability IC model A, B and C

Figure 2 presents the blocking probabilities for the IC for models A, B and C. The number of beds on the IC varies from 0 to 40 and the number of beds on the MC is zero. The blocking probabilities for the IC in models A and C are identical. Moreover if the hospital uses model B and the capacity of the MC is small, we see that the blocking probabilities for the IC are significantly higher. Instead of the 28 beds to achieve a maximum blocking probability of 5 percent now we need approximately 32 beds to achieve the same result. This is a difference of 4 beds which is a lot. The models differ the most in the range of approximately 16 to 32 beds on the IC. If the MC does not have the capacity to receive the patients coming from the IC, using model B will cause problems which result into a higher blocking probability on the IC.

In figure 3 we see the blocking probabilities of the MC for model A given a specified number of beds on the IC. If the number of beds on the IC ranges from 1 to 3, 7 beds are enough to have on the MC, although this is not a realistic situation. In model A the less IC admissions are blocked the more patients will become a patient that will enter the MC after their stay on the MC. Therefore the MC blocking probability becomes higher if the IC has more beds available. If the IC consists of a number of 31 beds or more the blocking probabilities of the MC will no longer increase and are close to zero and therefore the number of internal MC patients is at most and will not differ anymore.

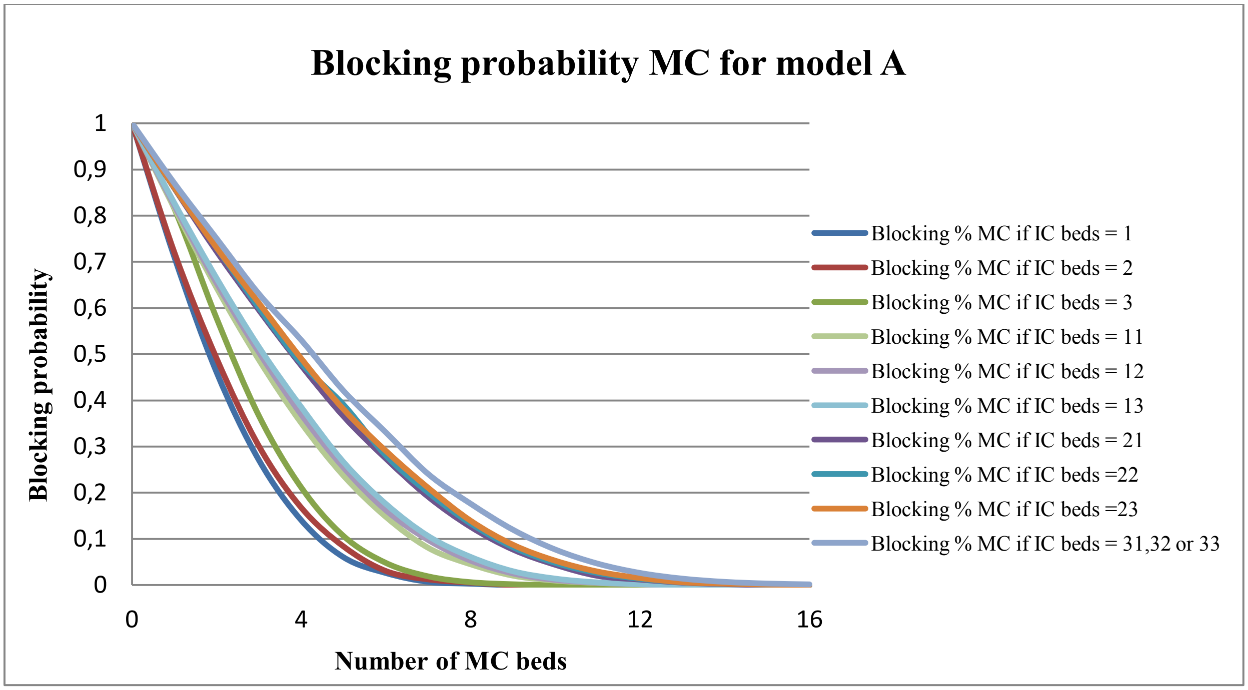

Figure 3 Blocking probability MC for model A without JW with internal patient flow

In figure 4 we see the blocking probabilities of the MC for model B given a specified number of beds on the IC. Note that the blocked MC patients partly involve IC patients that are medically ready for the MC, who find the MC full and therefore remain at the IC because of this the blocking probabilities of the MC when having a low number of beds behave quite differently.

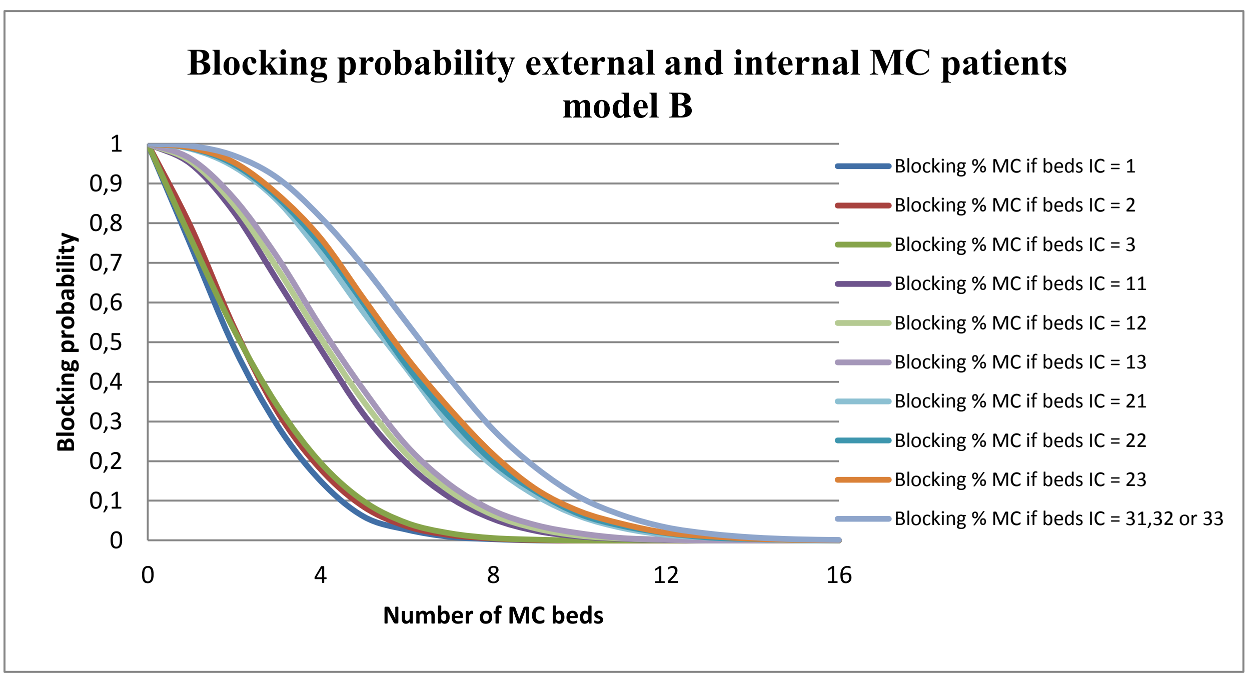

Figure 4 Blocking probability internal and external MC patients for model B

In figure 5 we see the blocking probabilities of the external MC patients for model B. For this model the lower the number of beds on the IC, the faster the blocking probability of the MC decreases. The higher the number of beds on the IC the more beds on the MC are needed to achieve the same blockings probability.

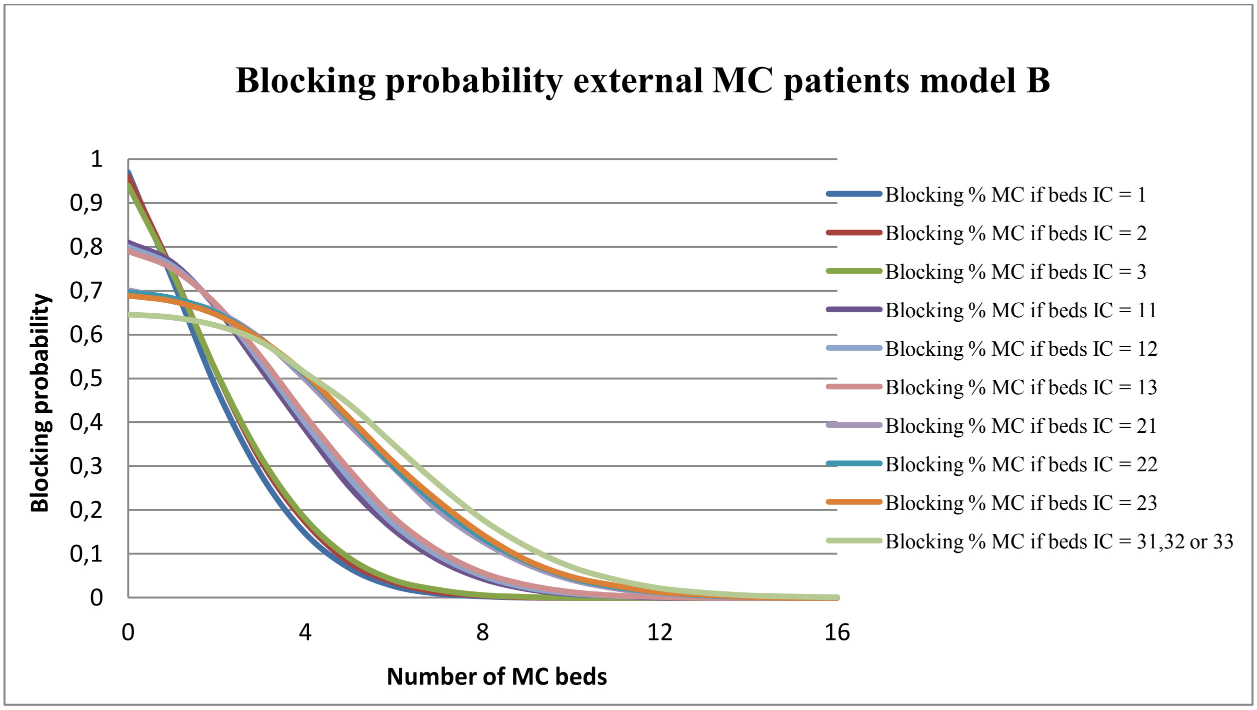

Figure 5 Blocking probability external MC patients for model B

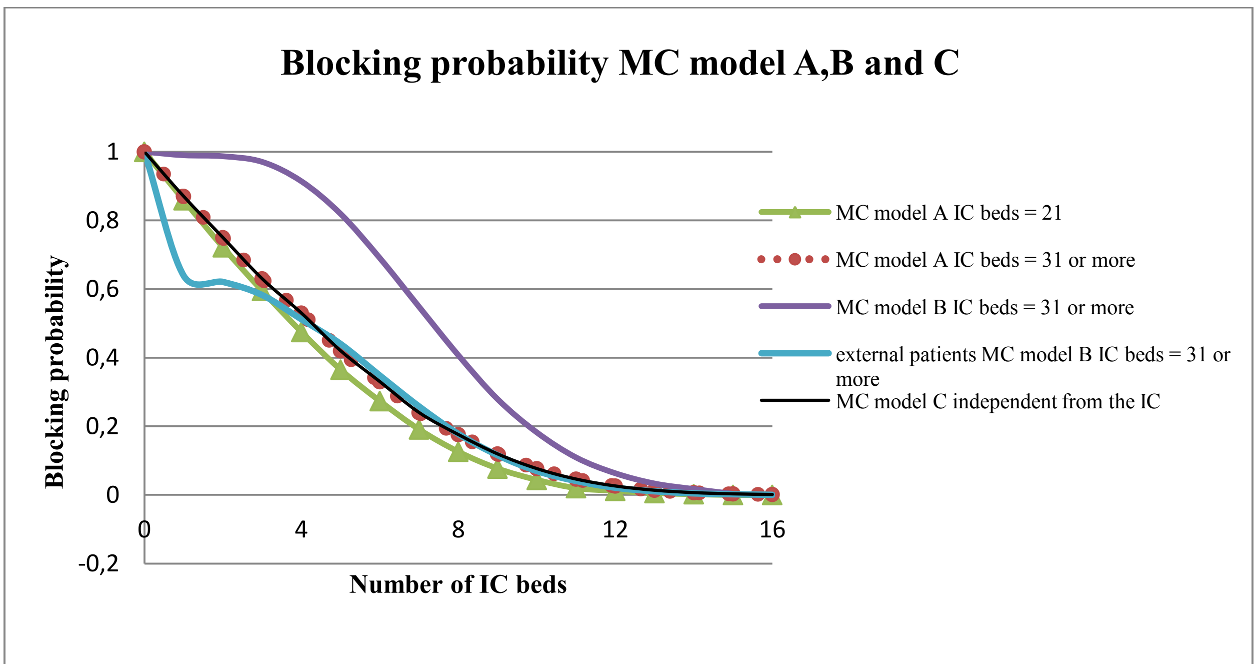

Figure 6 Blocking probability MC model A B, C

The blocking probabilities for the MC for models A, B and C are shown in figure 6. As mentioned above the more beds available on the IC, the higher the blocking probability for the MC in model A. The blocking percentage in model C for the MC does not depend on the number of available beds on the IC because the IC and MC are independent wards. Note that if the IC has 31 or more beds available in model A, then the blocking probability of the MC is the same for both models. This means that if the IC has 31 beds or more the blocking probabilities for the MC of model A can be compared with model C. Then the IC and MC in model A can be seen as having independent wards and the interaction between the IC and M can be neglected. However, the approximate blocking probability of the MC in model A with only 21 beds on the IC (see figure 6) approaches the line of 31 beds quite good. The blocking probability of only the external MC patients, (model B) can be compared with model A and C if the number of beds on the MC is 4 or more. Thus if internal MC patients remain at the IC if the MC is full and if the available beds on the MC is 4 or more, then the external MC patients will not be affected by the internal MC patients.

Number of beds			Blocking probabilities for the IC and MC			
IC	MC	JW	Model A IC	Model C IC	Model A MC	Model C MC
33	0	0	0,008	0,007	1,000	1,000
32	1	0	0,011	0,011	0,872	0,874
31	2	0	0,015	0,016	0,750	0,752
30	3	0	0,023	0,022	0,631	0,635
29	4	0	0,032	0,031	0,518	0,525
28	5	0	0,041	0,041	0,415	0,422
27	6	0	0,054	0,054	0,313	0,328
26	7	0	0,068	0,070	0,226	0,246
25	8	0	0,087	0,088	0,150	0,176
24	9	0	0,108	0,108	0,092	0,120
23	10	0	0,130	0,131	0,054	0,077
22	11	0	0,158	0,157	0,026	0,046
21	12	0	0,183	0,184	0,011	0,026
20	13	0	0,215	0,214	0,001	0,014
19	14	0	0,245	0,245	0,000	0,007
18	15	0	0,279	0,278	0,000	0,003
17	16	0	0,313	0,312	0,000	0,001
Average scenario ↑						
37	0	0	0,001	0,001	1,000	1,000
36	1	0	0,002	0,002	0,874	0,874
35	2	0	0,000	0,003	0,752	0,752
34	3	0	0,000	0,005	0,635	0,635
33	4	0	0,007	0,007	0,522	0,525
32	5	0	0,012	0,011	0,418	0,422
31	6	0	0,015	0,016	0,320	0,328
30	7	0	0,022	0,022	0,238	0,246
29	8	0	0,031	0,031	0,168	0,176
28	9	0	0,041	0,041	0,110	0,120
27	10	0	0,053	0,054	0,067	0,077
26	11	0	0,069	0,070	0,037	0,046
25	12	0	0,088	0,088	0,017	0,026
24	13	0	0,109	0,108	0,009	0,014
23	14	0	0,131	0,131	0,003	0,007
22	15	0	0,156	0,157	0,001	0,003
21	16	0	0,186	0,184	0,000	0,001
Wide scenario ↑						
29	0	0	0,030	0,031	1,000	1,000
28	1	0	0,041	0,041	0,872	0,874
27	2	0	0,054	0,054	0,745	0,752
26	3	0	0,070	0,070	0,622	0,635
25	4	0	0,087	0,088	0,503	0,525
24	5	0	0,108	0,108	0,387	0,422
23	6	0	0,131	0,131	0,289	0,328
22	7	0	0,158	0,157	0,197	0,246
21	8	0	0,183	0,184	0,124	0,176
20	9	0	0,213	0,214	0,073	0,120
19	10	0	0,244	0,245	0,036	0,077
18	11	0	0,279	0,278	0,016	0,046
17	12	0	0,313	0,312	0,006	0,026
16	13	0	0,348	0,348	0,002	0,014
15	14	0	0,384	0,384	0,001	0,007
14	15	0	0,421	0,422	0,000	0,003
13	16	0	0,460	0,460	0,000	0,001
Tight scenario ↑						

Table 1

- Model A IC = Model C IC
- Blocking probabilities for the MC in model C are higher than model A MC.
- The less blockings for the IC (wide scenario) the more model A and C show similarities between the blocking probabilities of the MC.
- For the most bed allocations the blocking probabilities for the MC for both models do not differ a lot. Therefore the product-form solution cannot be used for the exact calculations of the blocking probabilities for the MC but it is a good approximation.

In table 1 you can find the blocking probabilities of the IC and MC for models A and C for the three scenarios: *average, wide* and *tight*. The findings are presented next to this table. The outcomes of the performance measures for the IC in models A and C are exactly the same. Therefore we can use the product-form solution (Section 5.4.1) to calculate those blocking probabilities. For the MC we cannot use the formula for calculating the exact blocking probabilities but we can use it for the MC if we want an approximation of the blocking probabilities.

Number of beds			Blocking probabilities						
IC	MC	JW	Model A IC	Model A MC	Model B IC	Model B MC	Model B MC without	Model C IC	Model C MC
33	0	0	0,008	1,000	0,040	1,000	0,640	0,007	1,000
32	1	0	0,011	0,872	0,041	0,990	0,635	0,011	0,874
31	2	0	0,015	0,750	0,041	0,970	0,621	0,016	0,752
30	3	0	0,023	0,631	0,042	0,912	0,583	0,022	0,635
29	4	0	0,032	0,518	0,045	0,806	0,519	0,031	0,525
28	5	0	0,041	0,415	0,050	0,675	0,435	0,041	0,422
27	6	0	0,054	0,313	0,061	0,518	0,336	0,054	0,328
26	7	0	0,068	0,226	0,073	0,365	0,237	0,070	0,246
25	8	0	0,087	0,150	0,090	0,242	0,159	0,088	0,176
24	9	0	0,108	0,092	0,111	0,143	0,094	0,108	0,120
23	10	0	0,130	0,054	0,132	0,072	0,048	0,131	0,077
22	11	0	0,158	0,026	0,157	0,036	0,024	0,157	0,046
21	12	0	0,183	0,011	0,184	0,016	0,010	0,184	0,026
20	13	0	0,215	0,001	0,214	0,006	0,004	0,214	0,014
19	14	0	0,245	0,000	0,244	0,002	0,001	0,245	0,007
18	15	0	0,279	0,000	0,279	0,001	0,001	0,278	0,003
17	16	0	0,313	0,000	0,311	0,000	0,000	0,312	0,001

Average scenario ↑

37	0	0	0,001	1,000	0,012	1,000	0,632	0,001	1,000
36	1	0	0,002	0,874	0,013	0,995	0,630	0,002	0,874
35	2	0	0,000	0,752	0,013	0,973	0,616	0,003	0,752
34	3	0	0,000	0,635	0,013	0,916	0,580	0,005	0,635
33	4	0	0,007	0,522	0,014	0,824	0,520	0,007	0,525
32	5	0	0,012	0,418	0,015	0,691	0,441	0,011	0,422
31	6	0	0,015	0,320	0,019	0,545	0,346	0,016	0,328
30	7	0	0,022	0,238	0,024	0,398	0,254	0,022	0,246
29	8	0	0,031	0,168	0,032	0,274	0,176	0,031	0,176
28	9	0	0,041	0,110	0,042	0,173	0,111	0,041	0,120
27	10	0	0,053	0,067	0,055	0,095	0,060	0,054	0,077
26	11	0	0,069	0,037	0,069	0,049	0,030	0,070	0,046
25	12	0	0,088	0,017	0,088	0,025	0,016	0,088	0,026
24	13	0	0,109	0,009	0,108	0,011	0,007	0,108	0,014
23	14	0	0,131	0,003	0,131	0,004	0,002	0,131	0,007
22	15	0	0,156	0,001	0,156	0,001	0,001	0,157	0,003
21	16	0	0,186	0,000	0,183	0,000	0,000	0,184	0,001

Wide scenario ↑

29	0	0	0,030	1,000	0,096	1,000	0,650	0,031	1,000
28	1	0	0,041	0,872	0,097	0,994	0,650	0,041	0,874
27	2	0	0,054	0,745	0,098	0,965	0,630	0,054	0,752
26	3	0	0,070	0,622	0,103	0,896	0,588	0,070	0,635
25	4	0	0,087	0,503	0,109	0,779	0,515	0,088	0,525
24	5	0	0,108	0,387	0,120	0,625	0,414	0,108	0,422
23	6	0	0,131	0,289	0,140	0,460	0,313	0,131	0,328
22	7	0	0,158	0,197	0,159	0,311	0,209	0,157	0,246
21	8	0	0,185	0,124	0,186	0,187	0,128	0,184	0,176
20	9	0	0,214	0,073	0,215	0,101	0,070	0,214	0,120
19	10	0	0,244	0,036	0,245	0,048	0,034	0,245	0,077
18	11	0	0,279	0,016	0,278	0,020	0,014	0,278	0,046
17	12	0	0,313	0,006	0,313	0,007	0,005	0,312	0,026
16	13	0	0,348	0,002	0,347	0,002	0,002	0,348	0,014
15	14	0	0,384	0,001	0,384	0,001	0,000	0,384	0,007
14	15	0	0,421	0,000	0,421	0,000	0,000	0,422	0,003
13	16	0	0,460	0,000	0,460	0,000	0,000	0,460	0,001

Tight scenario ↑

Table 2

In table 2 we show, for the three different scenarios; *average, wide* and *tight*, again the blocking probabilities of the IC and the MC but now for all models. The yellow highlighted cells indicate the allocations which have a blocking probability less than 5 %. As you can see the three models do not differ much from each other. The IC blocking probabilities for the IC in model B only differ for a tight scenario. You can see in table 2 that having approximately 28 beds on the IC and around 11 beds on the MC gives a blocking probability of approximately 5 % for each scenario. Model C shows similar blocking probabilities for 28 beds on the IC and 11 beds on the MC. Therefore the product-form is a good approximation for calculating the required bed capacity.

Table 3 shows the optimal bed allocation values for the three scenarios. The values are obtained with the optimal bed allocation formula (see Section 5.5). This formula needs for each ward an alpha which indicates the importance of the ward. The alpha of the IC is twice as high as the alpha of the MC. The highlighted cells present the minimum value and therefore the optimal bed allocation solution. It is interesting to notice that for models A, B (internal and external blocked MC patients) and C the optimal bed allocation for all three scenarios show the same pattern. The most optimal solution to represent the reality of the hospital is to use model B (internal MC patients do not count as blocking persons), since this model has the lowest minimum value overall. An additional benefit if using this model is that the hospital needs less available MC beds than for other models.

Number of beds												
IC	MC	JW	Costs IC	Costs MC	costs JW	α IC	α MC	Model A	Model B	Model B without	Model C
33	0	0	2	1	0	2	1	1,005	1,027	1,188	1,005
32	1	0	2	1	0	2	1	0,954	1,014	0,841	0,955
31	2	0	2	1	0	2	1	0,906	0,997	0,818	0,908
30	3	0	2	1	0	2	1	0,862	0,968	0,780	0,863
29	4	0	2	1	0	2	1	0,820	0,925	0,724	0,822
28	5	0	2	1	0	2	1	0,782	0,875	0,660	0,784
27	6	0	2	1	0	2	1	0,746	0,819	0,592	0,752
26	7	0	2	1	0	2	1	0,717	0,766	0,531	0,724
25	8	0	2	1	0	2	1	0,694	0,726	0,489	0,703
24	9	0	2	1	0	2	1	0,678	0,697	0,461	0,688
23	10	0	2	1	0	2	1	0,670	0,678	0,445	0,679
22	11	0	2	1	0	2	1	0,670	0,672	0,446	0,675
21	12	0	2	1	0	2	1	0,671	0,673	0,455	0,677
20	13	0	2	1	0	2	1	0,679	0,680	0,468	0,682
19	14	0	2	1	0	2	1	0,689	0,689	0,484	0,691
18	15	0	2	1	0	2	1	0,701	0,701	0,504	0,701
17	16	0	2	1	0	2	1	0,714	0,713	0,522	0,714
Average scenario ↑											
37	0	0	2	1	0	2	1	1,001	1,008	1,173	1,001
36	1	0	2	1	0	2	1	0,950	0,998	0,825	0,950
35	2	0	2	1	0	2	1	0,900	0,981	0,801	0,901
34	3	0	2	1	0	2	1	0,852	0,954	0,762	0,855
33	4	0	2	1	0	2	1	0,809	0,915	0,710	0,810
32	5	0	2	1	0	2	1	0,769	0,862	0,643	0,769
31	6	0	2	1	0	2	1	0,730	0,807	0,575	0,732
30	7	0	2	1	0	2	1	0,698	0,752	0,512	0,700
29	8	0	2	1	0	2	1	0,671	0,708	0,464	0,674
28	9	0	2	1	0	2	1	0,649	0,671	0,428	0,653
27	10	0	2	1	0	2	1	0,634	0,645	0,405	0,638
26	11	0	2	1	0	2	1	0,626	0,630	0,397	0,629
25	12	0	2	1	0	2	1	0,623	0,625	0,399	0,626
24	13	0	2	1	0	2	1	0,625	0,625	0,407	0,626
23	14	0	2	1	0	2	1	0,629	0,629	0,418	0,630
22	15	0	2	1	0	2	1	0,636	0,636	0,433	0,637
21	16	0	2	1	0	2	1	0,647	0,645	0,449	0,646
Wide scenario ↑											
29	0	0	2	1	0	2	1	1,020	1,064	1,219	1,020
28	1	0	2	1	0	2	1	0,973	1,051	0,886	0,974
27	2	0	2	1	0	2	1	0,928	1,031	0,858	0,931
26	3	0	2	1	0	2	1	0,886	0,999	0,816	0,890
25	4	0	2	1	0	2	1	0,846	0,953	0,757	0,854
24	5	0	2	1	0	2	1	0,810	0,898	0,687	0,822
23	6	0	2	1	0	2	1	0,781	0,845	0,621	0,795
22	7	0	2	1	0	2	1	0,757	0,796	0,565	0,773
21	8	0	2	1	0	2	1	0,738	0,761	0,527	0,756
20	9	0	2	1	0	2	1	0,730	0,740	0,508	0,746
19	10	0	2	1	0	2	1	0,726	0,731	0,503	0,741
18	11	0	2	1	0	2	1	0,731	0,732	0,511	0,741
17	12	0	2	1	0	2	1	0,740	0,740	0,525	0,746
16	13	0	2	1	0	2	1	0,750	0,749	0,541	0,754
15	14	0	2	1	0	2	1	0,762	0,762	0,560	0,764
14	15	0	2	1	0	2	1	0,775	0,775	0,579	0,776
13	16	0	2	1	0	2	1	0,789	0,789	0,600	0,790
Tight scenario ↑											

Table 3 Optimal bed allocation

The most optimal bed allocation with optimal value of 0,397 (red highlighted cell) is, using table 3, 26 beds on the IC and 11 beds available on the MC, which can be achieved using model B with only counting the external MC patients as blockings. If we look at each scenario, the optimal bed allocation for the average scenario is on average 23 IC beds and 10 MC beds. For the wide scenario the results show an optimal bed allocation of 26 beds on the IC and 11 beds for the MC. The tight scenario has an optimal bed allocation of 19 beds for the IC and 10 on the MC.

I think the only realistic bed allocation is to have 26 beds on the IC and 10 beds on the IC. This result implies that the chosen alpha of 2 for the IC is probably too low. This indicates that the optimization task is hard to conduct since we keep asking ourselves “How to choose the correct alphas?”

6.1.2 Scenario 2 with joint ward:

Number of beds			Blocking probabilities						
IC	MC	JW	Model A IC	Model A MC	Model B IC	Model b MC	Model b MC without	Model C IC	Model C MC
23	10	0	0,131	0,052	0,132	0,076	0,051	0,131	0,076
22	10	1	0,132	0,031	0,131	0,041	0,028	0,132	0,050
22	9	2	0,109	0,040	0,111	0,051	0,034	0,112	0,056
21	9	3	0,112	0,030	0,112	0,036	0,024	0,113	0,043
21	8	4	0,093	0,041	0,095	0,049	0,033	0,091	0,052
20	8	5	0,093	0,034	0,096	0,041	0,027	0,097	0,046
20	7	6	0,079	0,045	0,086	0,053	0,035	0,085	0,055
19	7	7	0,079	0,042	0,084	0,048	0,052	0,085	0,052
19	6	8	0,071	0,052	0,076	0,058	0,039	0,077	0,060
18	6	9	0,072	0,051	0,077	0,056	0,038	0,077	0,059
18	5	10	0,065	0,058	0,073	0,064	0,044	0,072	0,064
Avarage scenario ↑									
26	11	0	0,071	0,036	0,070	0,051	0,033	0,070	0,046
25	11	1	0,069	0,020	0,069	0,026	0,017	0,071	0,027
25	10	2	0,055	0,024	0,055	0,014	0,019	0,056	0,030
24	10	3	0,055	0,014	0,056	0,019	0,012	0,056	0,020
24	9	4	0,043	0,020	0,045	0,024	0,015	0,045	0,023
23	9	5	0,043	0,016	0,044	0,017	0,011	0,045	0,018
23	8	6	0,035	0,020	0,036	0,023	0,014	0,037	0,022
22	8	7	0,036	0,018	0,037	0,019	0,012	0,037	0,020
22	7	8	0,030	0,022	0,031	0,023	0,015	0,032	0,023
21	7	9	0,030	0,021	0,033	0,023	0,015	0,032	0,022
21	6	10	0,027	0,024	0,028	0,025	0,017	0,029	0,024
Wide Scenario ↑									
19	10	0	0,245	0,037	0,245	0,048	0,034	0,245	0,076
18	10	1	0,245	0,023	0,246	0,029	0,021	0,246	0,053
18	9	2	0,217	0,033	0,216	0,042	0,029	0,217	0,066
17	9	3	0,215	0,028	0,216	0,034	0,024	0,219	0,057
17	8	4	0,189	0,045	0,193	0,055	0,038	0,194	0,075
16	8	5	0,190	0,042	0,191	0,049	0,034	0,194	0,071
16	7	6	0,167	0,062	0,173	0,072	0,052	0,173	0,091
15	7	7	0,171	0,059	0,172	0,068	0,049	0,174	0,090
15	6	8	0,153	0,080	0,157	0,088	0,063	0,158	0,108
14	6	9	0,154	0,075	0,158	0,084	0,062	0,158	0,107
14	5	10	0,141	0,092	0,147	0,104	0,077	0,147	0,121
Tight Scenario↑									

Table 4 blocking probabilities for models A,B and C

When taken into account a JW the blocking probabilities of the IC and MC of each model do not differ significantly from each other. However, for each scenario and model the more available beds on the JW the lower the blocking probability of the IC. The blocking probabilities of the IC for models A,B and C are very similar. Therefore for calculating these probabilities we might use the product-form solution.

When we look at the tight scenario we see that for model A,B and C the blocking probabilities of the MC differ more than for the IC. The wide scenario shows a significant difference in the blocking probabilities of the MC, however, the product-form could be a practical approximation for the blocking probabilities. Thus, if the number of beds on the IC is realistic and 26 or more

we can use the product-form for calculation of the blocking percentages.
Table 4 does not take into account the costs of a bed on the joint ward. To figure out what the costs of a joint ward might be, we obtained the results below. Table 5 and table 6 show the optimal bed allocation for different costs of a bed on the JW.

Number of beds												
IC	MC	JW	Costs IC	Costs MC	costs JW	α IC	α MC	Model A	Model B	Model B without	Model C
23	10	0	2	1	1	2	1	0,529	0,538	0,366	0,537
22	10	1	2	1	1	2	1	0,515	0,518	0,353	0,521
22	9	2	2	1	1	2	1	0,503	0,508	0,343	0,510
21	9	3	2	1	1	2	1	0,494	0,496	0,339	0,499
21	8	4	2	1	1	2	1	0,485	0,489	0,332	0,487
20	8	5	2	1	1	2	1	0,475	0,479	0,330	0,482
20	7	6	2	1	1	2	1	0,470	0,476	0,327	0,477
19	7	7	2	1	1	2	1	0,461	0,466	0,324	0,468
19	6	8	2	1	1	2	1	0,459	0,464	0,322	0,465
18	6	9	2	1	1	2	1	0,451	0,456	0,322	0,457
18	5	10	2	1	1	2	1	0,449	0,456	0,321	0,456
Average scenario ↑											
26	11	0	2	1	1	2	1	0,485	0,490	0,316	0,488
25	11	1	2	1	1	2	1	0,472	0,474	0,307	0,475
25	10	2	2	1	1	2	1	0,464	0,460	0,293	0,466
24	10	3	2	1	1	2	1	0,453	0,456	0,295	0,456
24	9	4	2	1	1	2	1	0,447	0,450	0,289	0,450
23	9	5	2	1	1	2	1	0,439	0,441	0,287	0,441
23	8	6	2	1	1	2	1	0,435	0,437	0,283	0,437
22	8	7	2	1	1	2	1	0,429	0,430	0,282	0,430
22	7	8	2	1	1	2	1	0,426	0,427	0,279	0,428
21	7	9	2	1	1	2	1	0,419	0,421	0,280	0,421
21	6	10	2	1	1	2	1	0,418	0,419	0,278	0,419
Wide scenario ↑											
19	10	0	2	1	1	2	1	0,589	0,593	0,428	0,602
18	10	1	2	1	1	2	1	0,576	0,579	0,422	0,587
18	9	2	2	1	1	2	1	0,561	0,563	0,408	0,572
17	9	3	2	1	1	2	1	0,549	0,552	0,406	0,562
17	8	4	2	1	1	2	1	0,538	0,544	0,399	0,551
16	8	5	2	1	1	2	1	0,529	0,531	0,395	0,541
16	7	6	2	1	1	2	1	0,520	0,527	0,392	0,534
15	7	7	2	1	1	2	1	0,513	0,517	0,390	0,525
15	6	8	2	1	1	2	1	0,508	0,513	0,386	0,521
14	6	9	2	1	1	2	1	0,498	0,504	0,386	0,512
14	5	10	2	1	1	2	1	0,495	0,503	0,385	0,509
Tight scenario ↑											

Table 5

If the costs of a bed on the joint ward equal the costs of a bed on the MC then we could have as much beds as possible on the Joint Ward. This is a very obvious result. This table was created to show the comparison of the optimization values between the different costs of a bed on the joint ward.

Number of beds											
IC	MC	JW	Costs IC	Costs MC	costs JW	α IC	α MC	Model A	Model B	Model B without	Model C
23	10	0	2	1	2	2	1	0,444	0,453	0,317	0,452
22	10	1	2	1	2	2	1	0,438	0,440	0,324	0,444
22	9	2	2	1	2	2	1	0,431	0,436	0,331	0,439
21	9	3	2	1	2	2	1	0,430	0,432	0,341	0,435
21	8	4	2	1	2	2	1	0,427	0,431	0,349	0,433
20	8	5	2	1	2	2	1	0,425	0,429	0,357	0,433
20	7	6	2	1	2	2	1	0,426	0,432	0,369	0,432
19	7	7	2	1	2	2	1	0,424	0,430	0,374	0,430
19	6	8	2	1	2	2	1	0,429	0,434	0,385	0,433
18	6	9	2	1	2	2	1	0,429	0,434	0,391	0,435
18	5	10	2	1	2	2	1	0,432	0,439	0,404	0,439
Average scenario ↑											
26	11	0	2	1	2	2	1	0,400	0,404	0,266	0,403
25	11	1	2	1	2	2	1	0,393	0,395	0,275	0,397
25	10	2	2	1	2	2	1	0,391	0,393	0,278	0,393
24	10	3	2	1	2	2	1	0,387	0,390	0,292	0,390
24	9	4	2	1	2	2	1	0,387	0,389	0,301	0,389
23	9	5	2	1	2	2	1	0,385	0,387	0,308	0,387
23	8	6	2	1	2	2	1	0,387	0,388	0,318	0,389
22	8	7	2	1	2	2	1	0,387	0,388	0,325	0,388
22	7	8	2	1	2	2	1	0,390	0,390	0,335	0,391
21	7	9	2	1	2	2	1	0,389	0,391	0,343	0,391
21	6	10	2	1	2	2	1	0,393	0,395	0,352	0,395
Wide scenario ↑											
19	10	0	2	1	2	2	1	0,507	0,510	0,383	0,520
18	10	1	2	1	2	2	1	0,502	0,505	0,396	0,513
18	9	2	2	1	2	2	1	0,494	0,496	0,398	0,505
17	9	3	2	1	2	2	1	0,491	0,493	0,409	0,503
17	8	4	2	1	2	2	1	0,486	0,492	0,417	0,499
16	8	5	2	1	2	2	1	0,485	0,488	0,423	0,498
16	7	6	2	1	2	2	1	0,484	0,491	0,434	0,497
15	7	7	2	1	2	2	1	0,485	0,489	0,439	0,498
15	6	8	2	1	2	2	1	0,487	0,493	0,450	0,500
14	6	9	2	1	2	2	1	0,486	0,492	0,455	0,500
14	5	10	2	1	2	2	1	0,490	0,498	0,467	0,504
Tight scenario ↑											

Table 6

If the costs for a bed on the JW equals the costs of a bed on the IC then it not profitable anymore to use as much beds as possible on the JW.

If the hospital uses model B (only external patients get blocked) they should not even have a joint ward since you can see in fact an IC bed as a bed on the joint ward. The red highlighted cell presents the lowest optimal value of this table. This means that we can say that it is approximately most efficient if we model the hospital with model B (when only external MC patients get blocked). If we look at the realistic wide scenario for models A and C we see that it is desirable to have 5 or 6 beds on the joint ward. The best solution if the hospital wants to use 5 or 6 beds on the JW is that a bed on the JW equals the price of a bed on the IC and it should be twice as high as a bed on the MC. As you can see in table 7 when having 5 (or 6) beds on the joint ward, the blocking probabilities stay under the 5%.

Number of beds			Blocking probabilities						
IC	MC	JW	Model A IC	Model A MC	Model B IC	Model B MC	Model B MC without	Model C IC	Model C MC
23	9	5	0,043	0,016	0,044	0,017	0,011	0,045	0,018
23	8	6	0,035	0,020	0,036	0,023	0,014	0,037	0,022

Table 7

6.2 Objective 2: "What is the influence of the MC on the IC?"

6.2.1 Scenario 1 without joint ward

Number of beds							
IC	MC	JW	# MC patients on IC in 1 year	Total time spend on IC in 1 year	Average days MC patient spend on the IC	Number of IC beds that are taken by MC patients	% of IC beds taken by MC patients
33	0	0	310,48	1646,53	5,303	4,511	13,67%
32	1	0	304,62	1283,33	4,213	3,516	10,99%
31	2	0	300,15	950,03	3,165	2,603	8,40%
30	3	0	282,82	657,84	2,326	1,802	6,01%
29	4	0	247,00	417,78	1,691	1,145	3,95%
28	5	0	206,03	259,26	1,258	0,710	2,54%
27	6	0	156,83	146,57	0,935	0,402	1,49%
26	7	0	108,00	77,15	0,714	0,211	0,81%
25	8	0	70,17	39,85	0,568	0,109	0,44%
24	9	0	40,93	19,35	0,473	0,053	0,22%
23	10	0	19,94	7,70	0,386	0,021	0,09%
22	11	0	9,57	3,01	0,315	0,008	0,04%
21	12	0	4,19	0,17	0,041	0,000	0,00%
20	13	0	1,26	0,36	0,290	0,001	0,00%
19	14	0	0,46	0,10	0,225	0,000	0,00%
18	15	0	0,16	0,03	0,191	0,000	0,00%
17	16	0	0,03	0,00	0,148	0,000	0,00%
Average scenario ↑							
37	0	0	320,53	1705,13	5,320	4,672	12,63%
36	1	0	318,49	1329,31	4,174	3,642	10,12%
35	2	0	311,57	992,89	3,187	2,720	7,77%
34	3	0	292,34	693,40	2,372	1,900	5,59%
33	4	0	264,49	472,78	1,788	1,295	3,93%
32	5	0	217,78	280,85	1,290	0,769	2,40%
31	6	0	173,08	171,43	0,990	0,470	1,52%
30	7	0	124,60	94,77	0,761	0,260	0,87%
29	8	0	85,12	51,78	0,608	0,142	0,49%
28	9	0	53,03	26,48	0,499	0,073	0,26%
27	10	0	28,24	11,27	0,399	0,031	0,11%
26	11	0	14,35	5,00	0,349	0,014	0,05%
25	12	0	7,26	2,21	0,304	0,006	0,02%
24	13	0	2,95	0,80	0,271	0,002	0,01%
23	14	0	1,02	0,24	0,238	0,001	0,00%
22	15	0	0,38	0,07	0,181	0,000	0,00%
21	16	0	0,11	0,02	0,171	0,000	0,00%
Wide scenario ↑							
29	0	0	292,27	1547,41	5,294	4,239	14,62%
28	1	0	290,71	1186,52	4,082	3,251	11,61%
27	2	0	281,95	855,46	3,034	2,344	8,68%
26	3	0	259,49	572,97	2,208	1,570	6,04%
25	4	0	222,32	350,90	1,578	0,961	3,85%
24	5	0	176,61	202,78	1,148	0,556	2,31%
23	6	0	128,37	108,57	0,846	0,297	1,29%
22	7	0	83,47	53,77	0,644	0,147	0,67%
21	8	0	48,23	24,00	0,498	0,066	0,31%
20	9	0	25,22	10,32	0,409	0,028	0,14%
19	10	0	11,90	4,12	0,346	0,011	0,06%
18	11	0	4,76	1,33	0,279	0,004	0,02%
17	12	0	1,63	0,41	0,252	0,001	0,01%
16	13	0	0,44	0,08	0,188	0,000	0,00%
15	14	0	0,10	0,01	0,134	0,000	0,00%
14	15	0	0,03	0,01	0,194	0,000	0,00%
13	16	0	0,00	0,00	0,000	0,000	0,00%
Tight scenario ↑							

Table 8

To find out what the influence is of the MC on the IC, when using a patient-flow in between and without a joint ward is, we created table 8. To obtain these results we had to use a model with patient flow between the IC and the MC and where internal MC patients remain at the IC if they encounter a full MC. Therefore we used model B because that is the only model where internal MC patients remain at the IC if they find the MC full. In table 8 you can find all values of the performance measures from model B, except for the blocking probabilities. The more available beds on the IC and the less available beds on the MC, the higher the percentages of IC beds which are taken by internal MC patients are. The number of beds on which are taken by MC is quite low because external MC patients do not enter the IC if the MC is totally occupied. The highlighted cells reflect the current situation in VUmc. The percentage of IC beds that are taken by MC patients is extremely low. You should expect that these numbers would be higher than one might anticipate. Based on these models, the influence of the MC on the IC seems limited.

6.2.2 Scenario 2 with joint ward

Number of beds							
IC	MC	JW	# MC patients on IC in 1 year	Total time spend on IC in 1 year	Average days MC patient spend on the IC	Number of beds that are taken by MC patients	% of IC beds taken by MC patients
23	10	0	21,045	7,804	0,371	0,021	0,000
22	10	1	11,383	4,135	0,363	0,011	0,000
22	9	2	14,323	5,968	0,417	0,016	0,000
21	9	3	10,275	4,152	0,404	0,011	0,000
21	8	4	13,968	6,453	0,462	0,018	0,000
20	8	5	11,530	5,262	0,456	0,014	0,000
20	7	6	15,405	7,895	0,512	0,022	0,000
19	7	7	13,663	7,170	0,525	0,020	0,000
19	6	8	16,293	10,534	0,647	0,029	0,000
18	6	9	14,820	8,879	0,599	0,024	0,000
18	5	10	17,103	12,881	0,753	0,035	0,000
Avarage scenario ↑							
26	11	0	14,873	5,190	0,349	0,014	0,000
25	11	1	7,650	2,561	0,335	0,007	0,000
25	10	2	9,135	3,485	0,382	0,010	0,000
24	10	3	5,645	2,081	0,369	0,006	0,000
24	9	4	7,325	3,166	0,432	0,009	0,000
23	9	5	5,513	2,148	0,390	0,006	0,000
23	8	6	7,218	3,350	0,464	0,009	0,000
22	8	7	5,973	2,595	0,434	0,007	0,000
22	7	8	6,973	3,693	0,530	0,010	0,000
21	7	9	7,015	3,818	0,544	0,010	0,000
21	6	10	7,798	4,900	0,628	0,013	0,000
Wide scenario ↑							
19	10	0	11,160	5,400	0,484	0,000	0,000
18	10	1	6,925	5,650	0,816	0,000	0,000
18	9	2	10,213	5,900	0,578	0,000	0,000
17	9	3	8,200	6,150	0,750	0,000	0,000
17	8	4	13,620	6,400	0,470	0,000	0,000
16	8	5	11,663	6,650	0,570	0,000	0,000
16	7	6	17,075	6,900	0,404	0,000	0,000
15	7	7	15,743	7,150	0,454	0,000	0,000
15	6	8	20,395	7,400	0,363	0,010	0,000
14	6	9	18,413	7,650	0,415	0,010	0,000
14	5	10	22,775	7,900	0,347	0,010	0,000
Tight scenario ↑							

Table 9

The number of beds that are taken by MC patients varies from 0,000 to 0,035, independent of the number of beds on the joint ward. For all bed allocations the percentage of IC beds that are taken by MC patients is approximately zero. The joint ward does not especially take care of decreasing the number of beds on the IC that are taken by MC patients.

7. Conclusion

Currently in VUmc the number of beds on the IC is 28 and the number of beds on the MC is 9. 0,215 % of the Intensive Care patients are transferred to the Medium Care. This care chain is important to investigate because the IC is a main cost-driver, and the IC needs to be accessible for patients who need acute and intensive attention. In this paper three models were created to investigate this patient-flow. These models all consist of the IC, MC and an additional joint ward. In model A the internal MC patients who come from the IC are blocked when encountering a full MC and JW. In model B internal MC patients who come from the IC and find both the MC and JW full, remain at the IC. Model C consist of two independent wards the IC and the MC and the additional JW.

The goal of this paper is to investigate the influence of the patient flow between the IC and the MC based on two objectives:

1. Do the blocking probabilities for the models show similar results?
2. What is the influence of the MC on the IC in Model?

One of the conclusion that can be drawn to give an answer on the first objective is that approximately 11 beds for the MC and 28 beds for the IC are needed to achieve a blocking probability of approximately 5 % when using the independent wards as in model C.
If we look at the scenario that the number of beds on the JW is zero then in model A the blocking probabilities of the IC are the same as in model C even if the number of beds on the MC is zero. However, for model B if the number of beds on the MC is zero, we see that the blocking probabilities for the IC are significantly higher. Instead of the 28 beds to achieve a maximum blocking probability of 5 percent you need approximately 32 beds to achieve the same result.
In model A the less IC patients are blocked the more patients will become a patient that will enter the MC after their stay on the IC. Therefore the blocking probability of the MC becomes higher if the IC gets more available beds. For most bed allocations the blocking probabilities for the MC for models A and C do not differ a lot. Therefore the product-form solution cannot be used for the exact calculations of the blocking probabilities for the MC but it is a good approximation. If the IC has 31 beds or more then all models roughly provide identical results. If the internal MC patients in model B are not seen as blocked patients when encounter a full MC then with 4 or more beds on the MC and 31 beds or more on the IC, the blocking probabilities of the IC can be compared with models A and C.
Therefore without a joint ward the product-form is a good approximation for calculating the blocking probabilities of realistic bed allocations for the IC and MC regardless which model you use.

With a joint ward for each scenario and model the more available beds on the JW the lower the blocking probability of the IC. The blocking probabilities of the IC for models A, B and C are very similar. The product-form can be used for calculating the blocking probabilities of the IC and for the MC only if we use a wide scenario. If we look at the blocking probabilities obtained with the product-form solution for the average and tight scenario we see that the approximation is a bit more worse than for the wide scenario but it is still useful.

For models A, B (all MC patients can be blocked) and C, if the costs of a bed on the joint equals the costs of a bed on the IC and are twice as high as a bed on the MC than it is desirable to have 5 or 6 beds on the JW. (Note that we do not take into account the costs for setting up the ward, train the nurses etc.). The blocking probabilities show desirable results (less than 5 %) with these bed allocations. However, when we use model B (only external MC patients can be blocked) we have seen that it is not necessary to use a joint ward.

The conclusion that we can drawn based on the results of the second objective, we obtained with model B, is that the influence of MC on the IC is negligible. The percentages of beds that are taken by internal MC patients are very low if we take a realistic bed allocation. The joint ward does not especially take care of decreasing the number of beds on the IC that are taken by MC patients. It is remarkable that the MC patients relatively speaking do not use beds on the IC very often, however the blocking probabilities of the external MC patients seems to improve a lot. This means that letting the internal MC patients stay at the IC when they encounter the MC and JW full is a good strategy to continue with.

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