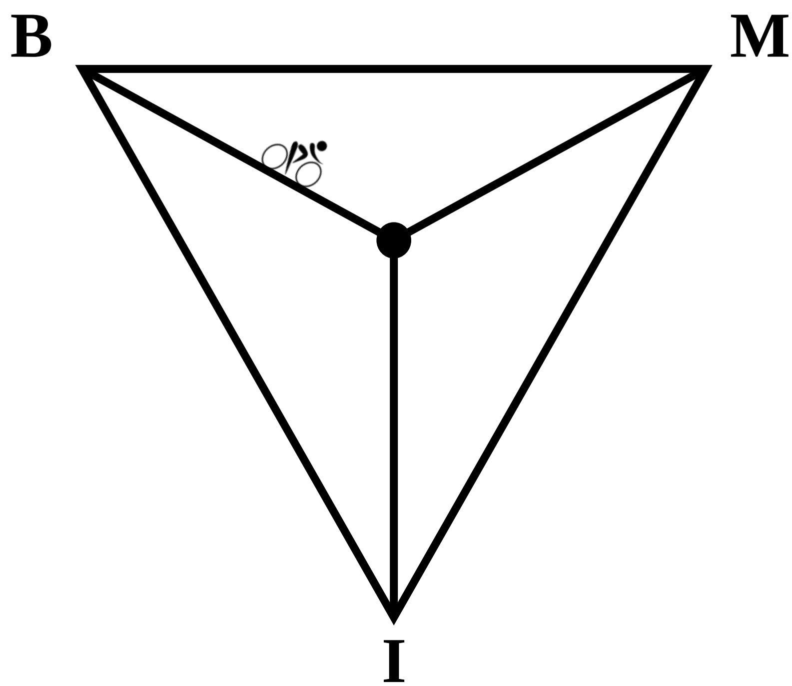
The Shortest Route

Developing an online bicycle route planner for the Netherlands



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Preface

This paper was written as an assignment for my Master in Business Mathematics & Informatics. The goal of the assignment is to write a paper on a research subject combining the business, mathematics and computer science aspects of the Master.

During my search for a suitable subject, Peter Kampstra mentioned that while there was a large amount of route planning software available, the Netherlands is still lacking an online bicycle route planner. Sure, some stand alone route planners have the option to plan bicycle routes and some provinces offer their own bicycle route planner, but an online planner covering all of the country is still not available.

Having encountered this issue myself in the past (I ended up taking the bus), my interest was awakened. Would it be possible to implement a bicycle route planner for the Netherlands with acceptable response times within the timeframe of a month? I took on the challenge and this paper presents the results of the research that I did, the choices that I made for the implementation and the resulting product.

I would like to thank Peter for proposing the subject and for all his useful suggestions as my supervisor. Furthermore, thanks to Sandjai Bhulai for agreeing to be the second reader, despite his busy schedule. And last but not least my thanks go out to Annemieke van Goor for not resting until I finally got started on the project.

Wouter Pepping,

March 2009.

Summary

In the past few years, route planners have rapidly become a part of every day life. All route planners depend on a routing algorithm to calculate the shortest or fastest route from a starting point to a destination. However, there is not one best algorithm that is used everywhere. Each route planner uses its own algorithm, based on a number of considerations: calculation time, preprocessing time, memory usage, complexity and whether the solution is optimal or suboptimal.

The most well-known routing algorithm is Dijkstra's algorithm. However, in terms of speed and memory usage, Dijkstra cannot compete with more recent methods. A* and bidirectional search try to improve Dijkstra's performance by only changing the routing algorithm, which results in small speedups. Geometric pruning, ALT algorithms and the multi-level approach use preprocessing to analyze the data. While adding preprocessing time and increasing complexity, these methods can greatly reduce calculation times, even up to a factor 1,000. Moreover, most methods can be combined to achieve even greater speedups. This means calculation times can be decreased to a few milliseconds. With calculation times this small, this will no longer form a bottleneck, but overhead like transmitting the data over a network or displaying the route will. For this reason, recent research indicates it is no longer interesting to accept suboptimal solutions in order to decrease calculation times.

During the course of the project, a study on existing route planners was performed. This study confirmed that there still is no Dutch route planner that offers decent support for planning bicycle routes and covers the entire Netherlands. The Dutch route planners that do offer the option to plan bicycle routes do not have bicycle paths in their data, resulting in inaccurate results.

Using the gained knowledge about routing algorithms, an online bicycle route planner was implemented for the Netherlands. In order to achieve acceptable processing times, the A* algorithm is used in combination with a specific data structure known as a Fibonacci heap. Using this method, even the longest routes can be calculated within acceptable time, while short routes take only a few milliseconds.

The speedup of A* over Dijkstra is impressive for shorter routes, but almost negligible for routes that cross the entire country. The most probably cause for this effect is that the borders of the dataset prevent unnecessary expansion of Dijkstra's search space for the latter.

Contents

1 Introduction

In the past few years, route planners have rapidly become part of every day life. Navigation devices appear in a growing number of cars and people that do not have them find their routes online. Many types of route planners exist, online or stand-alone, offering routes for cars, bicycles or pedestrians. However, a Dutch route planner that offers decent support for planning bicycle routes and covers the entire Netherlands is not yet available. The Dutch route planners that do offer the option to plan bicycle routes do not have bicycle paths in their data, resulting in inaccurate results. The aim of this project is to develop a Dutch bicycle route planner for the Netherlands.

Using a route planner is simple enough for everyone to understand. The technology behind it though, is not quite as simple. Finding a shortest route from one point to another can be solved by solving a shortest path problem. The shortest path problem means finding a shortest path from one node to another in a graph. Because maps can easily be translated to graphs, this problem applies to finding a shortest route on a map. However, when problems grow larger, algorithms for solving a shortest path problem can take a large amount of time and computer memory to come up with a solution.

One method to solve this type of problem is using Linear Programming (LP), but there are faster methods. A well-known algorithm for solving this type of problem is known as Dijkstra's algorithm. While the algorithm is faster than LP models, Dijkstra's original implementation still takes $O(n^2)$ time for a graph with n nodes. Fortunately, there are a number of ways to speed things up, but it will be clear that the fastest route to a solution is not as easy to find as the fastest route to a destination.

In Chapter 2 of this paper, the existing literature on the subject of shortest path problems is discussed. Several algorithms are presented, a data structure designed specifically to speed up this kind of algorithms is explained and a comparison of the performance of the algorithms is given.

In Chapter 3 a number of other route planners are discussed. These route planners will be compared based on algorithm, data source, user interface, language and whether or not they support bicycle routes.

Chapter 4 describes how a bicycle route planner for the Netherlands was implemented. It describes where the data that was used was obtained, what choices were made for the programming language, the algorithm and data structure and several other issues. The chapter ends with some statistics on the performance.

Chapter 5 concludes this paper. It presents a conclusion and a number of suggestions for future work.

2 Literature

Many articles have been written on the topic of solving shortest path problems. Many different algorithms exist, some optimal, some sub-optimal, one even faster than the other. This chapter will summarize what has been written on the subject, facilitating the choice of an algorithm in the next chapter.

This chapter is organized as follows. Section 2.1 describes the shortest path problem. First a graphical representation will be given, then a formal description is presented. In Section 2.2 an LP formulation of the problem is given. This LP model will solve any shortest path problem with nonnegative edge weights, but calculation time increases rapidly when problems grow large. Section 2.3 continues with Dijkstra's algorithm, which reaches a solution much faster than solving the LP formulation.

Still, Dijkstra is far from optimal in calculation time. Sections 2.4 to 2.8 all describe adaptations of Dijkstra's algorithm, meant to decrease either the calculation time, or the memory usage. Section 2.9 briefly mentions suboptimal algorithms. Finally, Section 2.10 discusses data structures used for implementations of the algorithms and Section 2.11 offers a comparison.

2.1 Shortest path problem

The problem of finding a route on a map can be represented in a simple form by using a graph, as in Figure 1. Here, the nodes (A to F) represent junctions and the edges represent streets between the junctions. Each edge has a weight that can represent the distance, or the estimated time it would take to travel this distance, whichever is more appropriate.

To find the shortest path from, e.g., A to F, one would need to find the collection of edges that connect A to F through any other nodes, for which the sum of the weights is as low as possible. Throughout this paper, the length of the shortest path from some node *s* to some node *t* will be referred to as the distance from *s* to *t*. Next, a formal description of the shortest path problem is provided.

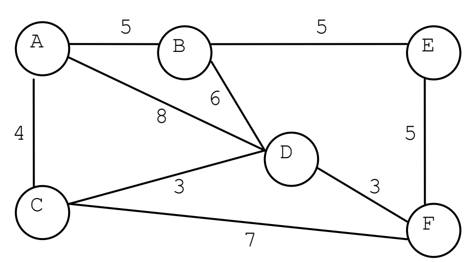

Figure 1: A graph with weighted edges.

Consider a directed graph $G = (V, E)$, where V is a finite set of nodes and E the set of edges between those nodes. The number of nodes $|V|$ is denoted by n and the number of edges $|E|$ is denoted by m. Each edge e has a weight $w(e)$. A path is defined by a sequence of nodes $(v_1, \dots, v_k)$ for which $(v_i, v_{i+1}) \in E$ for $1 \le i < k$. A node that is connected to some node v by some edge is referred to as a neighbor of v. A node that precedes some other node on a path is referred to as its parent on that path. Similarly, the successor of a node on some path is referred to as its child on that path.

Given a source node $s \in V$ and a target node $t \in V$, the shortest path is defined as the path (s , … , t) that minimizes the sum of the weights of all edges on the path. The length of the shortest path from s to node v is defined as $g(v)$ and is also referred to as the distance from s to v.

2.2 LP model

One way to solve a shortest path problem is using the linear programming model described in [1]. In order to formulate this model, the following variables need to be defined.

$$x_e = \begin{cases} 1 & \text{if edge } e \text{ is used in the optimal path} \\ 0 & \text{otherwise} \end{cases}$$

$$\delta_i^+ := \text{the set of edges entering } u_i$$

$$\delta_i^- := \text{the set of edges leaving } u_i$$

An LP model can now be defined as shown below.

$$\text{Minimize} \sum_{e \in E}^{n} x_e w(e)$$

Where

$$\sum_{e \in \delta_i^-} x_e - \sum_{e \in \delta_i^+} x_e = 0 \quad \forall i{:}v_i \in V \backslash \{s,t\}$$

$$\sum_{e \in \delta_s^-} x_e - \sum_{e \in \delta_s^+} x_e = 1$$

$$\sum_{e \in \delta_t^-} x_e - \sum_{e \in \delta_t^+} x_e = -1$$

$$x_e \geq 0 \quad \forall e \in E$$

The first constraint describes the demand that every node that is not s or t should be left as many times as it is entered. The second constraint describes that s should be left one time more than it is entered. The third constraint describes that node t be left one time less than it is entered. The fourth constraint describes that the decision variables should be larger than or equal to 0. The model itself ensures all decision variables take either the value 0 or the value 1, because any other values will lead to a non-optimal solution.

Route planners need to solve shortest path problems of substantial size. For this case, LP is not the most suitable method to find a solution. No matter how good the model, no matter how efficient the code, calculation times will grow too large if a problem specific approach is not used. The following sections describe a number of algorithms that offer a more efficient way to solve shortest path problems.

2.3 Dijkstra

In [2], Dijkstra describes an algorithm that solves a shortest path problem with nonnegative edge weights much more efficiently than LP. This algorithm is now known as Dijkstra's algorithm and has been documented thoroughly. For each node v, two properties are stored. The first is the length of the shortest path from s to v found so far, the second the node preceding v on that path. The algorithm will construct better paths in an iterative way, improving the solution in each step. Once finished, it will have found the shortest path from source node s to *all* other nodes in a graph.

2.3.1 The algorithm

In order to explain the algorithm, the following definitions are needed:

- $A :=$ the set of nodes v for which a *shortest* path from s to v has been found (also known as the closed set).
- $X :=$ the set of nodes v for which a path from s to v has been found of which it is uncertain if it is the shortest (also knows as the open set).
- $\hat{g}(v) :=$ length of the shortest path from s to v found so far (this can be seen as an estimate or upper bound for the shortest path from s to v).
- $\hat{g}(s) := 0$

Initialize the algorithm with:

- $A = \varnothing$
- $X = \{s\}$

Repeat the following steps until X equals the empty set.

1. Take node v from X for which $\hat{g}(v) = \min_{u \in X} \hat{g}(u)$, remove v from X and add it to A.
2. For each u for which $(v, u) \in$ E (each neighbor of v)

– If u is in A, do nothing.

– If u is in X

  – If $\hat{g}(v) + w(v,u) < \hat{g}(u)$, then set $\hat{g}(u) = \hat{g}(v) + w(v,u)$ and $parent(u) = v$.

– If u is not in A and not in X, add u to X and set $\hat{g}(u) = \hat{g}(v) + w(v,u)$ and $parent(u) = v$.

When the algorithm ends, all nodes that can be reached from s will be in A (including t) and the solution has been found. To save some calculation time, the algorithm can be stopped once t is moved to A, because at that point the shortest path to t has been found.

2.3.2 Proof of optimality

This section will show why Dijkstra always finds an optimal path. See the collection *A* as an expanding border around the source node s, where every node inside the border is closer to *s* than any node outside the border. *A* keeps expanding, until it contains the target node. This principle is illustrated in Figure 2.

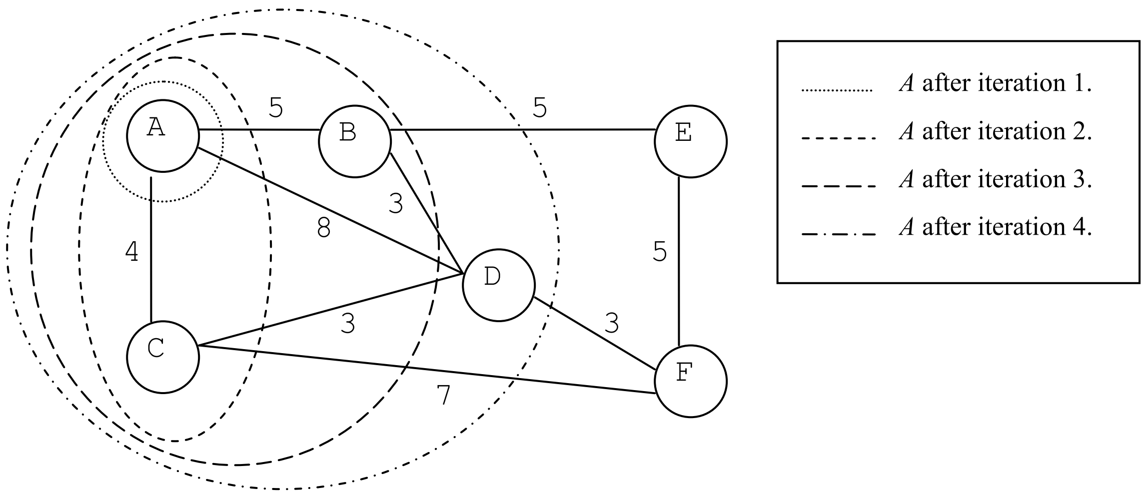

Figure 2: Expansion of *A* in Dijkstra's algorithm.

Define v_k as the node that is added to *A* in step *k*. Using induction it is shown that v_k is the node for which the length of the shortest path $g(v_k)$ is the shortest of all nodes not in *A*. This is trivial for step 1, because *s* is added with $g(s)=0$.

Say that this holds up to some step *k*, then after this step any node not in *A* will have a larger distance from *s* than any node in *A*. Necessarily, all neighbors of all nodes in *A*, will be in *X*. As a consequence, any node not in *A* and not in *X* will have a longer distance from *s* than at least one node in *X*, because you will have to travel through a node in *X* to get there. This means the node v_{k+1} that needs to be added to *A* in step $k+1$ will be in *X* and will therefore be found by the algorithm.

Because this holds for $k = 1$, this proves it holds for any $k \geq 1$, or any step in the algorithm. Because the shortest path to v_k is shorter than to any node outside of *A*, this path does not contain any nodes outside of *A* and therefore the shortest path found so far is the actual shortest path, or $\hat{g}(v_k) = g(v_k)$, making it valid to add it to *A*. A formal prove for Dijkstra is provided next.

When a node is added to *A*, the shortest path is found and the algorithm does not touch the node again. To prove that the path Dijkstra finds to each node is indeed the shortest path, we need to prove for each iteration *k*, there exists no path to v_k shorter than $\hat{g}(v_k)$, or $\hat{g}(v_k) = g(v_k)$. For this proof Lemma 1 provided in [3] is used.

Lemma 1
For any optimal path P from s to some node $u \notin A$, there exists a node $v \in X$ on P with $\hat{g}(v) = g(v_k)$.

Proof
If $s \in X$, this is trivially true, since $\hat{g}(s) = g(s) = 0$. Otherwise, let v^* be the node that was last added to A. Note that this cannot be the same node as v, because $v \notin A$. Let v be the successor of v^* on P. By definition of $\hat{g}$, $\hat{g}(v) \leq \hat{g}(v^*) + w(v^*, v)$. Because $v^* \in A$, $\hat{g}(v^*) = g(v^*)$ and because P is an optimal path, $g(v) = g(v^*) + w(v^*, v)$. Combining these results leads to $\hat{g}(v) \leq g(v)$, but in general $\hat{g}(v) \geq g(v)$. From this it follows that $\hat{g}(v) = g(v)$.

Lemma 1 shows that for each iteration there is a node $v_k \in X$ where $\hat{g}(v_k) = g(v_k)$, as long as there is a node $u \notin A$. When this node u does not exist, X equals the empty set and the algorithm is terminated. This proves that for every node that is added to A the shortest path has been found, so for every node $v \in A$, Dijkstra's algorithm has found the shortest path from s to v.

2.4 A*

When traveling to a certain destination, there is usually no point taking a road in the opposite direction. An algorithm would do well to give priority to roads that travel in the direction of the destination. That is where Dijkstra falls short. Dijkstra searches the search space in any direction, as indicated by the fact that the algorithm finds the shortest path from s to any other node. Clearly, the algorithm does not search in the direction of the target node.

Hart, Nilsson and Raphael [3] introduce the A* algorithm, that adds a heuristic to Dijkstra, making it more goal oriented. Instead of evaluating a node v using an estimate for the shortest path to this node $\hat{g}(v)$, Hart et al. introduce the following evaluation function.

$$f(v) = g(v) + h(v)\,.$$

The evaluation function $f(v)$ represents the length of the shortest path from the source node s to the target node t when traveling through node v. The $g(v)$ part is the distance from s to v and the $h(v)$ part is the distance from v to t. They also introduce an estimate for the evaluation function:

$$\hat{f}(v) = \hat{g}(v) + \hat{h}(v)\,.$$

The estimated distance from s to v, $\hat{g}(v)$, is determined in the same way as in Dijkstra's algorithm, the shortest path from s to v found so far. The estimated distance from v to t, $\hat{h}(v)$, is determined by using a heuristic. This heuristic can be any function and is often problem specific; there is no *best* heuristic. For route planning problems, a heuristic that is often used is the Euclidean distance from v to t.

The only alteration that A* makes in the algorithm shown in the previous section is that $\hat{g}(v) = \min_{u \in X} \hat{g}(u)$ is replaced by $\hat{f}(v) = \min_{u \in X} \hat{f}(u)$ in step 1.

Next, Hart et al. prove that the A* algorithm finds an optimal solution if

$\hat{h}(v) \leq h(v)$ for all v.

This is the same as saying the heuristic $\hat{h}(v)$ should always underestimate the true distance $h(v)$, or $\hat{h}(v)$ should give a lower bound on $h(v)$. This is known as an admissible heuristic.

2.4.1 Proof of optimality

The proof for A* is slightly more complicated than the proof Dijkstra. Russel and Norwig [4] try to give a feeling for the algorithm by describing the collection A as a contour around all nodes having $f(v)<c$ for some constant c, much like Dijkstra does for $g(v)$. With every iteration the value of c is increased and the contour is expanded.

Of course this does not provide a complete proof. Therefore the formal proof from [3] is given below. First the following corollary to Lemma 1 from the previous section is needed.

Corollary to lemma 1
Suppose a heuristic $h(v)$ is admissible and A* has not terminated. Then for any optimal path P from s to t, there exists a node $v \in X$ with $\hat{f}(v) \leq f(s)$.

Proof
By Lemma 1 there exists a node $v \in X$ on P with $\hat{g}(v) = g(v_k)$. Then:

$$
\begin{aligned}
\hat{f}(v) &= \hat{g}(v) + \hat{h}(v) && \text{(by definition of } \hat{f}\text{)} \\
&= g(v) + \hat{h}(v) && \text{(by lemma 1)} \\
&\leq g(v) + h(v) && \text{(because } \hat{h} \text{ is admissible)} \\
&= f(v) && \text{(by definition of } \hat{f}\text{)}
\end{aligned}
$$

Suppose A* terminates after finding a non-optimal route to t. Then $\hat{f}(t) = \hat{g}(t) > f(s)$. According to the corollary, just before termination, there was a note v on an optimal path with $\hat{f}(v) \leq f(s) < f(t)$. At this point the algorithm would have selected v at step 1 instead of t and the algorithm could not have terminated. This proves that A* cannot terminate after finding a non-optimal route to t.

2.4.2 Optimally efficient

It should be noted that A* is optimally efficient for any given heuristic function. This means no other optimal algorithm using the same knowledge is guaranteed to expand fewer nodes than A*. A short explanation for this is given in [4]: “any algorithm that does not expand all nodes in the contours between the root and the goal contour runs the risk of missing the optimal solution”. A formal proof is given in [3].

This observation does not mean there are no ways to get to a solution faster. As shown in the next sections, by improving the heuristic, or by performing some form of preprocessing, the calculation time can be decreased significantly.

2.4.3 Memory usage

One downside of both Dijkstra and A* is that when the search space grows large, memory usage increases rapidly for both methods. A search space may contain millions of nodes and for each node that is visited by the algorithm one integer $\hat{g}(v)$ and one pointer to the preceding node need to be stored in memory. Because of these memory requirements, methods were developed that can compete with A* in terms of performance, but use less memory. These algorithms are IDA*, MA* and SMA*.

IDA*

IDA* stands for Iterative Deepening A*. The method is based on Depth First Iterative Deepening. In each iteration, IDA* sets a threshold for the evaluation function $\hat{f}(v)$. It then searches one path until the evaluation function for this path exceeds the threshold. When the threshold is exceeded, the path is backtracked until a node is found with neighbors that have not been searched yet and the algorithm continues with a path from this neighbor. An iteration ends when all possible paths up to the threshold have been searched.

Once an iteration has finished, IDA* sets the threshold for the next iteration to the minimum of all values found that exceeded the current threshold. This process continues until the target node has been found.

Although it would seem that IDA* performs a lot of extra calculations by expanding nodes multiple times, the actual overhead this causes is rather small. The reason behind this is that the outer layer of the expanded nodes, expanded in the last iteration, contains by far the most nodes. The fact that other layers are expanded multiple times does not make much difference. IDA* returns an optimal solution under the same constraints as A*. A more detailed description of IDA* can be found in [11].

SMA*

The weakness of IDA* is that it effectively throws information away after each iteration. A method that tries to retain as much information as possible is known as Memory Bounded A*, or MA* [12]. Russel [13] introduces SMA* that improves MA* by letting even less information get lost and making the algorithm slightly easier to implement.

SMA* works roughly the same as A*, with the difference that it has a limit on the number of nodes that are allowed to be stored in memory. Once this limit has been reached, it will prune a leaf node only when space is needed for a better one. When a node has to be pruned, it will remove the shallowest node with the highest value of $\hat{f}(v)$.

In order to prevent unnecessary information loss, information from the successors is stored in preceding nodes. Once all successors of a node v have been expanded, the value of $\hat{f}(v)$ is replaced by the lowest value of all successors. This information is then updated for all predecessors of v.

Theorem 4 in [13] says that except for its ability to generate single successors, SMA* behaves identically to A* when the number of nodes generated by A* does not exceed the limit. SMA* returns an optimal path when enough memory is available to store the shallowest optimal solution.

2.5 Bidirectional search

Bidirectional search applies an arbitrary search algorithm both forward from the source node s and backwards from the target node t. This reduces the area that is searched and therefore the number of nodes that are searched, as illustrated in Figure 3.

Figure 3: The area of the one ellipse on the left is larger that the sum of the two small ones on the right.

For Dijkstra, applying bidirectional search is rather straightforward. The algorithm is initiated from both nodes and one iteration for both source and destination is executed at a time. Once the algorithm in one direction adds a node to A that is already in A for the other direction, the shortest path has been found.

For A* however, the method is not as simple. Because A* minimizes the evaluation function $\hat{f}(v)$, instead of the distance from the source node, there is no guarantee the minimum path has been found when the two closed sets meet. Kaindl and Kainz mention in [17] that previous research on bidirectional heuristic search indicates that the overhead of the method outweighs the benefit. Their own approach achieves some promising results, decreasing the running time by 30% in the most positive scenarios. However, this result is minimal in comparison to other optimizations that are presented in this paper. In general, the combination of A* with bidirectional search does not lead to impressive results.

2.6 Geometric pruning using bounding-boxes

Every road is only useful to get to a number of different places. For any other place, it is faster to take another road. If this information is known when planning a route, roads that are not in the shortest path to the destination can easily be ignored. Unfortunately, storing all this information takes up an enormous amount of memory.

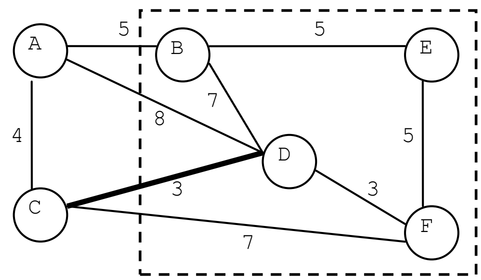

Figure 4: A bounding box for edge (C,D)

Wagner and Willhalm [9] introduce a method called geometric pruning. The idea behind the geometric pruning method is to store a set of nodes *S(e)* for each edge *e* during a preprocessing phase, containing all nodes that can be reached by a shortest path starting with *e*. Storing these sets for every edge would need *O(nm)* space. As mentioned, this is not feasible for large search spaces, so Wagner et al. store a geometric shape for each edge *e* that contains at least *S(e)*. In Figure 4 edge (C, D) is the first edge in a shortest path from C to D, E and F. Therefore, *S(C, D)* contains at least these three nodes.

These shapes contain a set of nodes *C(e)* called a container. A container is called consistent if $S(e) \subseteq C(e)$. If the container *C(e)* is consistent for all edges, then Dijkstra's algorithm with pruning will find the shortest path from *s* to *t*. The method makes a small change in Dijkstra's algorithm. When a node is added to *A* in step 1 of the algorithm, only the nodes *u* for which $t \in C(v,u)$ will be considered.

The proposed method needs a considerable amount of preprocessing, but can substantially decrease the number of visited nodes for a search. One of the most effective, yet simple geometric shapes is the so called bounding box, meaning the smallest square box containing all nodes in *S(e)*. Using bounding boxes, the number of nodes visited is decreased by 90 to 95 percent on road networks compared to Dijkstra, decreasing the running time by approximately a factor 10.

Although Wagner et al. only test this method with Dijkstra's algorithm, they mention in their conclusion their method can be easily combined with other methods. One of the methods mentioned is A*.

2.7 ALT algorithms

Goldberg and Harrelson [5] describe a method for improving performance of shortest path queries they call ALT algorithms, since they are based on A*, Landmarks and the Triangle inequality for shortest path distances (not Euclidean distances). The goal of the method is to find a better lower bound for the distance *h(v)* used by the *A** algorithm. Better lower bounds result in fewer nodes being searched.

An ALT algorithm selects a number of landmarks during preprocessing. For every node the distance to and from these landmarks is computed. Consider a landmark L and let $d(u,v)$ be the distance from any node u to any node v. Then by the triangle inequality, $d(u,L) - d(v,L) \leq d(u,v)$. Similarly, $d(L,v) - d(L,u) \leq d(v,u)$.

Since $d(w,L)$ and $d(L,w)$ were computed during preprocessing, this fact can then be used to compute a lower bound for the distance between two nodes. To achieve the tightest lower bound, the maximum over all landmarks is used. Once the lower bound is found, this can be used as the heuristic for $\hat{h}(v)$ in the A* algorithm.

Finding good landmarks is critical for the overall performance of ALT algorithms. Goldberg et al. describe three ways of selecting landmarks. If k is the number of landmarks that need to be selected, the simplest method described is selecting k nodes at random. However, there are better methods.

The second method described is called farthest landmark selection. It is initiated by picking one random node and picking the node farthest away from it as the first landmark. Then, for each new landmark that is to be selected, find the node that is farthest away from the current set of landmarks.

For road maps, having a landmark lying behind the destination tends to give good bounds. In this case, the method called planar landmark selection is a good choice. Find a node v near the center of the graph and divide the graph into k pie-slice sectors with v as their center, so that each slice contains approximately the same number of nodes. Then for each sector select the node farthest from c as a landmark. To avoid having two landmarks close to each other, if a landmark in sector A was chosen close to the border of the next sector B, skip the nodes in B that are close to the border with A.

For the number of landmarks that should be selected, Goldberg et al. suggest a moderate amount, e.g. 16, as the improvement when choosing more landmarks is relatively small compared to the extra calculation time. Finally, when calculating a shortest path from s to t, they suggest taking a small subset of landmarks (e.g. 4) that give the highest lower bounds on the distance from s to t.

Results show an improved efficiency of a factor 10 to 20 compared to Dijkstra when using ALT algorithms.

2.8 Multi-level approach

When traveling from one city to another, the largest part of the shortest path will be fixed, no matter what the exact starting point and destination of the journey is. It is obvious some paths are part of a large number of different shortest paths. The multi-level approach tries to take advantage of this fact.

Sanders and Schultes [6], [7] introduce a variant of this method they call highway hierarchies. Their method creates multiple levels for the original graph during preprocessing, where each higher level is an abstraction of a lower one. The original graph is the lowest level. When generating a new level, some nodes and edges may be removed and some edges may be added.

The multi-level approach tries to find so-called highways in the graph and add these to a new level. First a neighborhood around each node is determined. Highways consist of edges that are on a shortest path from some source node s to some target node t and have their beginning and end outside of both the neighborhoods of s and t and edges that either leave the neighborhood around s or enter the neighborhood around t, but not both.

Finally some steps are taken to clean up the newly created level. Paths consisting only of nodes with 2 neighbors are removed completely and replaced by a single edge connecting the start and end node of the path. Furthermore, all isolated nodes are removed.

Once a new level is created, the steps can be repeated to generate another level. When the desired number of levels has been created, all nodes in each level are linked to the corresponding ones in the level below to form the final graph. This step ensures that the shortest path is found; the algorithm can always take a step down to a more detailed level.

The queries on the final graph are performed by a slightly modified version of the A* algorithm and were shown to be between 500 and 2,000 times faster than Dijkstra's algorithm on real road networks.

2.9 Suboptimal algorithms

Some authors, for instance Botea, Müller, and Schaeffer in [10], have introduced suboptimal algorithms that lower calculation times significantly, while giving results within a small margin of the optimal solution. Although this could be interesting, Sanders and Schultes describe in [8] that shortest path algorithms have improved so much in terms of speed in the past years, they are actually becoming *too* fast.

With the latest methods, query times of only a few milliseconds can be achieved, causing overhead like displaying routes or transmitting them over a network to become the bottleneck. Considering this argument, it is no longer interesting to accept suboptimal solutions to decrease query times.

2.10 Priority queue

The data structure behind Dijkstra's algorithm, or variations of it, can have a significant effect on the performance. Because Dijkstra and A* will need to find the node in X with the lowest value for $\hat{g}$ or $\hat{h}$ respectively, implementations of Dijkstra, A* or one of their variations generally use a priority queue, or heap. This type of queue will always have the value with the highest priority (or in this case the lowest value for $\hat{g}$ or $\hat{h}$) available without needing to search the whole collection, thereby decreasing processing time.

Dijkstra's algorithm runs in $O(m)$ time plus the time required to perform the heap operations. Where Dijkstra's original implementation, using an array, needed $O(n^2)$, implementations using a Fibonacci heap [14] are able to decrease this to $O(m + n \log n)$. Under certain restrictions double buckets [15], or multi-level buckets [16] can achieve even greater speed-ups.

A priority queue consists of a set of items, each with its own real-valued key. This type of queue should support at least the following operations:

- Insert(h, x) - Insert item *x*, with predefined key, into heap *h*.
- Delete min(h) - Find the item with the minimum key in heap *h*, delete it from the heap and return it.
- Decrease key(h, x, d) - Subtract *d* from the key of item *x* in *h* and replace its key with this value.

2.10.1 Fibonacci heaps

A Fibonacci heap is designed to not only minimize the time needed for the *delete min* operation, but more importantly, the *decrease key* operation. Because the latter is generally performed multiple times in each iteration, this approach benefits the running times for Dijkstra and A*. How this goal is achieved will be explained in this section.

A Fibonacci heap consists of a number of trees, of which the roots are stored in a doubly linked circular list. The node *m* with the minimum key will always be in this list of roots and a pointer to this minimum node is kept. The children of each node are stored in a doubly linked circular list as well. There are no constraints on the number of trees or number of children, other than the constraints that are implicit in the definition of the heap operations. The number of children of a node is referred to as its rank. An example of a Fibonacci heap is shown in : . The arrow represents the pointer to the minimum.

Insert(h, x)

The operation insert(h, x) on a Fibonacci heap is performed by creating a new tree, consisting of one node with value *x*, and adding this node to the list of roots of *h*. If the key of *x* is lower than the current minimum key in *h*, the pointer is updated accordingly. All operations take $O(1)$ time. In : , the root with value 16 and without children could be the result of an insert operation of key 16.

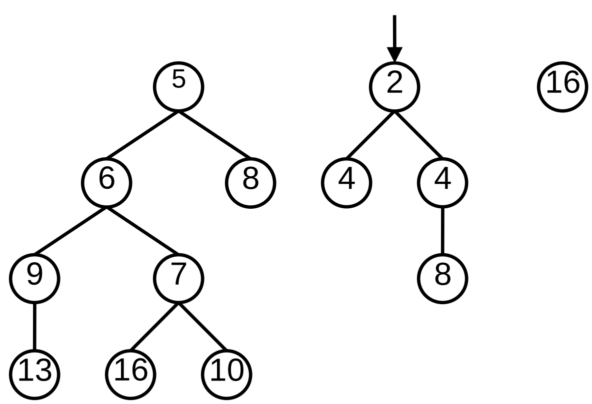

Figure 5: An example of a Fibonacci heap.

Delete min(h)
Delete min is the most time-consuming operation. The deletion is initiated by removing the minimum node *m* from *h*. Then the list of children of *m* is added to the list of roots of *h* without *m* and a *linking step* is performed.

The *linking step* finds two trees whose roots have the same rank and links them, until there are no two trees with roots of the same rank left. Linking two trees is performed by making the tree with the root with the larger key a child of the other and removing its tree from the list of roots. The entire operation on the Fibonacci heap of Figure 5 is illustrated in Figure 6.

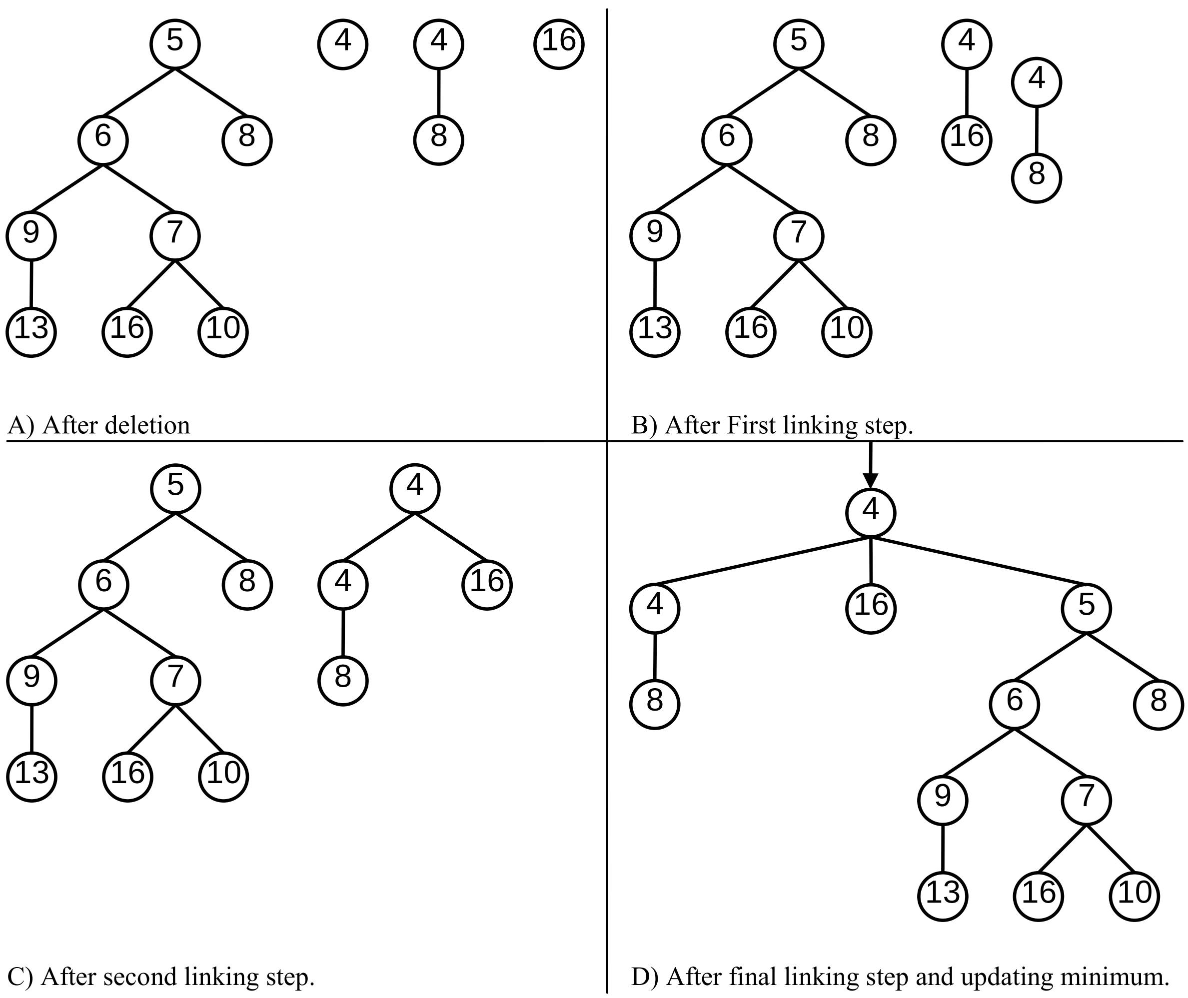

Figure 6: Effect of a delete min operation on the Fibonacci heap of Figure 5.

To perform the linking step, an array is used that is indexed by rank, from one up to the maximum possible rank. An attempt is made to insert each root into the appropriate position. Whenever a root needs to be inserted into a position that is already taken, these roots necessarily have the same rank and are linked. For the root of the resulting tree an attempt is made again to insert it into the array.

A property of a Fibonacci heap is that a node of rank *k* has at least $F_{k+2} \geq \varphi^k$ descendants, including itself, where F_k is the *k*-th Fibonacci number and $\varphi = (1 + \sqrt{5}) / 2$ is the golden ratio. Therefore, any root will have a maximum possible rank of $log_\varphi\ n$, where *n* is the total number of nodes in the heap.

When the linking is finished, the minimum is updated to point to the new minimum in the root list. The delete min operation takes $O(\log n)$, where *n* is the number of items in the heap.

Decrease key(h, x, d)
For the decrease key operation, it is assumed that the position of *x* in *h* is known. First, *d* is subtracted from the key of *x*. Next, the node *k* containing *x* is cut from its parent and added to the list of roots. This action forms a new tree with *k* as its root. If *k* already was a root, decrease key is performed by simply subtracting *d* from the key of *k*.

If the new key of *x* is smaller than the key of *m*, the pointer is updated to reflect this. A decrease key operation takes $O(1)$ time. After each decrease operation, cascading cuts may have to be performed, as described in the next section.

Cascading cuts
Finally, there is one implementation detail necessary to obtain the desired time bounds. After any operation that causes child nodes to loose their children, when a child node has lost two of its children, it is cut from its parent and added to the list of roots. Decrease key is such an operation. Another one is a delete operation, should this be implemented.

This operation is called a cascading cut. A single operation can result in a large number of cascading cuts. To keep track of cascading cuts, when a child is cut from a parent that is not a root, its parent is marked. When a child is cut from a parent that has already been marked, the parent is cut from its parent as well and it is unmarked. If a node is moved to the list of roots for any other reason and is later added as a child by the linking step, it is unmarked as well.

As an example, take the Fibonacci heap from Figure 5 and assume nodes 6 and 9 are marked as having lost a child. Figure 7 shows the heap after a decrease key operation has been performed on node 13, decreasing the key to 1 and causing two cascading cuts.

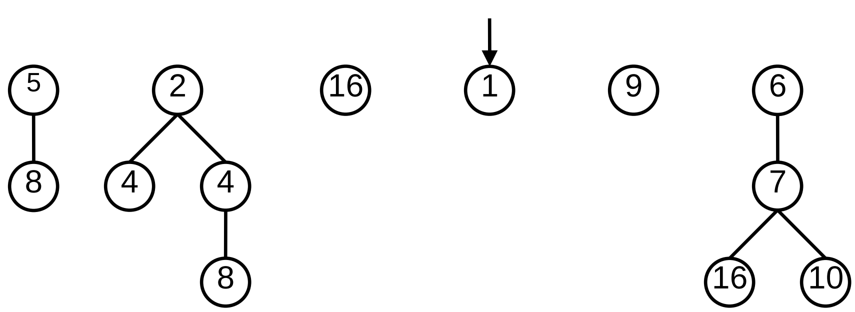

Figure 7: The Fibonacci heap of Figure 5 after a decrease key operation on node 13.

2.11 Comparison

Table 1 shows a comparison of the algorithms that were discussed in this chapter. Speedups are compared to the unidirectional Dijkstra algorithm, unless indicated otherwise.

Algorithm	**Preprocessing**	**Memory usage**	**Speedup**	**Optimal**
LP	None	Negligible	Slower than Dijkstra	yes
A*	None	Less memory usage because less nodes are searched	Around 2 times as fast, up to 10 times faster is favorable scenarios	yes
IDA*	None	Very little, only one path is stored in memory	Around a factor 2 slower than A*	yes
SMA*	None	Up to a set maximum amount	Slightly slower than A* when the memory limit is reached	yes
Bidirectional search	None	Less memory usage, because less nodes are searched	Around 2 times faster	yes
Geometric pruning	An all pairs shortest path computation	Stores a geometrical shape for each edge	10-20 times faster	yes
ALT algorithms	Around 16 global shortest path queries	High memory usage, stores a distance to all landmarks for each node	Around 10 times faster	yes
Multi-level approach	A local shortest path search around each node in the lower level is needed to form each new level	For each new level, a new, but smaller graph needs to be stored	Up to 2000 times faster in the most favorable scenario	yes

Table 1: Comparison of shortest path algorithms.

3 Other routing services

There are two main types of existing routing applications: offline, or onboard planners and online or web planners. Numerous applications of both types exist, this chapter will only name a few very popular ones and several that are in some way related to this project.

3.1 TomTom

TomTom is a large international company that offers stand alone navigation devices. Their devices are among the most popular ones, mainly because of the intuitive user interface, the speed with which it calculates routes and the accuracy of their data. The devices can calculate routes by car, by bike or on foot. Unfortunately, their routing algorithm and data source are not open to the public.

Recently TomTom has released an online version of their route planner as well, however, this version misses many of the features the stand alone device offers. Among other things, it lacks an option to plan routes by bike or on foot. Both the online version and the stand alone version are available in Dutch.

3.2 Google Maps

Google has recently started their own online map and routing service, named Google Maps. It is a very fast, free service. It is obvious that Google has performed some kind of preprocessing or caching to make their routing service this fast, but details or their routing algorithm stay within the company. Google Maps is available in Dutch, but only offers routes by car and on foot.

3.3 Via Michelin

Via Michelin is an online routing service based on the maps from the well-known publisher of road maps, Michelin. It is another impressively fast service. The service is free and seems to have only one purpose, as marking for Michelin. Again, some kind of preprocessing or caching would be necessary to achieve these calculation times, but this information is confidential. Via Michelin is available in Dutch and does offer an option to plan bicycle routes. However, the database lacks information on bicycle paths, so the results are often inaccurate.

3.4 Routenet.nl

Routenet.nl is a popular Dutch online routing service. The service is not quite as fast as the previous two, but the calculation times within the Netherlands are still very acceptable. It is a commercial service that is freely accessible and makes money from advertisements. Therefore, like TomTom and Google, their routing algorithm and data are not open to the public. Routenet.nl has recently added the option to plan bicycle routes, however, like Via Michelin, it lacks information on bicycle paths.

3.5 YourNavigation.org

YourNavigation.org is a demonstration website for the YOURS project. The goal of the project is to use OpenStreetMap (OSM) data to make a routing website utilizing as much other (opensource) applications as possible. It uses an opensource routing engine called Gosmore. The service is not very fast on longer routes and their website mentions that Gosmore is not designed for generating routes longer than 200km. The routeplanner offers an option to plan bicycle routes and the does have bicycle paths in their data source, but it is only available in English.

3.6 OpenRouteService.org

OpenRouteService.org is another online service that uses OSM data. It is a project of the Geography department of the University of Bonn in Germany. Like YourNavigation.org, it is a non-commercial service that is free and has no advertising on their website. Apart from the source code, almost every bit of information on the implementation is available on the website. The service uses A* and is not as fast on longer routes the commercial applications. OpenRouteService.org is only available in English, but can plan bicycle routes. It uses the same data source as YourNavigation.org, so it includes bicycle paths.

3.7 Comparison

Table 2 shows a comparison of the six routing applications that were discussed in this section. Unfortunately for the best performing applications the routing algorithms were confidential. The two that did reveal their routing algorithm both used A*. It is obvious that non-commercial routing services tend to stick with less complex algorithms, to limit the time that needs to be invested. Commercial applications have a lot depending on the performance of their service and can afford to invest more time and money in order to increase performance.

Name	**Algorithm**	**Data source**	**Selecting start and destination**	**Language**	**Supports bicycle routes**
TomTom	*confidential*	Own data	Addresses	English	Stand alone only
Google Maps	*confidential*	Own data	Both addresses and select on a map	English	No
Via Michelin	*confidential*	Michelin maps	Both addresses and select on a map	Dutch	Yes, but no bicycle paths.
Routenet.nl	*confidential*	Own data	Addresses	Dutch	Yes, but no bicycle paths.
YourNavigation	A*	OSM	Both addresses and select on a map	English	Yes
OpenRouteService	A*	OSM	Both addresses and select on a map	English	Yes

Table 2: Comparison of existing routing services.

4 Implementation

As a result of the study performed in Chapter 3, it can be concluded that there still is no Dutch route planner that offers decent support for planning bicycle routes and covers the entire Netherlands. Therefore, as part of this project, a bicycle route planner for the Netherlands will be implemented.

For the implementation, a number of decisions need to be made. First of all the decision needs to be made what programming language to use. Second, a suitable algorithm needs to be chosen. Based on the choice of algorithm, a suitable data structure may be needed to optimize the time for data storage and retrieval. Finally, this data needs to come from somewhere. A data source with data on all roads in the Netherlands is a requirement for the success of the project.

This chapter is organized as follows. Sections 4.1 to 4.4 describe these decisions and other details of the implementation. Section 4.5 shows the resulting application and some numbers on performance. Section 4.6 presents a validation of the routes the application finds.

4.1 Programming language

For the choice of programming language, the project implies a number of constraints. Because the application will likely need a substantial amount of processing time to calculate a route, the code should be compiled to benefit the speed of the calculations. Furthermore, to facilitate structured design, maintainability and extensibility of the application, an object-oriented language should be preferred over a non object-oriented language.

Taking these constraints into account, several options still remain. The choice was made to use ASP.NET, specifically C#, based on the availability and previous experience.

4.2 Algorithm

In Chapter 2 several algorithms were described for calculating the shortest route from one point to another. For the implementation, one algorithm, or a combination of several, needs to be chosen. The first constraint on this choice is the speed. The algorithm must find a solution within an acceptable timeframe.

Studies have shown that users start to believe an error has occurred when delays rise above 10 seconds and quality of service is rated as low for higher delays [18]. An online service should therefore aim for a response time below 10 seconds. The algorithm should at least be able to find a solution for any route within 10 seconds, preferably less, since transmitting results and displaying routes may increase the response time.

The second constraint for the choice of algorithm is complexity. Since there is only a limited amount of time available for the project, algorithms that are too complex may not be feasible.

The decision was made to use the A* algorithm with the Euclidean distance as heuristic, based on two arguments. First of all A* is one of the least complex algorithms, being only slightly more complex than Dijkstra. Second, methods such as geometrical pruning, ALT and multi-level approach can be combined with A* with only small adjustments to the algorithm. The major change for these methods is in the preprocessing phase. As a result, the application can be easily extended to support any of these methods at a later time.

The decision to use A* makes the choice for a data structure rather obvious. Because Fibonacci heaps have been created specifically for Dijkstra and A*, this should be a good choice.

4.3 Data source

As mentioned before, a data source that can supply accurate data on the complete road network in the Netherlands is a requirement for the success of the project. openstreetmap.org offers such a data source.

OpenStreetMap (OSM) is an openly available online world map. Anyone can contribute GPS data, which is used to form a dataset describing all the roads in the world. The data is by far not complete, but fortunately Automotive Navigation Data (AND) has donated a street network of the entire Netherlands. As a result the data is fairly complete for the Netherlands and a suitable source for the route planner.

The entire street network for the Netherlands (and other countries) can be downloaded as an XML file. This file contains node tags containing latitude and longitude and way tags containing the nodes it covers and several tags to indicate the type of road.

A road will be designated as allowing bicycles if at least one of the following tag/value combinations is included.

Tag	Value
highway	secondary tertiary unclassified residential livingstreet service track cycleway
cycleway	- any -
cycle	true yes 1

Table 3: Roads designated as allowing bicycles.

4.4 Other issues

When implementing a route planner, there are a number of issues to take into account. First of all there are several coordinate systems that can be used to specify a location on earth. Each system has its benefits and a decision needs to be made on which system to use. Also selecting a starting point and destination is not as trivial as it seems. Finally, a way to display the calculated route must be found. This section will describe these issues and the way they were handled.

4.4.1 Coordinates

The standard method to specify a location on earth is by specifying a longitude and latitude pair. Latitude is the angle between a point on the Earth's surface and the equatorial plane. Longitude is the angle east or west of a reference line that was chosen near Greenwich. The benefit of this sytem is that any location on earth can be pinpointed exactly. The downside however, is that calculating distances between two locations gets significantly more complicated. As a result, calculation times grow when distances need to be calculated often.

Fortunately, these coordinates can be approximated by coordinates in a plane[19], [20]. The exact width of one longitudinal degree on latitude φ can be calculated by

$$\frac{\pi}{180^\circ}\cos(\varphi)M_r$$

where M_r is the earth's average radius, which equals 6,367,449m. The width of one latitudinal degree can be approximated by

$$\frac{\pi}{180^\circ} M_r$$

By calculating longitude and latitude for each node in meters during startup, distances can be calculated in very little time.

4.4.2 Selecting a starting point and destination

In general, there are two methods of selecting a starting point and destination. Either have users enter an address, or let them select a location on a map. Unfortunately, both methods have their own issues.

The dataset from OSM does not specify a town for each way. This makes selecting an address difficult, because road names can be shared between towns. The second option, letting the user select a location on a map, creates the difficulty that the selected location will usually not be exactly on a node.

In the end the second method was implemented. To overcome the problem mentioned, the closest way to the selected location is determined by calculating the distance to all nearby ways. The closest node on that way is selected.

4.4.3 Displaying the route

For displaying the route there are again two options. Either use text to describe the route, or plot the route on a map. Since the map was already available for selecting the starting point and destination, this was the method that was chosen for the implementation.

4.5 Results

During the course of the project, a bicycle route planner for the Netherlands was implemented successfully. The application is called Cycle Route Planner. A class diagram describing the design of the implementation can be found in Appendix A. As mentioned, a Fibonacci heap is used to implement the open set. For the closed set, a Dictionary object as present in the C# API was used.

Figure 8 shows the application after a route has been plotted. A starting point can be selected by left clicking on the map and a destination point by right clicking on the map. When a user clicks “plan route”, both points are slightly adjusted to match an existing node and a route between both nodes is calculated and displayed on the map.

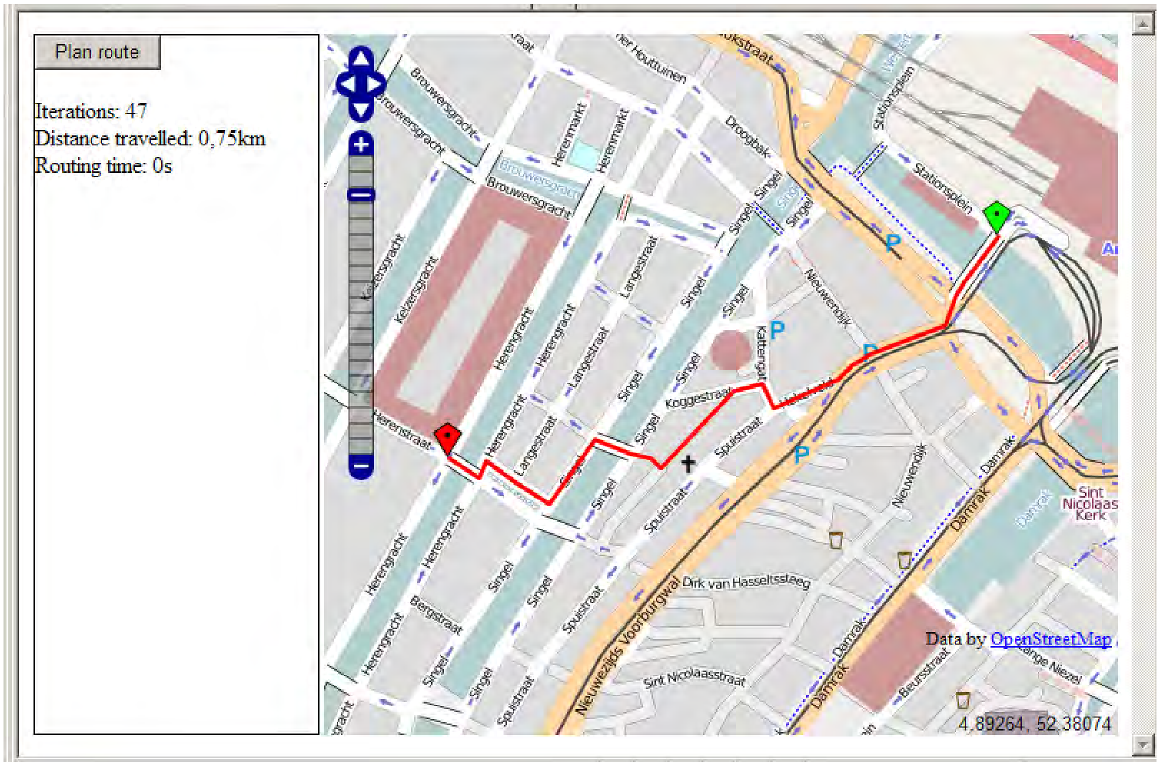

Figure 8: The Cycle Route Planner application.

4.5.1 Performance measurements

Table 4 shows some performance statistics for the application both when running A* and Dijkstra. Routes from the north-east corner to the south-west corner and from the north-west to the south-east corner of the country have been included, making the higher calculation times a worst case scenario. Most routes will be calculated in far less time.

The measurements were performed on a system with a Pentium Core 2 Duo E2160 processor running at 3Ghz and 4GB of RAM. All times were determined by calculating a route 3 times and taking the average of the 3 calculation times. Initially, the measurements were performed using 2GB of RAM, but the tests ran into memory limitations. This indicates memory usage is still an issue.

The statistics show that the constraint of staying below 10 seconds has been met with ease for the A* algorithm and only just for Dijkstra's algorithm. A* is faster in all of the tested cases.

The results also indicate that the relative speedup of A* compared to Dijkstra decreases when distances get longer. Apparently, when calculating a longer route, the number of nodes Dijkstra searches increases less rapidly than the number of nodes A* searches. It is likely that the cause of this is that a limited dataset is being used.

From	To	Distance (km)	A* (s)	Dijkstra (s)	Speedup
Amsterdam CS	Utrecht CS	44.18	0.13	1.36	10.5
Alkmaar	Breda	155.49	2.53	3.71	1.5
Leeuwarden	Rotterdam	211.39	2.15	5.28	2.5
Den Helder	Tilburg	215.18	3.24	7.05	2.2
Groningen	Tilburg	270.53	4.19	7.59	1.8
Alkmaar	Maastricht	301.67	6.85	9.23	1.3
Den Helder	Maastricht	342.51	7.4	9.04	1.2
Groningen	Middelburg	364.41	7.55	8.46	1.1

Table 4: Performance measurements.

When Dijkstra calculates a route from one end of the country to another, the area that is searched will not expand in all directions, simply because the borders of the dataset are reached. Because A* would have searched less nodes in the wrong directions anyway, it benefits less from this fact than Dijkstra does.

Taking this effect into account, it is probable that the lower speedup factor between A* and Dijkstra is not related to the absolute length of a route, but the length and position relative to the dataset.

4.6 Validation

In order to validate the results of the bicycle route planner, they were compared to the results of two international online route planners that use the same data source and have the option to plan bicycle routes. These route planners are OpenRouteService, available at http://www.openrouteservice.org/, and Your Navigation, available at http://www.yournagivation.org/. This section will show there are many differences between the results of these two route planners and the Cycle Route Planner application implemented during this project.

Figure 9 to Figure 11 show a route where there are no issues. All three route planners give the same route, which really is the shortest cycle route between those points. This is the ideal situation. Unfortunately, this is not always the case.

Figure 12 to Figure 14 show a route where OpenRouteService (ORS) gives a different result than the other two. It is obvious that the route found by ORS is the shortest. In real life the road chosen by ORS has a separate track for bicycles, so this is indeed the correct route. Analyzing the data shows that the road in question has type *primary highway*. This type is included in ORS and excluded in Your Navigation and the Cycle Route Planner application. The next example will illustrate why this decision was made.

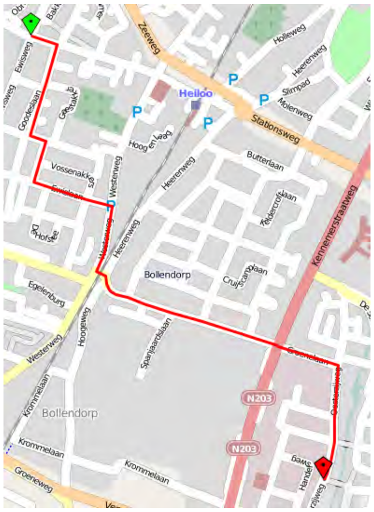

Figure 9: Cycle Route Planner

Figure 10: OpenRouteService

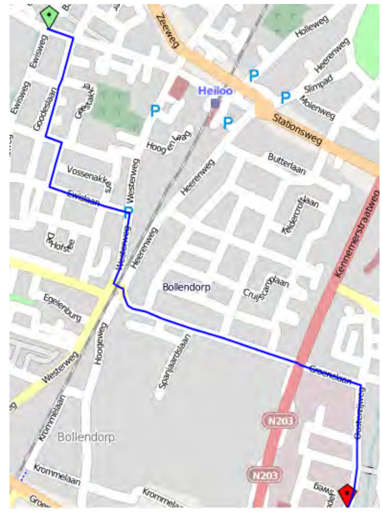

Figure 11: Your Navigation

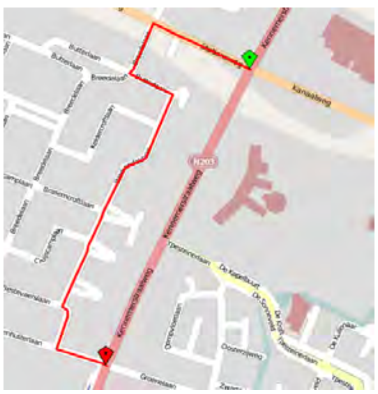

Figure 12: Cycle Route Planner

Figure 13: OpenRouteService

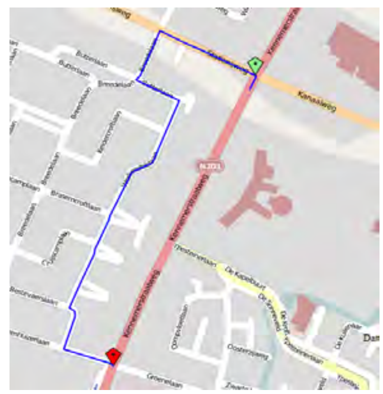

Figure 14: Your Navigation

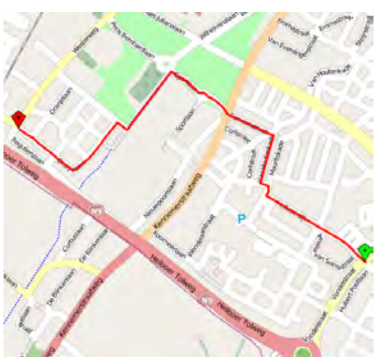

Figure 15: Cycle Route Planner

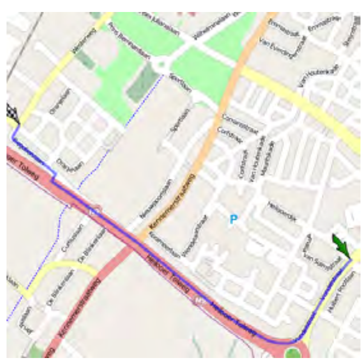

Figure 16: OpenRouteService

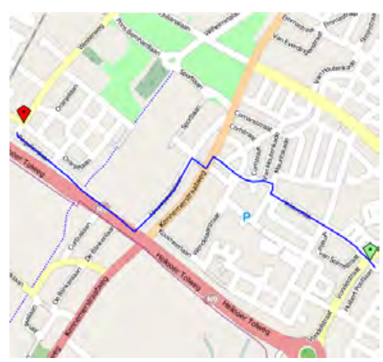

Figure 17: Your Navigation

Figure 15 to Figure 17 show a route that is different on all three route planners. ORS again chooses to take a primary highway and gives the shortest route. However, in this case the road does *not* allow bicycles. As a result, the route given by ORS cannot be taken by bike, although it should have planned a bicycle route.

The route that Your Navigation gives is still shorter than the one that the Cycle Route Planner gives. Closer analysis reveals that this route includes a pedestrian path to get to the destination. This means the Cycle Route Planner is the only planner that actually gives an admissible route.

The last example, Figure 18 to Figure 20, again gives three different routes. This time Your Navigation obviously gives the shortest route. What happens here is Your Navigation takes a pedestrian path again and the other two avoid it. OpenRouteService still manages to beat the Cycle Route Planner by taking a primary highway.

At first glance nothing new seems to happen here. However, surprisingly, Your Navigation gives the correct route. Although the dataset marks the path it uses as a pedestrian path, in real life it actually allows bicycles. The mistake is rather painful for the other two planners, as the path is in the middle of a long cycle track. The Cycle Route Planner and ORS both need to make very long detours to get to their destination, while they should just have taken the easiest route.

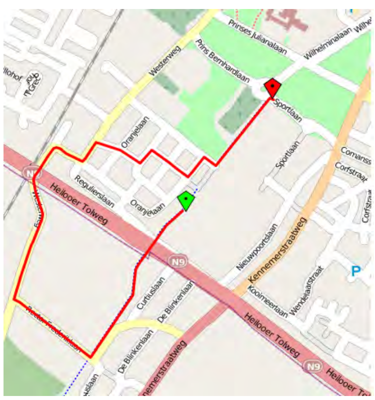

Figure 18: Cycle Route Planner

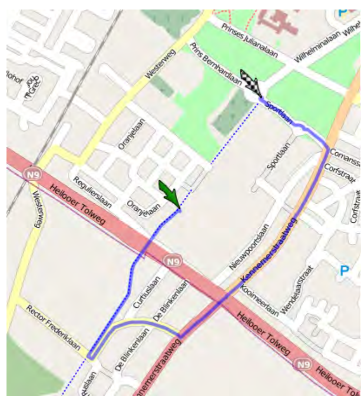

Figure 19: OpenRouteService

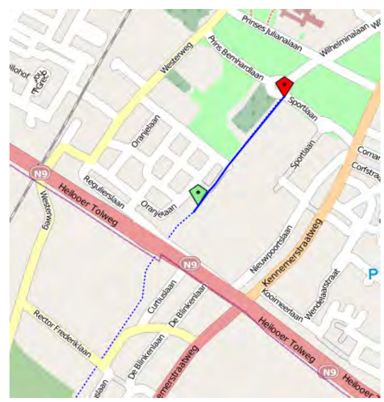

Figure 20: Your Navigation

To summarize, the differences can be explained by two factors. First, ORS allows primary highways for their bicycle routes. Second, Your Navigation allows pedestrian paths for their bicycle routes. The Cycle Route Planner allows neither.

The pedestrian path from the last example is almost certainly a one off mistake in the dataset. Therefore, pedestrian paths should not be allowed in cycle routes. Unfortunately there is no correct decision for primary highways. Whether primary highways allow bicycles or not varies between roads.

By deciding not to allow primary highways, the Cycle Route Planner may not give the shortest route in some cases, but it will always give an admissible route. This means users will never be surprised by a road they cannot take when following a route given by the application and this should be a priority when developing a route planner.

5 Conclusion & future work

5.1 Conclusion

The most well-known algorithm for solving a shortest path problem is Dijkstra's algorithm. However, when implementing an online service, where delays cannot be more than a few seconds, Dijkstra does not always suffice in terms of speed. Chapter 2 presented several methods to speed it up.

A* and bidirectional search are two methods that only adjust the route calculation algorithm and lead to small speedups. Geometric pruning, ALT algorithms and the multi-level approach use preprocessing to analyze the data. Preprocessing may take up to several hours during startup, but can decrease the query time by impressive amounts.

When using these methods, routes can be found thousands of times faster than when using Dijkstra. If necessary, most of these methods can even be combined to achieve even greater speedups. For online route planners, this means that the calculation time can be decreased to a few milliseconds. With calculation times this small, these no longer form a bottleneck, but overhead like transmitting the data over a network or displaying the route will.

Using the knowledge from Chapter 2, a bicycle route planner for the Netherlands was implemented. The route planner was written in C# and uses A* to calculate the routes, based on data from openstreetmap.org, which offers a complete road network for the Netherlands. In order to write the bicycle route planner application, a Fibonacci heap was implemented in C#. The Fibonacci heap uses general data types and can therefore be reused in alternate applications.

Performance statistics show that even the longest routes can be calculated within 8 seconds. Quality of service is only rated as low for response times over 10 seconds, so every route can be calculated with an acceptable QoS. The speedup of A* over Dijkstra is impressive for shorter routes, but almost negligible for routes that cross the entire country. Because of inconsistencies in the dataset, the bicycle route planner does not always give the shortest route. However, it will always give a route that can be taken by bike. This makes it stand out from the two route planners it was compared with during validation, where this was not always the case.

5.2 Future work

A relatively easy improvement for the bicycle route planner application would be adding another method for input and output. Another method for the location input would be to enter an address as text. This would have to be translated to longitude and latitude by means of reverse geocoding before it can be mapped to a node. Another method for the output would be to offer a written description that indicates where to go left or right and what the streets should be taken.

At the moment, the bicycle route planner supports only Dijkstra's algorithm and A*, using a Fibonacci heap for the open set. Implementing additional algorithms and/or data structures would result in two separate benefits. First, some of the algorithms discussed in the paper have been shown to decrease processing times substantially. Secondly, implementing several algorithms and data structures offers the opportunity to compare their performance.

When several methods are implemented, it should be relatively easy to combine them. This could lead to even greater speedups. Moreover, comparison of combinations of algorithms offers another subject of research. Finding the highest performing combination of algorithms and data structures in order to reach minimal processing times should offer an interesting challenge.

Finally, it was observed that the speedup of A* compared to Dijkstra grows smaller when the calculated routes grow larger. As mentioned, this effect is likely the result of the length and position of the route relative to the dataset. It would be interesting to confirm this effect for other datasets and see if similar effects can be found when comparing other methods.

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A Class diagram