



# Portfolio Optimisation

## *Modern Portfolio Theory*

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## **Table of contents**

<b>1</b>	<b>Introduction</b>	<b>1</b>
1.1	Introduction to BWI-thesis .....	1
1.2	Optimisation of IT-product portfolio.....	1
<b>2</b>	<b>Markowitz's Modern Portfolio Theory</b>	<b>2</b>
2.1	Introduction .....	2
2.2	Mean variance theory.....	2
<b>3</b>	<b>IT-product Portfolio</b>	<b>5</b>
3.1	Introduction .....	5
3.2	IT-product Portfolio .....	5
3.3	Optimisation .....	7
3.4	Assumptions .....	9
3.5	Optimisation with constraints .....	10
3.6	Heterogeneous portfolio .....	13
3.7	Conclusion.....	16
<b>4</b>	<b>Another application: loan portfolios</b>	<b>17</b>
4.1	Introduction .....	17
4.2	Loan Portfolio.....	17
4.3	Risk measurements .....	17
4.4	Optimise Portfolio.....	19
4.5	Optimising portfolio using mean-variance model.....	19
<b>5</b>	<b>Conclusions</b>	<b>20</b>
<b>6</b>	<b>Appendix</b>	<b>21</b>
6.1	Code macro for plotting Efficient Frontier .....	21
<b>7</b>	<b>References</b>	<b>22</b>



# 1 INTRODUCTION

## 1.1 Introduction to BWI-thesis

During my work at ABN-AMRO as part-timer and as an internee, I got involved in Credit-Portfolio Optimisation processes. During this exercise I got interested in this area and was advised by my supervisor (Prof. Bert Kersten) to write a separate BWI-thesis on the subject of Portfolio Optimisation. He also was kind to provide me an actual data set on which he was working on at LogicaCMG. With this small note I want to thank Professor B.Kersten for his great support and help during my internship at ABN AMRO BANK N.V.

## 1.2 Optimisation of IT-product portfolio

In an economic crisis it is – as in other situations – crucial for success to have a balanced and profitable product portfolio. Companies tend to neglect the balance in their portfolio, as they are adrift in growth, competition, production and delivery. From the domain of IT-portfolio management [I] and [II] (see Kersten, 2002, Kersten & Verhoef, 2002 and 2003) initiatives have been taken to apply quantitative techniques from portfolio methods to the IT-domain. Also the use of these techniques to “common” product portfolios of companies has been tested. Central starting point in all these experiments is Markowitz’s modern portfolio theory. The assumptions underlying this theory, which might be an obstacle for correct applications, are studied and solutions are found. The results are promising but as the results in the slipstream: portfolio management techniques contribute to transparency and decision making in the extensive IT-domain. An obvious example of the need for such an approach in IT is the Clinger-Cohen Act, which has been accepted in the Senate of the United States of America.

In this thesis Markowitz’s Modern Portfolio Theory is discussed. An implementation of the optimisation process on IT-product Portfolio based on Markowitz’s mean variance theory will be shown. Furthermore implementation of the theory on the credit portfolio is discussed.



## 2 MARKOWITZ'S MODERN PORTFOLIO THEORY

### 2.1 Introduction

The modern portfolio theory is constructed initially by Harry Markowitz and described in his book [111]. It is based on sophisticated investment decision approach that permits an investor to classify, estimate, and control both the kind and amount of expected risk and return. One of the essential parts of the modern portfolio theory is the quantification of the risk/return ratio.

The fundamental goal of Modern Portfolio theory is to optimally allocate investments between different assets.

### 2.2 Mean variance theory

#### *Mean variance model*

Modern Portfolio Theory is associated with mean variance return/risk analysis. A mean variance model minimises the portfolio risk for a given level of expected return. The volatility of an investment/asset is measured by the standard deviation of its return. Markowitz identifies the standard deviation of the portfolio return as the portfolio risk.

#### *Assumptions*

The mean-variance model has some underlying assumptions. First of all, the model is based on a single period model of investment. This means that the investor allocates its wealth among different assets in the beginning and harvests the returns at the end. Three measures are necessary for using the mean-variance model. The standard deviation of the return of each asset  $i$  (denoted by  $\mathbf{S}_i$ ), the expected return over a given time of period per asset and the correlation between each pair of assets are required. The variance or the standard deviation of an assets return over a given time of period is a standalone risk, also called the undiversified risk. The general idea is to minimise the portfolio's standard deviation of returns for a given level of expected portfolio return, considering the correlation of each pair of individual asset. The portfolio risk, standard deviation of portfolio returns, is denoted by  $\mathbf{S}_p$ , and is derived by

$$\mathbf{S}_p = \sqrt{\left( \sum_{i=1}^N X_i^2 \mathbf{S}_i^2 + \sum_{i=1}^N \sum_{\substack{j=1 \\ j \neq i}}^N X_i X_j \mathbf{S}_{ij} \right)}$$

$X_i$  represents the position of asset  $i$  in the portfolio.

$$\sum_{i=1}^n X_i = n$$

where  $n$  is the number of assets in the Portfolio

$$\mathbf{S}_{ij} = r_{ij} \mathbf{S}_i \mathbf{S}_j$$

$r_{ij}$  is the correlation between asset  $i$  and  $j$ .



### Efficient frontier

Modern portfolio theory assumes that for a specified expected portfolio return, a rational investor would choose the portfolio with the smallest possible risk and visa versa. A portfolio is said to be *efficient* if there is no portfolio having the same standard deviation with a greater expected return and there is no portfolio having the same return with a lesser standard deviation. The *efficient frontier* is the collection of all these efficient portfolios. An example of the efficient frontier is displayed beneath.

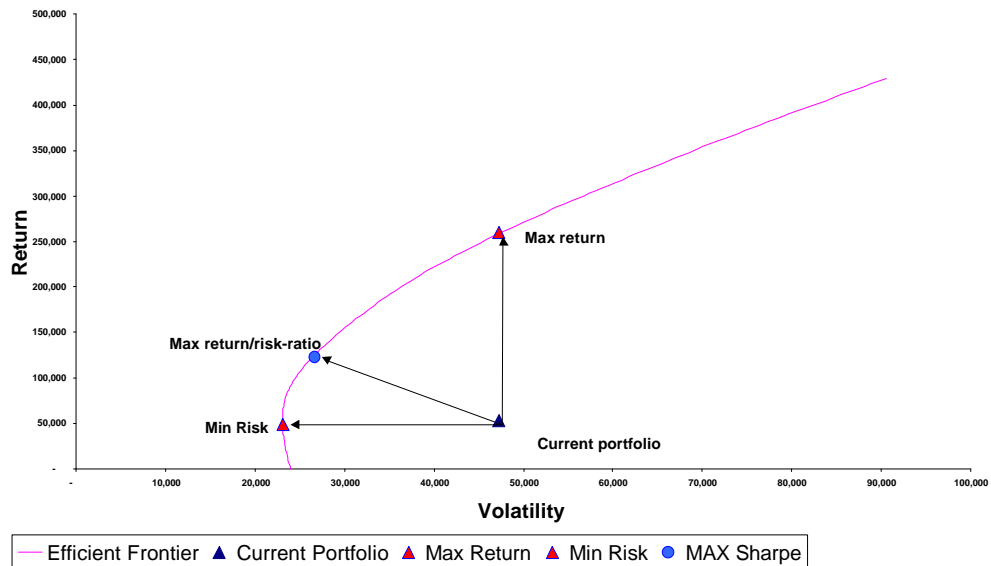


Figure i: An example of the efficient frontier, with three optimisation directions

The graph above is an example of an efficient frontier with the following four different Portfolio positions:

- ◆ Current portfolio: This is the point where the current portfolio is situated. The mean return and volatility is well below the efficient frontier.
- ◆ Max return: This point on the efficient frontier is the portfolio with the same volatility as the current portfolio. By optimal changing of positions in the portfolio an efficient portfolio with the same volatility and maximal return is given by this point on the efficient frontier.
- ◆ Min Risk: This point on the efficient frontier is the portfolio with the same mean return, but with the minimal volatility
- ◆ Max return/risk ratio (Sharpe): This ratio is a measurement for portfolio performance calculated as the mean return divided by the volatility of those returns. The higher the ratio the better the performance ratio.



Each portfolio on the efficient frontier offers the minimum possible risk for a given level of return. The general idea is to move the current portfolio in the direction of the efficient frontier. It's clear that for any given amount of return, you would like to choose a portfolio that gives you the least amount of risk. You want a portfolio that lies on the efficient frontier.

The following mean variance model [IV] is an example of a quadratic programming problem, which determines the efficient portfolio for a given level of expected portfolio return  $\mathbf{m}_p$ .

$$\text{Min } \sqrt{\left( \sum_{i=1}^N X_i^2 \mathbf{s}_i^2 + \sum_{i=1}^N \sum_{\substack{j=1 \\ j \neq i}}^N X_i X_j \mathbf{s}_{ij} \right)}$$

Subject to

$$\sum_{i=1}^N X_i = N \quad (1)$$

$$\sum_{i=1}^N X_i \mathbf{m}_i = \mathbf{m}_p \quad (2)$$

$$X_i \geq 0, \quad \text{for } i = 1, \dots, N \quad (3)$$

Where  $\mathbf{m}_i$  is defined as the expected return of asset  $i$ , and  $\mathbf{m}_p$  is the expected/desired return on the portfolio. The efficient frontier can be calculated by solving this problem for a number of portfolio returns.

Some additional restrictions could be applied to his model. An investor could constrain some asset's size with an upper- and lower bound. These constraints could be found necessary by the investor. Market specific information could make an investor to decide to limit some asset's size.



## 3 IT-PRODUCT PORTFOLIO

### 3.1 Introduction

In line with the Modern Portfolio Theory the mean variance model can be used to optimise an IT product Portfolio.

The goal is to minimise the risk on IT-product portfolio and increase the portfolio expected return.

### 3.2 IT-product Portfolio

Prof. Bert Kersten supplied the Portfolio, who is both working at the Free University as in LogicaCMG. Within LogicaCMG several business units operate with their own profit/loss responsibility. Amongst them is a business unit, which offers products and services in the domain of payroll and salary systems. The company is profitable in a high competitive market. In this market often opportunities arise which might yield to good results. Because of the many opportunities and the current portfolio of eighteen IT-products, the management chooses to use a portfolio approach. In preparing for this the expected return and volatility of each of these products is determined.<sup>1</sup> Due to confidentiality, the names of the products are replaced by  $P_1, \dots, P_{18}$ . The following tables give the monthly returns of each of product  $P_i$

Month	$P_1$	$P_2$	$P_3$	$P_4$	$P_5$	$P_6$	$P_7$	$P_8$	$P_9$
June	607,713	58,171	-6,861	-389	15,817	-72,520	21,604	33,714	13,255
July	710,581	69,573	-19,517	1,981	7,138	-63,656	5,986	61,343	10,292
August	435,793	62,288	-55,090	2,172	13,901	27,233	7,695	32,036	11,017
September	679,975	54,513	-30,337	481	11,480	-88,458	3,814	42,665	10,944
October	648,977	70,463	-10,772	2,144	10,019	-48,416	5,036	82,719	-14,832
November	732,679	64,867	4,553	2,081	9,294	73,063	4,785	34,817	13,634
December	1,143,622	80,822	-36,590	2,797	6,568	12,320	5,218	60,328	9,014
January	1,464,130	105,256	-24,251	575	7,412	-97,742	3,759	112,673	16,107
February	788,003	521,164	-27,537	2,215	12,807	-131,872	5,529	45,581	-513
March	702,673	56,760	-40,464	1,565	8,063	-36,699	4,537	43,906	-7,443
April	622,925	89,826	23,718	1,655	8,385	-127,196	4,780	52,787	-12,229
May	681,775	41,189	-69,999	2,991	12,308	-117,495	3,802	61,696	-9,294
June	675,692	58,566	-42,964	3,296	14,775	-172,369	6,614	59,636	-31,341
July	721,799	79,681	-45,885	2,980	10,122	-137,974	8,423	53,788	-13,641
August	628,581	52,164	-6,718	2,468	9,720	-122,108	22,567	75,008	-17,612
September	548,692	52,796	-21,851	2,611	8,958	-227,136	9,089	91,758	2,354
<b>Mean return</b>	737,101	94,881	-25,660	1,976	10,423	-83,189	7,702	59,028	-1,268

<sup>1</sup> Note that one of the assumptions underlining MPT might be an obstacle: tradability of products. However, the sense-of-urgency is that high that outsourcing and selling of products in the portfolio in a real option.



Month	$P_{10}$	$P_{11}$	$P_{12}$	$P_{13}$	$P_{14}$	$P_{15}$	$P_{16}$	$P_{17}$	$P_{18}$
June	-4,948	-22,845	-250,511	-26,460	-14,882	46,478	42,794	-68,274	-1,230
July	15,512	-28,632	-234,581	-35,780	-15,505	-15,328	71,246	-180,321	-26,404
August	3,954	-54,380	-223,717	-44,905	-21,983	38,002	24,939	-100,343	-3,274
September	3,798	-20,013	-185,654	-45,552	-15,305	-15,313	22,360	-142,448	101,822
October	4,050	-9,590	-222,239	-34,614	-19,313	24,733	48,562	-83,501	99,160
November	2,403	-8,338	-243,626	-44,895	-14,650	18,715	52,755	-80,559	79,891
December	2,163	-11,123	-172,501	-18,532	-54,727	-2,207	53,724	-31,769	-2,992
January	941	-11,742	-174,585	-24,395	-2,438	11,801	62,565	-79,668	41,194
February	1,757	-10,565	-178,679	-33,278	0	-17,780	27,125	-136,524	78,232
March	1,647	-23,299	-174,697	-40,319	0	-2,232	3,725	-115,973	38,705
April	769	-15,735	-410,180	-54,270	0	-14,552	22,852	-166,904	115,148
May	511	-16,655	-221,442	-51,349	0	14,213	28,113	-111,285	-285
June	537	-2,682	-150,413	-8,492	0	-10,705	27,955	-99,626	-14,191
July	21	-10,182	-185,309	-9,928	0	-13,684	13,444	-122,341	-19,196
August	503	-6,052	-147,301	127	0	-6,506	13,875	-96,620	37,369
September	1,807	2,891	-94,017	7,688	0	-28,011	48,463	-83,028	19,887
<b>Mean return</b>	<b>2,214</b>	<b>-15,559</b>	<b>-204,341</b>	<b>-29,060</b>	<b>-9,925</b>	<b>1,727</b>	<b>35,281</b>	<b>-106,199</b>	<b>33,990</b>

Table i: return per month of IT-products

The mean return for every product per month has been calculated. This mean return is also known as the expected monthly return per product. The sum of these expected returns represents the expected monthly portfolio return. From the data the expected return, volatility and correlation is calculated. The figures are displayed below.

Month	Volatility	Expected Return per month
$P_1$	235,088	737,101
$P_2$	111,161	94,881
$P_3$	22,699	-25,660
$P_4$	978	1,976
$P_5$	2,728	10,423
$P_6$	74,376	-83,189
$P_7$	5,655	7,702
$P_8$	21,621	59,028
$P_9$	13,737	-1,268
$P_{10}$	3,992	2,214
$P_{11}$	12,710	-15,559
$P_{12}$	66,346	-204,341
$P_{13}$	18,127	-29,060
$P_{14}$	14,124	-9,925
$P_{15}$	20,910	1,727
$P_{16}$	18,743	35,281
$P_{17}$	36,534	-106,199
$P_{18}$	45,930	33,990
<b>total</b>	<b>349,680</b>	<b>509,123</b>

Table ii: Volatility and mean return per month for every IT-product

The volatility is expressed in the standard deviation of the return.





The correlation between each pair of products are calculated and displayed in the table below.

	$P_1$	$P_2$	$P_3$	$P_4$	$P_5$	$P_6$	$P_7$	$P_8$	$P_9$	$P_{10}$	$P_{11}$	$P_{12}$	$P_{13}$	$P_{14}$	$P_{15}$	$P_{16}$	$P_{17}$	$P_{18}$
$P_1$	1.00	0.15	-0.03	-0.17	-0.49	0.11	-0.30	0.52	0.32	-0.03	0.26	0.17	0.09	-0.23	-0.05	0.44	0.33	-0.01
$P_2$	0.15	1.00	0.03	0.04	0.15	-0.15	-0.14	-0.11	0.04	-0.02	0.11	0.05	-0.07	0.16	-0.24	-0.06	-0.21	0.27
$P_3$	-0.03	0.03	1.00	-0.38	-0.27	0.05	0.26	0.06	0.09	-0.02	0.28	-0.50	-0.04	0.04	-0.05	0.25	-0.11	0.59
$P_4$	-0.17	0.04	-0.38	1.00	-0.11	-0.18	-0.22	0.11	-0.59	0.17	0.29	0.28	0.31	0.03	-0.38	-0.15	0.03	-0.32
$P_5$	-0.49	0.15	-0.27	-0.11	1.00	-0.15	0.35	-0.46	-0.17	-0.44	-0.23	0.02	-0.05	0.20	0.42	-0.33	0.05	-0.16
$P_6$	0.11	-0.15	0.05	-0.18	-0.15	1.00	-0.16	-0.45	0.50	0.21	-0.52	-0.27	-0.52	-0.65	0.59	0.21	0.27	0.08
$P_7$	-0.30	-0.14	0.26	-0.22	0.35	-0.16	1.00	-0.07	-0.08	-0.39	0.04	0.13	0.49	0.09	0.25	-0.16	0.25	-0.26
$P_8$	0.52	-0.11	0.06	0.11	-0.46	-0.45	-0.07	1.00	-0.12	0.07	0.51	0.37	0.48	0.18	-0.25	0.40	0.21	0.00
$P_9$	0.32	0.04	0.09	-0.59	-0.17	0.50	-0.08	-0.12	1.00	0.21	-0.40	-0.05	-0.27	-0.46	0.31	0.55	0.16	0.01
$P_{10}$	-0.03	-0.02	-0.02	0.17	-0.44	0.21	-0.39	0.07	0.21	1.00	-0.30	-0.03	-0.21	-0.22	-0.29	0.44	-0.51	-0.09
$P_{11}$	0.26	0.11	0.28	0.29	-0.23	-0.52	0.04	0.51	-0.40	-0.30	1.00	0.34	0.57	0.26	-0.49	0.10	0.25	0.20
$P_{12}$	0.17	0.05	-0.50	0.28	0.02	-0.27	0.13	0.37	-0.05	-0.03	0.34	1.00	0.71	0.03	-0.26	0.00	0.42	-0.38
$P_{13}$	0.09	-0.07	-0.04	0.31	-0.05	-0.52	0.49	0.48	-0.27	-0.21	0.57	0.71	1.00	0.10	-0.34	0.05	0.41	-0.42
$P_{14}$	-0.23	0.16	0.04	0.03	0.20	-0.65	0.09	0.18	-0.46	-0.22	0.26	0.03	0.10	1.00	-0.30	-0.43	-0.48	0.13
$P_{15}$	-0.05	-0.24	-0.05	-0.38	0.42	0.59	0.25	-0.25	0.31	-0.29	-0.49	-0.26	-0.34	-0.30	1.00	0.16	0.46	-0.09
$P_{16}$	0.44	-0.06	0.25	-0.15	-0.33	0.21	-0.16	0.40	0.55	0.44	0.10	0.00	0.05	-0.43	0.16	1.00	0.24	-0.13
$P_{17}$	0.33	-0.21	-0.11	0.03	0.05	0.27	0.25	0.21	0.16	-0.51	0.25	0.42	0.41	-0.48	0.46	0.24	1.00	-0.20
$P_{18}$	-0.01	0.27	0.59	-0.32	-0.16	0.08	-0.26	0.00	0.01	-0.09	0.20	-0.38	-0.42	0.13	-0.09	-0.13	-0.20	1.00

Table iii: return-correlation between each pair of products

### 3.3 Optimisation

First we calculate the efficient frontier by calculating the efficient set for every desired return  $m_p$ . In this picture the current position is displayed to give a general view of the distance to the efficient frontier. Then we calculate the positions of each investment for the following three directions of optimisation.

1. Minimise the risk, for a given level of return
2. Maximise the return for a given level of risk
3. Minimise the risk and maximise the return by maximising the risk/return-ratio.

The calculations were performed in Excel that provides a solver for non-linear quadratic problems.

The results are summarised in the following tables and graph.

Portfolio	Original	Min Risk, same return	Max Return, same risk	Max risk/return-ratio
Return	509,123	509,123	1,334,007	122,902
Volatility	349,680	86,524	349,680	9,886

Table iv: overview current situation versus the different optimisation directions



The following table shows the results after optimisation. The results of the three different optimisation directions are calculated and are shown in the next table.

	Original Portfolio		Min Risk		Max Return, same risk		Max return/risk-ratio	
	$X_i$	Expected Return	$X_i$	Expected Return	$X_i$	Expected Return	$X_i$	Expected Return
P <sub>1</sub>	1.00	737,101	0.33	245,579	1.26	931,713	0.05	33,805
P <sub>2</sub>	1.00	94,881	-	0	0.21	20,253	-	0
P <sub>3</sub>	1.00	-25,660	-	0	-	0	-	0
P <sub>4</sub>	1.00	1,976	-	0	-	0	9.75	19,267
P <sub>5</sub>	1.00	10,423	15.82	164,899	11.68	121,764	4.49	46,795
P <sub>6</sub>	1.00	-83,189	-	0	-	0	-	0
P <sub>7</sub>	1.00	7,702	-	0	-	0	1.30	10,047
P <sub>8</sub>	1.00	59,028	1.43	84,626	3.82	225,754	0.06	3,645
P <sub>9</sub>	1.00	-1,268	-	0	-	0	0.27	-337
P <sub>10</sub>	1.00	2,214	-	0	-	0	1.87	4,133
P <sub>11</sub>	1.00	-15,559	-	0	-	0	-	0
P <sub>12</sub>	1.00	-204,341	-	0	-	0	-	0
P <sub>13</sub>	1.00	-29,060	-	0	-	0	-	0
P <sub>14</sub>	1.00	-9,925	-	0	-	0	0.04	-418
P <sub>15</sub>	1.00	1,727	-	0	-	0	-	0
P <sub>16</sub>	1.00	35,281	-	0	-	0	-	0
P <sub>17</sub>	1.00	-106,199	-	0	-	0	-	0
P <sub>18</sub>	1.00	33,990	0.41	14,019	1.02	34,523	0.18	5,966
<b>Total</b>	<b>18.00</b>	<b>509,123</b>	<b>18.00</b>	<b>509,123</b>	<b>18.00</b>	<b>1,334,007</b>	<b>18.00</b>	<b>122,902</b>

Table v: comparison between current situation and different optimisation directions

	Original portfolio weights	Min Risk portfolio weights	Direction position $i$	Max return portfolio weights	Direction position $i$	Max return/risk-ratio portfolio weights	Direction position $i$
	$X_i$	$X_i$		$X_i$		$X_i$	
P <sub>1</sub>	1.00	0.33	-	1.26	-	0.05	-
P <sub>2</sub>	1.00	-	-	0.21	-	-	-
P <sub>3</sub>	1.00	-	-	-	-	-	-
P <sub>4</sub>	1.00	-	-	-	-	9.75	-
P <sub>5</sub>	1.00	15.82	-	11.68	-	4.49	-
P <sub>6</sub>	1.00	-	-	-	-	-	-
P <sub>7</sub>	1.00	-	-	-	-	1.30	-
P <sub>8</sub>	1.00	1.43	-	3.82	-	0.06	-
P <sub>9</sub>	1.00	-	-	-	-	0.27	-
P <sub>10</sub>	1.00	-	-	-	-	1.87	-
P <sub>11</sub>	1.00	-	-	-	-	-	-
P <sub>12</sub>	1.00	-	-	-	-	-	-
P <sub>13</sub>	1.00	-	-	-	-	-	-
P <sub>14</sub>	1.00	-	-	-	-	0.04	-
P <sub>15</sub>	1.00	-	-	-	-	-	-
P <sub>16</sub>	1.00	-	-	-	-	-	-
P <sub>17</sub>	1.00	-	-	-	-	-	-
P <sub>18</sub>	1.00	0.41	-	1.02	-	0.18	-
<b>Total</b>	<b>18.00</b>	<b>18.00</b>		<b>18.00</b>		<b>18.00</b>	

Table vi: change of each product position in the different optimisation portfolios



The whole optimisation and construction of the efficient frontier has been done in Excel. This application has a solver that is useful for small-scaled programming problems. To construct the efficient frontier a small macro has been written. The code can be found in the appendix. The macro is built in such a way that for every level of return, the minimum risk is determined by solving the optimisation problem. Plotting the results gives the following efficient frontier:

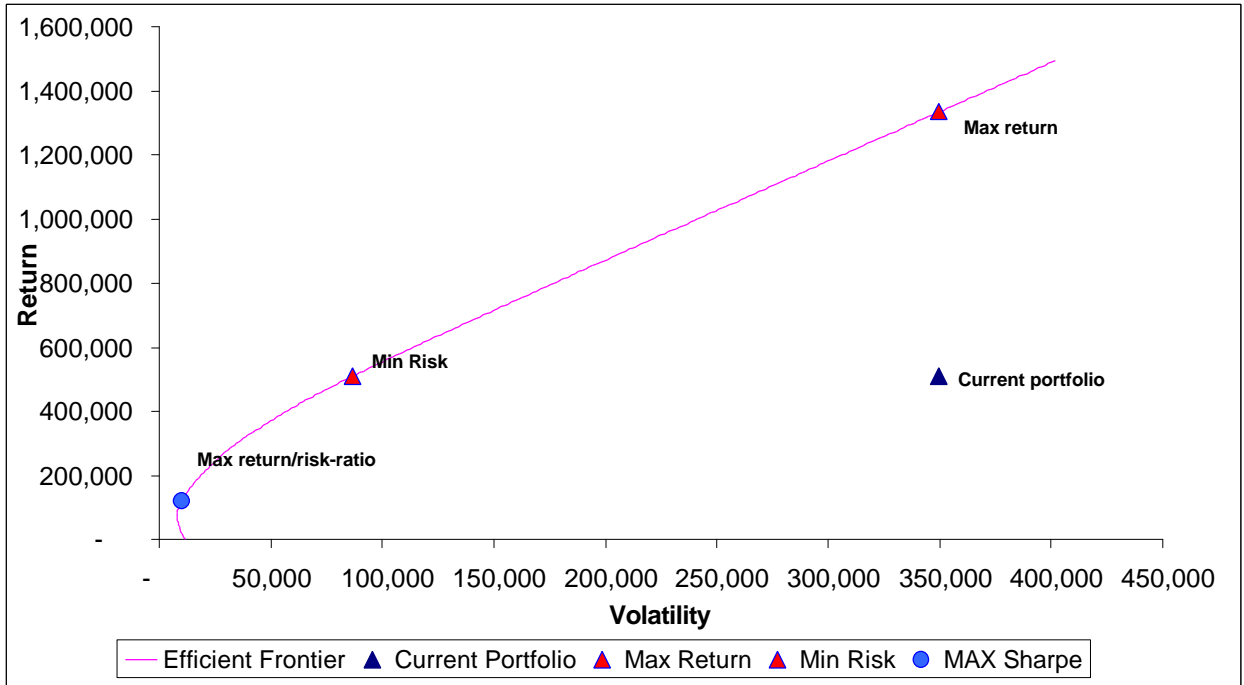


Figure ii: efficient frontier of the given portfolio

### 3.4 Assumptions

By calculating the efficient frontier no additional restrictions on product positions has been formulated. This means that for every product we assume the position is not bounded. Under market conditions some products positions could not so easily be expanded, due to market satiation or heavy competition.

The flexibility of reducing product positions is bound to contracts with customers. Therefore reducing the portfolio or the 'killing' of certain products is not that easy. A solution can be found in outsourcing or selling this part of the portfolio to another company who will take over the contracts.

All these constraints however can be implemented in the calculation of the efficient portfolio. The constrained efficient frontier will then move downwards compared with the unconstrained efficient frontier.

Another question mark could be placed by the calculation of the volatility and the correlation between products. The volatility is in fact the standard deviation of the return.



Correct information on these returns is critical in terms of calculating the volatility. Also the accuracy of the volatility is dependent on returns. Calculation of the volatility is more accurate if the information on the returns is looked over a longer period (which results in more return figures). The same goes for the calculation of correlation between each pair of products.

### 3.5 Optimisation with constraints

An additional analyse has been performed on the portfolio. The Company was asked to specify constraints for the products in the portfolio. They decided that no product should exceed 2 times its current size.

The portfolio optimisation is adjusted to the following equation, where every position  $X_i$  is constrained with an upper bound.

$$\begin{aligned} \text{Minimize} \quad & \sqrt{\left( \sum_{i=1}^N \sum_{j=1}^N X_i X_j \mathbf{s}_{ij} \right)} \\ \text{Subject to} \quad & \sum_{i=1}^N X_i = N \quad (1) \\ & \sum_{i=1}^N X_i \mathbf{m}_i = \mathbf{m}_p \quad (2) \\ & 0 \leq X_i \leq 2 \quad \text{for } i = 1, \dots, N \quad (3) \end{aligned}$$

Again the problem is solved for the three different optimisation directions. The results are summarised in the following table.

Portfolio	Original	Min Risk, same return	Max Return, same risk	Max risk/return-ratio
Return	509,123	509,123	1,102,088	484,244
Volatility	349,680	97,904	349,714	92,881

Table vii: comparison current portfolio with different optimal portfolios

With the restrictions on the change of positions, the calculated optimal portfolios are still out performing the original portfolio. For three times less risk, the same return can be achieved, while for the same risk an increase return can be doubled.

The following tables show the positions of the products after solving the problem with the three optimisation directions.



	Original Portfolio		Min Risk		Max Return		Max return/risk-ratio	
	$X_i$	Expected Return	$X_i$	Expected Return	$X_i$	Expected Return	$X_i$	Expected Return
P <sub>1</sub>	1.00	737,101	0.51	374,240	1.10	808,159	0.48	356,231
P <sub>2</sub>	1.00	94,881	0.42	40,136	0.82	77,891	0.41	38,595
P <sub>3</sub>	1.00	-25,660	0.36	-9,297	-	-	0.37	-9,517
P <sub>4</sub>	1.00	1,976	2.00	3,953	2.00	3,953	2.00	3,953
P <sub>5</sub>	1.00	10,423	2.00	20,846	2.00	20,846	2.00	20,846
P <sub>6</sub>	1.00	-83,189	-	-	-	-	-	-
P <sub>7</sub>	1.00	7,702	2.00	15,405	2.00	15,405	2.00	15,405
P <sub>8</sub>	1.00	59,028	0.89	52,472	1.61	95,162	0.86	50,552
P <sub>9</sub>	1.00	-1,268	1.16	-1,470	0.20	-259	1.20	-1,521
P <sub>10</sub>	1.00	2,214	2.00	4,428	2.00	4,428	2.00	4,428
P <sub>11</sub>	1.00	-15,559	0.91	-14,157	-	-	0.94	-14,596
P <sub>12</sub>	1.00	-204,341	-	-	-	-	-	-
P <sub>13</sub>	1.00	-29,060	0.82	-23,691	-	-	0.87	-25,380
P <sub>14</sub>	1.00	-9,925	1.76	-17,429	1.80	-17,875	1.74	-17,273
P <sub>15</sub>	1.00	1,727	1.41	2,438	1.84	3,168	1.40	2,415
P <sub>16</sub>	1.00	35,281	0.97	34,298	1.40	49,470	0.95	33,431
P <sub>17</sub>	1.00	-106,199	-	-	-	-	-	-
P <sub>18</sub>	1.00	33,990	0.79	26,951	1.23	41,741	0.78	26,678
<b>Total</b>	<b>18.00</b>	<b>509,123</b>	<b>18.00</b>	<b>509,123</b>	<b>18.00</b>	<b>1,102,090</b>	<b>18.00</b>	<b>484,244</b>

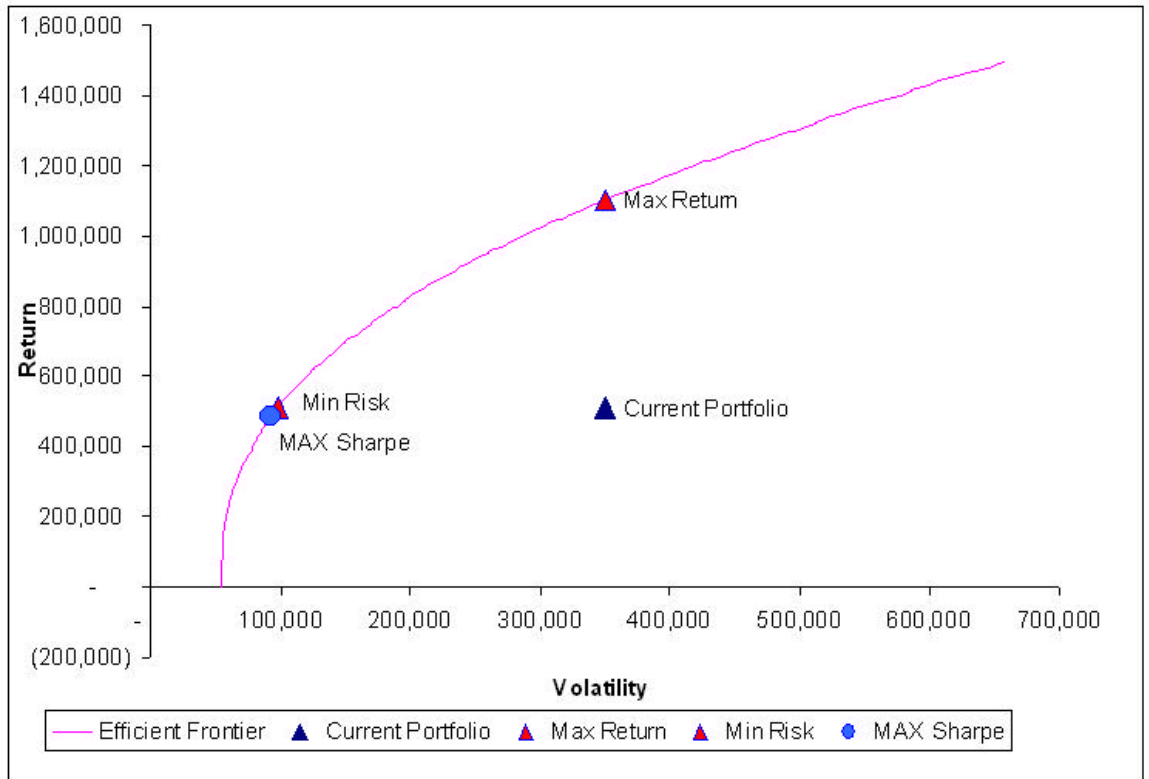
Table viii: comparison between current situation and different optimisation directions

	Original portfolio weights	Min Risk portfolio weights	Direction position i	Max return portfolio weights i	Direction position i	Max return/risk-ratio portfolio weights	Direction position i
	$X_i$	$X_i$		$X_i$		$X_i$	
P <sub>1</sub>	1.00	0.51	-	1.10	-	0.48	-
P <sub>2</sub>	1.00	0.42	-	0.82	-	0.41	-
P <sub>3</sub>	1.00	0.36	-	-	-	0.37	-
P <sub>4</sub>	1.00	2.00	-	2.00	-	2.00	-
P <sub>5</sub>	1.00	2.00	-	2.00	-	2.00	-
P <sub>6</sub>	1.00	-	-	-	-	-	-
P <sub>7</sub>	1.00	2.00	-	2.00	-	2.00	-
P <sub>8</sub>	1.00	0.89	-	1.61	-	0.86	-
P <sub>9</sub>	1.00	1.16	-	0.20	-	1.20	-
P <sub>10</sub>	1.00	2.00	-	2.00	-	2.00	-
P <sub>11</sub>	1.00	0.91	-	-	-	0.94	-
P <sub>12</sub>	1.00	-	-	-	-	-	-
P <sub>13</sub>	1.00	0.82	-	-	-	0.87	-
P <sub>14</sub>	1.00	1.76	-	1.80	-	1.74	-
P <sub>15</sub>	1.00	1.41	-	1.84	-	1.40	-
P <sub>16</sub>	1.00	0.97	-	1.40	-	0.95	-
P <sub>17</sub>	1.00	-	-	-	-	-	-
P <sub>18</sub>	1.00	0.79	-	1.23	-	0.78	-
<b>Total</b>	<b>18.00</b>	<b>18.00</b>		<b>18.00</b>		<b>18.00</b>	

Table ix: change of each product position in the different optimisation portfolios



The efficient frontier is constructed again, but this time the constraints are taken into account.



Note that the efficient frontier is closer to the current portfolio. This is due to the fact the portfolio is constrained. Nevertheless the portfolio is far from the efficient frontier.



### 3.6 Heterogeneous portfolio

The previous analyses were assuming that each product was equally weighted in the portfolio. That's fine if the products need the same amount or number of resources. In this paragraph we will demonstrate how we can optimise a IT-product portfolio considering that every product has a different weight.

LogicaCMG calculated the resources needed for each product. The following table summarises the result.

	Product costs	Resulting Weights
		$X_i$
$P_1$	7,503,120	26.45%
$P_2$	1,596,064	5.63%
$P_3$	1,195,111	4.21%
$P_4$	94,859	0.33%
$P_5$	133,546	0.47%
$P_6$	4,585,611	16.17%
$P_7$	10,574	0.04%
$P_8$	1,387,211	4.89%
$P_9$	276,407	0.97%
$P_{10}$	92,675	0.33%
$P_{11}$	148,427	0.52%
$P_{12}$	2,530,931	8.92%
$P_{13}$	318,581	1.12%
$P_{14}$	673,474	2.37%
$P_{15}$	1,418,486	5.00%
$P_{16}$	419,015	1.48%
$P_{17}$	3,854,142	13.59%
$P_{18}$	2,125,953	7.50%
<b>Total</b>	<b>28,364,187</b>	<b>100.00%</b>

The portfolio optimisation is adjusted to the following equation, where every position  $X_i$  is constrained with an upper bound.  $X_i$  is constrained to a shift of  $\pm 100\%$

$$\text{Minimize} \quad \sqrt{\left( \sum_{i=1}^N \sum_{j=1}^N X_i X_j \mathbf{s}_{ij} \right)}$$

$$\text{Subject to} \quad \sum_{i=1}^N X_i = 1 \quad (1)$$

$$\sum_{i=1}^N X_i \mathbf{m}_i = \mathbf{m}_p \quad (2)$$

$$0 \leq X_i \leq 2 * X_i^{Old} \quad \text{for } i = 1, \dots, N \quad (3)$$

Again the problem is solved for the three different optimisation directions. The results are summarised in the following table.



Portfolio	Original	Min Risk, same return	Max Return, same risk	Max risk/return-ratio
Return	509.123	509.123	1.023.053	1.789.036
Volatility	349.664	252.076	349.665	528.320

Table x: comparison current portfolio with different optimal portfolios

With the restrictions on the change of positions, the calculated optimal portfolios are still out performing the original portfolio.

The following tables show the positions of the products after solving the problem with the three optimisation directions.

	Original Portfolio		Min Risk		Max Return		Max return/risk-ratio	
	$X_i$	Expected Return	$X_i$	Expected Return	$X_i$	Expected Return	$X_i$	Expected Return
P <sub>1</sub>	0.26	737,101	0.15	426,978	0.29	814,309	0.52	1,452,873
P <sub>2</sub>	0.06	94,881	0.03	46,651	0.03	52,801	0.03	51,330
P <sub>3</sub>	0.04	-25,660	0.02	-9,925	0.02	-13,846	0.00	0
P <sub>4</sub>	0.00	1,976	0.01	3,953	0.01	3,953	0.01	3,953
P <sub>5</sub>	0.00	10,423	0.01	20,846	0.01	20,846	0.01	20,846
P <sub>6</sub>	0.16	-83,189	0.14	-72,781	0.17	-87,906	0.00	0
P <sub>7</sub>	0.00	7,702	0.00	15,405	0.00	15,405	0.00	15,405
P <sub>8</sub>	0.05	59,028	0.10	118,057	0.10	118,057	0.10	118,057
P <sub>9</sub>	0.01	-1,268	0.00	0	0.00	0	0.00	0
P <sub>10</sub>	0.00	2,214	0.01	4,428	0.01	4,428	0.01	4,428
P <sub>11</sub>	0.01	-15,559	0.00	0	0.00	0	0.00	0
P <sub>12</sub>	0.09	-204,341	0.00	0	0.00	0	0.00	0
P <sub>13</sub>	0.01	-29,060	0.00	0	0.00	0	0.00	0
P <sub>14</sub>	0.02	-9,925	0.05	-19,850	0.05	-19,850	0.05	-19,850
P <sub>15</sub>	0.05	1,727	0.10	3,453	0.10	3,453	0.10	3,453
P <sub>16</sub>	0.01	35,281	0.03	70,562	0.03	70,562	0.03	70,562
P <sub>17</sub>	0.14	-106,199	0.21	-166,634	0.03	-27,138	0.00	0
P <sub>18</sub>	0.07	33,990	0.15	67,980	0.15	67,980	0.15	67,980
<b>Total</b>	<b>1</b>	<b>509,123</b>	<b>1</b>	<b>509,123</b>	<b>1</b>	<b>1,023,053</b>	<b>1</b>	<b>1,789,036</b>

Table xi: comparison between current situation and different optimisation directions





	Original portfolio weights	Min Risk portfolio weights	Change in Max return portfolio weights %	Change in Max return portfolio weights i %	Change in Max return/risk-ratio portfolio weights %	Change in Max return/risk-ratio portfolio weights %	
	$X_i$	$X_i$	$X_i$	$X_i$	$X_i$	$X_i$	
$P_1$	0.26	0.15	-42%	0.29	10%	0.52	97%
$P_2$	0.06	0.03	-51%	0.03	-44%	0.03	-46%
$P_3$	0.04	0.02	-61%	0.02	-46%	0.00	-100%
$P_4$	0.00	0.01	100%	0.01	100%	0.01	100%
$P_5$	0.00	0.01	100%	0.01	100%	0.01	100%
$P_6$	0.16	0.14	-13%	0.17	6%	0.00	-100%
$P_7$	0.00	0.00	100%	0.00	100%	0.00	100%
$P_8$	0.05	0.10	100%	0.10	100%	0.10	100%
$P_9$	0.01	0.00	-100%	0.00	-100%	0.00	-100%
$P_{10}$	0.00	0.01	100%	0.01	100%	0.01	100%
$P_{11}$	0.01	0.00	-100%	0.00	-100%	0.00	-100%
$P_{12}$	0.09	0.00	-100%	0.00	-100%	0.00	-100%
$P_{13}$	0.01	0.00	-100%	0.00	-100%	0.00	-100%
$P_{14}$	0.02	0.05	100%	0.05	100%	0.05	100%
$P_{15}$	0.05	0.10	100%	0.10	100%	0.10	100%
$P_{16}$	0.01	0.03	100%	0.03	100%	0.03	100%
$P_{17}$	0.14	0.21	57%	0.03	-74%	0.00	-100%
$P_{18}$	0.07	0.15	100%	0.15	100%	0.15	100%
<b>Total</b>	<b>1</b>	<b>1</b>	<b>1</b>	<b>1</b>	<b>1</b>	<b>1</b>	

Table xii: change of each product position in the different optimisation portfolios

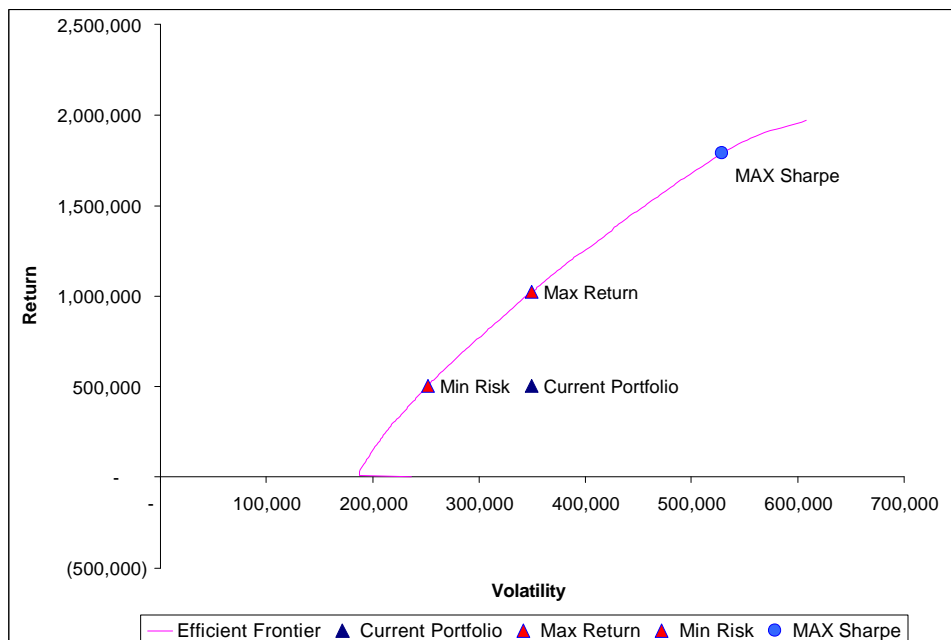


Figure iii: efficient frontier of the heterogeneous portfolio



### 3.7 Conclusion

With the mean variance theory constructed by Markowitz, the product portfolio can be improved. The results show a considerable decrease in risk, while maintaining the same return. Even with constraints applied on the portfolio and its products, the optimal portfolios perform far better.

The mean variance theory has proven its worthiness for an IT-product portfolio. By evaluating returns achieved in the past, portfolio selection is possible. A remark should be made: Returns from the past do not guarantee the future. The model can not foresee any event that could occur in the future. It only diversifies the portfolio by looking at the results of the past.



## 4 ANOTHER APPLICATION: LOAN PORTFOLIOS

### 4.1 Introduction

The mean variance analysis discussed in chapter 2 concentrates on decreasing the standard deviation of an asset-return Portfolio, maintaining the same return. This chapter discusses how we can apply this theory on a Loan Portfolio.

### 4.2 Loan Portfolio

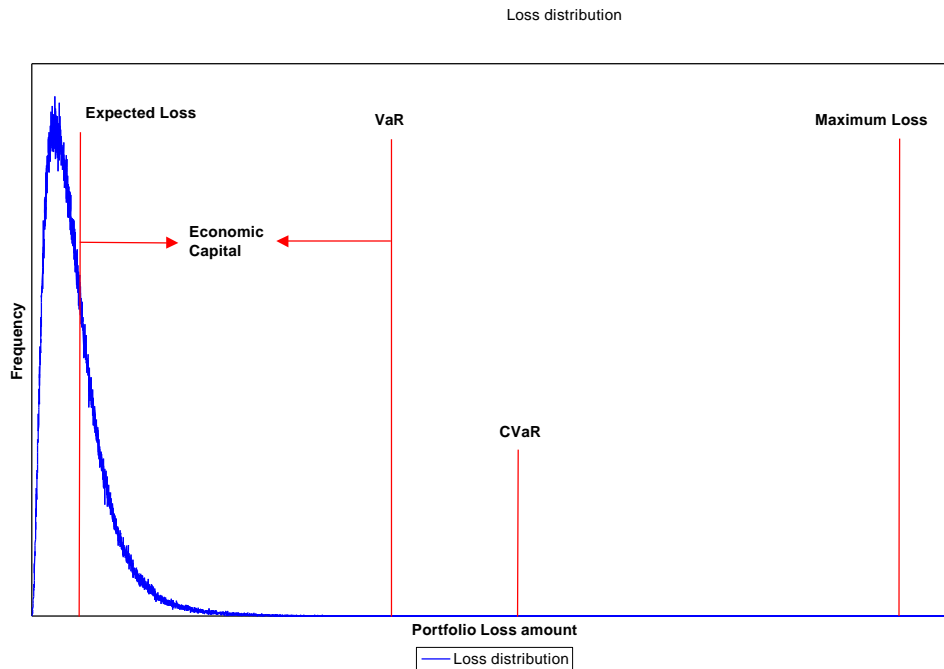
The general idea of optimising a loan portfolio is to reduce its risk, while maintaining the same level of return. Here again we can apply Markowitz's Modern Portfolio Theory.

The mean variance model assumes a normal return/loss distribution, but the actual loss distribution of a loan portfolio however is fat-tailed and heavily skewed. Our interest goes to relevant risk measures that describe the risk in the tail. The reduction of the tail risk is preferred. In the following paragraph we will discuss some risk-measures that can be used for optimisation purposes.

### 4.3 Risk measurements

<i>Expected Loss</i>	<p>Expected Loss is the expected annual loss for a facility. It represents the amount that a lender expects to lose for a given time of period (often over a one-year time horizon).</p> <p>Expected Loss is counterparty specific and is not influenced by correlation with other facilities/borrowers/relationships and does not benefit by diversification and does not worsen by concentration.</p>
<i>Unexpected Loss</i>	<p>Unexpected Loss, also called the volatility of losses, is commonly known as the standard deviation of the loss distribution. The Unexpected Loss of a facility is then the standard deviation of the facility loss. The portfolio Unexpected Loss is the standard deviation of the portfolio loss distribution.</p>
<i>VaR</i>	<p>Financial institutions are interested in risk measures that identify the risk in the fat and skewed tail of the loss-distribution. Value at Risk (VaR) was introduced as such a risk measure to define risk for a given portfolio. VaR is a maximum possible portfolio loss for a specified time horizon with a given confidence level <math>\beta</math> (i.e. <math>\beta=95\%</math>).</p>
<i>Economic Capital</i>	<p>Economic Capital is a risk measure quantifying how much capital must be allocated to cover possible losses in excess of the Expected Loss. The Economic Capital is acquired from the VaR and is given by <math>EC = VaR - EL</math>. The Economic Capital on facility level is known as the Economic Capital Contribution. It specifies the amount a facility contributes to the Portfolio Economic Capital. Economic Capital is calculated on portfolio level and has the property that diversification and correlation effects are taken into account.</p>





*Figure iv: Portfolio Loss distribution*

**CVaR** Another statistical measure is called Conditional Value at Risk (CVaR). This measure is also known as the Mean Excess Loss, Mean Shortfall, or Tail VaR. By definition, CVaR is the expected conditional on losses exceeding VaR. In this case it represents the expected loss in excess of confidence level  $\beta$ , or the expected loss given the fact that the loss is greater than VaR.

**Optimisation by VaR and CVaR**

Many publications are based on minimising VaR for a given level of return. But there are also downsides using VaR as the measure to optimise.

In the case of a finite number of scenarios, VaR is not a recommended measure of risk. VaR in this case is non-smooth, non-convex, and has a multi-extremum function, making it difficult to control and optimise. VaR also has the property as lacking of sub-additivity [V]. VaR also does not give any information about the excess loss (loss suffered in the tail).

CVaR, on the other hand, is considered a more coherent [VI] measure of risk than VaR. Unlike VaR it has the property of being sub-additive and convex. Another advantage is that CVaR can be optimised using linear programming techniques [VII].

Both VaR and CVaR could be implemented in the mean variance method. We would then minimise the portfolio VaR or CVaR for a given level of portfolio return.



#### 4.4 Optimise Portfolio

The main objective is to optimise the portfolio. Several restrictions and objectives should be taken into account. To optimise a portfolio one should analyse it first and improve it in such a way that it obtains for instance the smallest level of risk for a given level of return. So the mean-variance analyse could minimise the UL, VaR or CVaR for a given amount of return, or maximise the return amount for a given level of risk. For this thesis we will look at the standard deviation of the loss distribution, to maintain alignment with the mean variance theory.

#### 4.5 Optimising portfolio using mean-variance model

The basics of the mean-variance analyse is building an optimal loan portfolio using statistical measures for expectation and variance of loss. We will concentrate on minimising the risk on loss. In this case the standard deviation (square root of the variance) of a credit loan's loss is used to define the risk. We use the Mean Variance model below, where the standard deviation is minimised for a given level of return.

$$\text{Minimize} \quad \sum_{i=1}^n \sum_{j=1}^n x_i x_j \mathbf{S}_{ij}$$

$$\text{Subject to} \quad \sum_{i=1}^n r_i x_i = d \quad (1)$$

$$\sum_{i=1}^n x_i = n \quad (2)$$

$$x_i \geq 0 \quad \forall i = 1, \dots, n \quad (3)$$

Where

$x_i$  = position of facility  $i$

$r_i$  = return of facility  $i$

$d$  = desired return

$\mathbf{S}_{ij}$  =  $\mathbf{r}_{ij} \mathbf{S}_i \mathbf{S}_j$

$\mathbf{r}_{ij}$  = loss correlation between facility  $i$  and  $j$

For small loan portfolios, this problem could be solved by using the Excel-solver.



## 5 CONCLUSIONS

It is clear that the application of MPT to other domains than for which it was originally developed, yields interesting results. It introduces a quantitative approach to new domains, especially product portfolios and IT-portfolios. The application towards loan portfolios has been tried earlier.

The MPT results on the LogicaCMG data are satisfying in terms of applicability. However the importance of correct information in terms of returns is absolutely needed. It is highly recommended having as much return figures as possible. More expected periodically returns, would contribute to more accurate variance, volatility and correlation figures.



## 6 APPENDIX

### 6.1 Code macro for plotting Efficient Frontier

```
Sub Frontier()  
' Recorded by Serdar Ozdemir  
  
Range("W1:AA300").Select  
Selection.ClearContents  
Range("$W$1").Select  
current = Selection.Address  
  
For i = 1 To 300  
    SolverSolve (True)  
    Range("Desired_Return").Value = -5000 + i * 5000  
    ActiveCell.Value = Range("Return").Value  
    ActiveCell.Offset(0, 1).Value = Range("Target").Value  
    ActiveCell.Offset(0, 3).Value = Range("Desired_Return").Value  
    ActiveCell.Offset(1, 0).Activate  
Next  
Range(current).Select  
Application.ScreenUpdating = True  
End Sub
```



## 7 REFERENCES

- 
- I “IT Due Diligence” by H.M.P. Kersten
  - II “Quantitative IT-portfolio management” by C. Verhoef
  - III “Portfolio Selection Efficient: Diversification of Investment” by Harry M. Markowitz
  - IV “Portfolio Optimisation” by M.Pirbhai, M. Guertler and M.S. Mendi, *Optirisk white paper*, 2001.
  - V “Conditional Value-at-Risk: Optimization Algorithms and Applications” by S. Uryasev, *Financial Engineering News*, no 14, 1-5, (February 2000)
  - VI "Coherent Measures of Risk" by P. Artzner, F. Delbean, J.-M.Eber, D. Heath, *Mathematical Finance*, no 9, 203-228, (1999).
  - VII “Optimisation of Conditional Value-at-Risk” by S. Uryasev and R.T. Rockafellar, *Journal Risk*, no. 2, 21-41, (2000)

