Driver uncertainty during roadwork

The effect of driver uncertainty on the expected delay at an intersection

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Preface

This research paper is a mandatory part of the master program Business Analytics at the VU University Amsterdam. The goal of this research paper is to examine the influence of erratic behaviour and wrong path choice of vehicles at an intersection during roadwork, and the effect this behaviour has on the delay experienced.

I would like to thank my supervisor René Bekker for his help during the writing of this research paper.

Roan Meulblok, Amsterdam

Abstract

When a road at an intersection is closed, some drivers are unsure of what direction to go. They might switch lanes in front of the traffic light, causing delay. Or they might change their mind after they have chosen a direction and turn around to return to the intersection. In this paper we examine the effect of this uncertainty on the average delay incurred at an intersection, as well as the effect of the closed lane on the remaining open lanes.

This paper first discusses various models to estimate the delay incurred at an intersection under normal conditions. However, if more vehicles arrive over time than the intersection can handle, the intersection is oversaturated. Most models do not allow periods of oversaturation. To estimate a delay for periods of oversaturation, we look at a time-dependent model.

We add erratic behaviour and wrong path choice to the time-dependent delay model, with the assumption that the returning vehicles arrive according to a Poisson process. We also create a simulation to compare with the time-dependent model.

We find that by far the biggest contributor to increased delay at an intersection is the increased arrival rate at the other traffic lanes due to the road closure. However, because the intersection is continuously under high load, a small increase in load caused by erratic behaviour of a driver will have its effect multiplied on the average delay incurred at an intersection.

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1 Introduction

When driving home from Hoofddorp I was suddenly standing in a traffic jam before the highway. I wondered why this traffic jam occurred at 11 pm, a time far outside rush hour. As it turned out, the entry to the highway was blocked because of roadwork. Since there was no notification or directions given to drivers, a lot of them were confused. Cars were switching lanes multiple times in front of the traffic light; some cars turned around after choosing an, apparently wrong, direction. This made me think: how big is the effect of this uncertainty of the drivers, and did it cause a traffic jam at a quiet hour?

Lots of roadwork is done every year, some causing big traffic jams and others hardly affecting the flow of traffic. To minimize the delay that vehicles incur at this roadwork, and to appropriately time it, understanding the behaviour of vehicles is important. This led me to the following research goal:

How does erratic behaviour and wrong path choice affect delays at an intersection?

For this research there are 2 sub-questions to be answered:

- *How can delay at an intersection be quantified?*
- *How can erratic behaviour and wrong path choice be quantified, and what is the effect on delays/traffic jams?*

In Chapter 2 we explain some general information regarding delay models at an intersection. Chapter 3 describes some existing theoretical models that estimate the delay at an intersection, including time-dependent models that are able to estimate delay when the demand is larger than the capacity of an intersection. In Chapter 4 we describe additions to the model to account for erratic behaviour and wrong path choice. Also a description is given of a simulation of an intersection. In Chapter 5 a case study is done to examine the effects of various parameters, including erratic behaviour and wrong path choice. Conclusions are presented in Chapter 6.

2 Delay models for intersections

Intersections come in different varieties, those with traffic lights, and those without. Non-signalised intersections are difficult to model, because of the high dependency on human behaviour. Therefore the focus of this paper is on an isolated, signalised intersection as shown in Figure 1.

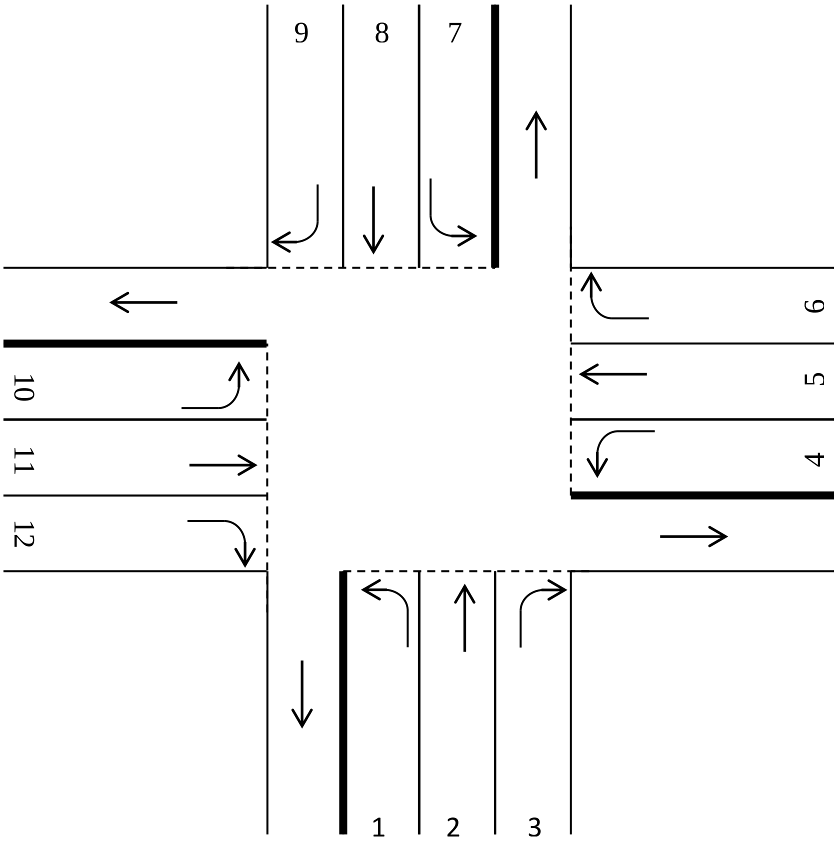

Figure 1 A 4-way intersection with 1 lane for every direction

Signalised means there are traffic lights on the intersection. The traffic lights turn green according to a schedule where non-conflicting lanes have a green light at the same time. There are 2 methods to control the traffic light schedule: a fixed-time signal scheme, and fully actuated control. These methods are further explained in Paragraph 2.1 and 2.2.

Isolated means the arrivals at this intersection are not influenced by other intersections: often vehicles arrive in platoons at a traffic light because the traffic light at the intersection before it turned green. Sometimes intersections are synchronised or communicate, allowing the time a traffic light turns green to depend on the intersection before it.

For simplicity we use a 12-lane symmetrical intersection as displayed in Figure 1. This differs from the intersection in Hoofddorp, which has 3 lanes instead of a single lane going towards the northern direction. Symmetrical means all lanes are identical: they have an equal vehicle arrival rate, an equal vehicle departure rate and equal effective green times. Because of this symmetry we can study the effects of changes on a single lane instead of all lanes, which decreases the number of parameters to study and allows us to focus more on the effects. Because conflicting lanes cannot have a green light at the same time, we need a cycle with at least 4 different periods. For example in period 1 all southern lanes (1, 2, and 3) have a green light, in period 2 all eastern lanes have a green light, etc.

2.1 Fixed-time signal scheme

With a fixed-time signal scheme the time each light stays green is fixed; it does not change depending on traffic. The delay of vehicles crossing the intersection is related to the time each light at the intersection stays green. The delays of the individual vehicles are estimated and an average delay per vehicle is calculated. Then it is estimated which green times have the shortest average expected delay at the intersection, often by using empirical data and an optimization model. Webster [1] was in 1958 one of the first to create an optimization model based on his own delay estimation. This estimation is explained in Chapter 3.

A variant of the fixed time schedule is pre-time control: changing the green times at the intersection based on the time of the day (e.g. longer green times during rush hour).

2.2 Fully actuated control

Fully actuated control is a system that is used most nowadays: there are loops in the road which measure arriving vehicles and the number of vehicles in queue. A traffic light will stay green until all vehicles have departed. This is called an exhaustive service: to serve all vehicles in a queue until the queue is empty. Fully actuated traffic lights also have a maximum green time, to ensure the system does not get stuck on a single light on moments of heavy traffic or sensor malfunction.

Studies on signalised intersections [2] have shown that intersections with light traffic and short queues benefit most from having fully actuated control, since it decreases the chance of a vehicle arriving just as the light turned red. The heavier the traffic is and the longer the queue is, the more a fixed-time schedule is preferable. When there is very heavy traffic on an intersection, the traffic lights often hit the maximum green time set, and will behave like a fixed-time schedule.

In this paper we are interested in heavy traffic: by closing down a lane we are increasing the load on the intersection. We assume fixed-time control, because it is easier to model and the preferable control in heavy traffic situations.

3 Calculating delay at an intersection

Over the years multiple models have been created to predict the average delay at an intersection; we discuss a few of the most common ones. But first we introduce some terminology.

g effective green time (seconds) – As shown in Figure 2 vehicles need some time to accelerate when the light turns green, and some time to stop when the light turns yellow. Therefore most calculations are done using the effective green time. This is the real green + yellow time of a traffic light in seconds with some time deducted from the start and end to account for the starting and stopping of vehicles.

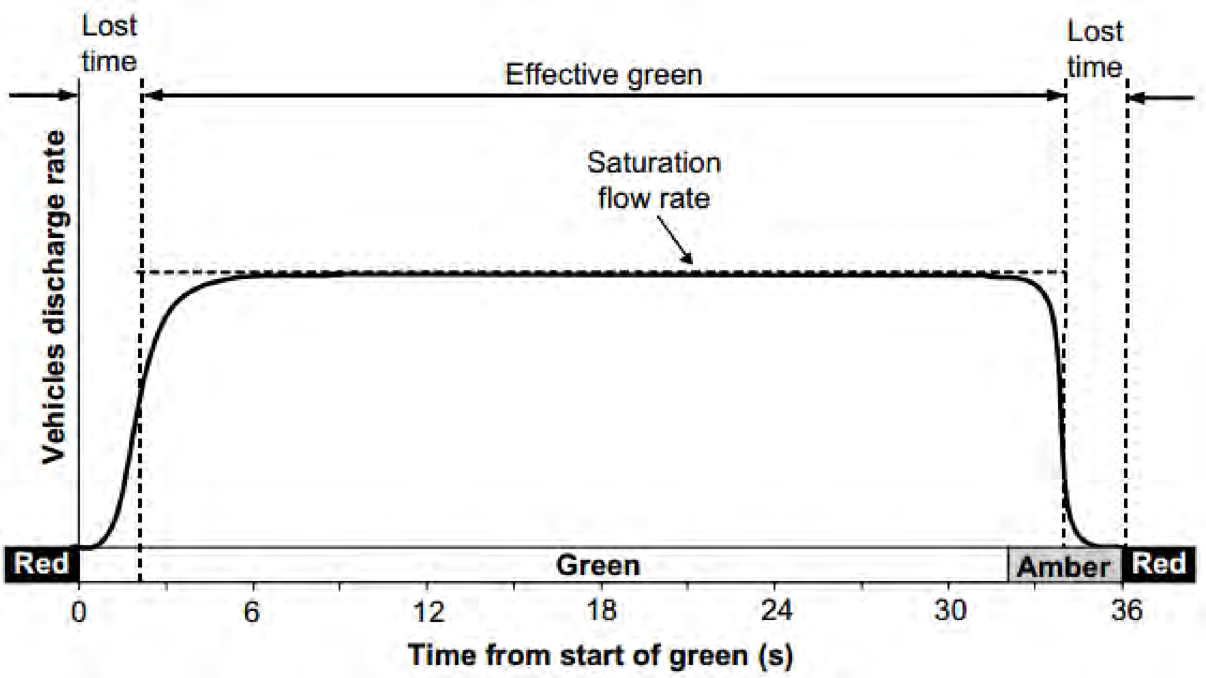

Figure 2 Effective green time [3]

Clearing time – the time between a light turning red and the next light turning green. This allows the last vehicles to safely depart.

C traffic signal cycle length (seconds) – the time in seconds it takes to complete a full cycle of all traffic lights. This consists of the sum of all disjoint effective green times and clearing times.

s saturation flow rate (veh/h) – the maximum speed at which vehicles can cross the traffic light, in other words the service rate in vehicles per hour.

v vehicle arrival flow rate (veh/h) – the rate at which vehicles arrive at the traffic light.

c $s \cdot g / C$ capacity of a traffic lane (veh/h) – the service rate weighted by the ratio green time-cycle time.

X v/c volume-to-capacity ratio or saturation – in other words the load weighted by the ratio green time- cycle time. If the saturation is below 1 the system is under-saturated: in the long run the system can handle all the vehicles that arrive. If the saturation is above 1 the system is oversaturated: more vehicles are arriving than the system can handle, and over time the queues will grow.

d average delay per vehicle in seconds.

3.1 Deterministic queueing model

In the deterministic queueing model the arrivals at the intersection are assumed to be of a deterministic nature. The model predicts the average delay when the saturation is less than 1: more vehicles are able to depart than there are arriving. Furthermore a couple of assumptions are made with this model:

1. The vehicles arrive at a uniform and constant rate.
2. The vehicles decelerate and accelerate instantaneously.
3. Vehicles queue vertically at the stop line – the distance between the vehicle and the traffic light is ignored in the delay calculation.

The average delay can then be derived from the following formula:

$$d = \frac{C\left(1-\frac{g}{C}\right)^2}{2\left(1-X\cdot\frac{g}{C}\right)} \tag{1}$$

The derivation of this formula is explained in Appendix A.

3.2 Steady-state stochastic delay models

Webster [1] was in 1958 the first to create a model that no longer assumes the arrivals to be deterministic. In his paper he proposed a formula that estimates delay in case of Poisson arrivals:

$$d = \frac{C\left(1-\frac{g}{C}\right)^2}{2\left(1-X\frac{C}{g}\right)} + \frac{X^2}{2v(1-X)} - 0.65\left(\frac{c}{v^2}\right)^{1/3} X^{2+\frac{g}{c}} \tag{2}$$

This formula consists of 3 parts. The first term is equal to the right-hand side of Equation (1) and gives an estimation of the average delay assuming uniform deterministic arrivals. The second term adds delay attributed to randomness of arrivals. And the third term is an empirical correction to let the delay be more consistent with simulation results. Over the following years other researchers have created variations of Webster's stochastic model, for example Miller (1963) [4], Newell (1950) [5], McNeil (1968) [6] and Heidemann (1994) [7]. These models all make the following assumptions:

1. The number of arrivals follows a known distribution, usually Poisson, and this distribution does not change over time.
2. The headways between departures are identical or follow a known distribution with a constant mean.
3. The system is under-saturated.
4. The system has been running long enough to settle into a steady state.

5. The vehicles decelerate and accelerate instantaneously.

3.3 Oversaturation

All of these models have the assumption that the saturation is below 1. If the capacity of the traffic light is below the vehicle arrival rate, the queue will grow and the delay eventually goes to infinity. We are able to determine the length of this queue for the deterministic queueing model: every cycle a number of vehicles remain in queue, equal to the difference between the arrival and departure rate. If the time is known, the length of the queue can be estimated because we know the amount of cycles that have passed. Based on that length we can create an estimation for the delay during oversaturation in seconds:

$$d = 900T\left((X-1) + \sqrt{(X-1)^2}\right), \quad (3)$$

where T is the duration of the analysis period in hours and X is the volume-to-capacity ratio. The derivation of this formula is further explained in Appendix B.

3.4 Time-dependent stochastic delay model

In Webster's model, the delay approaches infinity as the saturation approaches 1. This makes its use when the saturation is close to 1 unrealistic, and impossible for oversaturation. Unfortunately in practice oversaturation can occur during some periods of time, for example when it is rush hour. Therefore Robertson [8] conceived the idea to implement time-based delay in the stochastic delay model. In this model the delay should become tangent to the deterministic model once the saturation approaches and even surpasses 1, as shown in Figure 3 below.

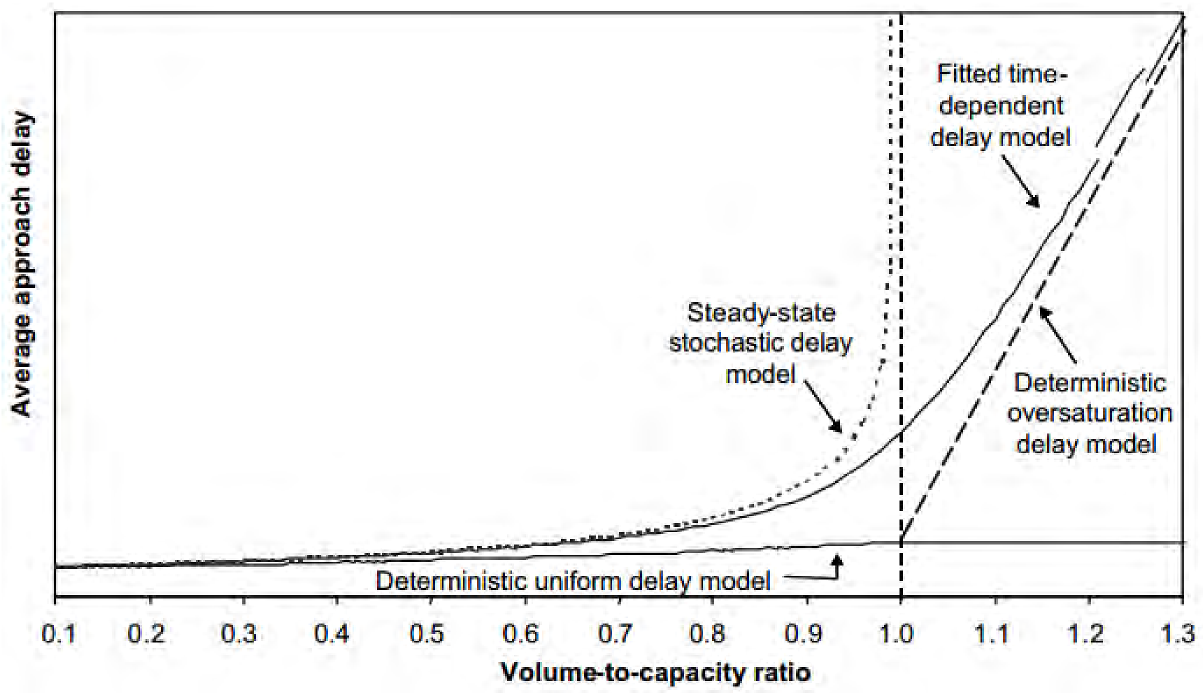

Figure 3 Estimated delay comparison [3]

In Figure 3 the delay is shown over a fixed time *T*, with no initial queue. We see that for saturations below 0.8 the model follows the same estimation as the steady-state stochastic model, for saturations above 1.2 it follows the deterministic oversaturation estimation, and between 0.8 and 1.2 the estimation is an interpolation between these two.

The theoretical basis for this approach is lacking (Hurdle, 1984) [9], however empirical evidence shows these models yield reasonable results. Over the years multiple formulas have been proposed, and some have been incorporated in capacity guides from countries such as the United States, Australia and Canada (Dion, et al. [3]).

The average delay over a fixed time T can be derived using the following formula:

$$d = d_1 . f_{PF} + d_2 + d_3 \cdot f_r \,, \tag{4}$$

with

$$d_1 = \frac{C\left(1-\frac{g}{C}\right)^2}{2\left(1-min(X,1)\frac{g}{C}\right)}, \tag{5}$$

$$f_{PF} = \frac{f_p(1-P)}{\left(1-\frac{g}{C}\right)}, \tag{6}$$

$$d_2 = 900X^nT\left((X-1)+\sqrt{(X-1)^2+\frac{mkI}{cT}\cdot(X-X_0)}\right), \tag{7}$$

where

d_1 delay based on a deterministic under-saturated queueing model, see Equation (2)
f_{PF} progression factor, consisting off:
P proportion of vehicles arriving during the effective green interval
f_p adjustment factor for batch arrivals
By adjusting these parameters the system can be set up to account for platoon arrivals. If a bigger proportion of vehicles arrive during the green light interval (P > g/C), the average delay will be shorter since vehicles do not have to wait as long during a red light, and the progression factor will be < 1. Lowering the progression factor means the estimated delay decreases.
d_2 delay based on a deterministic oversaturated queueing model of Equation (3), with an added component to match the model to empirical data
m, n model parameter used to match the model to empirical data
k, I adjustment factors for the effects of signal controller types and arrival patterns affected by upstream signalized intersections. For more information on these parameters see Paragraph 5.1.
X_0 volume-to-capacity ratio below which the overflow delay is negligible in capacity guide models
d_3 residual delay for oversaturation queues that may have existed before the analysis period

f_r adjustment factor for residual delay

This model makes the following assumptions:

1. Arrivals follow a Poisson distribution that remains constant over time
2. The headway between departures has a known distribution with a constant mean
3. The arrival and departure flow rates have been stationary for an indefinite period of time

The capacity guide models from various countries use different values for the parameters. For example, only the US model takes residual delay, the delay that might have occurred before the start of the model, into account. The Australian model handles batch arrivals by adjusting the parameter m, whereas the US model adjusts progression factor. For the case study we use the parameters as described in the Highway Capacity Model [10] of the US.

The US model allows multiple time-periods in the time-dependent model using the following equations:

$$d_3 = \frac{1800 Q_b \cdot (1+u) \cdot t}{cT}, \tag{8}$$

where

Q_b initial queue at the start of period T
u delay parameter
t the duration of the unmet demand in T (hours)
The duration of the unmet demand t is the time during the analysis period where the initial queue is being handled, and is calculated using Equation (9).

$$t = \begin{cases} 0, & Q_b = 0 \\ min\left(T, \dfrac{Q_b}{c \cdot \left(1 - min(1, X)\right)}\right), & else \end{cases} \tag{9}$$

If t equals T, the delay parameter u can be calculated using Equation (10).

$$u = \begin{cases} 0, & t < T \\ 1 - \dfrac{c \cdot T \cdot \left(1 - min(1, X)\right)}{Q_b}, & else \end{cases} \tag{10}$$

The initial queue in time period i can be calculated as shown in Equation (11).

$$Q_{b,i+1} = max\left(0, Q_{b,i} + c \cdot T \cdot (X_i - 1)\right) \quad i = 1,2, \dots, n \tag{11}$$

Finally, if there is an initial queue Equation (5) needs to be time-weighted as shown in Equation (12).

$$d_1 = d_s \cdot \frac{t}{T} + d_u \cdot f_{PF} \cdot \frac{(T - t)}{T}, \tag{12}$$

where d_s is the delay during which an oversaturation queue exists and has to be calculated using a value of $X = 1$, and d_u is the time during which no oversaturation queue exists and has to be calculated using the actual value of X.

3.5 Shock wave delay model

A different approach is to model the traffic flow through an analogy with fluid dynamics. First developed by Lighthill and Whitham (1955) [11], as well as Richards (1956) [12], the model describes the speed at which changes in traffic characteristics propagate along a roadway. It does this by dividing the road into zones, and then calculating the behaviour of traffic in that zone. In each zone the traffic flow rate is a product of the traffic density and the traffic flow rate. This can then be used to estimate delay. This estimation is complex and will not be used in this paper.

3.6 Microscopic simulation delay model

Another way to estimate the delay at an intersection is to create a microscopic simulation. This simulation will simulate the arrivals of each independent vehicle arriving at the intersection. One example of a simulation model is the cellular automaton model [13]: this model separates the road into small blocks. There can only be 1 vehicle in each block, and the behaviour of each vehicle is determined by its current speed and the presence of a vehicle in the block in front of it.

4 Adjustments to the model to account for roadwork

When doing a simulation, it is fairly straightforward to add wrong decisions or erratic behaviour to a model. However the literature on theoretical models seems to be lacking. We adjust the time-dependent stochastic delay model by using some assumptions.

4.1 Introducing wrong decisions to a model

When a driver makes a wrong decision it means he took a wrong turn and has to return to the intersection and choose a different direction. We assume that a vehicle will not return to the direction it originally came from.

The vehicles in the model are departing following a deterministic pattern, and there would be a correlation between the green time of a traffic light and the re-entry of vehicles into another lane. However the moment when a driver decides he has to turn around is most likely random. This randomness allows us to model the re-entry rate as a Poisson process. Because both re-entry arrivals and normal arrivals at a lane are Poisson they can be combined. This combination also constitutes a Poisson process with a new arrival rate:

*New arrival rate = normal arrival rate + arrival rate wrong decision lanes * chance of wrong decisions,*

where w*rong decision lanes* are the lanes with vehicles that, if they decide to return to the intersection, will return to the lane we are calculating the arrival rate for.

This modification does not work if the system is oversaturated – it would add re-entries to the lane that might not have been served during the period under consideration due to oversaturation. Therefore the model has to be adjusted to account for oversaturation.

*New arrival rate = normal arrival rate + min(arrival rate wrong decision lane, capacity wrong decision lane) * percentage of wrong decisions.*

To the best of our knowledge, no quantitative studies have appeared in the literature regarding the chance a driver makes a wrong decision. Furthermore, this chance is influenced by many factors, such as the layout and location of the intersection, if drivers are familiar in the area, etc. We investigate the behaviour of the model using multiple re-entry percentages.

4.2 Driver's erratic behaviour

If a driver is unsure about which way to go because of the road closure, he might switch lanes multiple times before actually departing. This assumption could be added to the model by adding randomness in the departure. Most traditional models assume arrivals are deterministic, except for the time-dependent stochastic delay model: this model only assumes that the headway between departures is of a distribution with a known mean. This allows us to add a random factor with a known mean to the departure:

New departure rate = departure rate + erratic behaviour factor.

4.3 Simulation

We choose, in addition to the time-dependent stochastic delay model, to run a simulation to determine the average expected delay. We create a discrete-time event simulation [14]. A discrete-time event simulation determines the time each event takes place and disregards the time in between because nothing changes.

We start off with a standard 4-way intersection – each side has 3 lanes, one for every direction, as shown in Figure 1. Each lane is simulated separately. To simulate the intersection we need 4 events:

1. The start of the green time – when the light turns green the first vehicle starts its departure. Also a time when the light will turn red again is scheduled, equal to *the current time + the green time* of this traffic light.
2. The end of the green time – no more vehicles can depart. The next time the light will turn green is scheduled, which is *the current time + the time of a full cycle - the green time of the traffic light*.
3. The arrival of a vehicle – Once a vehicle arrives a new arrival with rate λ can be scheduled. If there are no vehicles in queue and the light is green, the vehicle can immediately depart and thus a departure is scheduled at the current time.
4. The departure of a vehicle – If the light is green, the vehicle can depart with a service time of *1/s*.

This leads to the following basic algorithm:

```
While time < maximum run time {
  Handle event {
  case: green light  {
                       if (vehicle in queue) { schedule departure }
                       schedule turn light red
                     }
  case: red light    {
                       schedule turn light green
                     }
  case: arrival      {
                       add vehicle to queue
                       schedule next arrival with rate λ
                       if ( queue is empty & light is green ) {
                            schedule departure
                       }
                     }
  case: departure    {
                       if ( light is green ) {
                         lower the queue
                         schedule new departure at (time + 1/s)
                     } }
} }
```

To simulate closing a lane there are 3 changes we make to the current system:

1. Since an outgoing traffic lane is closed the lanes that are driving in that direction have their traffic distributed evenly over the other 2 lanes. For Figure 1 we consider the scenario in which we close off the northern lane. This means that the incoming traffic of lane 6 has to be divided over lanes 4 and 5, the traffic of lane 2 has to be divided over lanes 1 and 3 and the traffic of lane 10 has to be divided over lanes 11 and 12. As described in Chapter 4.1 we are able to combine two Poisson arrivals rates into a single arrival rate. An example using lane 4 when the northern outgoing lane is closed: the *new arrival rate* λ_4 *= arrival rate* λ_4 *+ 50% * arrival rate* λ_6.
2. To account for erratic behaviour, we add a random component to the departure time.

3. To account for a wrong choice, we have to add an arrival event to the lane where the car will return. For example, if a driver wrongly chooses lane 5, he will return to lane 12 to drive in the correct direction. As described Chapter 4.1, we add the arrival in case of under-saturation, and the capacity in case of oversaturation. Because both arrivals follow a Poisson process, we can add the two arrival processes together: *new arrival rate* λ_{12} *= arrival rate* λ_{12} *+ (chance vehicle returns to the intersection) * min(arrival rate* λ_5*,capacity* s_5*).*

Our new algorithm looks as follows:

```
While time < maximum run time {
   Handle event {
      case: green light  {
                            if (vehicle in queue) { schedule departure }
                            schedule turn light red
                         }
      case: red light    {
                            schedule turn light green
                         }
      case: arrival      {
                            add vehicle to queue

                            schedule next arrival with rate (λi + 50% * λj +
                            ∑(return chancek * min(λk, sk))
                            // λi is the original arrival rate
                            // λj is the arrival rate of the lane that will
                            no longer depart because of the closure
                            // λk, sk are the arrival rate and capacity of all
                            the lanes where the vehicles came from

                            if ( queue is empty & light is green ) {
                                 schedule departure
                            }
                         }
      case: departure    {
                            if ( light is green ) {
                                 lower the queue
                                 schedule new departure at (time + μ + random
                                 component)
                            }
                         }
} }
```

We will now use this simulation in a case study to compare with the time-dependent stochastic delay model.

5 Case study

In this case study the behaviour of the time-dependent stochastic delay model is studied and compared to simulation. For this we analyse a symmetrical intersection where all arrival and departure rates and effective green times are equal. Because of this symmetry, we only have to examine the results for a single lane. We start off by examining the behaviour of the model under normal conditions. We then close off the lane, after which we will add re-entries and erratic behaviour to the model. After that we examine the effect of the timing of roadwork, after which we will answer the research question.

5.1 Normal Conditions

To get an idea of the behaviour of the model, we first examine what happens without re-entry and erratic behaviour. There are several variables which can be examined, the most interesting being:

C the cycle time

g/C the effective green time / cycle time ratio

v the vehicle arrival flow rate

s the saturation flow rate

First, the influence of different cycle times is examined, while keeping the effective green time-cycle time ratio equal. Because the intersection is symmetric, the green time ratio is set at *¼*. We assume this is an isolated intersection, and can therefore set f_{PF} to *1*. We measure the delay for a timeframe of *T = 15 minutes = 0.25 hour*. For the parameters *m*, *k*, *I,* and *X0* we use the values as used in HCM 2000 [10]. This model, which has been in use for at least 16 years in the United States, specifies the values at respectively *8, 0.5, 1,* and *0.*

In Figure 4 the volume-to-capacity ratio is used on the horizontal axis. This ratio is set by changing the arrival rate, from *0.02 * 450 = 9* to *1.5 * 450 = 675* vehicles per hour. The only part of the model that changes with a different cycle time and an equal *g/C*-ratio is Equation (5), the estimation for the deterministic uniform delay. We can confirm this by drawing the delay for 3 different cycle times: *60, 120,* and *180 seconds*.

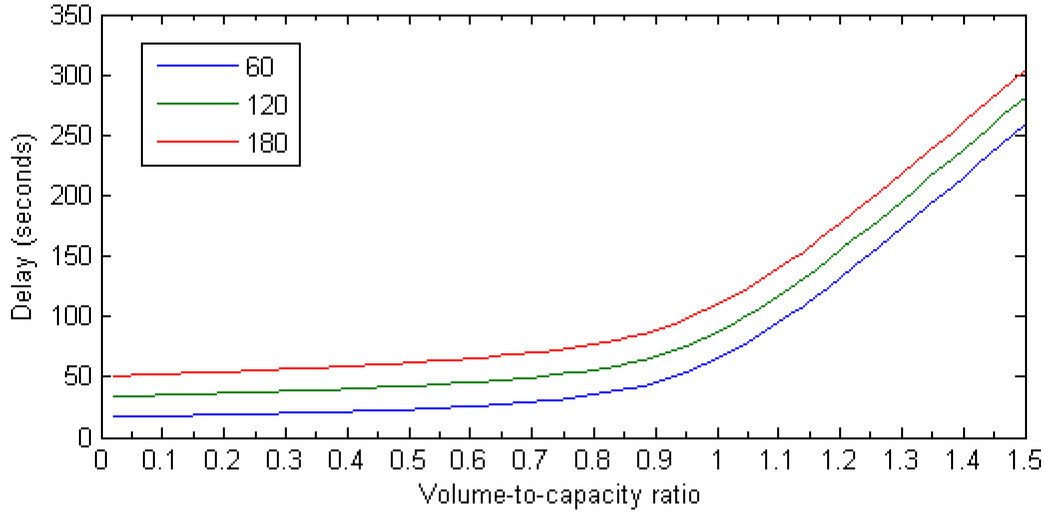

Figure 4 Estimated delay over 15 minutes using time-dependent stochastic delay model

In Figure 4 we confirm that the delays for the different cycle times are parallel. We run the simulation to compare with the model displayed in Figure 4:

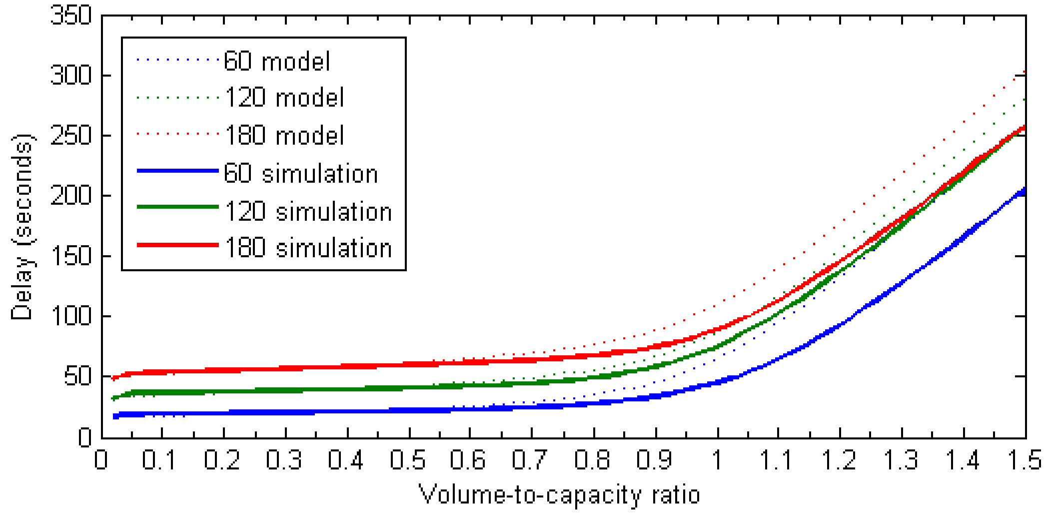

Figure 5 Estimated delay over 15 minutes, model (dotted lines) and simulation (solid lines)

As we can see, the shape of the diagram is fairly similar. However, when the volume-to-capacity ratio rises above 1, the model tends to overestimate the delay. Also the delay at $C = 120$ seconds increases faster than for the other 2 values. This is because of the discrete, deterministic departures in the simulation: at $C = 120$ the last vehicle departs *28* seconds after the light turned green, yielding a total of *15* departures or *7.5* departures per minute. However at $C = 60$ the last vehicle departs at *14* seconds after the light turned green, for a total of *8* departures per minute. For $C = 180$ this total is *23/3 = 7⅔* departures per minute. The theoretical models do not have this problem, because they use continuous values to determine the average delay.

For these graphs we kept an effective green time to cycle time ratio of *0.25*. But in practice, due to the start and stop time of vehicles, as shown in Figure 2, this ratio should be lower. To account for this effect, we estimate the combined start and stop time at *4* seconds. We create a new plot, where the effective green time is reduced by *4* seconds. We expect that the delay values will cross each other: on a cycle time of *60* seconds, cutting *4* seconds of the green time of *15* seconds will have more effect than reducing the green time of *45* seconds to *41* seconds at a cycle time of *180*. We expect that for longer timeframes the short cycle time will surpass the higher cycle time in estimated delay. Because adding the clearing time to the model changes the actual volume-to-capacity ratios between the models, we cannot compare equal volume-to-capacity ratios. Therefore, we plot against the arrival rates that were used to create the original volume-to-capacity ratio of *0.02* to *1.5* of Figure 4 and Figure 5.

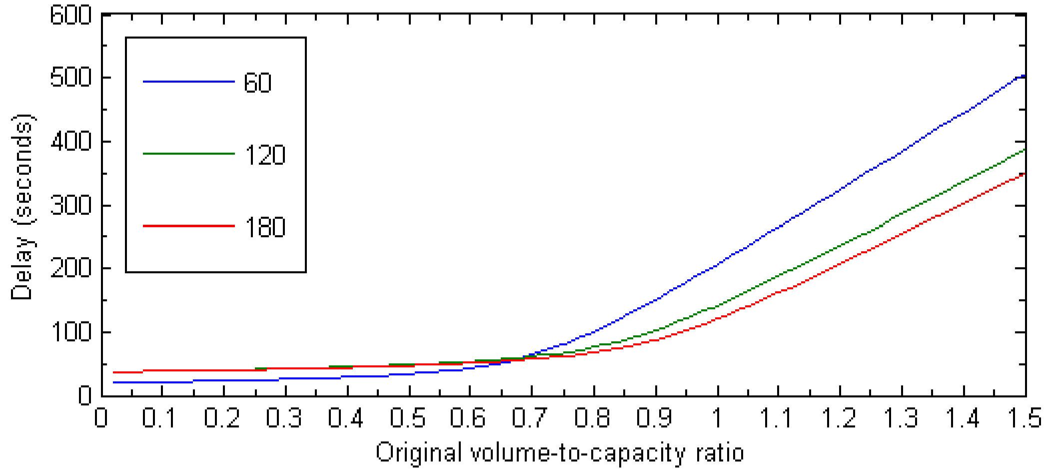

Figure 6 Estimated delay over 15 minutes with 4 second clearing times, model only

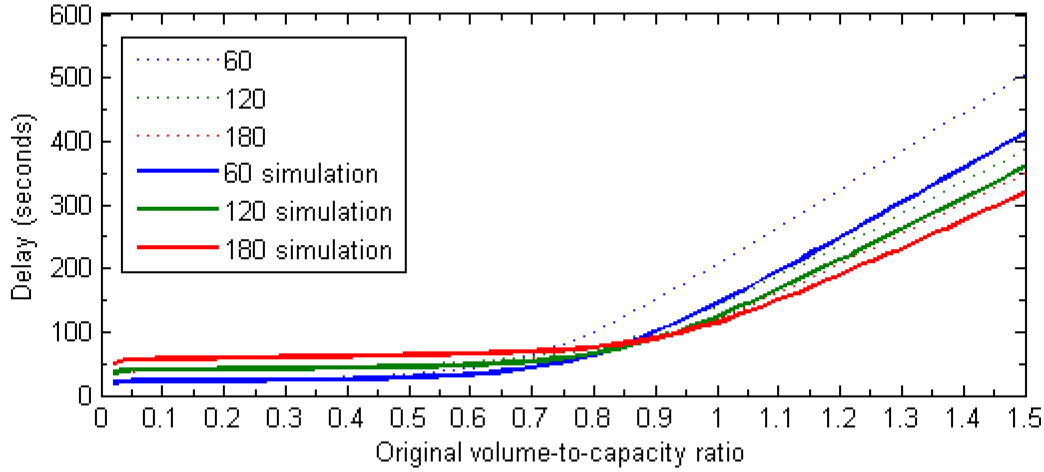

Figure 7 Estimated delay over 15 minutes with 4 second clearing times, model (dotted lines) and simulation (solid lines)

In this model we can see that higher cycle times indeed result in lower delays when the volume-to-capacity ratio is sufficiently large. In Figure 6 the delay of the lower cycle time intersects the higher cycle time between *0.7* and *0.8*, whereas in the simulation of Figure 7 the delay intersect between *0.85* and *0.95*. In general the simulation tends to have lower delay than the model, with the biggest difference being at a cycle time of *60* seconds. This difference is caused by the discrete values of the simulation: at a cycle time of *60* seconds, the model assumes an average of *11/2 = 5.5* departures, but in the discrete simulation *6* vehicles are able to depart, a difference of *9.1%.* At the cycle times of *120* and *180* this difference is resp. *0%* and *2.4%.*

Since the delay is estimated over a fixed timeframe we run the model with the same parameters using *T = 1* hour. We expect that the overall shape of the graph does not change, but the delay at saturations near and above 1 will increase because the queues increase over time when the system is oversaturated.

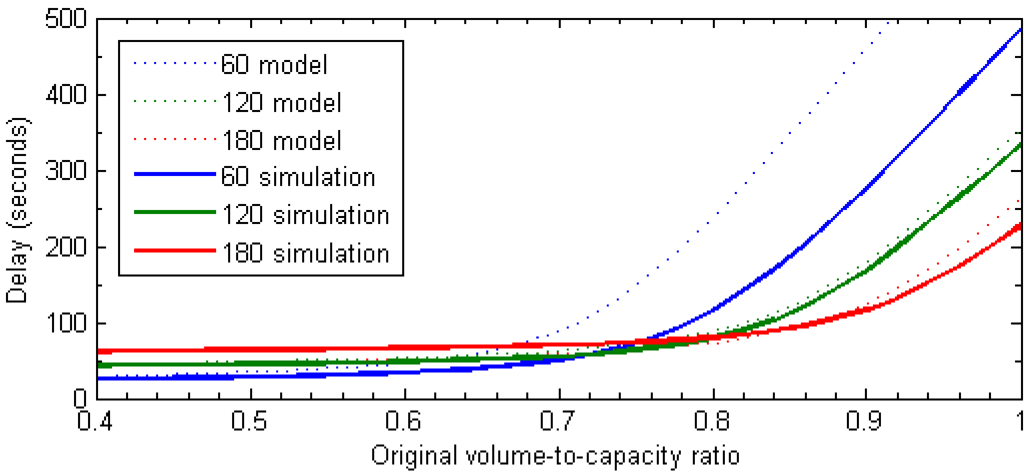

Figure 8 Estimated delay over 1 hour, model (dashed lines) and simulation (solid lines)

As we can see in Figure 8 our expectations are correct: the overall shape of the graph is the same, but the delay increases much faster at *v/c* ratios above *0.8* due to the increased time. Note that for better readability we have adjusted the ratio to go from *0.4* to *1*.

5.2 Closing a lane, adding wrong path choice and erratic behaviour

We continue our research using only one cycle time: *120* seconds. At this cycle time the simulation closely matches the model, there is less problems with discrete values, and it is the cycle time in use at the intersection in Hoofddorp as described in the introduction. Using a single cycle time means we are no longer comparing different cycle time/green times which change the *v/c* ratio, which allows us to plot using the volume-to-capacity ratio again. Because estimating the delay over a longer period of time does not seem to affect the overall shape of the figure, the time period is set at *T = 0.25 = 15* minutes to have more detail in the graph.

First, we investigate the effect of closing a lane. Closing a lane implies that traffic is diverted to the remaining open lanes, directly increasing the arrival rate by *50%*. Therefore, we expect each delay of Figure 7 to be reached at approximately *1/1.5* of the *v/c* ratio.

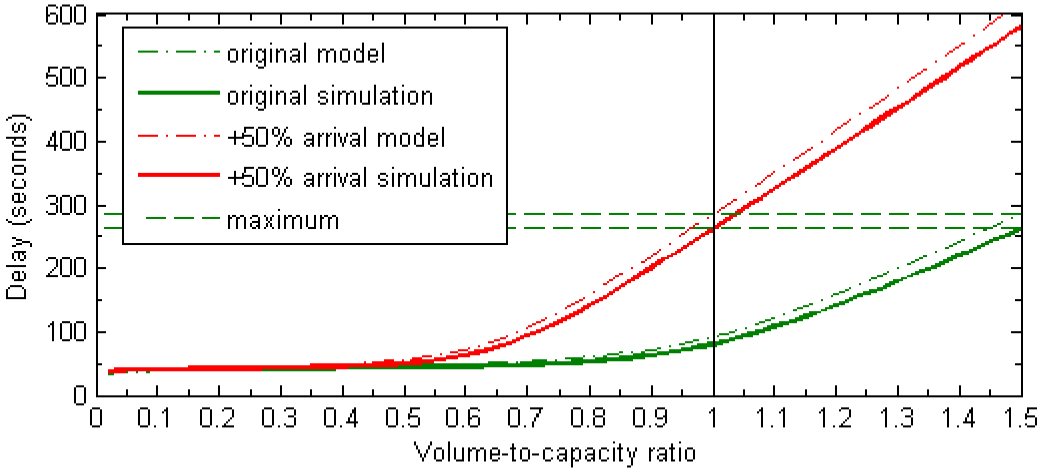

Figure 9 Estimated delay over 15 minutes after lane-closure, cycle time 120 seconds

As we can see in Figure 9 both the simulation and the model reach the delay of resp. *262* and *283* at a *v/c* ratio of *1* instead of *1.5*. Note that this plot and all subsequent plots draw the new delay against the *v/c* ratio of the original arrival and departure rates.

After we close down the lane we can check the effects of wrong decisions and erratic behaviour. To study a wrong decision we look at two re-entry rates: *5%* and *10%*. We expect only the redirected traffic to make a possible wrong choice, thereby having a third of the arrivals affected by a possible wrong decision. Because a re-entry influences the arrivals at a lane, the v/c ratio is directly influenced, which we expect to shift the graph by another *5% * ⅓ = 1.67%* and *10% * ⅓ = 3.33%*.

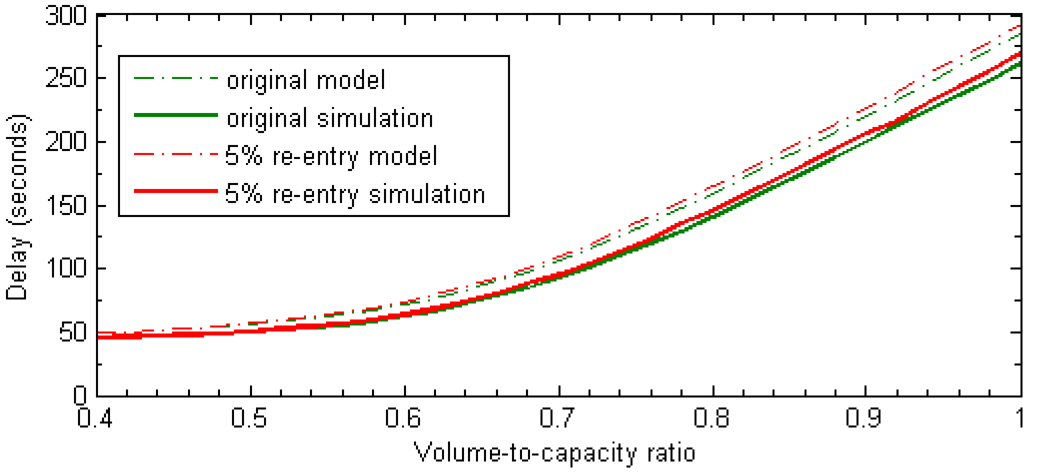

Figure 10 Estimated delay over 15 minutes with a vehicle re-entry rate of 5%

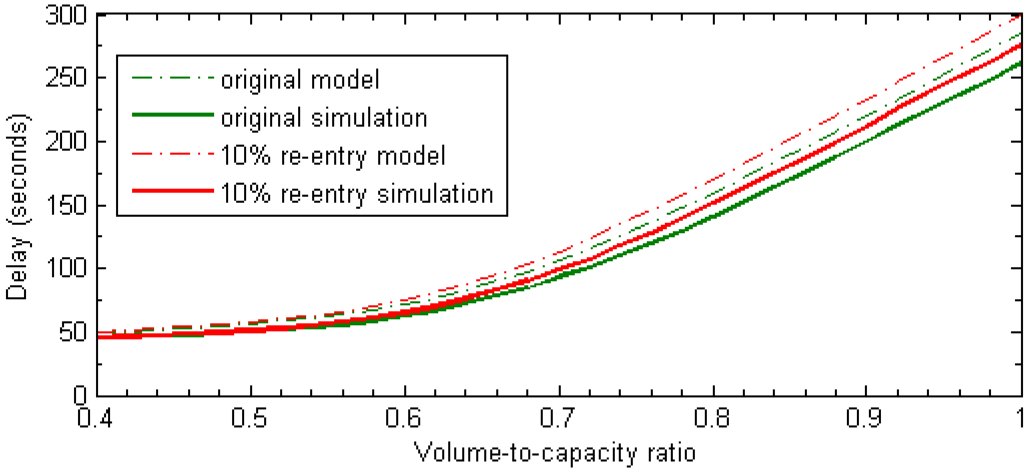

Figure 11 Estimated delay over 15 minutes with a vehicle re-entry rate of 10%

In Figure 10 and Figure 11 we see that the re-entry rate has a small but noticeable effect on the delay of vehicles. We graph the difference between the new and original model to further examine this effect.

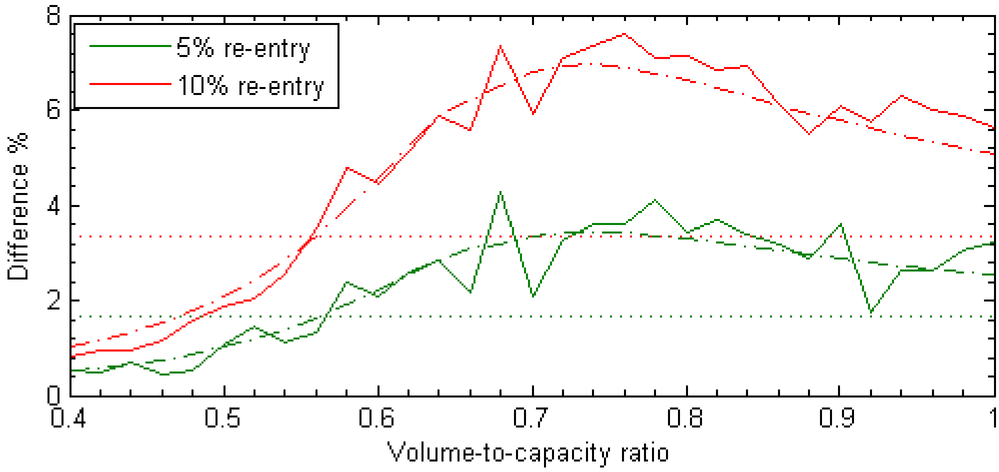

Figure 12 Difference estimated delay over 15 minutes with a vehicle re-entry rate of 5% and 10% (dotted line = v/c ratio increase)

The dotted lines in Figure 12 indicate the increase in *v/c* ratio. We can see that for both the *5%* and *10%* re-entry rate the delay increase intersects with the *v/c* ratio increase at roughly *0.56*, and reaches the highest value around *0.7*. This means that if an intersection has a *v/c* ratio of *0.56* or higher, the actual delay incurred by the vehicles at the intersection will be higher than the increase in *v/c* ratio. If the *v/c* ratio of an intersection is low, a large percentage of vehicles arrive as the first in line, causing the average delay to mainly depend on the arrival time. Therefore the increase in arrival rate due to re-entry has a much smaller effect on the increase in average delay.

Next we switch to adding randomness in departures to the model. We consider two scenarios for the additional departure delay: a *5%* chance to have an extra delay of *2* seconds (an increase of 100% of the original delay), and a *10%* chance to have an extra delay of *2* seconds at a departure. Since the change in *v/c* ratio is again *5% * ⅓ = 1.67%* and *10% * ⅓ = 3.33%,* we expect this change to have a similar effect on the delay.

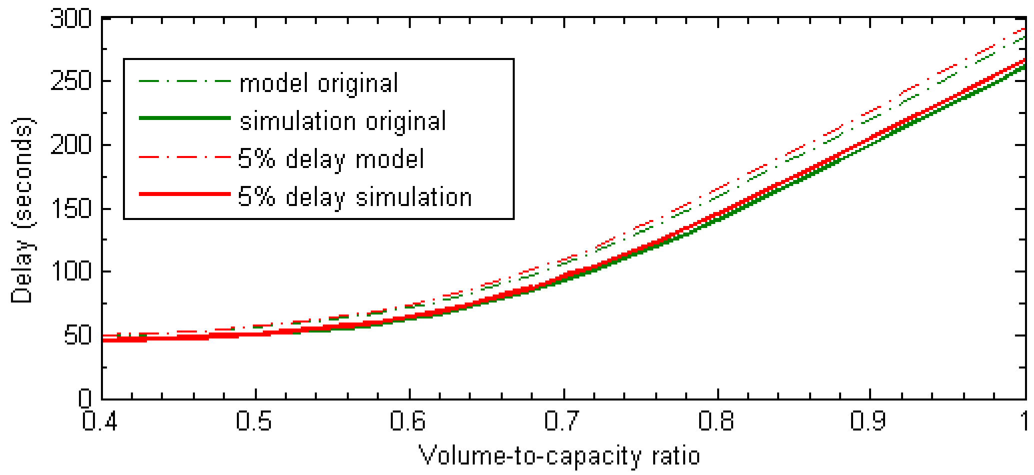

Figure 13 Estimated delay over 15 minutes with 5% chance of an extra 2 second delay

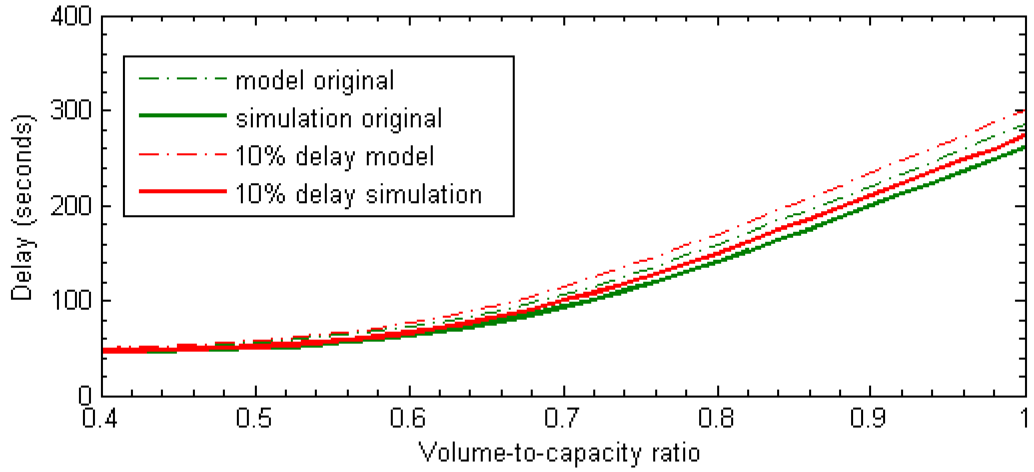

Figure 14 Estimated delay over 15 minutes with 10% chance of an extra 2 second delay

As expected the results in Figure 13 and Figure 14 are very similar to those in Figure 10 and Figure 11, respectively. To further confirm the similarity we examine the difference between the original delay and our new delay in Figure 15.

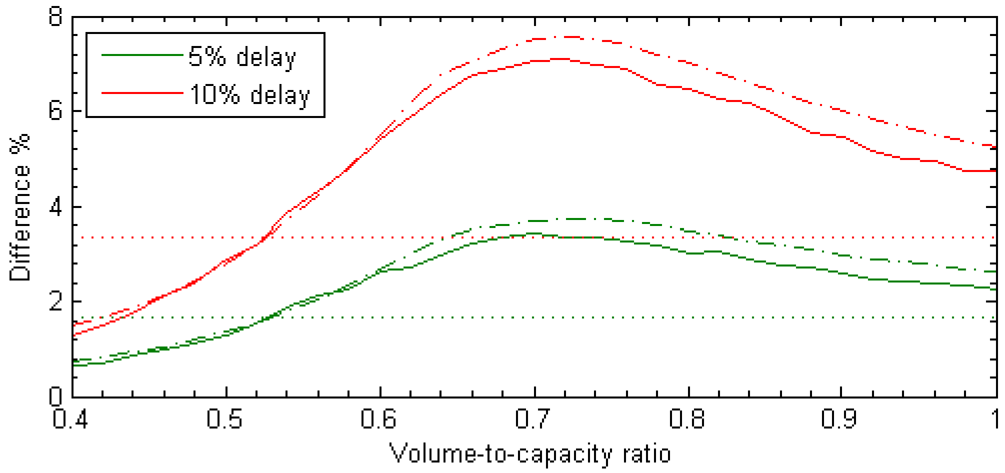

Figure 15 Difference estimated delay over 15 minutes with chance of increased delay of 5% and 10% (dotted line = v/c ratio increase)

Figure 15 is very similar to Figure 12, with the delay intersecting the *v/c*-ratio around 0.53 in this case. Also above a volume-to-capacity ratio of approximately *0.55* the increase in delay exceeds the increase in volume-to-capacity ratio. To see if this point of intersection changes with model duration we examine several different values for this duration *T*. We expect that the higher the duration of the model, the lower the *v/c* ratio of the intersection point will be.

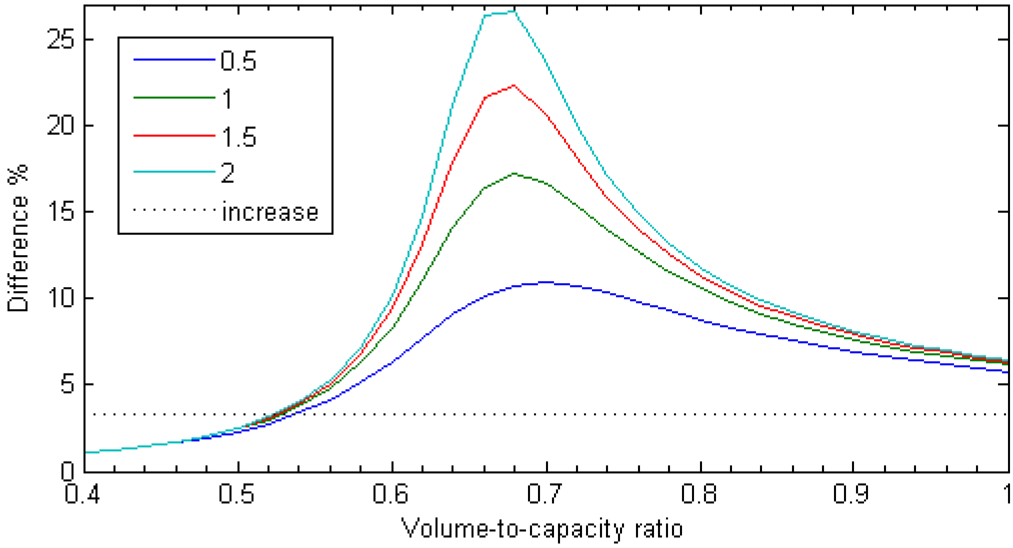

Figure 16 Difference estimated delay over multiple times T with an increase in arrivals of 10% * 1/3 = 3.33%

In Figure 16 we increased the arrivals by 10% * ⅓ = 3.33% and check its effect with duration T = 0.5, 1.0, 1.5 and 2.0 using the model. As we can see in Figure 16, the point of intersection seems to converge to a *v/c* ratio of approximately *0.52*. Also, an increase in time *T* causes a direct increase in the expected delay, with for all values a peak around *0.7*.

5.3 The effect over a full day

The previous simulations only ran for a single, relatively short time period. But for those in charge of planning road closures, it would be interesting to see what would happen during a full day in the event of a road-closure. This will help them decide if it is viable to plan a road closure during the week and if traffic needs to be redirected to other roads.

The arrival rate at an intersection varies during a day. To have an idea of the average arrival rates we examined some empirical data. We look at the data from a traffic analysis in Westzaan-Nauerna [15]. This analysis has empirical data on several roads. In Figure Figure 17 we chose a road that shows the difference between rush hour and normal traffic very well.

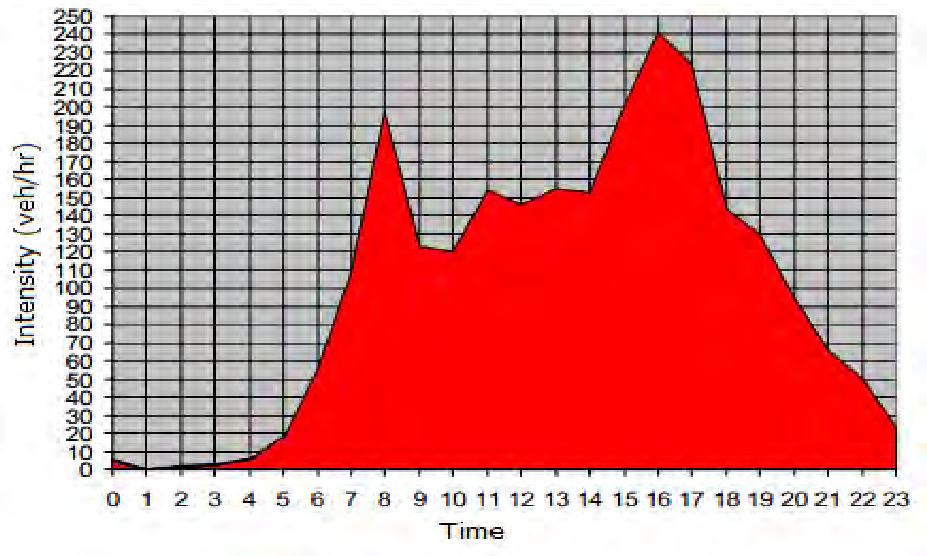

Figure 17 Average traffic intensity at the J.J. Allanstraat in Westzaan over 24 hours [15]

In Figure 17 we can see peaks in traffic intensity: morning rush hour at 08:00 and evening rush hour at 16:00 hours. Data on other roads show a similar pattern, with some difference in the height and time of the peaks.

We divide the day into time-periods of *15* minutes, for a total of *24 hours / 15 minutes = 96* time periods. We set the highest peak to have a volume-to-capacity ratio of *0.9* and set our arrival rate to match that ratio. For demonstration purposes we choose the remaining arrival rates such that our graph approximates a smooth variant of Figure 17.

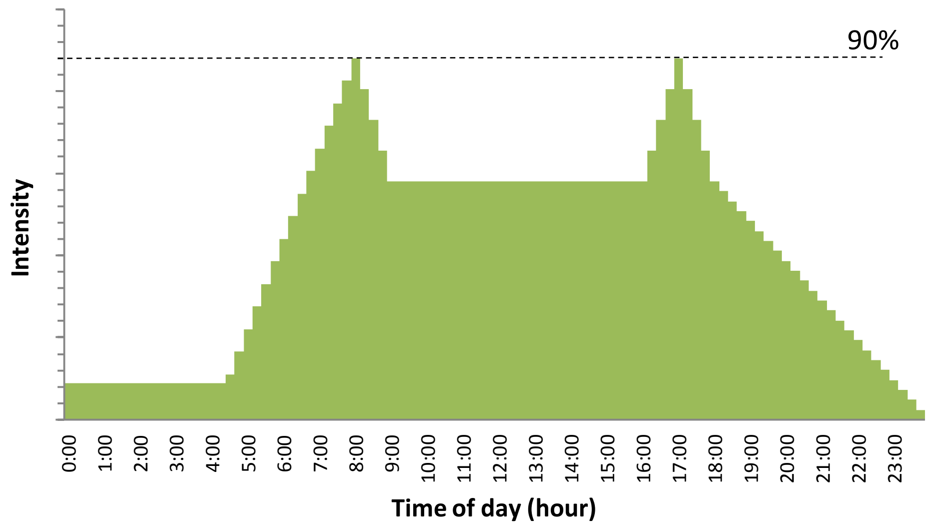

Figure 18 Traffic intensity used in model and simulation, time-periods of 15 minutes

We run the simulation and model for 3 different situations: the original intersection with normal arrival rates as shown in Figure 18, the intersection with a closed lane resulting in a 50% increase in arrival rate, and a closed lane with an added 10% re-entry rate. We expect the average delay around rush hour to increase by a large amount because the system is oversaturated for a long time. Also the difference between the re-entry and normal closed lane will be bigger compared to a single time-period.

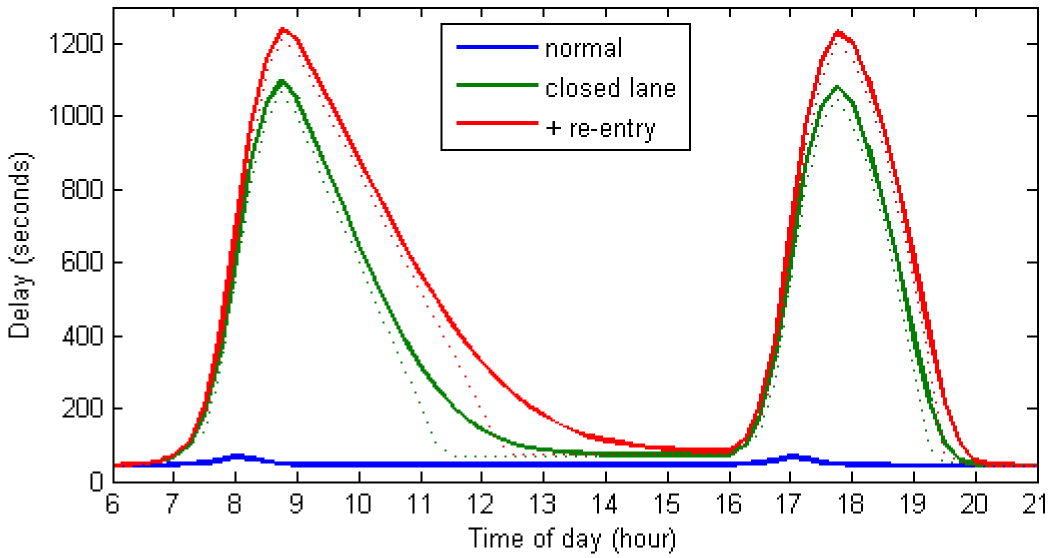

Figure 19 Average expected delay during the day (dotted line = model, solid line = simulation)

As we can see in Figure 19, the roadwork causes a huge delay, with an average of over *20* minutes in the worst case. The *3.3%* increase in arrival rates caused by the re-entry of vehicles causes the average expected delay to take over an hour longer to return to normal: where traffic returns to normal at around *13:00* hours with just a closed lane, it takes until well past *14:00* hours when re-entries are present. As we can see in Figure 19 the difference in delay seems to be around *200* seconds between the two. We also note that the model seems to underestimate the delay incurred when a time-period starts with an initial queue. To have a more accurate view, we take a look at the difference between the case with re-entries and the case without re-entries.

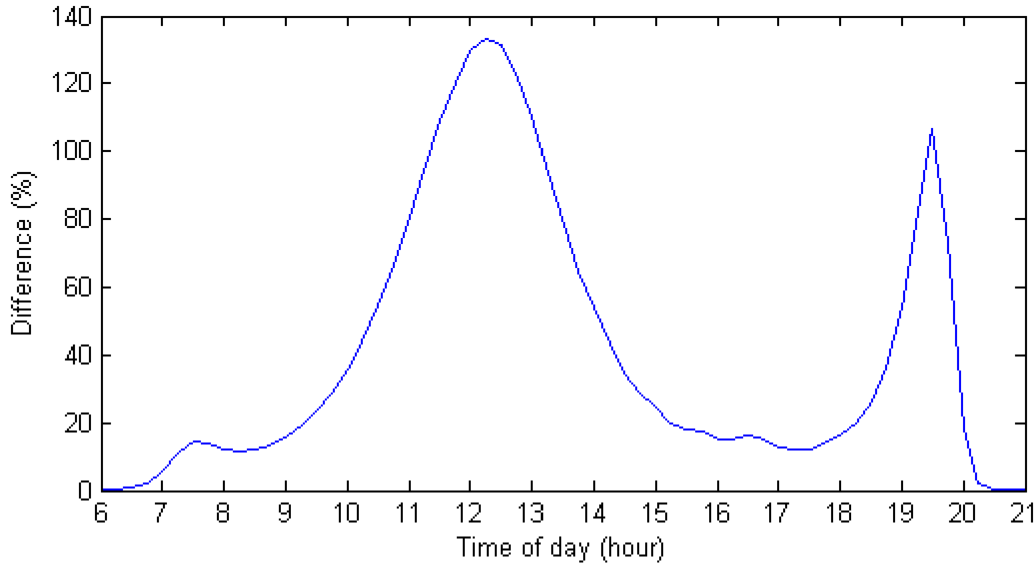

Figure 20 Difference in expected delay (simulation)

As we can see in Figure 20, the difference rises quickly to above *15%*, and rises to approximately *130%.* This peak occurs past the peak of rush hour, because the lower the delay becomes the bigger the relative difference is between the two cases. There is a much sharper decline in difference at the evening peak. This is due to the low arrival rate past *19:00* hours, causing queues to dissipate much faster compared to the morning rush hour peak.

At first we planned to run these models through the situation as it is for the concerning intersection in Hoofddorp. However the situation in Hoofddorp only differs in the number of lanes. This difference has a predictable effect: the redirected traffic increases the arrivals on the remaining lanes by approximately *150%*. Therefore we chose not to include this.

5.4 Extended roadwork

Roadwork on busy roads is often done in the weekend or on weekdays between 21:00-05:00. During this time period there is hardly any difference in expected delay with and without road closure, making this an ideal time for roadwork. It is interesting to see what would happen if the roadwork got delayed and finished later than expected. In Figure 21 the roadwork stops at 4 different times: *06:00, 07:00, 08:00,* and *09:00*. After this time the arrival rate returns to normal. We assume for this model all cars remain in their lane – they do not switch to the closed lane once it is re-opened.

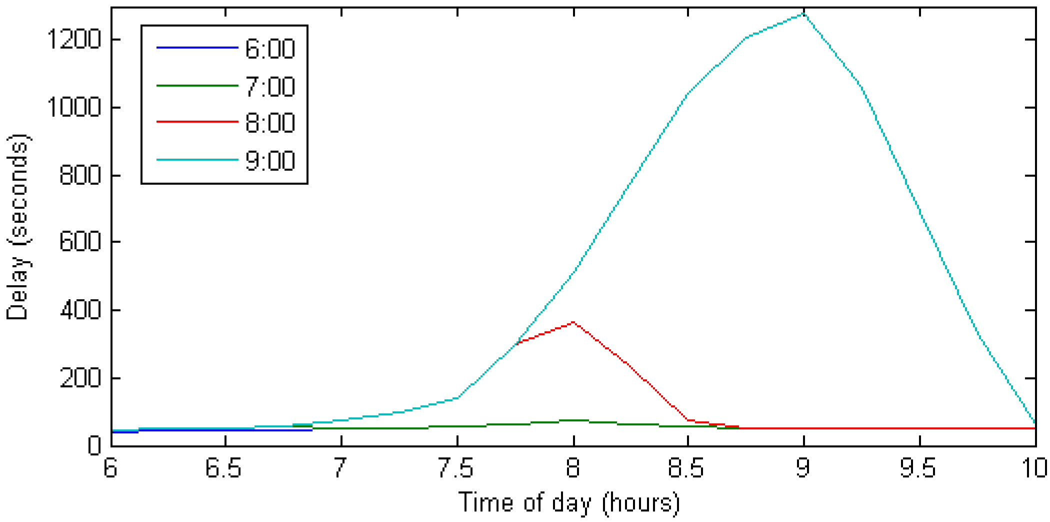

Figure 21 Expected delay when roadwork ends between 06:00 and 09:00 (model)

In Figure 21 we see that delays up to *07:00* hours do not seem to have much effect on the expected average delay. After *07:00* the delay starts to increase. If roadwork is delayed until *09:00* hours, the delay reaches a peak of over *20* minutes, as in Figure 19 for the morning rush hour.

We take the same traffic intensity but apply it to an intersection based on the situation in Hoofddorp as explained in Chapter 2. This intersection has 3 lanes that head towards the closed direction instead of one. Therefore the roadwork has much more impact: 3 lanes instead of 1 lane get redirected, causing an average increase of *3 x 50% = 150%* in arrival rates. We expect a delay in roadwork will have a much greater impact.

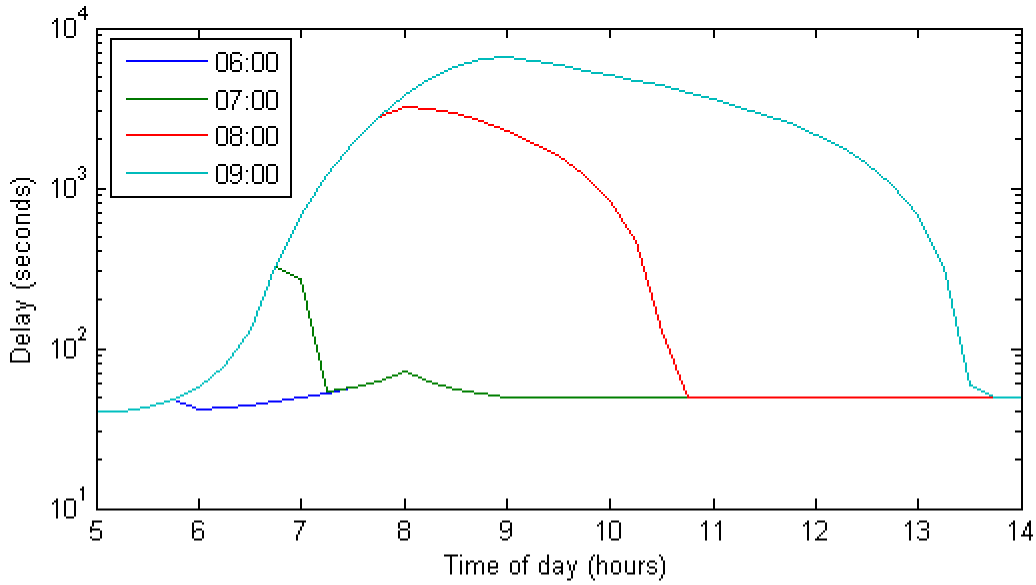

Figure 22 Expected delay at the intersection in Hoofddorp when roadwork ends between 06:00 and 09:00 (log scale y-axis, model)

We see in Figure 22 that indeed the delay of roadwork has a lot of impact. The delay at the start of the day is about 40 seconds, equal to the situation in Figure 21. But the delay at *07:00* hours has already increased to over *5* minutes. After *07:00* the delay increases to over *1* hour. With such high delays, our assumption that cars do not switch to the re-opened lane does not seem realistic: unless cars are physically unable to switch lanes, they will definitely switch lanes once it has re-opened.

Because roadwork has such a high impact if it is delayed, there should be an alternative available to drivers if this long delay situation occurs. Such alternatives could be a redirection of traffic or (if possible) the continuation of roadwork on another day.

5.5 Answer to research question

In the research question we asked what the effect of erratic behaviour and wrong path choice is on delays. Figure 12 and Figure 15 show that the source of the increase in load does not seem to have any effect. The effect of increased arrivals due to re-entry of vehicles and the increased departure time due to the erratic behaviour of drivers are very similar. For any intersection with a volume-to-capacity ratio above approximately 0.55 the increase in average delay is greater than the increase in arrivals or departure time and reaches its peak at approximately 0.72.

However the effect of re-entry and erratic behaviour is small compared to the impact the road closure itself has on the delay. This means the planning of roadwork is quite important: if roadwork is planned at a time with a very low v/c ratio, for example during the night, it has little effect on the delay. But if roadwork is done during rush hour it has a much bigger effect, and any increase in arrival rate or departure time causes an even greater increase in delay.

The model gives an accurate estimation on the average delay compared to the simulation. It can be used when planning roadwork to create an estimation of the expected delay. If the impact on the delay is big, alternatives should be planned.

6 Conclusion

A lot of theoretical models only allow modelling of intersections with a saturation below 1: as the volume-to-capacity ratio reaches 1, models tend to approach infinity. Multiple countries use a time-dependent delay model that is able to handle situations with saturations higher than 1. We applied the model that has been used in the United States for at least 16 years to create estimations on the average delay experienced by vehicles at an intersection.

Unfortunately, we could not find literature on wrong decisions and erratic behaviour of drivers. We took these features into account by imposing some assumptions on the time-dependent delay model. Both the increase of arrivals due to re-entry of vehicles and the increase of departure time due to the erratic behaviour of drivers have a similar effect on the increase in delay at the intersection. For any intersection with a volume-to-capacity ratio above approximately 0.55 the increase in delay is higher than the increase in arrivals or departure time.

However, by far the biggest influence on the increase of delay for vehicles is the increase of demand on the intersection by the closure of a lane. In case of a symmetrical intersection the load is increased by 50%. Because closing a lane puts such stress on the capacity of the intersections, any extra stress incurred by drivers making wrong decisions should be avoided. This could be done by providing clear and concise information of alternative routes to drivers near a road-closure and planning the roadwork on the least busy time of the day: approximately between 21:00 and 05:00 hour.

Bibliography

[1] F. Webster, „Traffic signal settings," Road Research Technical Paper No. 39, Road Research Laboratory, Her Majesty Stationary Office, London, UK, 1958.

[2] v. d. Broek, „Traffic signals: Optimizing and analyzing traffic control systems," *Master Thesis TU Eindhoven,* 2004.

[3] F. Dion, H. Rakha en Y. Kang, „Comparison of delay estimates at under-saturated and over-saturated pre-timed signalized intersections," *Transportation Research,* nr. B 28, pp. 99-122, 2004.

[4] A. Miller, „Settings for fixed-cycle traffic signals," *Operations Research Quarterly,* nr. 14, pp. 373-386, 1963.

[5] G. Newell, „Queues for a fixed-cycle traffic light," *Annals of Mathematical Statistics,* nr. 31 (3), pp. 589-597, 1960.

[6] D. McNeil, „A solution to the fixed-cycle traffic light problem for compound poisson arrivals," *Journal of Applied Probability,* nr. 5, pp. 624-635, 1968.

[7] D. Heidemann, „Queue length and delay distributions at traffic signals," *Transportation Research,* nr. B 28B, pp. 377-389, 1994.

[8] D. Robertson, „Traffic models and optimum strategies of Control," *Proceedings of the International Symposium of Traffic Control Systems,* pp. 262-288, 1979.

[9] V. Hurdle, „Signalized intersection delay models - A primer for the uninitiated," *Transportation Research Record,* nr. 971, pp. 96-105, 1984.

[10] National Research Board, „Highway Capacity Manual," United States of America, 2000.

[11] M. Lighthill en G. Whitham, „On kinematic wave II, a theory of traffic flow on long crowded roads.," *Proceedings of Royal Society of London,* nr. 229, pp. 317-345, 1955.

[12] R. Richards, „Shock waves on the highway," *Operations Research,* nr. 4, pp. 42-51, 1956.

[13] J. Esser en M. Schreckenberg, „Microscopic Simulation of Urban Traffic Based on Cellular Automata," *Computational Physics and Physical Computation,* nr. Volume 08, Issue 05, 1997.

[14] H. Tijms, „Operationele analyse," Utrecht, Epsilon, 2002, pp. 399-411.

[15] R. van der Leije, „Verkeersonderzoek Westzaan & Nauerna 2012," 2013.

[16] Y. Kang, „Delay, Stop and Queue Estimation for Uniform and Random Traffic Arrivals at Fixed-Time Signalized Intersections," Virginia Tech, Virginia, 2000.

Appendix A Delay deterministic queueing model

Figure 23 shows the queue length over time in a deterministic queueing model.

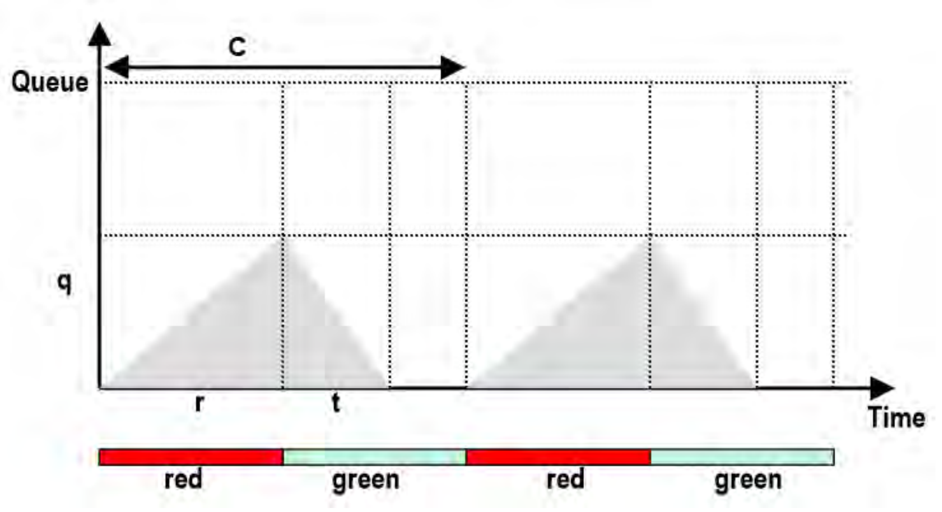

Figure 23 Queue length over time [16]

We see that during a red light the queue increases at rate *v*, until the light turns green. The queue will then decrease with rate *s-v* until 0 where it will remain until the light turns red again. The cycle then repeats itself.

We model the total delay during a cycle *C*, which is equal to the surface of the grey area:

$$Total\ delay = \frac{q \cdot r}{2} + \frac{q \cdot t}{2},$$

where $q = r \cdot v,\ t = \frac{q}{s-v} = \frac{rv}{s-v}$, and $r = C - g$. Then

$$Total\ delay = \frac{1}{2}q(r+t) = \frac{1}{2}r \cdot v\left(r + \frac{r \cdot v}{s-v}\right) = \frac{1}{2}r^2 \cdot v\left(1 + \frac{v}{s-v}\right) = \frac{1}{2}r^2 \cdot v\left(\frac{s}{s-v}\right).$$

$$Total\ arrivals = v \cdot C.$$

$$Average\ delay = {}^{Total\ delay}/_{Total\ arrivals} = \frac{1}{2}r^2 \cdot v\left(\frac{s}{s-v}\right)/v \cdot C = \frac{1}{2}r^2 \cdot \frac{1}{C}\left(\frac{s}{s-v}\right)$$

$$= \frac{(C-g)^2}{2 \cdot C}\left(\frac{s}{s-v}\right).$$

This formula can be rewritten using $= \frac{v}{c} = \frac{v}{s \cdot \frac{g}{C}}$.

$$Average\ delay = \frac{(C-g)^2}{2 \cdot C}\left(\frac{s}{s-v}\right) = \frac{1}{2}\frac{C^2 - 2gC - g^2}{C}\left(\frac{\frac{v}{X \cdot \frac{g}{C}}}{\frac{v}{X \cdot \frac{g}{C}} - v}\right)$$

$$= \frac{1}{2}C\left(1 - \frac{2g}{C} - \frac{g^2}{C^2}\right)\left(\frac{1}{\left(1 - X \cdot \frac{g}{C}\right)}\right) = \frac{1}{2}C(1 - \frac{g}{C})^2\left(\frac{1}{\left(1 - X \cdot \frac{g}{C}\right)}\right) = \frac{C(1 - \frac{g}{C})^2}{2\left(1 - X \cdot \frac{g}{C}\right)}$$

Appendix B Delay oversaturated deterministic queueing model

When the volume-to-capacity ratio is *X*, on average *X – 1* vehicles will be left in queue after each unit of time (with an equal number of green and red light times). The term $(X-1)+\sqrt{(X-1)^2}$ is 0 as long as *X* is below 1, and is *2(X-1)* when *X* is above 1. A vehicle arrives on average halfway in the measured period, at *½ T*. So the average delay in hours is $\frac{1}{2}T * \frac{1}{2}\left((X-1)+\sqrt{(X-1)^2}\right) = \frac{1}{4}T\left((X-1)+\sqrt{(X-1)^2}\right)$. Multiplying by 3600 gives the average delay in seconds instead of hours:

$$3600 * \frac{1}{4}T\left((X-1)+\sqrt{(X-1)^2}\right) = 900T\left((X-1)+\sqrt{(X-1)^2}\right).$$