Collateralized Commodity Obligations

Historical performance and rating approach via block bootstrap simulation

BMI Thesis

Author:

Dimitar Mechev

Supervisor:

Svetlana Borovkova

March 2010

vrije Universiteit amsterdam

Collateralized Commodity Obligations

Historical performance and rating approach via block bootstrap simulation

BMI Thesis

Author:
Dimitar Mechev

Supervisor:
Svetlana Borovkova

March 2010

VRIJE UNIVERSITEIT AMSTERDAM
Faculty of Sciences
Business Mathematics and Informatics
De Boelelaan 1081a
1081 HV Amsterdam

Contents

List of Figures

List of Tables

Summary

This paper is about Collateralized Commodity Obligations (CCO), performance evaluation of an example of a static synthetic CCO and implementation and evaluation of a rating approach via non-parametric bootstrap simulation procedures. The paper describes the structure of a CCO and discusses the differences and similarities between CCO and Commodity Debt Obligations (CDO). It examines the historical performance of a static synthetic CCO that was considered by Goldman Sachs International in 2006 focusing on the crisis period from 2008 up to now. In contrast to the rating agencies' parametric modeling and rating approaches, an alternative non-parametric simulation method is presented, implemented and evaluated.

Chapter 1

Introduction

Collateralized Commodity Obligations provide investors with a concept of spreads in the commodities markets adopting the structure of Collateralized Debt Obligation (CDO) products, which are created from a basket of underlying bonds, loans or credit default swaps. The underlying securities in a CCO, however, comprise around 100 derivatives known as commodities trigger swaps (CTS), based on up to 16 precious metal, base metal, agricultural and energy prices. CCO transform commodities to a ratable credit product and provide investors with a return that is linked to the performance of an underlying portfolio of commodities. These investors are mostly fixed income funds containing credit and interest rate products. The main advantage of adding commodity exposure to a fixed income portfolio is diversification as the commodity returns are uncorrelated with interest rate risk and credit risk. The biggest investors in CCOs, according to [14], are insurance companies, commercial banks and hedge funds in Europe, the USA and Asia.

After issuing, the commodity-linked notes are rated by an independent agency (Moody's, S&P, Fitch and etc.). The rating of a structured note is an indicator (often not very reliable) for the implied risk. Investment decisions cannot be based exclusively on ratings, since this often implies high model risk. Risk assessment of CCO can be improved by including an alternative rating approach via block bootstrap simulation. It is a non-parametric method for calculating probability of default of CCO tranches, which does not assume any parametric model for the underlying commodity prices. Under certain market conditions, parametric models can be very efficient, but also very unreliable when major changes in the prices' behavior are present. S&P is withdrawing their ratings from CCO in April 2010. This might be considered as a failure of its rating approach and definitely indicates the need of alternative methods towards CCO risk assessment.

Chapter 2

Mapping from CDO to CCO structure

2.1 Credit Default Swap

A Credit Default Swap (CDS) is a swap designed to transfer the credit exposure of fixed income products between parties. There are three parties involved in a CDS: the *reference entity*, the *protection buyer* and the *protection seller* [13]. The *buyer* of the CDS makes a series of payments to the *seller* and, in exchange, receives a payoff if the *reference entity* (credit instrument typically a bond or loan) undergoes a defined *credit event*, often described as a default (failure to pay). The default payoff is the difference between the par value and the market price of a specified debt obligation, thus a CDS has some *recovery rate*. Since the market for CDSs written on larger corporations is fairly liquid and the profitability of CDSs is barely affected by tax issues, CDSs are adopted as an underlying security for many complex credit derivatives.

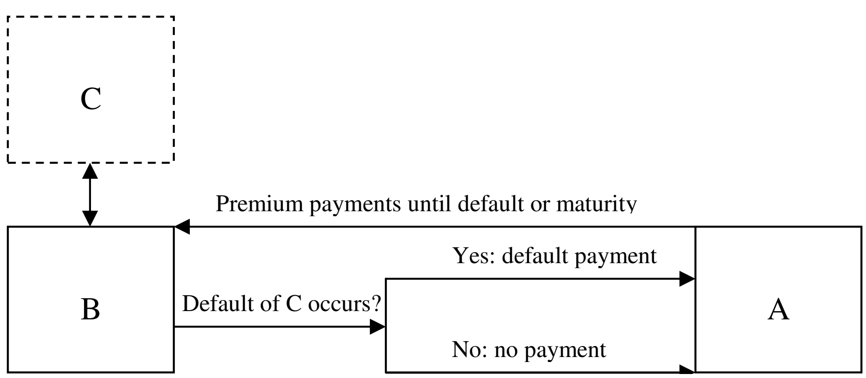


Figure 1. Basic structure of a CDS. Firm C is the reference entity, firm A is the protection buyer and firm B is the protection seller.

2.2 Synthetic Collateralized Debt Obligation

Collateralized Debt Obligations (CDO) is one of the most important classes of portfolio credit derivatives. A CDO is an instrument for the securitization of credit-risky securities related to a pool of reference entities such as bonds, loans or protection-seller positions in single-name CDSs. The basic structure of the different types of CDOs is identical. The structure of a synthetic CDO is considered as an example. The asset side of a synthetic CDO is formed by a portfolio of CDSs. The assets are sold to a so-called special-purpose vehicle (SPV). SPV is a company that carries out the securitization deal. In order to finance the purchase of the assets, SPV issues notes belonging to tranches of increasing seniority, which form the liability side. Losses on the asset side generated by credit events first affect the 'equity' tranche, then the 'mezzanine' tranches, and finally the 'senior' tranches. When tranches are issued they usually receive a rating from independent agencies (Moody's, S&P, and Fitch etc.).

2.2.1 Cash-flows of a synthetic CDO

On the asset side the SPV receives the premium payments on the CDSs in the asset pool and makes the corresponding default payments. On the liability side the SPV receives default payments from the noteholders, which are triggered by credit events in the asset pool, and makes periodic premium payments as compensation.

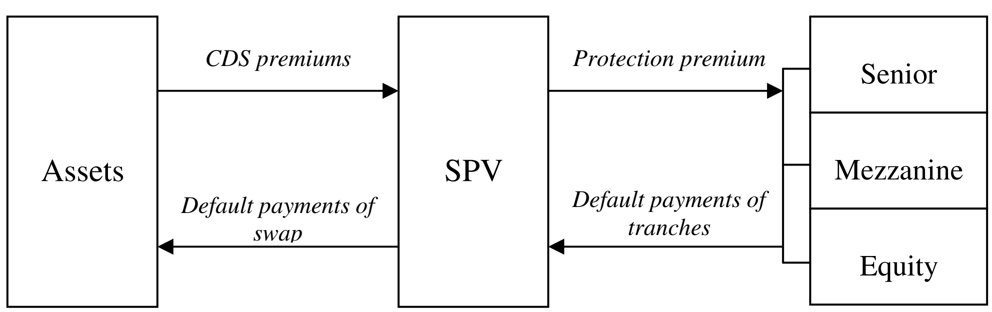


Figure 2. Schematic representation of the payments in a CDO structure.

2.2.2 Effect of *credit events* on the notional of tranches

Consider a portfolio of *m* different firms. The random vector $Y_t = (Y_{t,1} \cdots Y_{t,m})'$ describes the state of the portfolio at time $t \geq 0$, where $Y_{t,i} = 1$if firm *i* has defaulted up to time *t*, and $Y_{t,i} = 0$ otherwise. The cumulative loss of the portfolio up to time *i* is given by:

$$L_t = \sum_{i=1}^{m} I_i \delta_i Y_{t,i},$$

where I_i is the notional of CDS i and δ_i is the *loss given default* of firm *i*, which is equal to one minus the recovery rate of the corresponding CDS.

Consider a CDO with k tranches with indexes $\kappa \in \{1,..,k\}$ and *attachment points* $0 = K_0 < K_1 < \cdots < K_k \leq \sum_{i=1}^{m} I_i$. Initially the notional of a tranche κ is equal to $K_\kappa - K_{\kappa-1}$ and is reduced whenever a default event occurs so that the cumulative loss falls in the layer $[K_{\kappa-1,}K_\kappa]$.The notional of tranche κ at time *t* is given by:

$$N_\kappa(t) = f_\kappa(L_t) \text{ with } f_\kappa(l) = \begin{cases} K_\kappa - K_{\kappa-1}, & for\ l < K_{\kappa-1}, \\ K_\kappa - l, & for\ l \in [K_{\kappa-1,}K_\kappa], \\ 0, & for\ l > K_\kappa. \end{cases}$$

2.3 Commodity Trigger Swap

The structure of Commodity Trigger Swap (CTS) is identical to that of CDS. There are two main differences: first, the *reference entity* of CTS is some commodity price; and second the *credit event* (or so called *trigger event*) occurs when the corresponding commodity price falls (rises) below (over) some preset level at time of maturity T of the contract. This implies that the *protection buyer* has to make premium payments until time T. Important difference between CDS and CTS is also that the *recovery rate* of CTS is 0%, which implies 100% *loss given default*. CTS can be seen as a deep out-of-the-money European binary option [9].

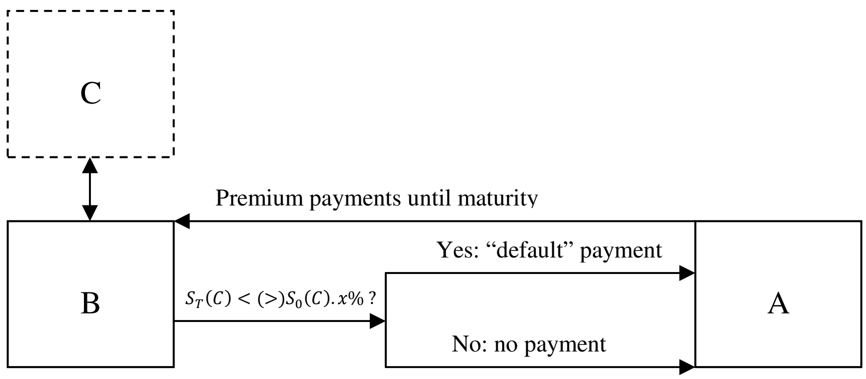


Figure 3. Basic structure of a CDS. Firm C is the reference entity, firm A is the protection buyer and firm B is the protection seller. $S_t(C)$ is the price of commodity C at time t and x is the preset level of the trigger event.

2.4 Synthetic Collateralized Commodity Obligation

The structure of a synthetic CCO is identical to that of a synthetic CDO. The asset side of a CCO is formed by a pool of around 100 commodity trigger swaps (CTS), based on up to 16 commodity prices. The protection sellers receive premiums until the time of maturity of the contract. CCO gives an investor exposure to commodity risk, with bond style payoff. The bond style payoff of the CCO consists of periodic interest payments of floating rate (e.g. LIBOR) plus additional premium. The additional premium depends on the tranche of the CCO. Besides the periodic interest payments, at the time of maturity the CCO also pays the principal minus losses generated by trigger events.

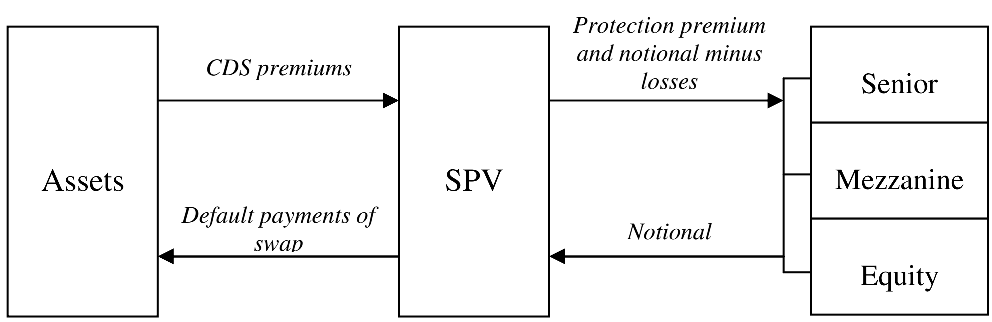


Figure 4. Schematic representation of the payments in a CCO structure.

2.4.1 Effects of *trigger events* on the notional of tranches

In contrast to CDO, where the losses are generated by defaults of firms and investors are exposed to credit risk, in CCO the losses are due to movement in the underlying commodity prices, thus related to market risk. The cumulative loss of the portfolio depends on the number of trigger events of the underlying CTSs that occur at time of maturity T. While notional of a CDO tranche is a function of the cumulative loss of the portfolio of CDSs, the notional of a CCO tranche depends on the total number of trigger events that occur in the portfolio. So L_T is defined as number of trigger events and is given by:

$$L_T = \sum_{i=1}^{m} I_i 1_{\left(S_0(C_i) \underset{(<)}{>} S_T(C_i) x\%\right)}$$

where $I_i$is the notional of CTS i and $S_t(C_i)$ is the price of commodity $C_i$at time *t*. *The* notional of tranche κ at time *t* is given analogously as for CDOs. The notional at maturity of CCO tranches depends exclusively on the total number of trigger events at maturity and equals [12]:

$$N_\kappa(T) = N_k(0) f_\kappa(L) \text{ with } f_\kappa(L) = \begin{cases} 1, & for\ L < K_{\kappa-1}, \\ \frac{K_\kappa - l}{K_\kappa - K_{\kappa-1}}, & for\ L \in \left[K_{\kappa-1,} K_\kappa\right], \\ 0, & for\ L > K_\kappa. \end{cases}$$

2.5 CDO-CCO comparison

2.5.1 Similarities between CDO and CCO

Both financial instruments are examples of structured asset-backed security (ABS) whose value is derived from a portfolio of underlying assets. The securities are split into tranches which represent different risk classes. Both are fixed-income style instruments, which means that they pay coupons and a notional (which can be lower than the initial notional). Interest and principal payments are made in order of seniority, so that junior tranches offer higher coupon payments to compensate for additional risk.

2.5.2 Differences between CDO and CCO

Although the basic idea and the product setup are the same, the instruments are significantly different in nature, because of the different underlying assets. In the case of synthetic CDO the underlying derivative is CDS, while a synthetic CCO has CTSs as underlying derivatives. CDS is a derivative contract that enables the transfer of ***credit risk*** related to the reference entity of the swap. With the use of CDSs, the synthetic CDO gains credit exposure to a portfolio of fixed-income assets without owing the assets themselves. Thus the risk associated with a synthetic CDO and its tranches is based on the credit quality of the CDSs' reference entities. In contrast to CDS, the underlying risk of CTS is ***market risk***, since the reference entity is some commodity price. It is of crucial importance that investors make clear distinction between the different structured notes and risks that underlie those.

Another important difference is found in the default payments: while a default event can occur before maturity influencing the premium payment to at least one tranche of a CDO, trigger events in a CCO are considered at maturity and all CCO tranches receive a constant premium to maturity.

Chapter 3
Example CCO implementation

This chapter presents an example of static synthetic CCO considered by Goldman Sachs International. The structure of the CCO is implemented in Matlab and a historical performance analysis is performed.

3.1 CCO structure

3.1.1 Asset side

The asset side of the CCO consists of portfolio of 100 CTSs in 12 different commodities with maturity of 5 years. For each commodity an upper trigger event strike and a lower trigger event strike are defined as a percentage of the price of the corresponding commodity at time zero. The trigger events (CTSs) are evenly distributed between the upper and the lower strike. Table 1 below shows the lower and upper strike for each commodity, as well as the distribution of CTSs among the different commodities.

No.	Commodity	Upper Trigger Event Strike	Lower Trigger Event Strike	# of Trigger Events (CTSs)
1	Aluminum	43%	35%	9
2	Tin	49%	42%	8
3	Zinc	25%	17%	9
4	Nickel	22%	14%	9
5	Silver	31%	24%	8
6	Platinum	30%	23%	8
7	Palladium	34%	27%	8
8	Corn	80%	74%	7
9	Wheat	59%	51%	9
10	Sugar	79%	72%	8
11	Copper	20%	12%	9
12	Lead	35%	28%	8

Table 1. Asset side of the CCO example: underlying commodities, upper and lower trigger strikes and number of CTSs per commodity.

3.1.2 Liability side

The liability side of the CCO consists of notes belonging to 5 tranches of increasing seniority: BBB, A, AA, AAA, and Super Senior. Each tranche has attachment and detachment points (Table 2). If the total number of trigger events at maturity is lower than the attachment point, then the notional of the corresponding tranche remains intact; if it is between the attachment and detachment point, then part of the notional is lost; and if the number of trigger events is bigger than the detachment point then the whole notional is lost.

Tranche	Attachment Point	Detachment Point	Indicative yield
BBB	12	18	LIBOR + 350 bp
A	18	21	LIBOR + 250 bp
AA	21	27	LIBOR + 190 bp
AAA	27	34	LIBOR + 115 bp
Super Senior	34	41	LIBOR + 60 bp

Table 2. Attachment and detachment points for all tranches of the CCO example.

Suppose that the prices of lead, copper and wheat fall to 30%, 13% and 55% of their value at time zero respectively, and the prices of all other commodities stay above their upper strikes. In this scenario the total number of trigger events would be 19. This will result in a loss of 100% of the notional of the BBB tranche and 33.3% of the notional of the A tranche.

3.2 Data

All data required for the analysis of the CCO from July 1st 1993 until January 31st 2010 is obtained from Thomson Datastream. Daily closing prices are used for the commodities. To ease the calculations, a five year interest swap rate is used. Due to incomplete data on the five year LIBOR rate, the daily five year US Constant Maturity Treasury rate (code: FRTCM5Y) is used for the coupons and discounting. Table 2 presents the list of commodities and their grade and quality specifications that are used in the CCO's underlying portfolio of commodity derivatives (CTSs).

Commodity	Code	Type/Grade
Aluminium	LAHCASH	LME-Aluminium 99.7% Cash U$/MT - A.M. OFFICIAL
Copper	LCPCASH	LME-Copper, Grade A Cash U$/MT - A.M. OFFICIAL
Lead	LEDCASH	LME-Lead Cash U$/MT - A.M. OFFICIAL
Tin	LTICASH	LME-Tin 99.85% Cash U$/MT - A.M. OFFICIAL
Zinc	LZZCASH	LME-SHG Zinc 99.995% Cash U$/MT- A.M. OFFICIAL
Nickel	LNICASH	LME-Nickel Cash U$/MT - A.M. OFFICIAL
Silver	SLVCASH	Silver Fix LBM Cash U$/Troy ounce
Platinum	PLATFRE	London Platinum Free Market $/Troy oz
Palladium	PALLADM	Palladium U$/Troy Ounce
Corn	CORNUS2	Corn No.2 Yellow U$/Bushel
Wheat	WHEATSF	Wheat No.2,Soft Red U$/Bushel
Sugar	WSUGDLY	Raw Sugar-ISO Daily Price U$/lb

Table 3. Technical description of the commodities.

3.3 Historical performance

The historical performance analysis of the CCO proposed by Goldman Sachs is based on a hypothetical daily issue of the CCO from 1st July 1993 up to 31st January 2005. The maturity of the contract is 5 years. The notional of tranches at maturity depends on the total number of trigger events generated by the portfolio of CTSs on the asset side of the CCO. The number of trigger events of issued CCOs will be examined and in order to indicate the performance of every CCO tranche the realized present value of all payments for every tranche will be calculated.

3.3.1 Commodity prices

Until 2003 most of the commodities that underlay the CTSs in the Goldman Sachs CCO follow a mean reverting process. In 2003 most of the prices gradually start to show strong positive trend and keep this trend until first quarter of 2008. By the end of 2008 all the prices of the commodities except for sugar drop significantly. Despite the major drop in the commodity market the prices drop on a level higher than the historical level before 2003.

3.3.2 Number of trigger events

The total number of trigger events generated by the commodity prices movement is calculated for the CCO issued every day from 1st July 1993 till 31st January 2005.

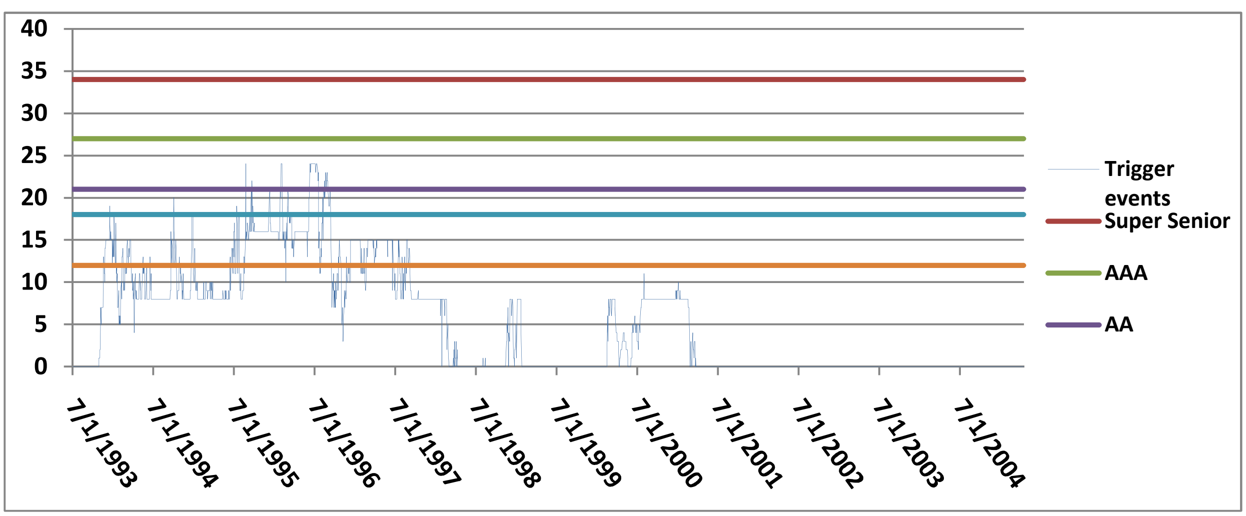


Figure 5. Number of trigger events combined with attachment points of the tranches.

In total, the investors in the BBB tranche lose some of their notional in 16.8% of the hypothetically issued CCOs. The investors in the A and AA tranches lose some of their notional in 2.49% and 1.42% of the issued CCO respectively. The investors in AAA and Super Senior tranches experience no lose. Since in 2002-2003 the commodity prices start to show a positive trend, no trigger events occur in the period after 2002. Even the CCOs issued after 2004 (for which the period from issue to maturity includes the crisis period) undergo no trigger events. This can be explained by the fact that the prices show a strong positive trend until 2008 and even after the major drop in the commodity market, the prices fall to a level higher than the historical level before 2003.

3.3.3 Realized present value

In order to get an overview of the historical performance of the example CCO, the realized present value for each tranche of each hypothetical issue of the CCO is calculated. The five year bond style payoff of the CCO consists of interest payments every three months of the three-month LIBOR rate plus an additional premium. The premium depends on the tranche of the CCO. Besides the interest payments, the CCO pays also the principal minus the generated losses at maturity. The realized present value of tranche *i* is calculated as follows:

$$PV_i = Discounted\ bond\ payments + Discounted\ principal - Discounted\ Losses,$$

In order to ease the calculation, a five-year interest swap is used so that the floating LIBOR rate can be substituted by constant 5-year rate. For the calculation of the realized present value the 5-year Constant Maturity Treasury (CMT) rate is used for the coupons and discounting:

$$PV_i = \frac{C_i}{(1+R)^{1/4}} + \frac{C_i}{(1+R)^{2/4}} + \cdots + \frac{C_i}{(1+R)^5} + \frac{P}{(1+R)^5} - \frac{P * f_i(L)}{(1+R)^5},$$

where C_i is the interest payment (CMT rate + premium) made each three months to the investor in tranche *i* and *R* is the CMT rate. Figure 5 below shows a pattern very similar to the plot of the number of trigger events. Before mid-1997 the present value of the tranches is quite volatile. The summary of the data for the realized present value shows that from a historical perspective the

BBB and A tranches are the least desirable, combining a lower average present value with a higher volatility.

1993-2005	BBB	A	AA	AAA	SS
Mean	1.07312	1.091157	1.077581	1.050065	1.026809
Median	1.148202	1.106539	1.081342	1.049822	1.026744
Standard Deviation	0.186831	0.103708	0.037065	0.000913	0.000211
Minimum	0.373167	0.33229	0.694311	1.048591	1.026348
Maximum	1.159459	1.113969	1.086675	1.052557	1.027537

Table 4. Descriptive statistics for CCO realized present value (1993-2005).

After mid-1997 the CCO performs significantly better and all of the tranches outperform the 5 year US Treasury bond. The descriptive statistics for the present value of the CCOs issued from 1998 till 2005 show a significantly higher average realized present value and lower risk for tranches BBB, A and AA. Based on these results it can be concluded that all tranches are performing well and even the most risky investors have made a profit.

1998-2005	BBB	A	AA	AAA	SS
Mean	1.152011	1.109049	1.083272	1.05105	1.02742
Median	1.151286	1.108375	1.082634	1.050455	1.026853
Standard Deviation	0.003667	0.002474	0.001759	0.000866	0.000225
Minimum	1.144216	1.103634	1.079284	1.048846	1.026367
Maximum	1.159459	1.113969	1.086675	1.052557	1.027537

Table 5. Descriptive statistics for CCO realized present value (1998-2005).

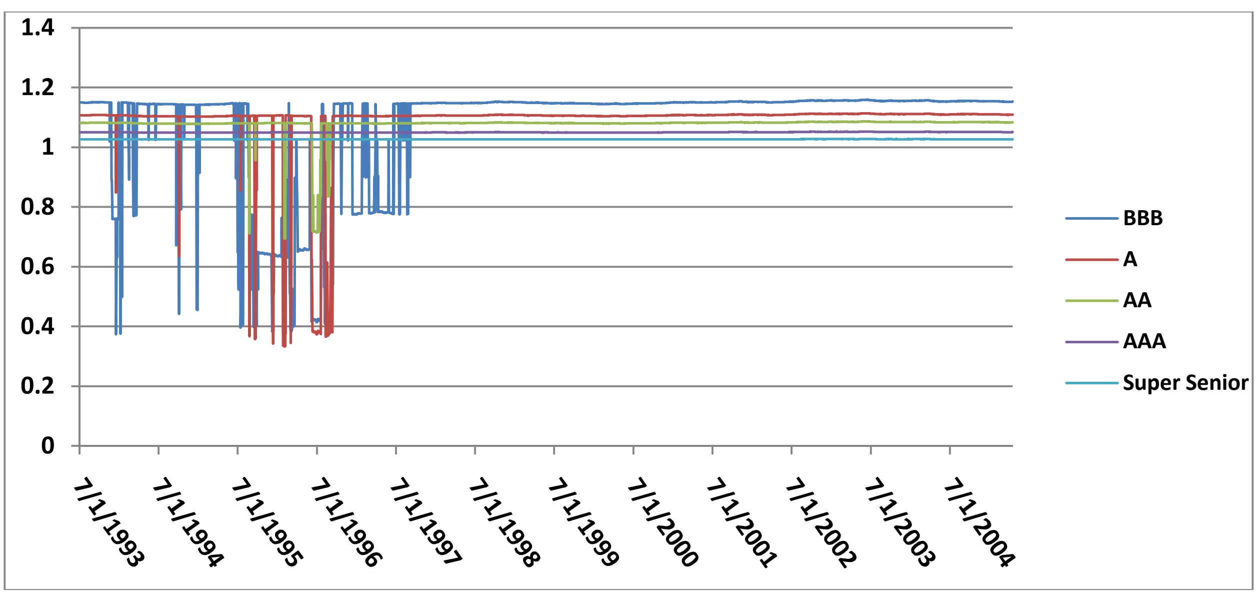


Figure 6. Realized present value of a tranche at a given issue date.

3.3.4 Conclusion

The historical performance analysis of the example CCO showed the examined period 1993-2005 can be divided in two significantly different periods: 1993-1998 and 1998-2005. Until 1998 the CCO is characterized by a high volatility of the realized present value of the tranches (especially BBB, A, and AA tranches). This may be caused by the fact that until 2002-2003 most of the commodity prices follow a mean reversion process. Since the commodity prices are correlated, mean reversion could result in a higher total number of trigger events generated by the portfolio of CTSs.

In the second period 1998-2005, the realized present value of the tranches is nearly constant. The CCO issued in this period clearly outperforms the 5 year US Treasury bond. Even investors in the CCO issued after 2004 (which means that the period till maturity includes the crisis period) do not lose their notional. This can be explained by the commodity prices behavior in the period 2002-2009. In 2002-2003 most of the commodities gradually start to show a strong positive trend which lasts until the beginning of 2008. Between the last quarter of 2007 and the first quarter of 2008 most of the commodity prices reach their absolute historical maximum. Even after the major decrease of the prices, they fall to a level higher than the historical level before 2003 (except for Sugar and Palladium).

Based on the historical simulation results it can be concluded that the AA and AAA tranches are the most desirable, when measuring risk versus reward. It combines relatively high return with low risk level.

Chapter 4

CCO modeling and rating

This section is an overview of the rating agencies' CCO modeling and rating approaches. When tranches are issued, they usually receive a rating from an independent agency. Each agency has a committee which ultimately determines the credit rating of the financial product. The outcome of their models is used as a guideline for the evaluation of the moving block bootstrap simulation rating approach.

4.1 S&P modeling and rating approach

S&P bases the method for rating CCO [4] on its already developed rating approach for CDO. First, for each individual CTS it determine the probability (PD) that a trigger is breached. In order to calculate PD it model the underlying commodity prices using mean-reversion jump diffusion (MRJD) process on log-prices. The model for commodity prices has the following general form:

$$\frac{dS}{S} = \beta(\xi - \ln S)dt + \sigma dW + dJ^{up} + dJ^{down}$$

which implies that the spot price S mean-reverts to the long-term level of e^{ξ} at a speed β. J denotes a Poisson process (a discrete time process) with intensity λ, which determines the frequency of the positive "up" and negative "down" jumps of a fixed size, and σ shows the volatility of prices. The jumps take effect for a maturity T ≤ 3 years. So for the example CCO with maturity 5 years considered in this paper, the MRJD process reduces to a standard mean-reverting process:

$$dx = (\theta - x)dt + \sigma dW$$

where $x = \ln S$ and $\theta = \xi - \frac{\sigma^2}{2\beta}$. The stochastic log-price *x* is normally distributed with mean:

$$E[x_t] = x_0 e^{-\beta \Delta t} + \theta\left(1 - e^{-\beta \Delta t}\right)$$

and variance:

$$V[x_t] = \frac{\sigma^2}{2\beta}\left(1 - e^{(-2\beta \Delta t)}\right)$$

In this case the PD can be solved analytically. The PD at any specified price level $K * S_{t=0}$ (where K is the strike level) is found by the cumulative lognormal probability distribution of S where:

$$x = \ln S \sim N(E[x], V[x])$$

After the default probability (or the probability that trigger is breached in the case of CTS) is calculated for every CTS, S&P use its CDO Evaluator with adjusted correlation table to calculate the default probability of the tranches. S&P defines the default probability of a tranche as the probability that the whole notional is lost at maturity. The CDO Evaluator of S&P is made to calculate loss distribution of N assets and uses Gaussian copula (method originally introduced by [11]) to model the dependency between the different assets.

In December 2009, Standard & Poor's announced that on 31st March 2010 it will begin withdrawing ratings from structured notes with variable principal payments linked to equity prices, commodity prices, or equity or commodity indices.

4.2 Fitch ratings modeling and rating approach

Fitch's modeling approach employs a Monte Carlo simulation of commodity returns to determine the joint portfolio loss distribution [5]. To arrive at the loss distribution, it is necessary to find the distribution of observed trigger events (and their respective loss severity). This is determined by simulating correlated commodity return paths. To simulate correlated return paths, a factor model for each commodity is obtained through Principal Components Analysis [16]. PCA identifies common factors contributing to each commodity's return process as well as quantifying the percentage of risk in the overall portfolio contributed by each factor. Each commodity's returns are a weighted sum of the principal component factors. The correlations among commodity returns are implicitly modeled by combining the factor loadings (weights) and their standard deviations.

Fitch chose a model for the factors that consists of an NGARCH process for the variance, a constant for the mean, and jumps in returns. The NGARCH process is appealing because it captures volatility clustering and, along with jumps, the leverage effect. The leverage effect refers to an increase in volatility resulting from a large, negative return or vice versa.

Given the simulated commodity return paths, Fitch calculates the cumulative return over the desired period, recording a loss whenever the cumulative returns for a commodity breach any of the pre-selected trigger levels. Fitch does not define PD of a tranche clearly, but most likely it is the probability that some part of the notional is lost.

4.3 Rating agencies implied probabilities of default

Credit rating	AAA	AA	A	BBB
P(default) in % by S&P	0.118	0.356	0.709	2.812
P(default) in % by Fitch	0.030	0.200	0.560	1.580

Table 6. Credit curve for tranches provided by S&P and Fitch for CDO with 5 years to maturity.

Chapter 5
CCO rating approach via block bootstrap

In contrast to the agencies' parametric rating approaches, this chapter presents a non-parametric alternative via moving block bootstrap simulation of CCO returns. Bootstrap methods in their classical form, as first proposed by Efron [7], are designed for application to samples of independent data. Under that assumption they implicitly produce an adaptive model for the marginal sampling distribution. During the last decade these approaches have been modified to suit the case of dependent data. One of the most popular bootstrap methods for dependent data is the block bootstrap method introduced by Hall [8], Carlstein [3] and Kuensch [10].

5.1 Moving block bootstrap

The moving block bootstrap consists of randomly selecting blocks of consecutive observations with replacement from a set of overlapping blocks. Given data $\mathrm{X} = \{X_i, 1 \leq i \leq n\}$ from a stationary time series, the method constructs a set of blocks $B_1, B_2, \dots, B_b$ of length l, where $B_i = \{X_{i1}, \dots, X_{il}\}$, $b = n - l + 1$ and $X_{ij} = X_{i+j-1}$. Blocks are drawn randomly from the set described above and joined together to form a simulated time series. The above procedure holds also for multivariate time series X. The block bootstrap relies on producing a compromise between preserving the dependence structure of the original data and corrupting it by supposing that the data are independent. Although blocks of data are dependent in the original time series, they are independent in the bootstrap version. This causes bias in the bootstrap variance which can be large if the dependence in the data is strong. As shown in the next section this drawback of the method is not of significant relevance when using moving block bootstrap to rate CCO tranches.

5.2 CCO rating approach via block bootstrap simulation

The rating of a CCO tranche is an indicator for the probability that the investor will lose (part of) the notional of the tranche. This probability can be calculated by simulating prices behavior and trigger events. Since the CCO performance depends on the total number of trigger events generated by cross-correlated commodity prices, any simulation method should take into account the correlation structure between the different commodities. The autocorrelation structure is of no significant importance because of the large time horizons of CCOs. The moving block bootstrap method applied to the multivariate series of returns is capable of preserving the cross-correlation structure of the commodities. This is done by selecting blocks of returns of different commodities with equal starting (and ending) time. As it was mentioned before, the drawback of the block bootstrap is the bias in the bootstrap variance for strongly dependent data. Since the returns are independent, the bootstrap drawback is of no concern when applied to CCO. The empirical evidence of volatility clustering implies autocorrelation of the squared returns. This property is of no significant importance, since the CCO performance depends almost exclusively on the commodity prices at time of issuing and maturity. Nevertheless, the autocorrelation structure is partly preserved by randomly selecting from overlapping blocks.

Once paths of commodity returns are simulated, the calculation of the total number of trigger events at maturity and PD is straightforward. In order to build a rating system "probability of default" should be defined. It can be defined as "the probability that the whole notional at maturity is lost" ($PD = P(N_T = 0)$) or "the probability that at least some part of the notional at maturity is lost" ($PD = P(0 \leq N_T \leq N_0)$).

5.3 Block bootstrap simulation of the example CCO

The block bootstrap procedure described in the previous section is used to simulate returns for the example CCO presented in Chapter 4. The simulation is carried out 500,000 times for several block lengths: 2, 5, 10, 20, and 30. Two periods of historical commodity returns are used: July 1993 - December 2007 and July 1993 - January 2010. In every run the notional at maturity is calculated for all 5 tranches. Both definitions for probability of default are used, namely: $PD = P(N_T = 0)$ and $PD = P(0 \leq N_T \leq N_0)$). In order to be compared with the implied default probabilities given by rating agencies and block bootstrap, PD is calculated for both definitions.

5.3.1 Results for Probability of default: $PD = P(N_T = 0)$

The probability of default as defined by Fitch is calculated for each tranche, time period and block length. The next tables show the results of the simulations in using both time periods.

Block length	BBB	A	AA	AAA	SS
2	2.2542	1.7344	0.1152	0.0122	0.0024
5	2.3918	1.8712	0.1212	0.0092	0.0014
10	2.6142	2.0394	0.1562	0.0116	0.0016
20	2.3404	1.8124	0.1518	0.016	0.0034
30	2.3912	1.8378	0.1598	0.0128	0.0022

Table 7. Block bootstrap simulation results for 1993-2007. PD as defined by Fitch.

In comparison to the implied probabilities given by Fitch, the block bootstrap simulation for 1993-2007 results in higher default rates for the BBB and A tranches and lower for the high rated tranches: AA and AAA.

Block length	BBB	A	AA	AAA	SS
2	5.1878	4.131	0.7704	0.2094	0.072
5	5.3762	4.3206	0.7916	0.2012	0.065
10	5.4348	4.3818	0.934	0.2576	0.0962
20	5.2578	4.2202	1.0568	0.3416	0.1466
30	5.116	4.1184	1.0702	0.3716	0.167

Table 8. Block bootstrap simulation results for 1993-2010. PD as defined by Fitch.

In order to get a grasp of the robustness of the block bootstrap approach, the set of historical returns is widened to include the highly volatile crisis period of the last years. Table 7 shows the results of the block bootstrap simulation for 1993-2010. The calculated probabilities are higher than the implied probabilities given by Fitch and the PD calculated via block bootstrap for 1993-2007, but still reasonable.

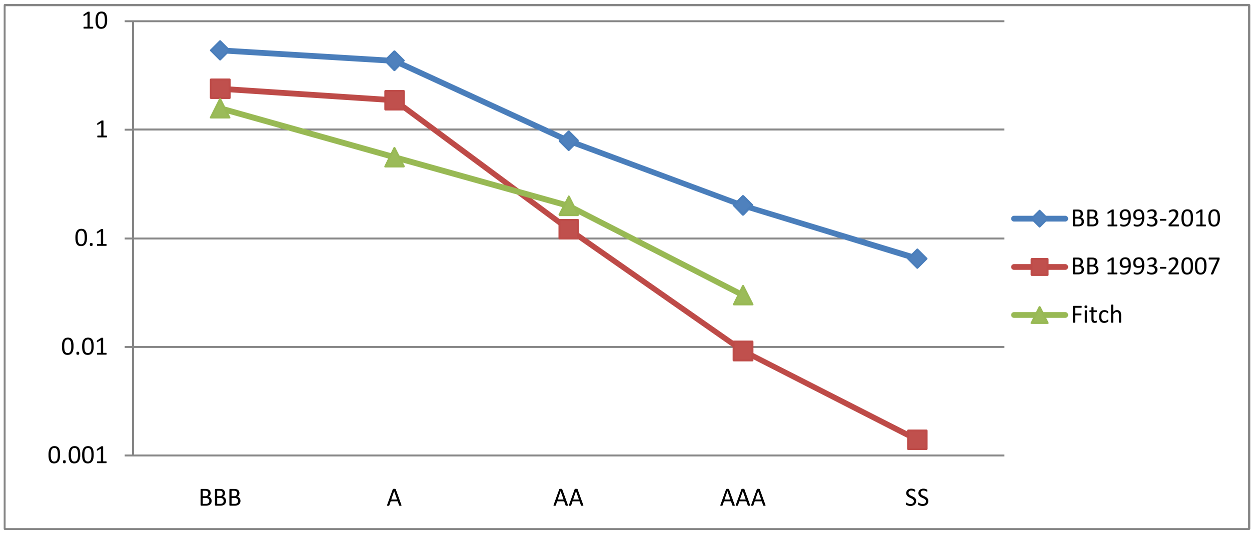


Table 9. Comparison plot of default probabilities for each tranche calculated by Fitch and block bootstrap with blocks of length 5 for 1993-2007 and 1993-2010.

5.3.2 Results for Probability of default: $PD = P(0 \leq N_T \leq N_0)$

The probability of default as defined by S&P is calculated for each tranche, time period and block length. The next tables show the results of the simulations in using both time periods.

Block length	BBB	A	AA	AAA	SS
2	12.0244	2.0656	1.5744	0.0986	0.01
5	12.0378	2.2066	1.7098	0.1036	0.0082
10	12.115	2.4124	1.8588	0.1368	0.0092
20	11.5966	2.1542	1.645	0.1304	0.0136
30	11.936	2.2006	1.6856	0.137	0.0102

Table 10. Block bootstrap simulation results for 1993-2007. PD as defined by S&P.

The default probabilities calculated by block bootstrap simulations are higher than the S&P's implied PD for all tranches except for the AAA tranche. The results suggest downgrading of the CCO tranches.

Block length	BBB	A	AA	AAA	SS
2	18.7658	4.8086	3.8178	0.6868	0.1804
5	18.6986	4.9946	4.0094	0.7054	0.1756
10	18.2934	5.0558	4.0682	0.836	0.2256
20	17.478	4.892	3.9176	0.9568	0.3042
30	17.1264	4.764	3.8204	0.9726	0.3316

Table 11. Block bootstrap simulation results for 1993-2010. PD as defined by S&P.

When sampling from a wider set of historical returns (1993–2010), the block bootstrap results in higher probabilities of default in comparison to those implied by S&P's ratings. The historical performance analysis of Goldman Sachs CCO in Section 4.3 shows that investors lost part of their notional in 16.8% (BBB), 2.49% (A) and 1.4% (AA). Although the set of historical data is not big enough to derive any reliable estimation of the PD of the tranches, the historical performance analysis suggests that the probabilities given by S&P might be too low.

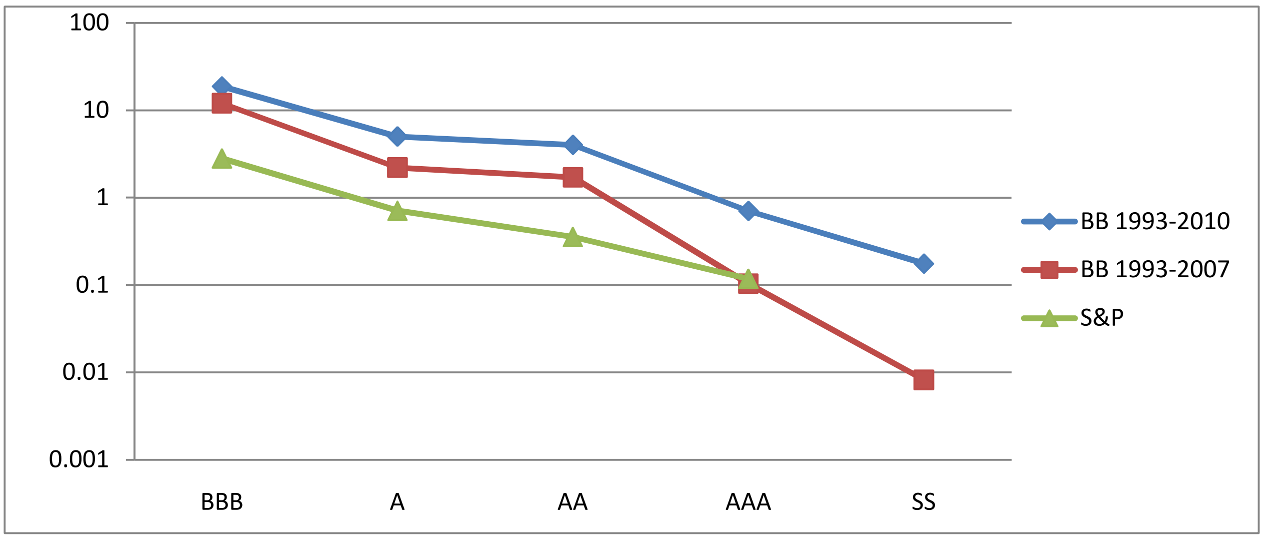


Table 12. Comparison plot of default probabilities for each tranche calculated by S&P and block bootstrap with blocks of length 5 for 1993-2007 and 1993-2010.

5.4 Block bootstrap method evaluation

Evaluation of the presented method is not straightforward. The main guidelines for evaluation are the agencies' rating methods and their implied probabilities of default. Both S&P and Fitch use different parametric models for commodity prices and it is hardly possible to check whether their approaches are right or wrong. Parametric models could be very efficient under a certain state of the market, but are also very unreliable when major changes in the prices behavior are present. While before 2003 most of the commodity prices follow a mean reverting process, after 2003 they show a strong positive trend. This change cannot be captured by the mean reversion parametric model (e.g. [15]), which results in an unrealistic PD for the Goldman

Sachs CCO (see [1]). The fact that S&P is withdrawing its ratings from commodity-linked notes also suggests that its rating method is not reliable. Rating approaches are very subjective in their nature and thus imply a significant amount of model risk for the investors. Investment decisions cannot and should not be based exclusively on agencies' ratings. In this sense the proposed non-parametric moving block bootstrap approach cannot be proved to be "the better approach" and does not intend to prove agencies' approaches right or wrong. The method is an alternative approach which is easy to implement and, most important of all, non-parametric, meaning that it does not assume an explicit commodity price model. This property is an advantage because it is more robust to major market changes, but at the same time it is a drawback because it does not assume any price behavior in the future (which sometimes can be preferable). Block bootstrap captures the most important property of CCO returns, namely the cross-correlation between different commodities and at the same time partly preserves their autocorrelation structure. The PD calculated by simulating the example CCO returns cannot be directly compared to the results from the historical performance analysis, because the historical data are enough only for around 3000 hypothetically issued CCOs. When sampling returns from 1993-2005, the acquired probabilities of default are very close to the implied PD of S&P and Fitch. The example CCO returns were also simulated from a wider set of historical returns which includes the volatile crisis period (1993-2010). In the latter case the PD given by the block bootstrap method is higher than the implied PD by the agencies' ratings. The values for the PD would suggest downgrading of the CCO tranches (SS to AA, AAA to A, and AA to B). Indeed it might have been the case, but unfortunately agencies' rating database is not freely available and the actual downgrades cannot be observed. As mentioned before, during 2008 most of the commodity prices decrease to a level higher than the historical level before 2003. Of course a reliable 5-year forecast for commodity prices cannot be given, but in general it is more risky to invest in a CCO when commodity prices are high, in this sense the higher PDs seem reasonable.

The influence of the block length on the probabilities of default of the tranches is also examined. The plots below show the probabilities of default with respect to block length. When sampling from historical returns of 1993-2010 an interesting effect is found: simulating with bigger block lengths results in higher PD for senior tranches and lower for the other tranches.

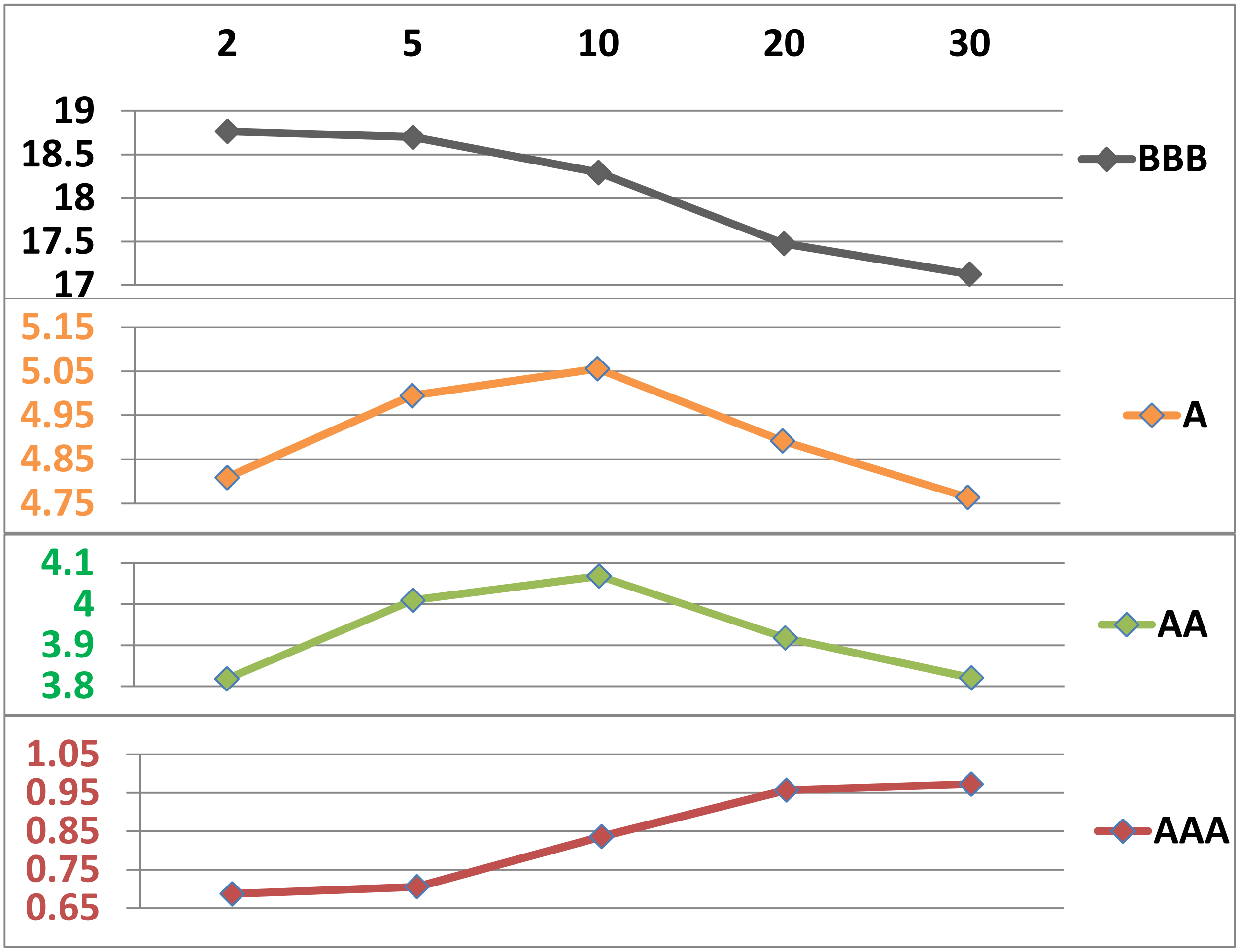


Figure 7. Plot of PD w.r.t. block length of the block bootstrap simulation.

Simulation with bigger blocks enables capturing clusters of simultaneous decrease of the prices (during the crisis period). Since the commodities are strongly correlated during extreme market situations and this dependence is well-captured, a high number of trigger events at maturity (>27) is more likely to occur. Although the difference in PD with respect to the block length is not significant, the choice of block length depends on the risk aversion of the investor. Since the underlying risk is market risk and the horizon is long, investors should be more cautious and safely choose the block length which gives the higher PD. Figure 7 suggests block length of 10 as a reasonable choice.

The evaluation of the moving block bootstrap simulation approach for rating CCOs is restricted by several factors:

- The main guidelines for evaluating the method are the agencies' ratings, which cannot be assumed as objective and reliable.
- Data on CCO ratings and rating migrations are not freely available.
- Scarce information on different CCO examples to test the method on.
- Historical data on commodities are not enough to give explicitly a comparable estimation for the PD.

Chapter 6

Conclusions

The paper describes the structure of a CCO and discusses the differences and similarities between CCO and Commodity Debt Obligations (CDO). It analyzes the historical performance of a static synthetic CCO that was considered by Goldman Sachs International in 2006 focusing on the crisis period from 2008 up to now. In contrast to the rating agencies' modeling and rating approaches, an alternative non-parametric simulation method is presented and evaluated.

The comparison between CDO and CCO shows that although the basic idea and the product setup are the same, the instruments are significantly different in nature, because of the different underlying assets. The risk associated with a synthetic CDO and its tranches is based on the credit quality of the CDSs' reference entities. In contrast to CDS, the underlying risk of CTS (and implicitly of CCO) is ***market risk***, since the reference entity is some commodity price. It is of crucial importance that investors take into account the different underlying risks indicated by the ratings of these structured notes.

The historical performance analysis of the example CCO shows that the examined period 1993-2005 can be divided in two significantly different periods. Until 1998 the CCO is characterized by high a volatility of the realized present value of the tranches (especially BBB, A, and AA tranches). In the second period 1998-2005, the realized present value of the tranches is nearly constant. In 2002-2003 most of the commodities gradually start to show a strong positive trend which lasts until the beginning of 2008. Between the last quarter of 2007 and the first quarter of 2008 most of the commodity prices reach their absolute historical maximum. Even after the major decrease of the prices, they fall to a level higher than the historical level before 2003 (except for Sugar and Palladium). The CCO issued in this period clearly outperforms the 5-year US Treasury bond.

When tranches are issued, they usually receive a rating from an independent agency. Each agency has a committee which ultimately determines the credit rating of the financial product. The paper reviews the rating methods of S&P and Fitch ratings. Both agencies have adjusted their CDO methodology to match the specific mechanics of the CCO. Although it is hardly possible to check if the rating agencies are right or wrong, their ratings are used as a guideline for the evaluation of the moving block bootstrap approach.

The paper presents the moving block bootstrap method as an alternative rating approach. The method simulates correlated commodity return paths randomly choosing blocks of historical returns. It is non-parametric and does not depend on any explicit model for commodity prices. This property is an advantage because it is more robust to major market changes, but at the same time it is a drawback because it does not assume any price behavior in the future (which sometimes can be preferable). When sampling returns from 1993-2005, the acquired probabilities of default are very close to the implied PD of S&P and Fitch. The example CCO returns are also simulated from a wider set (1993-2010) of historical returns which includes the volatile crisis period. In the latter case the PD given by the block bootstrap method is higher than the implied PD by the agencies' ratings. The values for the PD would suggest downgrading of the CCO tranches. Indeed it might have been the case, but unfortunately agencies' rating database is not freely available and the actual downgrades cannot be observed.

Ratings entail significant amount of model risk for the investors. Investment decisions cannot and should not be based exclusively on agencies' ratings. In this sense the proposed non-parametric moving block bootstrap approach cannot be proved to be "the better approach" and does not intend to prove agencies' approaches right or wrong. It should be seen as a sound rating method alternative for the investors.

References

[1] Borovkova, S., Bunk, H., De Goeij, W. & Veldhuizen, D. (2008), Collateralized Commodity Obligations uncovered, *Unpublished manuscript.*

[2] Carlstein, E., Do, K., Hall, P., Hesterberg, T., & Kunsch, H. (1998), Matched-block bootstrap for dependent data, *Bernoulli* 4(3), 305-328.

[3] Carlstein, E. (1986), The use of subseries values for estimating the variance of a general statistic from a stationary sequence. Ann. Statist., 14, 1171±1179.

[4] CDO Spotlight: Overview of Modeling Methodology for Commodity CDO Structures, Standard & Poor's (2006) available at: http://www2.standardandpoors.com/spf/pdf/fixedincome/110206_cco_quantSNAP.pdf?vregion=cn&vlang=en.

[5] Considerations for Rating Commodities-Linked Credit Obligations ("CCOs"), Fitch (2006) available at: http://www.fitchratings.com/dtp/pdf4-06/vcco1411.pdf.

[6] Default VECTOR 3.1 Model User Manual, Fitch (2004) available at:

http://www.fitchratings.com/web_content/product/vector/vector_release_notes.pdf.

[7] Efron, B. (1979), Bootstrap methods: another look at the jackknife. Ann. Statist., 7, 1-26**.**

[8] Hall, P. (1985), Resampling a coverage pattern. Stoch. Processes Applic., 20, 231±246.

[9] Hull, J.C. (2006), *Options, futures, and other derivatives 6th ed,* Upper Saddle River: Pearson/Prentice Hall.

[10] Kuensch, H.R. (1989), The jackknife and the bootstrap for general stationary observations. Ann. Statist., 17, 1217-1241.

[11] Li, D. (2000), On default correlation: A copula function approach, *Journal of Fixed Income* **9**, 43-54.

[12] Luscher, A. (2005), Synthetic CDO pricing using the double normal inverse Gaussian copula with stochastic factor loadings, Master thesis.

[13] McNeil, A. J., Frey, R., & P. Embrechts (2005), *Quantitative Risk Management*. Princeton: Princeton University Press.

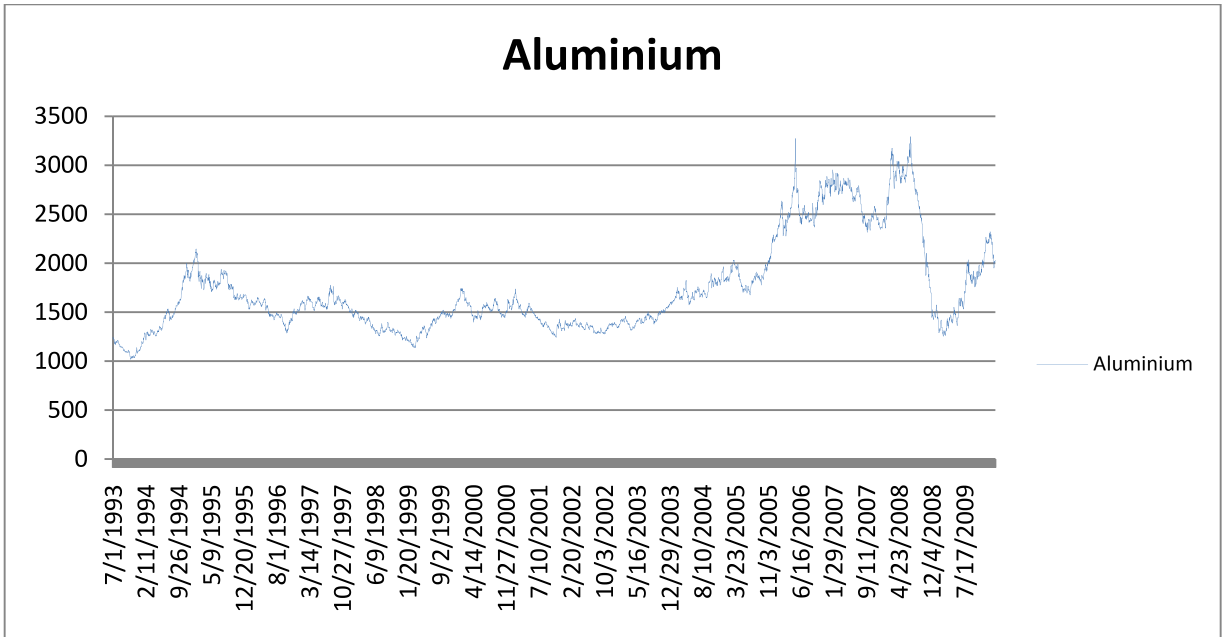
[14] Schwab, M. (2005), Collaterized Commodity Obligation 101, *bFinance*

available at: *http://www.bfinance.co.uk/article.do?docid=N12052&topId=17*.

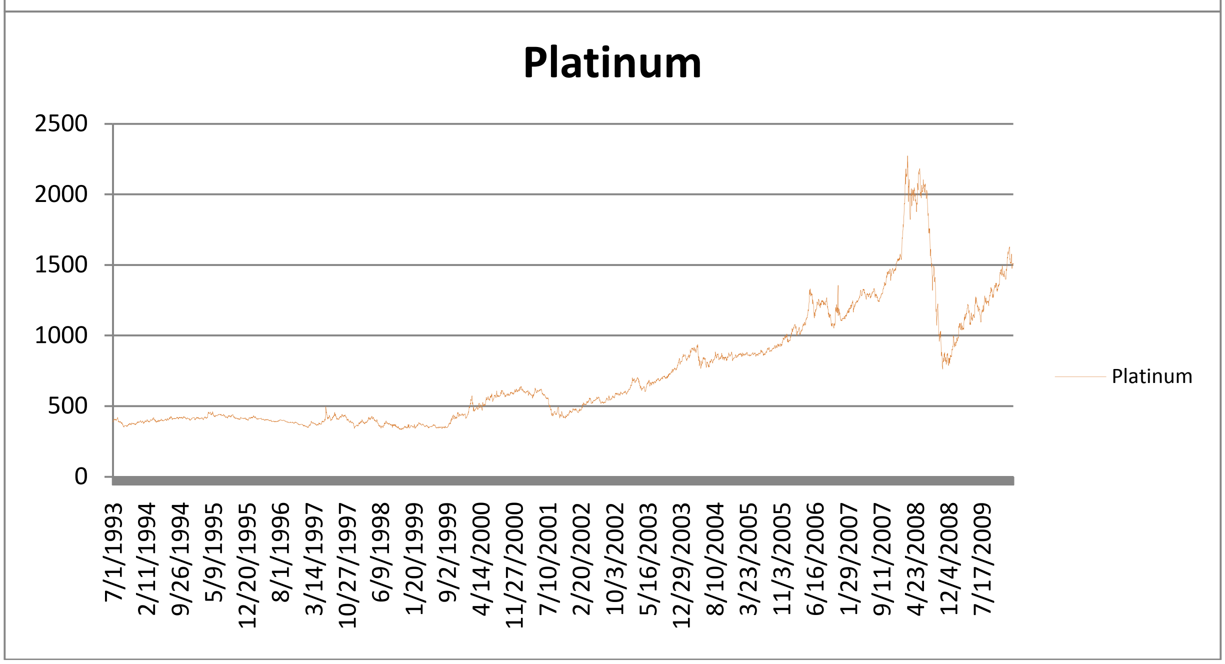
[15] Schwartz, E. (1997), The Stochastic Behavior of Commodity Prices: Implications for Valuation and Hedging, *Journal of Finance* **52**, 923-972.

[16] Smith (2002), A tutorial on Principal Components Analysis available at: *http://kybele.psych.cornell.edu/~edelman/Psych-465-Spring-2003/PCA-tutorial.pdf*.

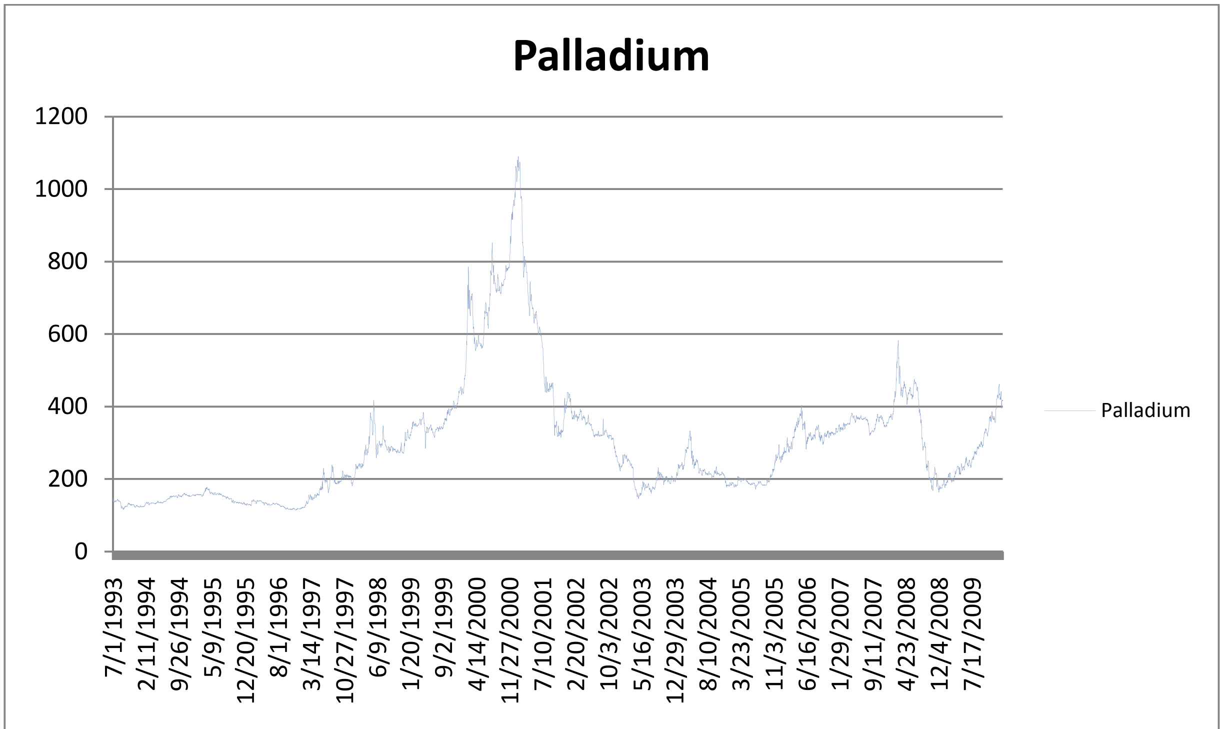
Appendix

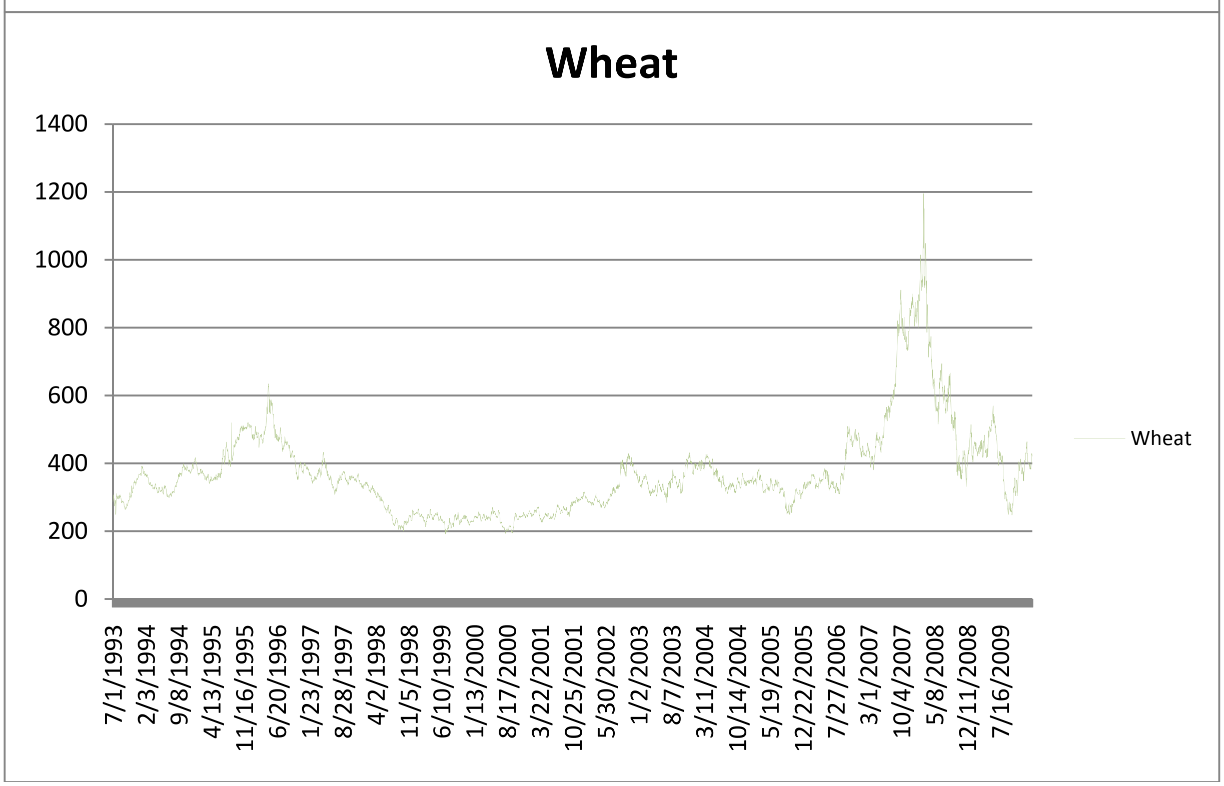


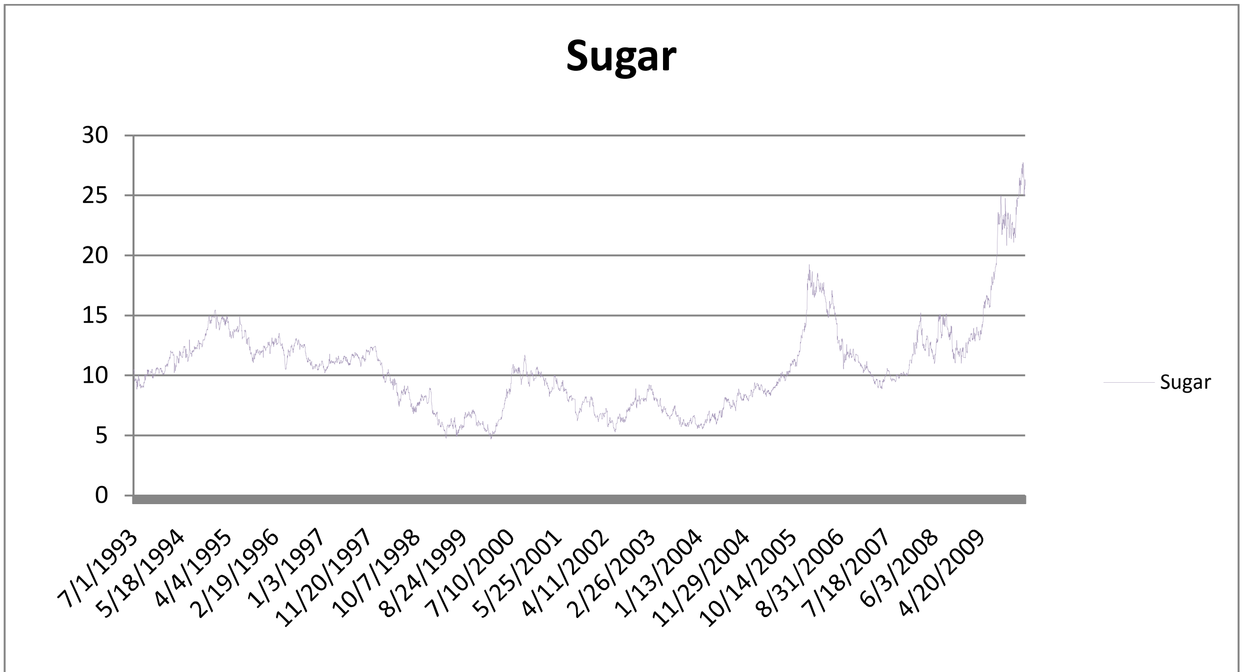
Aluminium
3500
3000
2500
2000
1500
1000
500
0
7/1/1993
2/11/1994
9/26/1994
5/9/1995
12/20/1995
8/1/1996
3/14/1997
10/27/1997
6/9/1998
1/20/1999
9/2/1999
4/14/2000
11/27/2000
7/10/2001
2/20/2002
10/3/2002
5/16/2003
12/29/2003
8/10/2004
3/23/2005
11/3/2005
6/16/2006
1/29/2007
9/11/2007
4/23/2008
12/4/2008
7/17/2009
Aluminium

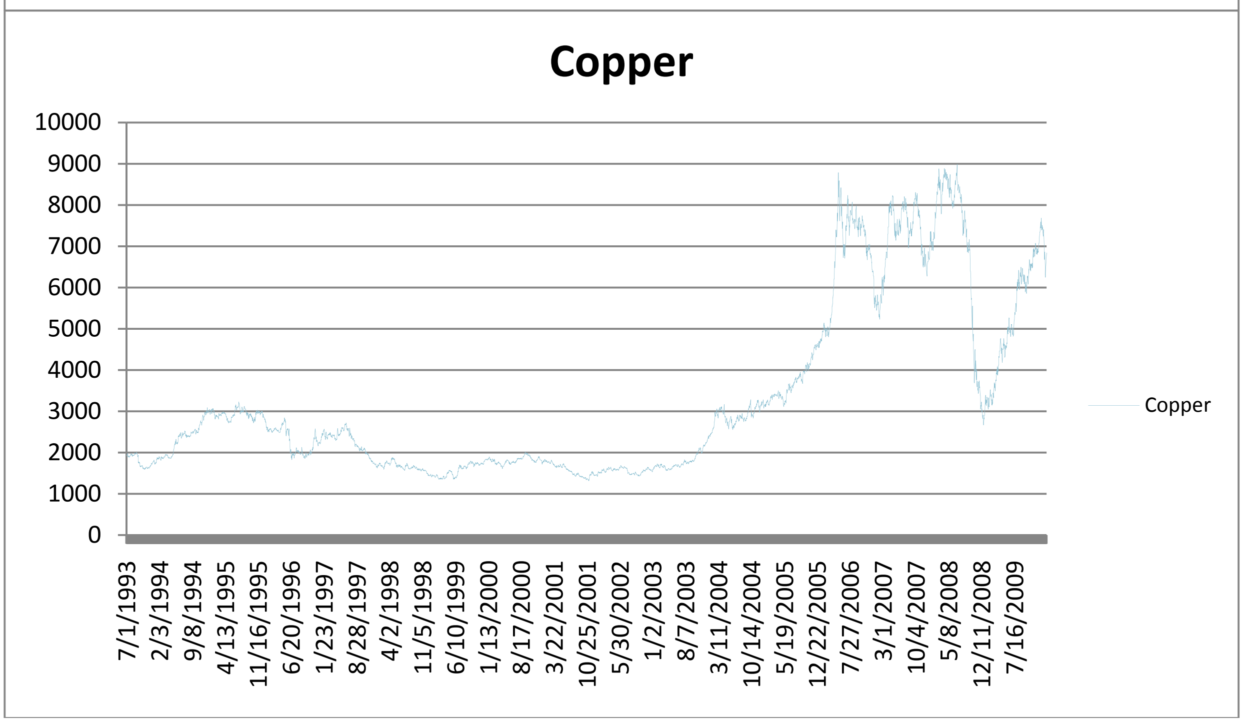


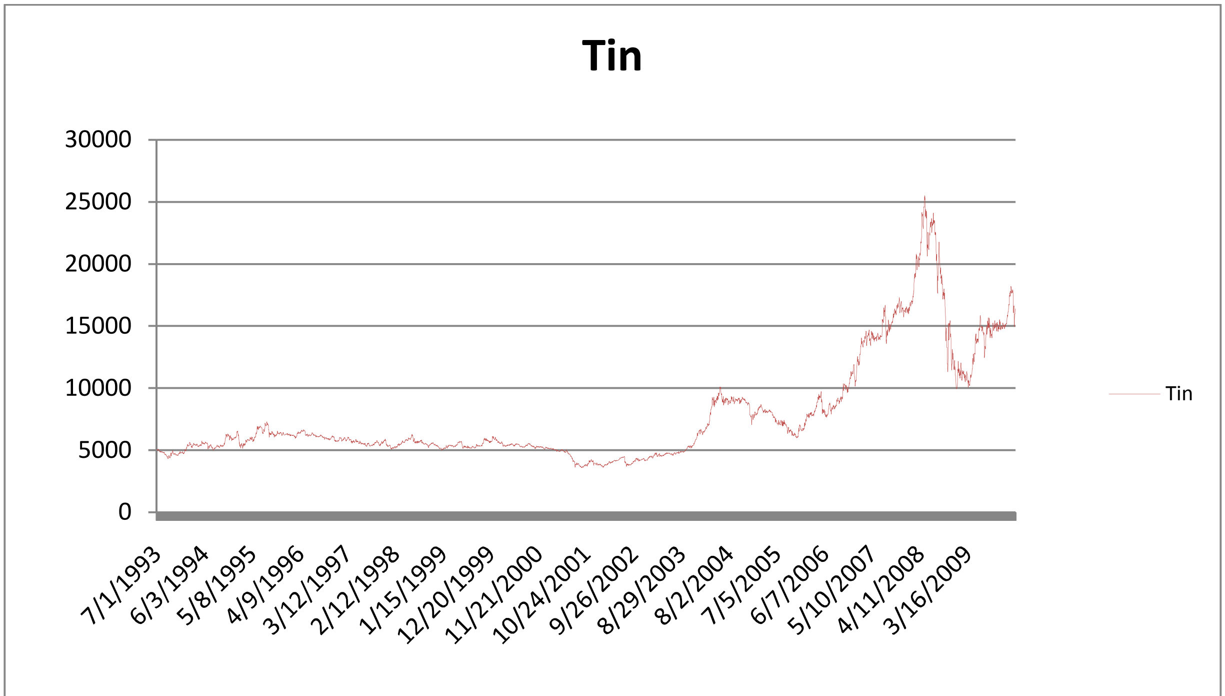
Platinum
2500
2000
1500
1000
500
0
7/1/1993
2/11/1994
9/26/1994
5/9/1995
12/20/1995
8/1/1996
3/14/1997
10/27/1997
6/9/1998
1/20/1999
9/2/1999
4/14/2000
11/27/2000
7/10/2001
2/20/2002
10/3/2002
5/16/2003
12/29/2003
8/10/2004
3/23/2005
11/3/2005
6/16/2006
1/29/2007
9/11/2007
4/23/2008
12/4/2008
7/17/2009
Platinum

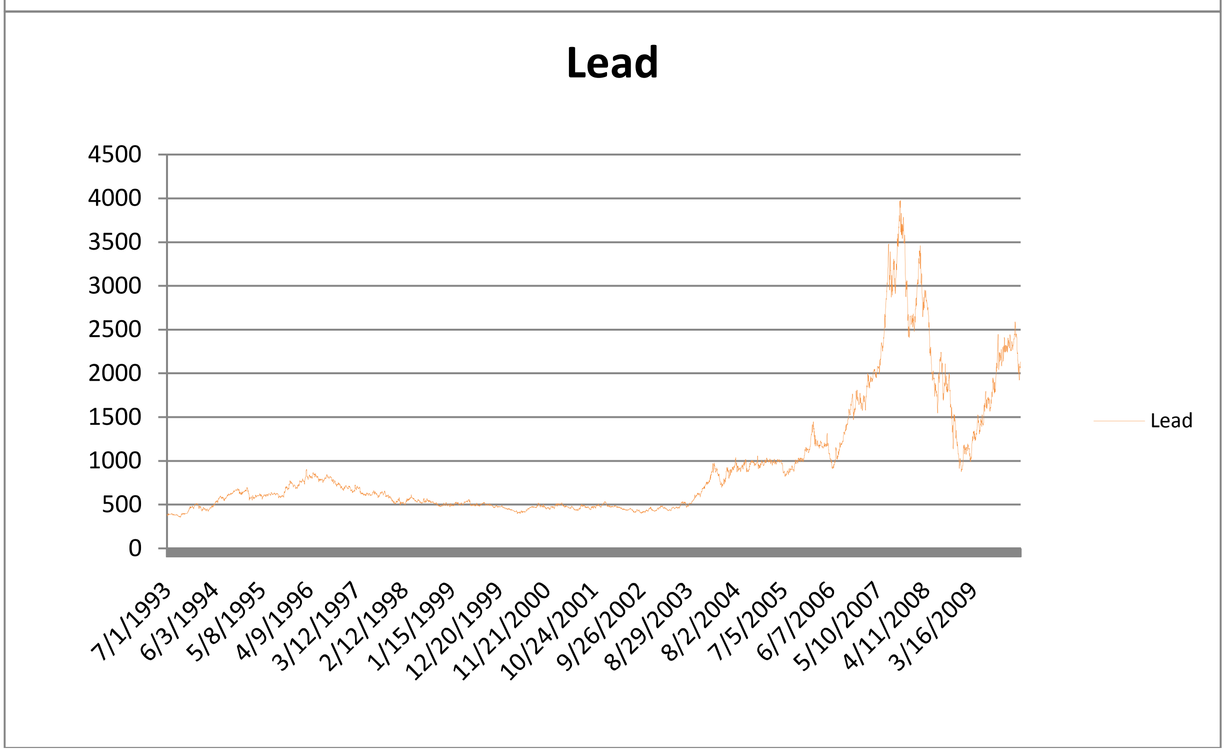

Palladium
1200
1000
800
600
400
200
0
7/1/1993
2/11/1994
9/26/1994
5/9/1995
12/20/1995
8/1/1996
3/14/1997
10/27/1997
6/9/1998
1/20/1999
9/2/1999
4/14/2000
11/27/2000
7/10/2001
2/20/2002
10/3/2002
5/16/2003
12/29/2003
8/10/2004
3/23/2005
11/3/2005
6/16/2006
1/29/2007
9/11/2007
4/23/2008
12/4/2008
7/17/2009
Palladium

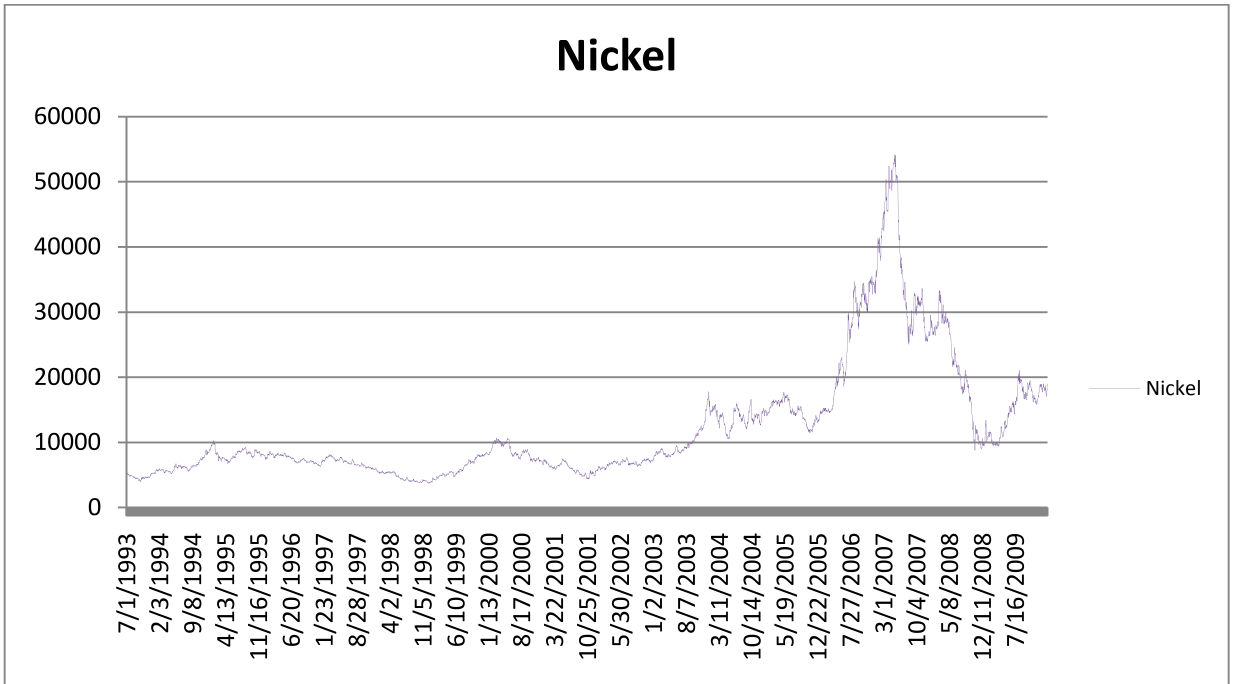

Wheat
1400
1200
1000
800
600
400
200
0
7/1/1993
2/3/1994
9/8/1994
4/13/1995
11/16/1995
6/20/1996
1/23/1997
8/28/1997
4/2/1998
11/5/1998
6/10/1999
1/13/2000
8/17/2000
3/22/2001
10/25/2001
5/30/2002
1/2/2003
8/7/2003
3/11/2004
10/14/2004
5/19/2005
12/22/2005
7/27/2006
3/1/2007
10/4/2007
5/8/2008
12/11/2008
7/16/2009
Wheat


Sugar
30
25
20
15
10
5
0
7/1/1993
5/18/1994
4/4/1995
2/19/1996
1/3/1997
11/20/1997
10/7/1998
8/24/1999
7/10/2000
5/25/2001
4/11/2002
2/26/2003
1/13/2004
11/29/2004
10/14/2005
8/31/2006
7/18/2007
6/3/2008
4/20/2009
Sugar

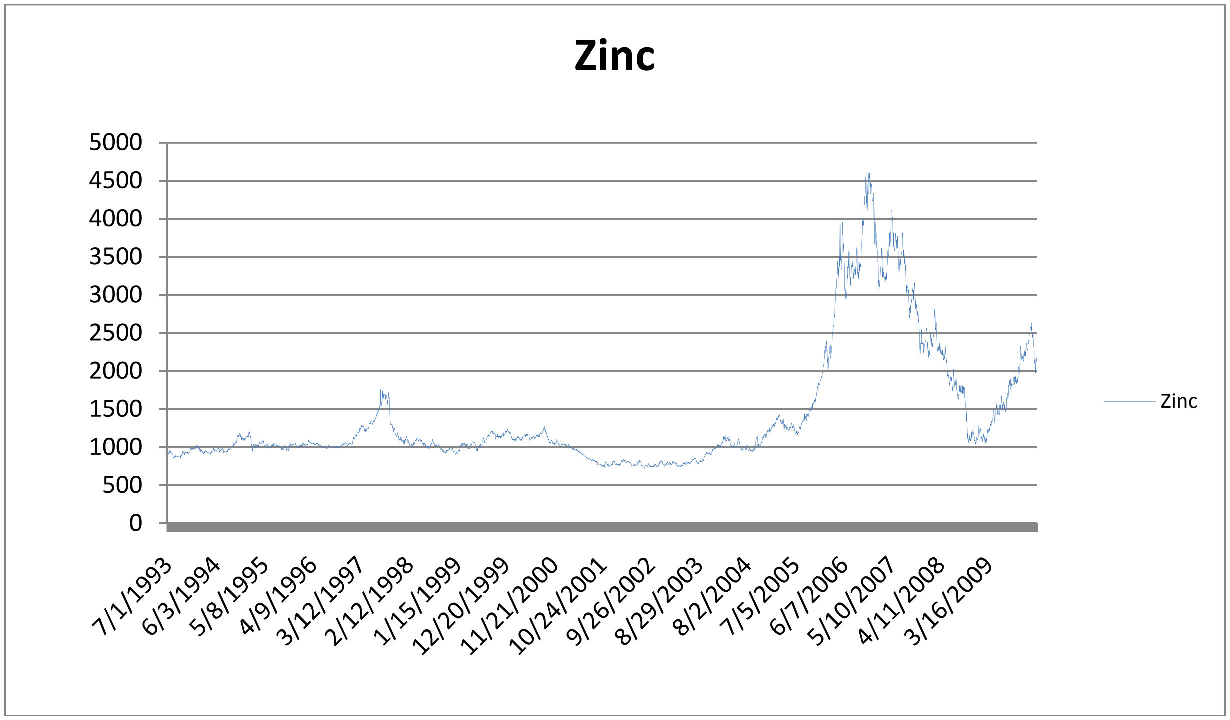

Copper
10000
9000
8000
7000
6000
5000
4000
3000
2000
1000
0
7/1/1993
2/3/1994
9/8/1994
4/13/1995
11/16/1995
6/20/1996
1/23/1997
8/28/1997
4/2/1998
11/5/1998
6/10/1999
1/13/2000
8/17/2000
3/22/2001
10/25/2001
5/30/2002
1/2/2003
8/7/2003
3/11/2004
10/14/2004
5/19/2005
12/22/2005
7/27/2006
3/1/2007
10/4/2007
5/8/2008
12/11/2008
7/16/2009
Copper


Tin
30000
25000
20000
15000
10000
5000
0
7/1/1993
6/3/1994
5/8/1995
4/9/1996
3/12/1997
2/12/1998
1/15/1999
12/20/1999
11/21/2000
10/24/2001
9/26/2002
8/29/2003
8/2/2004
7/5/2005
6/7/2006
5/10/2007
4/11/2008
3/16/2009
Tin


Lead
4500
4000
3500
3000
2500
2000
1500
1000
500
0
7/1/1993
6/3/1994
5/8/1995
4/9/1996
3/12/1997
2/12/1998
1/15/1999
12/20/1999
11/21/2000
10/24/2001
9/26/2002
8/29/2003
8/2/2004
7/5/2005
6/7/2006
5/10/2007
4/11/2008
3/16/2009
Lead


Nickel
60000
50000
40000
30000
20000
10000
0
7/1/1993
2/3/1994
9/8/1994
4/13/1995
11/16/1995
6/20/1996
1/23/1997
8/28/1997
4/2/1998
11/5/1998
6/10/1999
1/13/2000
8/17/2000
3/22/2001
10/25/2001
5/30/2002
1/2/2003
8/7/2003
3/11/2004
10/14/2004
5/19/2005
12/22/2005
7/27/2006
3/1/2007
10/4/2007
5/8/2008
12/11/2008
7/16/2009
Nickel


Silver
2500
2000
1500
1000
500
0
7/1/1993
6/3/1994
5/8/1995
4/9/1996
3/12/1997
2/12/1998
1/15/1999
12/20/1999
11/21/2000
10/24/2001
9/26/2002
8/29/2003
8/2/2004
7/5/2005
6/7/2006
5/10/2007
4/11/2008
3/16/2009
Silver


Zinc
5000
4500
4000
3500
3000
2500
2000
1500
1000
500
0
7/1/1993
6/3/1994
5/8/1995
4/9/1996
3/12/1997
2/12/1998
1/15/1999
12/20/1999
11/21/2000
10/24/2001
9/26/2002
8/29/2003
8/2/2004
7/5/2005
6/7/2006
5/10/2007
4/11/2008
3/16/2009
Zinc