# Admission planning for the IC unit at VUmc



Esther Louw BMI paper

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### Preface

This paper is written as part of the master program Business Mathematics & Informatics. The aim of the course is to give an answer to a certain problem by doing research. The course should at least cover two of the three components of Business Mathematics & Informatics.

The problem discussed in this paper is the determination of the admission planning for the IC unit at the VUmc. By determining how many patients of each type has to be admitted on every weekday we achieve an ideally occupancy of beds at the IC unit per weekday.

There are just two types of patients discussed, because they have most elective patients at the IC. The two types of patients discussed are Cardio Surgery (CCH) and General Surgery (HLK).

I would like to thank my supervisor dr. René Bekker for supporting me during the whole process of research and writing this paper.

### Abstract

In this paper we consider the admission planning of patients for the intensive care (IC) unit at VUmc. The bed occupancy at the IC depends on different resources, such as operating theatre (OT) time, IC beds and nursing staff. In this paper we assume there is infinite capacity of OT time, since we focus on the admissions by considering the number of occupied beds, the load. We are interested in a specific load for each weekday. We consider two types of patients, Cardio Surgery (CCH) and General Surgery (HLK). These two categories have most elective patients at the IC unit.

The length of stay (LOS) at the IC varies between one hour and 12 weeks. This makes it necessary to divide each specialism in different categories; patients admitted for just one day are more predictable than patients staying more than 3 days. The CCH patients are divided into 4 categories:

- 1. patients admitted for one day (short);
- 2. patients admitted for 2 or 3 days (middle);
- 3. patients admitted for more than 3 days (long);
- 4. emergency patients.

The HLK patients are divided into 3 categories:

- 1. patients admitted for one day (short);
- 2. patients admitted for more than one day (long);
- 3. emergency patients.

In 2008 the load of the CCH and HLK patients seems to be relatively stable already. But the admissions and load per weekday vary. By stabilizing the admissions per weekday the load will be more stabilized as well.

By using an optimization model for planning the admissions we want to achieve:

- no elective admissions during the weekend;
- a fixed load during the week and weekend;
- closing two CCH beds during the weekend.

The reason for closing two beds during the weekend is the availability of beds for elective CCH patients on Monday; more elective CCH patients can be admitted when the scheduled surgeries are resumed at the beginning of the week.

The objective function for each specialism is minimizing the sum of the squared deviations between the target utilization and the realized utilization, which can be modeled as the Quadratic Programming model. By taking the square of the differences between target and realized load, the model gives larger weight to larger deviations.

The results show that the load can be fixed during the week and weekend, with a small difference during the weekend for CCH patients; there are no admissions during the weekend, which prevents achieving the exact target load. Because the sum of the squared deviations is minimized, other weekdays (Friday and Saturday) have a slightly higher load.

The final conclusion is that most patients have to be admitted on Monday and Friday. During the weekend no admissions are planned, which makes the weekends inflexible. The load during the weekend has to be achieved by admitting patients, staying longer than one day, on Friday, which is the reason having more patients admitted on Friday for each specialism. On Monday two additional beds for CCH patients are available, which is the reason for admitting more CCH Patients on Monday. The reason for HLK patients having more patients admitted on Monday is due to the fact that patients admitted on Friday, to achieve the target load during weekend, are discharged on Monday.

## Content

Abstract	Preface
Content.71. Introduction.82. Problem formulation.103. Data analysis.113.1 Admissions per weekday.113.2 Length of stay (LOS).143.3 Bed occupancy.164. Mathematical models.184.1 Quadratic Programming model (QP model).194.2 Linear Programming model (LP model).205. Results.215.1.1 The scenarios.215.1.2 The results.225.2 Results for VUmc.245.2.1.1 Results from Quadratic Programming model.245.2.1.2 Results from Quadratic Programming model.265.2.2 HLK patients.275.2.2.1 Results from Quadratic Programming model.265.2.2 HLK patients.275.2.2 HLK patients <td< td=""><td>Abstract5</td></td<>	Abstract5
1. Introduction       8         2. Problem formulation       10         3. Data analysis       11         3.1 Admissions per weekday.       11         3.2 Length of stay (LOS)       14         3.3 Bed occupancy.       16         4. Mathematical models.       18         4.1 Quadratic Programming model (QP model)       19         4.2 Linear Programming model (LP model)       20         5. Results.       21         5.1.1 The scenarios       21         5.1.2 The results.       21         5.1.2 The results.       22         5.2 Results for VUmc       24         5.2.1.1 Results from Quadratic Programming model       25         5.2.1.2 Results from Quadratic Programming model       26         5.2.2.1 Results from Quadratic Programming model       28         5.2.2.1 Results from Quadratic Programming model	Content
2. Problem formulation       10         3. Data analysis       11         3.1 Admissions per weekday       11         3.2 Length of stay (LOS)       14         3.3 Bed occupancy       16         4. Mathematical models       18         4.1 Quadratic Programming model (QP model)       19         4.2 Linear Programming model (LP model)       20         5. Results       21         5.1 General       21         5.1.1 The scenarios       21         5.1.2 The results       22         5.2 Results for VUmc       24         5.2.1 CCH patients       24         5.2.1.1 Results from Quadratic Programming model       25         5.2.1.2 Results from Linear Programming model       26         5.2.2.1 Results from Quadratic Programming model       26         5.2.2.1 Results from Quadratic Programming model       26         5.2.2.1 Results from Linear Programming model       26         5.2.2.1 Results from Quadratic Programming model       27         5.2.2.1 Results from Quadratic Programming model       28         5.2.2.1 Results from Quadratic Programming model       28         5.2.2.1 Results from Quadratic Programming model       28         5.2.2.2 New Mode       14 <td>1. Introduction</td>	1. Introduction
3. Data analysis       11         3.1 Admissions per weekday.       11         3.2 Length of stay (LOS)       14         3.3 Bed occupancy.       16         4. Mathematical models       18         4.1 Quadratic Programming model (QP model)       19         4.2 Linear Programming model (LP model)       20         5. Results.       21         5.1 General       21         5.1.1 The scenarios       21         5.1.2 The results       22         5.2 Results for VUmc       24         5.2.1.1 Results from Quadratic Programming model       25         5.2.2.1.2 Results from Linear Programming model       26         5.2.2.1 Results from Quadratic Programming model       26         5.2.2.1 Results from Linear Programming model       26         5.2.2.1 Results from Quadratic Programming model       26         5.2.2.1 Results from Quadratic Programming model       28         5.2.2.1 Results from Quadratic Programming model       28         5.2.2.2 Results from Quadratic Programming model       28         5.2.2.1 Results	2. Problem formulation
3.1 Admissions per weekday.113.2 Length of stay (LOS)143.3 Bed occupancy.164. Mathematical models184.1 Quadratic Programming model (QP model)194.2 Linear Programming model (LP model)205. Results215.1 General215.1.1 The scenarios215.1.2 The results225.2 Results for VUmc245.2.1.1 Results from Quadratic Programming model255.2.2.1 Results from Quadratic Programming model265.2.2.1 Results from Quadratic Programming model265.2.2.1 Results from Quadratic Programming model275.2.2.1 Results from Quadratic Programming model285.2.2.1 Results from Quadratic Programming model285.2.2.2 Results from Quadratic Programming model285.2.2.3 Results from Quadratic Programming model285.3.3 Results from Quadratic Programming model285.3.3 Results from Quadratic Programming model285.3.3 Results from Quadratic Programming model285.3.4 Results from Quadratic Programming model285.3.5 Results from Quadratic Programming model285.3.5 Results from Quadratic Programming model385.3.5 Results from Quadratic Programming model385.3.5 Results from Quadratic Programming model315.3.5 Results f	3. Data analysis11
3.2 Length of stay (LOS)143.3 Bed occupancy.164. Mathematical models184.1 Quadratic Programming model (QP model)194.2 Linear Programming model (LP model)205. Results215.1 General215.1.1 The scenarios215.1.2 The results.225.2 Results for VUmc245.2.1 CCH patients245.2.1.1 Results from Quadratic Programming model255.2.2 HLK patients275.2.2.1 Results from Quadratic Programming model265.2.2 Results from Linear Programming model275.2.2 Results from Quadratic Programming model28	3.1 Admissions per weekday11
3.3 Bed occupancy.164. Mathematical models184.1 Quadratic Programming model (QP model)194.2 Linear Programming model (LP model)205. Results.215.1 General215.1.1 The scenarios215.1.2 The results.225.2 Results for VUmc245.2.1 CCH patients245.2.1.1 Results from Quadratic Programming model255.2.2 HLK patients275.2.2 HLK patients275.2.2.1 Results from Quadratic Programming model285.2.2 Dependence24	3.2 Length of stay (LOS)14
4. Mathematical models184.1 Quadratic Programming model (QP model)194.2 Linear Programming model (LP model)205. Results215.1 General215.1.1 The scenarios215.1.2 The results225.2 Results for VUmc245.2.1 CCH patients245.2.1.1 Results from Quadratic Programming model255.2.1.2 Results from Linear Programming model265.2.2 HLK patients275.2.2.1 Results from Quadratic Programming model285.2.2.1 Results from Quadratic Programming model285.2.2.2 Results from Quadratic Programming model285.2.2.3 Results from Quadratic Programming model285.2.2.4 Results from Quadratic Programming model285.2.2.5 Results from Quadratic Programming model285.2.2.1 Results from Quadratic Programming model285.2.2.2 Results from Quadratic Programming model285.2.2.3 Results from Quadratic Programming model285.2.2.4 Results from Quadratic Programming model285.2.2 Results from Quadratic Programming model285.3.3 Results from Quadratic Programming model315.3 Results from Quadratic Programming model325.3 Results from Quadratic Programming model31 <td< td=""><td>3.3 Bed occupancy</td></td<>	3.3 Bed occupancy
4.1 Quadratic Programming model (QP model)194.2 Linear Programming model (LP model)205. Results215.1 General215.1.1 The scenarios215.1.2 The results225.2 Results for VUmc245.2.1 CCH patients245.2.1.1 Results from Quadratic Programming model255.2.1.2 Results from Linear Programming model265.2.2 HLK patients275.2.2.1 Results from Quadratic Programming model28	4. Mathematical models
4.2 Linear Programming model (LP model)205. Results215.1 General215.1.1 The scenarios215.1.2 The results225.2 Results for VUmc245.2.1 CCH patients245.2.1.1 Results from Quadratic Programming model255.2.1.2 Results from Linear Programming model265.2.2 HLK patients275.2.2 Ne sults from Quadratic Programming model28	4.1 Quadratic Programming model (QP model)19
5. Results.215.1 General215.1.1 The scenarios215.1.2 The results.225.2 Results for VUmc245.2.1 CCH patients.245.2.1.1 Results from Quadratic Programming model255.2.1.2 Results from Linear Programming model265.2.2 HLK patients.275.2.2.1 Results from Quadratic Programming model28	4.2 Linear Programming model (LP model)
5.1 General215.1.1 The scenarios215.1.2 The results225.2 Results for VUmc245.2.1 CCH patients245.2.1.1 Results from Quadratic Programming model255.2.1.2 Results from Linear Programming model265.2.2 HLK patients275.2.2 I Results from Quadratic Programming model285.2.2 New We for Wind Control28	5. Results
5.1.1 The scenarios215.1.2 The results225.2 Results for VUmc245.2.1 CCH patients245.2.1.1 Results from Quadratic Programming model255.2.1.2 Results from Linear Programming model265.2.2 HLK patients275.2.2.1 Results from Quadratic Programming model285.2.2 NLK patients28	5.1 General
5.1.2 The results.225.2 Results for VUmc245.2.1 CCH patients.245.2.1.1 Results from Quadratic Programming model255.2.1.2 Results from Linear Programming model265.2.2 HLK patients.275.2.2.1 Results from Quadratic Programming model285.2.2 New Its from Quadratic Programming model28	5.1.1 The scenarios
5.2 Results for VUmc       24         5.2.1 CCH patients       24         5.2.1.1 Results from Quadratic Programming model       25         5.2.1.2 Results from Linear Programming model       26         5.2.2 HLK patients       27         5.2.2.1 Results from Quadratic Programming model       28         5.2.2 New last from Quadratic Programming model       28	5.1.2 The results
5.2.1 CCH patients       24         5.2.1.1 Results from Quadratic Programming model       25         5.2.1.2 Results from Linear Programming model       26         5.2.2 HLK patients       27         5.2.2.1 Results from Quadratic Programming model       28         5.2.2 New list from Quadratic Programming model       28	5.2 Results for VUmc
5.2.1.1 Results from Quadratic Programming model       25         5.2.1.2 Results from Linear Programming model       26         5.2.2 HLK patients       27         5.2.2.1 Results from Quadratic Programming model       28         5.2.2 New Work       28         5.2.2 New Work       29	5.2.1 CCH patients
5.2.1.2 Results from Linear Programming model       26         5.2.2 HLK patients       27         5.2.2.1 Results from Quadratic Programming model       28         5.2.2 Decision       20	5.2.1.1 Results from Quadratic Programming model
5.2.2 HLK patients	5.2.1.2 Results from Linear Programming model
5.2.2.1 Results from Quadratic Programming model	5.2.2 HLK patients
	5.2.2.1 Results from Quadratic Programming model
5.2.2.2 Results from Linear Programming model	5.2.2.2 Results from Linear Programming model
6. Conclusions and recommendations	6. Conclusions and recommendations
References	References

### **1. Introduction**

There is an increasing interest for research on optimizing resource usage in hospitals, like operating theatre planning and bed planning. This is especially true for the Intensive Care unit, due to the awareness that the number of patients suffering serious malfunctions of one of the organs are still increasing. A second issue is competition between the hospitals. Patients nowadays do not accept long waiting times. Also service quality aspects in health care delivery play an important role in patient satisfaction. And the last but not least are the costs; the operating theatres and the intensive care are cost-drivers, which make them often real bottlenecks.

The IC unit is an important department at the VU Medical Centre (VUmc) in Amsterdam. Most of the patients at the IC unit have undergone a heavy surgery and need special (intensive) care 24 hours per day. Most patients at the IC unit are suffering malfunctions of the heart. Next to these patients, there is another specialism which is quite common at the IC unit, General Surgery.

The patients at the IC unit can be distinguished in elective patients and emergency patients. We mainly consider elective patients, because we do not have any influence on emergency patients. The flow of intensive care patients is shown in Figure 1.



Figure 1. Flow of intensive care patients

In this paper we focus on the admission planning for the Intensive Care (IC) unit at the VUmc. We determine the mix of patients that ideally needs to be admitted at each day of the week within a cyclic planning period of one week to optimize the use of beds at the IC unit, taking into account some restrictions in planning combinations of patients and the availability of beds.

Previous work of Vissers, Adan and Bekkers [ref 5] involved modeling admission planning at tactical level with multiple resources and constraints, using data from general surgery and cardiothoracic surgery. The model is based on a deterministic length of stay corresponding to an estimate for the average. By taking into account multiple resources the results probably lead to a more efficient use of the available resources and an increase in the number of treated patients.

Another study of Adan, Bekkers, Dellaert, Vissers and Yu [ref 1] involves the investigation of the impact of stochastic length of stay for IC and MC units on the performance of the admission planning at tactical level. They also considered the impact on the proposed admission profile for the planning period generated by the model. They used the same setting as mentioned in the article above for comparison of the difference in results.

Another consideration they made is showing the effect on performance for IC and MC units by taking an infinite OT capacity. They found that further improvement is limited.

This paper is organized as follows: in Chapter 2, the problem is described in more detail and the approach to optimize the admission planning is introduced. Chapter 3 shows the data analysis of the patients admitted in 2008 at the IC unit of VUmc. The mathematical model used for optimizing the mix of patients is described in Chapter 4. Chapter 5 shows the results of the optimal solution. Finally, we draw some conclusions in Chapter 6.

### 2. Problem formulation

Patients at the IC unit need special care 24 hours per day. That results in a higher workload (nursing hours) per bed than at other wards; having one more bed occupied at the IC unit results immediately in a higher demand for nurses. During this paper the load represents the number of occupied beds and not the nursing workload.

An important issue during this paper is that no admissions take place during the weekend, but the target load still needs to be achieved at each day of the week, including the weekend. This issue emphasizes the importance of planning the admissions correctly.

Since the length of stay differs from 1 hour to 12 weeks and a patient admitted for just one day is more predictable than a patient admitted for more than 3 days, it is important to categorize the patients per specialism into 3 or 4 different categories:

- 1. patients admitted for one day;
- 2. patients admitted for 2 or 3 days (extra category for CCH patients);
- 3. patients admitted for more days;
- 4. emergency patients.

Category 4 consists of emergency patients. We cannot plan emergency patients; we do not know when and how many will arrive. HLK has more emergency patients than CCH. The length of stay (LOS) per category depends on the specialism.

To determine the load we use the queueing model  $M/D/\infty$ . This model assumes a Poisson arrival process of patients, deterministic distributed LOS and an infinite capacity of beds. For the queueing model we use the number of arrivals,  $\lambda$ , of 2008. The admissions per category are determined by using the admission data of 2008.

The number of elective admissions depends on the available beds. Normally the number of elective admissions also depends on the capacity of the operating theatre, but in this case we assume an infinite capacity of operating theatre hours to be able to focus completely on the bed capacity at the IC unit.

### 3. Data analysis

To determine the ideal mix of patients it is important to know which categories of patients are interesting to consider. A certain amount of elective patients is of course necessary, as we are not able to plan emergencies. There are 3 categories which dominate the number of admissions at the IC in 2008:

- Cardio Surgery (CCH) (39%);
- General Surgery (HLK) (19%);
- Neuro Surgery (NCH) (9%).

Figure 2 clearly shows that CCH and HLK admit most of the elective patients at the IC unit, respectively 72% and 17%. The number of NCH patients at the IC unit is reasonably large, but just a few of them are elective, only 2% of all elective patients at the IC. This makes it very difficult to optimize the admissions per weekday for NCH patients. During this paper we focus on CCH and HLK.



Figure 2. Distribution of patients at IC unit 2008

### 3.1 Admissions per weekday

For the admission planning, we plan the number of elective patients per day of the week. In this section we consider the current situation for admissions per weekday.

Figure 3 shows the average and the standard deviation of the number of admissions per weekday at the IC.

The standard deviation per weekday is determined by:

$$\sqrt{\frac{\displaystyle\sum_{i=1}^{n} \left(x_{i}-\overline{x}\right)^{2}}{n-1}}$$
, with

- n equal to 52 weeks;

- $x_i$  the number of admissions on a specific day of the week in week i;
- and  $\overline{x}$  the average number of admissions on a specific day of the week.

The elective admissions are roughly equally spread from Monday until Friday. But the standard deviation shows that the number of arrivals per weekday varies. There are hardly any elective admissions during the weekend, which we also expect; no operations are planned during the weekend.



Figure 3. Elective admissions per weekday at IC in 2008

Figures 4 and 5 show the distributions of admissions for CCH and HLK. The pattern for CCH, shown in Figure 4, is similar to the total number of admissions at the IC (Figure 3). This can be due to the fact that 72% of the elective patients at the IC are CCH patients.



Figure 4. Elective admissions per weekday for CCH in 2008

The admissions of HLK patients have a different pattern. The average number of admissions on Tuesday and Thursday are higher than on other weekdays. In this picture it is very difficult to recognize any pattern. This is also caused by the fact that the number of elective admissions for HLK (118 patients) is much lower than for CCH (497 patients).



Figure 5. Elective admissions per weekday for HLK in 2008

Figure 6 shows the number of elective admissions per week for CCH and HLK in 2008. The number of admissions per week for each specialism shows a large variation.



Figure 6. The number of elective admissions per week for CCH and HLK in 2008

### 3.2 Length of stay (LOS)

The length of stay at the IC varies between 1 hour and 12 weeks. This makes it necessary to work with subcategories for each specialism; patients admitted for just one day are more predictable than patients staying more than 3 days. Figure 7 shows the LOS for all CCH patients at the IC unit in 2008. The first two categories are respectively 1 day and, with certain probabilities, 2 or 3 days. Most patients left the IC within 4 days. When a patient stays longer than 3 days, it is very difficult to know in advance how long the patient will stay. It can be either 2 weeks or 6 weeks. To approximate the LOS of the third category, we take the ALOS of all patients staying longer than 72 hours at the IC unit, which is 13 days.



Figure 7. LOS of CCH patients at the IC unit in 2008

In Figure 8, the LOS for all HLK patients is shown. Figure 2 clearly shows that most HLK patients at the IC are emergencies (174 patients). There are just 118 elective patients. The small number of elective patients makes it more difficult to get a good estimation of the ALOS of the elective patients. Another issue is that HLK patients can be all kinds of patients, in contrast with CCH. In Figure 8 you can see that most elective HLK patients stay no longer than 24 hours at the IC. But when the patient stays longer than 1 day, it is again not predictable how long the patient will stay. For HLK we only have two categories, because the LOS of all patients staying more than one day is quite spread. The ALOS of the second category is provided by the ALOS of all patients staying longer than 24 hours, which is 10 days.



Figure 8. LOS of HLK patients at the IC unit in 2008

For each specialism we estimate the LOS of emergency patients by taking the average. The values we use in the optimization model for the different categories are shown in Table 1.

	length of stay (LOS) in days						
patient category	short	middle	long	emergency			
ССН	1	2 or 3	13	5			
HLK	1	-	10	7			

Table 1. LOS per category

### 3.3 Bed occupancy

As we have analyzed the number of admissions per weekday and the LOS, we can determine the number of beds occupied, which is called the load. Tables 2 and 3 show the load per weekday per specialism. For the loads in Tables 2 and 3 we use the deterministic LOS per category as mentioned in Table 1.

ССН	emergencies	short elective	middle elective	long elective	total
Monday	1.44	1.12	0.55	2.10	5.20
Tuesday	1.31	1.27	1.04	2.06	5.67
Wednesday	1.08	1.19	1.18	1.92	5.38
Thursday	1.31	1.31	1.12	2.04	5.77
Friday	1.65	0.79	1.17	2.19	5.81
Saturday	1.50	0.04	0.74	2.19	4.47
Sunday	1.33	0.02	0.16	2.00	3.51

Load per day 2008

Table 2. Bed occupancy of CCH patients

Table 2 shows that during the weekend approximately 2 beds for CCH patients are less occupied than during the week. The reason is that more beds are available on Monday, so elective CCH patients can be admitted on Monday. The load during the week seems to be stable already, which is good. But the admissions per weekday vary, as shown by the standard deviation in Figure 4 and the realization in Figure 6.

Load per day 2008							
HLK	emergencies	short elective	long elective	total			
Monday	3.35	0.10	1.77	5.21			
Tuesday	3.35	0.27	1.81	5.42			
Wednesday	3.35	0.19	1.83	5.37			
Thursday	3.35	0.25	1.92	5.52			
Friday	3.35	0.12	1.94	5.40			
Saturday	3.35	0.02	2.00	5.37			
Sunday	3.35	0.02	1.81	5.17			

Table 3. Bed occupancy of HLK patients

Table 3 shows that for HLK the emergency patients occupy most beds. Similar to CCH, the load during the week is quite stable, but the number of admissions varies, as shown by the standard deviation in Figure 5.

### 4. Mathematical models

To optimize the number of admissions of elective patients, we translate the planning problem into a mathematical model. There are two models we use, Quadratic Programming and Linear Programming. The difference between the two models is the way of minimizing the over- and underutilization of beds.

First of all we describe how the planning problem of the admissions can be translated into a mathematical model. We introduce the following notation. T denotes the cycle length in days of the admission schedule; we assume here that T is equal to 7 days. The patients per specialism are categorized according to their length of stay. N denotes the number of categories and  $D_c$  denotes the number of days the patient of category c stays at the IC.

The number and mix of patients admitted on each day during the cycle are the decision variables in the planning problem.  $X_{cd}$  denotes the number of patients of category c admitted on day d:

$$X_{cd} \in \{0,1,2,\ldots\}, \qquad c = 1,\ldots, N, \quad d = 1,\ldots, T,$$

with the restriction that the total admissions per category during a cycle should be equal to the target patient throughput per category:

$$\sum_{d=1}^{T} X_{cd} = TPT_c, \qquad c = 1, \dots, N.$$

A second constraint is added for the fact that there are no elective admissions during the weekend; no operations are scheduled during the weekend:

$$\sum_{d=6}^{7} X_{cd} = 0, \qquad c = 1, ..., N.$$

We let d = 1 correspond to Monday.

We want to choose the values of  $X_{cd}$  such that the differences between the target utilization on day d " $\rho_d$ " and the realized utilization on day d " $x_d$ " is minimized. These differences are divided into two variables:

 $Z_d^+$  Overutilization of beds on day d;

 $Z_d$  Underutilization of beds on day d,

that is,  $Z_d^+ = \max(x_d - \rho_d, 0)$  and  $Z_d^- = \max(\rho_d - x_d, 0)$ .

The realized utilization can never exceed the available capacity of beds on day d,  $C_d$ . Thus, for the LP model we consider

$$\rho_d + Z_d^+ \le C_d, \qquad d = 1, \dots, T.$$

The number of patients at the IC unit on day d are the patients admitted on day d, d - 1, ..., d  $-D_c + 1$ . So the total number of patients on day d is equal to

$$\sum_{c=1}^{N} \sum_{t=1}^{D_c} X_{c,d-t+1}, \qquad d = 1,..., T.$$

For the LP model we have the following restriction:

$$\rho_d - Z_d^- \le \sum_{c=1}^N \sum_{t=1}^{D_c} X_{c,d-t+1} \le \rho_d + Z_d^+, \qquad d = 1, ..., T.$$

For CCH, we slightly extend the model for the patients of the category with an ALOS of 2 or 3 days: with a probability of 20% they admit for 3 days and with a probability of 80% they admit for 2 days.

Input parameters	Description	Variables	Description
Т	Cycle length (days)	X <sub>c,d</sub>	Number of patients
	= 7		of category c
			admitted on day d
TPT <sub>c</sub>	Target patient	$Z_d^+$	Overutilization of
	throughput of		beds on day d
	category c		
C <sub>d</sub>	Available capacity of	$Z_{d}$	Underutilization of
	beds at IC unit on		beds on day d
	day d		
$\rho_{d}$	Target utilization of	$x_d$	Load of patients on
	beds at IC unit on	u	day d
	day d		
D <sub>c</sub>	Length of stay at the		
	IC unit of a patient		
	of category c		

Both programming models use the parameters and variables mentioned in Table 4.

Table 4. Parameters and variables used in QP and LP model

#### 4.1 Quadratic Programming model (QP model)

We formulate the QP model as follows:

$$\min\sum_{d=1}^7 (\rho_d - x_d)^2,$$

such that

$$\begin{aligned} x_{d} &= \sum_{c=1}^{N} \sum_{d=1}^{7} X_{c,d-t+1}, \qquad d = 1,....,7, \\ \sum_{d=1}^{7} X_{c,d} &= TPT_{c}, \qquad c = 1,....,N, \end{aligned}$$

$$\sum_{d=6}^{7} X_{c,d} = 0, \qquad c = 1,...,N,$$
$$X_{c,d} \ge 0, \qquad c = 1,...,N, \qquad d = 1,...,7.$$

### 4.2 Linear Programming model (LP model)

We formulate the LP model as follows:

$$\min\sum_{d=1}^{7} \left( Z_d^- + Z_d^+ \right),$$

such that

$$\sum_{d=1}^{7} X_{c,d} = TPT_{c}, \qquad c = 1,...,N,$$

$$\rho_{d} + Z_{d}^{+} \le C_{d}, \qquad d = 1,...,7,$$

$$\rho_{d} - Z_{d}^{-} \le \sum_{c=1}^{N} \sum_{t=1}^{D_{c}} X_{c,d-t+1} \le \rho_{d} + Z_{d}^{+}, \qquad d = 1,...,7,$$

$$\sum_{d=6}^{7} X_{c,d} = 0, \qquad c = 1,...,N,$$

$$X_{c,d} \ge 0, \qquad Z_{d}^{-}, Z_{d}^{+} \ge 0, \qquad c = 1,...,N, \qquad d = 1,...,7.$$

We solve the optimization problems using the solver of Excel. In the next chapter we show the results.

### 5. Results

First we show some results for an IC unit in general. The second paragraph gives specific results for the IC at the VUmc. The numbers of admissions per week in the first paragraph are larger than in the second paragraph, because we consider all patients admitted at the IC instead of each specialism separate, which is the case at the second paragraph. On average 24 beds are occupied per week at the IC unit at VUmc in 2008. For the general optimization we take this into consideration.

### 5.1 General

To get an insight of the effects of the LOS, mix of patients and the number of treated patients per week, we show outcomes of different situations. As written above we work with a situation where 24 beds are occupied. During the weekend only 22 beds are occupied. In each situation we deal with three categories of patients:

- patients admitted for one day (category 1 (short));
- patients admitted for more than one day (category 2 (long));
- emergency patients (category 3).

The situations we are interested in are:

- the average admissions and load in 2008;
- change in patient mix (scenario 2);
- production increase (scenario 3);
- increase in LOS (scenario 4).

We optimize with the Quadratic Programming model. The different scenarios are shown in Table 5.

### 5.1.1 The scenarios

As mentioned before we are interested in the outcomes of some different situations. We consider the following scenarios:

	admitted patients per week			ALOS in days		
	emergency	short	long	emergency	short	long
scenario 1	16	7	5	7	1	9
scenario 2	16	16	4	7	1	9
scenario 3	16	14	5	7	1	9
scenario 4	16	7	5	7	1	16

Table 5. different scenarios

The numbers of the first scenario are based on the average admissions and LOS of patients at the IC unit in 2008. In the second scenario we reduce the number of patients from category 2 (patients admitted for more than one day) by one. Obviously you can admit 9 additional

patients of the first category as the ALOS is 9 days for patients from category 2. In scenario 3 we show the effect on the number of admitted patients by extending the load by one per day. In total 7 additional patients per week can be admitted. Per year this makes a difference of 365 patients. However, this only concerns 'easy' patients with a LOS of at most 24 hours. For the last scenario we choose to change the LOS of the patients from category 2. The LOS of patients at the IC is between one hour and 12 weeks, so 16 days seems reasonable. Then, of course, the number of occupied beds per day increases by 5 (every admitted patient of this category stays one week extra). This can have an enormous effect on the nursing workload, especially at the IC where patients need special care 24 hours per day.

### 5.1.2 The results

Table 6 shows the number of admissions and load per day for the first scenario. The patients admitted from category 2 (admitted for more than one day) are spread over the 5 weekdays, with the admissions on Tuesday and Friday a bit larger. The reason for having more patients admitted on Friday is because that is the only possibility having an extra patient during the weekend, as the LOS of patients from category 2 is 9 days. Patients from category 1 (admitted for one day) just stay for one day, so no patients from category 1 will be at the IC during the weekend. On Saturday only patients from category 3 (emergency patients) and from category 2 (admitted during last week and the Friday the week before) are at the IC unit. That extra patient is discharged on Sunday, which is the reason for having a lower load on Sunday than on Saturday. This brings us straight to the reason why Monday has the highest amount of admissions from category 1: more beds are available after the weekend.

	Admi	tted patients p		La	oad per day			
Scenario 1	emergencies	short elective	long elective	total	emergencies	short elective	long elective	total
Monday	2.29	2.36	0.81	5.45	16.00	2.36	5.81	24.17
Tuesday	2.29	1.08	1.28	4.65	16.00	1.08	7.09	24.17
W ednesday	2.29	1.11	0.77	4.17	16.00	1.11	7.05	24.17
Thursday	2.29	1.42	0.97	4.68	16.00	1.42	6.74	24.17
Friday	2.29	1.03	1.17	4.48	16.00	1.03	7.14	24.17
Saturday	2.29	0.00	0.00	2.29	16.00	0.00	6.17	22.17
Sunday	2.29	0.00	0.00	2.29	16.00	0.00	5.00	21.00
				28.00				

Table 6. Number of admissions and load per day for scenario 1

For the second scenario the number of admissions on Friday for the second category increases, see Table 7. As we require that the load during the week is 24 and during the weekend 22, it is necessary to admit more patients from category 2 (admitted for more than one day) on Friday to increase the load during the weekend. Because patients from category 1 are only admitted for one day and patients from category 3 are unscheduled, the only category left is category 2. The number of patients admitted from category 2 decreases by one in comparison to scenario 1, which results in a lower load on Sunday. Because the goal function is minimizing the total mean squared error, the load increases equally for the rest of week.

Admitted patients per day						Lo	oad per day	
Scenario 2	emergencies	short elective	long elective	total	emergencies	short elective	long elective	total
Monday	2.29	4.24	0.10	6.62	16.00	4.24	4.10	24.33
Tuesday	2.29	3.27	0.97	6.52	16.00	3.27	5.06	24.33
Wednesday	2.29	2.80	0.57	5.65	16.00	2.80	5.54	24.33
Thursday	2.29	3.73	0.03	6.05	16.00	3.73	4.60	24.33
Friday	2.29	1.97	2.33	6.59	16.00	1.97	6.36	24.33
Saturday	2.29	0.00	0.00	2.29	16.00	0.00	6.33	22.33
Sunday	2.29	0.00	0.00	2.29	16.00	0.00	4.00	20.00
				36.00				

Table 7. Number of admissions and load per day for scenario 2

Table 8 shows the results for scenario 3, where the load is extended by one per day. The spread in the number of admissions is comparable with the second scenario. Again Friday has most admitted patients from category 2. The load needs to be increasing by one each day of the week, including the weekend. Having more admissions on Friday gives a higher load on Saturday, as explained before. Furthermore Monday has most admissions of patients from category 1, which is also the case at scenario 1 and 2.

	Admitted patients per day					Lo	oad per day	
Scenario 3	emergencies	short elective	long elective	total	emergencies	short elective	long elective	total
Monday	2.29	3.71	0.62	6.62	16.00	3.71	5.62	25.33
Tuesday	2.29	2.71	1.01	6.00	16.00	2.71	6.63	25.33
W ednesday	2.29	2.59	0.74	5.61	16.00	2.59	6.75	25.33
Thursday	2.29	3.29	0.30	5.88	16.00	3.29	6.04	25.33
Friday	2.29	1.70	2.33	6.32	16.00	1.70	7.63	25.33
Saturday	2.29	0.00	0.00	2.29	16.00	0.00	7.33	23.33
Sunday	2.29	0.00	0.00	2.29	16.00	0.00	5.00	21.00
				35.00				

Table 8. Number of admissions and load per day for scenario 3

The last scenario, where we change the LOS of patients from category 2 from 9 days to 16 days, shows fewer differences between the weekdays for the second category. Because patients from category 2 have an ALOS of 16 days, the load during the weekend is already high, so the admissions on Friday do not have to be much more than the rest of the week.

Clearly the results show that patients from category 2 are leading for determining the admission planning. The patients of the first category are more flexible, because these patients just stay for one day.

Admitted patients per day						Lo	oad per day	
Scenario 4	emergencies	short elective	long elective	total	emergencies	short elective	long elective	total
Monday	2.29	2.14	1.03	5.45	16.00	2.14	11.03	29.17
Tuesday	2.29	1.01	1.12	4.42	16.00	1.01	12.15	29.17
W ednesday	2.29	1.30	0.74	4.33	16.00	1.30	11.86	29.17
Thursday	2.29	1.49	0.94	4.71	16.00	1.49	11.68	29.17
Friday	2.29	1.06	1.17	4.51	16.00	1.06	12.11	29.17
Saturday	2.29	0.00	0.00	2.29	16.00	0.00	11.17	27.17
Sunday	2.29	0.00	0.00	2.29	16.00	0.00	10.00	26.00
				28.00				

Table 9. Number of admissions and load per day for scenario 4

### 5.2 Results for VUmc

In this part, we consider each specialism separately.

### 5.2.1 CCH patients

The CCH patients are divided into 4 categories:

- category 1: patients admitted for one day (short electives);
- category 2: patients admitted for 2 or 3 days (middle electives);
- category 3: patients admitted for more than 3 days (long electives);
- category 4: emergency patients.

The optimization program for the CCH patients consists of two parts; first we optimize the first and second category and then we separately optimize the third category.

The LOS of the first and second category is quite predictable and the number of admissions per week is more than the number of admissions from the third category, which makes the influence larger of these categories on the admission planning. Therefore we want to schedule these patients first, to guarantee the admissions of these patients. The optimization of the first and second category takes into consideration that two beds are closed during the weekend. By closing two beds during the weekend, more CCH patients can be admitted on Monday. CCH has a large amount of treatable patients per year; therefore it is important having extra beds available for elective admissions during the week.

Patients from the third category stay for a long (variable) time. The number of admissions for the third category is on average only one per week. It is not clear when the patient will be discharged, so you do not know when the bed is available again. To be able to optimize the third category, we consider also for this category the ALOS as the LOS in the optimization model (as mentioned in Table 1). We optimize to a target load of 2 beds per weekday for category 3 to be able to admit one patient per week.

We start by showing the average number of admissions and loads per week in 2008 to compare these with the optimization of the solver. Tables 10 and 11 show the values for the CCH patients. Table 11 shows the same results as Table 2. The loads are determined using the deterministic LOS mentioned in Table 1.

Cardio surgery	emergencies	short elective	middle elective	long elective	total
Monday	0.33	1.12	0.54	0.23	2.21
Tuesday	0.27	1.27	0.50	0.13	2.17
Wednesday	0.21	1.19	0.58	0.17	2.15
Thursday	0.40	1.31	0.44	0.31	2.46
Friday	0.44	0.79	0.62	0.19	2.04
Saturday	0.17	0.04	0.04	0.04	0.29
Sunday	0.10	0.02	0.00	0.04	0.15
sum	1.92	5.73	2.71	1.12	11.48

Admitted patients per day 2008

Table 10. Admissions per weekday for CCH patients in 2008

ССН	emergencies	short elective	middle elective	long elective	total		
Monday	1.44	1.12	0.55	2.10	5.20		
Tuesday	1.31	1.27	1.04	2.06	5.67		
Wednesday	1.08	1.19	1.18	1.92	5.38		
Thursday	1.31	1.31	1.12	2.04	5.77		
Friday	1.65	0.79	1.17	2.19	5.81		
Saturday	1.50	0.04	0.74	2.19	4.47		
Sunday	1.33	0.02	0.16	2.00	3.51		

Load per day 2008

Table 11. Load per weekday for CCH patients in 2008

The patients from category 2 stay with a probability of 80% for 2 days and with a probability of 20% for 3 days. The reason for not choosing an extra category is the size of each group. The groups separately are too small to be able to optimize the admission planning. Another reason is that the LOS for patients staying no longer than 3 days is predictable, after three days it is difficult to predict what the exact LOS of the patient will be.

### 5.2.1.1 Results from Quadratic Programming model

Tables 12 and 13 show the optimized distribution of admissions and loads for each category of CCH patients, using the QP model.

		- Alliva	s per udy		
ССН	emergencies	short elective	middle elective	long elective	total
Monday	0.27	1.13	1.13	0.22	2.76
Tuesday	0.27	0.78	0.36	0.22	1.63
Wednesday	0.27	0.77	0.92	0.22	2.18
Thursday	0.27	1.28	0.00	0.22	1.77
Friday	0.27	1.78	0.31	0.22	2.58
Saturday	0.27	0.00	0.00	0.00	0.27
Sunday	0.27	0.00	0.00	0.00	0.27
					11.48

Arrivak ner dav

Table 12. Optimized admission planning for CCH patients using the QP model

	Loau per uay							
ССН	emergencies	short elective	middle elective	long elective	total			
Monday	1.37	1.13	1.13	2.01	5.65			
Tuesday	1.37	0.78	1.49	2.01	5.65			
Wednesday	1.37	0.77	1.50	2.01	5.65			
Thursday	1.37	1.28	0.99	2.01	5.65			
Friday	1.37	1.78	0.49	2.23	5.87			
Saturday	1.37	0.00	0.31	2.23	3.91			
Sunday	1.37	0.00	0.06	2.01	3.44			

I and nor day

Table 13. Load per day which belongs to the admission planning of Table 12

As mentioned before, we keep the load stable for patients admitted more than 3 days at 2 beds per day. The weekend is inflexible, because no admissions take place during the weekend. So during the weekend the difference between the target utilization and the load of category 3 and 4 together can only be minimized by adding patients from category 2 on Friday. Because, with a probability of 80%, patients of category 2 are admitted for 2 days, the load on Sunday is lower than on Saturday (patients admitted on Wednesday). The difference between the current situation at the IC unit and the optimized admission planning is:

- The stability of the load;
- No elective admissions during the weekend;
- More admissions on Monday and Friday and less on Thursday.

5.2.1.2 Results from Linear Programming model

Tables 14 and 15 show the results by using the LP model for the optimization of the admission planning for CCH patients.

		Alliva	s per uay		
ССН	emergencies	short elective	middle elective	long elective	total
Monday	0.27	2.01	0.30	0.19	2.77
Tuesday	0.27	2.01	0.00	0.23	2.51
Wednesday	0.27	0.00	2.25	0.23	2.75
Thursday	0.27	0.00	0.06	0.23	0.56
Friday	0.27	1.71	0.10	0.23	2.31
Saturday	0.27	0.00	0.00	0.00	0.27
Sunday	0.27	0.00	0.00	0.00	0.27
					11.44

Arrivals per day

Table 14. Optimized admission planning for CCH patients using the LP model

Load per day								
ССН	emergencies	short elective	middle elective	long elective	total			
Monday	1.37	2.01	0.30	2.00	5.68			
Tuesday	1.37	2.01	0.30	2.00	5.68			
Wednesday	1.37	0.00	2.31	2.00	5.68			
Thursday	1.37	0.00	2.31	2.00	5.68			
Friday	1.37	1.71	0.60	2.23	5.91			
Saturday	1.37	0.00	0.11	2.23	3.71			
Sunday	1.37	0.00	0.02	2.04	3.43			

Table 15. The load per day which belongs to the admission planning of Table 14.

The difference between QP and LP is that QP gives weights to each deviation by taking the squares of the differences between the target utilization and the realized utilization. The deviation by using QP becomes larger by having bigger differences. Therefore the more the difference is spread over the week, the smaller the sum of deviations. With LP, it does not matter how the deviation is spread, the sum of over- and underutilizations is always the same, because no weights are given to the over- or underutilization per day. This makes the solution of QP more robust than LP. LP gives more optimal solutions with the same value for the minimized deviation, which makes it difficult for knowing which answer is the desired one. Therefore, we do not add any comments to the outcomes mentioned in Tables 14 and 15.

### 5.2.2 HLK patients

The HLK patients are divided into 3 categories:

- Category 1: patients admitted for one day (short electives);
- Category 2: patients admitted for more than one day (long electives);
- Category 3: emergency patients.

The optimization model for HLK patients uses one optimization step.

In 2008 16 HLK patients were admitted during the weekend. The optimization model does not allow admissions during the weekend. Furthermore, the optimization model solves for a fixed load, which is equal every weekday. To be able to compare the solutions of the model, Tables 16 and 17 show the average admissions and loads per week in 2008. The loads are determined by using the deterministic LOS mentioned in Table 1.

Admitted patients per day								
HLK	emergencies	short elective	long elective	total				
Monday	0.48	0.10	0.19	0.77				
Tuesday	0.42	0.27	0.21	0.90				
Wednesday	0.52	0.19	0.12	0.83				
Thursday	0.48	0.25	0.29	1.02				
Friday	0.52	0.12	0.23	0.87				
Saturday	0.46	0.02	0.17	0.65				
Sunday	0.46	0.02	0.10	0.58				
sum	3.35	0.96	1.31	5.62				

Table 16. Admissions per weekday for HLK patients in 2008

Load per day 2008								
HLK	emergencies	short elective	long elective	total				
Monday	3.35	0.10	1.77	5.21				
Tuesday	3.35	0.27	1.81	5.42				
Wednesday	3.35	0.19	1.83	5.37				
Thursday	3.35	0.25	1.92	5.52				
Friday	3.35	0.12	1.94	5.40				
Saturday	3.35	0.02	2.00	5.37				
Sunday	3.35	0.02	1.81	5.17				

#### .

Table 17. Load per weekday for HLK patients in 2008

#### 5.2.2.1 Results from Quadratic Programming model

Table 18 shows the results for the optimization of admissions and load per weekday, using the QP model.

		Arrivals per day	ÿ			Load per day		
HLK	emergencies	short elective	long elective	total	emergencies	short elective	long elective	total
Monday	0.48	0.40	0.30	1.18	3.35	0.40	1.60	5.35
Tuesday	0.48	0.09	0.31	0.88	3.35	0.09	1.92	5.35
Wednesday	0.48	0.09	0.00	0.57	3.35	0.09	1.92	5.35
Thursday	0.48	0.39	0.00	0.86	3.35	0.39	1.62	5.35
Friday	0.48	0.00	0.70	1.18	3.35	0.00	2.01	5.35
Saturday	0.48	0.00	0.00	0.48	3.35	0.00	2.01	5.35
Sunday	0.48	0.00	0.00	0.48	3.35	0.00	2.01	5.35
				5.62				

Table 18. Optimized admission planning and load per weekday for HLK patients using the QP model

The ALOS for HLK patients from category 2 is 10 days. The weekends can only reach their target utilization by admitting patients who stay more than one day, because during the weekend no elective patients are admitted. Patients from category 2, admitted on Friday, are staying for two weekends. Therefore, the optimized admission planning, in Table 18, shows that most patients from category 2 are admitted on Friday.

The difference in number of admissions per day between the current situation at the IC unit and the optimized admission is on Monday relatively large. The model shows more admissions on Monday to be able to achieve to target load on Monday. Furthermore there is a difference in the number of admitted patients per day per category; currently each day patients of each category are admitted, but the optimized version shows more admissions of each category on specific days. Furthermore the load is stable during the week at the optimized admission planning, which makes planning easier. And last but not least: no more admissions during the weekend.

### 5.2.2.2 Results from Linear Programming model

Table 19 shows the optimized admission planning and load for HLK patients using the LP model.

		Arrivals per da	У			Load per day		
HLK	emergencies	short elective	long elective	total	emergencies	short elective	long elective	total
Monday	0,48	0,70	0,00	1,18	3,35	0,70	1,31	5,36
Tuesday	0,48	0,09	0,61	1,18	3,35	0,09	1,92	5,36
Wednesday	0,48	0,09	0,00	0,57	3,35	0,09	1,92	5,36
Thursday	0,48	0,09	0,00	0,57	3,35	0,09	1,92	5,36
Friday	0,48	0,00	0,70	1,18	3,35	0,00	2,01	5,36
Saturday	0,48	0,00	0,00	0,48	3,35	0,00	2,01	5,36
Sunday	0,48	0,00	0,00	0,48	3,35	0,00	2,01	5,36
				5,64				

Table 19. Optimized admission planning and load for HLK patients using the LP model

As mentioned before LP is less robust than QP. Because the target function for HLK can be achieved exactly, QP and LP show the same result; target function is equal to zero.

### 6. Conclusions and recommendations

In this paper we presented an optimization model for developing an admission planning for the CCH patients and HLK patients at the IC unit for VUmc. The main issue by determining the admission planning is that no admissions take place during the weekend, but the weekend does have a target load. Therefore the main question is: how can we admit the patients during the week such that the target load during the week and during the weekend is realized?

At the moment the admissions and the load per weekday at the IC of VUmc varies. Each day a certain load has to be realized. The present situation shows that on Monday the occupancy can be lower than on other days. To prevent these kinds of situations it is necessary to plan the admissions based upon the target load, which is stable.

In this paper, we applied an optimization model giving the number of admissions per day, depending on their LOS, such that the difference between the realized and target load is minimized.

The results for CCH show that most admissions take place on Monday and Friday. Because no admissions are allowed during the weekend, the target load on Saturday and Sunday can only be realized by admitting patients on Friday staying for 2 or 3 days. Because CCH has to treat a certain number of elective patients per year, 2 beds are closed during the weekend to be able to admit more patients on Monday. This is one of the constraints in the optimization model for CCH patients. For this reason the results of the optimization model show that more patients are admitted on Monday.

The results for HLK also show that most admissions take place on Monday and Friday. Again no admissions are allowed during the weekend. The target load on Saturday and Sunday can only be reached by admitting patients on Friday staying more than 1 day. Because the LOS of patients from category 2 is 10 days, the patients are discharged on Monday; therefore more patients can be admitted on Monday.

We advise further research to include other departments in the optimization model, like the operating theatres and the medium care unit. These departments have influence on the occupancy at the IC unit. Furthermore predictability and implementation are difficult issues by optimizing the admission planning.

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