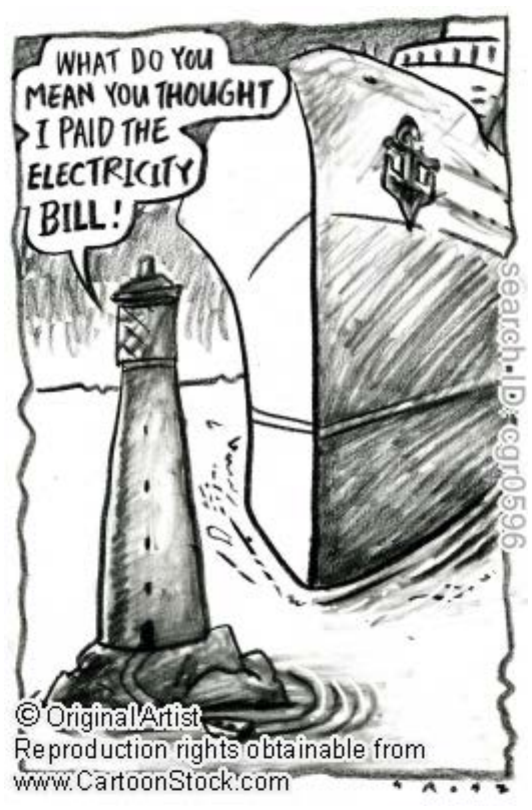
BMI paper

Weather routeing for cargo ships

An investigation of the influence of weather conditions on time optimal ship routes.



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Preface

This paper is part of the Master programme Business Mathematics and Informatics. The goal of the project is that the student learns how to do an investigation of a business problem on his own and to present the outcome of this investigation in a correct manner, on paper as well as in an oral presentation.

I considered it as a positive fact that there was no predetermined subject that had to be investigated. This way, I could choose a part of business and a problem that I am really interested in.

The whole process of writing this paper was a nice experience and I have learned a great deal from it. There was a lot of diversity in things that needed to be done: the search for data, the programming part, the part where the programme was used and the results obtained and the writing part. Already the search for data was challenging and interesting. It gave me the opportunity to contact people that work in the field of my interest. I am very thankful to Rob Grin of the Dutch maritime research organisation Marin, who gave me all the data that I needed for this paper and who answered the questions that I had about the data.

I would like to thank Dennis Roubos as well, who supervised the process of writing this paper. He helped me a lot by answering questions if I had any and by giving me feedback on my work.

Contents

Contents 3
1. Introduction 4
2. Different methods of weather routeing 5
    Calculus of variations 5
    The isochrone method 5
    Dynamic programming 7
    Dijkstra's algorithm 8
    Conclusion of the discussion of the different techniques 9
3. The solution method and its implementation 9
    How should the weights be assigned to the different parts of the sea? 10
    How should the grid be built? 14
    Implementation issues 14
4. Results 15
5. Conclusion 19
6. References 20

1. Introduction

Ever since ships were used to transport goods from one place to another, people have to think about the question how to arrive safely at the destination within a reasonable time span. In today's globalised world, transportation of goods at sea is of great relevance: more than ninety percent of the global trade is carried over sea [1] by a world merchant fleet that has a deadweight of more than 1.12 billion tonnes (beginning of 2008) [2]. Considering the fact that the market of sea logistics is very competitive and that the profit margins are small, it is important that the ships are optimally used. For example, even medium sized container carriers can have daily operating costs of tens of thousands of dollars [3], so suboptimal usage can cost a lot of money.

Ship routeing considers with the problem how to make optimal use of a fleet of ships. There are different points of view in this problem. Assigning the ships to different trading routes can be viewed as a part of the tactical level of the problem. The determination of the order in which different harbours should be visited by a particular ship belongs to this level as well. Another problem, which is on a lower level, is to determine the optimal route of a ship at see when it is already known what the points of departure and destination are. This is the problem of ship routeing on which this paper focuses.

The shortest path between two points on the planet is along the greatest circle that connects these points. For ships, it is not always possible to sail along a great circle in the first place. There might be land or other obstacles like oil platforms or offshore windmill parks on this path. Shallowness of water or governmental rules can be problems that prevent a ship from sailing along a great circle as well. And even if it is theoretically possible to sail along a great circle, it is not always optimal to do this. The weather at sea is an important factor when considering the routeing of a ship. This kind of ship routeing is called weather routeing. Wind and the characteristics of waves are the most important weather conditions that influence the speed of a ship [4]. If it is known what these weather conditions are at the predetermined route, it can be better to make a small detour and save time and fuel. In this paper, it is investigated how to determine the optimal route considering the weather conditions. The optimal route depends on the objective that is chosen. In times of an economical boom most of the shipping companies want the ships to arrive as early as possible, because there are a lot of orders that have to be carried out. In an economical crisis like the one that we are in now, the objective can be to sail a route with minimal fuel consumption, within a certain amount of time of course.

The main question that is answered in this paper is:

Under which weather conditions is it optimal to deviate from the predetermined optimal course?

The predetermined optimal course is defined here as the optimal course under ideal weather, that is sailing along the great circles as much as possible, with respect to the obstacles that may lie on these circles as described above. Ideal weather means that there is no wind and that there are no waves.

To arrive at an answer to this problem, the paper is structured in the following way. In chapter two, four common solution techniques for the ship routeing problem are briefly described and discussed. The chapter ends with a conclusion about which technique is best suited for the problem stated in this paper. This technique is explained in greater detail in chapter three, where the implementation of the chosen technique is described as well. The results of different practical cases using real weather data are shown in chapter four. The investigation of the main question is also done in this chapter, by synthetically varying the parameters of the weather and see the results on the optimal route. The paper ends with a conclusion in chapter five, where the main question is answered and suggestions for future research are given.

2. Different methods of weather routeing

The problem of how to route a ship from its departure point to its arrival point based on weather conditions, is investigated for years. When summarizing the literature, there are four main solution techniques for this problem: calculus of variations, dynamic programming, the isochrone method and Dijkstra's algorithm. All of the methods rely heavily on a good weather forecast.

Calculus of variations

The technique which uses calculus of variations is an analytical method that views ship routeing as a continuous optimization problem. It assumes that a function which assigns travel time according to the severeness of the weather conditions to different positions and points in time. The positions of departure and arrival are boundary conditions. The course, position and time are assumed to be deterministic.
The method starts with an arbitrary guess of the optimal route. The real optimal route is found by refining the gradients of the objective function, such that the error at the end point is reduced. Of course, this is done under some constraints which make sure that the route is a realistic one.
The solution can be found in two ways: with a set of linear differential equations or the Euler Hamiltonian equations that need to be solved.

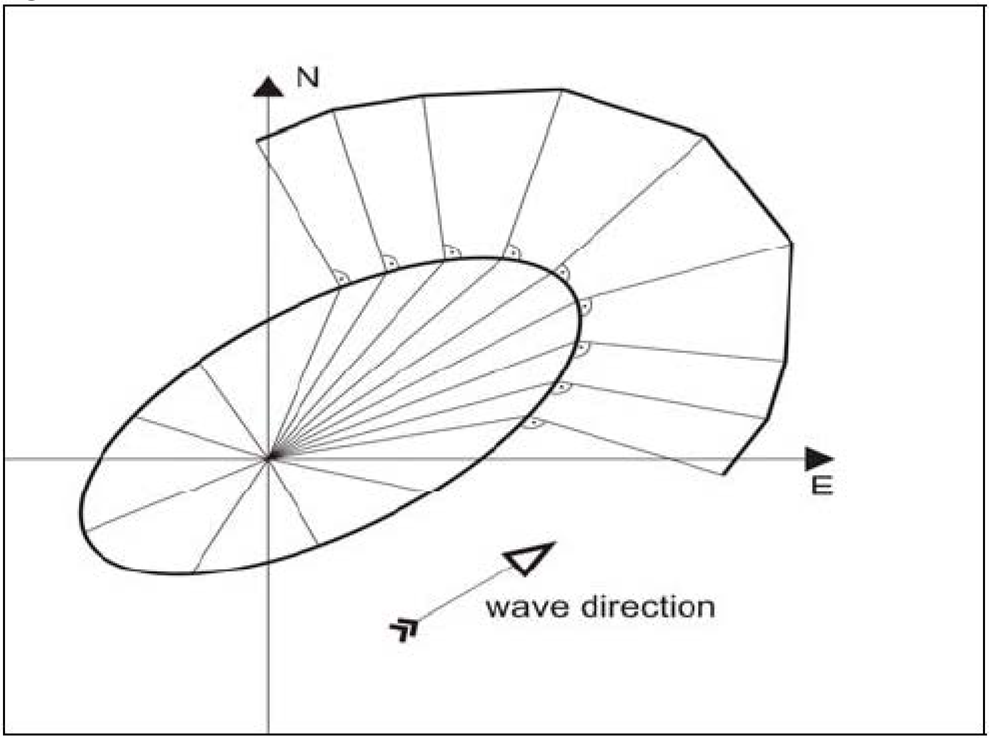
Advantages
Calculus of variations is a mathematically elegant method, since its objective is to solve the problem exactly.
It is a quite general method, so it can be used for a lot of applications.

Disadvantages
The first disadvantage of this method lies in the fact that it assumes a lot of variables to be deterministic, while they are not in reality. As is often the case for an analytical method, a lot of simplification is needed.
It is not possible to deliberately decrease sailing speed with this method. Sailing at the maximum possible speed, may not always be optimal, especially when the main objective is to save as much fuel as possible and the time of arrival is only a constraint.
If there are errors in the predicted weather conditions –and it is assumable that there are- the use of differentials might result in inaccuracies in the solution.
The partial derivatives of the ship's speed with respect to the position of the ship might have a dependency relationship on each other which will result in a convergence problem.
The initial route has to be chosen carefully, otherwise the method can converge to a local optimum. [5][6]

The isochrone method

One of the older techniques for solving the ship routeing problem is the isochrone method [7]. This method is based on the distance that a particular ship can cover within a time unit. A distance boundary is created at each time unit, where the starting point of a possible movement of a ship in a new time unit is the boundary of the previous time unit. A time front, the boundary where a ship can be after a certain amount of time, is iteratively created in this way. The shape of this time front depends on the weather conditions of the considered sea. It is obvious that the distance that a ship can cover in a time unit is smaller in case of a severe storm than in case of friendly weather. To prevent routeing over land or other obstacles, the speed of a ship at land should be chosen to zero. *Figure 1* visualizes the algorithm.

Figure 1 First two isochrones [8]



The first time front shows the boundary where the ship can be after the first time unit. This depends on the direction of the waves relative to the course of the ship. To generate the second time front, some points on the first time front are chosen and a perpendicular line to the tangent of the first time front, with a length that represents the maximum distance that a ship can cover in the respective part of the sea is drawn. The next time fronts are generated in the same manner. When the arrival point is reached, the optimal route is found and the procedure can be stopped.

Advantages
An advantage of the isochrone method is that it can be used manually. Although this was an important feature a few decades ago, in the modern, computerized world it is not feasible to do these kinds of calculations by hand.
Another advantage is the fact that changes in weather conditions during a travel can dynamically be updated in this algorithm.
When implemented in a modified way, this algorithm can be relatively fast.

Disadvantages
A disadvantage is that the basic isochrone method does not work flawlessly when implemented in a computer programme. When computing many isochrones, which is desirable and can be done with the help of a computer programme, there is a significant probability that 'isochrone loops' appear. It is possible that an isochrone does not have a regular shape at a certain point due to the non convexity of the speed characteristic of weather

situations. Such an irregularity is called an isochrone loop. New isochrones cannot come out of a loop and there is no realistic time front created in that particular direction.
Other problems are concerned about avoiding land. The algorithm can get stuck when there is a small water way between two coasts. If there is only one point generated within this straight, it might be difficult for the algorithm to find a passage. This problem can be solved by generating more points that represent a time front, but this will increase the amount calculation power significantly.
The other problem with landmass is that if a part of the time front is assigned to a point on land, new isochrones will not be created from this part. It is possible that an optimal route surrounding the land cannot be found.
As for the method of calculus of variations, a disadvantage of the isochrone method is that it is assumed that the ship will always sail at maximum speed if this is possible.
It is possible to deal with some of these problems by modifying the algorithm. There exists literature about this ([8] ,[9]) and the number of problems were reduced, but it turned out that it is still not feasible to fully rely on this method to find a solution to the proposed ship routeing problem.

Dynamic programming

A method that relies on a recursive algorithm is dynamic programming. It is based on Bellman's principle of optimality. The algorithm makes use of a grid that represents the geographical position and its weather conditions in a given sea. Based on this grid, a discrete optimization problem is formulated and can be solved by a recursive equation.
The position can be specified with two variables: degrees longitude and degrees latitude. Since the position is continuous and the algorithm can work only with discrete variables, a grid is built that discretizes the position.
Also the time has to be discretized and the same problem arises of defining how many time steps should be taken.
The state that is used in this method has to contain the geographical position of the ship in the grid and the time. The distance that can be covered within a time unit depends on the state of the sea and how a particular ship responds to this factor. This can be specified by the heading, the power output of the ship and a constraint vector which reflects the sea keeping characteristics of the ship; the maximum motions that the ship can handle. The state of the sea can be represented by a random vector and is assumed to be constant within each part of the grid.
The objective is to find a path from the starting point to the target location, while minimizing the costs, which can be divided in the costs of the ship arriving late at the target location and the costs of travelling which are largely fuel costs inflicted by time and weather conditions. The constraints should be met, of course. Note that there is no time of arrival constraint when posting the problem in this way, only costs of arriving late.
This problem can be solved by carrying out a recursive computation procedure based on bellman's Principle of optimality. Other possibilities to solve the problem exist as well, like linear programming and successive relaxations.

Advantages
An advantage of using dynamic programming as a way to solve the ship routeing problem is that it tries to capture the randomness of the weather when formulated as a stochastic optimization problem.
Another advantage is that is faster than the analytical method of calculus of variations.

Unlike the method of calculus of variations, a decision mechanism can be implemented in the technique. It can be decided to wait on the grid point for better weather conditions or maybe sail slower instead of not sailing at all.
Like calculus of variations, dynamic programming can be used for a lot of problems.

Disadvantages
The possibility to include the randomness of the weather is an advantage of dynamic programming. However, it is assumed that the weather changes according to a Markov chain. When modelling this in a realistic way, the complexity of the method is increased. At every grid point, a weather condition has to be defined and the transition probabilities have to be chosen in a way that the process of the waves over time and space are sufficiently approximated.
A disadvantage of this method is that it still requires a lot of calculation power. The accuracy of the solution is based on the fineness of the grid. If the accuracy is to be increased, the grid should be finer which results in a greater demand of calculation power and time.
[10][11][12]

Dijkstra's algorithm

Apart from the techniques that are discussed above, younger attempts to solve the ship routeing problem seem to use Dijkstra's algorithm (1959) more often. The algorithm was designed to find the shortest path of a network which consists of directed arcs that have known positive weights that indicate the resistance (or length) of an arc. Actually, it is a simple form of dynamic programming as it relies on a recursive procedure. In the rest of this paper, the term 'dynamic programming' is used for the more sophisticated dynamic programming methods and the method that uses Dijkstra's algorithm is viewed as a separate technique.
If the ship routeing problem is viewed as a deterministic problem where the weather conditions do not change, this algorithm can be used. The geographical space has to be discretized, which results in a grid. Each smallest rectangle of the grid represents an area in which the weather conditions are assumed to be constant. A weight that represents the added resistance due to the weather conditions is given to each of these rectangles. Dijkstra's algorithm can now find the shortest path from the starting point to the end point, by considering the distance between the centres of the rectangles and the added resistance given by the weights.

Advantages
An advantage of this method is its simplicity which makes it very appealing people that should use the results of the method. It can be implemented without great difficulties as well.

Disadvantages
A disadvantage of this method is that it assumes that the weather conditions do not change, because they do.
This method requires the generation of a grid, which is a discretization of reality. An issue concerning the grid is its fineness. Dijkstra's algorithm gives better approximations if the grid is finer, but this also requires more calculation power.
The disadvantage that it is not possible to deliberately reduce speed applies also for this method.
[13][14][15]

Conclusion of the discussion of the different techniques

Each technique has its advantages and disadvantages. The only real analytical method is the method of calculus of variations, the rest of the methods give approximations to the optimal solution. But because of the fact that the objective function of the calculus of variations method is based on empirical data and on simplifications, the added value of the analytical concept is reduced. A lot of problems can be encountered when using this method. The other methods are faster alternatives.
The isochrone method is faster, but still has a lot of pitfalls. The biggest is that problems are encountered when implementing this technique in a computer programme.
The two grid based methods remain. They both share the disadvantage of the usage of a grid. The method of dynamic programming is somewhat more sophisticated than the method which uses Dijkstra´s algorithm. In dynamic programming, it is tried to deal with the randomness of the weather. But if this is to be correctly done, the method becomes rather complicated which makes it hard to solve. The strength of the method that uses Dijkstra´s algorithm lies in its simplicity. It can be easily implemented and probably easier adapted by people who have to work with its results. Understandable methods are usually better adapted in practice. Because the aim is that the method is going to be used in practice, this paper will focus on Dijkstra´s algorithm and how it can be used for the ship routeing problem.

3. The solution method and its implementation

When using Dijkstra's algorithm for the described ship routing problem, an important question is how to arrive at the needed weights and the grid. This chapter gives an answer to the following main questions:

- How should the weights be assigned to the different parts of the sea?
- How should the grid be built?

After a more precise description of the algorithm, these questions are described and answered in the subparts of this chapter. It ends with a few important implementation issues.

Dijkstra's algorithm needs a directed graph with nonnegative weights as input. The output is the shortest route from a starting point to any other node in the graph. The algorithm consists of a number of iterations. In each iteration, it is known which nodes can be reached by taking one step. The node that has the shortest route (the smallest sum of weights) from this node to the starting point is determined, the predecessor of this node and the value of the route (the sum of the weights) is memorised. If the destination point is reached, the procedure can be stopped and the route can be found by tracking the predecessors while the value of the route was already memorised.

This dynamic programming algorithm can be described mathematically as follows:
Let

a be the starting node
V_i be the set of visited nodes in the i'th iteration, where $i = 0,1,\ldots$. So $V_0 = a$
$d(y, V_i)$ denote the distance from node y to a, with the information of V_i
$w(x, y)$ denote the distance from node x to y, where y can be reached from x in 1 step

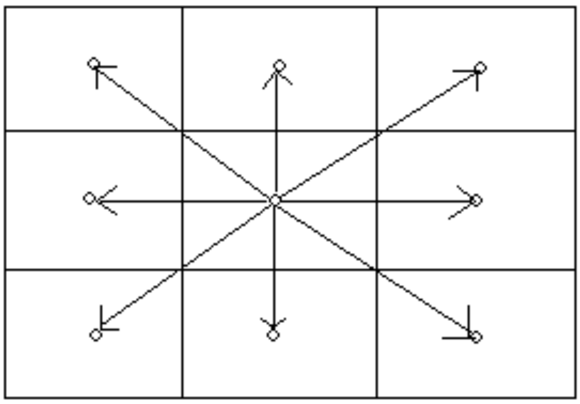
Then the recursive step is described by:

$$d(y \mid V_i) = \begin{cases} 0 & \textit{for } y = a \\ \min\limits_{x}\{w(x,y) + d(x \mid V_i)\} & \textit{for } y \neq a,\ x \in V_i \\ \text{infinity} & \text{otherwise} \end{cases}$$

How should the weights be assigned to the different parts of the sea?

As described in the previous chapters, there can be different objectives for ship routing. If the demand for transportation is greater than the supply, it is very important to make sure that a ship arrives as soon as possible, so that it can handle a next assignment quickly. If the demand is not so high, fuel costs become more important. Although the safety of the crew, ship and cargo are also important factors, these safety issues are not considered in this paper, because they are more difficult to quantify and depend on the sailing style of the captain. The objective of finding the minimum sailing time is investigated in this paper. The minimal fuel consumption objective is more complicated to analyse, because beside the weather conditions of the sea, the time constraint which is imposed on a ship is also an important factor. This factor depends on each transportation order individually and also on the possibility of arrivals of new orders for the ship. In this paper, the focus is on the impact of the weather conditions. Let us now consider the objective where the sailing time of a ship is to be minimized. In reality, a ship can cross an ocean in infinitely many ways. When using Dijkstra's algorithm, the continuous space of the sea is discretized into a grid where it is assumed that a ship can only sail from one centre point of a grid to another. This means that the ship can sail to the centre points of the eight different neighbouring rectangles, see *figure 2*.

Figure 2 Different possible directions



The time that it takes for a ship to travel such a path is determined by the speed of the ship and the distance between the two centre points. In the case of minimum sail time as the objective, the power of the engine is held constant at a maximum power. The fuel consumption per unit of time is constant, independent of the weather conditions that the ship encounters. The speed v of the ship is constant between two centre points, because the weather conditions are assumed to be constant within the part of the sea that has to be traversed.

The time that a ship needs to travel from one centre point to another is calculated by:

$$t_{ij} = \frac{d_{ij}}{v_{ij}}$$

where t_{ij} is the time needed to sail from point i to point j in hours, d_{ij} the distance between point i and point j in nautical miles and v_{ij} the speed of the ship between point i and point j in nautical miles per hour (knots).
The total time that a ship needs to get to the destination is the sum of the t_{ij}'s from the departure point to the destination point. This sum is minimized with Dijkstra's algorithm, which considers the t_{ij}'s as weights.

Distance between two centre points
The distance between the two centre points is smallest along a great circle, since our planet is a sphere. This distance in nautical miles can be calculated with the help of the haversine formula:

$$d = 2r \cdot \arcsin(\sqrt{\sin^2(\frac{\phi_2 - \phi_1}{2}) + \cos\phi_1 \cos\phi_2 \sin^2(\frac{\lambda_2 - \lambda_1}{2})},$$

Where

d is the distance between two points on a sphere in nautical miles
r is the radius of the sphere in sea miles
The radius of the earth is approximately 3440 nautical miles.
ϕ_1 is the latitude of the position of the starting point in radians
ϕ_2 is the latitude of the position of the destination point in radians
λ_1 is the longitude of the position of the starting point in radians
λ_2 is the longitude of the position of the destination point in radians

This formula is chosen over other formulas that can be used to calculate distance on a sphere because of its sufficient accuracy and relative simplicity.

Speed between two centre points
The weights, which represent the sailing times between the different centre points, also depend on the speed of the ship and therefore on the weather conditions. This means that it is necessary to know the weather conditions of different parts of the considered sea and how a ship responds to these conditions to define the different weights. These questions are not trivial, since the weather conditions as well as the ship's response can be described in great detail. Ocean currents, fog, precipitation, wind and waves are examples of factors that influence the speed of a ship. But according to literature [4], if one only takes wind and wave characteristics into account, a good approximation to the optimal route is obtained because these factors add by far the most resistance. The goal of the model that is used in this paper is not to find a solution that is as close to the truth as possible, but to find a good approximation that is still efficient and simple. Therefore, it is reasonable to include only characteristics of the wind and waves.
Wind adds resistance when it hits the surface of the ship. It is obvious that the relative direction of the wind compared to the direction of the ship matters. If the wind blows in the opposite direction of the ship's course, the speed reduction is greater than in case of a side wind.
But the wind has even more impact: it is the cause of the generation of waves. Although there are different kinds of waves, most of the added resistance can be described by the wave height

and the wave direction relative to the ship's direction. And because of the fact that the relationship between the wind and the wave height is linear, the sum of the influence of waves and wind can be shown in a single graph where the speed loss is related to the wave height. [16]
When a ship crosses an ocean, the weather conditions obviously change during the voyage. The dynamic character of the weather is not included in the model of this paper. The data of the wave height and wave direction that are used are satellite observations of the actual conditions in the North Atlantic Ocean as they were on the following days:
January the 4th, the 6th, the 8th and 10th in 2004. The data are obtained by the courtesy of the Dutch maritime research organization Marin. The data can be found in *appendix A*. For each day, the wave height in metres can be found in the different rectangles. The position of the rectangles is defined by the degrees latitude and longitude of the centre of the rectangle. These can be found in the y-axis and the x-axis respectively. Note that only degrees are used, not minutes or seconds. Half a degree equals 30 minutes. Negative values on the x-axis represent the number of degrees west longitude while positive values represent degrees east longitude. The wave direction is shown in the grid below the first one in the same appendix. A direction of 0 degrees means that the waves come from the North and 180 degrees that the waves come from the south. Any values in between indicate that the waves have a direction coming from the west. Negative values represent directions coming from the east. *NaN* means that there is landmass in the centre of the particular rectangle in both tables. *Appendix C* shows the wave height in the different parts of the ocean graphically. The centre points of the rectangles from *appendix A* are now represented as the intersection of the lines. The parts that are coloured black are on land.
Each ship responds differently to the weather conditions. The research in this paper is carried out for a Panamax container vessel of 200 metres long and 32.2 metres wide, capable of sailing approximately 22.1 knots. This ship frequently crosses the Northern Atlantic Ocean from New York to Le Havre and back. The relation between the ship's speed and the wave height for five different relative wave directions can be found in *table 1* and *figure 3* on the next page. The direct influence of waves to the ship's speed as well as the influence that the wind has when such waves are observed are included in this figure. The wave directions relative to the ship's direction are shown in *figure 4.* The ship is symmetrical: the speed loss values for the directions on portside are the same as those for starboard, where the directions on portside correspond to 360˚ minus the direction on starboard. The use of more directions does not add much to the accuracy of the model. In fact, in literature often only three directions are used: head seas (180˚), beam seas (90˚) and following seas (0˚) [12] [13] [14].

An interesting observation is that the waves that come from an angle of 45 degrees result in a greater speed reduction than waves that come from an angle of 90 degrees. The reason for this is that waves on the quarter make the ship pitch, while waves from the side do not. Pitching has a negative influence on the speed of the ship. [13]

The relative direction of the waves -and thus the weights- depend on the course of the ship. A ship can enter a certain rectangle of the grid from various positions. For each of these positions, the speed of the ship and as a consequence the sailing time between the neighbouring centre points have to be calculated to be able to use Dijkstra's algorithm. There is only data for the five directions that are shown in *table 1* and *figure 3*. The relative of the waves have to be assigned to one of these directions. That is done in such a way that the nearest value of the five directions is chosen. A relative direction of 22 degrees, for example, is closest to 0 degrees, so the ship will get the speed that belongs to the wave height and a relative wave direction of zero degrees. Also the wave heights are divided in such a way: the wave height of a rectangle is compared to those in *table 3* and the closest value is used for further analysis.

Table 1 Ship speed in knots for different wave heights and relative wave directions [Marin]

	Relative wave direction				
Wave height in metres	0 deg	45 deg	90 deg	135 deg	180 deg
0,5	22,12	22,11	22,12	22,1	22,1
1,5	22,12	22,025	22,08	21,93	20,8335
2,5	22,1	21,82	21,97	21,54	19,386
3,5	22,06	21,465	21,85	20,87	17,7395
4,5	21,98	20,865	21,72	19,75	15,8
5,5	21,86	19,98	21,57	18,1	13,575
6,5	21,69	19,04	21,37	16,39	11,473
7,5	21,52	18,07055	21,15	14,6211	9,503715
8,5	21,36	17,0782	20,96	12,7964	7,67784
9,5	21,25	16,08465	20,88	10,9193	5,6551
10,5	21,12	15,05645	20,84	8,9929	3,6054

Figure 3 Ship speed in knots for different wave heights and relative wave directions [Marin]

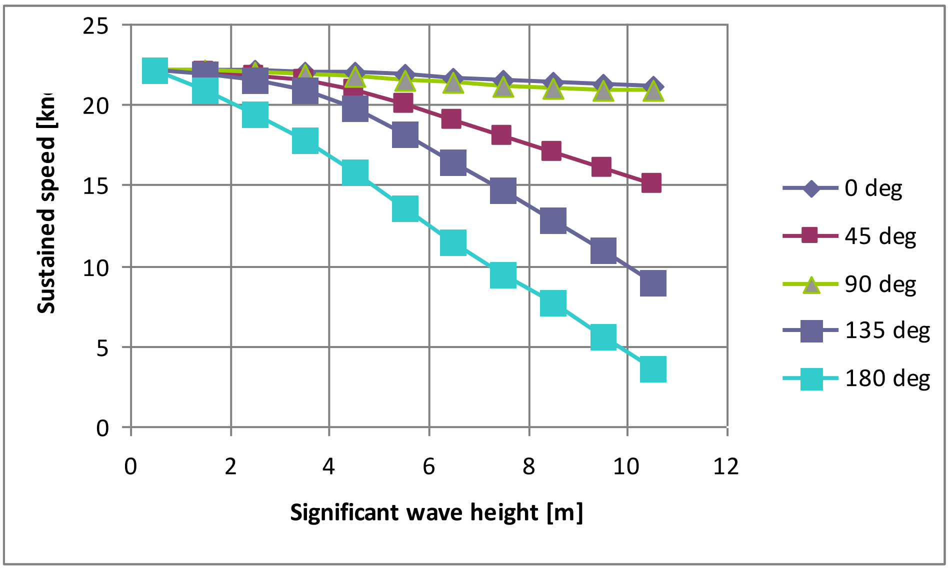
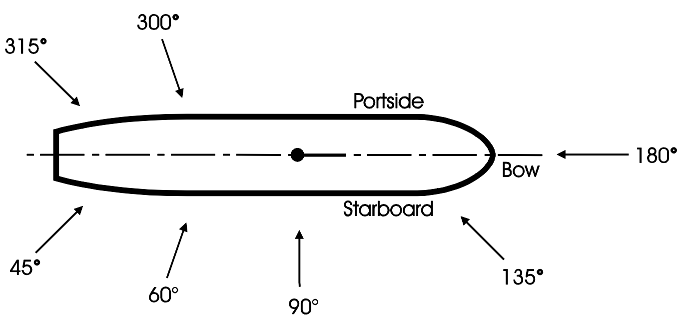


Figure 4 Wave directions relative to ship's direction [Marin]



How should the grid be built?

Before building the grid, it is important to determine the number of reachable neighbouring centre points and also the fineness of the grid. These two characteristics influence both the accuracy and the complexity of the model. The number of reachable neighbouring centre points can also be viewed as the number of different directions a ship can sail. If only two centre points that lie in the direction towards the destination point can be reached, the result will not be very realistic. Each extra direction will increase the accuracy, but this has the consequence that the needed calculation power increases exponentially, since there is an extra option at each step.
A same kind of argument applies for the fineness of the grid. The finer the grid, the better reality is approximated, but at a cost of more calculation power. In fact, an infinitesimal fine grid would be an exact representation of reality, but this cannot be done in practice.
The model is chosen to be as accurate as possible with the data that is available. Each rectangle has a width of 1.25 degrees longitude and a length of 1 degree latitude. The position of New York is approximately 40.5 degrees north latitude and 74 degrees west longitude while the position of Le Havre is approximately 49.5 degrees north latitude and 0 degrees west longitude. So there is a difference of approximately 60 rectangles in width and 10 rectangles in length. Of course, some clearly infeasible routes in the opposite directions are also computed, but still the optimal route can be found in a reasonable amount of time.

Implementation issues

To obtain results, the programming language C++ is used to implement Dijkstra's algorithm. The programme is tailored to the data from *appendix A*.

In the implementation of the algorithm, the rectangles with landmass get the maximum possible value of the programming language for the wave height. Then, it is infeasible for a ship to cross landmass. Another thing that has to be considered during the implementation is the fact that a ship crosses two rectangles when sailing from one centre point to another. This is implemented in such a way that the distance between the centre points is calculated first, and then half of this distance has to be traversed in the weather conditions of the first rectangle and the other half in the conditions of the other. The two sailing times are added to arrive at a sailing time that is needed for the travel between the two centre points. This is not completely correct, since the grid is built for a flat surface and not for a surface on a sphere like the earth, which has the consequence that the borders of the rectangles are slightly misplaced. Because of the fact that weather conditions in two neighbouring states do not differ that much, the resulting error will be very small and it is chosen not to change the solution method for this.

As mentioned above, the particular ship investigated in this paper frequently sails between New York and Le Havre. This route is taken as a test case for the programme. The approximate position of New York (40.5˚ latitude, -74˚ longitude) should be the starting point for an eastbound travel. However, the grid is a discretization of the space, so the nearest rectangle has to be found. There are two nearest options: (40˚ latitude, -73.75˚ longitude) and (41˚ latitude, -73.75˚ longitude). The second option has a centre point on land mass, which is not unlogical since New York is built on land. But the first option is chosen, because the ship has to be able to reach the centre of the rectangle. The same applies for Le Havre, which has an approximate position of (49.5˚ latitude, 0˚ longitude). The rectangle in the grid has to be the one with centre point (50˚ latitude, 0˚ longitude). It should be noted that this means that the route that is calculated does not actually lead the ship to the harbour of the city, but rather to the nearest possible point in sea that is available in the obtained data. This is not a problem,

since the ship cannot sail at full speed when leaving the starting harbour or approaching the destination harbour. So, the waiting time and the sailing time that is needed to leave or enter the harbours should be added to the sailing time that the programme presented in this paper calculates.
The code of the resulting programme can be found in *appendix E*.

4. Results

The obtained programme can be used to find the optimal routes for the different days. These routes can be found in *appendix B*. The yellow route represents the travel from New York to Le Havre, the red route represents the travel from Le Havre to New York and the turquoise route represents the route that best approximates the great circle route, which is the same in both directions and for all the different days. If the three routes go through the same rectangles, the respective rectangles are coloured green. If the great circle route crosses one of the other routes, the numbers in the rectangles representing wave height or direction are coloured turquoise. The route that best approximates the great circle route (later referred to simply as 'great circle route') is found by choosing all the waves to have a height of zero metres. This means that there is no influence of the weather, so the ship has to sail the shortest possible route, which is the route that lies on the great circle connecting the beginning and end point. Of course, if there is landmass on the great circle, this land is avoided.

The sailing times of the different routes can be found in *table 2*. The route under *Ideal weather* has been added. Ideal weather means that there are no waves at all. The result is the great circle route. Under *Optimal route*, the sailing time that the programme found is shown. The number under *Great circle route* is the sailing time that would be needed if the great circle route was sailed under the weather conditions of that day.

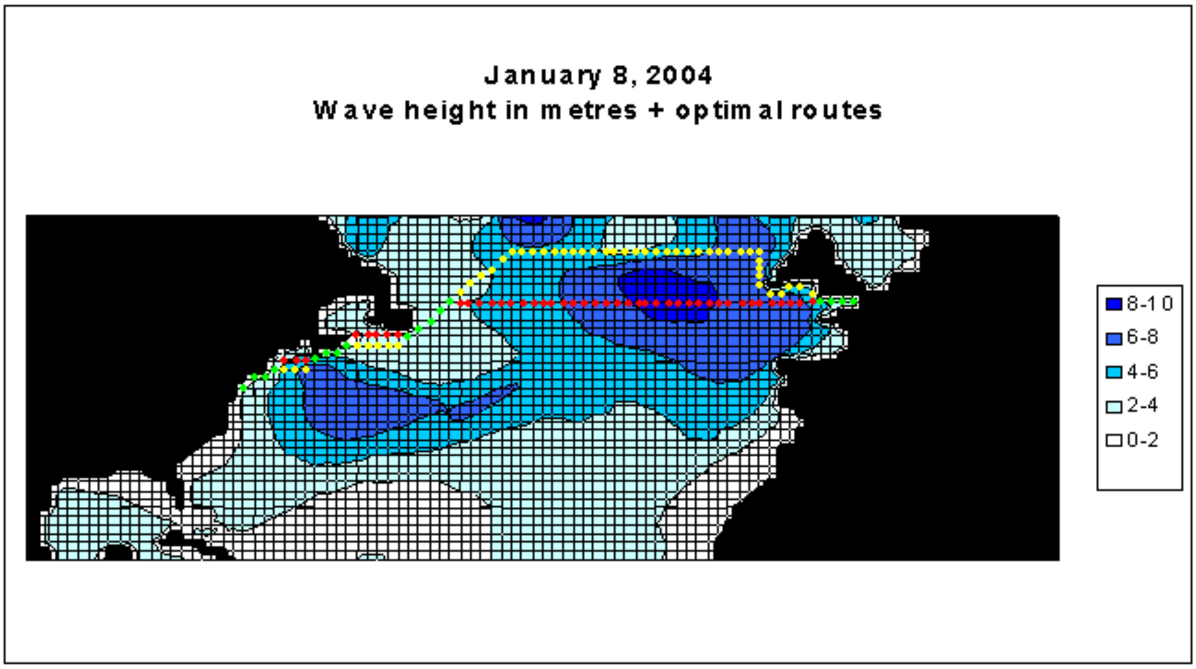
Table 2 Sailing times of the routes in hours

Sailing time in hours	Optimal route		Great circle route	
	New York – Le Havre	Le Havre – New York	New York – Le Havre	Le Havre – New York
Ideal weather	145.638	145.584	145.638	145.584
4 January 2004	160.608	148.013	162.565	148.114
6 January 2004	163.321	151.482	174.18	152.414
8 January 2004	190.3	148.494	242.809	148.494
10 January 2004	187.849	150.21	211.455	153.099

The small difference between the sailing times of the different directions under ideal weather is caused by the fact that the programme considers waves with a height from 0 to 0.9 metres the same. So the ship is still subject to a small resistance. But on the whole journey, a difference of 0.054 hours is negligible.

One thing that can be noticed is that the travel from New York to Le Havre (eastbound) takes more time in all the cases than the same journey in the other direction (westbound), apart from the ideal weather case of course. If one considers the fact that for all of the days the majority of the wave directions in the rectangles in the area of the great circle route have negative values (coming from an eastern direction), this is not a surprising observation. It can also be seen that the sailing time of the westbound optimal route does not differ much from the sailing time of the westbound great circle route. For January 8, it is even the same value. *Appendix B* shows that for all the days the great circle route (turquoise) does not differ much from the optimal westbound route (red). The small waves of January 4 do not make much of a difference for this observation compared to the bigger waves of January 8 and 10. So if the wave direction is largely the same as the direction of the ship, it seems that it is not optimal to deviate from the great circle route a lot. This is in line with the data from *figure 3*, which shows that the graph representing a relative direction of zero degrees does not drop quickly for bigger wave heights. But if the wave direction is in the opposite direction of the ship's direction, which generally is the case for the eastbound journeys, then the wave height has more impact. The yellow route in *appendix B* is far more north for the last two days than the one for the first two days where the waves are lower. In the north, the waves do not directly come from the opposite direction and the waves are lower than on the great circle route or south of this route. So it seems that if the wave direction is largely the opposite than the ship's direction, higher waves make it more feasible to deviate from the great circle route. This is also in line with *figure 3*, which shows a graph that quickly drops when the wave height gets bigger for the relative direction of 180 degrees. The sailing time for the eastbound optimal route is clearly less than the eastbound great circle route, especially for the days which showed higher waves. The greatest difference can be found for January 8, where more than 52 hours can be won on the journey from New York to Le Havre by deviating from the great circle route. The optimal routes together with the wave height in different parts of the ocean are visualized in *Figure 5*. Note that intersections in this figure represent the centre points of the grids that can be found in *appendix B*.

Figure 5 The eastbound route avoids high waves



Again, the westbound route is red and the eastbound route is yellow. The black parts are landmass or irrelevant sea. For a graphical overview of the wave directions in the Atlantic Ocean on January 8, 2004, see *appendix D*.
The last part of the route is the most interesting when considering deviation from this route. This part can be found in *figure 6*, where the upper table represents the wave heights and the lower table the wave directions. *Figure 7* combines the information about the wave height, the direction of the waves (the arrows) and the optimal routes for the same part of the route as *figure 6*. The arrows can have eight different directions in the figure: only multiples of 45

degrees. The arrow that has a direction closest to the real wave direction is shown. Note again that the intersections in *figure 7* represent the centre points in *figure 6*.

Figure 6 Eastern part of routes in numbers

Wave height

lat/lon	-13,75	-12,5	-11,25	-10	-8,75	-7,5	-6,25	-5	-3,75	-2,5	-1,25	0	1,25
57	6,60	6,70	6,60	6,30	5,70	4,50	NaN	NaN	NaN	NaN	3,10	3,40	3,60
56	6,80	6,90	6,70	6,20	5,50	4,00	NaN	NaN	NaN	NaN	3,20	3,70	3,70
55	6,90	7,00	6,80	6,00	4,30	NaN	NaN	NaN	NaN	NaN	2,50	3,80	3,90
54	7,00	7,10	6,80	5,60	NaN	NaN	NaN	NaN	NaN	NaN	NaN	2,90	3,70
53	7,30	7,10	7,00	5,70	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	2,40
52	7,50	7,40	7,20	NaN	NaN	2,70	3,40	3,00	NaN	NaN	NaN	NaN	NaN
51	7,60	7,50	7,40	7,10	6,80	6,30	5,80	4,30	NaN	NaN	NaN	NaN	NaN
50	7,70	7,60	7,50	7,40	7,30	6,90	6,70	5,10	4,00	3,90	3,70	3,50	NaN
49	7,70	7,60	7,60	7,50	7,30	7,00	6,80	6,20	4,70	2,80	NaN	NaN	NaN

Wave direction

lat/lon	-13,75	-12,5	-11,25	-10	-8,75	-7,5	-6,25	-5	-3,75	-2,5	-1,25	0	1,25
57	-132	-130	-128	-126	-123	-119	NaN	NaN	NaN	NaN	149	160	169
56	-126	-124	-123	-120	-116	-109	NaN	NaN	NaN	NaN	142	154	167
55	-121	-119	-118	-113	-105	NaN	NaN	NaN	NaN	NaN	133	147	162
54	-115	-114	-114	-103	NaN	NaN	NaN	NaN	NaN	NaN	NaN	140	153
53	-109	-109	-109	-102	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	148
52	-105	-105	-105	NaN	NaN	-139	-126	-122	NaN	NaN	NaN	NaN	NaN
51	-100	-101	-101	-102	-104	-106	-108	-106	NaN	NaN	NaN	NaN	NaN
50	-96	-97	-97	-99	-101	-102	-105	-108	-118	-127	-136	-148	NaN
49	-93	-94	-94	-95	-100	-103	-105	-106	-104	-109	NaN	NaN	NaN

Figure 7 Vizualization of all the information for the eastern part of the routes

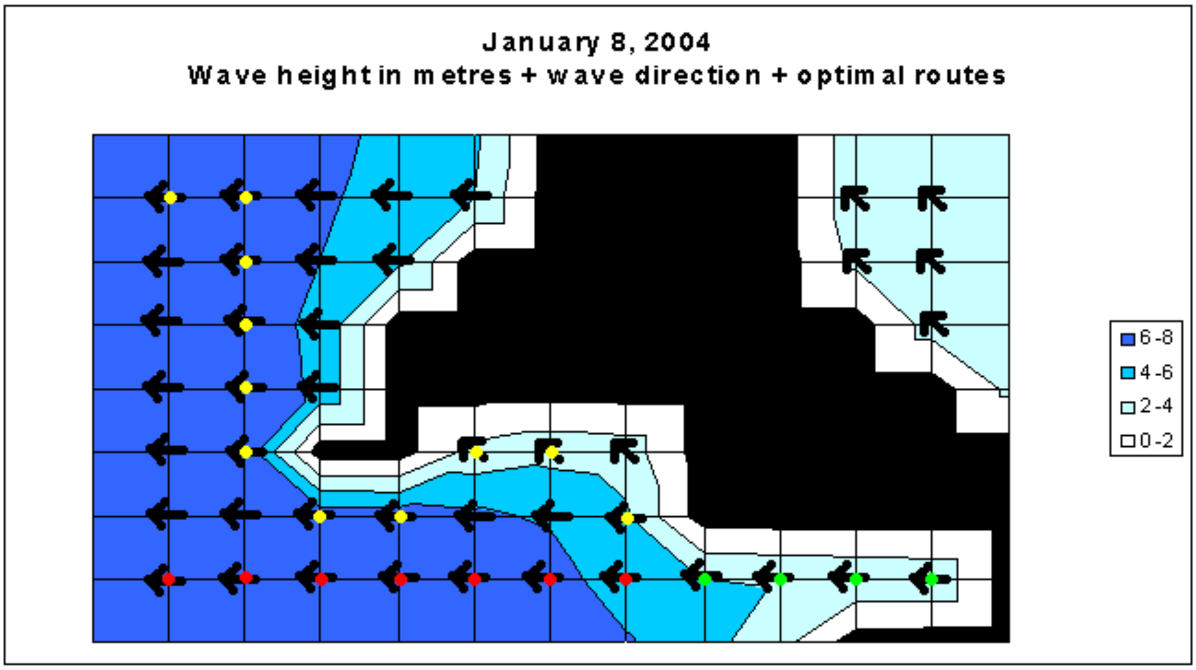


Figure 6 shows that the first part of the westbound route (red) is indeed simply the great circle route (turquoise numbers). The eastbound route comes from far more north than the great circle route. It can be seen that the directions on the great circle route are less favourable for a

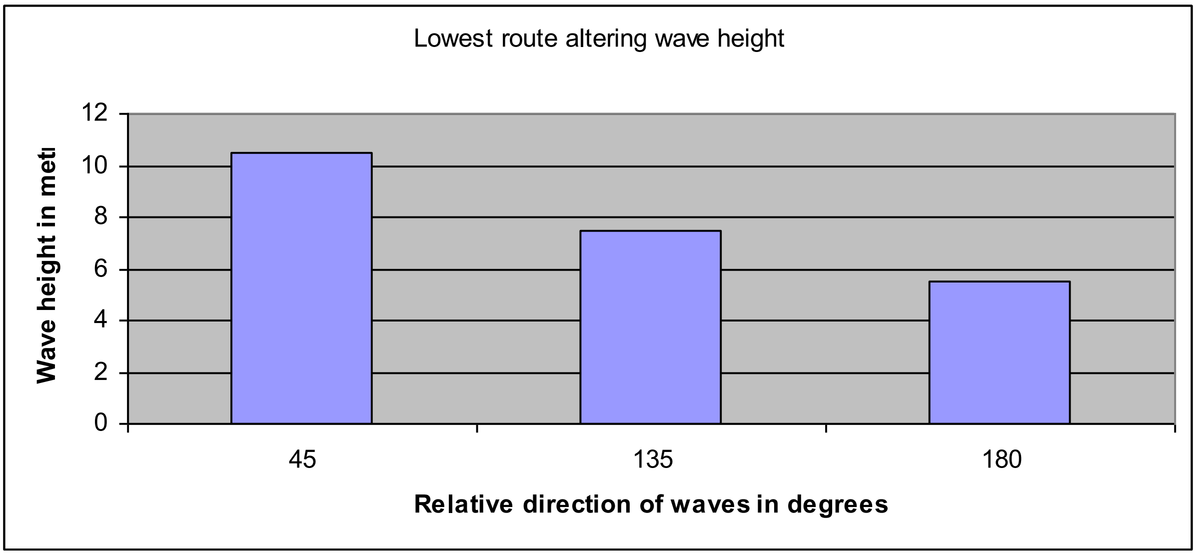
ship heading east than is the case in the north. In the part where the ship is heading south, the waves come only from the side, which does not result in such a great reduction of speed. In the part where the ship is heading east near the British coast, the waves come from the front diagonally. The same applies for the part where the ship is heading southeast. *Figure 3* showed that waves coming from the front diagonally result in less speed reduction than waves that come directly from the front. The fact that the waves are lower at the coast is also an important factor.
So although the ship has to sail a greater distance, its increased speed more than compensates the lost time.

The question now rises when it is optimal to sail around a specific rectangle. The programme can be used to get an answer to this question. A point on the great circle route is chosen (50, -27.5) where the wave directions and heights are synthetically altered, while the waves of all the other rectangles are defined to be zero. Of course, this is not a realistic situation. It is extremely unlikely that the wave heights in one rectangle are negligible while the wave heights in the neighbouring rectangle are 10 metres higher. But the results can give insight in the influence of weather conditions on the optimal route of a ship. These results can be found in *table 3* and *figure 8*.

Table 3

Relative direction of waves	Lowest height that results in difference with great circle route in metres
180	5.5
135	7.5
90	-
45	10.5
0	-

Figure 8



The wave heights for the relative directions of 0˚ and 90˚ are left out of *figure 8*, since even waves with a height of 12 metres do not have an impact on the optimal route. These results are in line with the observations that were made earlier: waves coming from the same direction as the ship do not make the ship sail a different route than the great circle route and if the waves are coming from the opposite direction, higher waves will lead to a deviation from this route. The table also makes clear that waves coming from the front diagonally can lead to deviation as well, only higher waves are needed. Waves coming from the back diagonally have less impact and waves coming from the side not at all when considering one rectangle. Of course, in reality there are greater parts of the sea that have similar weather conditions and then it could be better to sail around such areas even when there are smaller waves. This can be seen in *Appendix B* or *figure 5,* where the yellow route, where the ship generally sails in the opposite direction as the waves, clearly surrounds the largest waves.

5. Conclusion

This paper started with a discussion about the concept of weather routeing and why this is important for charterers of cargo ships. Four main techniques that are used in weather routeing for ships were then analysed and compared based on their effectiveness. Although the isochrone method, methods based on calculus of variations and dynamic programming all have their advantages, it was decided to use the method that uses Dijkstra's algorithm for the implementation because of its simplicity. This algorithm was described in more detail and was used to build a programme which is able to find the time shortest route of a ship that sails at full speed between two points that lie near the coast of the Atlantic Ocean where weather conditions in this ocean are known. The main considerations about the implementation were described. Important issues were the discretization of the ocean, the conversion of weather conditions and distances to weights that are needed for the algorithm and how to deal with the fact that the earth is a sphere. As weather conditions, only the main factors that have an influence on a ship's speed are considered, which are the wave heights and wave directions. The code is programmed in C++.

The resulting programme was used to find the optimal routes between Le Havre and New York for both directions for four days in the past: January 4, 6, 8 and 10 in the year 2004. These routes and the needed sailing time were shown, discussed and compared with the routes that a ship would sail under ideal weather conditions. This optimal route is referred to as the 'great circle route', which follows the great circle between the two cities as much as possible; land is avoided.

It is shown in this paper that it can be optimal to deviate from the great circle route. Depending on the direction of the waves, the wave height can have a lot of influence. Deviation from the great circle route can lead to reductions in sailing time in cases where the waves generally come from the opposite as the ship's direction. Higher waves on the great circle route lead to higher reductions of sailing time when avoiding this route. The influence of wave direction and wave height was further investigated by synthetically creating a calm ocean and varying direction and height of waves in one particular part of the ocean which lies on the great circle route. The minimal wave heights that led to deviation from the route are reported and compared for different wave directions.

In this paper, the stochastic character of the weather was not taken into account. This would make the problem far more complicated and would require a different solution technique, for example dynamic programming. Another interesting question is how to optimally route a ship when total costs have to be minimized. Fuel consumption plays an important role here. The complexity of the problem will increase dramatically, since a lot of other factors need to be considered. Examples are the question if the terminal in the destination harbour is available at the moment that the ship would arrive, or even at a higher level, if there is enough new demand for the ship after arriving at the destination harbour. These are examples of questions that could be a subject for future research.

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