## The Pension Simulator

Which factors determine the level of pension?

Bastiaan Kars<br>Research Paper Business Analytics<br>(Supervisor VU: André Ran)

Faculty of Sciences
VU University Amsterdam
De Boelelaan 1081

1081 HV Amsterdam, the Netherlands

Focus Orange
Rooseveltlaan 2-4, 1078 NH
Amsterdam, the Netherlands

## 1 Foreword

This paper will give the reader some insights in the pension arrangements in the Netherlands and extensively describes the methods and results of the Pension Simulator produced for the consultancy Focus Orange. The paper is a compulsory part of the study Business Analytics at the VU University and the goal of the Research Paper is to do an individual investigation of a relevant business problem in the mathematical- and the computer science domain.

Focus Orange is a consultancy firm supporting companies with all relevant HR-issues. The Pension Simulator is currently used by the retirement department of Focus Orange to assist companies in the selection of pension providers. The Pension Simulator gives insight in the expected pension and related risks of the pension providers.

Hereby, I want to thank my colleagues of Focus Orange and especially my supervisors, Prof. dr. André Ran of the VU University, Dirk Jonker of Focus Orange and President of the Royal Actuarial Association Jan Kars, for their support and feedback. I would also like to thank Strategeon for their time in helping me improve the Pension Simulator.

## 2 Abstract

The pension landscape has changed in the Netherlands over the years. In the past, employers faced the risks that occur with a Defined Benefit plans while nowadays, the employees face the uncertainty. The Defined Contribution agreement is currently the most used pension arrangement. With a DC-agreement, the company pays for every employee a yearly pension premium that is invested into an individual portfolio according to a life cycle. At retirement, the portfolio is sold to buy an annuity. So the level of the pension at retirement depends on the insurance tariffs at retirement but most of all, on the returns of the investments.

To give employees and employers insight in the risks, returns and costs of the different life cycles pension providers offer, Focus Orange created the Pension Simulator. This simulator simulates the returns on the investments to compare the expected returns and risks of the life cycles. In this paper, the methods used to calculate the expected pensions are explained.

## 3 Table of Contents

1 Foreword22 Abstract ..... 3
4 Introduction ..... 6
4.1 Problem Definition ..... 6
4.2 Goal ..... 6
4.3 Relevant Information ..... 6
4.4 Structure of the Report ..... 6
5 The accumulation of pension ..... 7
5.1 The pension premium ..... 7
5.1.1 The pension agreement ..... 7
5.1.2 Franchise ..... 9
5.2 Investments for DC-plans ..... 10
5.2.1 Life cycles ..... 10
5.2.2 Investment categories for DC-plans ..... 11
5.2.3 Interest Match ..... 11
5.3 Annual Pension Payment. ..... 13
5.3.1 Annuity factor ..... 13
5.4 Costs ..... 14
5.4.1 TER ..... 14
5.4.2 Buying and selling expenses ..... 14
5.4.3 Rebalance costs ..... 14
6 The Pension Simulator. ..... 15
6.1 Input ..... 15
6.1.1 Employees-File ..... 15
6.1.2 Retirement age ..... 15
6.1.3 DC premiums ..... 16
6.1.4 Franchise ..... 16
6.1.5 Salary Increase Percentages ..... 16
6.1.6 Maximum Salary ..... 16
6.1.7 Mortality Tables ..... 16
6.2 Pension Providers ..... 17
6.2.1 Life cycles. ..... 17
6.2.2 Costs ..... 17
6.2.3 Fixed return ..... 18
6.3 Methodology ..... 19
6.3.1 Model ..... 19
6.3.2 Objects ..... 20
6.3.3 Calculate returns per year per simulation ..... 21
7 Results ..... 31
7.1 Boxplot ..... 31
7.2 Scatter plot ..... 32
7.3 Costs ..... 33
8 Conclusions ..... 34
8.1 Investments ..... 34
8.2 Life cycle ..... 34
8.3 Costs ..... 34
9 Improvements ..... 35
9.1 Method 4 ..... 35
10 Bibliography ..... 36
11 Attachments ..... 37
11.1 Providers ..... 37
11.1.1 Costs ..... 37
11.1.2 Life Cycles ..... 38

## 4 Introduction

### 4.1 Problem Definition

Focus Orange is a consultancy firm specialized in human resources. A large part of the business is related to the retirement department, this department supports and advises companies on their pension plan, especially the so-called defined contribution plans (DC). This department is specialized in the selection process for pension providers for those DC-plans. To give the best advice to the customer, the following questions need to be answered:

- What factors determine the level of pension?
- What is the expected pension per employee at retirement?
- What are the worst-case scenarios?
- Which costs are involved in the process?


### 4.2 Goal

The goal of the research described in this paper was to build a program that can simulate the pension for the employees provided by the different pension providers. This program needs to give employees and companies insight in the different DC-plans and their expected pension outcomes and corresponding risks.

### 4.3 Relevant Information

The pension legislation has changed a lot the last few years and new changes are expected. For example, as of January 2015, the maximum pensionable salary of $€ 100,000$ will be introduced. Therefore, it may be possible that some information in this paper needs to be adjusted due to new pension legislation.

### 4.4 Structure of the Report

This paper begins with an introduction in the three different pension agreements and an extensive explanation of the Defined Contribution agreement. Next, the operation of the Pension Simulator and its methods will be covered. Finally, we will discuss some actual results of the Pension Simulator.

This paper contains some screenshots (in Dutch) to give the reader some more insight in the operation of the Pension Simulator.

## 5 The accumulation of pension

Pension is the second greatest reward for employees but does not always get as much attention as it deserves. Most employees do not know what kind of pension agreement they have, what their estimated pension at retirement will be and what their current accrued pension capital is.

There are many aspects and rules in the accrual of the pension, some apply to the employee and others to the corresponding company. There are rules for survivor pension, orphan's pension, incapacity for work, costs for the employees and costs for the company. However, in this research paper the main focus will be on the (variable) costs effective for the employee, the accrual of the pension and the investment possibilities and returns. Together they determine the capital used to purchase an annuity at retirement.

### 5.1 The pension premium

The pension premium is a monthly or annual deposit done by the company and the employees to the pension provider. This deposit is used to invest in bonds, shares, real estate or even in saving accounts and for the ongoing costs of the pension provider. The amount of the premium depends on the type of the pension agreement and is in most cases paid jointly by the employee and the company.

### 5.1.1 The pension agreement

In the Netherlands, there are three types of pension agreements: a Defined Contribution (DC) arrangement, a Defined Benefit (DB) arrangement and a Collective Defined Contribution (CDC) arrangement. Most current pension agreements are based on a DB arrangement, however, now most new agreements are DC arrangements. Usually, with a DB arrangement the risk of the pension lies for the most part with the company, whereas with a DC arrangement or a CDC arrangement the risk lies for the most part with the employee.

### 5.1.1.1 DC Arrangement

The DC arrangement is an upcoming way to fill in the pension plan for the employees and the companies. DC stands for Defined Contribution. With this arrangement, the employees and the companies pay a predefined premium per year or per month. This premium is a percentage of the pensionable salary and depends on the age of the employee but also has a maximum set by the government. This limit has been constant for many years but is recently lowered twice because of the economic crisis, the decreasing mortality probability and because the pension premium is deductible from the annual taxes. (Koninkrijk der Nederlanden, 22 februari 2013)

The maximum premium percentages per age are captured in a pension rates table. There are currently eight DC-rates approved by the Dutch tax authority, four for an actuarial interest rate of 3\% and four for an actuarial interest rate of $4 \%$. The actuarial interest rate is the expected return on the invested premiums. An actuarial interest rate of 4\% assumes a higher return at retirement than a 3\% rate. So the premium rates with a $4 \%$ interest rate are lower than the rates with a $3 \%$ interest rate because less money is needed for the same pension. In case companies would like to use lower rates, they need to make sure the rates per age still be in proportion to the maximum rates. In the table below, the maximum rates are shown for 2013, 2014 and 2015 for the two actuarial interest rates.

| Maximum Rates approved by Dutch Tax Authority |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Age | $\mathbf{2 0 1 3}$ (4\%) | $\mathbf{2 0 1 3}$ (3\%) | $\mathbf{2 0 1 4}$ (4\%) | $\mathbf{2 0 1 4}$ (3\%) | $\mathbf{2 0 1 5}$ (4\%) | $\mathbf{2 0 1 5}$ (3\%) |
| $20-25$ | $6,0 \%$ | $10,2 \%$ | $5,3 \%$ | $9,0 \%$ | $4,7 \%$ | $8,0 \%$ |
| $25-30$ | $7,3 \%$ | $11,8 \%$ | $6,4 \%$ | $10,5 \%$ | $5,7 \%$ | $9,3 \%$ |
| $30-35$ | $8,9 \%$ | $13,8 \%$ | $7,8 \%$ | $12,2 \%$ | $6,9 \%$ | $10,8 \%$ |
| $35-40$ | $10,9 \%$ | $16,0 \%$ | $9,5 \%$ | $14,2 \%$ | $8,4 \%$ | $12,5 \%$ |
| $40-45$ | $13,3 \%$ | $18,6 \%$ | $11,6 \%$ | $16,5 \%$ | $10,2 \%$ | $14,6 \%$ |
| $45-50$ | $16,3 \%$ | $21,7 \%$ | $14,2 \%$ | $19,2 \%$ | $12,5 \%$ | $17,0 \%$ |
| $50-55$ | $20,0 \%$ | $25,5 \%$ | $17,4 \%$ | $22,4 \%$ | $15,4 \%$ | $19,8 \%$ |
| $55-60$ | $24,8 \%$ | $30,1 \%$ | $21,5 \%$ | $26,4 \%$ | $18,9 \%$ | $23,3 \%$ |
| $60-65$ | $31,1 \%$ | $35,9 \%$ | $26,8 \%$ | $31,4 \%$ | $23,6 \%$ | $27,7 \%$ |
| $65+$ | - | - | $31,5 \%$ | $35,7 \%$ | $27,7 \%$ | $31,5 \%$ |

Table 1: Maximum Rates approved by Dutch Tax Authority

### 5.1.1.2 DB Arrangement

The DB arrangement is still the most used arrangement in the current pension plans. DB stands for Defined Benefit. With a DB agreement, the pension at retirement is defined and the level of the premium per year depends on the age of the employees and the funded states of the pension fund or insurance contract. If the investments are lower than expected, an extra premium is needed to reach the agreed pension at retirement. In practice, the difference is paid by the company so the company is faced with the investment risks.

### 5.1.1.3 CDC Arrangement

CDC stands for Collective Defined Contribution. With this agreement, a pension at retirement is agreed and with this pension, a yearly premium for the whole group of employees is determined. But when the investments made with the premium are lower than expected, the pensions at retirement are reduced for every participant in the plan. So with this agreement, the yearly contributions are defined and there is a defined benefit, but the employees are still faced with the investment risks.

### 5.1.2 Franchise

As said, the pension premium per year depends on the age of the employee and the kind of pension agreement. However, this premium is not a percentage of the whole salary but just a part of the salary. The part of the salary which won't be taken into account for the pension arrangement is called the franchise. This part of the pension is expected to be taken care of by the government. In the Netherlands, this is called an AOW-payment and every person gets this payment per year at retirement, now at age 67 . So the pension built up with the premium is an addition to the AOWpayment. The minimal legal franchise is determined by the government every year and depends on the level of the AOW-payment.

### 5.2 Investments for DC-plans

The pension providers invest the deposited money with the ultimate goal to give retirees a decent pension. The investment aspect plays a huge role in the level of the eventual pension. However, an economic crisis, such as the one a few years ago, can result in huge losses on investments and to a lower pension for the retired employees. On the other hand, investments with a low risk, such as a savings account, can lead to lower return on the investments and thus a lower pension at retirement. To counter this problem, pension providers have life cycles that indicate the investment portfolio per age.

### 5.2.1 Life cycles

A life cycle is an investment plan per age or per year till retirement that indicates the percentage that needs to be invested in a certain fund or asset. The main goal of this plan is to get the highest return on the investments with the lowest risk. There are many different life cycles, but most life cycles have one thing in common: the risky investments with the highest expected return are done at the beginning of the life cycle while the safe investment with a lower expected return are done at the end of the life cycle.

The invested capital increases per year, so the most important year is the last year of a life cycle. Thus a loss on the investments in the last year has a far greater impact on the level of the pension than a loss at the beginning of a life cycle. Taking into account the fact that we cannot afford too much risk, it is important to lower the investment risk at the end of the life cycle and make the risky investments at the beginning.

An example of a neutral life cycle from a pension provider is shown below.


Figure 1: A plot of a common neutral Life Cycle
In this life cycle, $76 \%$ of the capital is invested in shares, $7 \%$ in real estate and $18 \%$ in short term bonds in the first part of the life cycle. As of age 42, the percentage invested in shares and real estate decreases per year and the percentage invested in short term bonds increases. As of age 52, the percentage in safe investments increases by using long term bonds until at age $66,95 \%$ is invested in long term bonds and only $5 \%$ in shares. So, with the above life cycle, most risk lies at the beginning of the life cycle and the safest investments are done at the end of the life cycle.

### 5.2.2 Investment categories for DC-plans

When we look at the investments made by the pension providers, we can divide the investments in the following investment categories:

### 5.2.2.1 Shares

Highest expected return, high risk.
Investments in shares are usually made in the beginning of a life cycle. Possible losses can be compensated with gains later on during the life cycle.

## Real Estate

High expected return, highest risk.
Investments in real estate were usually seen as a low risk investment with a relative high expected return. However, since the recent economic crisis the risk in these investments has increased while the expected return did not. So currently not all pension providers invest in real estate but it still can be used to create a diverse portfolio.

### 5.2.2.2 Short Term Bonds

Low expected return, low risk
Investments in short term bonds are usually investments in corporate bonds or bonds of developing countries with a duration of about five years. These investments are usually made throughout the whole life cycle to have a relative save investment with a higher return than zero risk bonds.

### 5.2.2.3 Long Term Bonds <br> Lowest expected return, lowest/no risk

Long term bonds are usually investments made in government bonds with an AA+ rating and with a duration of more than ten years. These investments are made to get a low risk return at the end of the life cycle. Long term bonds also play a role to offset a falling interest rate at retirement.

### 5.2.3 Interest Match

During the life cycle, the portfolio changes from high risk investments with a short duration to a low risk portfolio with a long duration. It is important to have a long duration at the end of the life cycle to match with the insurance tariffs at retirement. This is due to the fact that the value of investments made in bonds decrease if the interest rate in zero coupon bonds increases and vice versa. However, an increasing interest at retirement results in a higher annuity while a decreasing interest can cause a lower annuity. A portfolio at retirement with a duration that matches the length of the expected retirement period offsets this effect and this is called matching. The matching principle is explained in the following example with a zero coupon bond and two scenarios.

A zero coupon bond is a bond that does not pay coupon payments but pays an amount of money at maturity: the face value. The value of a zero coupon bond is discounted to the present value with the following formula:

$$
\text { Zero coupon bond value }=\frac{\text { Face value of bond }}{(1+\text { yield })^{\text {time to maturity }}}
$$

In the following example the value of a zero coupon bond is calculated per year for two scenarios, one with an increasing yield and one with a decreasing yield. The zero coupon bond is used as an investment and is sold at retirement to buy an annuity.

### 5.2.3.1 Interest matching example



As one can see, if the interest rate increases the value of the bond decreases and vice versa. The bond in scenario 2 has a return of more than $50 \%$ in five years while the bond in scenario 1 only has a return of $7 \%$ in the same period. However, the annuity that can be bought with both bonds is approximately the same. ${ }^{1}$

The risk on an increasing or decreasing interest rate can be matched by the right duration of the investments at retirement. With an expected life time of fifteen years from retirement, a duration of about 7,5 years can match the risk on a decreasing or increasing interest rate. This is because a 15 year periodic payment roughly corresponds with an one-time payment in 7,5 years.

[^0]
### 5.3 Annual Pension Payment

At retirement, the portfolio with the investments is sold to buy an annuity. This annuity will be paid each year until the pensioner dies. The level of this annuity depends on a number of factors including the expected remaining lifetime, the current interest rates and the value of the accumulated portfolio. The annuity is calculated by dividing the capital at retirement by an annuity factor used by insurance companies.

The formula for the annuity factor that will be used in this report was introduced in the Dutch book 'Actuariële Wiskunde' by (Heer \& Sattler, 1954). An important remark is the fact that the formula for the annuity factor in the book uses only one interest rate and no evolving mortality rates while the formula that is used in this paper uses evolving yields and mortalities.

### 5.3.1 Annuity factor

The formula for the annuity factor uses the following. Let us define 'age' as the age of retirement, we shall use 67 in this report. Also put ' $j$ ' for the current year. Then define
$p l_{x, i, j}:$ The chance that a man still lives at age $i$ in year $j$
$p l_{y, i, j}:$ The chance that a woman still lives at age $i$ in year $j$
$p d_{x, i, j}:$ The chance that a man dies at age $i$ in year $j$
$p d_{y, i, j}$ The chance that a woman dies at age i in year $j$
$r_{i, j}:$ The interest rate for duration $i$ at year $j$
$p c_{x}:$ The percentage of men, ussualy $75 \%$
$p c_{y}:$ The percentage of women, ussualy $25 \%$

Then the annuity factor can be written with the following formula:

$$
\text { Annuity factor }=p c_{x} *\left(a_{x}+0,70 * a_{x \mid y}\right)+p c_{y} *\left(a_{y}+0,70 * a_{y \mid x}\right)
$$

where:

$$
\begin{aligned}
a_{x} & =\sum_{i=a g e}^{120} p l_{x, i+1, j+i-\text { age }} *\left(1+r_{i-\text { age }+1, j+i-\text { age }}\right)^{-i+\text { age }-1}, \text { pension man } \\
a_{y} & =\sum_{i=\text { age }}^{120} p l_{y, i+1, j+i-\text { age }} *\left(1+r_{i-\text { age }+1, j+i-\text { age }}\right)^{-i+\text { age }-1}, \text { pension woman }
\end{aligned}
$$

$$
a_{x \mid y}=\sum_{i=\text { age }}^{120} p d_{x, i, j+i-\text { age }} * a_{y, i-3} *\left(1+r_{i-\text { age }+1, j+i-\text { age }}\right)^{-i+\text { age }-1}, \text { survivors pension woman }
$$

$$
a_{y \mid x}=\sum_{i=\text { age }}^{120} p d_{y, i, j+i-a g e} * a_{x, i+3} *\left(1+r_{i-\text { age }+1, j+i-\text { age }}\right)^{-i+\text { age }-1}, \text { survivors pension man }
$$

The lower the annuity factor, the higher the resulting pension. The level of the factor is for a great part determined by the level of the interest rate. This is due to the fact that insurers buy bonds with the accumulated capital and use the return on these bonds to pay the annual pension. A high interest rate leads to a higher return on bonds, so a higher pension but a low interest leads to a lower pension. That is why matching your investment portfolio at retirement is so important.

### 5.3.1.1 Assumptions

In this formula, we assume that the chance to live at age 120 is exactly zero. We also assume that female spouses are 3 years younger than male spouses to calculate the survivor's pension. Finally, we assume that the payments take place at the end of the year.

### 5.4 Costs

Pension providers use several costs, some are paid by the employees and some are paid by the companies. There are constant costs like administration costs, risk premiums for partner and orphans pension and disability pension and variable costs like management costs for the investments and costs resulting from purchases and sales of investments. These variable costs are normally paid by the employees by taking a percentage of the investments value or by taking a percentage of the deposit.

### 5.4.1 TER

The costs for managing the portfolio are called the Total Expense Ratio and these costs are dependent on the value of the investments. The TER that pension providers use vary often between $0,20 \%$ and $0,50 \%$ per year.

So if an employee has a portfolio with a value of $€ 100$ at the end of a year and the TER has a value of $0,50 \%$, then the resulting pension costs are $€ 0,50$ in that year. These costs are obtained by the pension provider by selling some of the investments with the same value or by subtracting the costs from a deposit.

### 5.4.2 Buying and selling expenses

Besides management costs, there are also buying and selling costs. These costs are variable and are applicable when investments are sold or purchased. Most of the time, these costs are made because of the trading process but it could also happen that the pension provider takes from every deposit some percentage for its own. These costs often vary between $0,00 \%$ and $0,50 \%$.

### 5.4.3 Rebalance costs

Other variable costs that could occur are rebalance costs. Rebalance costs are costs made by allocating the portfolio via the lifecycle. So it could happen that $50 \%$ of the portfolio must be sold to buy other investments to decrease the risk. However, often investments are traded between employees within in the same pension provider which lead to no switching costs.

It often occurs that a pension provider uses different costs for the different investment categories. In common, shares and real estate have a higher TER and trading costs than bonds, which could even have no trading costs. A lower TER on bonds is preferred because, usually, the investment portfolio consists of almost only bonds and has the highest value at retirement. That is why it is important not to look at the average TER but at the TER for each category.

The final pension is strongly influenced by the costs. A $0.5 \%$ higher TER can reduce the pension by $9 \%$. For an employer that offers a guaranteed pension, this could lead to $9 \%$ higher pension premiums.

## 6 The Pension Simulator

The goal of the pension simulator is to give companies and employees a view of expected pension outcomes and the corresponding costs for the different pension providers. This program can be used to compare pension providers not only on the estimated pension but also on the possible risks

In general, there are three pension arrangements: A defined benefit-, a defined contribution- and a collective defined contribution arrangement. The pension simulator is built to compare DC arrangements.

With a DC arrangement, the employees face the investments risks because the value of the investments made at retirement determine the resulting annual pension. So the most import part of a DC arrangement are the investments and the related costs. This is the reason we only look at variable costs

### 6.1 Input

There are a lot of factors that have influence on the pension outcome with a DC-arrangement. Besides the DC-premiums and the franchise, also the mortality rates and the salary increase play a huge role in the resulting pension. In this chapter, the input of the model will be briefly discussed.


Figure 2: A screenshot of the Pension Simulator

### 6.1.1 Employees-File

In this file all employees (or just one or two that reflect the workforce) are stored. The employees have the following attributes: ID, Name, Salary, Age, FTE (part-time percentage), sex and capital already built up.

### 6.1.2 Retirement age

The retirement age is the age at which the government starts paying its state pension (AOW) and the regular age to retire. The current retirement age is 65, however, people born later than 1950 have to work one month longer each year. At 2023, the retirement age will be 67, which is assumed in the model for all employees.

### 6.1.3 DC premiums

With a DC-arrangement, the company pays a defined premium per year to the pension provider. The premium is a percentage of the salary minus the franchise. This premium depends on the used interest rate, usually $3 \%$ or $4 \%$. A higher interest rate implies a higher expected return on the deposit which results in lower premiums. The used DC rates needs to be approved by the government in order to receive important tax benefits. The Dutch government has approved four rates, two with an actuarial $3 \%$ interest rate and two with a $4 \%$ interest rate.

### 6.1.4 Franchise

With the current pension arrangements, we assume a franchise of $€ 13.449$ and a yearly increase of $2 \%$.

### 6.1.5 Salary Increase Percentages

The level of the salary determines for a large part the annual pension premium. Because of inflation or promotion, the salary of employees can change every year. In the model, we assume that the salary of each employee increases with two percent per year. In this file, additional salary increases can be stored.

For the additional increases, the following percentages will be used:

| Age | Annual salary increase (On <br> top of fixed increase) |
| :--- | :--- |
| $20-24$ | $4 \%$ |
| $25-34$ | $3 \%$ |
| $35-44$ | $2 \%$ |
| $45-54$ | $1 \%$ |
| $55-67$ | $0 \%$ |

Table 2 : Table with the annual salary increase percentages (On top of fixed increase)

### 6.1.6 Maximum Salary

Currently, there is a legislative proposal done by Dutch government that limits the pensionable salary. This limit implies that no pension is accrued above the salary limit and it should be active as of 2015 with a level of $€ 100.000$. The limit will be yearly indexed because of inflation and resulting increasing salaries. (Ministeries van Financiën en van Sociale Zaken en Werkgelegenheid, 15-04-2014)

Currently, we assume a pension limit of $€ 100.000$ with a yearly increase of $2 \%$.

### 6.1.7 Mortality Tables

A mortality table contains the mortality rates per gender, age and year. These tables are produced by The Royal Actuarial Association, the professional association of actuaries and actuarial specialists in the Netherlands. With these tables and the current interest rate, the expected pension per year is calculated which results in an annuity at retirement.

### 6.2 Pension Providers

To compare the different pension providers, information about the life cycles and costs are needed. However, not all providers use the same costs or life cycle structure. So, to compare the providers, all costs and life cycles need to be generalized to the four investment categories.

### 6.2.1 Life cycles

Most pension providers have different life cycles which can be used: A defensive-, neutral- or offensive life cycle. Some providers even offer the possibility to participants to choose their own investments. However, in $90 \%$ of the cases, participants do not change their life cycle. So, to compare the pension providers, we only use their neutral life cycle.

The pension simulator assumes that there are only four different investment categories: Shares, short term bonds, long term bonds and real estate. However, most providers have their own funds and investments, so all investments need to be categorized in the four groups. Eventually, we have per pension provider for the ages 20 till 67 the investment allocation.

### 6.2.2 Costs

Besides the life cycle, also information about the costs per investment category are needed. These costs are a percentage of the value of the investments or the annual premium. The pension simulator uses three types of costs: acquisition- , disposal-, and TER costs.


Figure 3: A screenshot of the provider input screen of the Pension Simulator

### 6.2.2.1 Acquisition costs

The acquisition costs are the costs that are made when an investment is bought. In most cases, these costs are only the broker costs and vary between 10 and 20 base points. However, some providers also take a percentage of each deposited premium as management fee. In general, the purchase costs for shares and real estate are more expensive than bonds but we assume one percentage for all investments. It can also happen that providers do not mention the acquisition costs or even say that these costs are zero in their offer. In these cases, we assume that the acquisition costs are 15 base
points because there are always transaction costs. So, in the pension simulator the acquisition costs are the broker costs plus any provider fee.

### 6.2.2.2 Disposal costs

The disposal costs are the costs that are made when an investment is sold. In most cases, these costs are only the broker costs and vary, just like the acquisition costs, between 10 and 20 base points. The disposal costs only play a role when the life cycle changes and the allocation between the investment groups change. However, in most cases the investments are traded with other participants within the same fund with zero costs. Some providers also subtract a percentage of each disposal as management fee. It can also happen that providers do not mention the disposal costs or even say that these costs are zero in their offer. In these cases, we assume that the disposal costs are 15 base points. Because the disposal costs are hard to program, these costs are only used at the end of the life cycle. So when someone retires and he sells his investments, a percentage of his capital is subtracted. This percentage is the average of the disposal costs of the four investment categories over the whole life cycle plus any management fee.

### 6.2.2.3 TER costs

The TER costs are the costs that are made for managing the investment portfolio. TER stands for Total Expense Ratio and is an annual percentage of the value of the investments. The TER can vary between the different investment categories and in general the TER for shares and real estate are higher than for bonds. In contrast to the acquisition- and disposal costs, one TER for all investment categories is not desirable. This is due to the fact that, in most life cycles, the percentage of the investment portfolio in long term bonds at the end of the life cycle is higher than the other investments. So a low TER for long term bonds is much more desirable than a low TER for shares because the value of the portfolio is the highest at the end of the life cycle.

It can also happen that pension providers apply other variable costs like management costs while other providers apply fixed management costs. In the case of fixed management costs, the costs are paid by the company. With variable management costs, those are subtracted as an annual percentage of the total value of the portfolio. So the TER costs are the TER costs per investment category plus any other annual variable costs.

### 6.2.3 Fixed return

Many employees are afraid to invest their pension premiums and rather put their pension deposit in a savings account with a fixed return and no risk. However, no risks means low investment result means low pension.

To give employees this insight, the pension simulator has a function that adds an extra pension provider with no costs and a fixed return on the investments.

### 6.3 Methodology

The main purpose of the pension simulator is to compare the pension providers on the level of the resulting pension. The resulting pension is obtained by 'walking through' the life cycle of a pension provider. Each year, the company deposits a premium, receives a return on the invested premiums and costs are subtracted from his investments. At retirement, the investments are sold and an annuity is bought. The pension could be invested in four categories: shares, real estate, short term bonds and long term bonds.

The deposited premium is the same for every pension provider while the life cycle and costs vary per provider. The return on the investments differ per employee and provider, however, the return per investment category is the same for all providers in the same year and simulation.

The pension simulator uses the following model to simulate the accumulation of the pension:

### 6.3.1 Model <br> Calculate returns per simulation per year

## Per simulation

Per pension provider

1) Franchise $=$ Starting Franchise
2) Maximum Salary = Maximum Salary

Per year
Per employee
If Age (employee) < retirement age

1) Determine return $r$ (year, simulation, life cycle)
2) Deposit $=$ Premium - acquisition costs (provider)
3) Capital + = Deposit + capital* $r+$ deposit* $^{*}\left((1+r)^{\wedge} 0.5-1\right)$
4) Capital -= Capital * TER (life cycle, provider)
5) Capital -= Rebalance costs(life cycle, provider)
6) Age $+=1$
7) Salary +=Salary * salary increase percentage (age)

If Age (employee) $=$ retirement age

1) Capital -= Capital * disposal costs(provider)
2) Determine Annuity factor (year)
3) Annuity = Capital/ annuity factor

If Age (employee) > retirement age

1) Age $+=1$
2) Franchise += Franchise * franchise increase percentage
3) Maximum Salary += Maximum Salary * salary increase percentage

## Generate output

### 6.3.2 Objects

To calculate the pensions, a number of objects are used. The most important two are the object Person and the object Provider. These objects have several attributes that can be adjusted to keep track of his pension and of the total costs that are made.

### 6.3.2.1 Person

Main attributes: ID, Age, Salary, Capital, FTE, Annuity Factor
The object Person is the most important object of the pension simulator. This object is used to keep track of the person's salary, age and to adjust his capital over time. Instead of using a portfolio to keep track of someone's investments, we use a capital that is adjusted per year with the return on the investments and the deposited premium. We also assume that the FTE will stay the same throughout the simulation.

### 6.3.2.2 Provider

Main attributes: Life cycle, TER, Acquisition and Disposal costs, Made costs, Simulations
Main Functions: Give average/standard deviation/median/min/max/percentile of capital,
The object Provider contains the attributes Life cycle, TER, Acquisition- and Disposal costs to calculate the return and costs per employee per year. This object also contains the attribute Made costs where the costs per year made by this provider are saved. These costs are eventually divided by the number of simulations to obtain the average costs per year. The attribute Simulations is a matrix with the simulations versus the employees and is used to save the persons and it's attributes per simulation.

The object Provider also has a few functions to calculate some statistics of the capital of all employees of one simulation, to calculate the statistics of one employee of all simulations or to calculate the statistics of all employees of all simulations. The functions can calculate the average, standard deviation, median, minimum value, maximum value or the $k^{\text {th }}$ percentile of an array of values.

### 6.3.3 Calculate returns per year per simulation

The most important and yet most difficult part of the model is calculating the returns on the investments. The return on the investments determine for the greatest part the level of the resulting pension. To calculate the returns on investments, we have used three different methods. At this moment, the pension simulator uses method 3 .
6.3.3.1 Method 1: Expectations and Standard Deviations


Figure 4: Screenshot of the investments input screen of the pension simulator in method 1
The first method we used to calculate the return on the investments uses a normal distribution. The user could give the expectation and standard deviation per investment category. Then the return per category, per year and per simulation was calculated and stored in a three dimensional matrix with the following formula:

$$
r_{s, j, i}=n * s t d_{i}+a v g_{i}
$$

where

$$
\begin{aligned}
& s=\text { the simulation with } s=1, \ldots, S \\
& j=\text { the year with } j=1, \ldots, J \\
& i=\text { the investment categorie with: } \\
& i=1, \\
& i=2, \\
& i=3, \\
& i=3 \text { Shorest Term Bonds } \\
& i=4, \\
& l o n g \text { Term Bonds } \\
& i=
\end{aligned}
$$

with:

$$
\begin{aligned}
& r_{s, j, i}=\text { return category } i \text { in simulation } s \text { and year } j \\
& n \sim N(0,1) \\
& \text { std }_{i}=\text { standard deviation investment category } i \\
& \text { avg }_{i}=\text { average investment category } i
\end{aligned}
$$

With this method, we used the following averages, standard deviations and durations:

|  | Average return | Standard Deviation | Durations |
| :--- | :--- | :--- | :--- |
| Shares | $7.13 \%$ | $17.90 \%$ | - |
| Short Term Bonds | $3.25 \%$ | $1.93 \%$ | 5 years |
| Long Term Bonds | $4.25 \%$ | $3.83 \%$ | 15 years |
| Real estate | $6.90 \%$ | $22.17 \%$ | - |

Table 3: Table with the used average, standard deviations and durations for the investments in method 1
These values are obtained from an Asset Liability Management Study performed by Aegon. (Aegon, 2013) In this method, we did not use the durations. The durations were only used to categorize the different funds per provider into the four investment categories.

Eventually, the returns per category per year per simulation are used to calculate one return for a person in a certain life cycle. This return is the return on the investments and is added to the capital for the employee together with the premium minus costs. This is done with the following formulas:

$$
\begin{aligned}
& r_{p, s, j, e}=\sum_{i=1}^{4} l_{p, a_{e}, i} * r_{s, j, i} \\
& C_{e, p, s, j}=C_{e, p, s, j-1} *\left(1+r_{p, s, j, e}\right)+D C_{e, j} *\left(1+\left(\left(1+r_{p, s, j, e}\right)^{0.5}-1\right)-A_{p}\right)
\end{aligned}
$$

where:
$p=$ The pension provider
$s=$ The simulation
$j=$ The year
$e=$ The employee
$i=$ The investment categorie
$a_{e}=$ The age of employee $e$
with:
$r_{p, s, j, e}=$ The return for person $e$ and provider $p$ in simulation $s$ and year $j$
$l_{p, e_{a}, i}=$ The life cycle percentage for investment category $i$, age $a_{e}$ and provider $p$
$r_{s, j, i}=$ The return of category $i$ in simulation $s$ and year $j$
$C_{e, p, s, j}=$ The capital of employee $e$ and provider $p$ in simulation $s$ and year $j$
$D C_{e, j}=$ The premium of person $e$ in year $j$
$A_{p}=$ The acquisition costs of provider $p$

In the above formula, it is assumed that the deposited premium is only a half year in the capital because, usually, the premiums are paid monthly. So the deposited premiums do not get the whole annual return. At the end of the year, also the management costs for the current provider are subtracted from the total of capital, deposit and returns. At retirement, the capital is sold to buy an annuity. With this transaction, selling expenses are subtracted.

### 6.3.3.2 Method 2: Cholesky Decomposition



Figure 5: Screenshot of the investments input screen of the Pension Simulator in method 2
The second method uses the Cholesky Decomposition method to calculate the returns on the investments. The disadvantage of the first method was the fact that there was no correlation between the returns on the investments in the same year. With the Cholesky Decomposition method, the returns are drawn with a Multivariate normal distribution. This draw returns random normally distributed correlated returns.

The Cholesky decomposition was discovered by André-Louis Cholesky and is a way to simulate multiple correlated variables. More information about algorithms using Cholesky can be found in the paper Modified Cholesky Algorithms: A Catalog with New Approaches (Fang \& O'Leary, 2008).

The multivariate normal distribution can be written in the following notation:

$$
\begin{aligned}
& x=\left[x_{1}, x_{2}, \ldots, x_{n}\right] \\
& x \sim N(\boldsymbol{\mu}, \boldsymbol{C}) \\
& \mu=\left[E\left(x_{1}\right), E\left(x_{2}\right), \ldots, E\left(x_{n}\right)\right] \\
& \boldsymbol{C}=\left[\operatorname{Cov}\left[x_{i}, x_{j}\right]\right], \quad i=1,2, \ldots, n ; j=1,2, \ldots, n
\end{aligned}
$$

A multivariate normal distribution works with a vector with $n$ means and anxncoriance matrix. With this distribution, the covariance matrix must be positive definite because, otherwise, one investment category is sufficient. The covariance matrix can be constructed with the given correlation matrix and the standard deviation with the following formula:

$$
\operatorname{Cov}(i, j)=\operatorname{Cor}(i, j) * \operatorname{std}_{i} \operatorname{std} d_{j}
$$

for:

$$
i, j=\text { The investment categorie }
$$

with:

$$
\text { std }_{i}=\text { The standard deviation of investment category } i
$$

In this method, the same averages, standard deviations and durations of method 1 are used. However, also the correlation matrix from the same ALM study was used (Aegon, 2013).

|  | Average return | Standard Deviation | Durations |
| :--- | :--- | :--- | :--- | :--- |
| Shares | $7.13 \%$ | $17.90 \%$ | - |
| Short Term Bonds | $3.25 \%$ | $1.93 \%$ | 5 years |
| Long Term Bonds | $4.25 \%$ | $3.83 \%$ | 15 years |
| Real estate | $6.9 \%$ | $22.17 \%$ | - |

Table 4: Table of the averages, standard deviations and durations of the investments in method 2
Correlation matrix:

|  | Shares | Short Term Bonds | Long Term Bonds | Real Estate |
| :--- | :--- | :--- | :--- | :--- |
| Shares | 1 | -0.1 | -0.1 | 0.6 |
| Short Term Bonds | -0.1 | 1 | 0.9999 | 0 |
| Long Term Bonds | -0.1 | 0.9999 | 1 | 0 |
| Real estate | 0.6 | 0 | 0 | 1 |

With the correlation matrix and the standard deviations, the following covariance matrix can be constructed:

|  | Shares | Short Term Bonds | Long Term Bonds | Real Estate |
| :--- | :--- | :--- | :--- | :--- |
| Shares | 0.032041 | $-3.4547 \mathrm{E}-4$ | $-6.8557 \mathrm{E}-4$ | 0.02381058 |
| Short Term Bonds | $-3.4547 \mathrm{E}-4$ | $3.724900000 \mathrm{E}-4$ | $7.391826081 \mathrm{E}-4$ | 0 |
| Long Term Bonds | $-6.8557 \mathrm{E}-4$ | $7.391826081 \mathrm{E}-4$ | 0.00146689 | 0 |
| Real estate | 0.02381058 | 0 | 0 | 0.04915089 |

Table 6: Covariance matrix of the investments in method 2
The rest of the method remains the same as method 1.

### 6.3.3.3 Method 3: Yield Curve



Figure 6: Screenshot of the investments input screen in method 3
The third method we used to calculate the returns on the investments uses a yield curve. This curve consists of the current spot rates that show the spotted return of zero coupon bonds per maturity. With the spot rates, the forward rates can be calculated. The forwards show the returns per maturity per year that should occur in an economic perfect world. The forwards are used to determine the return of bonds per year in the future.

### 6.3.3.3.1 Nelson-Siegel-Svensson method

The spot rates are the rates of zero coupon bonds per maturity that the market is willing to offer at the moment. The spot rates can be estimated with the Nelson-Siegel-Svensson with the use of a number of parameters. The curve produced by the Nelson-Siegel-Svensson method tries to fit a curve to the observed spot rates. The European Central Bank publishes new parameters of the Nelson-Siegel-Svensson model every day. With these parameters, the yield curve of the workday before can be produced.

The Nelson-Siegel model was first introduced by Nelson and Siegel in the paper: Parsimonious Modeling of Yield Curves The purpose of the article was to introduce a simple, parsimonious model that is flexible enough to represent the range of shapes generally associated with yield curves: monotonic, humped and S shaped (Nelson \& Siegel, 1987).

Svensson added an extra 'hump' to the formula in the paper: Estimating and Interpreting Forward Interest Rates. In this paper, Svensson demonstrated the use of forwards as monetary policy indicator using Sweden 1992-1994 as an example. He estimated the forward rates with an extended and more flexible version of the Nelson-Siegel model (Svensson, 1994). The Nelson-Siegel-Svensson model is widely seen as one of the best ways to model the yield curve and for estimating forward rates.

The Nelson-Siegel-Svensson method uses the following formula to produce the spot rates with maturity m:

$$
y(m)=\beta_{0}+\beta_{1} \frac{\left[1-e^{\left(-\frac{m}{\tau_{1}}\right)}\right]}{m / \tau_{1}}+\beta_{2}\left(\frac{\left[1-e^{\left(-\frac{m}{\tau_{1}}\right)}\right]}{\frac{m}{\tau_{1}}}-e^{\left(-\frac{m}{\tau_{1}}\right)}\right)+\beta_{3}\left(\frac{\left[1-e^{\left(-\frac{m}{\tau_{2}}\right)}\right]}{\frac{m}{\tau_{2}}}-e^{\left(-\frac{m}{\tau_{2}}\right)}\right)
$$

with:
$y(m):$ The yield of a zero coupon bonds with maturity $m$
$\beta_{0}:$ The long - term component
$\beta_{1}:$ The short - term component
$\beta_{2}:$ The medium - term component
$\beta_{3}:$ The extra medium component introduced by Svensson
$\tau_{1}:$ The decay factor
$\tau_{2}:$ The extra decay factor introduced by Svensson

### 6.3.3.3.2 Forwards

With the spot rates, the forward rates can be calculated. Forwards are the future yield of zero coupon bonds that should apply in a perfect economic world in the future. We now denote the spot rates $y(m)$ calculated with the Nelson-Siegel-Svensson (NSS) method by $f_{m, 0}$. Then the forward rates $f_{m, y}$ with maturity $m$ in year can be calculated with the following formula:

$$
f_{m, y}=\frac{\left(1+f_{m+1, y-1}\right)^{\frac{1}{m}+1}}{\left(1+f_{1, y-1}\right)^{\frac{1}{m}}}-1
$$

with:

$$
\begin{aligned}
& m: \text { The maturity } \\
& y: \text { The year }
\end{aligned}
$$

and:
$f_{m, y}:$ The forward rate with maturity $m$ in year $y$
$f_{m, 0}:$ The spot rates calculated with the NSS method

The idea behind forwards is that the return in a certain time period on zero coupon bonds is always the same. For example, if you have a bond in year 1 with maturity 2 and a certain return, then the return on a bond in year 1 with maturity 1 and a bond in year 2 with maturity 1 should give the same return. So, for example:

$$
\begin{aligned}
& f_{m=1, y=1}=1.00 \% \text { and } f_{m=1, y=2}=1.50 \% \\
& \left(1+f_{m=2, y=1}\right)^{2}=(1+1 \%) *(1+1.5 \%) \\
& f_{m=2, y=1}=\sqrt[2]{(1+1 \%) *(1+1.5 \%)}-1 \approx 1.250 \%
\end{aligned}
$$

### 6.3.3.3.3 Expected return bonds

With the forwards, the expected return on zero coupon bonds can be calculated per year. In this paper, we assume that there are only two bonds, a short term bond with a duration of five years and a long term bond with a duration of fifteen years. With a DC-agreement, it is assumed that the deposited premiums are invested according to the current life cycle of an employee. So every year, a new bond contract is bought with a certain maturity and added to the portfolio. Eventually, the duration of all the short term bonds should be on average five years and with the long term bonds on average fifteen years. However, when all purchased bonds have a maturity of respectively five and fifteen year, the average duration is not equal to the desirable duration. This is due to the fact that the maturity of the bond decreases every year until the bond expires. This can be shown in the following example for the short term bonds:

|  | Year 1 | Year 2 | Year 3 | Year 4 | Year 5 | Year 6 | Year 7 | Year 8 | Year 9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Bond 1 | M 5 | M 4 | M 3 | M 2 | M 1 | M 5 | M 4 | M 3 | M 2 |
| Bond 2 | - | M 5 | M 4 | M 3 | M 2 | M 1 | M 5 | M 4 | M 3 |
| Bond 3 | - | - | M 5 | M 4 | M 3 | M 2 | M 1 | M 5 | M 4 |
| Bond 4 | - | - | - | M 5 | M 4 | M 3 | M 2 | M 1 | M 5 |
| Bond 5 | - | - | - | - | M 5 | M 4 | M 3 | M 2 | M 1 |
| Duration | 5 | 4.5 | 4 | 3.5 | 3 | 3 | 3 | 3 | 3 |

Table 7: Table of average duration of a portfolio consisting of short term bonds
As one can see, the average duration will converge to a duration of three years while a duration of five years for the short term bonds is desirable. The desired average duration of five years for the short term bonds and fifteen years for the long term bonds can be accomplished by investing every year in bonds with a maturity of respectively nine- and twenty-nine years. This is shown for the short term bonds in the following table.

|  | Year 1 | Year 2 | Year 3 | Year 4 | Year 5 | Year 6 | Year 7 | Year 8 | Year 9 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Bond 1 | M 9 | M 8 | M 7 | M 6 | M 5 | M 4 | M 3 | M 2 | M 1 |
| Bond 2 | - | M 9 | M 8 | M 7 | M 6 | M 5 | M 4 | M 3 | M 2 |
| Bond 3 | - | - | M 9 | M 8 | M 7 | M 6 | M 5 | M 4 | M 3 |
| Bond 4 | - | - | - | M 9 | M 8 | M 7 | M 6 | M 5 | M 4 |
| Bond 5 | - | - | - | - | M 9 | M 8 | M 7 | M 6 | M 5 |
| Bond 6 | - | - | - | - | - | M 9 | M 8 | M 7 | M 6 |
| Bond 7 | - | - | - | - | - | - | M 9 | M 8 | M 7 |
| Bond 8 | - | - | - | - | - | - | - | M 9 | M 8 |
| Bond 9 | - | - | - | - | - | - | - | - | M 9 |
| Duration | 9 | 8,5 | 8 | 7,5 | 7 | 6,5 | 6 | 5,5 | 5 |

The expected return on the bonds is obtained by taking the average of the forwards in that year. This average is used in the Cholesky Decomposition of method two with a certain standard deviation to obtain the correlated returns in a year.

### 6.3.3.3.4 Changing yield curve

In the model, the spot rates for zero coupon bonds per maturity are calculated with the Nelson-Siegel-Svensson method. With the spot rates, the forward rates of zero coupon bonds are calculated per year per maturity in such way there is time-independence. This results in the following average expected returns per bond:


Figure 7: Plot of average expected return of bonds per year
As one can see, the returns fluctuate over time and this has impact on the value of the bonds. An increasing return results in a decreasing value of the bonds and a decreasing return results in an increasing value. This is due to the fact that when a bond is bought, a constant annual interest rate is determined. However, when the interest rates of zero coupon bonds increase, a higher return can be accomplished for the same price and this results in a decreasing value of the bonds already owned. On the other hand, when the interest rates of zero coupon bonds decrease, the return on bonds already owned is higher than the return on the current bonds and this results in an increase in the value of the bonds already owned.

This phenomena is programmed in the following way:

$$
\hat{r}_{i, j, s}=r_{i, j, s}-\left(a v g_{j, d}-a v g_{j-1, d}\right) * \text { duration }_{i}
$$

where:

$$
\begin{aligned}
& i: \text { The investment cate gory (short - or long term bonds) } \\
& j: \text { The year } \\
& s: \text { The simulation } \\
& d: \text { The maturity in which is invested (9 or } 29)
\end{aligned}
$$

with:
$\hat{r}_{i, j, s}$ : The adjusted return of investment category $i$ in year $j$ and simulation $s$
$r_{i, j, s}$ : The observerd return of investment category $i$ in year $j$ and simulation $s$ $a v g_{j, d}$ : The average of the forwards with maturity 1 to $d$ in year $j$ duration $_{i}$ : The duration of investment category $i$ (5 or 15)

For example, an absolute increase of $0.20 \%$ of the interest results in a decrease in the value of $1 \%$ for the short term bonds and $3 \%$ for the long term bonds. So the return on short term bonds is less dependent on the fluctuating yield curve than long term bonds.

### 6.3.3.3.5 Risk Premium

In reality, pension providers do not only invest in zero coupon bonds like triple-A government bonds but also in bonds with a risk on default like corporate bonds and developing country bonds. To compensate the extra risk, a risk premium is added to the return obtained via the NSS-method. The level of the risk premium was obtained by comparing the average returns of this method with the expected return of the Asset Liability Management Study performed by Aegon.

### 6.3.3.3.6 Summarized

The return on investments is obtained in the following way:

1) Calculate the spot rates with the Nelson-Siegel-Svensson method
2) Calculate the forward rates with the spot rates
3) Simulate the return per investment category per simulation per year For each simulation

## For each year

a) Get sample with Multivariate Normal distribution using standard deviations and means of return investments
i) Means shares and real estate are deterministic
ii) Standard Deviations are deterministic
iii) Mean short term bonds for year j are obtained by taking the average of forwards year $j$ with maturity 1 to 9 year
iv) Mean long term bonds for year $j$ are obtained by taking the average of forwards year $j$ with maturity 1 to 29 year
b) Adjust return bonds in year j by the difference of the average return forwards of year j-1 times the duration
c) Add risk premium to return bond
4) Return year j in simulation s is weighted with the lifecycle

### 6.3.3.3.7 Input returns

In this method, the same averages, standard deviations and durations of method 1 are used, just as the correlation matrix of method 2 (Aegon, 2013). However, in this method the risk premium per investment category and the Nelson-Siegel-Svensson parameters are added.

|  | Average return | Standard Deviation return | Durations | Risk Premium |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Shares | $7.13 \%$ | $17.90 \%$ | - | - |
| Short Term Bonds | $*$ | $0.5 \%$ | 5 years | $1.5 \%$ |
| Long Term Bonds | $*$ | $0.25 \%$ | 15 years | $2.0 \%$ |
| Real estate | $6.9 \%$ | $22.17 \%$ | - | - |

Table 9: Table with averages, standard deviations, durations and risk premiums of investments in method 3

* Average return is determined with the forwards

Correlation matrix:

|  | Shares | Short Term Bonds | Long Term Bonds | Real Estate |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Shares | 1 | -0.1 | -0.1 | 0.6 |  |
| Short Term Bonds | -0.1 | 1 | 0.9999 | 0 |  |
| Long Term Bonds | -0.1 | 0.9999 | 1 | 0 |  |
| Real estate | 0.6 | 0 | 0 | 1 |  |
|  | Table 10: Correlation matrix of investments in method 3 |  |  |  |  |

With the correlation matrix and the standard deviations, the following covariance matrix can be constructed:

|  | Shares | Short Term Bonds | Long Term Bonds | Real Estate |
| :--- | :--- | :--- | :--- | :--- |
| Shares | 0.032041 | $-3.4547 \mathrm{E}-4$ | $-6.8557 \mathrm{E}-4$ | 0.02381058 |
| Short Term Bonds | $-3.4547 \mathrm{E}-4$ | $3.724900000 \mathrm{E}-4$ | $7.391826081 \mathrm{E}-4$ | 0 |
| Long Term Bonds | $-6.8557 \mathrm{E}-4$ | $7.391826081 \mathrm{E}-4$ | 0.00146689 | 0 |
| Real estate | 0.02381058 | 0 | 0 | 0.04915089 |

Table 11: Covariance matrix of investments in method 3
The Nelson-Siegel-Svensson parameters are obtained from the ECB website ${ }^{2}$.

| Parameter | Value |
| :--- | :--- |
| $\boldsymbol{\beta}_{\mathbf{0}}$ | 1.048912 |
| $\boldsymbol{\beta}_{\mathbf{1}}$ | -0.969635 |
| $\boldsymbol{\beta}_{\mathbf{2}}$ | 24.505947 |
| $\boldsymbol{\beta}_{\mathbf{3}}$ | -21.232726 |
| $\boldsymbol{\tau}_{\mathbf{1}}$ | 6.448378 |
| $\boldsymbol{\tau}_{\mathbf{2}}$ | 4.935579 |

Table 12: Table with NSS-parameters in method 3
The rest of the method remains the same as method 2 .

[^1]
## 7 Results

The pension simulator simulates the accumulated pension at retirement for one or more pension providers and for one or more persons. To compare the expected pension and corresponding risks per provider, the pension simulator produces a number of graphs. At Focus Orange, we usually compare the result per provider for the following two straw man:

| Straw Man | Age | Annual Salary | FTE |
| :--- | :--- | :--- | :--- |
| $\mathbf{1}$ | 30 | $€ 40.000$ | $100 \%$ |
| $\mathbf{2}$ | 40 | $€ 65.000$ | $100 \%$ |

Table 13: Table with used straw man
The straw man can vary per selection process and should reflect the current population of the corresponding company. The output is based on real data from real pension providers. The costs and life cycles per provider are listed in the attachments. For the two straw man, the following insights are produced:

### 7.1 Boxplot

The boxplot shows the spread of the annual pension at retirement for one or more employees. The Boxplot gives a good view of the expected pension and the best- and worst case scenarios.

To put the results of the providers in perspective, another pension provider is added. This provider has no costs and no investments are made but there is a steady annual $2.0 \%$ return.

The boxplot shows the $5^{\text {th }}, 25^{\text {th }}, 50^{\text {th }}, 75^{\text {th }}$ and $95^{\text {th }}$ percentile of the sorted results per provider.

Boxplot: Straw Man 1


Figure 8: Example of boxplot


Figure 9: Boxplot of pension for straw man 1
The boxplot above shows the annual pension for straw man 1. This straw man starts with a salary of $€ 40,000$, ends with a salary around $€ 120,000$ and an average salary around $€ 80,000$ according to the used salary increase percentages.

As one can see, the steady $2 \%$ return provider gives an annual pension around $€ 30.000$, the lowest compared to the other providers. The median of the other providers lies around $€ 45.000$ but the spread can differ a lot per provider. So one could conclude that a steady $2.0 \%$ return gives an assurance on a low pension and is in almost all the cases the worst option.

It is hard to tell which provider gives the best pension. Provider A and Provider F are the providers with respectively the lowest and highest spread, but the median is approximately the same. So the conclusion could be drawn the risk on a low pension is higher with Provider F and Provider A is preferred. However, Provider F has a high upward potential while its downward potential is much smaller.

### 7.2 Scatter plot

To give better insight in the risk and the expected pension, the program also produces another graph, a scatter plot with the expected pension versus the standard deviation. The provider with the steady $2.0 \%$ return is excluded from the graph because the boxplot already crossed out the option.


Figure 10: Scatterplot of expected pension versus risk for straw man 1
This plot gives another perspective to the same results of the boxplot for straw man 1 . While the median of all providers seemed to be approximately the same, the average pension differs a lot per provider. On the basis of this plot, Provider E and Provider G would not be selected. Respectively Provider A and Provider B give approximately the same expected pension with a lower standard deviation. The same applies also to Provider F and Provider D, Provider D offers a slightly lower expected pension than $F$ while the risk at $F$ is much higher. However, this plot does not show the potential upside of a lifecycle that Provider $F$ has.

### 7.3 Costs

The expected pension at retirement already considers the costs that are made with the investments. However, in contrast to the lifecycles, the costs are often negotiable. So to give the employees and the company more insights in the costs, the following graphs are produced:


Figure 11: Plot of yearly- and cumulative costs for straw man 1

The costs per provider consists of four expenses: The Total Expense Ratio(TER), the acquisition-, the disposal and rebalance costs. The TER are the annual management costs, the acquisition costs are the costs made to buy investments, the disposal costs are the costs that occur when selling investments and the rebalance costs are the costs that are made when the life cycle is reallocated.

Some of the costs per provider per year are not smooth but consist of peaks and dips. These peaks occur when the life cycle is rebalanced to reduce the risk of the investments. It can also occur that some providers are cheaper than other providers in the beginning of the life cycle but are more expensive at the end. This is due to the fact that most life cycles begin with shares as largest investment and end with bonds. When the TER of shares are lower than the average but the TER of bonds are higher than the average, the provider will be cheaper at first but will be more expensive at the end of the life cycle.

As one can see, the cumulative costs per provider can vary between $€ 30,000$ and $€ 50,000$ at retirement for straw man 1 . With an annuity factor of 20 , a decrease in costs of $€ 20,000$ can result in an increase of the annuity at retirement of $€ 1,000$.

## 8 Conclusions

The main goal of the Pension Simulator is to select the best provider for the employees and the company with a Defined Contribution (DC) agreement. However, the Pension Simulator focusses on maximizing the expected pension at retirement and less on minimizing the costs for the company or the quality of the whole service. So what is the best pension arrangement for an employee?

### 8.1 Investments

With a DC agreement, the return on the investments determines for a large part the pension at retirement. We already saw that a steady $2.0 \%$ annual return on the deposited premiums does not provide a significant pension, so the deposited premiums should be invested.

The highest return on investment can be obtained with shares and real estate, however, these investments also have the highest risk. To reduce the risk, one should also invest in bonds but bonds have a lower expected return than shares and real estate. However, investing in too much different funds can lead to high rebalancing costs.

An important remark: "results from the past do not give guarantees for the future." However, most providers note in their offers the past returns of the investments. To compare the investments of the different providers, the Pension Simulator uses general expected returns and standard deviations per investment category. These assumptions can be adjusted with every ALM-study to match the expectations of the market and the client.

### 8.2 Life cycle

The allocation of the investments per age is captured in a life cycle. The goal of a life cycle is to get the highest expected return with the lowest risk. In general, all life cycles begin with a high percentage in shares and a low percentage in bonds and end with a high percentages in bonds and a low percentage in shares. So the highest risk and highest return lie at the beginning of the life cycle and the lowest risk and lowest return at the end of the life cycle.

To match possible falling interest rates, and to reduce the investment risk, it is important to have a significant proportion of bonds in the portfolio a number of years before retirement. However, starting too early with the reduction of the percentage of shares in the life cycle will result in a low return on the invested premiums and a low pension at retirement.

### 8.3 Costs

In general, the management costs are the greatest expense of a DC agreement. The management costs can differ between the investment categories and in most cases the TER of shares is higher than bonds. This is preferable because the value of the portfolio is the highest at the end of the life cycle when most of the portfolio consists of bonds.

## 9 Improvements

The Pension Simulator gives insight in the expected pension and corresponding risks for Defined Contribution pension agreements. The level of the pension is determined for a large part by the return on the investments and the interest rate at retirement. The simulation of the returns is the hardest part to program and could be improved in several ways.

In the current method to simulate the returns, method 3, the returns of shares and real estate are determined with a multivariate normal distribution. However, in reality the returns of shares and real estate follow a skewed normal distribution. With this distribution, the chance on a loss remains the same but the level of the loss increases.

The zero coupon interests are obtained using the forward rates determined with the Nelson-SiegelSvensson model. These forwards are the interest rates that should apply in a perfect economic world but in reality the forwards can vary a lot. Because only one scenario for the zero coupon bonds is used, the yield at retirement is constant so the annuity factor is constant for all simulations. It would be better to use another scenario for each simulation. These scenarios could be created by using different parameters for the Nelson-Siegel-Svensson model per simulation or by adjusting the current one with a random factor.

### 9.1 Method 4

Currently, I am working on a new method in cooperation with Strategeon. Strategeon is a consultancy company advising pension funds on investment strategies. Strategeon currently helps Focus Orange in the valuation of the investment strategy of pension providers by also using use the results of the Pension Simulator.

This new method uses four time series as input instead of the means, standard deviations and correlations per investment category. Per simulation, the returns on the investments are forecasted with an Auto Regressive model. The advantage of this new method is the fact there is not only correlation between the investments in one year but there is also correlation with previous years. Another benefit is the fact that, with this method, the zero coupon yields used to calculate the return on bonds and the annuity factor differ per simulation. This results in a different annuity factor per simulation and a better simulation of the interest match.

### 9.2 DB and CDC

At this moment, I am also programming the other two pension arrangements, the DB- and CDC arrangement in the Pension Simulator. With this expansion, Focus Orange can give companies insight in the expected pension, corresponding risks and costs between the three pension agreements.

## 10 Bibliography

Aegon. (2013). Verzekerde oplossing: schets van toegevoegde waarde. Amsterdam.
European Central Bank. (n.d.). Euro area yield curve. Retrieved from Website of the European Central Bank: http://www.ecb.europa.eu/stats/money/yc/html/index.en.html

Fang, H.-r., \& O'Leary, D. P. Modified Cholesky Algorithms: A Catalog with New Approaches. Math. Program 115 (2008), no. 2, Ser. A, 319-349

Heer, D. W., \& Sattler, J. (1954). Actuariële Wiskunde. 's-Gravenhage: Martinus Nijhoff.
Koninkrijk der Nederlanden. (22 februari 2013). Loonheffingen. Pensioenen; beschikbarepremieregelingen en premie- en kapitaalovereenkomsten. Staatscourant.

Ministeries van Financiën en van Sociale Zaken en Werkgelegenheid. (15-04-2014). Wet verlaging maximumopbouw- en premiepercentages pensioenen en maximering pensioengevend inkomen.

Nelson, C. R., \& Siegel, A. F. (1987). Parsimonious Modeling of Yield Curves. Journal of Business, 473489.

Svensson, L. (1994). Estimating and Interpreting Foreward Interest Rates: Sweden 1992-1994. Papers, 579 - Institute for International Economic Studies.

## 11 Attachments

### 11.1 Providers

### 11.1.1 Costs

The costs per provider as a percentage of the premium or the value of the investments. The costs are equal over time.

|  | TER Costs |  |  |  |  | Acquisition Costs |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | Bonds (S) | Bonds(L) | Real Estate | All Categories | All Categories |  |
| Provider A | $0,41 \%$ | $0,47 \%$ | $0,28 \%$ | $0,71 \%$ | $0 \%$ | $0 \%$ |
| Provider B | $0,37 \%$ | $0,37 \%$ | $0,27 \%$ | $0,37 \%$ | $0,17 \%$ | $0,17 \%$ |
| Provider C | $0,38 \%$ | $0,33 \%$ | $0,35 \%$ | $0,43 \%$ | $0,15 \%$ | $0,14 \%$ |
| Provider D | $0,33 \%$ | $0,43 \%$ | $0,24 \%$ | - | $0,25 \%$ | $0,14 \%$ |
| Provider E | $0,46 \%$ | $0,24 \%$ | $0,35 \%$ | - | $0,20 \%$ | $0,09 \%$ |
| Provider F | $0,45 \%$ | $0,32 \%$ | $0,32 \%$ | - | $0 \%$ | $0 \%$ |
| Provider G | $0,39 \%$ | $0,51 \%$ | $0,39 \%$ | $0,69 \%$ | $0 \%$ | $0 \%$ |

### 11.1.2 Life Cycles

The allocation of the life cycle per provider per age per investment category:




[^0]:    ${ }^{1}$ The annuity factor in this example is calculated without using mortality chances. The factor is calculated based on the interest rate at retirement and a duration of fifteen years. The formula for the annuity factor is explained in chapter 5.3.

[^1]:    ${ }^{2}$ Obtained from the ECB website at 3 June 2014: (European Central Bank, sd)

