The Price of a Mortgage

The influence of the embedded options and
the market interest rate on the price of a mortgage

BMI-paper
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Preface

The paper that lies before you is the final report of a literature research that is an element of the study Business Mathematics & Informatics at the Free University of Amsterdam. The study is a combination of three fields: Economics, Mathematics and Computer Science.
The research considers the price of a mortgage. It consists of all of these three fields: Mortgage is an economic topic, mathematical aspects that are needed to determine the price of a mortgage are described and as an example a simulation program is written to determine the price of a mortgage.

During my research Dr. Harry van Zanten supervised me. I would like to thank him for his time, advice and critics.

Yonina Karg,
Amstelveen, June 2004

Abstract

This paper describes certain mathematical aspects that are needed to determine the price of a mortgage that financial institutions offer their clients. The price of a mortgage depends on the daily fluctuations of the market interest rate and also on the embedded options. The embedded options are certain rights that clients have. This paper discusses these options with special attention to the prepayment option. There are several interest models in the literature to forecast the development of the market interest rates.

As an illustration of how to determine the price of a mortgage, we describe a simulation technique that uses the Hull & White interest model and includes the prepayment option.

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Introduction

The mortgage market is one of the largest sectors of the debt market in the world. In the Netherlands, the number of house owners is increasing enormously. In most cases, a great part of the money to buy a house must be borrowed in the form of some sort of mortgage. The financial institutions that provide mortgages run risks from the moment the offer for a mortgage has been made till the last term of redemption. One of the causes for this is that the clients have certain rights, usually called *options*. So when determining the price of the mortgage, these options should also be considered. Another reason why financial institutions run risks, is because they have to deal with the daily fluctuations in the market interest rate. There are many models to predict the course of the interest rate. This paper concerns the influence of these two aspects on the price of a mortgage. In Chapter 1 an introduction of a mortgage is given including a description of interest and the embedded options in a mortgage. In Chapter 2 and Chapter 3 further attention will be paid to respectively the prepayment option and interest rate models. Chapter 4 describes a way to determine the price of a mortgage by means of a Hull & White interest rate model while considering the prepayment option. Finally, this paper will be ended in Chapter 5 with a conclusion and some recommendations.

CHAPTER 1
Mortgage: An Introduction

What is a mortgage?

Most homebuyers have to borrow money from financial institutions (e.g. banks, insurance companies, pension funds) to be able to buy their home. The home loan they receive is called a *mortgage*. Generally, a mortgage is a loan of money to the homeowner secured on a property; the home is collateral for the loan. If the client does not pay the debt, the lender has the right to take back the property and sell it to cover the debt. The mortgage contract is a legal contract in which is stated that the debt will be paid, with interest and other costs, typically over 15 to 30 years. To repay the debt, the client makes monthly instalments or payments that include the principal and the interest. Other costs like taxes and insurance (e.g. life insurance) are also included, but will not be discussed in this paper.

Principal -- The principal is simply the sum of money that is borrowed to buy the home.

Interest -- Usually expressed as a percentage called the interest rate. Interest is what the lender charges the client for borrowing the money. The interest rate depends on the market rate at the time the mortgage is closed, and on the length of the period for which the interest rate is determined.

Fixed-rate period -- The interest rate is determined for a fixed period (1-20 years). This period is called the fixed-rate period.

1.1 Types of mortgages

In the past there were mostly two types of mortgages, an annuity mortgage and a mortgage based on linear redemption. These mortgages still exist, but seemed to be not very flexible. That is why many other mortgages have arisen. Nowadays, a client can choose from many different types of mortgages [www.homeinvest.nl]. For all mortgages it holds that the client borrows an amount of money that has to be repaid in a certain period. During this period interest is being paid over the outstanding debt. There are many ways that a client can pay off the mortgage loan plus interest. Several mortgages are described below:

Mortgages where the loan is being paid off during the mortgage term by periodical payments. Each payment includes a small amount of redemption, so the debt becomes less during the term. The interest that has to be paid monthly decreases also.

- *Annuity mortgage*

  During the mortgage term the client pays a fixed amount every month to the financial institution; the annuity. The annuity includes the interest and redemption. In the beginning the annuity includes a large amount of interest and a small amount of redemption. During the mortgage term, the amount of interest that is being paid becomes less and the amount of redemption becomes more.

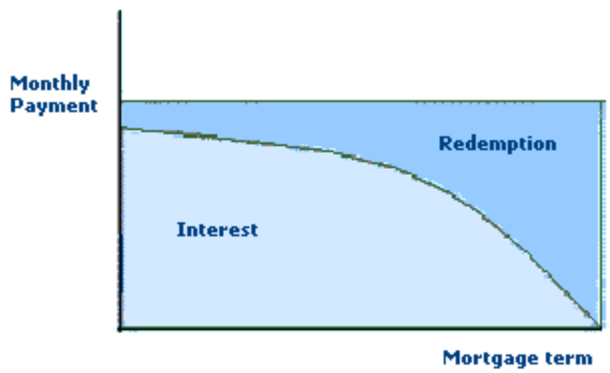

Figure 1.1: Annuity mortgage

- *Linear mortgage*

  The client pays off a fixed amount of the mortgage during the period. This amount is equal to the loan divided by mortgage term. If the term is 30 years (=360 months) the clients pays each month 1/360 part of the loan.

  Besides the fixed redemption, the client has to pay interest over the rest of the debt. Because the debt decreases, the interest decreases. So in the beginning the monthly payments are high but become less during the term.

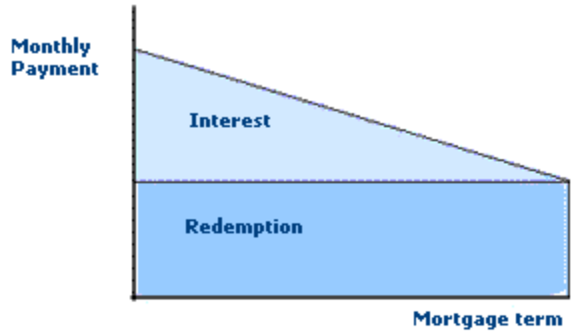

Figure 1.2: Linear mortgage

2 Mortgages where the loan is paid off at the end of the mortgage term. The final redemption is arranged by closing a life insurance at the same time the mortgage is being closed, or by making an investment.

- *Life mortgage*

  The client borrows money from a financial institution and at the same time he closes a life insurance. During the term the loan is not being paid off. The monthly payments include the interest and a premium for the life insurance. In this premium a savings part, insurance and certain costs is included. The savings part is being invested by the financial institution. At the end of the mortgage term, the savings part is used to pay off the mortgage loan. The amount of the saving is not at hand fixed, because this depends on the profit of certain investment funds. So it is not guaranteed that the savings amount will be 100% of the mortgage loan.

  An advantage of the mortgage loan though, is that when the client dies before the end of the term, the mortgage will be paid off and the house will be debt-free for the relatives.

- *Savings mortgage*

  During the mortgage term the loan is not being paid off. As in case of a life mortgage, the monthly interest is constant. When closing a savings mortgage, a savings account is closed. Each month a premium is paid which includes the saving and certain costs. The saving is deposited into the savings account. The interest rate that is gained of this savings account is equal to the interest rate that is paid for the mortgage.

  At the end of the mortgage term it is guaranteed that the sum that is saved is enough to pay off the total mortgage. This sum is exact the mortgage loan, nothing more and nothing less.

- *Investment mortgage*
  The monthly payments include only interest and a premium. No redemption is being paid during the term of the mortgage. Part of the premium is to invest in selected funds or stocks. The client selects the funds himself. During the period, the client can sell or buy the stocks. This is the client's full responsibility. The profit of the investment can be higher than the interest costs for the mortgage. But it can also go the other way. Investments always come with risks. This causes an uncertainty in the sum of the money at the end of the mortgage period. It is not guaranteed that the profit will be enough to pay off the mortgage loan.

- *Stocks-mortgage*
  At the start of the mortgage period money is invested in stocks. This is a once-only action. No redemption is being paid during the mortgage term, only interest. At the end of the period the mortgage is paid off with the profit of the investment. Because investments always come with risks, it is not guaranteed that the profit will be enough to for the redemption.

3 Mortgages where the loan is paid off at the end of the mortgage term but where the final redemption is not arranged.

- *Redemption-free mortgage*
  The mortgage loan is not being paid off during the mortgage term. The monthly payments include only interest, which are constant. This type of mortgage has the lowest monthly mortgage costs. But at the end of the term the client has to pay off the loan and this redemption is not arranged.

4 A combination of different types of mortgages.

- *Combination mortgage*
  This is a mortgage that is a combination of the given types of mortgages. It is mostly a combination of a savings-/life-/investment mortgage where part of it is redemption-free. The advantage of the redemption-free part is a lower monthly payment, but the disadvantage is that part of the mortgage loan is not being paid off.

This is just a selection of the number of types of mortgages there are in the market. The mortgage that is most suitable for a client depends on the situation of the client, for instance, his financial obligations and how long he intends to keep the house. Usually a combination of these types is the best type. Finally, a mortgage is chosen that is both best for the client *and* the financial institution. This paper is written from the financial institutions' point of view; what is the profit for these institutions when lending money? In other words, how much is the mortgage worth to the financial institution: *the price of the mortgage.*

1.2 The price of a mortgage

While the interest that the client has to pay is fixed, there is an uncertainty in the cash flow of the financial institution. Financial institutions enter into a number of mortgages daily. They have to finance the money they lend to clients themselves and have to deal with the market *interest* rate that fluctuates daily. This is one of the causes for the uncertainty in the cash flow. Many *options* embedded in the mortgage also create risk on the mortgage value.

1.2.1 Interest

If we lend money or deposit money at a bank, then we receive interest for that money. Let $p(t,T)$ be the amount of money that has to be deposit at time t ($t < T$) if one wants to save an amount of one unit at time T. The amount $p(t,T)$ is known as the *price of a discount* or a *zero-coupon bond*.

Long rate

The relationship between the price $p(t,T)$ of a zero-coupon bond at time t and the zero-coupon interest, $R(t,T)$ can be given by:

$$R(t,T) := -log\ p(t,T) / (T-t) \qquad (1.1)$$

Where $(T-t)$ denotes the length of the term.

$R(t,T)$ are known as the *long rates*. Because mortgage rates are usually fixed for a longer period (1 to 20 years), so called *long rates* should be considered. The term "long rate" implicates the profit on long-term loans.

Short rate

When a financial institution lends money to a client, interest is received over a number of periods. If the lengths of each period are taken to be very short, this interest rate is known as the *short rate*.

$$r_t = \lim_{T \downarrow t} R(t,T) , \tag{1.2}$$

where r_t is the short rate at time t. Many models have been developed to predict the values of r_t. In Chapter 3 a few of them will be discussed.

In case of the Hull & White model a linear relationship exists between the long rates $R(t,T)$ and the short rates r_t . The relationship is as follows:

$$R(t,T) = A(t,T) + B(t,T) r_t \tag{1.3}$$

where r_t is the short rate at time t and the function $B(t,T)$ is given by:

$$B(t,T) = \frac{1 - e^{-\theta(T-t)}}{\theta(T-t)} \tag{1.4}$$

and the function $A(t,T)$ is determined by:

$$A(t,T) = \log\left(\frac{p(0,T)}{p(0,t)}\right) + B(t,T) f(0,t) - \frac{\sigma^2}{4\theta}(1 - \exp(-2\theta t)) B(t,T)^2 \tag{1.5}$$

θ stands for the mean reversion speed and is the tendency to be "pulled back" to a certain mean value. The greater the value of this parameter, the faster the rates will reach a mean rate value. The volatility of the interest, σ, describes the stability of the market. In a market with great changes, this parameter will have a large value. In a stable market, the value will be small.
$f(0,t)$ are the forward rates.

In the following figures examples of long rates are given. Figure 1.3 describes the development of the 5-years mortgage rate since 1960 and Figure 1.4 describes the development since 1995 [www.hypotheker.nl].
For a more detailed explanation on short and long rates, see [Boshuizen en Spreij, 2002].

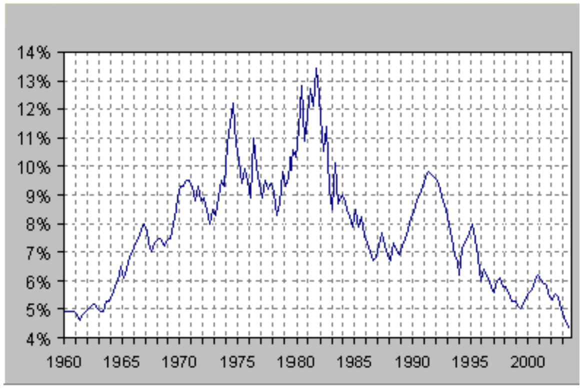

Figure 1.3: The development of the 5-years mortgage rate since 1960

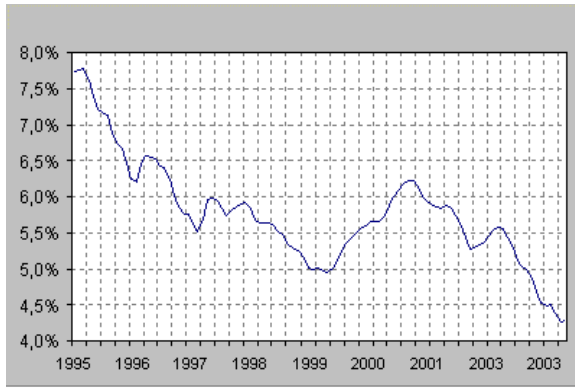

Figure 1.4: The development of the 5-years mortgage rate since 1995

1.2.2 Embedded options in the Mortgage contract

An embedded option is an option that is combined with an underlying asset, such as a loan. An embedded option in the mortgage contract means that this option is an inseparable part of this contract. From the moment the offer for a mortgage has been made, the client has certain rights, the embedded *options*.

The offer option -- When an offer has been made, the client has a certain period in which he can decide if he wants to accept the offer. This period is usually about 8 weeks. The benefit of this option for the client is that when the market rate increases, the client has the right to accept the offer with the lower interest rate. When the market rate drops, this lower rate will be taken in the offer.

The moving option -- When the client decides to move, the mortgage must be paid completely, for the house is no longer a security. If the client buys a new house and therefore needs a new mortgage, then the financial institution offers the client the option to either take the old mortgage with him or to close another mortgage offer. The client will get another mortgage offer if the current interest rate for the new home is lower then the old interest rate.

The prepayment option -- During the mortgage-period, the client has the right to prepay the mortgage. This means that when a mortgage-contract has been closed, this does not mean that the financial institution is certain of its cash flow. When a mortgage is being prepaid, the financial institution loses the interest rate payments for the rest of the period. This option will be further discussed in the next chapter.

The rate-time for reflection -- An interest rate is determined for just part of the mortgage-period. At the end of each fixed-rate period a new interest-rate should be determined for the next fixed-rate period. The client usually has the benefit of an interest rate–time for reflection. In this time the client has the option to choose the new interest rate. He can choose this rate at any time during the interest rate-time for reflection. So the client is not under an obligation to take the interest rate at the end of the fixed-rate period. Because of this rate-time for reflection option, the client can make an early decision about the new mortgage rate for the next fixed-rate period. This can be favourable for the client if he thinks that the current rate is very low and can only increase.

CHAPTER 2
Prepayment option

Prepayment risk is the chance that borrowers prepay their mortgages faster than contractually agreed. This can affect the yield of the investment for the lender.

When a client prepays his mortgage, the financial institution can reinvest the prepaid amount. This does not have any negative consequences for the financial institution, when the current market interest rate is the same or higher than the interest rate on the existing contract. But when interest rates have dropped, the financial institution makes a loss on the reinvestment. It is therefore important for a financial institution to understand the risk drivers behind prepayment and be able to model the prepayment rate so it can be taken into account when valuating the price of a mortgage.

2.1 The risk drivers

The prepayment option is a very important subject of mortgage pricing, since it is difficult to model human behaviour. There have been many studies on prepayment behaviour. Many models have been built that try to forecast the prepayment rate over time based on a number of variables. The question is, which variables should be taken into account? What influences a client's decision to prepay the mortgage?

Refinancing incentive

Mortgagers have the option to prepay their mortgages at any time. So It is most likely to prepay when interest rates drop, for they can then refinance at the current lower rate. A popular and simple model [Boshuizen and Spreij, 2002] that is based on the development of the interest rate is:

$$v_t = \alpha + \beta\,(r_t^{ma10y} - r_t^{now10y}) + \gamma\ \max\{r_t^{ma10y} - r_t^{now10y}, 0\} \tag{2.1}$$

where v_t is the prepayment rate at time t (as percentage of the debt at time t), r_t^{now10y} and r_t^{ma10y} are respectively the 10-years interest rate at time t and the 10-years moving average of the 10-years interest rate at time t. α is the average prepayment rate; $\alpha > 0$. β and γ imply the influence on the prepayment rate of a drop or an increase in the market interest rate; β , γ > 0.

This model states that a drop in the current market interest rate causes the prepayment rate to move up.

A fall in the interest rate does not necessarily imply that a client will prepay. This, because the contract includes certain penalties for prepayment. In order to decide whether to exercise this option, they must compare the cost of continuing the monthly payments to the costs of refinancing at the current rate.

A client's prepayment decision depends upon the relationship between rates at which the mortgage may be refinanced and the contract rate on the mortgage. If an available refinancing rate is less than the contract rate, there exists an incentive to prepay. The *refinancing incentive* is defined as the incentive that homeowners have to refinance their existing contract. Many prepayment models developed so far, mention this refinancing incentive as the most important factor.

As a lender has to finance the contract at the origination of the mortgage, the refinancing incentive is at that moment per definition zero. To determine the prepayment rate of that contract you would then need to make assumptions about the expected interest rate development.

Burnout, Seasoning and Seasonality

In most US literature on the prepayment subject other important variables are: *burnout, seasoning and seasonality*. They were first used in the model by Kang and Zenios in 1992 and are still included in most currently applied models.

- The burnout effect is an aging effect where the older the mortgage, the lower the prepayment rate. The idea is that if a mortgage hasn't been prepaid after a number of years it is unlikely it will be prepaid going further.

- Seasoning is a seemingly opposite effect where prepayment rates increase over time until they reach a stable or "seasoned" level. The American Public Securities Association model [Gulko, 1996](also known as "the PSA aging ramp") on prepayment rates assumes that prepayment rates increase linearly in the first thirty months of the contract before levelling off.

- Seasonality measures the correlation between prepayment rates and the month of the year. This cyclical behaviour is linked to the cyclical behaviour observed in house sales. During spring and early summer, house sales tend to be much higher then in the winter.

Moving

As mentioned, the mortgage contract includes certain penalties for prepayment. So one must also consider cases of prepayment without these penalties. In the Netherlands, the most important cause of penalty-free prepayment is the case that the owner of the house moves or passes away. As *moving* is the most important reason for penalty-free prepayment, this should be an important factor for financial institutions to focus on. The borrowers current status in life with respect to age, income level, where he lives, marital status and other related factors, will determine whether he is likely to move.

CHAPTER 3
Interest rate models

As mentioned in Chapter 1, one of the causes of the risks that financial institutions deal with when closing a mortgage contract, is the daily fluctuations in the market interest rate. Because a mortgage has interest rate sensitive cash flows, for the construction of valuation models for mortgages, it is important to be able to model the behaviour of the interest rates through time. An interest rate model is a probabilistic description of the future development of interest rates. Based on today's information, future interest rates are uncertain: An interest rate model is a characterization of that uncertainty. There are several models in the literature. This chapter discusses several of these interest rate models. A detailed explanation of these models will not be given here, but can be found in [Brigo and Mercurio, 2001] and [Kwok, 1998].

3.1 One-factor short rate models in general

In general, one-factor short rate models contain two parts. One part specifies the average rate of change of the short rate at each time step, known as the *drift*. The other part specifies the instantaneous *volatility* of the short rate. The general notation for this is

$$dr(t) = \mu(r,t)dt + \sigma(r,t)dz(t) \tag{3.1}$$

The left-hand side of this equation represents the change in the short rate over the next time step. The first term on the right is the drift multiplied by the size of the time step. The second is the volatility multiplied by a normally distributed random increment (Wiener process). Differences between models arise from different dependences of the drift and the volatility terms on the short rate.

3.2 Vasicek and Cox-Ingersol-Ross

The two most popular models for the short rate are the *Vasicek mean reversion* model (1977) and the *Cox-Ingersol-Ross(CIR) square root process* model (1985). Both have the same form for the drift term, namely a tendency for the short rate to rise when it is below the long-term mean, and fall when it is above.

Vasicek was the first to give an explicit characterization of the term structure.

Vasicek mean reversion model

The Vasicek mean reversion model is given by:

$$dr_t = \theta(\alpha - r_t)dt + \sigma dW_t \tag{3.2}$$

where r_t is the short rate at time t, α is a constant, θ a non-negative constant (the mean-reversion speed), σ the volatility of the interest and dW_t the Wiener increment $dW_t = W_{t+dt} - W_t$.
The above process is sometimes called the *elastic random walk* or *mean reversion process*.

However, this model has several drawbacks. First, the model can lead to negative rates. Second, the volatility is supposed to be constant. And third, the model supposes that all bond prices are driven by the short-term interest rate.

The problem of negative rates was solved by Cox, Ingersoll and Ross. They suggested a square root model. The two models also differ in the rate dependence of the volatility: in the Vasicek model it is constant (when expressed in points per year), and in the CIR square root process model it is proportional to the square root of the short rate.

Cox-Ingersoll-Ross model

Cox, Ingersoll and Ross (1985) suggested a square root model:

$$dr_t = \theta(\alpha - r)dt + \sigma\sqrt{r_t}\, dW \tag{3.3}$$

With an initially non-negative interest rate, the short interest rate *r(t)* will never be negative.

3.3 One factor versus Multi factor

Besides the Vasicek and Cox-Ingersoll-Ross models, there is also a number of other one-factor interest rate models in the literature. In one-factor interest rate models the interest rate movement depends on a single stochastic variable only: the short rate. Examples of one-factor interest rate models are:

- Dothan model (1978)
- Brennan-Schwartz model (1979)
- Cox-Ingersoll-Ross variable rate model (1980)
- Constant elasticity of variance model (Cox and Ross, 1976)

Some argued that the assumption of a short rate process as the only factor determining the whole term structure was too unrealistic. Multi-factor models involve the short rate together with other factors, such as long rate. In principle it is straightforward to move from a one-factor model to a multi-factor model. In practice, though, implementations of multi-factor valuation models can be complicated and slow. They require estimation of many more volatility and correlation parameters than are needed for one-factor models. So there may be some benefit to using a one-factor model when possible.

In Chapter 4 we will introduce the one-factor Hull & White model, which is an extension of the Vasicek mean reversion model. We will use this model in an illustration of a simulation technique that determines the price of a mortgage.

CHAPTER 4
The Hull & White simulation

In this chapter a simulation technique is described to value the price of a mortgage with the prepayment option. This simulation technique considers an annuity mortgage and is based on the Hull & White model (see Chapter 3) and the prepayment model (2.1).

4.1 Annuity mortgage

An annuity mortgage is a mortgage where clients, during the fixed rate period, yearly pay a fixed amount. This fixed amount includes the mortgage rate plus the redemption of the mortgage to the financial institution (see also Chapter 1).

We presume the following:

B := the amount that the client borrows from the financial institution, the principal.

N := the number of years in which B should be paid.

r := the fixed interest rate per year (in percentage).

i_n := the paid interest rate in year n, $n = 0, 1, ..., N$

a_n := the redemption in year n, $n = 0, 1, ..., N$

The sum of i_n and a_n is a fixed amount for every year n. This fixed amount is called the *annuity*. Let S_n be the remaining debt in month n. Then $S_0 = B$ and $S_N = 0$. The value of i_n and a_n can be determined as follows:

$$i_n = rS_{n-1} \qquad \text{and} \qquad a_n = a - i_n\,,$$

where a is the annuity which can be computed by:

$$a = \frac{r(1+r)^N S_0}{(1+r)^N - 1} \tag{4.1}$$

In case of no prepayment, the yearly cash flow to the bank is equal to the annuity:

$$CF_n = a_n + i_n. = a, \qquad n = 0, 1, ..., N \tag{4.2}$$

If the prepayment option is considered, the yearly cash flow to the bank becomes:

$$CF_n = a_n + v_n S_{n-1} + i_n \tag{4.3}$$

where v_n is the prepayment rate in year n based on formula (2.1). Note that if the mortgage loan has been prepaid, the remaining debt is zero, the cash flows to the bank, CF_n, are zero. S_n can now be recursively computed by:

$$S_n = S_{n-1} - a_n - v_n S_{n-1} \tag{4.4}$$

By use of interest rate models the value of the uncertain cash flows to the bank can be valued. In theory the value of the mortgage is given by:

$$V_T = E\left(\sum_{t=1}^{T} \exp(-\int_0^t r_s ds) CF_t (r_s, s \leq t) \right) \tag{4.5}$$

V_T is the value of the mortgage at time T and CF_t are the uncertain cash flows at time $t = 1, \ldots, T$.

4.2 The Hull and White model

The Hull & White model includes the Wiener process, which is introduced below. After introducing the Hull & White model, a discrete approximation of this model is given, which is needed for simulation.

Wiener process

The Wiener process is also known as the Brownian motion. In Figure 4.1 an example of the Wiener process is given. This stochastic process has the following properties:

1. The Wiener increment $dW_t = W_{t+dt} - W_t$ over a time step dt is a random variable drawn from a normal distribution with zero mean and a time-step variance dt: $dW_t \sim N(0,dt)$.
2. The Wiener increment is independent of the past.

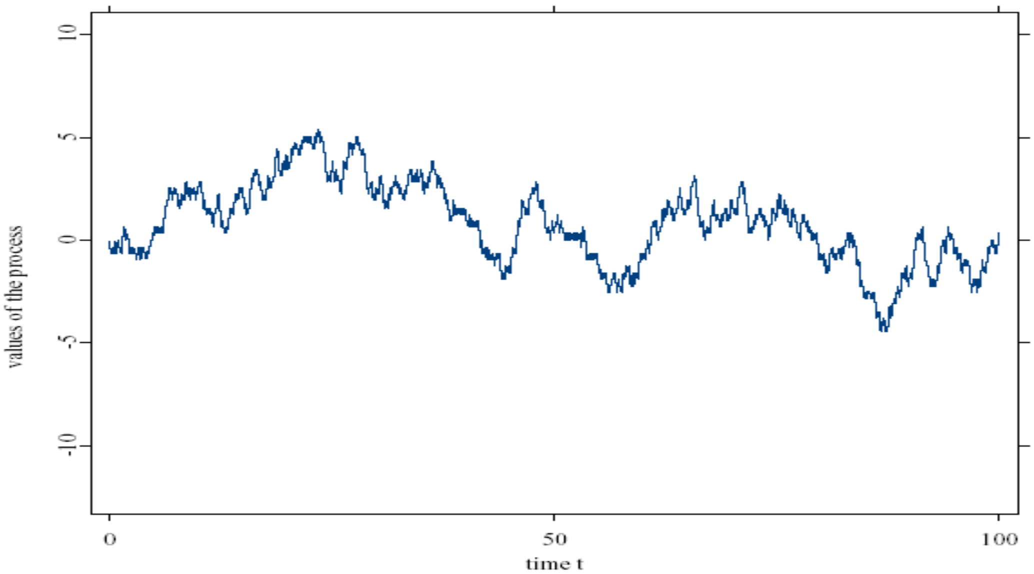

Figure 4.1: Example of a Wiener process

4.2.1 Hull & White interest rate model

Consider the time period $[0,T]$ with as start time the time the client accepts an offer. In the Hull & White model the interest is described by the solution of the following equation:

$$dr_t = (\alpha(t) - \theta r_t)dt + \sigma dW_t \tag{4.6}$$

where r_t is the short rate at time t, $\alpha(t)$ a given function at $[0,T]$, θ a non-negative constant (the mean-reversion speed), σ the volatility of the interest and dW_t the Wiener increment $dW_t = W_{t+dt} - W_t$. The function $\alpha(t)$ is based on the current price of a zero-coupon bond in the market. And the values for the parameters θ and σ can be derived from the prices of interest options [Boshuizen en Spreij, 2002].

4.2.2 Discrete approximation

For simulation of the Hull & White equation (4.6), a discrete version of this equation is needed. Consider equation (4.6):

$$dr_t = (\alpha(t) - \theta r_t)dt + \sigma dW_t .$$

Let h be a small time interval. By approximation it holds that:

$$r_{t+h} - r_t = (\alpha(t) - \theta r_t)h + \sigma(W_{t+h} - W_t). \tag{4.7}$$

Let t now repeatedly be a multiple of h, say $t = nh$, then the following equation is obtained:

$$r_{n+1}^h = r_n^h + (\alpha_n^h - \theta r_n^h)h + \sigma \Delta W_{n+1}^h, \tag{4.8}$$

where $r_n^h = r_{nh}, \alpha_n^h = \alpha(nh)$ and $\Delta W_{n+1}^h = W_{(n+1)h} - W_{nh}$.

This last equation gives a solution for r_n that can be computed recursively. Note that ΔW_{n+1}^h is normally distributed with mean equal to zero and variance equal to h.

4.3 Steps for simulation

The steps for the simulation technique are as follows:

1. Simulate a series of short rates with h = 1/12 (steps of 1 month) with formula (4.8).
2. Evaluate the long rates with formula (1.3).
3. Evaluate the prepayment percentages v_t with formula (2.1), based on the long rates computed in step 2.
4. Evaluate the cash flows CF_n with formula (4.3). Note that S_n and a_n are zero if S_n becomes negative in formula (4.4).
5. Approximate the value of the cash flows, taking the simulated short term rates into account:

$$\exp(-\sum_{i=1}^{n} hr_i^h)CF_n(r_1^h,\ldots,r_n^h)$$

   and compute the sum of these cash flows. (This step is an approximation of V_T in (4.5)).
6. Repeat steps 1 till 5 a large number of times, say 1000, and take for the estimation of the value of the mortgage portfolio the average of the sum of the cash flows computed in step 5.

4.4 Illustration and Results

As an illustration we consider an annuity mortgage that has to be paid off in 30 years (see Table 4.1). We vary the volatility of the interest. In Table 4.2 the values of the parameters θ and σ for the Hull & White model are given. We look at the case where there is no prepayment and several cases with prepayment. Table 4.3 gives several values for the parameters α, β and γ for the prepayment model (2.1).

The program that is written for this illustration can be found in the Appendix.

Type of mortgage	Annuity
Principal	100
Fixed-interest rate (per year)	9%
Mortgage term	30 years

Tabel 4.1: Illustration of simulation.

	θ	σ
Base	0.1	1.0%
High	0.1	3.0%
Low	0.1	0.5%

Tabel 4.2: Parameters of the Hull & White Model

	α	β	γ
Base	7%	0.1	0.7
High	14%	0.2	1.4
Low	3.5%	0.05	0.35

Tabel 4.3: Parameters of the prepayment model

In case of no prepayment, the parameters of the prepayment model are all 0.

In Table 4.4 the results of the simulation are given.

Interest rate	**No prepayment**	**Prepayment 1 (Base)**	**Prepayment 2 (High)**	**Prepayment 3 (Low)**
Base	112.4	103.6	101.8	107.4
High	108.6	102.1	100.4	105.8

Low	113.9	104.9	102.4	108.3

Tabel 4.4: Results of the simulation

4.4.1 Discussion

The value of the mortgage is in all cases higher than the principal. This, because the profit for the borrower is included in the value of the mortgage. In the case of no prepayment the value of the mortgage is quite high. The reason for this is because the fixed interest rate is higher than the average interest rate of the mortgage period. This average interest rate is circa 7 %- 8% and the fixed interest rate is 9%. Lower market interest rates lead to a higher value for the discounted cash flows (step 5). This will lead to a higher value of the mortgage.

Furthermore, we see that as the volatility of the interest rate increases, the value of the mortgage decreases. The higher the volatility, the less stable the market, which causes the uncertainty in the cash flows to increase. See in Figure 4.2 an example of the course of the cash flows in case of no prepayment. Here, the yearly cash flow to the bank is equal to the annuity.

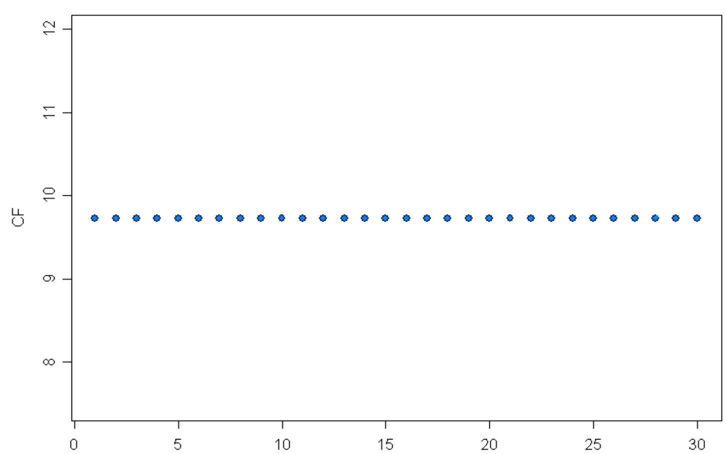

Figure 4.2: Cash flows in case of no prepayment

In the case of prepayment, we see that the value of the mortgage is less than in the case of no prepayment. This is just as what we expected; if the client prepays the mortgage loan, the borrower misses the profit (interest) for the remaining period. See in Figure 4.2 an example of the cash flows in case of prepayment. In this example the client prepays the mortgage loan after 12 years.

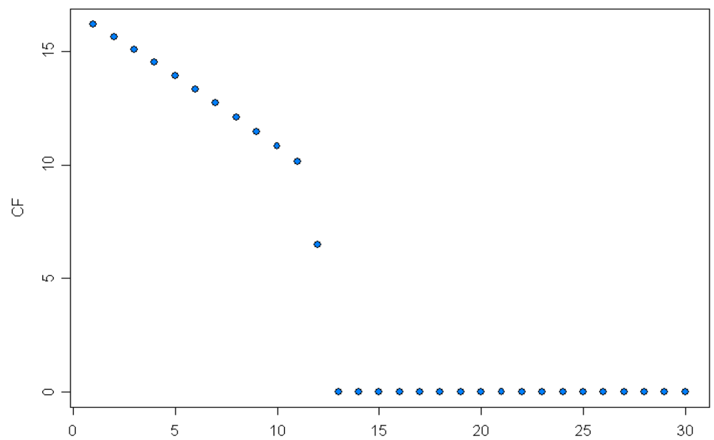

Figure 4.3: Cash flows in case of prepayment

In case the parameters β and γ increase (Prepayment 2 “High”), the value of the mortgage becomes less and vice versa. The higher (lower) these values, the more (less) influence a drop of the interest rate has on the prepayment rate. Clients will pay off their mortgage and refinance because the interest is low.

A high α is profitable in case the market interest rate is high, but is unprofitable in case the interest is lower than the agreed fixed interest rate. If the current market rate is higher than the agreed fixed interest rate, the borrower can reinvest the prepaid amount for a higher interest rate. In the other case, the borrower will have a loss in the reinvestment.

CHAPTER 5
Conclusions and Recommendations

5.1 Conclusions

In this paper we have discussed the valuation of a mortgage. The cash flows that a mortgage generates are uncertain for the financial institution. These cash flows depend on the fluctuations of the market interest rate and on certain embedded options. The embedded options are certain rights that the clients have, such as the right to prepay the mortgage.
To determine the price of a mortgage one should forecast the development of the interest rates through time. For this, interest models are used.

In this paper we have focused on the prepayment option. To forecast the prepayment rate over time, one should analyse the drivers that influence a client's decision to prepay the mortgage. Several risk drivers for prepayment are:

- Refinancing incentive
- Burnout, seasoning and seasonality
- Moving

By means of simulation techniques one can determine the price of a mortgage. A simulation technique is described in this paper to value the price of a mortgage. This program is based on the Hull & White interest model and includes the prepayment option. The prepayment model that is used is based on the refinancing incentive; how likely it is for a client to prepay the mortgage when the interest rate drops so he can refinance the mortgage.

5.2 Recommendation

In this research we selected the Hull & White interest model to predict the development of the interest rate. There are many more interest models that might make a better prediction of the development. Some of these models are introduced in Chapter 3. Experiments with other interest rate models are recommended.
The interest rate models introduced in Chapter 3 are all based on only one factor. To predict the development of the interest rate more accurately, one might consider more factors.
In the simulation technique that we have described, we only included the prepayment option. More research should be done on the behaviour of clients, so other options can also be modelled. This will likely lead to a better prediction of the price of a mortgage.

To determine the prepayment rate, a model has been used which only takes the interest rate factor into account. While moving is one of the most important reasons for penalty-free prepayment, this should be one of the main factors for financial institutions to focus on. The borrowers current status in life with respect to age, income level, where he lives, martial status and other related factors, will determine whether he is likely to move. More studies based on certain factors should be done, when trying to forecast the prepayment rate.

Appendix
Simulation Program

```
### ------------------------------------------------------------------------------------------------------------------- ###
### ------------------------------------------------------------------------------------------------------------------- ###
###         Title: Hull & White Simulation                                                                            ###
###         Author: Yonina Karg                                                                                       ###
###         Date: June 2004                                                                                           ###
###                                                                                                                   ###
###         This simulation program values the price of a mortgage with prepayment option.                           ###
###         The simulation technique considers an annuity mortgage and is based on the                               ###
###         Hull & White model (Chapter 3) and the prepayment model (Chapter 2).                                     ###
###         The program is written in S-Plus.                                                                         ###
### ------------------------------------------------------------------------------------------------------------------- ###
### ------------------------------------------------------------------------------------------------------------------- ###

runs<- 1000
result <- numeric(runs)

for (j in 1:runs) {

###---------------------------------------- ###
###----------   Short rates    --------- ###
###---------------------------------------- ###

### Parameters Hull & White model ###

S <- 10                              # we consider the 10-years interest rate
r <- numeric(361)                    # short rates
y <- numeric(361)                    # mean-zero stochastic process
theta <- 0.1                         # mean reversion speed
sigma <- 0.01                        # volatility
h <- 1/12                            # steps of one
deltaW <- rnorm(360,0,1)             # Wiener process


y[1] <- 0


alpha <- c( 0.052249592, 0.052836811, 0.053974941, 0.055070946, 0.056126051,
            0.057141477, 0.058118431, 0.059058103, 0.059961673, 0.060830297,
            0.061665113, 0.062467243, 0.063237771, 0.063977779, 0.064688314,
            0.065370399, 0.066025032, 0.066653194, 0.067255829, 0.067833863,
            0.068388196, 0.068919695, 0.069429224, 0.069917591, 0.070385601,
            0.070834027, 0.071263619, 0.071675104, 0.072069182, 0.072446536,
            0.072807825, 0.073153678, 0.073484699, 0.073801498, 0.074104643,
            0.074394674, 0.074672133, 0.074937528, 0.075191353, 0.075434084,
            0.075666177, 0.075888075, 0.076100201, 0.076302964, 0.076496755,
            0.076681953, 0.076858921, 0.077028005, 0.077189543, 0.077343856,
            0.077491253, 0.077632031, 0.077766475, 0.077894858, 0.078017443,
            0.078134481, 0.078246214, 0.078352874, 0.078454683, 0.078551853,
```

0.078644588, 0.078733084, 0.078817529, 0.078898102, 0.078974974,
0.079048313, 0.079118269, 0.079185341, 0.079248649, 0.079309353,
0.079367246, 0.079422453, 0.079475096, 0.079525291, 0.079573148,
0.079618774, 0.079662271, 0.079703733, 0.079743256, 0.079780927,
0.079816831, 0.079851049, 0.079883659, 0.079914734, 0.079944344,
0.079972559, 0.079999441, 0.080025052, 0.080049452, 0.080072697,
0.080094839, 0.080115931, 0.080136021, 0.080155156, 0.080173386,
0.080190736, 0.080207265, 0.080223005, 0.080237994, 0.080252266,
0.080265855, 0.080278793, 0.080291112, 0.080302842, 0.080314005,
0.080324633, 0.080334751, 0.080344381, 0.080353548, 0.080362274,
0.080370578, 0.080378482, 0.080386004, 0.080393163, 0.080399975,
0.080406458, 0.080412627, 0.080418497, 0.080424082, 0.080429396,
0.080434452, 0.080439263, 0.080443839, 0.080448193, 0.080452335,
0.080456275, 0.080460023, 0.080463588, 0.080466985, 0.080470205,
0.080473274, 0.080476192, 0.080478967, 0.080481607, 0.080484117,
0.080486504, 0.080488774, 0.080490933, 0.080492986, 0.080494938,
0.080496795, 0.080498568, 0.080500238, 0.080501833, 0.080503353,
0.080504793, 0.080506164, 0.080507468, 0.080508707, 0.080509885,
0.080511005, 0.080512077, 0.080513082, 0.080514044, 0.080514959,
0.080515828, 0.080516654, 0.080517439, 0.080518186, 0.080518895,
0.080519569, 0.080520213, 0.080520819, 0.080521398, 0.080521948,
0.080522472, 0.080522967, 0.080523439, 0.080523887, 0.080524313,
0.080524718, 0.080525103, 0.080525469, 0.080525816, 0.080526146,
0.080526461, 0.080526758, 0.080527041, 0.080527313, 0.080527565,
0.080527808, 0.080528038, 0.080528258, 0.080528466, 0.080528663,
0.080528851, 0.080529031, 0.080529199, 0.080529363, 0.080529513,
0.080529658, 0.080529796, 0.080529927, 0.080530051, 0.08053017,
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0.080530764, 0.080530846, 0.080530925, 0.080530999, 0.08053107,
0.080531137, 0.080531281, 0.080531261, 0.080531318, 0.080531372,
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0.080531953, 0.080531972, 0.080531994, 0.080532014, 0.080532033,
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0.080532134, 0.080532144, 0.080532156, 0.080532168, 0.08053218,
0.080532191, 0.080532201, 0.080532211, 0.080532226, 0.080532229,
0.080532237, 0.080532245, 0.080532253, 0.080532261, 0.080532267,
0.080532273, 0.080532288, 0.080532285, 0.080532291, 0.080531974,
0.080531979, 0.080531984, 0.080531989, 0.080531993, 0.080531997,
0.080532001, 0.080532004, 0.080532008, 0.080532011, 0.080532014,
0.080532017, 0.080532024, 0.080532023, 0.080532025, 0.080532027,
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0.080532047, 0.080532048, 0.080532049, 0.080532051, 0.080532052,
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0.080532068, 0.080532068, 0.080532068, 0.080532068, 0.080532068,

```
    0.080532069, 0.080532069, 0.080532069, 0.080532069, 0.080532069,
    0.080532069, 0.080532069, 0.080532071, 0.080532076, 0.08053207,
    0.080532079, 0.080532075, 0.080532075, 0.080532072, 0.08053207,
    0.080532077, 0.080532076, 0.080532074, 0.080532071, 0.080532071,
    0.080532071, 0.080532071, 0.080532071, 0.080532071, 0.080532071,
  0.080532071, 0.080532071, 0.080532071, 0.080532071, 0.080532071,
  0.080532071, 0.080532071, 0.080532071, 0.080532071, 0.080532071,
  0.080532071, 0.080532071, 0.080532071, 0.080532071, 0.080532071,
  0.080532071)


for(i in 1:360) {
y[i+1] <- y[i] + (-(theta*y[i]))*h + sigma*deltaW[i]*sqrt(h)
}
r <- y + alpha

###---------------------------------------- ###
###------------ Long rates  ----------- ###
###---------------------------------------- ###

t <- 0:360
B <- (1/(theta*S)) * (1-exp(-theta*S))


p0tPlus10 <- c( 0.4717, 0.4686, 0.4654, 0.4623, 0.4592, 0.4562, 0.4531, 0.4501, 0.4471, 0.4441,
    0.4411, 0.4382, 0.4353, 0.4324, 0.4295, 0.4266, 0.4237, 0.4209, 0.4181, 0.4153,
    0.4125, 0.4098, 0.4070, 0.4043, 0.4016, 0.3989, 0.3962, 0.3936, 0.3910, 0.3884,
    0.3858, 0.3832, 0.3806, 0.3781, 0.3755, 0.3730, 0.3705, 0.3681, 0.3656, 0.3632,
    0.3607, 0.3583, 0.3559, 0.3535, 0.3512, 0.3488, 0.3465, 0.3442, 0.3419, 0.3396,
    0.3373, 0.3351, 0.3328, 0.3306, 0.3284, 0.3262, 0.3240, 0.3218, 0.3197, 0.3175,
    0.3154, 0.3133, 0.3112, 0.3091, 0.3071, 0.3050, 0.3030, 0.3009, 0.2989, 0.2969,
    0.2949, 0.2930, 0.2910, 0.2891, 0.2871, 0.2852, 0.2833, 0.2814, 0.2795, 0.2777,
    0.2758, 0.2740, 0.2721, 0.2703, 0.2685, 0.2667, 0.2649, 0.2631, 0.2614, 0.2596,
    0.2579, 0.2562, 0.2545, 0.2528, 0.2511, 0.2494, 0.2477, 0.2461, 0.2444, 0.2428,
    0.2412, 0.2395, 0.2379, 0.2364, 0.2348, 0.2332, 0.2316, 0.2301, 0.2286, 0.2270,
    0.2255, 0.2240, 0.2225, 0.2210, 0.2195, 0.2181, 0.2166, 0.2152, 0.2137, 0.2123,
    0.2109, 0.2095, 0.2081, 0.2067, 0.2053, 0.2039, 0.2025, 0.2012, 0.1998, 0.1985,
    0.1972, 0.1959, 0.1946, 0.1933, 0.1920, 0.1907, 0.1894, 0.1881, 0.1869, 0.1856,
    0.1844, 0.1832, 0.1819, 0.1807, 0.1795, 0.1783, 0.1771, 0.1759, 0.1747, 0.1736,
    0.1724, 0.1713, 0.1701, 0.1690, 0.1679, 0.1667, 0.1656, 0.1645, 0.1634, 0.1623,
    0.1612, 0.1601, 0.1591, 0.1580, 0.1570, 0.1559, 0.1549, 0.1538, 0.1528, 0.1518,
    0.1508, 0.1498, 0.1488, 0.1478, 0.1468, 0.1458, 0.1448, 0.1438, 0.1429, 0.1419,
    0.1410, 0.1400, 0.1391, 0.1382, 0.1372, 0.1363, 0.1354, 0.1345, 0.1336, 0.1327,
    0.1318, 0.1309, 0.1301, 0.1292, 0.1283, 0.1275, 0.1266, 0.1258, 0.1249, 0.1241,
    0.1233, 0.1224, 0.1216, 0.1208, 0.1200, 0.1192, 0.1184, 0.1176, 0.1168, 0.1160,
    0.1153, 0.1145, 0.1137, 0.1130, 0.1122, 0.1115, 0.1107, 0.1100, 0.1092, 0.1085,
    0.1078, 0.1071, 0.1063, 0.1056, 0.1049, 0.1042, 0.1035, 0.1028, 0.1022, 0.1015,
    0.1008, 0.1001, 0.0994, 0.0988, 0.0981, 0.0975, 0.0968, 0.0962, 0.0955, 0.0949,
    0.0942, 0.0936, 0.0930, 0.0924, 0.0918, 0.0911, 0.0905, 0.0899, 0.0893, 0.0887,
    0.0881, 0.0875, 0.0870, 0.0864, 0.0858, 0.0852, 0.0847, 0.0841, 0.0835, 0.0830,
    0.0824, 0.0819, 0.0813, 0.0808, 0.0802, 0.0797, 0.0792, 0.0786, 0.0781, 0.0776,
    0.0771, 0.0765, 0.0760, 0.0755, 0.0750, 0.0745, 0.0740, 0.0735, 0.0730, 0.0725,
```

```
          0.0721, 0.0716, 0.0711, 0.0706, 0.0702, 0.0697, 0.0692, 0.0688, 0.0683, 0.0678,
          0.0674, 0.0669, 0.0665, 0.0660, 0.0656, 0.0652, 0.0647, 0.0643, 0.0639, 0.0634,
          0.0630, 0.0626, 0.0622, 0.0618, 0.0613, 0.0609, 0.0605, 0.0601, 0.0597, 0.0593,
          0.0551, 0.0547, 0.0544, 0.0540, 0.0536, 0.0533, 0.0529, 0.0526, 0.0522, 0.0519,
          0.0515, 0.0512, 0.0508, 0.0505, 0.0502, 0.0498, 0.0495, 0.0492, 0.0488, 0.0485,
          0.0482, 0.0479, 0.0475, 0.0472, 0.0469, 0.0466, 0.0463, 0.0460, 0.0457, 0.0454,
          0.0450, 0.0447, 0.0444, 0.0441, 0.0439, 0.0436, 0.0433, 0.0430, 0.0427, 0.0424,
          0.0421)

p0t <- c( 1, 0.9957, 0.9912, 0.9867, 0.9822, 0.9776, 0.9729, 0.9681, 0.9633, 0.9585,
          0.9536, 0.9487, 0.9437, 0.9388, 0.9337, 0.9287, 0.9236, 0.9185, 0.9134, 0.9083,
          0.9032, 0.8980, 0.8928, 0.8877, 0.8825, 0.8773, 0.8721, 0.8670, 0.8618, 0.8566,
          0.8514, 0.8463, 0.8411, 0.8360, 0.8308, 0.8257, 0.8206, 0.8155, 0.8104, 0.8053,
          0.8003, 0.7953, 0.7902, 0.7852, 0.7802, 0.7753, 0.7703, 0.7654, 0.7605, 0.7556,
          0.7508, 0.7459, 0.7411, 0.7363, 0.7316, 0.7268, 0.7221, 0.7174, 0.7127, 0.7081,
          0.7035, 0.6989, 0.6943, 0.6897, 0.6852, 0.6807, 0.6763, 0.6718, 0.6674, 0.6630,
          0.6586, 0.6543, 0.6500, 0.6457, 0.6414, 0.6372, 0.6329, 0.6288, 0.6246, 0.6205,
          0.6163, 0.6123, 0.6082, 0.6042, 0.6002, 0.5962, 0.5922, 0.5883, 0.5844, 0.5805,
          0.5766, 0.5728, 0.5690, 0.5652, 0.5614, 0.5577, 0.5540, 0.5503, 0.5466, 0.5430,
          0.5393, 0.5357, 0.5322, 0.5286, 0.5251, 0.5216, 0.5181, 0.5147, 0.5112, 0.5078,
          0.5044, 0.5011, 0.4977, 0.4944, 0.4911, 0.4878, 0.4845, 0.4813, 0.4781, 0.4749,
          0.4717, 0.4686, 0.4654, 0.4623, 0.4592, 0.4562, 0.4531, 0.4501, 0.4471, 0.4441,
          0.4411, 0.4382, 0.4353, 0.4324, 0.4295, 0.4266, 0.4237, 0.4209, 0.4181, 0.4153,
          0.4125, 0.4098, 0.4070, 0.4043, 0.4016, 0.3989, 0.3962, 0.3936, 0.3910, 0.3884,
          0.3858, 0.3832, 0.3806, 0.3781, 0.3755, 0.3730, 0.3705, 0.3681, 0.3656, 0.3632,
          0.3607, 0.3583, 0.3559, 0.3535, 0.3512, 0.3488, 0.3465, 0.3442, 0.3419, 0.3396,
          0.3373, 0.3351, 0.3328, 0.3306, 0.3284, 0.3262, 0.3240, 0.3218, 0.3197, 0.3175,
          0.3154, 0.3133, 0.3112, 0.3091, 0.3071, 0.3050, 0.3030, 0.3009, 0.2989, 0.2969,
          0.2949, 0.2930, 0.2910, 0.2891, 0.2871, 0.2852, 0.2833, 0.2814, 0.2795, 0.2777,
          0.2758, 0.2740, 0.2721, 0.2703, 0.2685, 0.2667, 0.2649, 0.2631, 0.2614, 0.2596,
          0.2579, 0.2562, 0.2545, 0.2528, 0.2511, 0.2494, 0.2477, 0.2461, 0.2444, 0.2428,
          0.2412, 0.2395, 0.2379, 0.2364, 0.2348, 0.2332, 0.2316, 0.2301, 0.2286, 0.2270,
          0.2255, 0.2240, 0.2225, 0.2210, 0.2195, 0.2181, 0.2166, 0.2152, 0.2137, 0.2123,
          0.2109, 0.2095, 0.2081, 0.2067, 0.2053, 0.2039, 0.2025, 0.2012, 0.1998, 0.1985,
          0.1972, 0.1959, 0.1946, 0.1933, 0.1920, 0.1907, 0.1894, 0.1881, 0.1869, 0.1856,
          0.1844, 0.1832, 0.1819, 0.1807, 0.1795, 0.1783, 0.1771, 0.1759, 0.1747, 0.1736,
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          0.1508, 0.1498, 0.1488, 0.1478, 0.1468, 0.1458, 0.1448, 0.1438, 0.1429, 0.1419,
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          0.1318, 0.1309, 0.1301, 0.1292, 0.1283, 0.1275, 0.1266, 0.1258, 0.1249, 0.1241,
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          0.080532071, 0.080532071, 0.080532071, 0.080532071, 0.080532071,
          0.080532071)

A <- log(p0tPlus10/p0t) + (B*f0t) - (((sigma^2)/(4*theta))*(1-exp(-2*theta*t))*(B^2))

PtT <- exp(A-B*r)

R <- -log(PtT)/S          #The long rates




### ----------------------------------------###
### --------- Prepayment rates ------###
### ----------------------------------------###

###  Parameters prepayment model  ###

alphav <- 0.07
betav  <- 0.1
gammav <- 0.7


Rnow <- numeric(30)
Rma <- numeric(30)


for (i in 1:30)
{
      Rnow[i] <- R[i*12]
}


for (i in 1:10)
{
      Rma[i] <- mean(Rnow[1:i])
```

```
}


for (i in 11:30)
{
      Rma[i] <- mean(Rnow[(i-9):i])
}


v <- numeric(30)

for(i in 1:30)
{
      v[i] <- alphav + (betav * (Rma[i]- Rnow[i])) + (gammav * max((Rma[i]- Rnow[i]),0))
}


### ------------------------------------------------------###
### ---------- Discounted Cash Flows    --------- ###
### ------------------------------------------------------###

fixedrate <- 0.09
debt <- 100
annuity <- (fixedrate * ((1+fixedrate)^30) * debt) / ( ((1+fixedrate)^30) - 1)


interest <- numeric(30)
redemption <- numeric(30)
CF <- numeric(30)


for (i in (1:30))
{
            interest[i] <- fixedrate*debt
            redemption[i] <- annuity -interest[i]
            if (redemption[i] + v[i]*debt >= debt)
                        {
                        CF[i] <- debt + interest[i]
                        debt <- 0
                        annuity <- 0
                        }
            else
                        {
                        debt <- debt - redemption[i] - v[i]*debt
                        CF[i] <- redemption[i] + v[i]*debt + interest[i]
                        }
}
help <- numeric(30)
discountfactor <- numeric(30)

for (i in (1:30))
{
```

```
		help[i] <- sum(r[1:(12*i)])
}
discountfactor <- exp(-h*help)

result[j] <- sum(discountfactor*CF)
}

mean(result)
```

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Links

www.hypotheker.nl; rentebarometer; historisch renteverloop.

www.homeinvest.nl