The number of urgent calls per nurse

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Preface

One of the compulsory parts of the Master's program Business Analytics at the VU University Amsterdam is writing a research paper. The paper consist of a small research conducted in the field of Business Analytics and reporting the results. I would like to thank R. Bekker of the VU University for his time and feedback.

Abstract

A proficiency guideline is under development by the Dutch society of medical managers ambulancecare which will describe when a nurse is qualified. A part of this guideline is a national standard (NS) for the number of urgent calls an ambulance nurse should do per year. The number of urgent calls per nurse of UMCG Ambulancezorg varies between 100 and 500 for the year 2014. It was unclear what causes this variation but it was suspected that it depends on how often and where a nurse works.

A model was developed which explains the number of urgent calls per nurse by the number of shifts a nurse worked. An important part of this model is that the number of urgent calls per shift is roughly Poisson distributed where the mean number of calls can vary per different shift. Because the number of urgent calls per shift is modeled as a Poisson distribution, the number of urgent calls per nurse is also Poisson distributed. This is used to calculate upper and lower bounds for the expected number of urgent calls per nurse. The Poisson distribution slightly overestimates the chance of large numbers of urgent calls per shift and therefore statistical tests reject the null hypothesis that the model explains the number of urgent calls per nurse. When the number of urgent calls per shift are bootstrapped instead of modeled as a Poisson distribution, the model passes the statistical tests. Because the outcomes of the model with Poisson distribution barely differ from the model with bootstrapping, the model with Poisson distribution is a reasonable choice to describe the number of urgent calls per nurse.

By assigning nurses to different shifts, the number of urgent calls per nurse can be influenced such that each nurse satisfies the national standard. Questions that arise then are: what is the maximum value of the national standard, what is the influence of the national standard on the commuting time of the nurses and what is the influence of part time employees on the total commuting?
To answer these questions the assignment of nurses to different shifts is modeled as an assignment problem with two versions. The commuting time can be defined as the commuting time of all nurses (overall commuting time) or as the commuting time of the nurse with the largest commuting time (nurse commuting time). The first version of the assignment problem minimizes the overall commuting time while the second version minimizes the nurse commuting time first and then minimizes the overall commuting time.
Based on 2014 data of UMCG Ambulancezorg the maximum value of the national standard with and without part time employees is 250 and 298, respectively. Using the first version of the assignment problem the overall commuting time increases while the nurse commuting time fluctuates when the national standard increases, independent whether there are part time employees. Using the second version of the assignment problem the overall commuting time increases while the nurse commuting time remains the same when the national standard increases, independent whether there are part time employees. The situation with part time employees results in a higher overall commuting time compared to the situation with (almost) no part time employees.
Setting the national standard equal to the maximum value increases the commuting time with roughly 4% while a standard of 200 results in almost no increase.

Content

1 Introduction

1.1 Motivation/occasion

The Medical Manager Ambulance Care (MMA) is a licensed physician affiliated to a Regional Ambulance Service (RAV). Within this RAV the MMA has final responsibility for the medical care and has four main tasks. The first task is monitoring the competencies and skills of ambulance care providers in the context of the individual. The second task consists of assisting in the formulation of medical policy, monitoring the implementation of medical policy and applying protocols within ambulance care. The third and fourth task consists of ensuring medical coordination with chain partners and monitoring the implementation of the rights and obligations of the patient and care provider laid down in the Medical Treatment Act (WGBO).

For the first task a proficiency guideline is under development which will describe when a nurse is qualified. A part of this guideline is a national standard (NS) for the number of urgent calls per year a nurse should do. Figure 1 shows that the number of urgent calls per nurse of the ambulances services UMCG Ambulancezorg for the year 2014 varies between roughly 100 and 500: there are some exceptions; there was one nurse with almost 600 calls and 5 nurses with less than 100 calls. Although the MMA suspects that the number of urgent calls per nurse depends on how often and where someone works, it is unclear whether this fully explains the variation. When this presumption holds, than the number of urgent calls per year per nurse can be affected by assigning a nurse to other locations such that the national standard is met. A side effect could be a longer commuting time between home and work for a nurse. A question which arises then, is how large is the influence of the national standard on the commuting time.

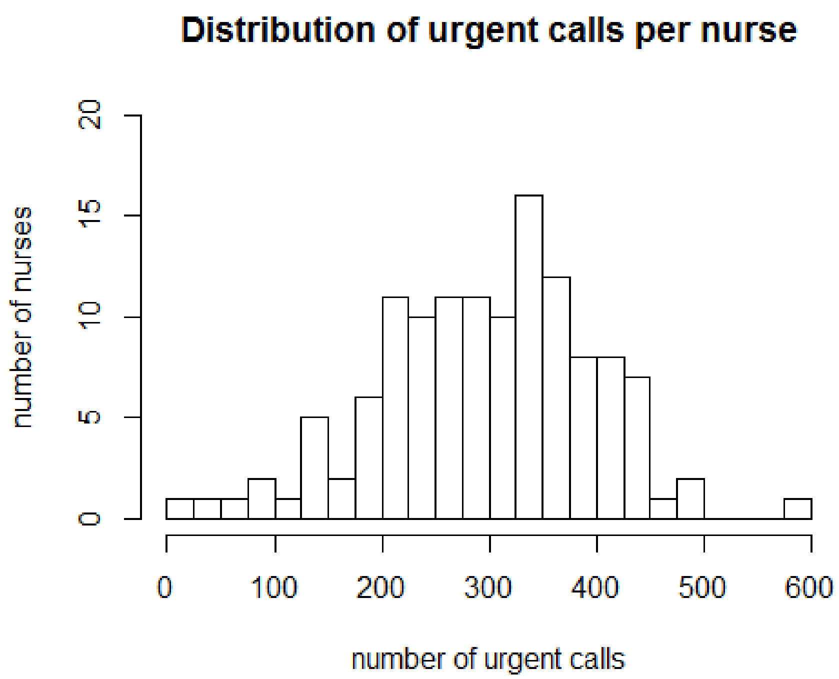

Figure 1 Distribution of the number of urgent calls per nurse for the year 2014.

1.2 Research question

With regard to the national standard there are some questions. For instance, does the suspicion of the MMA fully explains the number of urgent calls per nurse. Further, could every nurse meet the national standard? But also, is the maximum value of the national standard larger when there are only fulltime employees? Therefore the main research question is:

- *What causes the variation in the number of urgent calls per nurse and can every nurse meet the national standard?*

To answer this question the following sub question are defined:

- Can the variation be explained by the frequency and the location someone works?
- What is the maximum value of the national standard?
- What is the influence of the national standard on the commuting time?
- What is the influence of only full time nurses on the national standard and the commuting time?

The objective is to answer the above questions by developing a statistical model for the first question, an optimization model for the other questions, and implement and solve this optimization model in a software program.

1.3 Paper outline

The outline of this paper is as follows. It starts with describing the essential parts of ambulance care in chapter 2. A model for the number of urgent calls per nurse is developed in chapter 3. Chapter 4 gives an overview of the assignment problem. In chapter 5 the models from chapters 3 and 4 are used to develop a model to influence the number of urgent calls per nurse. Finally, the results are reported in chapter 6 and the conclusions are in chapter 7.

2 Ambulance care

UMCG Ambulancezorg provides ambulance care in the northern part of the Netherlands for the province of Drenthe and a part of the province of Friesland including the three islands Vlieland, Terschelling and Schiermonnikoog. This section briefly explains what ambulance care is.

2.1 Ambulance care process

Ambulance care consists of, among others, a dispatch centre and ambulances. The dispatch centre receives phone calls requesting for ambulance care. If these calls require ambulance care, the dispatch centre assigns ambulances to these calls. When an ambulance is assigned to a call, the ambulance drives to the patient, treats the patient and, when necessary, transports the patient to a hospital. After the patient is treated and possibly transported, the ambulance returns to his post unless it is assigned to a new call.

2.2 Calls urgency

Ambulances can be assigned to calls with different urgency. Urgent calls consists of calls with the urgency A1 and A2. A1 urgency calls are life-threatening, whereas A2 urgency calls are not immediately life-threatening but may involve (serious) damage to the patient's health. Therefore, the ambulance should arrive as soon as possible on site. The guideline is that in 95% of the calls with urgency A1 and A2 the ambulance should arrive on site within 15 and 30 minutes, respectively. Scheduled transportation calls consists of calls with the urgency B1 and B2. Of these calls the pick-up and delivery location and the pick-up or delivery time of a patient are known in advance.

2.3 Types of ambulances

Different types of ambulances are used for serving different types of calls. An ALS ambulance contains a driver and a nurse and has transport capacity, whereas a solo ambulance is a car or motor without transport capacity driven by a nurse only. A care ambulance contains two persons who can provide low-complexity care (care providers) and has transport capacity. Solo ambulance are assigned only to urgent calls and care ambulances only to B2 calls. Although ALS ambulances could be assigned to all urgencies, B2 calls are assigned preferably to care ambulances.

2.4 Posts and shifts

UMCG Ambulancezorg has 22 posts at the mainland and a single post at the islands Vlieland, Terschelling and Schiermonnikoog. Figure 4 shows the location of the post on a map. From these posts ambulances are stationed to response to a call and provide ambulance care. A shift defines when, where and at which post what kind of an ambulance is available. Table 1 shows for the different shift types their start time and duration, ambulance type, staff type and timetable hours. During a D24 shift the crew is 24 hours available but within the timetable this shift counts for 17.7 hours because the crew is allowed to sleep between 23:00 and 8:00. The crew of a D24 Island shift is also 24 hours available but within the timetable this shift only counts for 10 hours because of the low ambulance demand on the islands. During a year a full time nurse should work 1530 hours which means this nurse can work $1530 \div 17.7 \approx 86$ D24 shifts a year on the mainland and therefore he/she would be $86 \times 24 = 2064$ hours available if a nurse would only work D24 shifts.
The base timetable consist of 37 shifts on the mainland who are scheduled on a daily or (two) weekly basis. Table 2 shows the base timetable of 2014 for all shifts with an ALS or solo ambulance, their post, shift type, number of shifts per year and the average number of urgent calls per shift, which will be used in chapter 3. The fourth column of Table 2 states the number of shift per year. For instance,

26*5 means that on a two weekly basis this shift is scheduled during weekdays, whereas 52*5*2 means that every week during weekdays two shifts are scheduled. Although the base timetable consist of the same shift types at different posts, the average number of urgent calls for the same shift type is not the same. Note that the base timetable only contains unique shifts while the same shift type can occur more than once.

Table 1 shift types and their characteristics

Shift type	Shift start and duration	Type of ambulance	Staff type	Hours timetable
D24	08:00 – 24 hours	ALS ambulance	Driver and nurse	17.6
Day8	08:00 – 9 hours	ALS ambulance	Driver and nurse	9
Day7	07:00 – 8 hours	ALS ambulance	Driver and nurse	8
Day10	10:00 – 8 hours	ALS ambulance	Driver and nurse	8
Evening	15:00 – 8 hours	ALS ambulance	Driver and nurse	8
Night	23:00 – 8 hours	ALS ambulance	Driver and nurse	8
Solo day	08:00 – 8 hours	Solo	nurse	8
Solo evening	14:00 – 8 hours	Solo	nurse	8
D24 Island	08:00 – 24 hours	ALS ambulance	Driver and nurse	10
D12	08:00 – 12 hours 16:00 – 12 hours	ALS ambulance	Driver and nurse	12
Care	08:00 – 9 hours	Care ambulance	Two care providers	9

Table 2 all shifts of the base timetable 2014

Shift	Post	Shift type	Number of shifts per year	Average number of urgent calls per shift
An24	Annen	D24	365	3,51
As24	Assen	D24	2*365	5,23
AsD	Assen	Day8	52*5	1,48
AsDC	Assen Centrum	Day8	365	2,67
AsSO	Assen	Solo day	52*5	1,44
Be24	Beilen	D24	365	3,31
Bo24	Borger	D24	365	3,75
Bp24	Buitenpost	D24	365	4,05
BpD	Buitenpost	Day8	26*5	1,53
Co24	Coevorden	D24	365	4,25
D12Bu	Bergum	D12	26*2	2,39
D24B	Bolsward	D24	52*5	3,68
D24K	Koudum	D24	365	2,08
DBu	Bergum	Day8	26*5	1,73
Di24	Dieverbrug	D24	365	2,46
Djo	Joure	Day8	365	1,53
Em24	Emmen	D24	365	6,1
EmD	Emmen	Day8	52*5*2	1,61
EmN24	Emmen Noord	D24	365	5,24
EmSO	Emmen	Solo day	52*5	1,18
HoD	Hoogeveen	Day8	365	2,01
Ho24	Hoogeveen	D24	365	6,02
Kl24	Klazienaveen	D24	365	5,31
LeD24	Leeuwarden	D24	26*7	5,36

LeD	Leeuwarden	Day8	26*2	2,69
LeD10	Leeuwarden	Day10	26*5	1,82
Le24	Leeuwarden/Stiens	D24	26*7	4,05
LeD7	Leeuwarden	Day7	26*12	2,08
LeL1	Leeuwarden	Evening	26*7	2,53
LeN2	Leeuwarden	Night	26*7	2,32
LESA	Leeuwarden	Solo evening	26*5	2,03
Me24	Meppel	D24	365	4,97
Me24	Meppel	Day8	52*5	1,74
Ro24	Roden	D24	365	4,48
Sn24	Sneek	D24	365 + 52*2	4,2
SNSA	Sneek	Solo evening	52*5	0,84
WeD	Westerbork	Day8	365	1
Islands24	Islands	D24 Islands	1460	-

3 The number of urgent calls per nurse

Figure 1 shows that in 2014 the number of urgent calls per nurse ranges roughly from hundred to five hundred calls. In this section the variation of the number of urgent calls per nurse is explained by the total hours a nurse works and where and when a nurse works.

3.1 Available data

The data consists of all urgent calls of the year 2014. Of these calls the time and location of the call is available, which shift served the call and which nurse was assigned to this shift. With this data the distribution of the number of urgent calls per shift and the number of urgent calls per nurse can be calculated. Furthermore, the number of shifts a nurse has worked of a certain shift can be extracted from the timetable. For each unique shift, the shift duration and how many hours a shift counts in the timetable is also available. Finally, the number of hours available (contract size) and the number of hours worked in 2014 is known of each nurse.

3.2 Hours worked

A possible explanation for the variability of the number of urgent calls per nurse is the size of the labour contract of a nurse. This explanation excludes unavailability because of prolonged illness, pregnancy and the recruitment, transfer and departure during a year. By looking at the total number of hours worked according to the timetable, the above mentioned factors have no influence on the number of urgent call per nurse. The left plot of Figure 2 shows that the number of available hours and the number of urgent calls per nurse are correlated, but do not fit perfectly. The 7 points at the right lower corner do not follow this correlation. The right plot shows the same relation as the left plot but now for the hours worked. The hours worked seems to be a better variable to explain the number of urgent calls than the available hours but both variables do not completely explain the variation in the number of urgent calls per nurse.

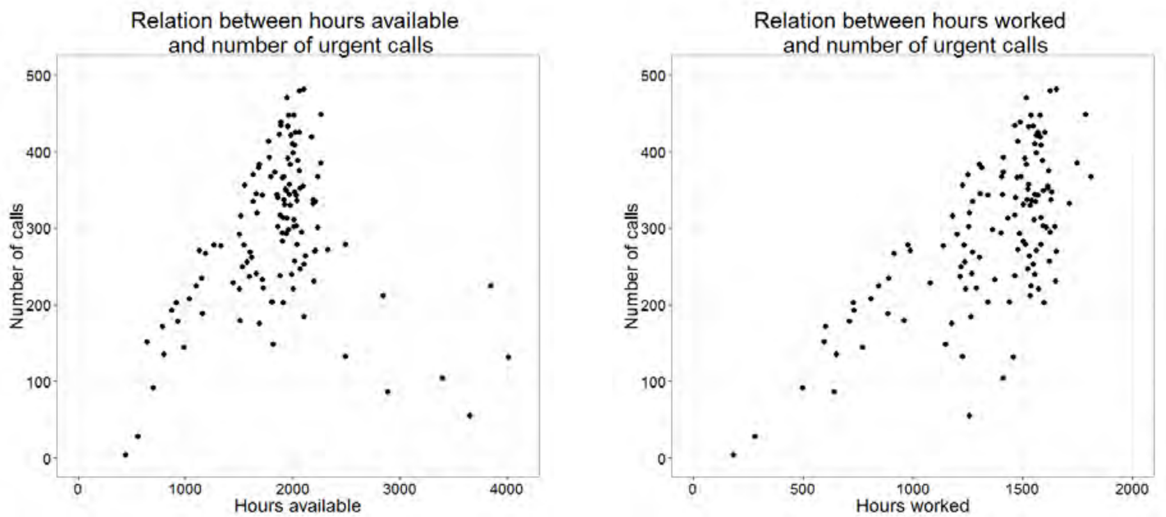

Figure 2 Relation between the hours available/worked (left/right plot) and the number of urgent calls for the year 2014

3.3 Location and time

Figure 3 shows that the average ambulance demand depends on the hour of the day. Furthermore the demand per hour is roughly the same for each day of the week, except for the Friday and Saturday nights which have a slightly higher demand than the other days. Figure 4 shows the post of UMCG Ambulancezorg and the number of urgent calls per square of 5x5 kilometres per year. The colour of the squares vary from light blue (0 calls) to blue (750 calls). Figure 4 shows that the ambulance demand is location dependent. Because most of the time the closest available ambulances is assigned to a call and the ambulance demand is time and location dependent, it is likely that the number of calls per shift depends on the location of the shift and the start and end time of the shift. Therefore the number of urgent calls per nurse is likely to depend on the location of the shift and the start and end time of the shift.

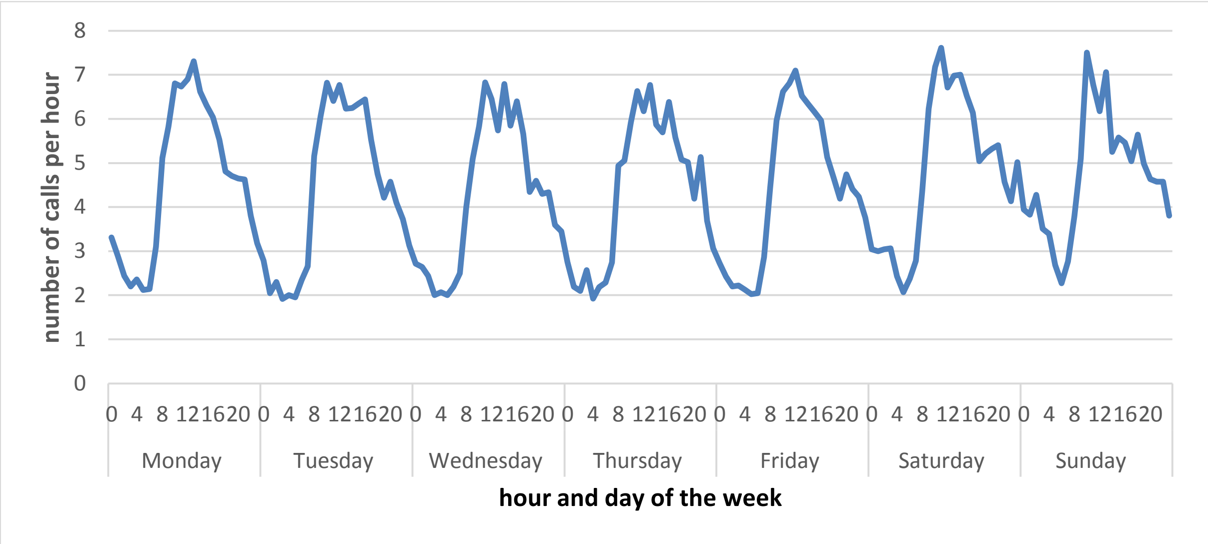

Figure 3 average number of urgent calls per hour per day of the week 2014 of UMCG Ambulancezorg

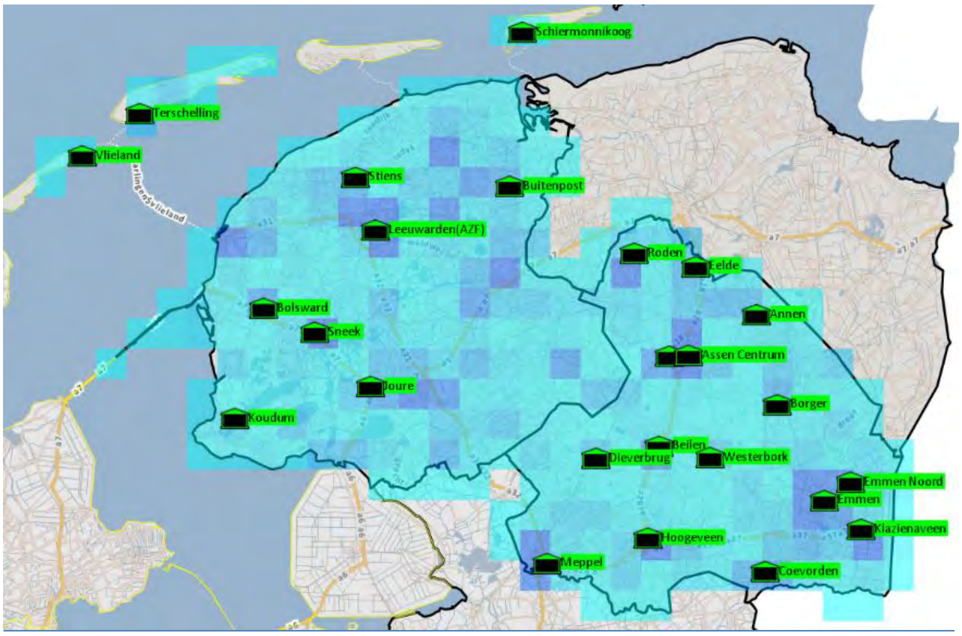

Figure 4 posts UMCG Ambulancezorg

The plots from 9.1 of the appendix show for all 37 shifts at the mainland the empirical distribution of the number of urgent calls per shift (bar) as well the Poisson distribution with lambda equal to the average number of urgent calls per shift. Figure 5 shows the same plots as 9.1 of three different shifts. The mean number of urgent calls per shift, the total number of shifts for the year 2014 and the p-value of the Chi-Square test are stated above each plot. These plots indicate that the Poisson distribution has a reasonable fit with the empirical distribution for the number of urgent calls per shift. The Chi-Square test is used to test the null-hypothesis that the number of urgent calls per shift is Poisson distributed with lambda equal to the average number of urgent calls per shift. The Chi-Square test statistic is based on the comparison of the expected number and the empirical number of observations per bin, in this case the number of urgent calls per shift. Because the expected number of observations per bin should be larger than 5, bins with a lower expected number of observations are combined with other bins such that the expected number of observations is larger than 5. Specifically, this may hold for the left and right tail of the distribution. See Table 3 for more details about the test.

Section 9.2 of the appendix contains for all 37 shifts at the mainland the results from the Chi-Square test with a confidence level of 95%. The first table makes no distinction between day of the week while the second table distinguishes between day of the week. When no distinction is made between days of the week, 13 of the 37 null-hypothesis are rejected, whereas with this distinction between day of the week, only 25 of the 207 null-hypothesis are rejected. Although 13 null-hypothesis are rejected, the plots show that the Poisson distribution and the empirical distribution do not differ much and therefore the Poisson distribution without distinction between day of the week seems still a reasonable choice to describe the number of urgent calls per shift.

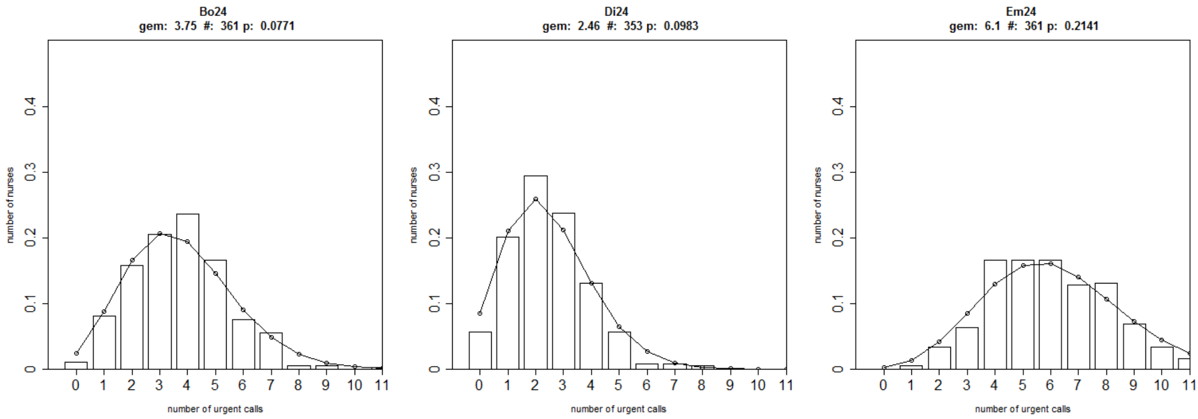

Figure 5 The empirical distribution (bar) of the number of urgent calls per unique shift and the Poisson distribution (line)

Table 3 Chi-Square test

1. $H_0: the\ number\ of\ urgent\ calls\ per\ shift\ is \sim Poisson(\lambda)$
   $with\ \lambda = mean(number\ of\ urgent\ calls\ per\ shift)$
   $H_1: the\ the\ number\ of\ urgent\ calls\ per\ shift\ is\ not \sim Poisson(\lambda)$
   $with\ \lambda = mean(number\ of\ urgent\ calls\ per\ shift)$
2. The test statistic $T = X^2 = \sum_{ii=0}^{kk} \frac{[Z_{ii} - zp_{ii}]^2}{zp_{ii}}$ is used which is χ^2_{kk-2} distributed.
   $Z_{ii}: number\ of\ shifts\ with\ ii\ urgent\ calls\ per\ shift$
   $kk: number\ of\ bins$
   $p_{ii}: probability\ of\ ii\ urgent\ calls\ per\ shift$
3. Calculate the critical region $K = \{\geq x\}$ with $P_{H_0}(T \geq x) = 0{,}05$
4. Reject the null hypothesis when $T \in K$, accept the null hypothesis

3.4 Seasonality

The figures in section 9.3 of the appendix shows for all 37 shifts at the mainland of UMCG Ambulancezorg the number of urgent calls per day per shift of the year 2014 (black line) and the moving average for 31 days of the number of urgent calls per shift. For some shifts the moving average is a fairly straight line while for other shifts the moving average has some peaks. These peaks and could be caused by local seasonal fluctuation in the demand for ambulance care but the stochastic nature of the number of urgent calls per shift could also be a cause. Because only data from 2014 is available and there is visually no clear indication of a seasonal pattern, seasonal patterns are assumed to be absent. Therefore the number of urgent calls per shift is modelled without seasonal influence as a Poisson random variable with lambda the average number of urgent calls per shift.

3.5 The number of urgent calls per nurse explained

Section 3.2 shows that there is a relation between the number of urgent calls per nurse and the amount of hours worked per nurse. Section 3.3 shows that number of urgent calls per post and shift type can differ significant and can be described with a Poisson distribution. Combining the observations of these sections, leads to the model that the number of urgent calls per nurse over a longer period depends on the number of shifts a nurse works of a certain type at a certain post and that the number of urgent calls per nurse is Poisson distributed. This model is mathematically formulated below and uses the fact that the sum of two Poisson random variables is again a Poisson random variable.

$$X_j: number\ of\ urgent\ calls\ per\ shift\ j$$

$$X_j \sim Poisson(\lambda_j)$$

$$\lambda_j = mean\ number\ of\ urgent\ calls\ for\ shift\ j\ based\ on\ dataXj)$$

$$u_{ij}: number\ of\ shift\ j\ worked\ by\ nurse\ i$$

$$k: total\ number\ of\ different\ shifts$$

$$Y_i: total\ number\ of\ urgent\ calls\ for\ nurse\ i$$

$$\bar{\lambda}_i = \sum_{j=1}^{k} [u_{ij}\ \lambda_j]$$

$$Y_i \sim Poisson(\bar{\lambda}_i)$$

To test if the model above explains the number of urgent calls per nurse, the model is calculated using data of the year 2014 in the following five steps:

1. Y_i For each nurse *i* in the timetable of 2014, the number of urgent calls served in 2014 is counted.
2. λ_j The mean number of urgent calls per shift *j* is calculated for each shift in the timetable of 2014.
3. u_{ij} For each nurse *i* the number of shifts worked in the timetable of 2014 of a specific shift *j* is counted.

4. $\bar{\lambda}_i$ For each nurse *i* the number of shifts from step three is multiplied with the mean number of urgent calls per shift *j* from step 2 and summed.
5. Per nurse *i* a lower bound (LB) and upper bound (UB) is calculated for the number of urgent calls such that $P(X < LB) \leq 0.025$ and $P(X > UB) \leq 0.025$ with $X \sim Poisson(\lambda)$ and λ equal to the result of step 4.

The blue region of Figure 6 indicates the space between the lower and upper bound of the number of urgent calls based on the model above for the year 2014. The empirical number of urgent calls per nurse are indicated by a green dot when these calls lie between the lower and upper bound and by a red dot otherwise. The dots are ordered based on the expected number of urgent calls per nurse. The left plot makes a distinction between the day of the week, whereas the right plot does not make such a distinction. For both versions of the mathematical model, the empirical number of urgent calls per nurse of only 1 of the 126 nurses does not lie between the lower and upper bound of the expected number of urgent calls per nurse.

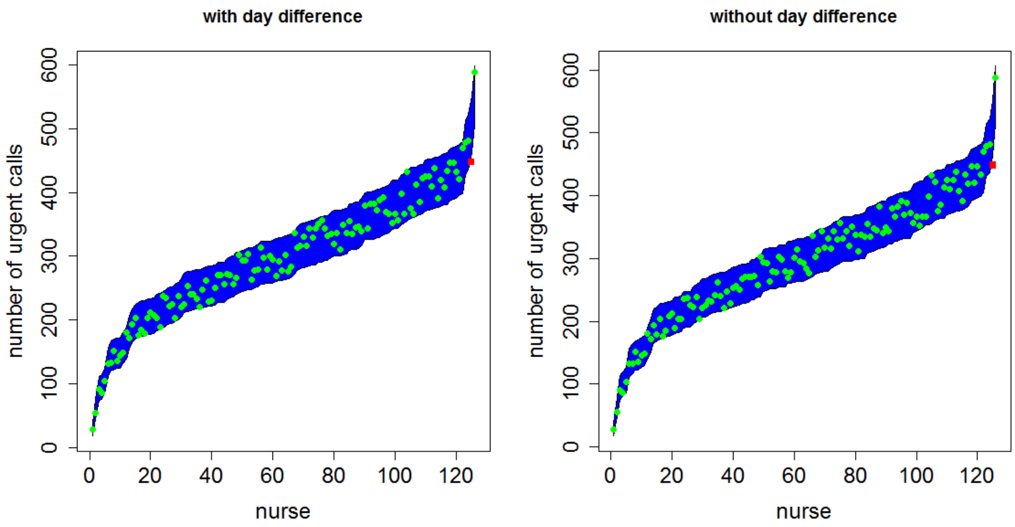

Figure 6 Lower and upper bound of the number of urgent calls per nurse based on the model and the timetable of 2014.

Assuming that the model is correct, the chance that the empirical number of urgent calls per nurse lies between the lower and upper bound of the number of urgent calls per nurse is 0.95. Therefore the number of dots outside(inside) the interval, marked by the lower and upper bound, is binomial distributed. Using this fact, the expected number of dots outside this interval is $126 \times 0.05 = 6.25$ while for both versions only 1 dot is outside the interval. The binomial two-sided test is used to test the null-hypothesis that the number of urgent calls per nurse can be described by the mathematical model. The hypothesis testing of Table 4 rejects the null-hypothesis.

Table 4 Hypothesis testing for the model of table 1 based on data of the year 2014

1. H_0: *the number of urgent calls per nurse can be described by the mathematical model*
   H_1: *the number of urgent calls per nurse can't be described by the mathematical model*
2. The test statistic $T = number\ of\ nurse\ outside\ the\ interval\ marked\ by\ the\ LB\ \&\ UB = 1$ is used and $T \sim \text{binomial}(n, p)$ with p=5/100 and n=126.
3. The binomial two-sided test is used and the null hypothesis is rejected when $P_{H_0}(T \geq t) \leq 0{,}025 \cup P_{H_0}(T \leq t) \leq 0{,}025$.
4. p-value: $p = 2 \times Min\{P_{H_0}(T \geq 1), P_{H_0}(T \leq 1)\} = 2 \times 0.01191 = 0.02382 < 0{,}05$ and therefore the null hypothesis is rejected

This could mean that the number of urgent calls per nurse over a longer period does not depend on the number of shifts a nurse works of a certain type at a certain post. A small(large) number of dots outside the interval and thus rejecting the null-hypothesis, could also indicate that the interval is too wide(narrow). Section 3.3 shows that Poisson distribution is a reasonable choice to describe the number of urgent calls per shift but it overestimates the chance of large number of calls per shift. This results in a larger variance of the number of urgent calls per shift compared to the empirical variance which causes a lower(higher) lower(upper) bound of the number of urgent calls per nurse. Using the empirical distribution of the number of urgent calls per shift for bootstrapping the lower and upper bound of the number of urgent calls per nurse results in Figure 7. Using the null-hypothesis procedure of Table 4 with $t = 4$, $p = 5/100$ and $m = 126$ there is no reason to reject the null-hypothesis for the bootstrap method (p-value=0.5354). The mean interval width for the model and bootstrap method with(without) day difference is 66.7(66.8) and 57.9(60.5), respectively. This indicates that the Poisson distribution leads to a higher upper bound and a lower lower bound and therefore it describes the number of urgent calls per nurse less accurate than when the empirical distribution is used. Although the mean interval width for the model with Poisson distribution and the bootstrap method differ, the difference is small and therefore the model is a reasonable choice to describe the number of urgent calls per nurse.

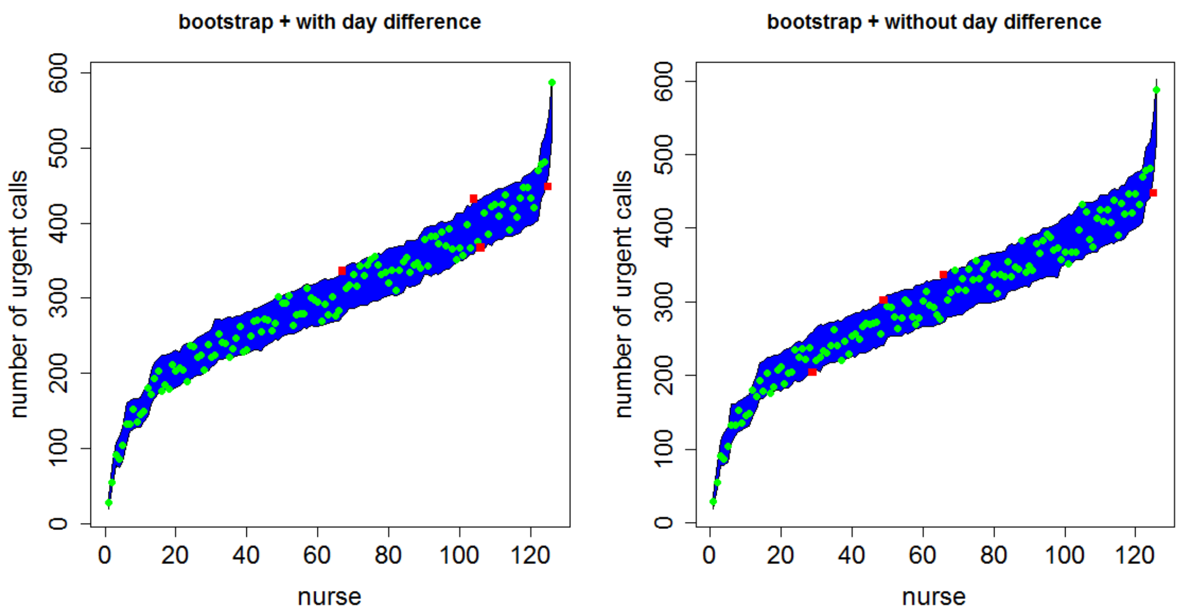

Figure 7 Lower and upper bound of the number of urgent calls per nurse based on bootstrapping and the timetable of 2014

4 Assignment problem

This chapter gives a short overview of the assignment problem based on the paper Pentico (2007). Techniques from Bertsimas et al (1997) are used to reformulate the mathematical models that are non linear as Integer Linear Programming (ILP) models.

4.1 The classic assignment problem

For this problem there are n tasks and m agents and $n = m$. Each task j needs to be assigned to one agent i and each assignment has a cost c_{ij}. The objective is to minimize the total cost of the assignments.

The mathematical model is as follows:

$$Minimize \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$$

$$\sum_{j=1}^{n} x_{ij} = 1 \quad i \in \{1 \dots m\}$$

$$\sum_{i=1}^{m} x_{ij} = 1 \quad i \in \{1 \dots m\}$$

$$x_{ij} \in \{0,1\}$$

If agent i is assigned to task j then $x_{ij} = 1$ and else $x_{ij} = 0$. The first constraint ensures that every agent is assigned to one task while the second constraint ensures that every task is assigned to one agent. If the model is solved with linear programming then the third constraint is always satisfied because a fractional solution is never a basis feasible solution (Papadimitriou et al, 1998).
For the next problems the same notation is used as above unless stated otherwise.

4.2 The bottleneck assignment problem

The classic assignment problem minimizes or maximizes the total costs of assigning tasks to agents. This could result in low costs for all assignments except for one assignment having very large costs. The bottleneck assignment problem (BAP) minimizes the maximum (or maximizes the minimum) individual assignment costs.

$$Minimize \ \max_{ij}\{c_{ij}x_{ij}\}$$

$$\sum_{j=1}^{n} x_{ij} = 1 \quad i \in \{1 \dots m\}$$

$$\sum_{i=1}^{m} x_{ij} = 1 \quad j \in \{1 \dots n\}$$

$$x_{ij} \in \{0,1\}$$

There are standard tricks for reformulating this min-maxproblem into an ILP. Introduce the variable q, representing the maximum costs. By adding the constraint

$$q \geq c_{ij}x_{ij}$$

$$i, j \in \{1 \dots n\}$$

and changing the objective function in

$$Minimize\ q$$

the variable q will be equal to the maximum cost. These adjustments make the objective function linear and therefore the problem can be modelled as an ILP.

4.3 The semi-assignment problem

In the classic assignment problem every agent and task is unique which means that each row and column of the cost matrix c_{ij} is different. However, the cost of several agents and/or tasks could be the same despite that the agent and/or the task is unique. An example is "in the area of Navy personnel assignment [in which] a particular ship may require several radio operators with the same rank and skill level" (Kennington et al., 1992). If there are *m* agents, *n* task groups $(n < m)$ with each d_j tasks and the total number of tasks is $\sum_{j=1}^{n} d_j = m$ and each agent can be assigned to at most one task, then the assignment problem is as follows:

$$Minimize \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$$

$$\sum_{j=1}^{n} x_{ij} = 1 \quad i \in \{1 \dots m\}$$

$$\sum_{i=1}^{m} x_{ij} = d_{ij} \quad j \in \{1 \dots n\}$$

$$x_{ij} \in \{0,1\}$$

The first constraint ensures that every agent is assigned to a task and the second constraint ensures that every task within each task group is assigned an agent. The third constraint ensures that there are no fractional solutions.

4.4 Combining multiple criteria into one

In the classic assignment problem the overall costs is minimized, whereas the bottleneck assignment problem minimizes the cost of the assignment with the maximum cost. Both objectives can be combined as follow

$$Minimize \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} + F \times \max_{ij}\{c_{ij}x_{ij}\}$$

$$Minimize\ F \times \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} + \max_{ij}\{c_{ij} x_{ij}\}$$

The first objective minimizes the maximum individual assignment cost first and then it minimizes the overall costs by choosing $F > \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$ for all feasible solutions. The second objective minimizes the maximum individual assignment cost after the overall costs is minimized by choosing $F > \max_{ij}\{c_{ij} x_{ij}\}$ for all feasible solutions.

4.5 The assignment problem with side constraints

All the problems discussed above optimize an objective function subject to two sets of constraints. These constraints ensure that all tasks are assigned to an agent and each agent is assigned to one task. There are, however, problems which have additional constraints. Mazzola et al. suggest the following general model for adding side constraints to the classic assignment problem:

$$\sum_{i=1}^{m} \sum_{j=1}^{n} r_{ijk}\, x_{ij} \le b_k$$

with r_{ijk} is the amount of resource *k* it takes for assigning agent *i* to task *j* and b_k is the total amount of resource *k* that is available.

4.6 The generalized assignment problem (GAP)

For all the previous discussed problems, the number of tasks assigned to an agent is exactly one. The generalized assignment problem (GAP) allows multiple tasks assigned to an agent without the restriction of using all the capacity of that agent. Each assignment of task j to agent i uses an amount a_{ij} of the total available capacity b_i of agent i. Just as with the classic assignment problem, a job can only be assigned once. The mathematical model is as follows:

$$Minimize \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

$$\sum_{i=1}^{m} x_{ij} = 1 \quad j \in \{1 \dots n\}$$

$$\sum_{j=1}^{n} a_{ij} x_{ij} \le b_i \quad i \in \{1 \dots m\}$$

$$x_{ij} \in \{0,1\}$$

The first constraint ensures that every task is assigned to one agent whereas the second constraint ensures that every agent is assigned at most his capacity.

4.7 The bottleneck GAP

The general assignment problem can be adjusted to a bottleneck GAP in the same way as the classic assignment problem can be adjusted to a bottleneck assignment problem. For the bottleneck GAP there are two options for the objective function. The first option minimizes the maximum assignment cost per task which is the same as for the BAP model. The second option minimizes the maximum total cost per agent with the following minimization objective

$$Minimize \max_{i \in \{1 \dots m\}} \left\{ \sum_{j=1}^{n} c_{ij} x_{ij} \right\}$$

The objective functions of both options are non linear. Because the first option has the same objective function as the BAP model, the first option can be formulated as a linear program by applying the adjustments from the BAP model to the GAP model. The objective function of the second option is equivalent to minimizing the variable q which is larger or equal than $\sum_{j=1}^{n} c_{ij} x_{ij}$ for all $i \in \{1 \dots m\}$. By adding the constraint

$$q \geq \sum_{j=1}^{n} c_{ij} x_{ij} \;\; i \in \{1 \dots m\}$$

and changing the objective function in

$$Minimize\ q$$

to the GAP model the variable q is equal to the total cost per agent of the agent with the maximum cost.

5 A model for assigning nurses to shifts

In this chapter the previous topics are used to create an ILP model which can be solved to answer the research question. To accomplish this a couple of steps are made. First, the research question and the assignment problem are connected to formulate a model. Second, this new model and the model for the number of urgent calls per nurse, are linked. Finally, data is applied to this new model.

5.1 The research question modelled as an assignment problem

Section 3 shows that the number of urgent calls per nurse depends on the number of shifts a nurse works of a certain shift. The number of urgent calls per nurse can be influenced by assigning more or less shifts of a certain shift to a nurse. Assigning nurses to different shifts can be formulated as a generalized assignment problem by modelling the shifts as tasks, the nurses as agents, the commuting time as cost and minimizing the commuting time as the optimization objective.

Table 5 shows the generalized assignment problem as an integer linear programming (ILP) model. Here c_{ij} is the travel time from the home of nurse *i* to the post of shift *j*, the decision variable x_{ij} is 1 when nurse *i* is assigned to shift *j* and else 0, a_j is the shift duration of shift j, b_i the number of contract hours a nurse *i* should work and C a constant. There are m nurses and n shifts. The objective of the model is to minimize the total commuting time of all nurses. The first constraint ensures that every shift is assigned to an agent. The second and third constraint ensures that the difference between the total hours assigned to a nurse and his labour agreement is no more than a constant C. The fourth constraint ensures that a nurse is not assigned partially to a shift.

Table 5 assignment problem for minimizing the total commuting time

$$Minimize \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$$

$$\sum_{i=1}^{m} x_{ij} = 1 \quad j \in \{1 \dots n\}$$

$$\sum_{j=1}^{n} a_j x_{ij} \leq b_i + C \quad i \in \{1 \dots m\}$$

$$\sum_{j=1}^{n} a_j x_{ij} \geq b_i - C \quad i \in \{1 \dots m\}$$

$$x_{ij} \in \{0,1\}$$

The model from Table 5 has a decision variable for each shift whereas the same shift is carried out multiple times per year. This observation is used in Table 6 which is a reformulation of Table 5. In this model, variable x_{il} is the number of shifts (non-negative integer) nurse *i* is assigned to of shift *l* and Q_l is the number of shifts available per year of shift *l* of the total *k* different shifts.

Table 6 reformulated assignment problem minimizing the total commuting time

$$Minimize \sum_{i=1}^{m}\sum_{l=1}^{n} c_{il}x_{il}$$
$$\sum_{i=1}^{m} x_{il} = Q_l \quad l \in \{1 \dots k\}$$
$$\sum_{l=1}^{n} a_l x_{il} \le b_i + C \quad i \in \{1 \dots m\}$$
$$\sum_{l=1}^{n} a_l x_{il} \ge b_i - C \quad i \in \{1 \dots m\}$$
$$x_{il} \in \mathbb{N}^+$$

The ILP's will be implemented and solved with AIMMS 4.9. AIMMS is "A comprehensive platform providing decision support for everyone" and it can use, among others, CPLEX and Gurobi to solve LP and ILP models. With our parameter setting, AIMMS always finds a feasible solution for the model of Table 6 while this is not the case for the model of Table 5. Therefore the model of Table 6 will be used.

5.2 including the number of urgent calls per nurse

The previous formulated ILP model minimizes the commuting time, ensures that every shift is assigned to a nurse and that the total duration of assigned worked per nurse is roughly the same as his labour agreement. Because there is no constraint that ensures that every nurse carries out a certain number of urgent calls per year, the observed variation of Figure 1 could still occur. This could be incorporated in the model of Table 6 by adding the following constraint:

$$\sum_{l=1}^{k} r_l x_{il} \ge s$$

$$i \in \{1 \dots m\}$$

Here r_l is the expected number of urgent calls for shift *l* and *s* is a lower bound for the expected number of urgent calls per nurse. The above constraint ensure that the expected number of urgent calls for every nurse is greater or equal to s and section 3 shows that the number of calls per nurse is roughly Poisson distributed. The value of s could be chosen as follows. Consider the random variable *X* with

$$\mathrm{X} \sim Poisson(s)$$

Choosing s such that

$$P(X \le NS) \le 0.05$$

ensures that the chance that the number of urgent calls per nurse per year is less than the national standard (NS), is at most 0.05. By fixing the value of *s,* the expected number of urgent calls per nurse can easily be calculated, and vice versa.

5.3 Model variations

The ILP model of Table 6 minimizes the total commuting time of all nurses. Table 7 shows a variation which first minimizes the commuting time of the nurse with the maximum commuting time and then minimizes the overall commuting time. The first constraint ensure that the commuting time of every nurse is less or equal than the commuting time of the nurse with the maximum commuting time. Note that $\sum_{i=1}^{m}\sum_{l=1}^{k} c_{il}x_{il}$, the overall commuting time of all the nurses, is less than 1.000.000. Table 6 and Table 7 minimize the commuting time for a give s. However, there is a value of s for which the models of Table 6 and Table 7 are solvable while for larger values of s, these models are not solvable. Table 8 calculates this maximum value of s. For this new ILP Table 6 is adjusted by turning the constant s into a variable and maximizing s.

Table 7 minimizing the commuting time of the nurse with the maximum commuting

$$Minimize \sum_{i=1}^{m}\sum_{l=1}^{k} c_{il}x_{il} + 1.000.000 \times q$$

$$\sum_{l=1}^{k} c_{il}x_{il} \leq q \quad i \in \{1 \ldots m\}$$

$$\sum_{i=1}^{m} x_{il} = Q_l \quad l \in \{1 \ldots k\}$$

$$\sum_{l=1}^{k} a_l x_{il} \leq b_i + C \quad i \in \{1 \ldots m\}$$

$$\sum_{l=1}^{k} a_l x_{il} \geq b_i - C \quad i \in \{1 \ldots m\}$$

$$\sum_{l=1}^{k} r_l x_{il} - s \geq 0 \quad i \in \{1 \ldots m\}$$

$$x_{il} \in \mathbb{N}^+$$

Table 8 maximizing the expected number of urgent calls of the nurse with the lowest expected number of urgent calls.

$$Maximize\ s$$

$$\sum_{i=1}^{m} x_{il} = Q_l \quad l \in \{1 \ldots k\}$$

$$\sum_{l=1}^{k} a_l x_{il} \le b_i + C \quad i \in \{1 \ldots m\}$$

$$\sum_{l=1}^{k} a_l x_{il} \ge b_i - C \quad i \in \{1 \ldots m\}$$

$$\sum_{l=1}^{k} r_l x_{il} \ge s \quad i \in \{1 \ldots m\}$$

$$x_{il} \in \mathbb{N}^+$$

5.4 Model instances

In practice, nurses from the islands work sometimes on the mainland and nurses from the mainland sometimes work on the islands, but there are no rules or guidelines which determines the number of shifts that should be worked at the island or mainland. Because there are only 4 nurses who live on an island and they also mainly work on the islands, nurses from the islands are only assigned to shifts on the islands. These nurses therefore can be excluded from the model. Because all the nurses from the islands together can do only 110 shift of the 1460 shifts at the islands per year, the other 1350 shifts are assigned to nurses from the mainland. By setting the expected number of urgent calls per shift and the commuting time to zero for shifts at the islands, these shifts do not influence the number of urgent calls per nurse and the total travel time. To prevent that only a few nurses are assigned to shifts on the island, the number of shifts on the islands per nurse are constraint by 20.

The ILP models from Table 6, Table 7 and Table 8 have the same constraints but different objective functions. The ILP from Table 6 minimizes the overall commuting time of all the nurse, the ILP from Table 7 minimizes the commuting time of the nurse with the maximum commuting time while Table 8 calculates the maximum value of the national standard. These three models are initialized as follow:

variable	Description
k	There are 38 unique shifts. No distinction is made for the day of the week because it results not in a better model and it increase the number of decision variables.
m	There are 122 nurses at the mainland.
n	There are 38 unique shifts scheduled on a daily or on a (two) weekly basis. Therefore the total number of shifts is 12550.
x_{il}	There are 38 unique shifts and 122 nurses . Therefore there are 4636 decision variables.
c_{il}	The commuting time is the travel time from the home location of nurse *i* to the station of shift *l*. The commuting time is calculated using the API of the demo server

	of project OSRM and is rounded up to ten.
Q_l	The number of shifts available of shift *l* for the year 2014; see Table 2
a_l	The duration of shift *l* in hours is based on the column *hours timetable* from Table 1.
b_i	The number of hours that nurse *i* should work is equal to the FTE of the labour agreement multiplied by 1530 where a full time employed nurse is 1530 hours available. See section 9.4 of the appendix.
C	The maximum difference between the labour agreement and the planned hours is set to 40 hours per year.
r_l	The expected number of urgent calls per shift *l* is equal to the mean number of urgent calls per shift *l* in 2014 as mentioned in Table 2.
s	Different values for the minimum expected number of urgent calls per nurse are used.

6 Results

This section presents the results of the sub questions 2, 3 and 4. The results are obtained by solving the models from Table 6, Table 7 and Table 8 using the software package AIMMS and GUROBI 6.0 as solver.

6.1 Maximum value national standard

The maximum value of the national standard is equal to the maximum number of urgent calls of the nurse with the lowest number of urgent calls. This maximum is calculated by solving the model from Table 8. The influence of (almost) only full time employee is modeled by removing the nurses 112 until 122 from the model and setting the FTE of almost all the other nurses to 1. This ensures that the number of hours needed to carry out all shifts of 2014 is roughly the same as the number of hours available of all nurses. The number of hours a nurse should work, b_i in the model, is based on the tables from Section 9.4. For both versions of the model holds that the national standard is independent of the FTE of a nurse, because a part time and full time nurse should be equally competent. Table 9 shows the maximum expected value and the lower bound of the national standard. The lower bound is the number of urgent calls every nurse will do with 95% probability. The minimal number of urgent calls per nurse for the model with only full time employees is almost 20% higher compared to the model with part time employees.

Table 9 maximum expected number of urgent calls per nurse and maximum value of the national standard

	With part time employees	Only full time employees
Maximum expected value national standard	277	327
Maximum value lower bound	250	298

6.2 Influence of the national standard on the commuting time

The models from Table 6 and Table 7 have the same constraints but different objective functions. While the first model minimizes the overall commuting time, the second minimizes the commuting time of the nurse with the largest commuting time, abbreviated in the following to nurse commuting time. This difference will probably result in a larger overall commuting time for the second model compared to the first model while this will be the other way around for the nurse commuting time. Therefore the overall commuting time and the nurse commuting time is measured for each model. It is important to observe that Table 7 minimizes both the overall commuting time and the nurse commuting time but the latter is minimized first because the overall commuting time is smaller than one million. Figure 8 shows for different values of s (the minimum expected number of urgent calls per nurse) the overall commuting time (solid lines) and the nurse commuting time (dotted lines) for the models of Table 6 (blue lines) and Table 7 (red lines). For the overall commuting time the blue and red line are more or less parallel lines with a distance of roughly 2000 minutes between them. The nurse commuting time of the dotted blue and red line are not parallel lines. The dotted red line remains almost the same while the dotted blue line fluctuates. The fluctuation of the blue dotted line is a consequence of only minimizing the overall commuting time. The red lines show that minimizing both measurements results in fluent lines. While the travel time only increases slightly when the standard moves from 100 to 200, it increases rapidly for standards larger than 200.

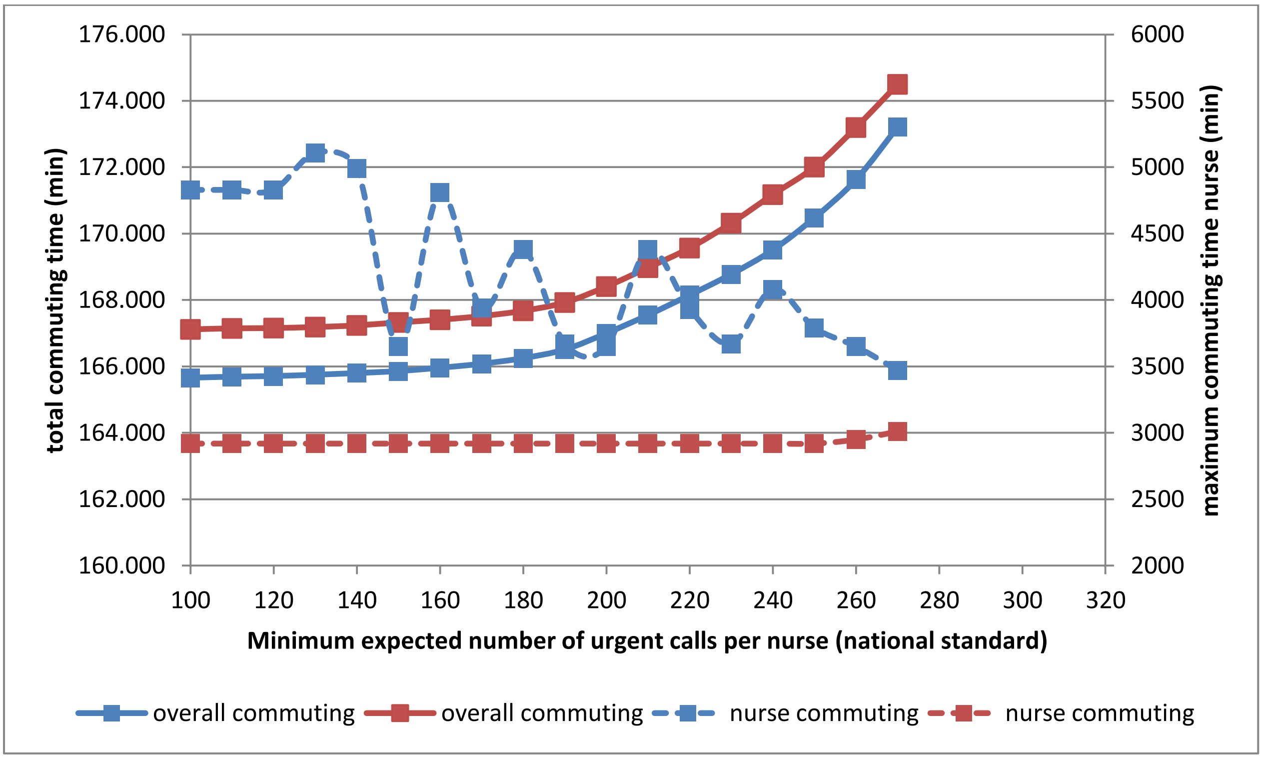

Figure 8 the total commuting time and the maximum commuting time with part time employees.

6.3 Influence of part time employee on the commuting time

Figure 9 is similar to Figure 8 except that for Figure 9 the models from Table 6 and Table 7 are solved with (almost) no part time employees. See section 6.1 how this is modelled. The total travel time of the red line is roughly 1200 minutes more than the blue line for most points. Except for larger values of s, the commuting time of the nurse with the maximum commuting time of all nurses remains almost the same when minimizing the nurse commuting time, while this fluctuates when minimizing the overall commuting time. Both observations are a result of the double minimization of Table 7.
Figure 8 and Figure 9 show comparable behavior of the lines. In both figures the red and blue line run roughly parallel and increase for larger values of the national standard. Further, the red dotted line is almost a straight line and the blue dotted line fluctuates for both figures. The figures differ from each other when the overall commuting time are compared. When minimizing the overall (nurse) commuting time and a national standard of 100, the overall commuting time of Figure 8 and Figure 9 are respectively 165.660 (167.120) and 160.470 (161.620) minutes, a difference of 5.000 (5.500) minutes. This indicates that part time employee results in more overall commuting time. To meet the national standard a part time nurse could need to work more busy shifts and less calm shifts than a full time employee. Although living in the same city, this could result in a larger commuting time for the part time employee because of the larger commuting time to a busy shift.
Comparing both figures shows that the commuting time of the nurse with the maximum commuting time could be insensitive for the number of part time employees. Consider a nurse with a large commuting time to the closest shift. Although increasing the number of part time employees, the commuting time could remain the same for this nurse because the closest shift provides enough urgent calls to meet the standard and still have the largest commuting time of all nurses. This is also the case for Figure 9. As in Figure 8 the travel time increases rapidly for values of the standard larger than 200, although not so rapidly as in Figure 8.

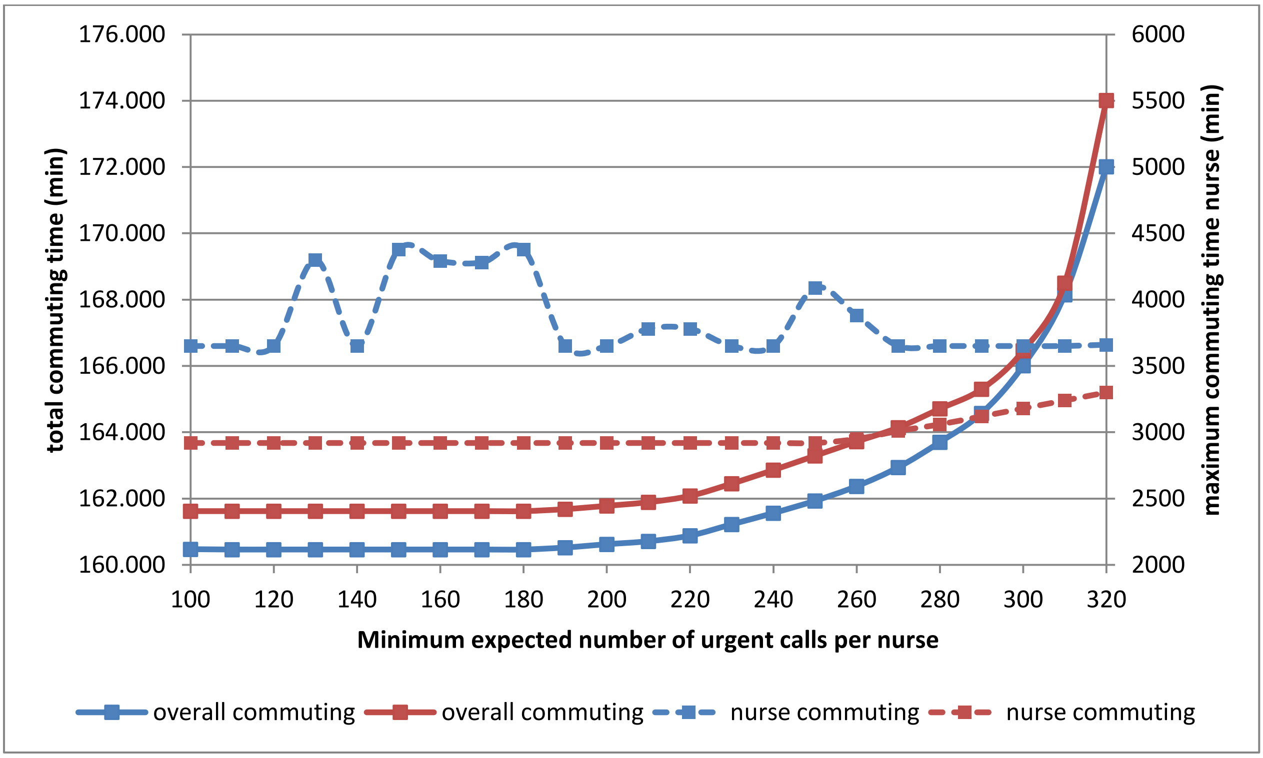

Figure 9 the total commuting time and the maximum commuting time without part time employees.

7 Conclusion

The variation in the number of urgent calls per nurse can be explained by the number of shifts a nurse works of a certain shift. Modelling the number of urgent calls per shift as a Poisson distribution results in a model for the number of urgent calls per nurse with loose lower and upper bounds. Using bootstrap instead of the Poisson distribution improves these bounds but leads to longer computation and a less flexible model. Distinguishing between the day of the week does not improve the model. For the situation of 2014 the maximum value of the national standard is 250 and 298, with and without part time employees respectively.
Independent whether there are part time employees or not, the national standard influences the overall commuting time of all the nurses but it has almost no influence on the commuting time of the nurse with the maximum commuting time. Especially large values of the national standard result in large increase of the overall commuting time.
The overall commuting time is smaller when there are only full time employees while the commuting time of the nurse with the maximum commuting time is the same for the situation with and without part time employees. The national standard is larger for the situation with only full time employees compared to the situation with part and full time employees. A national standard of 200 would only increase the commuting slightly, while for larger values the commuting time increases rapidly.

8 References

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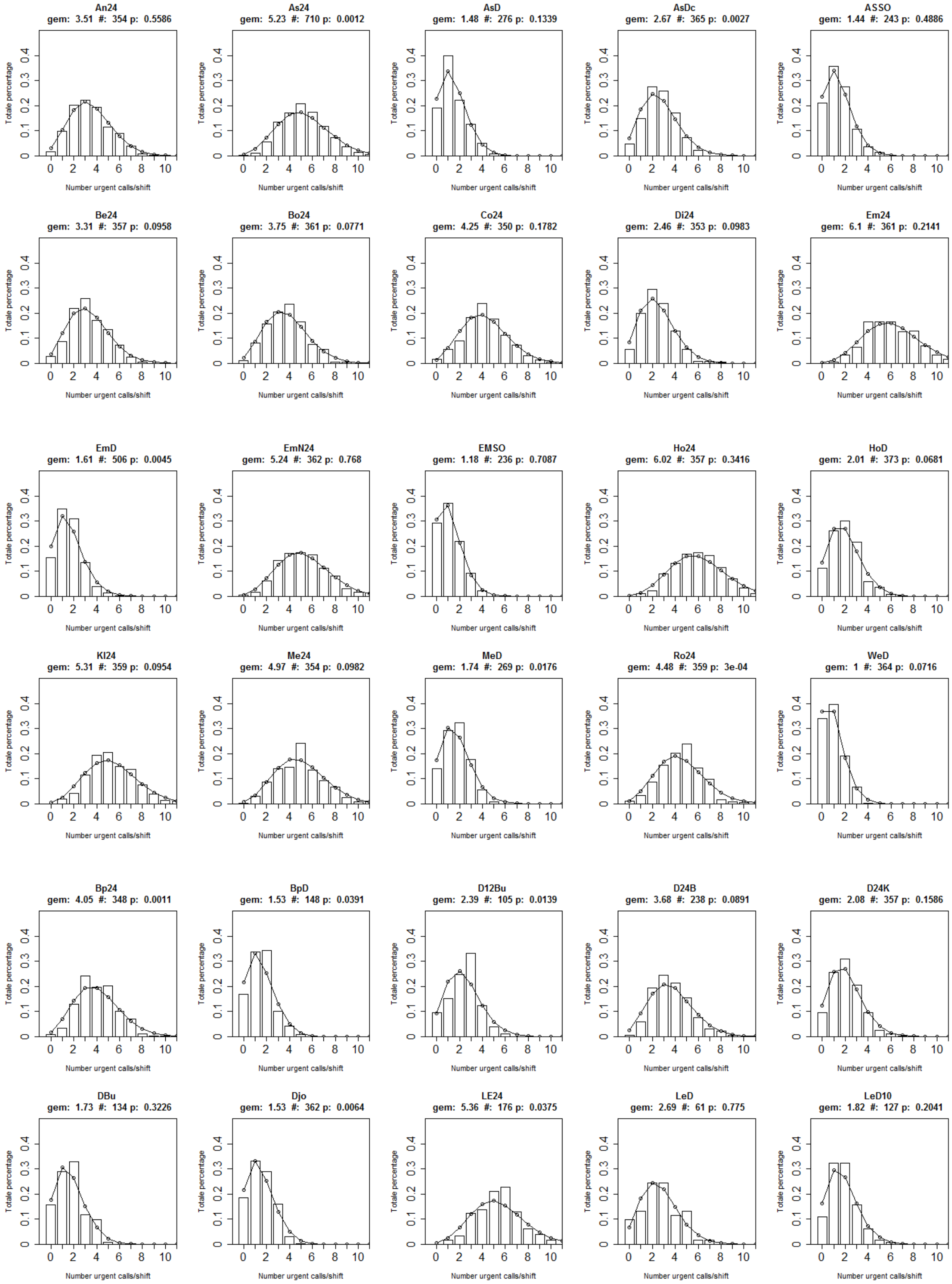
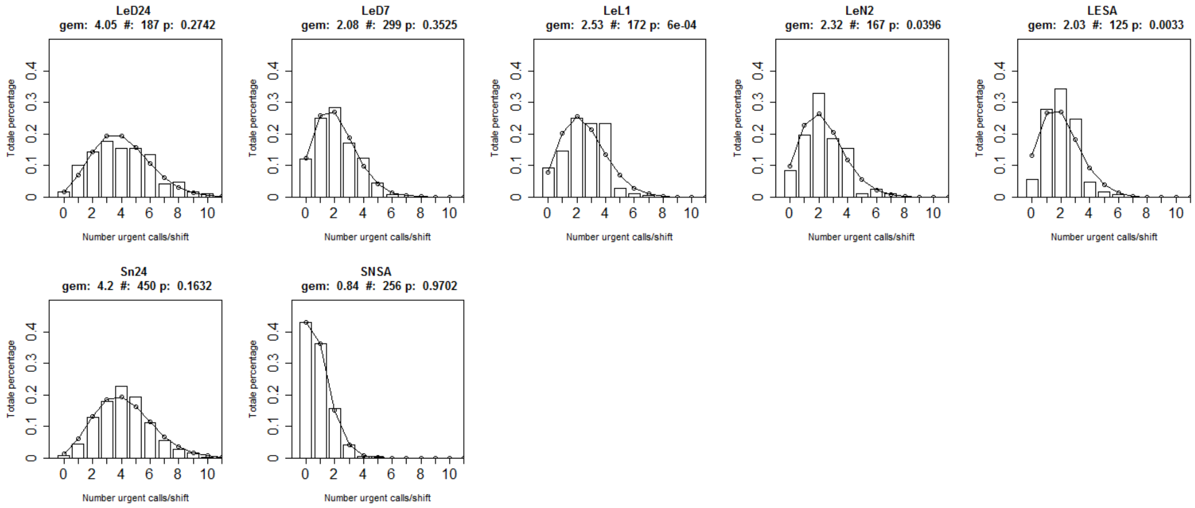
9 Appendix

9.1 Empirical distribution





9.2 Chi-Square Test

No distinction made for the day of the week

Shift	p	df	Chi	maxchi
An24	0.5586	7	5.8393	14.0671
As24	0.0012	10	29.0755	18.3070
AsD	0.1339	3	5.5804	7.8147
AsDc	0.0027	6	20.0965	12.5916
ASSO	0.4886	3	2.4270	7.8147
Be24	0.0958	7	12.1469	14.0671
Bo24	0.0771	8	14.1818	15.5073
Co24	0.1782	7	10.1867	14.0671
Di24	0.1198	5	8.7411	11.0705
Em24	0.2141	10	13.1745	18.3070
EmD	0.0045	4	15.0973	9.4877
EmN24	0.7680	9	5.7157	16.9190
EMSO	0.7087	3	1.3863	7.8147
Ho24	0.3416	10	11.2073	18.3070
HoD	0.0681	5	10.2651	11.0705
Kl24	0.0954	9	14.8393	16.9190
Me24	0.0982	8	13.4202	15.5073
MeD	0.0176	4	11.9662	9.4877
Ro24	0.0003	8	29.1587	15.5073
WeD	0.0716	3	7.0109	7.8147
Bp24	0.0011	8	25.9690	15.5073
BpD	0.0391	3	8.3592	7.8147
D12Bu	0.0139	4	12.5237	9.4877
D24B	0.0891	7	12.3666	14.0671
D24K	0.1586	5	7.9566	11.0705
DBu	0.3226	3	3.4859	7.8147

Djo	0.0064	4	14.2981	9.4877
LE24	0.0375	8	16.3616	15.5073
LeD	0.7750	3	1.1087	7.8147
LeD10	0.2041	3	4.5937	7.8147
LeD24	0.2742	6	7.5354	12.5916
LeD7	0.3525	5	5.5498	11.0705
LeL1	0.0006	5	21.8510	11.0705
LeN2	0.0396	5	11.6677	11.0705
LESA	0.0033	4	15.7877	9.4877
Sn24	0.1632	9	12.9878	16.9190
SNSA	0.9702	2	0.0604	5.9915

distinction made for the day of the week

For day of week 0 means Sunday while 6 means Saterday.

shift	Day of week	p	Df	chi	maxchi
An24	0	0.7575	4	18.815	94.877
An24	1	0.5867	4	28.301	94.877
An24	2	0.3998	3	29.472	78.147
An24	3	0.7675	4	18.272	94.877
An24	4	0.1965	4	60.359	94.877
An24	5	0.9346	4	0.8282	94.877
An24	6	0.9343	4	0.8309	94.877
As24	0	0.5236	7	61.393	140.671
As24	1	0.0016	6	213.408	125.916
As24	2	0.1998	6	85.616	125.916
As24	3	0.3227	6	69.803	125.916
As24	4	0.1787	6	89.107	125.916
As24	5	0.0909	6	109.209	125.916
As24	6	0.0632	8	147.998	155.073
AsD	1	0.5304	2	12.683	59.915
AsD	2	0.2016	2	32.027	59.915
AsD	3	0.7923	2	0.4657	59.915
AsD	4	0.8948	2	0.2223	59.915
AsD	5	0.1444	2	38.697	59.915
AsDc	0	0.0283	3	90.782	78.147
AsDc	1	0.3553	3	32.453	78.147
AsDc	2	0.2075	3	45.542	78.147
AsDc	3	0.3959	3	29.722	78.147
AsDc	4	0.2489	3	41.186	78.147
AsDc	5	0.1122	3	59.886	78.147
AsDc	6	0.3349	3	3.393	78.147
Be24	0	0.1828	4	62.267	94.877
Be24	1	0.1343	4	70.307	94.877

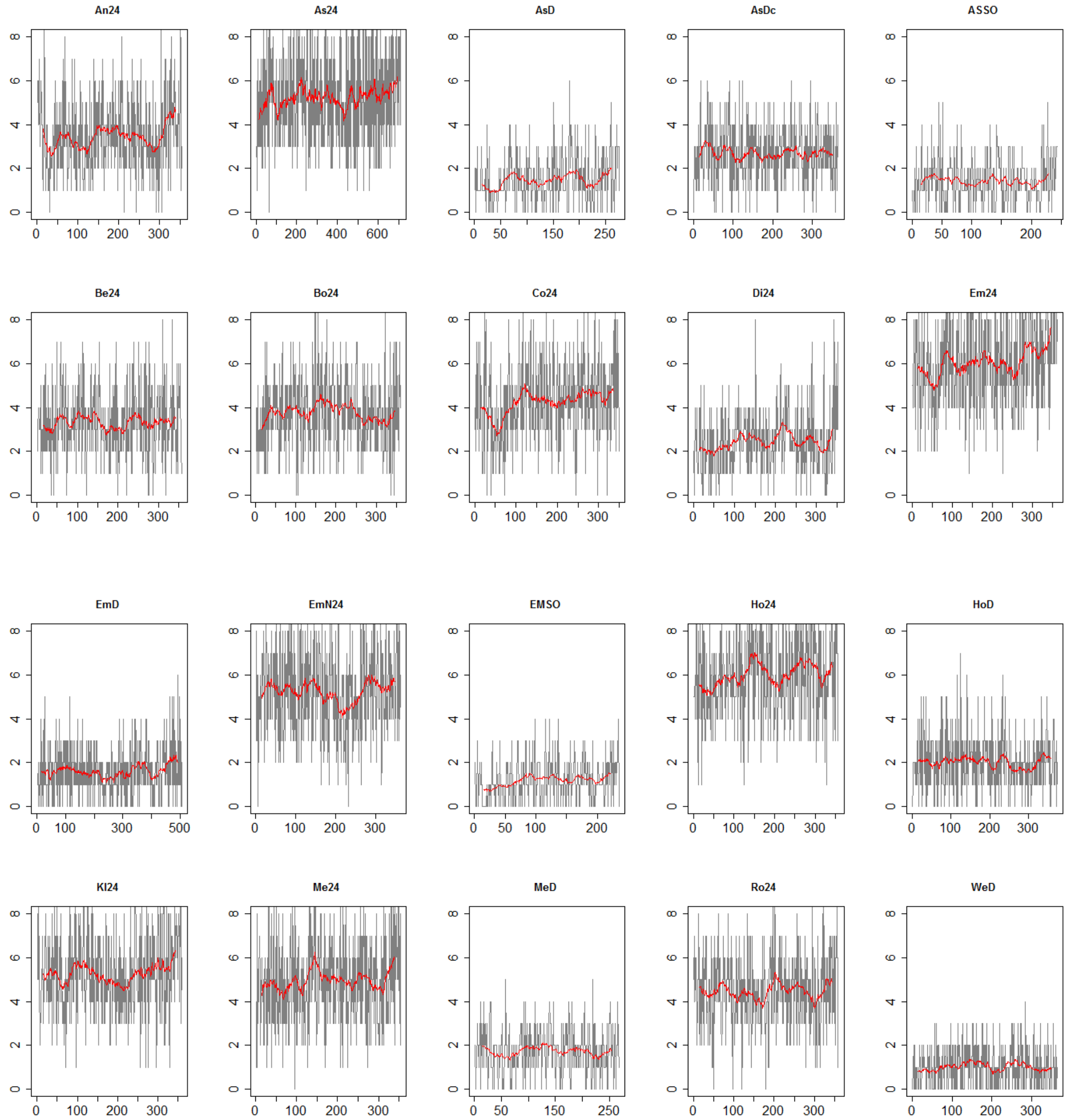
Be24	2	0.605	4	27.242	94.877
Be24	3	0.2318	3	42.901	78.147
Be24	4	0.5867	3	19.319	78.147
Be24	5	0.1563	4	66.375	94.877
Be24	6	0.0483	4	95.695	94.877
Bo24	0	0.0906	4	8.026	94.877
Bo24	1	0.8182	4	15.475	94.877
Bo24	2	0.9394	4	0.7931	94.877
Bo24	3	0.4318	4	38.138	94.877
Bo24	4	0.653	4	24.533	94.877
Bo24	5	0.7604	4	18.662	94.877
Bo24	6	0.4428	4	3.737	94.877
Bp24	0	0.0461	4	96.846	94.877
Bp24	1	0.5858	4	2.835	94.877
Bp24	2	0.0476	4	96.052	94.877
Bp24	3	0.2314	4	55.966	94.877
Bp24	4	0.04	4	10.025	94.877
Bp24	5	0.3453	4	44.767	94.877
Bp24	6	0.3972	4	40.654	94.877
BpD	1	0.2027	2	31.925	59.915
BpD	2	0.1731	1	18.562	38.415
BpD	3	0.7661	2	0.5328	59.915
BpD	4	0.6609	1	0.1924	38.415
BpD	5	0.0264	1	49.308	38.415
Co24	0	0.813	4	15.765	94.877
Co24	1	0.2642	4	52.336	94.877
Co24	2	0.1439	4	68.521	94.877
Co24	3	0.1123	4	74.875	94.877
Co24	4	7,00E-04	4	191.386	94.877
Co24	5	0.3506	4	44.329	94.877
Co24	6	0.3164	5	58.964	110.705
D12Bu	0	0.5482	3	21.183	78.147
D12Bu	6	0.0055	3	126.281	78.147
D24B	1	0.1155	4	74.153	94.877
D24B	2	0.2541	4	53.401	94.877
D24B	3	0.144	4	68.506	94.877
D24B	4	0.0636	3	72.765	78.147
D24B	5	0.3004	4	48.746	94.877
D24K	0	0.4224	3	28.067	78.147
D24K	1	0.2849	3	37.914	78.147
D24K	2	0.2232	3	43.806	78.147
D24K	3	0.131	3	56.306	78.147
D24K	4	0.2606	2	26.894	59.915
D24K	5	0.4563	3	2.607	78.147
D24K	6	0.037	3	84.817	78.147

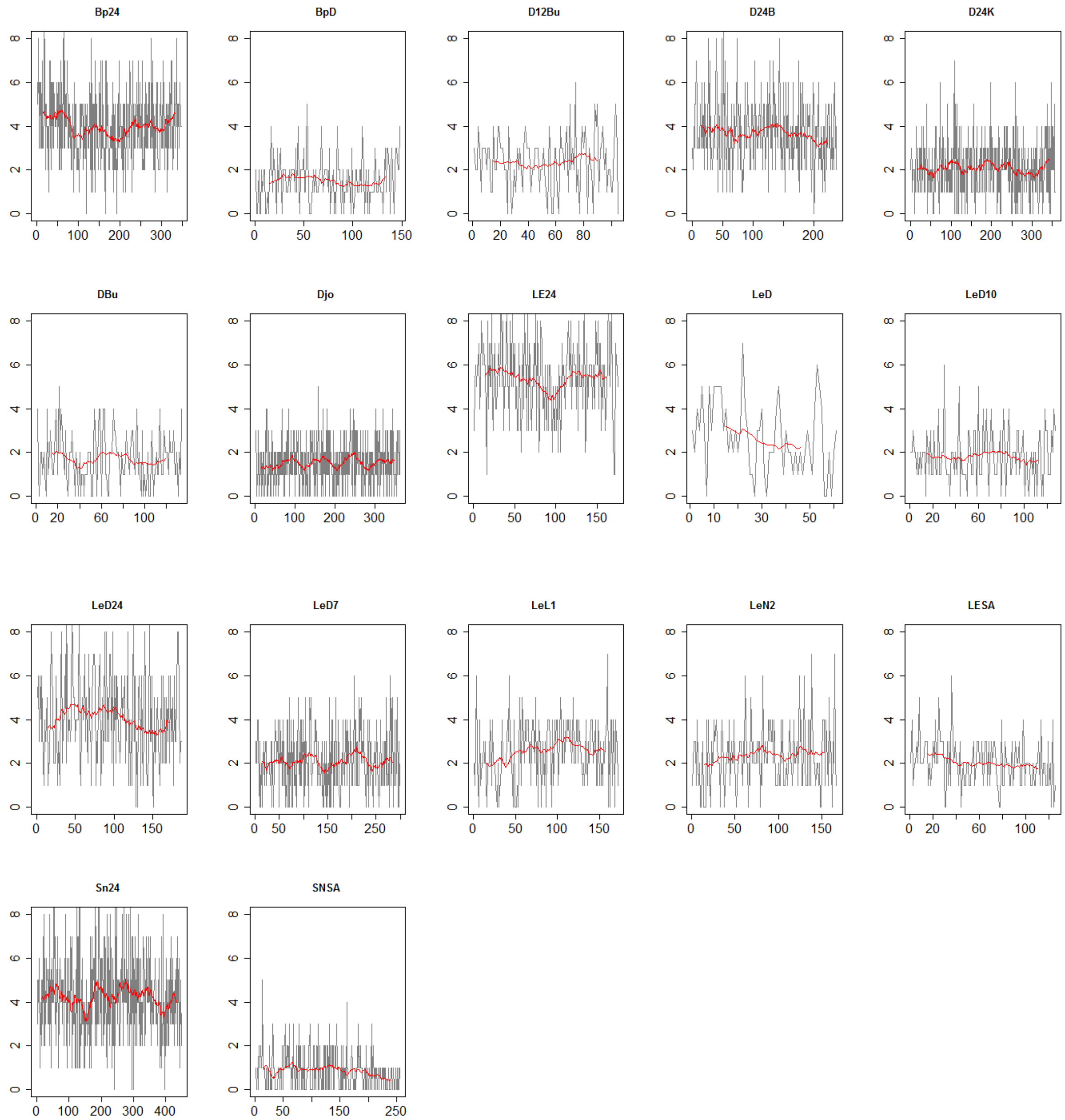
DBu	1	0.0011	1	106.619	38.415
DBu	2	0.0711	1	32.582	38.415
DBu	3	0.4874	1	0.4822	38.415
DBu	4	0.2421	1	13.685	38.415
DBu	5	0.6434	1	0.2143	38.415
Di24	0	0.1026	3	61.931	78.147
Di24	1	0.2983	3	36.787	78.147
Di24	2	0.0482	2	6.066	59.915
Di24	3	0.3406	2	21.538	59.915
Di24	4	0.5577	3	20.715	78.147
Di24	5	0.5708	3	20.079	78.147
Di24	6	0.0076	3	119.272	78.147
Djo	0	0.1959	2	32.605	59.915
Djo	1	0.6899	2	0.7426	59.915
Djo	2	0.6559	2	0.8434	59.915
Djo	3	0.3236	2	22.566	59.915
Djo	4	0.9612	2	0.0792	59.915
Djo	5	0.0539	2	58.398	59.915
Djo	6	0.8458	3	0.8151	78.147
Em24	0	0.1515	6	94.167	125.916
Em24	1	0.6083	5	36.003	110.705
Em24	2	0.0922	5	94.553	110.705
Em24	3	0.6582	5	32.715	110.705
Em24	4	0.0843	5	96.977	110.705
Em24	5	0.0054	5	165.525	110.705
Em24	6	0.0502	6	125.807	125.916
EmD	1	0.5452	3	21.331	78.147
EmD	2	0.0935	3	6.404	78.147
EmD	3	0.0447	3	80.661	78.147
EmD	4	0.5868	3	19.312	78.147
EmD	5	0.0587	3	74.554	78.147
EmN24	0	0.3134	6	70.811	125.916
EmN24	1	0.511	5	42.719	110.705
EmN24	2	0.0634	4	89.078	94.877
EmN24	3	0.6651	5	32.267	110.705
EmN24	4	0.107	5	90.516	110.705
EmN24	5	0.0838	4	82.214	94.877
EmN24	6	0.0213	5	132.262	110.705
Ho24	0	0.648	5	33.379	110.705
Ho24	1	0.2928	6	73.141	125.916
Ho24	2	0.9422	5	12.275	110.705
Ho24	3	0.0028	5	180.995	110.705
Ho24	4	0.4213	5	49.555	110.705
Ho24	5	0.2246	5	69.471	110.705
Ho24	6	0.4976	5	43.687	110.705

HoD	0	0.2118	3	45.056	78.147
HoD	1	0.3812	3	30.681	78.147
HoD	2	0.2874	3	37.696	78.147
HoD	3	0.4037	2	18.141	59.915
HoD	4	0.3041	3	36.314	78.147
HoD	5	0.3836	3	30.527	78.147
HoD	6	0.6503	3	16.404	78.147
Kl24	0	0.0422	5	115.062	110.705
Kl24	1	0.6783	5	31.405	110.705
Kl24	2	0.0999	5	92.379	110.705
Kl24	3	0.2802	5	62.769	110.705
Kl24	4	0.506	5	43.075	110.705
Kl24	5	0.8215	5	21.956	110.705
Kl24	6	0.2567	5	65.451	110.705
LE24	0	0.0063	3	123.357	78.147
LE24	1	0.2224	3	43.896	78.147
LE24	2	0.5937	2	10.426	59.915
LE24	3	0.9882	3	0.1288	78.147
LE24	4	0.4911	3	24.138	78.147
LE24	5	0.2157	2	30.675	59.915
LE24	6	0.2575	3	40.373	78.147
LeD	0	0.5644	2	1.144	59.915
LeD	6	0.8093	2	0.4232	59.915
LeD10	1	0.4738	1	0.513	38.415
LeD10	2	0.3209	1	0.9852	38.415
LeD10	3	0.6484	1	0.208	38.415
LeD10	4	0.0439	1	40.616	38.415
LeD10	5	0.6669	2	0.8102	59.915
LeD24	0	0.3773	2	19.496	59.915
LeD24	1	0.6596	3	15.992	78.147
LeD24	2	0.1367	2	39.806	59.915
LeD24	3	0.913	3	0.5265	78.147
LeD24	4	0.6782	2	0.7766	59.915
LeD24	5	0.7347	3	12.764	78.147
LeD24	6	0.559	3	20.653	78.147
LeD7	0	0.2655	2	2.652	59.915
LeD7	1	0.3639	2	20.216	59.915
LeD7	2	0.7334	2	0.62	59.915
LeD7	3	0.2682	3	39.383	78.147
LeD7	4	0.732	2	0.6239	59.915
LeD7	5	0.9211	3	0.4901	78.147
LeD7	6	0.9059	1	0.014	38.415
LeL1	0	0.2497	2	27.747	59.915
LeL1	1	0.5267	2	12.824	59.915
LeL1	2	0.5591	2	1.163	59.915

LeL1	3	0.5722	2	11.166	59.915
LeL1	4	0.3706	1	0.8016	38.415
LeL1	5	0.2562	2	27.234	59.915
LeL1	6	0.892	2	0.2285	59.915
LeN2	0	0.4934	2	14.127	59.915
LeN2	1	0.3423	1	0.9019	38.415
LeN2	2	0.1472	1	2.101	38.415
LeN2	3	0.0209	1	53.313	38.415
LeN2	4	0.3995	1	0.7099	38.415
LeN2	5	0.11	1	25.539	38.415
LeN2	6	0.2566	2	27.207	59.915
Me24	0	0.0599	5	105.991	110.705
Me24	1	0.0511	4	94.357	94.877
Me24	2	0.0734	5	100.671	110.705
Me24	3	0.9531	5	11.108	110.705
Me24	4	0.5338	4	31.455	94.877
Me24	5	0.2011	4	59.741	94.877
Me24	6	0.0737	6	115.143	125.916
MeD	1	0.2207	3	44.076	78.147
MeD	2	0.4606	2	15.503	59.915
MeD	3	0.3515	3	32.721	78.147
MeD	4	0.5834	2	10.776	59.915
MeD	5	0.6769	3	15.234	78.147
Ro24	0	0.0116	5	147.221	110.705
Ro24	1	0.3638	4	43.247	94.877
Ro24	2	0.6683	4	23.688	94.877
Ro24	3	0.2033	4	5.945	94.877
Ro24	4	0.5423	4	30.936	94.877
Ro24	5	0.8051	4	16.205	94.877
Ro24	6	0.0015	5	196.508	110.705
Sn24	0	0.0406	6	131.574	125.916
Sn24	1	0.3992	4	40.508	94.877
Sn24	2	0.9165	4	0.9556	94.877
Sn24	3	0.1803	4	62.644	94.877
Sn24	4	0.8153	4	15.637	94.877
Sn24	5	0.4391	4	37.626	94.877
Sn24	6	0.0618	6	120.053	125.916
WeD	0	0.7354	2	0.6146	59.915
WeD	1	0.2495	1	1.326	38.415
WeD	2	0.078	1	31.066	38.415
WeD	3	0.5956	1	0.2817	38.415
WeD	4	0.3449	1	0.8923	38.415
WeD	5	0.0257	1	49.729	38.415
WeD	6	0.2823	1	11.559	38.415

9.3 Seasonality




Bp24
BpD
D12Bu
D24B
D24K
DBu
Djo
LE24
LeD
LeD10
LeD24
LeD7
LeL1
LeN2
LESA
Sn24
SNSA

9.4 Nurses

With part time employees

nurse	fte	nurse	fte	nurse	fte
1	1	42	1	83	0,94
2	0,94	43	0,91	84	1
3	1	44	1	85	0,78
4	0,78	45	0,75	86	1
5	1	46	1	87	1
6	0,9	47	1	88	1
7	0,9	48	0,98	89	1
8	0,95	49	0,91	90	0,94
9	1	50	0,91	91	1
10	0,83	51	0,85	92	0,74
11	0,5	52	1	93	0,98
12	0,95	53	0,94	94	1
13	0,98	54	1	95	0,87
14	0,94	55	0,94	96	1
15	1	56	0,91	97	1
16	0,8	57	1	98	0,98
17	1	58	0,5	99	0,8
18	1	59	0,8	100	1
19	1	60	1	101	0,75
20	0,83	61	1	102	0,94
21	1	62	0,92	103	1
22	1	63	1	104	1
23	0,94	64	0,75	105	0,9
24	1	65	0,91	106	0,75
25	1	66	1	107	0,8
26	1	67	1	108	0,67
27	0,7	68	0,8	109	0,93
28	0,73	69	0,78	110	0,92
29	0,7	70	0,8	111	1
30	0,65	71	1	112	0,94
31	1	72	0,78	113	1
32	0,8	73	1	114	1
33	1	74	0,95	115	0,65
34	1	75	1	116	1
35	1	76	0,95	117	0,92
36	1	77	0,8	118	1
37	0,78	78	0,75	119	1
38	1	79	0,8	120	1
39	1	80	0,78	121	1
40	0,78	81	0,78	122	1
41	1	82	1		

With only full time employees

nurse	fte	nurse	fte	nurse	fte
1	1	41	1	81	1
2	1	42	1	82	1
3	1	43	1	83	1
4	1	44	1	84	1
5	1	45	1	85	1
6	1	46	1	86	1
7	1	47	1	87	1
8	1	48	1	88	1
9	1	49	1	89	1
10	1	50	1	90	1
11	1	51	1	91	1
12	1	52	1	92	1
13	1	53	1	93	1
14	1	54	1	94	1
15	1	55	1	95	1
16	1	56	1	96	1
17	1	57	1	97	1
18	1	58	1	98	1
19	1	59	1	99	1
20	1	60	1	100	1
21	1	61	1	101	1
22	1	62	1	102	1
23	1	63	1	103	1
24	1	64	1	104	1
25	1	65	1	105	1
26	1	66	1	106	1
27	1	67	1	107	1
28	1	68	1	108	1
29	1	69	1	109	1
30	1	70	1	110	0,8
31	1	71	1	111	0,7
32	1	72	1	112	0,7
33	1	73	1		
34	1	74	1		
35	1	75	1		
36	1	76	1		
37	1	77	1		
38	1	78	1		
39	1	79	1		
40	1	80	1		

9.5 Results

Minimizing the total travel time

Expected number of urgent calls per nurse	Total travel time	Max travel time
100	165660	4830
110	165690	4830
120	165710	4830
130	165750	5110
140	165800	4990
150	165860	3650
160	165960	4810
170	166080	3940
180	166250	4380
190	166510	3670
200	167000	3650
210	167550	4380
220	168150	3930
230	168780	3670
240	169510	4080
250	170480	3791
260	171630	3650
270	173220	3470

Minimizing the maximum commuting time of the nurse with the largest commuting time of all nurses

Expected number of urgent calls per nurse	Total travel time	Max travel time
100	167120	2920
110	167150	2920
120	167160	2920
130	167190	2920
140	167240	2920
150	167330	2920
160	167410	2920
170	167520	2920
180	167680	2920
190	167930	2920
200	168410	2920
210	168980	2920
220	169570	2920
230	170320	2920
240	171180	2920
250	172010	2920
260	173200	2950
270	174500	3010

Minimizing the total travel time

Model with only full time employees

Expected number of urgent calls per nurse	Total travel time	Max travel time
100	160470	3650
110	160460	3650
120	160460	3650
130	160460	4300
140	160460	3650
150	160460	4380
160	160460	4290
170	160460	4280
180	160460	4380
190	160520	3650
200	160620	3650
210	160710	3780
220	160880	3780
230	161220	3650
240	161560	3650
250	161930	4090
260	162370	3880
270	162940	3650
280	163700	3650
290	164570	3650
300	166010	3650
310	168150	3650
320	174010	3300

Minimizing the commuting time of the nurse with the largest commuting time of all nurses

Model with only full time employees

Expected number of urgent calls per nurse	Total travel time	Max travel time
100	161620	2920
110	161620	2920
120	161620	2920
130	161620	2920
140	161620	2920
150	161620	2920
160	161620	2920
170	161620	2920
180	161620	2920
190	161680	2920
200	161780	2920
210	161890	2920
220	162080	2920
230	162450	2920

240	162860	2920
250	163290	2920
260	163720	2950
270	164140	3010
280	164710	3060
290	165300	3120
300	166460	3180
310	168500	3240
320	174010	3300