Faculteit der Bètawetenschappen Master: Business Analytics
Optimizing Patient Appointment Scheduling using Variable Interval Time slots

Research Paper MSc. Business Analytics

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## Abstract

In the introduction to this research paper, the necessity of an optimally functioning appointment scheduling system that could, for instance, improve patient satisfaction by reducing the expected waiting time in hospitals, is explained. Chapter 2 explains that a good scheduling system accounts, among other things, for unpunctuality from both doctors and patients, the heterogeneity of patients and the presence of walk-ins. The relevance of choosing a fair performance measure in which each patient's expected waiting time should meet certain constraints is also emphasized in this chapter. Chapter 3 illustrates the setup of such a scheduling system. Initially, this was done by generating data and simulating a base scheduling system with fixed interval time slots. However, this schedule proved to be unfair and as a result, it was then optimized to produce the desired scheduling system with variable interval time slots. This was done by successively re-scheduling the appointment times of the appointment slots in the base schedule until the expected waiting times for patients scheduled in that time slot sufficed the set constraints. After analyzing the results in chapter 4 , it was concluded that compared to the base scheduling system, this optimized appointment scheduling system, which uses variable intervals to schedule individual patients in a time slot, is fair. In addition to this, it does not negatively affect the idle time and overtime of doctors. The conclusion is therefore that hospitals should implement a scheduling system with variable interval time slots to schedule its patients, in order to reduce expected waiting times.

## Preface

This research paper is part of the curriculum of the master Business Analytics at the Vrije Universiteit Amsterdam (VU). Ger Koole, a researcher and professor in the Mathematics Department of the VU, has been my supervisor throughout this research. This study focuses on the unfairness of waiting times for scheduled patients in hospitals. The expected waiting time of these scheduled patients will be optimized through the design of an appointment scheduling system that has variable interval time slots. I would like to thank Ger Koole very much for his guidance and support throughout this research.

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## Chapter 1

## Introduction

### 1.1 Background

Back in 1952, Bailey and Welch were the first scientists to address the widely perceived problem of long waiting times in hospitals, and they were determined to find a solution [5]. At that time, a common strategy for doctors was to schedule all patients from the time the clinic opened each day. This meant that patients were served on a first come first serve (FCFS) basis. The reason why doctors implemented the "all patients arrive at the same time system" was that doctors did not like to sit idle. This happened when patients arrived late or, even worse, did not show up at all.

By using this scheduling system, doctors knew that the patients could be served consecutively without there being any pauses in the work. However, in terms of waiting time, especially for the patients who were seen at the end of a clinical session, this scheduling system did not work at all. To start with, in order to see a doctor, these patients had to take an entire day off.

Big waiting rooms are good for the amount of space they provide for patients. But when they are full they tend to cause patients to think they will have to wait longer, which in turn decreases patient satisfaction [3]. In the healthcare sector, reducing (perceived) patient waiting time is one of the primary objectives for improving the quality of healthcare. In order to improve the quality of healthcare by increasing patient satisfaction, a better appointment scheduling system is required.

Bailey and Welch argued that space was scarce and expensive and therefore allocating a big room to fit all waiting patients is an unnecessary waste of a hospital's resources. Instead, this could be used to increase the quality of healthcare in that facility in the form of, for instance, an additional treatment room.

### 1.2 Literature

There is a considerable amount of literature on patient appointment scheduling in hospitals. The earliest study by Bailey and Welch suggests that clinics should make a distinction between types of consultations, with different average consultation times. Secondly, there should be two patients scheduled at the start of each clinical session. Finally, every next appointment should be scheduled to the average consultation time rounded off to the nearest minute later. Their study showed that an imbalance between expected patient waiting time and expected doctor idle and overtime exists; an attempt to decrease either one will result in an increase of the other. Therefore, they suggest that a trade-off should be made in terms of expected waiting and expected idle time.

Their proposed scheduling system, though effective, has paved a way for mathematicians, and especially queuing theorists, to contribute numerous alternative scheduling systems using analytical models and simulations.

A large number of existing studies in the broader literature have examined the inclusion of the number of appointments per clinical session, service times, number of doctors, unpunctuality of patients, lateness and interruption levels of doctors, arrival processes and queue disciplines into their algorithms [1]. Prior research by Milhiser strongly suggests that the interarrival times between appointments should not be equally spaced [4].

### 1.3 Problem Statement

Appointment scheduling has gradually become an area of focus and over time there have been many studies that have attempted to optimize patient appointment scheduling. Most of them, however, fail to identify and address discrepancies between reality and the output of those frameworks.

One of the main topics that should be investigated in this field is the actual performance of appointment scheduling system models. Currently, the performance measure of most models is based on averages [3]. The problem with this is that an average waiting time of 10 minutes could mean that there are patients who do not have to wait at all, while others might have to wait for 20 minutes. This remains a problem that needs to be solved in the area of appointment scheduling. A new approach is therefore needed to schedule appointment time slots.

The aim of this research is to develop an appointment scheduling system that schedules the patients in such a way that none of the scheduled patients is faced with waiting time that exceeds set constraints. In addition, this schedule should not have a negative effect on a doctor's idle and overtime. One approach to solve this problem involves the use of variable interval time slots to schedule the patients. As far as we know, this strategy has not yet been used in previous proposed solutions and will solve several problems simultaneously.

In Chapter 2, the model that will be used to create an appointment scheduling system is described and the performance measure will be clarified. Next, the development of the scheduling system is explained in Chapter 3. Following this, the results are shown and explained in Chapter 4. Finally, Chapter 5 consists of the conclusion and recommendations regarding this appointment scheduling system.

## Chapter 2

## Model

A previous study exists that contains a detailed description of the steps that should be taken when designing an appointment scheduling system and handling the assumptions [1]. These are briefly described in this chapter.

### 2.1 Problem Definition and Formulation

First, we check the features that should be present in an appointment scheduling system and argue what, with regard to the objective of this paper, reasonable assumptions would be.

### 2.1.1 Nature of Decision Making

The appointment scheduling system will be static, therefore the appointment times will be determined and communicated to the patients before the start of a clinical session. There are also cases where a dynamic schedule would suffice, however, this is more common in high stochastic service types, often with a lot of inpatients as these are easier to (re-)schedule.

### 2.1.2 Modeling of Clinic Environments

## Number of Services

A patient could be going through a trajectory concerning multiple service types, while passing through the same service several times. However, most literature focuses on a single service system as a multi service system is often too complex to model. This paper also assumes a single service type, where the walk-in probability could compensate the lack of multiple service types by anticipating a probability that more patients should be served during a clinical session than actually scheduled.

## Number of appointments per Clinical Session

The number of available time slots in the schedule will be equal to the minimum number of patients the doctor should be able to serve during a clinical session. Due to walk-ins and no-shows, the number of actual appointments per session could vary.

## Number of Doctors

Several service types do not require a patient-doctor relationship, for instance, in the radiology department. Here, it is already practice that whenever an X-ray room becomes available, the next patient in line will be treated. However, since it is known that multiple servers for a single queue already leads to shorter waiting times [2], this paper does not address this specific situation. Instead, it focuses on situations in which a single doctor is responsible for seeing patients in one queue.

### 2.1.3 Arrival Process

## Unpunctuality

In general, patients do not enter the waiting room dead on time for their appointment. In this paper, we consider voluntary early arrival. The model will take the probability into account of patients arriving after their scheduled appointment time. The decision has been made to give the unpunctuality of late patients a Bernoulli distribution.

## No-shows

Besides patients who show up later than scheduled, there is also the possibility of patients not showing up at all. This scenario is also accounted for in the design of the appointment scheduling system. Each patient is given a probability of not showing up for their scheduled appointment.

## Walk-ins

Some hospitals deal with emergency situations or walk-in patients entering the waiting room without having an appointment scheduled. This scheduling system will take the effect of these walk-ins into account by assigning a uniform-probability of an arbitrary number of walk-ins entering during a clinical session.

### 2.1.4 Service Times

Although it has been shown that doctors tend to take their time at the beginning of their shift and that their consultations get shorter towards the end, in this paper the decision was made to keep the service time distribution the same throughout the day. The distribution of service times can best be represented by a log-normal, unimodal right skewed distribution [1].

### 2.1.5 Lateness and Interruption level of Doctors

In this paper, it is assumed that doctors do not always arrive exactly at the agreed starting time of a clinical session. Also, doctors may need breaks or deal with ancillary activities in-between their service sessions. The scheduling system allows for lateness and interruptions based on the Bernoulli distribution.

### 2.1.6 Queue Discipline

In practice, there are scenarios that involve two or more queues, where for instance the walk-ins, perhaps emergency cases, are treated with priority. In this paper, the choice has been made for one queue with scheduled patients, where walk-ins randomly enter this queue and are seen on a basis of FCFS.

### 2.1.7 Patient Classification

The majority of the articles related to scheduling systems assumes homogeneity of patients [4]. However, a heterogeneity approach has been opted for in this paper. This means that in calculating the service times for individual patients, older patients with limited mobility (rollator, wheelchair), who require on average considerably more time than the younger and walking patients, are taken into consideration. Here too, the Bernoulli distribution divides each patient into distinctive groups.

### 2.1.8 Performance Measure

It is essential for a realistic performance measure to be present before concluding the effectiveness of an appointment scheduling system. There are several ways to measure the performance of an appointment scheduling system.

## Cost-based

A cost-based evaluation method can be (a subset) of the following formula:

$$
\begin{equation*}
\operatorname{minimize} E[T C]=c_{w} E[W]+c_{i} E[I]+c_{o} E[O]+c_{f} E[F] \tag{2.1}
\end{equation*}
$$

Where,
$-\mathrm{E}[\mathrm{TC}]=$ Expected Total Costs
$-\mathrm{E}[\mathrm{W}]=$ Expected Waiting Time: "Begin Service Time - Appointment Time"
$-\mathrm{E}[\mathrm{I}]=$ Expected Idle Time: Sum of "The expected idle time between the consultation of two consecutive patients" during a clinical session
$-\mathrm{E}[\mathrm{O}]=$ Expected Over Time: "The expected time a consultation continues after the end of a clinical session"
$-\mathrm{E}[\mathrm{F}]=$ Expected Facility Costs: "In theory not so much used, can for instance be the expected costs of machines in idle time"
$-c_{x}=\mathrm{A}$ (constant) function that indicates the weights of the respective variable
Using a cost-based evaluation method might not be the most optimal performance measure. The cost function $c_{x}$ is hard to estimate [1], and since it uses an average for the waiting time, it does not account for the fairness of waiting time for all patients.

## Time-based

In time-based performances measures, the purpose of the scheduling system is to reduce the average waiting, idle and overtime. However, this measure also lacks the ability to identify congestion, in other words, situations in which patients wait longer (or shorter) than is acceptable, and is therefore also not an effective performance measure.

## Congestion-based

This performance measure identifies and tries to tackle congestion that tends to build up during a clinical session. In essence, it demands a fair schedule in which the probability of a patient waiting longer than can be considered acceptable is minimal. However, it excludes idle and overtime in determining the performance of an appointment schedule.

A combination of these three performance measures has been adopted in the scheduling system that will be designed in this paper. The appointment schedule should see fit to handle the fact that over time, congestion tends to build up and waiting times get longer. Furthermore, it should take into consideration that the idle and over time are acceptable. (2.2)

$$
\begin{equation*}
\min E[T C]=\forall_{n} c_{w} E\left[W_{n}<x\right]+c_{i} E[I]+c_{o} E[O] \tag{2.2}
\end{equation*}
$$

The mathematical formula of the performance measure is shown above, where the parameter $n$ represents the patients and x is a time based parameter.

## Chapter 3

## Methodology

In this chapter, the steps that were taken to develop a fair appointment scheduling system are explained. The experiment was performed using Python 3. This experiment is designed in such way that the scheduled patients have an expected waiting time of approximately 9 to 10 minutes, and is conducted in two stages. In the first stage, data is generated and simulated for 25 days in order to design an appointment scheduling system with fixed interval time slots. The resulting appointment schedule is used as a base schedule. In the second phase, the base schedule is optimized so that it is fair in the sense that all scheduled patient are expected to have to wait for between 9 and 10 minutes.

### 3.1 Data Generation

### 3.1.1 Patient Data

The first step in developing the base appointment scheduling system is to create patient data. Per day, up to 40 patients should be scheduled for an appointment. Each patient has a probability of not showing up, being on time or being late (see: table 3.1)..

| Punctuality | Probability (\%) |
| :---: | :---: |
| no show | 5 |
| on time | 45 |
| 3 min late | 35 |
| 5 min late | 15 |

Table 3.1: Arrival Process

The service times are all log-normal(2, 0.03), which suggests that the expected service time is about 7.24 minutes, with a standard deviation of 13 seconds. However, since we assumed heterogeneity, there are three different types of patients, each with their own addition to the expected service time (see: table 3.2).

| Heterogeneity level | Probability (\%) |
| :---: | :---: |
| service time | 30 |
| $1.3^{*}($ service time $)$ | 30 |
| $1.7^{*}($ service time $)$ | 40 |

Table 3.2: Patient Classification

An example of the patient data is shown in figure 3.1 below.

### 3.1.2 Doctor Data

The second step is the data generation for a single doctor with two variables: lateness and interruption. Each day, the doctor can arrive on time or late, furthermore, after each patient the doctor serves, he can be interrupted (see: table 3.3).

|  | Time (min) | Probability (\%) |
| :---: | :---: | :---: |
| Lateness | 0 | 80 |
|  | 5 | 15 |
|  | 10 | 5 |
| Interruption | 0 | 60 |
|  | 2 | 30 |
|  | 10 | 10 |

Table 3.3: Lateness and Interruption of Doctors

|  | Day | Patient | Patient Late Arrival (min) | Patient Mobility | Total Service Time (min) |
| ---: | ---: | ---: | ---: | ---: | ---: |
| $\mathbf{0}$ | 1 | 1 | 0 | semi-mobile | 9.036646 |
| $\mathbf{1}$ | 1 | 2 | 1 | semi-mobile | 9.425452 |
| 998 | 25 | 39 | 0 | not mobile | 11.197665 |
| 999 | 25 | 40 | 1 | semi-mobile | 9.268206 |

Figure 3.1: Patient Data

### 3.2 Design Appointment Scheduling System

Next, the data of the patients and the doctor is combined. ${ }^{1}$

|  | Lateness (min) | Patient | Interruption (min) | Patient Late Arrival (min) | Patient Mobility | Total Service Time (min) |
| :---: | :---: | :---: | :---: | :---: | ---: | ---: |
| $\mathbf{1}$ | 0 | 1 | 2 | 0 | semi-mobile | 9.036646 |
| $\mathbf{5}$ | 5 | 40 | 0 | 1 | semi-mobile | 9.268206 |

Figure 3.2: Combined Data
The design of the base appointment schedule is as follows. The first patient is scheduled at the beginning of a clinical session, which is determined to be at 9 am . The second to the last patient are scheduled with fixed scheduling intervals according to the formula below:

$$
\begin{equation*}
(n-1) *(E[\text { Lateness }]+E[\text { Service Time }]+E[\text { Interruption }]) \tag{3.1}
\end{equation*}
$$

[^0]
### 3.3 Adding Walk-ins

The following step is the addition of walk-ins. Each day, a random number of between 0-6 patients enters the system at a random time between the beginning and the end of the clinical sessions at 5 pm . The parameters, heterogeneity and service time for the walk-ins are the same as for the scheduled patients. However, the interruption time concerning the doctor data is slightly different as there is no Bernoulli distribution here. Instead, the interruption time is equal to the mean interruption time of the simulated scheduled patients.

|  | Day | Patient | Patient Actual Arrival Time | Total Service Time (min) | Type |
| ---: | ---: | ---: | ---: | ---: | ---: |
| 0 | 1 | 41 | $16: 52: 00$ | $00: 09: 22$ | Walk-in |

Figure 3.3: Example walk-in

Since the queue is FCFS, walk-ins should enter the queue directly behind the patient (or walk-in) who joined the queue before this walk-in. The start and end times of each patient's consultation are then added to the scheduling system via the formulas below.

For the first patient:
$(\text { Begin Service })_{1}=\left(\right.$ Appointment Time $_{1}+\max \left(\right.$ Lateness $_{1}+$ Interruption $_{1},\left(\right.$ Patient Late Arrival $\left._{1}\right)$
$(\text { Begin Service })_{n}=\max \left[(\text { End Service })_{n-1}+\right.$ Interruption $\left._{n},(\text { Patient actual Arrival Time })_{n}\right]$
Where $1<n \leq 40$.

$$
\begin{equation*}
(\text { End Service })_{i}=(\text { Begin Service })_{i}+(\text { Total Service Time })_{i} \tag{3.4}
\end{equation*}
$$

Where $1 \leq i \leq 40$.
The resulting appointment schedule can be found in the column "Appointment Time (base)" in figure 3.5.
This information can be used to calculate the waiting times of the patients and the idle time of the doctor, the results of which are shown in figure 3.4. Also, overtime can be computed as the positive difference between the actual end time of a clinical session and the scheduled end time at 5 pm .

|  | Appointment Time | Day | Interruption (min) | Lateness (min) | Patient | Patient <br> Actual <br> Arrival Time | Patient Late <br> Artival (min) | $\begin{array}{r} \text { Total } \\ \text { Service } \\ \text { Time (min) } \end{array}$ | Type | start_service | end_service | idle_time | waiting_time |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 09:00:00 | 1 | 0.0 | 0.0 | 1 | 09:00:00 | 0.0 | 00:11:20 | Scheduled | 09:00:00 | 09:11:20 | 00:00:00 | 00:00:00 |
| 1 | 09:11:30 | 1 | 0.0 | 0.0 | 2 | 09:14:30 | 3.0 | 00:09:03 | Scheduled | 09:14:30 | 09:23:33 | 00:03:10 | 00:00:00 |
| 2 | 09:23:00 | 1 | 0.0 | 0.0 | 3 | 09:24:00 | 1.0 | 00:07:20 | Scheduled | 09:24:00 | 09:31:20 | 00:00:27 | 00:00:00 |
| 3 | 09:34:30 | 1 | 0.0 | 0.0 | 4 | 09:34:30 | 0.0 | 00:11:18 | Scheduled | 09:34:30 | 09:45:48 | 00:03:10 | 00:00:00 |
| 4 | 09:46:00 | 1 | 0.0 | 0.0 | 5 | 09:47:00 | 1.0 | 00:08:54 | Scheduled | 09:47:00 | 09:55:54 | 00:01:12 | 00:00:00 |
| 5 | 09:57:30 | 1 | 10.0 | 0.0 | 6 | 09:57:30 | 0.0 | 00:07:30 | Scheduled | 10:05:54 | 10:13:24 | 00:00:00 | 00:08:24 |
| 6 | 10:09:00 | 1 | 2.0 | 0.0 | 7 | 10:10:00 | 1.0 | 00:08:51 | Scheduled | 10:15:24 | 10:24:15 | 00:00:00 | 00:05:24 |
| 7 | 10:20:30 | 1 | 0.0 | 0.0 | 8 | 10:21:30 | 1.0 | 00:11:07 | Scheduled | 10:24:15 | 10:35:22 | 00:00:00 | 00:02:45 |
| 8 | NaN | 1 | 1.3 | 0.0 | 45 | 10:30:00 | 0.0 | 00:12:22 | Walk-in | 10:36:40 | 10:49:02 | 00:00:00 | 00:06:40 |
| 9 | 10:32:00 | 1 | 2.0 | 0.0 | 9 | 10:33:00 | 1.0 | 00:08:55 | Scheduled | 10:51:02 | 10:59:57 | 00:00:00 | 00:18:02 |

Figure 3.4: Schedule after added walk-in

| Patient | Appointment Time (base) | Waiting Time base (min) | Patient | Appointment Time (base) | Waiting time base (min) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 09:0000 | 3.3 | 21 | 12:59:20 | 12.58 |
| 2 | 09:11:58 | 223 | 22 | 13.1118 | 12.53 |
| 3 | $09.23: 56$ | 3.07 | 23 | 13.23.16 | 13.33 |
| 4 | 09.35:54 | 431 | 24 | 13:35:14 | 13.34 |
| 5 | 09.47:52 | 3.56 | 25 | 13.47 .12 | 15.02 |
| 6 | 09.59:50 | 431 | 26 | 13:59:10 | 14.24 |
| 7 | 10:11:48 | 447 | 27 | 14.11:08 | 14.36 |
| 8 | 10.23.46 | 501 | 28 | 14.23:06 | 17.57 |
| 9 | 10:35:44 | 532 | 29 | 14.35 .04 | 11.45 |
| 10 | 10:47:42 | 607 | 30 | 14.47.02 | 11.42 |
| 11 | 10.59.40 | 532 | 31 | 14.59:00 | 11.27 |
| 12 | 11:11:38 | 55 | 32 | 15:10:58 | 12.56 |
| 13 | 11,23,36 | 7.43 | 33 | 15:22:56 | 12.11 |
| 14 | 11:35:34 | 705 | 34 | 15:34:54 | 10.23 |
| 15 | 11:47:32 | 8.05 | 35 | 15:46:52 | 10.32 |
| 16 | 11:59:30 | 831 | 36 | 15.58 .50 | 10.23 |
| 17 | 12:11:28 | 9,32 | 37 | 16:10:48 | 10.5 |
| 18 | 12:23:26 | 10.32 | 38 | 16:22:46 | 932 |
| 19 | 12:35:24 | 11.52 | 39 | 16.34.44 | 10.02 |
| 20 | 12:47:22 | 12.56 | 40 | 16:46:42 | 10.2 |

Figure 3.5: Appointment Times and Waiting Times base schedule

### 3.4 Analysis Appointment Scheduling System

The waiting times for both the scheduled patients (\# 1-40) and the walk-in patients (\# 41-46) are aggregated over the simulation time of 25 days. This aggregation is illustrated in figure 3.6.


Figure 3.6: Average Waiting time per patient

In the base schedule, the average waiting times for patients is 10 minutes, which seems reasonable. However, this present study confirms the findings that suggest an increasing trend in waiting time for scheduled patients due to congestion that builds up during the day. Figure 3.6 shows that the average waiting time of the first 16 patients is shorter than the lower boundary of acceptable waiting time, which is 9 minutes. This suggests that they could be scheduled earlier. Scheduling them earlier contributes to a fairer scheduling system, in which each patient has similar estimated waiting times.

However, from patient number 18 onwards, waiting times begin to exceed the upper boundary of the acceptable average waiting time of 10 minutes. This implies that the base schedule is actually scheduling those patients too early on average, and that these patients should be scheduled later. A scheduling system that does not schedule patients with fixed interarrival intervals allows such appointment schedule.

### 3.5 Optimize Appointment Scheduling System

In the optimized appointment schedule, the expected waiting times for all scheduled patients except the first patient should be not shorter than 9 minutes, but also not longer than 10 minutes. In the base schedule, it was observed that the expected waiting time for patient 2 is equal to 2.23 min , which is below the lower boundary of the acceptable expected waiting time limit. The expected waiting time of patient 2 is altered by re-scheduling the appointment time of patient 2 in the base schedule to 30 seconds earlier. Following this, the new expected waiting time of patient 2 is calculated and this process is repeated until the expected waiting time of patient 2 is between 9 and 10 minutes.

If, however, the expected waiting time of a particular patient in a time slot exceeds the acceptable waiting time limit of 10 minutes, the appointment time is re-scheduled 30 seconds later, after which the new expected waiting time for patients in that specific time slot is calculated until their expected waiting time is also between 9 and 10 minutes. This process is repeated for each time slot in ascending order. Figure 3.7 shows a diagram of this algorithm.


Figure 3.7: Algorithm optimization Appointment Scheduling
The optimized appointment schedule can be found in figure 3.8

| Patient | Appointment Time (optimal) | Wating Time optimal (min) | Pationt | Appointment Time (optimal) | Waiting Time optimal (min) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 09:00:00 | 3.3 | 21 | 12:59:50 | 9.42 |
| 2 | 09.03:28 | 904 | 22 | 13.11 .48 | 955 |
| 3 | 09.15:26 | 9.9 | 23 | 13.25.16 | 9.42 |
| 4 | 09:26.24 | 9.3 | 24 | 13.39 .14 | 9.02 |
| 5 | 09.36.22 | 9.44 | 25 | 13:53.12 | 909 |
| 6 | 09.48 .50 | 9.58 | 26 | 14.04:10 | 958 |
| 7 | 10:01.48 | 924 | 27 | 14116:38 | 9.43 |
| 8 | 10:12:16 | 9.27 | 28 | 142836 | 917 |
| 9 | 10:25:14 | 9.35 | 29 | 14.37.34 | 956 |
| 10 | 10.36.42 | 917 | 30 | 14.4832 | 959 |
| 11 | 10:48:40 | 9.14 | 31 | 15:01:00 | 927 |
| 12 | 10.59:38 | 935 | 32 | 15-14.58 | 928 |
| 13 | 11:14:06 | 938 | 33 | 15.25:56 | 9.46 |
| 14 | 11:25:04 | 958 | 34 | 15.35 .54 | 955 |
| 15 | 11:39:32 | 9.21 | 35 | 15.47.52 | 942 |
| 16 | 11:51:30 | 953 | 36 | 15.59.50 | 953 |
| 17 | 12:04:58 | 9.55 | 37 | 16:11:48 | 9.43 |
| 18 | 12:17:56 | 949 | 38 | 16.2216 | 935 |
| 19 | 12,34.24 | 9.11 | 39 | 16:34:14 | 9.57 |
| 20 | 12:47:52 | 948 | 40. | 16-47-42 | 915 |

Figure 3.8: Appointment Times and Waiting Times optimal schedule

## Chapter 4

## Results

In this chapter, we describe the findings with regard to waiting, idle and overtime.

### 4.1 Waiting Time

By analyzing the waiting times in figure 4.1, a few things become clear. The total average waiting time of all patients together has neither decreased nor increased but is more evenly spread over the scheduled patients. This demonstrates clear support for designing an appointment schedule using variable interval time slots to promote fairness regarding expected waiting times for each scheduled patient throughout a clinical session.

Walk-in patients do have a longer waiting time than the scheduled patients. This can be explained by the fact that they enter the optimized scheduling system unexpectedly. Due to the attempt to make each patient wait for 9-10 minutes, a scheduled patient would arrive around the time the preceding patient is still waiting for their turn or has just entered service (see: table 3.2). Therefore, if the walk-in patient enters the queue shortly before this scheduled patient, it is expected that the walk-in will have to wait $9-10$ minutes. But if the walk-in arrives shortly after the scheduled patient, he or she is expected to have to wait an additional 9-10 minutes. This causes the expected waiting time of a walk-in to fluctuate between once and even twice as long as that of a scheduled patient.


Figure 4.1: Optimized average Waiting Time per Patient

### 4.2 Idle Time

Figure 4.4 confirms that the optimized appointment scheduling system, which uses variable interval time slots, does not influence the doctor's average idle times per day. In both the base and the optimized appointment schedule, the idle time is 35 minutes.


Figure 4.2: Base schedule Idle Times


Figure 4.3: Optimized schedule Idle Times

Figure 4.4: Comparison Idle Times

### 4.3 Overtime

As discussed in chapter 2, there might be an increase in the overtime due to the trade-off mentioned by Bailey and Welch. In fact, the implementation of the variable interval time slot scheduling system has decreased the average overtime of doctors from 8 to 7 minutes. This can be seen in figure 4.7.


Figure 4.5: Base schedule Overtimes


Figure 4.6: Optimized schedule Overtimes

Figure 4.7: Comparison Overtimes
One of the possible causes of over time, can be attributed to walk-ins arriving just before the end of the clinical session.

### 4.4 Cost Function

The applicability of these new results were then tested via the measurement of performance formula 2.2. The total cost function is presented below in order to illustrate the impact of the optimized scheduling system compared to the base scheduling system. In these equations the waiting time, idle time, and over time have equal weights.

$$
\begin{equation*}
\text { Base schedule }: E[T C]=10 \mathrm{~min}+35 \mathrm{~min}+8 \mathrm{~min}=53 \mathrm{~min} \tag{4.1}
\end{equation*}
$$

$$
\begin{equation*}
\text { Optimized schedule }: E[T C]=10 \min +35 \min +7 \min =52 \mathrm{~min} \tag{4.2}
\end{equation*}
$$

All the results combined indicate, that the optimized appointment scheduling system is fair and regardless of the weights of the variables in the cost function is able to decrease the expected total costs.

### 4.5 Additional Experiments

This experiment has been repeated with success on two separate occasions. In these experiments the input variables; $\mathrm{E}[$ Lateness], $\mathrm{E}[$ Interruption], and E [Service Time] which determine the appointment timeslots in the base schedule were different than those in Chapter 3. The results for these experiments are shown in table 4.1 and provide more support for the use of variable interval timeslots as the waiting times for the scheduled patients were unbiased in contrast to the base schedules and on top of that in both optimal schedules the idle time and overtime decreased.

|  | Experiment 1 |  | Experiment 2 |  |
| :--- | :--- | :--- | :--- | :--- |
|  | Base | Optimal | Base | Optimal |
| Waiting Time | 9 | 9 | 8 | 10 |
| Idle Time | 44 | 43 | 51 | 39 |
| Overtime | 31 | 30 | 34 | 22 |
| Total Costs | 84 | 82 | 93 | 71 |

Table 4.1: Experiments

## Chapter 5

## Conclusion

This paper concludes by arguing that an appointment scheduling system using variable interarrival time slots to schedule individual patients outperforms a scheduling system that uses fixed inter-arrival time slots. The latter scheduling system will result in congestion during the clinical sessions, which in turn will lead to undesired short or long waiting times for multiple patients. By considering the possibility of re-scheduling individual appointment slots and allowing them to have variable intervals, an appointment scheduling system can be designed that performs better. The optimized appointment scheduling system that has been designed in this paper tops the base scheduling system in terms of fairness of the expected waiting times for every scheduled patient. There is also no additional expected idle time and a slightly decreased expected overtime for doctors.

### 5.1 Recommendations

With regard to the design of the schedule, as the expected waiting times for walk-in patients still exceed the upper limit of 10 minutes, it could be improved. Since, the cause of exceeding this limit is a result of the absence of an appointment, a hospital might want to discourage (nonemergency) walk-in patients entering the queue and require them to schedule an appointment instead. Furthermore, other uncertainties that are implemented in scheduling systems that can lead to inaccuracy should be avoided, such as no-shows and late arrivals of both doctors and patients. Finally, in every hospital or service type, the variables influencing the expected costs differ, and therefore each service type per hospital should analyze the dependent variables in their own context in order to find their optimal appointment schedule. This is because the actual service time of a particular service type might have different parameters for the log-normal distribution, or because walk-ins entering the queue happens more or less frequently.

## Bibliography


[^0]:    ${ }^{1}$ Patient Late Arrival (min) : -1 , represents a no-show
    Patient Mobility determines the heterogeneity level

