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Comparison of theory and practice of revenue management with undifferentiated demand

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Abstract

Revenue management is a growing field with many theories and techniques to increase profits without increasing supply. However, in the smaller to medium sized hotel chains the theoretical approaches are barely used. In this research paper I examine several theoretical methods for revenue management with undifferentiated demand. I also explore the manner in which a real hotel chain with only one type of room (making demand undifferentiated) decides their prices. I then compare both the theoretical methods with each other and the theoretical methods with the method of the hotel chain for different demand rate structures, allowing me to show the difference in total revenue, variability and robustness.

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INTRODUCTION

Revenue management is the practice of maximizing revenue without increasing supply. Many businesses have a limited supply of goods, some well known examples are airlines and hotels. In this paper the focus will be on hotels. A hotel only has a limited number of rooms to offer and so increasing revenue can only be done by asking higher prices for the rooms. However, if hotels increase their prices too much, they will loose potential customers and thereby only decrease their revenue. Finding the right balance is exactly what revenue management does. It tries to optimize the price a hotel should ask for a room at any given time in order to maximize revenue. This optimization does not necessarily lead to a higher occupancy rate [1], meaning an optimization of the prices often leads to less rooms been booked. This occurs as it is more profitable to have higher prices and less customers, then have a sold out hotel for low prices. Revenue management is therefore not always the best tool for customers.

In this paper a comparison will be made between the revenue obtained with theoretical methods and the revenue obtained with a practice used in hotel chains today. As such, the paper will first clearly outline the problem parameters, before discussing several theoretical solutions to the problem. The paper then explains the process of price management at a real hotel which corresponds to the problem parameters. Both theoretical and practical solutions are then implemented in order to compare their revenues under different scenarios. The paper closes with an opinion on whether it is realistically possible to implement theoretical solutions in real hotel chains and a reasoned conclusion on whether this would significantly increase the revenue of the hotel.

PROBLEM STATEMENT

The most important assumption made in this paper of the hotel is that it has undifferentiated demand. Undifferentiated demand means customers have no specific preference for what room they book. The customer will happily pay less, even if it means taking a less luxury room, but will never pay more than his/her maximum. A simple scenario in which this occurs in real life is when a hotel only has one type of room and this paper assumes this is the case from now on.

Next, it must be mentioned that the hotel has N price classes, which in this case only corresponds to different rates for the same type of room. Each price class has its own demand and its own price, the latter represented by f_j for price class j, $j = 1, \ldots, N$. This price could be time dependent, but in this paper the prices are assumed constant throughout time. The price classes are ordered such that $f_{j-1} > f_j$, $j = 1, \ldots, N$, meaning class 1 is the most expensive class and class N the least expensive. The hotel has C rooms in total and rooms are booked as a unit, meaning it makes no difference to the room price whether it is occupied by 1 or 2 people.

Lastly, an important part of revenue management is the prediction of demand per price class. In this paper it is assumed that the demand is known or already predicted, but many papers and books give in-depth explanations of techniques and pitfalls of demand prediction.

Having clarified the problem parameters, the optimization problem corresponding to the revenue management goal now becomes:

Maximizing the total revenue of a hotel with undifferentiated demand, knowing the demand and price per class, without exceeding supply.

This will lead to theoretical solutions to this still theoretical hotel. Another interesting question is how the theory can be translated to real life hotels and therefore the paper also tries to answer:

Can theoretical solutions to the optimization problem be translated to real life applications and if they can, does it increase revenue compared to currently used methods?

THEORETICAL OPTIMIZATION

In the last 15 years much research has been done into revenue management for undifferentiated demand. Several algorithms have been created and proven effective for this problem. In this paper, three algorithms will be discussed. All three algorithms use a dynamic programming algorithm, with a Markov process idea behind it.

The idea is to translate the optimization problem to a Markov Decision Process. As a supply of a hotel is the number of rooms available for a particular date, the Markov decision process will only look at how to optimize the revenue of bookings for a particular arrival date. The process will have $C+1$ different states, with each state representing the number of rooms left for the specified arrival date. Denote a state by x. The time step, represented by τ, between states is constant and small enough that at most one arrival of a potential customer will occur in a time step.

After a time step has occurred, an action needs to be chosen for the next time step. Actions in this Markov decision process refer to the choice about which price classes to open for a potential new customer. As there is undifferentiated demand, opening price class j, essentially also opens price classes 1 through $j-1$ as they have higher prices and the customer will therefore buy down to price class j. As such, choosing to open price class j means setting the price for the room at f_j. As the action corresponds to opening a particular price class, there are N actions possible.

The transition probabilities of the process depend on both the action and on the demand. Having defined the action, demand needs to be defined. Let $d_j(t)$ represent the expected demand for class j in τ time at time t and define $d_j(t) = \lambda(t)p_j(t)$. Then $\lambda(t)$ represents the probability that a customer arrives in τ time and $p_j(t)$ the probability that the arrival is for class j at time t. The transition probabilities then easily follow from the idea that if there is an arrival and the open price class is k, then the arrival books a room if he/she is from a price class j with $j \leq k$. It is also possible no arrival occurs in the time step, namely with probability $1 - \sum_{j=1}^{N} d_j(t)$.

To complete the translation to a Markov decision process the rewards need to be defined. Define $V_t(x)$ as the expected revenue to be made with x capacity left from time t onwards, where $t = 0, \ldots, T$ with time T the arrival date and time 0 the time the rooms become available for this arrival date. Clearly, the objective is to maximize this expected revenue for all t and x, as $V_0(C)$ represents the total expected revenue and is dependent on the other $V_t(x)$. How $V_t(x)$ is calculated depends on the method chosen below.

Solution 1: Dynamic Programming DownSell (DPDS)

This solution utilizes the given translation to a Markov Decision Process. In each time step the objective is to choose the right price class to open to possibly allow an arriving customer to book a room. The idea is to capture this objective in an expression for $V_t(x)$. If price class k were to be opened, then either a booking is made or it is not. A booking is made if there is an arrival in the next time step from price class j, with $j \leq k$, resulting in a probability of $\sum_{j=1}^{k} d_j(t)$ of a booking occurring. The new state would then be $V_{t+1}(x-1)$ and a revenue of f_k would have been made. On the other hand, if there is an arrival from a price class j with $j > k$ or no arrival at all, then no booking is made and the new state would be $V_{t+1}(x)$. This occurs with probability $\sum_{j=k+1}^{N} d_j(t) + \left(1 - \sum_{j=1}^{N} d_j(t)\right) = 1 - \sum_{j=1}^{k} d_j(t)$. So, if price class k is opened it holds that:

$$V_t(x) = \sum_{j=1}^{k} d_j(t)(f_k + V_{t+1}(x-1)) + (1 - \sum_{j=1}^{k} d_j(t))V_{t+1}(x) \ .$$

As the objective is to maximize $V_t(x)$ the best action is to open the price class for which this expression is maximized. This now leads to the recursion step of the algorithm, namely:

$$V_t(x) = \max_k \left(\sum_{j=1}^{k} d_j(t)(f_k + V_{t+1}(x-1)) + (1 - \sum_{j=1}^{k} d_j(t))V_{t+1}(x) \right) \ . \tag{1}$$

Having found the recursion step, all that remains is to set the boundary conditions. The boundary conditions are $V_T(x) = 0 \ \forall x \leq C$, as no more revenue can be made after the arrival date and $V_t(0) = 0 \ \forall t \leq T$ as no more revenue can be made if the hotel has no more rooms left.

The algorithm eventually results in a table of C by (T/τ), where entry (x,t) corresponds to the best price class to open if x rooms are left at time t. There is no need to have state $x = 0$ represented in this reference table as no more actions can be taken at that point and therefore C rows are enough. This table can then be used as a reference when considering a real situation. At each time, see how many rooms are left, go to the right table entry and act accordingly.

Solution 2: DPDSup

This solution does not allow a cheaper class to be opened than the previously opened class. Meaning if at the last time step price class k was open, the next time step may only choose a price class j with $j \leq k$. This might be beneficial as opening cheap price classes close to the arrival date encourages late bookings. [1] This means the hotel cannot plan ahead. The benefit of this method is therefore in the long run predictability, but not in the short term revenue, as the extra restriction compared to solution 1 will mean the algorithm should always perform worse.

The solution again utilizes the given translation to a Markov decision process, however it increases the dimension of the state space. To implement that no cheaper class may be opened, it is necessary to know which price class was chosen before, and as Markov processes may only use the previous state as information, this information needs to be added to the state. The state space, x, will therefore consist of two dimensions, one is the number of rooms left, denoted by y and one is the highest (cheapest) class available, denoted by k. This means $x = (y, k)$ and the number of states is $(C + 1)N$. The rest of the translation remains the same, except that the expected revenue now depends on a state space of two dimensions: $V_t(y, k)$.

Having redefined the state space, the algorithm uses the same maximization expression for $V_t(y, k)$ as in equation (1), except now only $j \leq k$ may be selected. This results in the recursion step [1]

$$V_t(y,k) = \max_{l:l\leq k}\left(\sum_{j=1}^{l} d_j(t)(f_l + V_{t+1}(y-1,l)) + (1 - \sum_{j=1}^{l} d_j(t))V_{t+1}(y,l)\right) . \tag{2}$$

The boundary conditions are very similar to those of the DPDS, namely $V_T(y, k) = 0\ \forall y \leq C,\ k \leq N$ and $V_t(0, k) = 0\ \forall t \leq T,\ k \leq N$. The algorithm then produces a table of CN by (T/τ), where entry $(y * N + k, t)$ corresponds to the best price class to open if y rooms are left, k was the last price class to be opened and there is t time left. This table can then again be used as a reference when considering a real life situation.

Solution 3: Marginal revenue transformation (MRT)

Showing the relationship between DPDS and the marginal revenue

This solution again translates the problem to the given Markov decision process. However, the solution does make some changes to the problem first. With this solution the actions are not limited to choosing the cheapest class to open. Instead, any combination of classes can be opened at any time. This however results in a set of 2^N actions, which from now on will be denoted by A. Before the dynamic algorithm can defined, A must be reduced to a smaller set.

The reduction of the action set A is done per time step. To reduce the set of actions, first calculate the expected revenue (R) and demand probability (D) per time step for each $Z \in A$. Here $D_t(Z) = \sum_{j\in Z} d_j(t)$ and $R_t(Z) = D_t(Z)\min_{j\in Z}\{f_j\}$. The expected revenue formula follows from the fact that the customers will always buy down as it is undifferentiated demand. Having calculated $R_t(Z)$ and $D_t(Z)$ for each $Z \in A$, plot the results in a scatter-plot of D_t vs R_t. The upper boundary of the convex hull of this scatter-plot is the efficiency frontier. Let the actions on the efficiency frontier be denoted by $a_t^0, \ldots, a_t^M$ and ordered such that $D_t(a_t^k) \leq D_t(a_t^{k+1})$ for all $k \leq M$ and $R_t(a_t^k) \leq R_t(a_t^{k+1})$ for all $k \leq M$. These $M + 1$ actions are now the new reduced action set, A'_t, and are called efficient actions. [2]

Finding the reduced action set A'_t is usually done by a recursion. If a_t^k has been found, then

$$a_t^{k+1} = \underset{a\in A:\, D_t(a)\geq D_t(a_t^k),\, R_t(a)\geq R_t(a_t^k)}{\operatorname{argmax}} \frac{R_t(a) - R_t(a_t^k)}{D_t(a) - D_t(a_t^k)} .$$

The fraction in this recursion step is known are the marginal revenue, which is where the name of this solution comes from. Let a_t^0 be equal to the empty set, meaning that action is equal to no price classes open at all. The recursion step is then capable of finding the set A'_t as it traces the convex hull. The recursion stops when no action $a \in A$ complies with $D_t(a) \geq D_t(a_t^k)$ and $R_t(a) \geq R_t(a_t^k)$, when a_t^k was the last action found. [3]

If this procedure was to be followed, it turns out that the reduced action set is equal to the action set defined in the original translation to the Markov decision problem. This means that a_t^k is equal to opening price classes 1 through k for any t. Therefore, it is shown that this original action set is the set with all efficient actions for any time.[2] Thus let $a_k = a_k^t$ and $A' = A'_t$ for all t. Then an expression for $V_t(x)$ can be found. If action a_k is chosen, then a booking is made with probability $D_t(a_k)$ and has expected revenue $R_t(a_k)$. If a booking is made, the new state would be $V_{t+1}(x-1)$. With action a_k, no booking occurs with probability $1 - D_t(a_k)$ and the new state would be $V_{t+1}(x)$ in that case. So if action a_k is chosen, it holds that

$$\begin{aligned} V_t(x) &= D_t(a_k)f_k + D_t(a_k)V_{t+1}(x-1) + [1 - D_t(a_k)]V_{t+1}(x) \\ &= D_t(a_k)[f_k + V_{t+1}(x-1)] + [1 - D_t(a_k)]V_{t+1}(x) \end{aligned}$$

As $V_t(x)$ is the expected total revenue, it has to be maximized and so:

$$V_t(x) = \max_{a_k \in A'} \left(D_t(a_k)[f_k + V_{t+1}(x-1)] + [1 - D_t(a_k)]V_{t+1}(x) \right).$$

This recursion is equivalent to the recursion step found for DPDS in equation (1). It shows that this method gives the same action set as I presumed was correct to take in the DPDS and DPDSup methods, with this way of looking at the problem even leading to the same recursion step as the DPDS.

DPIP with marginal revenue transformation

The method derived back to DPDS above is one way of using marginal revenue for optimization under undifferentiated demand. By using the fact that the efficient actions are equal for all t and nested, it is possible to use a direct application of the marginal revenue to solve the optimization problem.

The solution redefines the price of each price class. Take a_k as the action of opening classes 1 through k and define $D_t(a_k) = \sum_{j=1}^{k} d_j(t)$ and $R_t(a_k) = D_t(a_k)f_k$, which are equivalent definitions to those given before. The price for price class k is then redefined as

$$f'_k(t) = \frac{R_t(k) - R_t(k-1)}{D_t(k) - D_t(k-1)} = \frac{R_t(k) - R_t(k-1)}{d_k(t)}.$$

By applying this transformation, called the marginal revenue transformation, the demand becomes differentiated. [2] This means that customers will never buy down, but always buy from their price class.This is very useful as there is a wide range of solutions for solving these types of problems. It allows me and real hotels to explore those methods in order to find out if they are better or more easily applied.

In this paper I choose to use a dynamic algorithm similar to DPDS: the DPIP. This algorithm is better known as the traditional dynamic program for revenue management and assumes differentiated demand. After the transformation of the prices as discussed, this method can now be used. It can again be seen in the Markov decision process framework. The idea is again to find an expression for $V_t(x)$. Assume action a_k is chosen, then with probability $\sum_{j=1}^{k} d_j(t)$ a booking occurs and the new state becomes $V_{t+1}(x-1)$. However, this time it matters for which class the booking is made for the revenue. The expected revenue is therefore equal to $\sum_{j=1}^{k} d_j(t)f'_j(t)$ by definition of expectation. If action a_k is chosen, no booking occurs with probability $1 - \sum_{j=1}^{k} d_j(t)$. This leads to an expression for $V_t(x)$ when a_k is chosen of

$$\begin{aligned} V_t(x) &= \sum_{j=1}^{k} d_j(t)V_{t+1}(x-1) + \sum_{j=1}^{k} d_j(t)f'_j(t) + [1 - \sum_{j=1}^{k} d_j(t)]V_{t+1}(x) \\ &= \sum_{j=1}^{k} d_j(t)[V_{t+1}(x-1) + f'_j(t)] + [1 - \sum_{j=1}^{k} d_j(t)]V_{t+1}(x). \end{aligned}$$

Again $V_t(x)$ must be maximized and therefore the recursion step becomes:

$$V_t(x) = \max_{k \leq N} \left(\sum_{j=1}^{k} d_j(t)[V_{t+1}(x-1) + f'_j(t)] + [1 - \sum_{j=1}^{k} d_j(t)]V_{t+1}(x) \right) . \tag{3}$$

The boundary conditions for the DPIP are $V_T(x) = 0 \; \forall x \leq C$ and $V_t(0) = 0 \; \forall t \leq T$, just like with DPDS.

This dynamic program again results in a table of C by (T/τ), where entry (x, t) corresponds to the best action a_k to take if x rooms are left at time t. This table can then be used as a reference when considering a real situation.

METHOD USED BY REAL HOTEL

Method of hotel X

The company studied in this paper will be called hotel X. This is a chain of hotels, which each only have one type of room. They therefore conform to the assumption of undifferentiated demand, when looking per hotel. Their method for deciding the price at any moment differs greatly from the theoretically based methods discussed above.

In the theoretical methods discussed above, revenue management consists of two main steps. The first step involves estimating the demand rates per price class, this part of the process is not discussed in this paper but assumed to be known or estimated already. The second step is to optimize the policy of when to open which price classes for which I have just given 3 theoretical methods. To accomplish step 1 the company must decide which prices to ask and estimate the demand rate per price class. There are many factors influencing this step, such as

Benchmark The company compares their hotel prices to the prices of the hotels in the neighborhood. This gives an indication of what their competition asks, which is important as a similar hotel with a lower price could severely lower demand for their hotel. However, their only tool for comparison is that they get a benchmark, which is an average of the hotels in the neighborhood, while the hotels in the neighborhood may be of a very different quality than their hotel.

Events The company also tries to keep track of any events near their hotels, as that also influences the demand. Namely, a higher price can be asked if a large event is close to the hotel, as the amount of people that need a hotel room in the area is much higher.

Date The company also uses the date to determine demand. For instance, when it is Christmas, less people tend to stay in hotels, as they are usually with family and friends.

Brand The company sets a maximum price the hotel room may cost. If they don't and the price goes extremely high, they might have high short term revenue, but their brand is damaged. This leads to reduced revenue in the future as negative reviews and experiences of customers comparing price to product will cause lower demand in the future.

Past experience The company uses that it has been running for several years. It goes through the old data to see how much rooms had been booked a certain amount of time before the date of arrival. They specifically look at how fast the rooms were booked in previous years for the same arrival date. However, the company doesn't always want to repeat the past and the other factors mentioned that it might have created year-specific results.

The company uses all these factors to determine which prices to ask and how high demand might be. They make a prediction curve, which says how many rooms they expect to have left a specific number of weeks before the arrival date. This is their way of predicting. So in contradiction to the theoretical situation where we predict demand per price class, the company predicts how many rooms are booked a specific number of weeks before arrival.

As the way the company predicts is so different, their prediction is not suitable for the theoretical methods suggested for step 2. It highlights a problem with the theoretical methods, namely the rigid nature of the input. A company must be able to determine price classes and their corresponding price and demand or the dynamic algorithm doesn't work. Especially predicting demand per price is a difficult thing to do for a company. As a result, the prediction of the company is given differently and therefore also used very differently. In the theoretical situation we optimize over all the different price classes to determine a policy which will be followed to the letter. In practice, hotel X is developing a machine learning algorithm which takes the predicted curve of how many rooms are still free at any time before T, from now on called the pick up curve, as input. (Or all the different factors, after which it determines the pick up curve by itself). It then checks per week whether the real life situation is behaving according to the pick up curve. If it is not, it adjusts the expectation to the new information in the sense that the set of open price classes is adjusted. For instance, if demand turns out to be higher, the algorithm adjusts its expectation to a higher demand and therefore increases the price as a higher demand means enough people will want to book a room, even now that a higher price is being asked. This way the real life situation conforms back to the pick up curve.

The factors and the final method with which they determine the price show that the company is not only focused on revenue maximization. This is an assumption commonly made with the theoretical problems. The company is looking towards the future, and apart from indeed trying to maximize revenue, they also want to maintain a good brand and predictability in their revenue. These other objectives and the rigid nature of the input of the theoretical methods, where demand probabilities are needed per price class, are clear reasons why the company chooses to use a different technique for choosing their price at any moment in time then the theoretically optimal one.

Algorithmic interpretation of the pick up curve

The goal of this paper is to compare the theoretical methods with the method of hotel X, which requires the algorithms to use the same input. It is therefore necessary to not make the pick up curve from the factors mentioned above, like past experience and date, but to create a pick up curve which is comparable to the demand rates used as input for the theoretical methods. I create the pick up curve by using simulation. Setting which price classes are open and using the same demand rates and price classes as the theoretical methods, I 'create' 1000 samples of a pick up curve by running a simulation. I register when a reservation took place, allowing me to create a declining array showing the number of rooms left at each time step. Averaging and rounding the result of all these simulations forms the pick up curve used in this paper.

This pick up curve can then be used as reference in real situations, by checking every set time interval if the number of rooms left is above or below the pick up curve and adjusting which price classes are open accordingly. If more rooms are still available than the pick up curve suggests there should be at this moment in time, then a cheaper price class can be opened. On the contrary, if more rooms have been booked then the pick up curve suggests, the cheapest fare class can be closed.

RESULTS

Approach 1

So far I have described how to obtain the reference tables of the theoretical methods and how to create a corresponding pick up curve as reference. I now need to simulate using these methods in real situations. I present the situation I chose in the theoretical framework. I take $C = 100$, $\tau = 1$ hour, T= 8 weeks, $N = 7$ with prices $(140, 130, 120, 110, 100, 90, 80)$ and $p_j(t) = 1/7\ \forall j, t$. Lastly, I choose to have 3 different $\lambda(t)$'s to investigate, all of which change with time. They are represented in figure 1 and are naturally named low, medium and high demand rates.

I now have three different scenarios, all completely the same except for the probability that an arrival occurs. I can calculate the reference tables and pick up curve for all three scenarios. The rate used to create a reference table or pick up curve is called a predicted rate. However, in practice it could occur that the prediction is wrong and the rate experienced in the real situation is different. This rate will be referred to as the realized rate.

To illustrate how an optimal path could look during a simulation, I have illustrated single simulation paths taken under the various methods, shown in figure 2. From this figure it is clear that the DPDS and DPIP with MAR method variate a lot with the price classes, constantly closing and opening classes. This is in clear contrast with DPDSup, which can only close classes due to the extra restriction placed

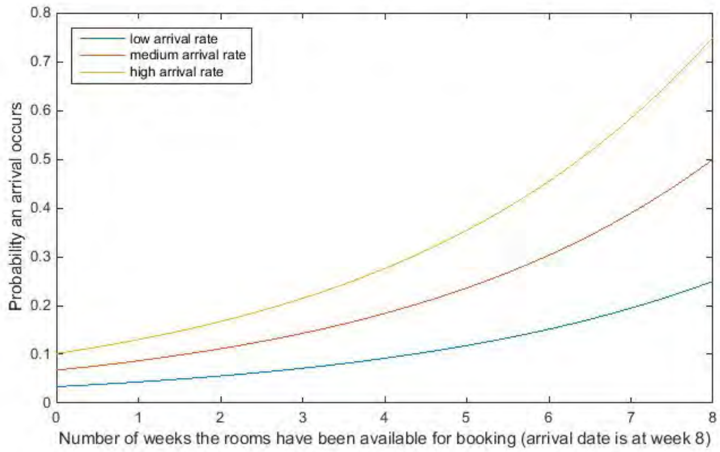

Figure 1: The three options for $\lambda(t)$

upon it and therefore has much less variation in price. All three methods do share that only with low demand rates the higher and cheaper price classes are opened, meaning that the rooms are only available for low prices if demand is low. The pick up curve method shows interesting results, even with the high demand rate the method opens very cheap classes. This is most likely due to the nature of the method, it doesn't look at the potential still ahead, but only at what it would expect at that moment. As this is a single run, it can deviate from the average quite a bit, causing the lower classes to open even though the demand is high enough that this isn't necessary.

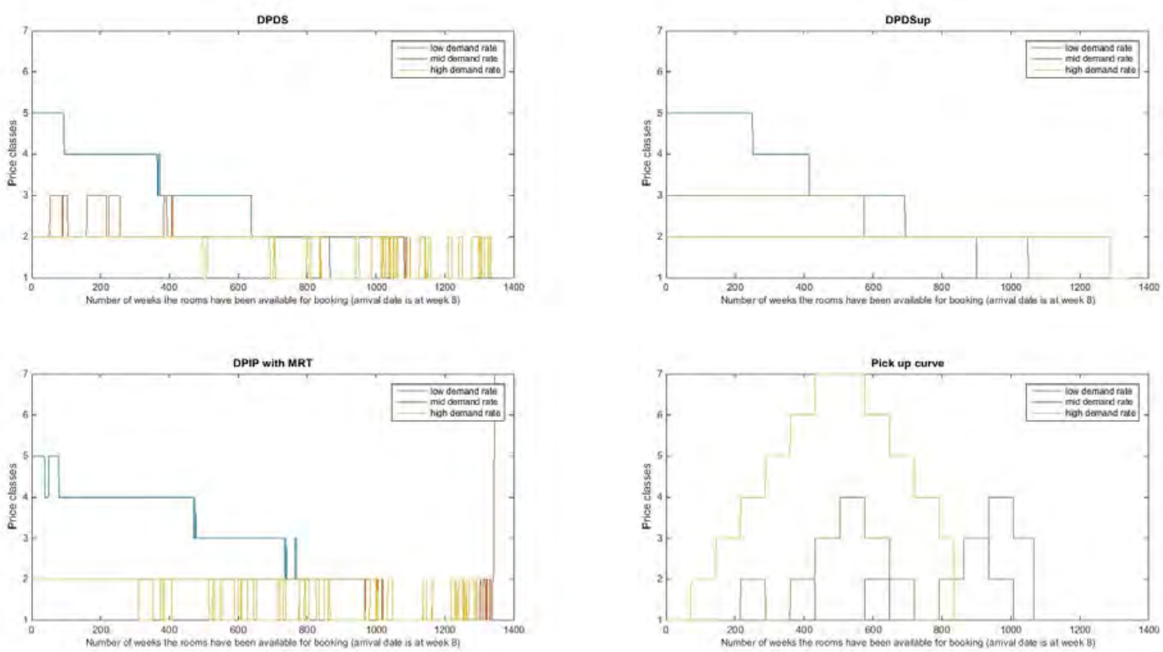

Figure 2: Optimal path of all four methods during a single simulation

Results approach 1

Before comparing theory to the hotel X method, I first investigate the three theoretical methods discussed. It isn't always desirable to have a high expected revenue if the variability is very high. This could mean that the hotel expects to make X revenue, but experiences a much lower Y revenue, maybe creating financial difficulty if the expected profits are already reinvested. A good method therefore combines high revenue with low variability. In figure 3 the three methods are shown for all three rates proposed with an error bar indicating their standard deviation. Each scenario was simulated 10000 times to create this average revenue and standard deviation.

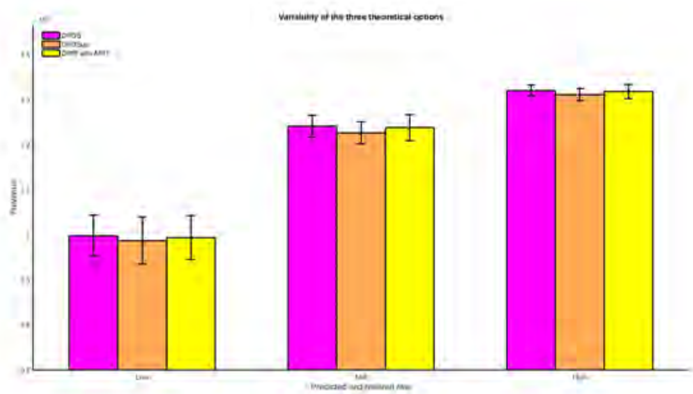

Figure 3: Mean and standard deviation of the revenue with 10000 simulations

From the results shown in figure 3 it is clear that the biggest factor in variability is the demand rate. With a lower demand rate the standard deviation increases, this however is not a factor that can be controlled in real life. For all predicted and realized rates the difference in standard deviation and mean revenue for the three methods isn't very large and most likely statistically insignificant. It is therefore impossible to conclude one method better than the others. It should be noted that the DPDSup method is indeed performing worse than the DPDS method, as already expected.

It is also interesting to see if the expected revenue as predicted by the dynamic programming algorithm falls in the confidence interval created by these simulations. Table 1 shows the simulation mean revenue, confidence interval and expected revenue for each of the three methods for each of the three demand rates.

Table 1: Comparing simulation results with expected revenue

Method	Predicted and realized rate	Simulation-mean revenue	95%-confidence interval for revenue	Expected revenue $[V_0(C)]$
DPDS	low	9988.4	(9979.5,9997.3)	9985,1
DPDS	medium	12414	(12409,12418)	12415
DPDS	high	13210	(13208,13213)	13209
DPDSup	low	9881.7	(9871.4,9891.9)	9890.4
DPDSup	medium	12270	(12265,12275)	12273
DPDSup	high	13118	(13115,13121)	13115
DPIP with MRT	low	9951.3	(9941.7,9960.9)	5405
DPIP with MRT	medium	12379	(12374,12385)	8792.2
DPIP with MRT	high	13188	(13185,13191)	10907

Table 1 shows that the confidence intervals of DPDS and DPDSup are quite accurate, with all expected revenues falling inside or on the border of their corresponding confidence interval. The result assures that the theoretical methods are implemented well and indeed give a good prediction of the expected revenue for the company.

The fact that the expected revenue of DPIP with MRT is so far outside of the corresponding confidence intervals is not surprising. The DPIP with MRT method uses adjusted prices, based on the marginal revenue transformation, making the prices much lower or even negative. These prices are therefore effective in creating a reference table, but do not lead to an accurate expected revenue.

Having investigated the three theoretical methods, I can see that the performance of the three theoretical methods is quite similar. No theoretical method is clearly better than the other two. I therefore compare the pick up curve strategy with all three theoretical methods. I now run 10000 simulations of each combination of predicted and realized demand rates for the theoretical methods and the pick up curve method. The results of this are shown in figure 4.

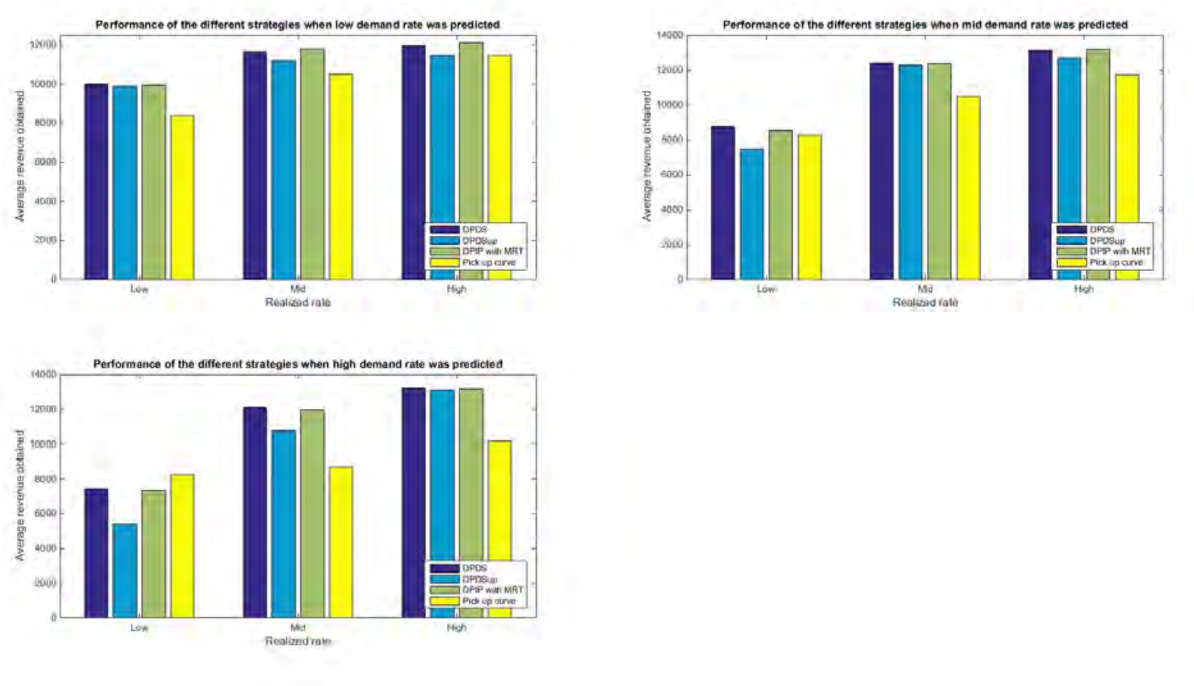

Figure 4: Comparison of all four methods on revenue

I first look at the performance of the pick up curve compared to the theoretical methods when the realized rate is the predicted rate. In that case the theoretical rates perform quite equally as seen before, but the pick up curve method performs much worse. The mean revenue is between 1500 to 3000 less when using the pick up curve method. Actually, the pick up curve consistently performs worse than the theoretical methods, except when a high demand rate was predicted and a low demand rate was realized. This is most likely because the pick up curve will have a much lower number of rooms left each time the comparison is made forcing more price classes to open, therefore allowing a lot of rooms to be booked even though the demand rate is low as prices become low. This effect is most likely less quick when using the theoretical methods which keep the price classes closed for a long time expecting high demand rates and therefore enough time to book the rooms at a higher price at a later time before the arrival date.

The cross comparison of the predicted and realized demand rates shows some interesting effects. The different theoretical methods respond differently to wrongly predicted demand rates, with DPDSup producing very bad results in comparison with the other methods if the predicted demand rate is higher than the realized demand rate. This is most likely due to the restriction that a class with a lower price, say price class 7 with price 80, cannot be opened anymore after it has been closed. Higher demand rates suggest that price classes with higher prices can be used, therefore closing the price classes with lower prices early. However, if the demand is lower than expected this leads to many rooms being left unbooked at the end, resulting in a lower revenue. This same rigid restriction also seems to cause a slightly worse performance than the other methods if the predicted demand rate is lower than the realized demand rate.

Due to the low predicted demand rate the cheaper classes will stay open, causing more bookings at lower prices than necessary.

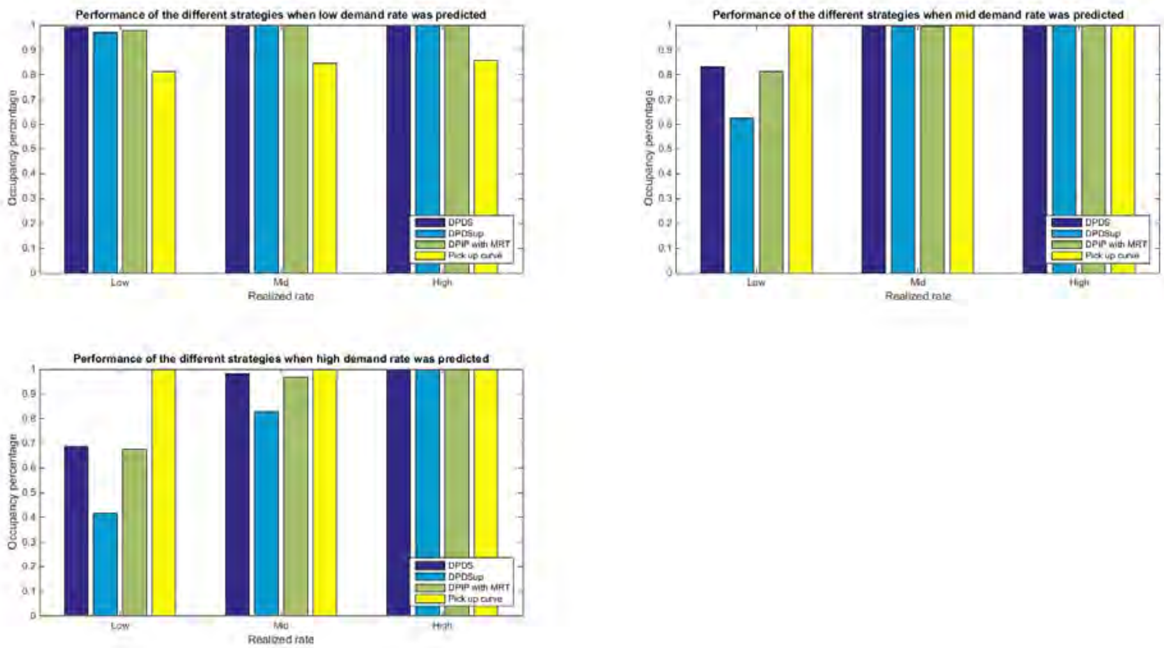

Figure 5: Comparison of all four methods on occupancy

The idea that the restriction of DPDSup causes a low occupancy and therefore a low revenue when the predicted rate is higher than the realized rate is confirmed in figure 5. In this figure the occupancy for all four methods used is shown and the occupancy of DPDSup indeed decreases dramatically if the realized rate is lower than the expected rate. For instance, the lowest occupancy found for all methods is for DPDSup when high was predicted and low was realized.

Overall, the occupancy levels are high, indicating that maybe the hotel is somewhat small for the rates used, meaning the hotel is easily filled for high prices. However, the effects of wrong predictions are still clear with occupancy levels falling severely for the theoretical methods when the predicated rate is higher than the realized rate. The pick up curve method on the other hand maintains a high occupancy even if the predicted rate is too high, due to the quick response of the method. It quickly opens cheaper price classes, balancing out the low demand. Surprisingly, when low demand is predicted, the pick up curve occupancy stays at around 0.8, even though the theoretical methods all have much higher occupancy levels. This is most likely because the method is trying to keep to an expectation, namely the pick up curve. Therefore, if the pick up curve only reached an 0.8 occupancy level, the pick up curve method will not sell out the hotel.

Approach 2

Having examined the results of the four methods (three theoretical methods and the pick up method), it is clear to see a difference between theory and practice when it comes to revenue. However, the theoretical methods still perform relatively similar. To examine those three methods a little more and also confirm the comparison between theory and practice again, I examine a second situation. I again present the situation in the theoretical framework. I take $C = 100$, $\tau = 1$ hour, $T=$ 8 weeks, $N = 2$ with prices $(100, 60)$ and $\lambda(t)$ again dependent on t with exponential behavior. Lastly, I either take $p_j(t) = 1/2\ \forall j, t$ or the $p_j(t)$ declines for j=1 (the higher price class) when I get near the arrival date. The precise effect is shown in figure 6 where the $d_j(t)$ for both price classes is shown. The first scenario with $p_j(t) = 1/2\ \forall j, t$

will be called the exponential (exp) scenario and the other scenario the realistic (real) scenario.

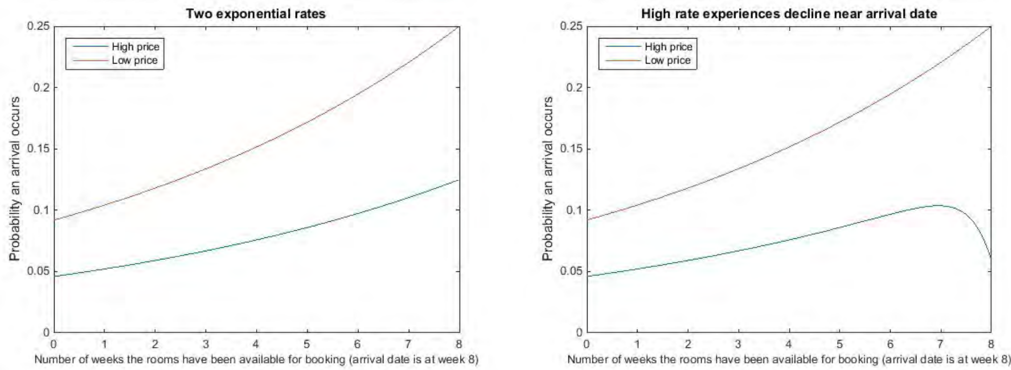

Figure 6: The two options for $d_j(t)$

The reason for this comparison is a theory that even though most simulations or applications use exponential demand rates, it might actually be that the price classes with the higher prices experience a drop in their demand rates when closer to the arrival date. This would be because of competition with other hotels, which tend to lower prices near the end in order to attract last minute customers. If the customer can get a similar room for a cheaper price at a different hotel, it will no longer be willing to pay the higher price.

Results approach 2

Having explained the practical relevance of these two scenarios and their comparison I produce the reference tables and pick up curves related to both scenarios. I can then run 10000 simulations to investigate the performance of each method in a real situation. The result of these simulations for all combinations of realized and predicted rates are shown in figure 7.

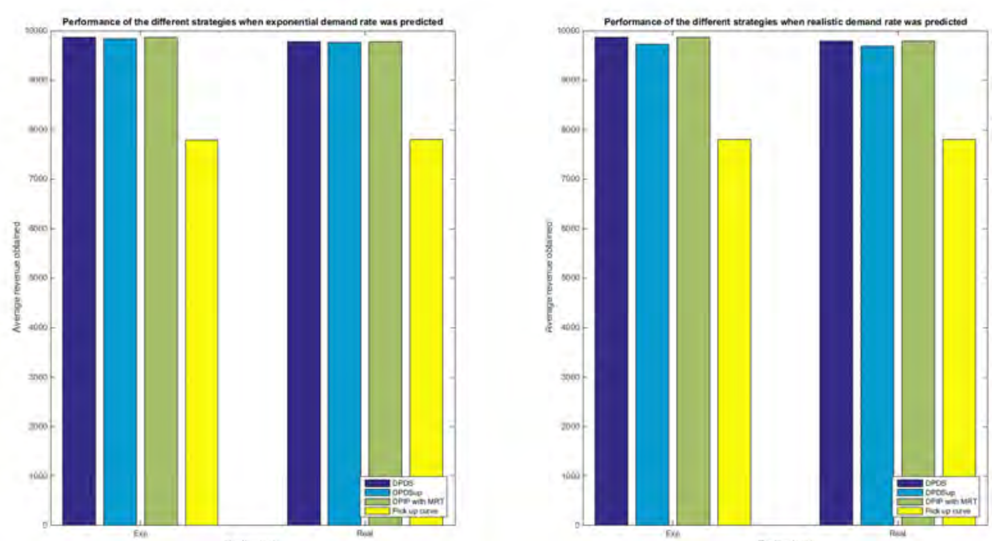

Figure 7: Comparison of the four methods

The first thing that is clearly noticeable is the confirmation that indeed the pick up curve performs quite a bit worse than the theoretical methods, with about a difference of 2000 in revenue each time.

Furthermore, the pick up curve shows very little difference in mean revenue over all four scenarios. It seems the pick up method is quite insensitive to a drop in demand rate like experienced in the realistic scenario.

When the predicted rate is exponential, no matter whether the realized rate is exponential or realistic, the theoretical methods performance is quite similar. No method seems to be better than the other two. However, when the predicted rate is realistic, the DPDSup method performs much worse than the other two theoretical methods no matter what the realized rate is. This while the other two methods, namely DPDS and DPIP with MRT, perform very similar throughout all four scenarios. The reason DPDSup performs worse than the other two methods under a realistic prediction rate is because the method keeps the lower class open for longer, expecting the drop and not being allowed to reopen the class. The other methods are allowed to reopen classes and can therefore close the cheaper class temporarily. This increases the revenue as the hotel rooms are then booked at a higher price during that time.

CONCLUSION AND DISCUSSION

Conclusion

I have shown several theoretical methods that can be used for maximizing the revenue of a hotel. The methods DPDS and DPIP with MRT give nearly interchangeable results, which isn't an unexpected result. The marginal revenue transformation simply translates the problem into one with differentiated demand and both DPDS and DPIP solve the problem optimally under their respective assumptions. The theory therefore already suggests that they should perform equally.

More interesting was the behavior of DPDSup. This method could theoretically never do better than DPDS, as it was the same problem with an extra restriction. The results show that the method indeed performs worse then DPDS in terms of revenue. When the predicted rate is also the realized rate, the DPDSup still performed quite well in either approach. However, the cross-examination of predicted and realized rates in approach 1 shows the lack of flexibility the method has, which causes a lack of adaptability if the prediction is wrong. This can severely impact the revenue obtained, as clearly shown in figure 4. This lack of flexibility again produces lesser results in approach 2, when the predicted rate is real. The only way in which the method can respond is by keeping the cheaper price class open until the end, booking many rooms for an unnecessary low price.

Combining these results, I can only conclude that without tangible evidence of the long term benefits of the DPDSup method, the best theoretical method to use out of the three is DPDS (as DPIP with MRT requires an extra calculation). Approach 2 also shows that it is advisable to use exponential demand rates, as these methods adapt well to a possible deviation. Obviously, the true rate gives the best results, but when in doubt an exponential predicted rate is a good choice.

Having established the best theoretical method and demand rate, it remains to compare the theory to the practice. The results clearly show the loss of revenue by using the pick up curve method, with the pick up curve almost consistently performing worse than theory. The comparative nature of the method clearly shows its shortcomings in the occupancy rates for low predicted rates. The method here caps off at 0.8 occupancy, just because the pick up curve does, even though demand may be high enough to get a higher occupancy at reasonably high prices leading to a higher revenue. However, the method does show adaptability in case of wrongly predicted rates, which might be quite beneficial if the hotel chain is having trouble producing predictions.

To conclude, the pick up curve method clearly under-performs compared to the theoretical methods, making a change to more theoretical approaches beneficial. However, the large difference in how a prediction is made in theory or in practice makes implementing the theoretical methods difficult and maybe even impossible. Plus, the difficulty could cause large errors in the demand estimation, leading to worse results in the end. The rigid nature of the required input of the theoretical methods may therefore make them unaccessible.

Discussion

Having answered the goals of the paper there are many arguments left to consider. As already mentioned in the comparison of the theoretical methods, the DPDSup results do not reflect the possible long run benefits of not promoting last minute bookings. More research into the tangible results should be done before the method is rejected. All that the results in this paper show, is that the effect should be considerable in order to make DPDSup a viable option as it performs quite a bit worse than the other two methods. It will be difficult however to create tangible results about this impact.

Clearly the pick up curve under-performs in terms of revenue, but the conclusion does not take into account the other objectives a hotel has: predictability and brand. Though brand could be protected by choosing appropriate price classes to use in the optimization, it is clear from figure 4 that the difference in revenue seen by theoretical methods is larger than that of the pick up curve method when the predicted rate is higher than the realized rate. This loss of predictability when changing to more theoretical methods should be considered. Especially as predicting the demand rate per price class is even more difficult than predicting a pick up curve and therefore increases the chances the predicted rate is different from the realized rate.

The algorithm used for the pick up method could be tested further, to see if the results might improve. It might be interesting to see how different time periods between comparison with the pick up curve influences the result. This paper used a time period of 3 days, but changing this means changing how responsive the real situation is to the pick up curve. Due to the high variability in arrivals, it might be beneficial to take a longer time period. However, the responsiveness of the algorithm declines with a longer time period, so making the time period too long could negate the effects of the method.

The time period is not the only aspect of the pick up curve method that could be investigated further. It is questionable whether the pick up curves produced in this paper are comparable to the pick up curves produced in real life. This could strongly affect the results. Investigating different methods of producing the pick up curve could create better results for the pick up curve method that possibly reflects the real scenario better. Another approach could be to use real data and extrapolate the demand probability per price class and pick up curve from that. This way the pick up curve is realistic, but the demand probabilities are estimated and therefore possibly wrong. It would be interesting to see the comparison in revenue between theory and practice, if the uncertainty of prediction is captured in the theoretical methods, instead of the practical methods like this paper does.

A last remark is about the parameters of the problem. In this paper the occupancy results show that the demand probabilities are quite high for the size of the hotel. It would be interesting to see how the different methods perform for a much larger hotel or a hotel with much lower demand probabilities. This forces the occupancy to be lower and the choice between selling more rooms at a lower price or fewer rooms at a higher price becomes more evident. I suspect that the results remain quite similar, but it would be interesting to try and confirm this intuition.

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