Vrij Universiteit Amsterdam

Can you still beat the dealer?

Hans Hellemons | BMI paper

Vrije Universiteit *Amsterdam*
Business Mathematics and Informatics

Faculty Sciences
Business Mathematics and Informatics
De Boelelaan 1081A
1081 HV Amsterdam

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Hans Hellemons

hhs800@few.vu.nl

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“It is remarkable that a science which began with the
consideration of games of chance should have become
the most important object of human knowledge...
The most important questions of life are, for the
most part, really only problems of probability.”
Pierre Simon, Marquis de Laplace
ThéorieAnalytique des Probabilitiés, 1812

Preface

The last part of the Master program Business Mathematics and Informatics is writing a thesis. Business Mathematics and Informatics is a multidisciplinary programme, based upon mathematics, computer science and business management, aimed to improve business processes. These three disciplines will be integrated in this thesis.

In this theses, I will give an introduction to the game of blackjack and use the existing literature to give multiple strategies about how to play the game. Most of these strategies date back to the 60's of the last century, since then some rules of the game have been changed. The impact of these changes are tested in a self-written simulation program.

In summary, I will present the reader an overview of strategies used in blackjack to beat the dealer and about the winning chances of using these strategies under the rules that are currently used in casinos. After reading this thesis the reader will thus be able to answer the question "can you still beat the dealer?".

Finally, I would like to thank my supervisor dr. René Bekker for his help during with this thesis.

Hans Hellemons
Amsterdam, March 2011

Summary

Blackjack is one of the only casino card games where the cards are not shuffled between rounds, instead these cards are set aside. A good player can take advantage of this by keeping track of the cards dealt in previous rounds and use that information in the upcoming round. Some academics specialized in mathematics were particularly intrigued by this and started a search for a winning strategy.

The first to succeed was a mathematics professor, Edward Thorp in the early 1960's. With access to one of the fastest computer at the time at MIT, he managed to find two winning strategies. His book "Beat the dealer" has had an immense influence on the game. All of the sudden there was a way to make certain money at a casino. The casino reacted by taking countermeasures, as far as changing the rules of the game. Since the publication of the book by Edward Thorp, the rules in the casinos around the world have been altered slightly to make card counting more difficult. However, also the strategies have been improved. The main question of this thesis is "can you still beat the dealer?".

A fast approach to give an answer to this question is the use of simulation. For six types of rules, including the classic rules and five rules used nowadays around the world, and for five different strategies a simulation was done.

From this simulation it follows that a blackjack player can still have an advantage over the dealer under the current rules for certain strategies. However, the advantage shrunk considerable under the current rules compared to the classic rules. The Atlantic City rules are the most favourable to the blackjack player nowadays. But still on the long run the player can still beat the dealer for all current rules, except for the casinos using a card shuffling machine. Card shuffling machines make card counting impossible as every new round is again an independent event.

However, the casino nowadays keep a close watch on players winning money at the blackjack table. They train their dealers how to count cards which enables them to spot the card counters more easily. And once spotted as a card counter the name and picture of the player are uploaded in a blacklist which is shared between casinos around the world. After which it becomes nearly impossible to beat the dealer after that. To conclude, it is still possible to beat the dealer if a player only plays occasionally and keeps a low profile. However, beating the dealer on a regular basis is nearly impossible as it is inevitable that a regular player will be marked as a card counter.

Contents

Chapter 1

"There are three subjects you can count upon a man
to lie about: sex, gas mileage, and gambling."

R.A. Rosenbaum

Introduction

The exact origin of the game of blackjack or twenty-one is unknown. There are many stories going around about where the game was invented. The first written reference about the game is from the hand of the Spanish author Miquel de Cervantes. The main characters in his book "Rinconete y Cortadillo" are a couple of cheaters at the game "ventiuna" (Spanish for twenty-one). This book was written around 1600, which implies that the game was played since the beginning of the 17^{th} century or earlier.

Whereas the first reference to the game of blackjack comes from Spain, the French are generally credited the invention of the game, as the game is believed to be derived from old French card games like "Chemin de Fer" and "French Ferme". The game "vingt-et-un" (French for twenty-one) made its first appearance in the French casinos around 1700. Thanks to French colonists it gradually spread to the United States, where it soon was played all over the North American continent.

The game was still named "twenty-one" when it gained popularity in the United States. In 1931, Nevada was the first state legalize gambling and as a result the popularity of the game increased to even larger proportions. It was also in Nevada where the game acquired its now common name, blackjack. This name was associated with a bonus that could be won in some casinos in order to draw more people to the game, the players received this bonus when they were dealt a jack and ace of spades as their first two cards. This hand was called a "blackjack". Although the casinos later dropped this bonus, the name "blackjack" stuck to the game. In the modern game, a "blackjack " refers to a hand of an ace and a 10-count card, regardless of whether the cards are suited.

Today the game of blackjack is one of the most popular games worldwide, played in practically every casino around the world. Millions of people play this game every day and its popularity is still growing. With around a billion dollar revenue per year in the state Nevada, United States alone blackjack is a big business.

1.1 Research objectives

In most casino card games the cards are shuffled after each round of play, the cards that were played in the last round of play are thus again available in the next round. As a consequence, every round of play is an independent event and does not have a "memory" of the previous rounds. The reason that makes blackjack an interesting research area is the feature that the cards are not returned after each round, but are set aside. Consequently, the rounds of play are not independent events and the cards in a new round do actually have a "memory" of the previous rounds. This feature of the game of blackjack can be used to the player's advantage if it can be calculated what the effect is of a certain card being removed from the deck.

The main objective in this thesis is to provide the reader with an answer to the question "can you still beat the dealer?". The approach for this will be to first present some pre-existing research on winning strategies in the game of blackjack. Most of these strategies date back to the 60's of the last century. As some rules of the game have been changed since then, these strategies will be tested for the rules that casinos apply nowadays.

The first part of this thesis will be about the rules of blackjack. It is important for the reader to understand the effect of each rule and each variation in order to fully capture the remainder of the thesis.

The remainder of the thesis is organized as follows. After mastering the rules of blackjack in chapter 2, the 3rd chapter consists of a mathematical derivation of the first strategy, the basic strategy. An overview of the four most important strategies from existing literature are given in chapter 4. Chapter 5 is about computer simulation. In this chapter, the presented strategies are tested with a self-written simulation program. This program will test these strategies on six different sets of rules that are currently used around the world. The program performs two sorts of simulations. The first and most important aim of the simulations will be to simulate the player's long run expectation and thus answers the question whether or not the dealer can still be beaten. Secondly, the program will simulate a player playing at a casino for a certain time ranging from one night to playing one whole week. In order to come up with a density function of the player's capital after that specific time unit.

Chapter 2

"You have to learn the rules of the game.
And then you have to play better than anyone else."

Albert Einstein

The rules of blackjack

In order to fully understand the strategies presented in chapter 3 it is essential that the reader fully masters the rules of blackjack first. However, knowledge of the rules does not suffice; the reader should understand the consequences of the rules .

Whereas most casinos use the same basic rules, the rules in some casinos differ slightly. This thesis will first concentrate on the most common rules.

Number of players

The blackjack game consists of a dealer and from one to seven spots for players. A player is allowed to play at more than one spot simultaneously. If there are no other players, the player can even play all seven spots at the same time. The player left of the dealer receives his cards first, and this spot is called "First base". The player on the dealer's right receives cards last and is called "Final base".

The pack

Originally the game was played using one ordinary 52-card deck. However when the first winning strategies were introduced, the casinos increased the number of decks to make card counting more difficult. The casinos in Las Vegas, Nevada, United States nowadays use 1 to 8 decks. In chapter 4 it will be seen that increasing the number of decks cuts the player's advantage slightly.

Some casinos like the Holland Casino in the Netherlands use card shuffling machines. Shuffling the cards after each round of play, thus making each hand an independent event. The cards now don't have any "memory", which makes card counting useless.

Numerical value of the cards

The value of an ace can be either 1 or 11, this is up to the player. The numerical value of a face card is 10, and the numerical value of all other cards are just their face values. Because the value of an ace can differ, I will distinguish between two kinds of hands.

1. Soft hand: this hand contains an ace and that ace can be counted as 11 without causing the total to exceed 21.
2. Hard hand: all other hands.

The distinction between hard and soft hands is important, because the strategy for a certain total of a hard hand can differ from the same total holding a soft hand. This will be further shown in chapter 3.

Betting

The players must make their bets before any cards are shuffled, except for 'insurance' (discussed later). The bet can range between the minimum and maximum bet as defined by the casino.

Objective of the player

Each player will try to obtain a total value of the hand that is greater than the total of the dealer, but does not exceed 21.

The deal

At the beginning of each round the dealer first "burns" a card (placed face up on the bottom of the deck). The card may or may not be shown; in this thesis I assume that the card is shown. Next the dealer deals two cards to himself, one card face up and one card face down. Consequently the dealer deals two cards to each of the players, both face up and are called his "hole cards".

If the first two cards dealt to the dealer or player consist of an ace and a 10-value card, it is called a "blackjack". If the player has a blackjack and the dealer doesn't, the player receives 1.5 times his bet above his initial bet. If the dealer has a blackjack and the player doesn't, the player loses his bet. If both the player and the dealer have a blackjack, no money is exchanged.

The draw starts at the player most left of the dealer and then proceeds clockwise. A player looks at his hole cards and has two options. Either the player can "stand" (draw no additional cards) or the player can choose to request extra cards from the dealer, which are also dealt face up.

If the player "busts" (goes over 21), he immediately loses his bet. After all players have drawn additional cards, the dealer turns up his face down card. If his total is 16 or less, he must draw additional cards until his total is 17 or more, at which point he must stand. If the dealer receives an ace and when counting this ace as 11 will bring his total to 17 or more without exceeding 21, then he must count the ace as 11 and stand.

The settlement

If the player's total does not exceed 21 and the dealer busts, the player wins his bet. If neither the player nor dealer busts, then the one with the higher total hand value wins the bet. If the dealer and the player have the same total hand value, no money is exchanged.

Splitting pairs

If the two hole cards are numerically identical, they are called a pair. The player may choose to treat them as his initial cards in two separate hands. This is known as splitting a pair. The original bet goes to one of the split cards and the same amount should be bet on the other card. After splitting the player directly receives an extra open card on each of the split card. Both hands will be played, one at a time, as though they were ordinary hands, with the following exceptions. In the case where the player splits two aces, the player receives only one more card on each ace. Furthermore, if a 10-value card is dealt on one of the split aces it does not count as blackjack, but only as ordinary 21. Similarly, if the player splits a 10-value pair and then draws an ace on one of the split 10-value card it also just counts as an ordinary 21. Lastly, if the player splits a pair and receives a third card of the same value, he is not permitted to split again.

Doubling down

After looking at the hole cards, a player may choose to "double down" (double his initial bet). When a player chooses to double down, he may only draw *one* more card. A player who splits any pair, except aces, may, after receiving the additional cards from splitting, double down on one or both of his split hands.

Insurance

If the dealer's up card is an ace and the player does not have a blackjack, then the player is permitted to make a side bet of at most half his original bet. After the player has decided to do this, the dealer checks his face down card. If the dealer has a blackjack, the side bet wins twice its amount. If the dealer does not have a blackjack, the side bet is lost and the play continues. Thus the side bet could be seen as an insurance against a blackjack of the dealer.

Chapter 3

"All men can see these tactics whereby I counter, but what none can see is that strategy out of which victory is evolved."

Sun Tzu

The basic strategy

The idea of the perfect strategy to win large amounts of money began with a Las Vegas legend. In the early 1950's there was a blackjack player known as "Greasy John". He got his nickname from never playing without a basket of fried chicken next to him. This made him so repulsive that no other players could stand sharing the blackjack table with him. He played for hours on end, just him at all seven player spots against the dealer. According to the legend he won night after night, so clearly he had a system, but nobody could figure out what it was. Before he was able to tell anyone his system, he suffered a massive heart attack and he took his secret to the grave. Whoever could figure out his strategy would have the key to enormous wealth.

3.1 The derivation of the basic strategy

The basic strategy was first described by Baldwin et al. [1]. The strategy is based on the hole cards (up cards of the player) and the up card of the dealer. Based on these cards a strategy was calculated. For all possible combinations the best option was computed; either to split, double down, draw or stand. In the this section the derivation of the basic strategy is given.

3.1.1 The player's decisions

As described in chapter 2, the game begins with certain preliminaries. In the basic strategy it is assumed that the game is played with 1 deck of cards. When the game starts, first the players are seated, the dealer shuffles the deck and burns the first card. After this, the players have to place their bets and the dealer gives two cards to each player and himself.

At this point the player has to make some decisions. The principal choices are whether to split, to double down or not, and whether to stand or draw. These decisions depend on the cards of the player and on the up card of the dealer. Thus all cards displayed in previous rounds are ignored. This makes that the basic strategy is the best possible way to play the game, with only the information about these three cards.

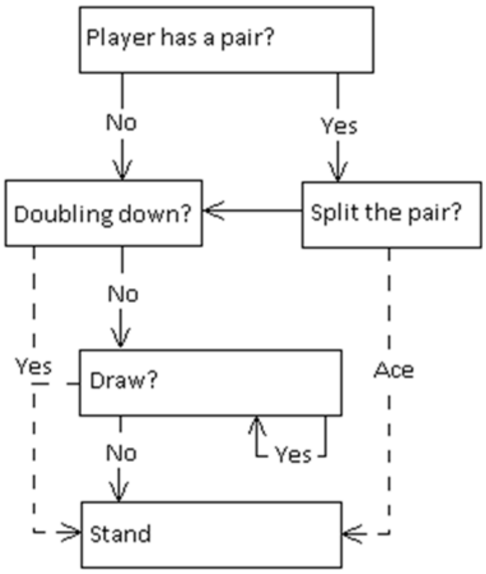


Figure 3.1 – The player's key decisions

The player's key decisions and the order in which he makes them are illustrated in Figure 3.1. Remember if the player decides to split his pair of aces he only receives one more card on each of the aces and is then forced to stand on both hands. Also when the player decides to double down he can at most draw one additional card. The player also has the possibility to buy insurance when the dealer's up card is an ace or a ten. The possibility to buy insurance is at the beginning of the game when the initial cards have just been dealt.

3.1.2 Definitions

In the basic strategy the approach is to determine the gaming decision that will optimize the expectation that the player will win the game at each stage of the game. The following definitions will be needed in this analysis.

1. W is a random value representing the player's earnings.
2. x represents the player's total value at the time of his decision, $x \in \{1,2,3, \dots ,20\}$. The total does not exceed 20 because if the player's total value is 21 it is trivial to stand. In any other case ($x > 21$) the player busts. In case that the player has a soft hand such that the player's total value has two possible values not exceeding 20, x is defined as the larger total.
3. D represents the numerical value of the dealer's up card, $D \in \{2,3, . . ,10, (1,11)\}$.
4. $M(D)$ represents the minimum standing number for *hard* hands. A player holding $x \geq M$ should stand and a player holding $x < M$ should draw.
5. $M^*(D)$ is defined in the same way, for $soft$ hands.
6. T represents the dealer's final total value.
7. J represents the total value obtained by a player after drawing one card to whatever he currently holds.
8. $E_S^x[W] = E(W|Player\ stands\ with\ x)$ represents the expectation of the player who stands on a total of x.
9. $E_D^x[W] = E(W|Player\ draws\ one\ card\ to\ x)$ represents the expectation of the player who draws one more card on a total of x.

Both $E_S^x[W]$ and $E_D^x[W]$ are conditional expectations, conditioned on a player having total x and using a particular strategy. The S and D in respectively $E_S^x[W]$ and $E_D^x[W]$ stands for "Stand" and "Draw" (thus the D is unrelated to the dealer's up card here). In the situation where $x < 12$ it is trivial that the player always draws as there is no chance of busting. At this time theorem 3.1 is introduced.

Theorem 3.1

For $x \geq 12$, $E_D^x[W] - E_S^x[W]$ is a non-increasing function of x.

From this it follows that for $x \geq 12$ the player should draw if $E_D^x[W] - E_S^x[W] > 0$ and stand if $E_D^x[W] - E_S^x[W] < 0$ the player should stand. In the case that the expectations are equal, the player's decision doesn't matter.

Proof 3.1

For a total x_1 where $x_1 \geq 12$.

It follows that $P(J|x_1) = P(J|x_1 + 1)$ for $J = \{(x_1 + 2), \dots , 21\}$

But $P(J > 21|x_1 + 1) > P(J > 21|x_1)$

In other words the probability of ending up with a total of J is equal for x_1 and $(x_1 + 1)$, but the probability of busting is larger for $(x_1 + 1)$. So in this way, a situation is created where there are more ways of losing without creating new ways of winning.

It follows from induction that this is true for all $x \geq 12$. And therefore $E_D^x[W] - E_S^x[W]$ is a non-increasing function of x.

3.1.3 Deriving the decision equation

For simplicity, the assumption is made, without loss of generality, that the player's bet equals 1 unit. Considering the expectation of the player standing on a total of x gives:

$$E_S^x[W] = 1 \cdot P(T > 21) + 1 \cdot P(T < x) - 1 \cdot P(x < T \leq 21)$$
$$= P(T > 21) + P(T < x) - (1 - P(T \leq x \cup T > 21))$$
$$= P(T > 21) + P(T < x) - \left(1 - \left(P(T \leq x) + P(T > 21)\right)\right)$$
$$= P(T > 21) + P(T < x) - \left(1 - P(T \leq x) - P(T > 21)\right)$$
$$= P(T > 21) + P(T < x) - 1 + P(T \leq x) + P(T > 21)$$

This can be rewritten to get the following expression,

$$E_S^x[W] = 2P(T > 21) + 2P(T < x) - 1 + P(T = x) \tag{1}$$

Similarly, it is possible to find an expression for the expectation of the player drawing one card on a total of x. For this calculation the random value J is needed. So,

$$E_D^x[W] = E_D^x[W|J < 17] \cdot P(J < 17) + \sum_{j=17}^{21} E_D^x[W|J = j] \cdot P(J = j) + E_D^x[W|J > 21] \cdot P(J > 21)$$

These are all conditional expectations that are straightforward to compute.

$$E_D^x[W|J < 17] = 1 \cdot P(T > 21) - 1 \cdot P(T \leq 21)$$
$$E_D^x[W|J = j] = 1 \cdot P(T > 21) + 1 \cdot P(T < j) - 1 \cdot P(j < T \leq 21)$$
$$E_D^x[W|J > 21] = -1$$

Combining these terms, gives

$$E_D^x[W] = P(J < 17)\left(1 \cdot P(T > 21) - 1 \cdot P(T \leq 21)\right)$$
$$+ \sum_{j=17}^{21} P(J = j)\left(1 \cdot P(T > 21) + 1 \cdot P(T < j) - 1 \cdot P(j < T \leq 21)\right) - P(J > 21)$$
$$= P(J < 17)\left(P(T > 21) - \left(1 - P(T > 21)\right)\right)$$
$$+ \sum_{j=17}^{21} P(J = j)\left(P(T > 21) + P(T < j) - P(j < T \leq 21)\right) - P(J > 21)$$

This yield to the following general expression,

$$E_D^x[W] = P(J < 17)(2P(T > 21) - 1) - P(J > 21)$$
$$+ \sum_{j=17}^{21} P(J = j)\left(P(T > 21) + P(T < j) - P(j < T \leq 21)\right) \tag{2}$$

Now combining (1) and (2) and performing some straightforward algebraic manipulation and using the fact that J and T are independent, gives the most general form of the decision equation, which is equal to:

$$E_D^x[W] - E_S^x[W] = -2P(T < x) - P(T = x) - 2P(T > 21)P(J > 21)$$
$$+ \sum_{t=17}^{21} P(T = t)\left(2P(t < J \leq 21) + P(J = t)\right) \tag{3}$$

To check if this equation holds, let's consider the two baseline cases: $x(hard) < 12$ and $x(soft) < 17$, for which it's obvious that the player should draw. We verify that the decision equation (3) agrees with these trivial decisions. Since the dealer never stands on totals smaller than 17, the first two terms are zero. Also because J can never exceed 21, the term $P(J > 21)$ is also equal to zero. This leaves only the sum of probabilities, which must all be greater than zero. Hence, $E_D^x[W] - E_S^x[W] \geq 0$, thus the decision is to draw, which corresponds with the obvious decision. Therefore it also follows that $M(D) > 11$ and $M^*(D) > 16$ for all values of D.

Now consider the evaluation of the decision equation when $12 \leq x(hard) \leq 16$ where the decision to draw is not trivial. The first two terms in the decision equation are zero for the same reason as above. This leaves the following decision equation:

$$E_D^x[W] - E_S^x[W] = -2P(T > 21)P(J > 21) + \sum_{j=17}^{21} P(T = t)[2P(t < J \leq 21) + P(J = t)]$$

Recall that under the basic strategy it is assumed that each card has an equal chance of being drawn by the player. Thus the probability distribution of $J - x$, the single card drawn by the player, is given by $P(J - x = 10) = \frac{4}{13}$ and $P(J - x = i) = \frac{1}{13}$ for $i = \{2,3,..,9,(1,11)\}$. With this assumption,

$$P(J > 21) = \frac{1}{13}(x - 8) \quad for\, x(hard) > 12$$

$$P(t < J \leq 21) = \frac{1}{13}(21 - t) \quad for\, 17 \leq t \leq 21$$

$$P(J = t) = \frac{1}{13} \quad for\, 17 \leq t \leq 21$$

So, the decision equation becomes (still for $12 \leq x(hard) \leq 16$),

$$E_D^x[W] - E_S^x[W] = -\frac{2}{13}(x - 8)P(T > 21) + \sum_{j=17}^{21} P(T = t)\left[\frac{43}{13} - \frac{2t}{13}\right] \tag{4}$$

It's easier now to just set $E_D^x[W] - E_S^x[W] = 0$, rather than to compute $E_D^x[W] - E_S^x[W]$for all values of $12 \leq x(hard) \leq 16$. Since the function decreases linearly with$x$, it's possible to obtain a single solution $x = x_0$:

$$x_0 = 8 + \frac{\sum_{t=17}^{21}\left(\frac{43}{2} - t\right)P(T = t)}{P(T > 21)} \tag{5}$$

Now if $x_0 < 12$ than $M(D) = 12$; if $x_0 > 16$ than $M(D) > 16$; and if $12 \leq x_0 \leq 16$ than $M(D) = [x_0] + 1$ where $[z]$ is defined as the largest integer not greater than z. As expected, the greater the probability that the dealer will bust, the lower the player's minimal standing number. Calculating x_0 will be done later on in this chapter.

First consider the case where $x(hard) = 17$. Only the first term in the decision equation equals zero in this case. Thus the decision equation can be rewritten to:

$$E_D^{17}[W] - E_S^{17}[W] = -P(T = 17) - 2P(T > 21)P(J > 21)$$

$$+P(T = 17)\big(2P(17 < J \leq 21) + P(J = 17)\big) + \sum_{t=18}^{21} P(T = t)\big(2P(t < J \leq 21) + P(J = t)\big)$$

Now using that:

$$P(J > 21) = \frac{9}{13}\ ,\ P(17 < J \leq 21) = \frac{4}{13} \text{and } P(J = 17) = 0$$

Gives the following decision equation for $x(hard) = 17$:

$$E_D^{17}[W] - E_S^{17}[W] = -\frac{18}{13}P(T > 21) - \frac{5}{13}P(T = 17) + \sum_{j=18}^{21} P(T = t)\left[\frac{43}{13} - \frac{2t}{13}\right] \tag{6}$$

Later in this chapter, we will show that $E_D^{17}[W] - E_S^{17}[W] < 0$ for all D, therefore $M(D) \leq 17$. It is unnecessary to evaluate the decision equation further for unique hands.

The only remaining use of the decision equation is now for the situation of $x(soft) = 17$. In this case the terms $P(T < x)\, and\, P(J > 21)$ are equal to zero, this gives the following decision equation:

$$E_D^{17}[W] - E_S^{17}[W] = -P(T = 17) + \big(2P(17 < J \leq 21) + P(J = 17)\big)P(T = 17) + \sum_{t=18}^{21} P(T = t)\big(2P(t < J \leq 21) + P(J = t)\big)$$

Now using that:

$$P(J = 17) = \tfrac{4}{13} \text{ and } P(17 < J \leq 21) = \tfrac{4}{13}$$

This gives the following decision equation for $x(soft) = 17$:

$$E_D^{17}[W] - E_S^{17}[W] = -\frac{1}{13}P(T = 17) + \sum_{j=18}^{21} P(T = t)\left[\frac{43}{13} - \frac{2t}{13}\right] \tag{7}$$

Calculations later in this chapter show that the $E_D^{17}[W] - E_S^{17}[W] > 0$ for all values of D (dealer's up card), proving that $M^*(D) > 17$.

3.1.4 Evaluation of the dealer's probabilities

In order to be able to calculate x_0, the expression for $x(hard) = 17$ and for the expression $x(soft) = 17$, it is needed to calculate the dealer's probabilities. Notice that the dealer's probabilities should be conditioned on the knowledge of the dealer's up card. In fact, the decision we make must be based solely on the value of this card, D.

Thus for $T = \{17,18,19,20,21\}$, the conditional probability $P(T = t|D)$ should be computed for each possible value of D. For example, $P(T = 17|D = 10)$ is the probability that the dealer ends up with a total of 17 given that the dealer's up card is 10. To calculate this probability exactly it is needed to evaluate all possible ways to get from 10 to 17. All these different options and corresponding probabilities are displayed in table 3.1.

Cards dealt										# of cards	# of possibilities	Probability
A	2	3	4	5	6	7	8	9	10			
						1				1	1	$\frac{1}{13} = 0{,}0769231$
1					1					2	1	$\frac{4 \cdot 4}{52 \cdot 51} = 0{,}00603318$
	1			1						2	2	$\frac{4 \cdot 4 \cdot 2}{52 \cdot 51} = 0{,}01206637$
		1	1							2	2	$\frac{4 \cdot 4 \cdot 2}{52 \cdot 51} = 0{,}01206637$
1	1		1							3	4	$\frac{4 \cdot 4 \cdot 4 \cdot 4}{52 \cdot 51 \cdot 50} = 0{,}0019306$
	2	1								3	3	$\frac{4 \cdot 3 \cdot 4 \cdot 3}{52 \cdot 51 \cdot 50} = 0{,}0010860$
1		2								3	2	$\frac{4 \cdot 4 \cdot 3 \cdot 2}{52 \cdot 51 \cdot 50} = 0{,}0007240$
2	1	1								4	6	$\frac{4 \cdot 3 \cdot 4 \cdot 4 \cdot 6}{52 \cdot 51 \cdot 50 \cdot 49} = 0{,}0001773$
1	3									4	3	$\frac{4 \cdot 4 \cdot 3 \cdot 2 \cdot 3}{52 \cdot 51 \cdot 50 \cdot 49} = 0{,}0000443$
3	2									5	4	$\frac{4 \cdot 3 \cdot 2 \cdot 4 \cdot 3 \cdot 4}{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48} = 0{,}000004$
Total												$P(T = 17\|D = 10) = 0{,}1144$

Table 3.1 – The evaluation of the conditional probability of dealer ending up with 17, given that the up card is 10.

An explanation of table 3.1 might be needed:

- Row 1: $P(7) = \frac{4}{52} = \frac{1}{13}$
  - This is the probability of receiving a 7.
- Row 2: $P(6, ace) = \frac{4}{52} \cdot \frac{4}{51}$

- This is the probability of receiving a 6 and then receiving an ace, and only in this order, because when the ace was dealt first the dealer would have a blackjack and thus stands.
- Row 5: $P(2, ace, 4) = \frac{4}{52} \cdot \frac{4}{51} \cdot \frac{4}{50} \cdot 4$
  - These cards can be received in 4 different ways; $(2, ace, 4)$, $(2, 4, ace)$, $(4, ace, 2)$ and $(4, 2, ace)$. Again if the ace were to be dealt first the dealer would obtain a blackjack thus stands.

The probabilities for the other values of T when $D = 10$ were computed in the same way and are displayed in table 3.2.

D	$T = 17$	$T = 18$	$T = 19$	$T = 20$	$T = 21$	Blackjack	$T > 21$
10	0.114418	0.112879	0.114662	0.328879	0.036466	0.078431	0.214264

Table 3.2 – The conditional probabilities $P(T = t|D = 10)$.

The conditional probability of a blackjack is the probability that the dealer has an ace as his hole card, which is $\frac{4}{52}$. The conditional probability of the dealer busting is given by:

$$1 - \sum_{t=17}^{21} P(T = t|D = 10).$$

This evaluation was done for all values of D, and the conditional probabilities for all values of D are displayed in table 3.3.

D	$T = 17$	$T = 18$	$T = 19$	$T = 20$	$T = 21$	Blackjack	$T > 21$
2	.138976	.131762	.131815	.123948	.120526	0	.352973
3	.130313	.130946	.123761	.123345	.116047	0	.375588
4	.130973	.114163	.120679	.116286	.115096	0	.402803
5	.119687	.123483	.116909	.104694	.107321	0	.428905
6	.166948	.106454	.107192	.100705	.097879	0	.420823
7	.372345	.138583	.077334	.078897	.072987	0	.259854
8	.130857	.362989	.129445	.068290	.069791	0	.238627
9	.121886	.103921	.357391	.122250	.061109	0	.233442
10	.114418	.112879	.114662	.328879	.036466	.078431	.214264
A	.126128	.131003	.129486	.131553	.051565	.313726	.116540

Table 3.3 – Dealer's probabilities given the dealer's up card, D.

However, when the dealer's up card is a 10 or an ace, he looks immediately at his down card. If the dealer holds a blackjack, he announces this at once and proceeds with the settlement. This means that when the dealer holds a blackjack, the player is not able to either draw, split or double down. Therefore the dealer's probabilities for when the up card D is 10 or ace are given by $P(T = t|T \neq Blackjack)$, the conditional probability that the dealer ends up with t given that the player does not have a blackjack. These probabilities are given in table 3.4 below.

D	$T = 17$	$T = 18$	$T = 19$	$T = 20$	$T = 21$	Blackjack	$T > 21$
10	.124156	.122486	.124421	.356869	.039570	0	.232499
A	.183786	.190890	.188680	.191692	.075137	0	.169815

Table 3.4 – Dealer's probabilities given that the dealer does not have a blackjack.

These probabilities are calculated in the same way as before, only it's given that the first card the dealer receives for the up card D=10 is not an ace and not a 10 for D=ace.

3.1.5 Calculating the minimum standing numbers for hard totals

As the dealer's probabilities are known, the valuesx_0 for all different D's can now be calculated using equation (5). These values are displayed in table 3.5.

D	2	3	4	5	6	7	8	9	10	A
x_0	12,709	12,252	11,780	11,436	11,783	17,655	17,723	16,651	15,972	21,497
M(D)	13	13	12	12	12	≥ 17	≥ 17	≥ 17	*	≥ 17

Table 3.5 – Calculations of M(D) for 12≤ x(hard) ≤ 16

For D=10, the value of x_0 is very close to 16, thus an extra evaluation is needed. In equation (5) it was assumed that all cards have an equal probability of being drawn from the deck. But since the dealer has already drawn a 10-counting card from the deck, the probability of drawing another 10-counting card is smaller than assumed. Adjusting this probability yields a more realistic result. In that case the value of $x_0$is bigger than 16, therefore $M(D) \geq 17$.

Thus for the dealer's up card $D = (7{,}8{,}9{,}10, ace)$ a further evaluation is needed in order to find the minimal standing numbers. The calculations for$x(hard) = 17$ of the decision equation (6) are given in table 3.6.

D	$E_D^{17}[W] - E_S^{17}[W]$	M(D)
7	-0,3748206	17
8	-0,1143663	17
9	-0,1437788	17
10	-0,1704675	17
A	-0,0804428	17

Table 3.6 – Calculations of M(D) for x(hard) = 17

Since the expectation of a player who draws one more card is smaller than the expectation of a player who stands on a total $x(hard) = 17$ for all values of D, $M(D) = 17$. An overview of the minimal standing numbers strategy for hard total is given in table 3.7.

x	D									
	2	3	4	5	6	7	8	9	10	A
17						■	■	■	■	■
16										
15										
14										
13	■	■								
12			■	■	■					

■ Standing numbers

Table 3.7 – Standing numbers for hard totals

3.1.6 Calculating the minimum standing numbers for soft totals

The decision equation for $x(soft) = 17$ is given by equation (7), and can be calculated using the dealer's probabilities. The results for all values of D are given in table 3.8.

D	$E_D^{17}[W] - E_S^{17}[W]$	M(D)
2	0,148831	≥ 18
3	0,145477	≥ 18
4	0,133501	≥ 18
5	0,134665	≥ 18
6	0,116476	≥ 18
7	0,099545	≥ 18
8	0,256304	≥ 18
9	0,216952	≥ 18
10	0,189656	≥ 18
A	0,211235	≥ 18

Table 3.8 – Calculations of M*(D) for x(soft) = 17

As for all values of D, the expectation of a player who draws one more card is larger than the expectation of a player who stands on a total $x(soft) = 17$, therefore$M^*(D) \geq 18$.

An analysis of the minimal standing numbers for soft hands requires to compare the mathematical expectation of two players using a slightly different strategy. Both players will use the same strategy, except when they have a soft total. In case of a soft total the first player will stand and the second player continuous to draw a card until his total is less than M(D). This comparison is equal to comparing $E_S^x[W]$,the expectation of the player standing on total x, with $E_{D^*}^k[W]$,the expectation of a player with a soft total x who draws one more card and follows the M(D) policy for drawing or standing. This requires a new definition:

10. $E_{D^*}^{x}[W]$ =E(W|Player with soft total x who draws one card and then applies policy M(D)). Represents the expectation of the player with a soft total x who draws one card and then draws or stands according to M(D).
11. $P(H = h|H_p = h_p)$ is the conditional probability that the player obtains a final total of h $\{h \geq M(D)\}$ given that the player has a partial total of h_p $\{h_p < M(D)\}$.

Using the fact that $E_S^x[W] = -1$ for $x > 21$, $E_{D^*}^{x}[W]$ is given by:

$$E_{D^*}^{x}[W] = \sum_{j \geq M(D)} P(J = j)\, E_S^j[W] + \sum_{j < M(D)} P(J = j) \sum_{h \geq M(D)} P(H = h|H_p = j) E_S^h[W]$$

One might wonder why x doesn't appear on the right hand side of the equation, but remember that its value affects $P(J = j)$. The two terms in the equation for $E_{D^*}^{k}[W]$ might need some intuitive explanation about what it represents. The equation consists of two parts, the first part is where J is equal or greater than M(D) and therefore stands, the second part is where J is smaller than M(D) and draws another card. The second part of the equation is then multiplied by the conditional probability that the player obtains a total greater or equal than M(D) in which case he will stand.

The results of the calculations $E_{D^*}^{x}[W] - E_S^x[W]$ for $x(soft) = 18$ are given in table 3.9.

D	2	3	4	5	6	7	8	9	10	A
$\boldsymbol{E_{D^*}^{18}[W] - E_S^{18}[W]}$	< 0	< 0	< 0	< 0	< 0	< 0	< 0	> 0	> 0	< 0
$\boldsymbol{M^*(D)}$	18	18	18	18	18	18	18	≥ 19	≥ 19	18

Table 3.9 – Calculations of M*(D) forx(soft) = 18

We still need two unknown minimal standing numbers for soft totals, namely where $D = 9$ or $D = 10$. This requires the calculations $E_{D^*}^{x}[W] - E_S^x[W]$ for $x(soft) = 19$. These results are displayed in table 3.10.

D	$\boldsymbol{E_{D^*}^{19}[W] - E_S^{19}[W]}$	$\boldsymbol{M^*(D)}$
9	< 0	19
10	< 0	19

Table 3.9 – Calculations of M*(D) forx(soft) = 19

Since $E_S^{19}[W] > E_{D^*}^{19}[W]$ for all values of D all the minimal standing numbers for hard and soft totals are known. An overview of the strategy for soft hands is given in table 3.11.

x	D									
	2	3	4	5	6	7	8	9	10	A
19								■	■	
18	■	■	■	■	■	■	■			■

■ Standing numbers

Table 3.11 – Standing numbers for soft totals

3.1.7 Doubling down

Remember that if the players chooses to double down, he then doubles his initial bet and only receives at most one more card before he must stand. Thus the expectation of a player who chooses to double down on a total of x, is equal to $2 \cdot E_D^x[W]$. In order to determine if this is a good decision it's needed to compute the expectation of a player with a total x who follows the drawing strategy given by $M(D)$ and $M^*(D)$. Again there is a need for new definitions:

12. $E^x_{M,M^*}[W]$= E(W| Player with a total x who follows the drawing strategy given by $M(D)$ and $M^*(D)$). Represents the expectation of the player with a total of x and then follows the drawing strategy given by $M(D)$ and $M^*(D)$.
13. $X = X(D)$is the set of values of x for which the player should double down, given the dealer's up card D.

Thus the player will choose to double down if $2 \cdot E^x_D[W] - E^x_{M,M^*}[W] > 0$. It is easy to see that $E^x_{M,M^*}[W] \geq E^x_D[W]$, therefore:

$$2 \cdot E^x_D[W] - E^x_{M,M^*}[W] \leq 2 \cdot E^x_D[W] - E^x_D[W] = E^x_D[W]$$

It follows from this equation that if $E^x_D[W] < 0$ then $2 \cdot E^x_D[W] - E^x_{M,M^*}[W] < 0$. Since $E^x_D[W]$ is easy to compute, it immediately shows that doubling down is a poor strategy for:

- $x(hard) > 11$ for all values of D.
- $x(hard) < 8$for all values of D.
- $x(soft) < 17$ for $D \in \{6,7,8,9,10,(1,11)\}$.

However, this leaves many other possible combinations for x and D that needs to be computed separately. In order to compute $2 \cdot E^x_D[W] - E^x_{M,M^*}[W]$ it is first needed to come up with an equation for $E^x_{M,M^*}[W]$, this equation is given by:

$$E^x_{M,M^*}[W] = \begin{cases} E^x_S[W] & if\ x(hard) \geq M(D) \\ E^x_S[W] & if\ x(soft) \geq M^*(D) \\ \sum_{h \geq M(D)} P(H = h | H_p = x) E^h_S[W] & otherwise \end{cases}$$

The results of computing $2 \cdot E^x_D[W] - E^x_{M,M^*}[W]$ for all possible combinations of x and D are given in table 3.12 for hard hands and in table 3.13 for soft hands.

x	D									
	2	3	4	5	6	7	8	9	10	A
11	■	■	■	■	■	■	■	■	■	■
10	■	■	■	■	■	■	■	■		
9	■	■	■	■	■					

■	Double down
	Do not double down

Table 3.12 – Doubling down strategy for hard totals

x	D									
	2	3	4	5	6	7	8	9	10	A
A,7		■	■	■	■					
A,6	■	■	■	■	■					
A,5			■	■	■					
A,4			■	■	■					
A,3			■	■	■					
A,2			■	■	■					
A,A				*	*					

■	Double down
	Do not double down
*	Double down if pair splitting is not permitted

Table 3.13 – Doubling down strategy for soft totals

3.1.8 Splitting pairs

The decision to split a pair is made at the beginning of the game. When a player decides to split a pair the original bet goes to one of the split cards, and an equal amount should be bet on the other card. After this the game continues as normal, except for splitting an ace in which case the player only receives one more card on both aces and stands. Some new definitions are needed again:

14. When a player holds a pair, the value of the two cards is denoted by y.
15. $E^y_{no-split}[W]$ = E(W| Player with a total $2y$who does not split). Represents the expectation of the player with a total $2y$that does not split his pair.

16. $E^{y}_{split}[W]$ = E(W| Player with a total $2y$ splits his pair). Represents the expectation of the player with a total $2y$who splits his pair.
17. $Y = Y(D)$ as the set of values of y for which the player should split a pair of y's, given the dealer's up card D.

The player should split his pair of y's if and only if$E^{y}_{split}[W] > E^{y}_{no-split}[W]$. These mathematical expectations are given by:

$$E^{y}_{no-split}[W] = \begin{cases} 2 \cdot E^{2y}_{M,M^*}[W] & 2y \in X \\ E^{2y}_{M,M^*}[W] & 2y \notin X \end{cases}$$

$$\frac{1}{2}E^{y}_{split}[W] = \sum_{j \in X} P(J=j) 2 \cdot E^{j}_{M,M^*}[W] + \sum_{j \notin X} P(J=j) E^{j}_{M,M^*}[W]$$

For all combinations of y and D the equation $E^{y}_{split}[W] - E^{y}_{no-split}[W]$ should be computed. In the special where y is an ace, the player is only allowed to draw one more card and then stands. Thus the expectation for splitting an ace is given by:

$$\frac{1}{2}E^{ace}_{split}[W] = \sum_{\forall j} P(J=j)\, E^{j}_{S}[W]$$

The results for pair splitting are displayed in table 3.14.

x	D									
	2	3	4	5	6	7	8	9	10	A
A,A	■	■	■	■	■	■	■	■	■	■
10,10	□	□	□	□	□	□	□	□	□	□
9,9	■	■	■	■	■	□	■	■	□	□
8,8	■	■	■	■	■	■	■	■	■	■
7,7	■	■	■	■	■	■	■	□	□	□
6,6	■	■	■	■	■	■	□	□	□	□
5,5	□	□	□	□	□	□	□	□	□	□
4,4	□	□	□	■	□	□	□	□	□	□
3,3	■	■	■	■	■	■	□	□	□	□
2,2	■	■	■	■	■	■	□	□	□	□

■ Split pair
□ Do not split pair

Table 3.14 – Pair splitting strategy

3.1.9 Insurance

If the dealer's up card is an ace, an additional wager is allowed before the draw. This additional wager is at most half of the initial bet. When the player decides to do this, the dealer checks his hole card. If the dealer has a blackjack, the side bet wins twice its amount. If the dealer does not have a blackjack the side bet is lost and the game continues as normal. To decide if it is a good strategy for a player do buy this insurance it is needed to calculate the probability that the dealer has a blackjack given that the dealer's up card is an ace. This probability was already calculated in the section "Evaluation of the dealer's probabilities" and equals 31,4%. As the side bet only wins twice its amount, the probability should be higher than 50% to be profitable. Thus buying insurance is not a good decision for the basic strategy.

3.1.10 Overview of the basic strategy

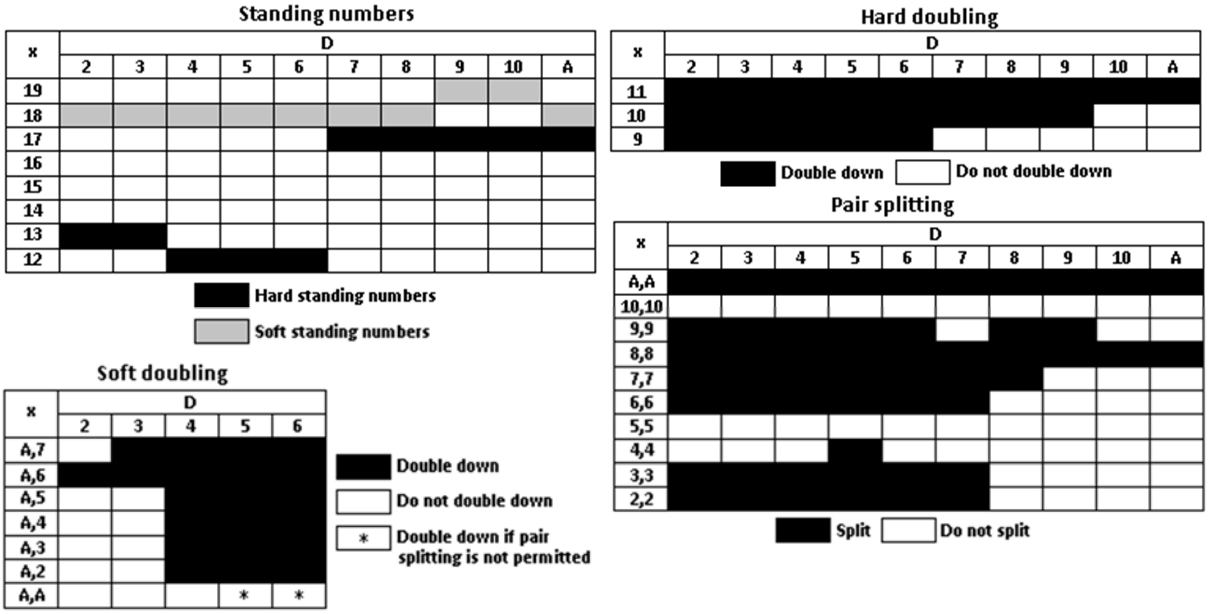


Table 3.14 – An overview of the basic strategy

3.2 Expectation of the basic strategy

In order to calculate the expectation of a player using the basic strategy one more definition is needed:

18. $E[W_D]$ = E(W| The amount a player wins given the dealer's up card D).

Now the mathematical expectation of the player is given by:

$$E[W] = \frac{1}{13}\sum_{D\neq 10} E[W_D] + \frac{4}{13}E[W_{10}]$$

In the two special cases $D = 10$ and $D = ace$, the expectation must be calculated under two conditions, when the dealer has a blackjack and when the dealer does not have a blackjack. To obtain $E[W_D]$ in these cases, the conditional expectation for both situations are multiplied by the probability of the conditions.

In order to obtain $E[W_D]$ the probabilities for various hands formed by the players two hole cards need to be calculated. It's assumed that these hole cards are drawn from a complete deck except for one D-counting card.

For the case that D=10 or D=ace and the dealer has a blackjack, $E[W_D]$ is given by:

$$E[W_D] = -1 \cdot \big(1 - P(player's\ hole\ cards\ form\ a\ blackjack)\big)$$

For all other the cases, $E[W_D]$ is given by:

$$\frac{3}{2}P(player's\ hole\ cards\ form\ a\ blackjack)$$
$$+\sum_{y\in Y} P(player's\ hole\ cards\ are\ a\ pair\ of\ y's) \cdot E_{split}^{y}$$
$$+\sum_{j\in X} P(player's\ hole\ cards\ total\ j) \cdot 2 \cdot E_D^j$$
$$+\sum_{j\notin X} P(player's\ hole\ cards\ total\ j) \cdot E_{M,M^*}^j$$

In the last two terms it is understood that the hole cards of the player do not form a natural nor a pair of y's with $y \in Y$. The results of computing $E[W_D]$ for all different values for the dealer's up card are given in table 3.15.

D	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**A**
$\boldsymbol{E[W_D]}$	.090	.123	.167	.218	.230	.148	.056	-.043	-.176	-.363

Table 3.15 – The player's expectation given the dealer's up card D

This gives an overall expectation of -0.006 (-0.6%). The player´s disadvantage comes from the fact that if both the player and the dealer busts, the dealer wins.

Chapter 4

"The only way to make money at a casino game,
is to steal chips when the dealer is not looking."

Albert Einstein

Winning blackjack strategies

Since the publication of the basic strategy by Baldwin et al. appeared in 1956 a lot of research was performed in order to improve this strategy. The first successful attempt was by Allan Wilson of San Diego State University, who programmed a simulation of hundred thousand hands of blackjack to receive the most accurate strategy up to that date. However, this new improved strategy was still a break-even system and not a winning system for blackjack.

One person who was also particularly interested in the technical paper by Baldwin et al. was Edward Thorp, an American mathematics professor at the time. Like Allan Wilson, he also had access to a sophisticated computer. This computer enabled Thorp to dispense the approximations that were made in the original basic strategy. Which led to improved results, the player's expectation rose from -0.6% to +0.13%. Meaning that blackjack became a casino game where the player has an edge over the house if he sticks to the basic strategy! However, the edge remained very small. An example of the slight improvements that Edward Thorp made to the basic strategy was to double down on hard totals of 8 when the dealer's up card is 5 or 6, except when the player holds (6,2).

The winning strategies presented in this chapter depend largely on the fact that, as the composition of the deck changes during the play, also the advantage shifts between the player and the dealer. As successive rounds continue to be played from an increasingly depleted deck, the advantage shifts back and forth between the player and the dealer. The idea now is to make large bets when the advantage is in favour of the player and to make small (minimum) bets when dealer has the advantage. In the long run this will result in a considerable net profit, as the player wins most of his large favourable bets and loses most of his small bets.

4.1 Five count strategy

Baldwin et al. had already suggested that the development of winning strategies might be possible if the player could somehow keep track of the cards that were already dealt. However, it was Edward Thorp who used this suggestion and came up with the first winning strategy for blackjack, the five count strategy.

4.1.2 Influence of removing cards from the deck

Thorp used his high speed computer again to calculate the player's expectation for different kind of subsets of the deck. This made it possible to take into account the cards that became visible during the game. This is essential for determining a winning strategy. In table 4.1 the player´s expectation are given for different kind of subsets; this table is adopted from [8].

It can be seen from table 4.1 that the effect of removing all the fives from the deck is the greatest on the player's expectation. More precise the player's advantage over the house is equal to 3.6% when the deck is depleted of its fives. Why does removing all the fives from the deck increase the player's expectation? This can intuitively be explained by the fact that if the player receives one five as his initial two cards, the player is most likely to have an unfavourable total. An unfavourable total being a low total to stand on and from which the probability of busting is considerable, like a total of 15. One might wonder why the effect of the fives mostly influences the player and not the dealer. The reason for this is that the player is first to act. Thus when the player busts, the dealer automatically wins.

Description of the subset	Player's expectation (in %)	Description of the subset	Player's expectation (in %)
complete	0.13	$Q(10) = 12$	-1.85
$Q(A) = 0$	-2.42	$Q(10) = 20$	1.89[2.22]
$Q(2) = 0$	1.75	$Q(10) = 24$	3.51[4.24]
$Q(3) = 0$	2.14	$Q(10) = 28$	5.06[6.10]
$Q(4) = 0$	2.64	$Q(10) = 32$	6.48[7.75]
$Q(5) = 0$	3.58	$Q(10) = 36$	7.66[9.11]
$Q(6) = 0$	2.40	$Q(9) = Q(10) = 0$	9.92
$Q(7) = 0$	2.05	$Q(8) = \cdots = Q(10) = 0$	19.98
$Q(8) = 0$	0.43	$Q(5) = \cdots = Q(10) = 0$	78.14
$Q(9) = 0$	-0.41	½ deck	0.85[0.93]
$Q(10) = 0$	1.62	2 decks	-0.25
$Q(10) = 4$	-2.14	4 decks	-0.41
$Q(10) = 8$	-3.13	5000 decks	-0.58

$Q(X) = Y$ means that a particular subset was altered by changing the quantity Q of cards that have numerical value X, so that there are Y of such cards left. And the values in the [] brackets is the advantage of insurance.
For example, $Q(5) = 0$ means that only all the fives are removed from one deck.

Table 4.1 – Player's expectation for certain subsets

Another interesting fact from table 4.1 is the influence of removing the aces from the deck, it gives the dealer an advantage over the player of 2.42%. This can be explained by the fact that when in the remaining rounds no aces can appear; there will be no blackjacks, no soft hands, and no splitting aces (splitting aces is highly favourable to the player).

The odds of the dealer drawing all four fives at the beginning of a new shuffled deck is about once in 300.000 shuffled decks. Therefore some might wonder if the player's advantage of 3.58% also holds for a depleted deck that contains no fives but also misses other cards. Thorp proves [8] that this situation (assuming the deck contains enough cards for a next round of play) is mathematically identical to the subset of a complete deck except that the fives are removed.

Thorp used his high speed computer to compute a strategy for the situation when the deck is depleted of its fives, this strategy is displayed in table 4.2.

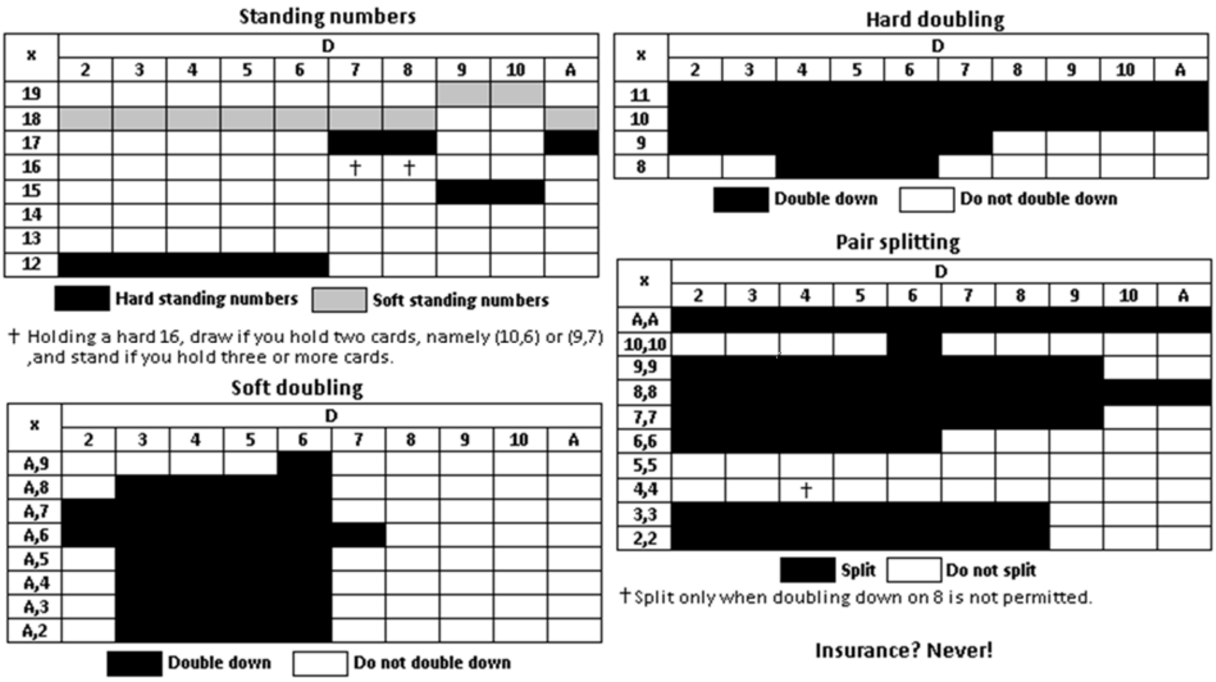


Table 4.2 – The five count strategy table

Table 4.2 is read in the same way as the basic strategy tables. Thorp also calculated what would happen if the player only applies the standing numbers of table 4.2 and neglects the changes in splitting and doubling down and continuous to use the basic strategy for these options. In this case the player's expectation only drops from 3,6% to an 3,4% edge over the house.

4.1.2 Occurrence of favourable situations

The five count strategy depends on the situation of no fives remaining in the deck occurring. The question that now arises is how often does this favourable situation happen. The probability that the fives are all gone in the beginning of the deck is pretty small as seen before. Suppose now that the player does not only keeps track of the fives but also of the remaining unseen cards in the deck. Then it is possible to simply calculate if the deck if "five-poor" or "five-rich". The way to do this is to divide the number of unseen cards U by the number of unseen Fives F. At the beginning of a new game of play the ratio is equal to U/F = 13. When U/F is greater than 13, the deck is five-poor and favourable for the player.

The larger U/F, the larger the player's edge over the house is, thus the player should bet more when U/F is large and less when U/F is smaller than 13 in which case the house has the edge over the player. In order to determine the frequency of these different situations occurring, a simulation of hundred thousand of shuffled decks was done. For each of these simulated decks the ratio U/F was calculated after a certain amount of cards were dealt. The results of this simulation are displayed in table 4.3.

Ratio		Number of cards dealt									
From	**Below**	**5**	**10**	**15**	**20**	**25**	**30**	**35**	**40**	**45**	**Average**
0	**6.5**	-	-	-	-	6.44	19.88	9.66	22.36	45.02	**11.48**
6.5	**13**	66.08	41.48	67.58	50.04	66.18	37.25	30.04	43.91	-	**44.73**
13	**18.5**	29.83	42.35	25.78	24.89	-	-	41.05	-	-	**18.21**
18.5	**24**	3.92	14.25	-	-	-	32.81	-	-	-	**5.67**
24	**∞**	0.17	1.92	6.64	15.06	27.38	10.06	19.25	33.73	54.98	**18.80**
No fives		-	0.07	0.47	1.80	4.57	10.06	19.25	33.73	54.98	**13.88**
The values in this table are given in percentage. The values on each column should therefore add up to 100%.											

Table 4.3 – Frequency of favourable situations in the five count strategy

Table 4.3 is read as follows. When there are 30 cards dealt off the top of the deck and the player has kept track of the count, the player will find that a ratio between 6.5 and 13 will occur 37.25% of the time. And a ratio of 13 or more will occur 42.87% of the time. (This last percentage is calculated by adding up all values in the column 30 cards dealt from row 13-18.5 up to the row 24-∞, namely, 32.81 + 10.06 = 42.87%.)

In the row "No fives" of table 4.3 the frequency of the situation that there are no more fives left in the deck are given. It shows that this most favourable situation does not happen that frequently. Only in the case were there is only one player at the table he has 54.98% chance of having no fives in the last round of play, and only in one third of the time (33.73%) there will be no fives dealt in the last *two* hands. Table 4.3 also shows that most of the time there are still fives in the deck. The player should switch between basic strategy table accordingly.

Whenever the deck is five-rich and unfavourable for the player, the player should keep to the basic strategy table for all decision taking. Only when the ratio is above 13 and thus five-poor, the player should play according the five count strategy table.

4.1.3 Betting scheme

The betting strategy would be to bet small when the deck is "five-rich", $U/F \leq 13$, and bet large when the deck is "five-poor", $U/F > 13$. Finally bet maximum when there are no more fives remaining in the deck. However, the player should also take into account the available number of cards left to deal, it is unadvisable to bet the maximum when there is only one card left to be dealt, as the dealer will shuffle the deck and the ratio will be 13 again. Therefore the player should always take into account the number of cards left to be dealt and adjust his bet accordingly.

The minimum number of cards left for a new round of play depends on the number of players at the table and on the average cards a player and dealer draws per round of play. Thorp calculated these numbers and came up the minimum number of remaining cards given the number of players at the table. The results are displayed in table 4.4.

Number of players	Minimum of remaining cards
1	7
2	11
3	14
4	18
5	21
6	25
7	28

Table 4.4 – Minimum cards needed for a new full round of play

At this point, three types of players are introduced that will come back throughout this thesis. The first player is a conservative better that at all cost does not want to raise the casino's suspicion, the second player is a moderate better that takes his chances but also tries to keep casino's suspicion as low as possible, and the third player is a risky better who has high variety in his bet spread and therefore takes the chance of being asked to leave the table by the casino floor manager. The betting scheme for the three different kind of players are displayed in table 4.5. When there are not enough cards left for another round of play, all three players bet the minimum amount of 1 unit.

Ratio (U/F)	Conservative better (units)	Moderate better (units)	Risky better (units)
< 13	1	1	1
13 to 20	2	2	2
> 20	2	3	5
No fives	5	10	20

Table 4.5 – Betting scheme for the five count strategy

In chapter 5 the player's expectation and the winning probabilities for the three types of players using the five count strategy are simulated.

4.2 Ten count strategy

The five count strategy is a good strategy for a beginning player to experiment with counting. However, if a player actually wants to make a profit from blackjack, he needs to make very large bets and thus have a high starting capital. But in general the number of favourable situations in the five count strategy are too scarce to make a profit for the average player. Thus this strategy should only be learned by players that don't want to go to the trouble of learning a more complex strategy, but on the other hand don't like losing money to a casino either.

For the players that *do* want to learn a more complex and more profitable strategy, it was again Edward Thorp in his first edition of "Beat the dealer" [8], who came up with an appropriate strategy, the ten count strategy.

4.2.1 Influence of the 10-counting cards

One might wonder, looking at table 4.1, how counting the tens will give a better result than counting the fives. As the table shows that for adding four tens to the normal deck ($Q(10) = 20$) only increases the player's expectation to 1.89%, whereas for removing all the fives from the deck increases the player's expectation to 3.58%. Card for card, fives have more effect than tens. The solution is that there are 16 tens and only 4 fives in a deck. Therefore the probability that the deviation of the number of tens differs from the average number of tens is larger than it is for the fives.

The richer the deck is in tens, the higher the player's expectation for the ten count strategy is, generally. This can be intuitively explained by the fact that for the player receiving at least one ten as his initial cards , the player is likely to have a favourable total to play with. Also the probability of busting is small when the player holds at least one ten. If the player only stand on totals higher than 17 (plays like a dealer), table 3.3 shows that the probability of busting is 21.4%. Where it is 42.9% for holding a five and standing on totals of at least 17.

Like in the five count strategy the player should again keep track of all the cards and count them. The deck will be divided into two types of cards "tens" and "others". During the play the player will keep track of both the number of tens as well as the number of others. With these two numbers the "ten-richness" of the deck can be determined, by computing the ratio, others divided by the tens. For example, for a complete deck, the count for the number of tens is 16 and the count for the others is 36. The corresponding ratio for a complete deck is thus given by 36/16 = 2.25. Edward Thorp calculated the approximate player's expectation corresponding to the different ratios; these are displayed in table 4.6.

Ratio Others/Tens	Player's expectation
3.00	-2.0
2.25	0.1
2.00	1.0
1.75	2.0
1.63	3.0
1.50	4.0
1.35	5.0
1.25	6.0
1.16	7.0
1.08	8.0
1.00	9.0

Table 4.6 – Player's expectation (%) for the ten count strategy

The values in table 4.6 show that when there are as much tens left as there are others left in the deck, thus the corresponding ratio equals 1.00, the player has an edge of 9% over the house. The advantage of 9% for the ten count strategy is significantly larger than the maximum advantage of 3.6% for the five count strategy.

4.2.2 Occurrence of favourable situations

Just like the five count strategy, this strategy also depends on the occurrence of favourable situations. The frequency of these situations were obtained by simulating five hundred thousand of shuffled decks and by keeping track of the ratio after a certain amount of cards were dealt. The results of this simulation are displayed in table 4.7.

Ratio		Number of cards dealt									
Above	**To**	**5**	**10**	**15**	**20**	**25**	**30**	**35**	**40**	**45**	**Average**
0	**0.5**	-	-	-	-	-	-	-	0.04	2.30	**0.26**
0.5	**1.0**	-	-	-	-	0.06	0.18	1.89	2.49	9.84	**1.61**
1.0	**1.1**	-	-	-	0.01	0.43	0.95	-	7.66	-	**1.01**
1.1	**1.2**	-	-	-	0.10	-	-	5.57	-	-	**0.63**
1.2	**1.3**	-	-	-	0.87	2.09	3.70	-	-	-	**0.74**
1.3	**1.4**	-	-	0.13	-	-	-	-	-	24.74	**2.76**
1.4	**1.5**	-	-	1.29	3.84	6.65	9.87	13.31	17.76	-	**5.86**
1.5	**1.6**	-	-	-	-	-	-	-	-	-	**-**
1.6	**1.7**	-	1.64	6.21	10.52	-	-	-	-	-	**2.04**
1.7	**1.8**	-	-	-	-	14.43	18.07	-	-	-	**3.61**
1.8	**1.9**	-	9.41	15.89	-	-	-	22.23	-	-	**5.28**
1.9	**2.0**	14.45	-	-	19.46	-	-	-	-	-	**3.77**
2.0	**2.1**	-	23.01	24.39	-	22.07	-	-	26.60	-	**10.67**
2.1	**2.2**	36.14	-	-	-	-	23.56	-	-	-	**6.63**
2.2	**2.3**	-	29.65	-	24.23	-	-	-	-	-	**5.99**
2.3	**∞**	49.41	36.29	52.10	40.97	54.28	43.69	57.00	45.45	63.12	**49.15**

The values in this table are given in percentage. The values on each column should therefore add up to 100%.

Table 4.7 – Frequency of favourable situations in the 10-count strategy

Table 4.7 is read in the same manner as table 4.3. Note that from table 4.7 follows that the player has an advantage of 1% or more around 27.5% of the time (the ratio is below 2.0). It turns out that the advantage ranges from 1% for the player to 1% to the dealer about 22.5% of the time. And that the house has an advantage of 1% or more remaining 50% of the time.

This means that in order to make this strategy profitable, the player will need to adjust his betting scheme. Such that the player bets large when he has the advantage and bets low otherwise.

4.2.3 Strategy table

For this strategy there is a complication in giving the strategy table. Because for the best play, the table should be adjusted for various ratios. For each ratio there is an optimal strategy. Fortunately, Edward Thorp managed to combine all these separate charts into on single chart, given in table 4.8. The player using table 4.8 should keep a running count during the game. The running count enables the player to play his hands with great precision.

Standing numbers

x	D									
	2	3	4	5	6	7	8	9	10	A
19										2.2*
18										2.2 3.1*
17										3.1
16	3.9	4.5	5.3	6.5	4.6		1.2	1.7	2.2	1.4
15	3.2	3.6	4.1	4.8	4.3			1.4	1.9	1.3
14	2.7	2.9	3.3	3.7	3.4			1.1	1.6	1.2
13	2.3	2.5	2.6	3.0	2.7				1.3	1.1
12	2.0	2.1	2.2	2.4	2.3				1.1	1.0

Hard standing numbers · Soft standing numbers

*Against an Ace, the soft standing number is 19 if the ratio is **above** 2.2. It is 18 if the ratio is 2.2 or less. The hard standing number against an ace is 18 when the ratio is above 3.1. It is 17 when the ratio is 3.1 or less.

Soft doubling

x	D					
	2	3	4	5	6	7
A,9	1.3	1.3	1.5	1.6	1.6	
A,8	1.4	1.7	1.8	2.0	2.0	
A,7	2.0	2.2	3.3	3.8	3.5	
A,6	2.1	2.5	3.2	4.8	4.8	1.1
A,5	1.6	1.9	2.5	3.1	4.0	
A,4	1.6	1.9	2.4	3.0	3.2	
A,3	1.5	1.8	2.3	2.9	3.0	
A,2	1.5	1.7	2.1	2.6	2.7	

Hard doubling

x	D									
	2	3	4	5	6	7	8	9	10	A
11	3.9	4.2	4.8	5.5	5.5	3.7	3.0	2.6	2.8	2.2
10	3.7	4.2	4.8	5.6	5.7	3.8	3.0	2.5	1.9	1.8
9	2.2	2.4	2.8	3.3	3.4	2.0	1.6			0.9
8	1.3	1.5	1.7	2.0	2.1	1.0				
7	0.9	1.1	1.2	1.4	1.4					
4,2			1.0	1.2	1.3					
3,2			1.0	1.1	1.1					

Pair splitting

x	D									
	2	3	4	5	6	7	8	9	10	A
A,A	4.0	4.1	4.5	4.9	5.0	3.8	3.3	3.1	3.2	2.6
10,10	1.4	1.5	1.7	1.9	1.8					
9,9	2.4	2.8	3.1	3.7	3.2	1.6		4.2		1.5
8,8									1.6*	4.8
7,7										1.4
6,6	2.4	2.6	3.0	3.6	4.1	3.4				
5,5										
4,4	1.3	1.6	1.9	2.4	2.1†					
3,3						1.1*	2.4*	4.2*	5.3*	
2,2	3.1	3.8				1.1*	3.8*			

* Numbers followed by * are read in reverse fashion, thus split on a certain total is the ratio is above that number.

† Only split (4,4) against 6, when the ratio is 2.1 or less and doubling down is not permitted.

Insurance: If ratio ≤ 2.0

Table 4.8 – Strategy table for the ten count strategy

Table 4.8 has the same format as the strategy table from the previous two strategies, with one exception, some of the squares now have numbers instead of being black, blank or grey. These squares with numbers for doubling down and pair splitting are to be interpreted as follows. If the ratio is equal or less than the number in the square, consider the square black, thus split the pair or double down. If the ratio is larger than the number in the square, consider the square blank. There are some numbers with an asterisk (*). These numbers are interpreted in the opposite way, if the ratio is larger than such a number, consider the square black, otherwise consider it blank.

The standing numbers of table 4.8 are to be interpreted as follows. For the decision on soft standing numbers there is only one change compared to the basic strategy, namely when the dealer's up card is an ace. In this case it is 18 when the ratio is 2.2 or less and 19 when the ratio is greater than 2.2. The hard standing numbers against the dealer's ace is 17, if the ratio is equal or less than 3.1 (but above 1.4). If the ratio is above 3.1, the hard standing number against an ace is 18.

The hard standing numbers for the dealer's up card 2 through 10 and for an ace when the ratio is less than 1.4, are interpreted from table 3.23 as follows. For a given ratio, consider that all the squares where the ratio is equal or greater than this ratio are black. Then the lowest black square is the correct standing number. For example, consider the dealer showing a 5. The standing numbers are: 12 for ratios of 2.4 or less; 13 for ratios above 2.4 but less than or equal to 3.0; 14 for ratios above 3.0 but less than or equal to 3.7; and so on until the ratio is greater than 4.6, then the minimal standing number is 17.

An important change to the basic strategy is that the player should incorporate the option insurance in his play. Whenever the ratio is below 2.0, take insurance if the opportunity (dealer's up card is an ace) is there. Do not insure when the ratio is 2.0 or more. The reasoning behind simple, if the deck is ten-rich, the dealer is more likely to have a blackjack.

Because in this strategy a running count of all tens and others is being kept at all time, it is possible to calculate the expectation of insuring. As an example assume the case with a single player where the new round of play is dealt from a complete deck and the dealer's up card is an ace. Since the dealer's ace is visible, there are 51 possibilities for his hole card (this example is illustrative, thus for simplicity the two cards of the player are not taken into account), 16 of these are tens. The player's expectation at this time is now:

$$2 \cdot P(dealer's\ hole\ card\ is\ a\ ten) - P(dealer's\ hole\ care\ is\ not\ a\ ten) = 2 \cdot \frac{16}{51} - \frac{35}{51} = -\frac{1}{17}$$

If now the two cards of the player and the burnt card are taken into account, there are now four cases to explore: no tens dealt, one ten dealt, two tens dealt and three tens dealt.

- No tens dealt: 2×16/48 – 32/48 = 0
- One ten dealt: 2×15/48 – 33/48 = -0.0625
- Two tens dealt: 2×14/48 – 34/48 = -0.125
- Three tens dealt: 2×13/48 – 35/48 = -0.1875

In the case were no tens were dealt, the ratio was equal to 32/16 = 2.0. For this situation the player's expectation of insuring is zero. Thus there is in the long run no gain nor loss on the average. The only advantage of insuring in this instance is that insuring will reduce the fluctuation in the player's capital. Thus a player with minimal capital should insure when the ratio is exactly 2.

4.2.4 Betting scheme

For this strategy again the three different types of players are used; the conservative, moderate, and risky better. The betting scheme for the three players according to the ratio are displayed in table 4.9. Just like for the five count strategy, when there are not enough cards left for another round of play, all three players will bet the minimum of 1 unit. The minimum amount of cards left for another round of play are assumed to be the same as for the five count strategy (table 4.4).

Ratio	Player's expectation (%)	Conservative better (units)	Moderate better (units)	Risky better (units)
> 1.75	≤ 2.00	1	1	1
1.75-1.50	2.00 – 4.00	2	3	5
1.50-1.25	4.00 – 6.00	3	5	8
1.25-1.00	6.00 – 9.00	5	8	15
≤ 1.00	≥ 9.00	5	10	20

Table 4.9 – Betting scheme for the ten count strategy

In chapter 5 the player's expectation and the winning probabilities for the three types of players using the ten count strategy are simulated.

4.3 Point count strategy

This strategy was first announced to the scientific public by Harvey Dubner at a panel session of a computer conference in Las Vegas, Nevada. This panel session was about the use of computers to study games of chance and skill. Harvey Dubner proudly presented here the full point count system based on own calculations. Also his plays in the casinos during the conference were very successful.

This aroused the interest of the other panel members. In particular the panel member Julian Braun of the IBM Corporation. Working at IBM Corporation meant that he had access to the fastest computer at that time. This enabled Braun to perform detailed calculations, which showed that although there were some inaccuracies in the calculations of Dubner, the complete point count system was a powerful and effective winning blackjack strategy.

Edward Thorp used the computer calculations by Julia Braun on the complete point count system and published this strategy in the second edition of his book "Beat the dealer" [8].

4.3.1 Counting the cards

When the deck is poor on high valued cards (tens and aces), table 4.1 shows that this negatively influences the player's expectation. The same table shows that when the deck is poor on low valued cards (2,3,4,5,6,7), it influences the player's expectation positively. This suggests a system that measures whether the deck has an excess of high cards (good) or low cards (bad), and bet accordingly. The way to do this is to count low cards as +1 as they are seen, and to count high cards as -1 as they fall. Sevens, eights and nines are not counted. The count starts at 0. Table 4.10 gives an overview of the counting values.

2	3	4	5	6	7	8	9	10	A
+1	+1	+1	+1	+1	0	0	0	-1	-1

Table 4.10 – Counting values for the complete point-count system

One might wonder why the sevens are not included in the count, as table 4.1 clearly shows that when the deck is depleted of all the sevens it gives the player an advantage of 2.05%. The reason for this is to keep the count "balanced". The count system is said to be balanced when there is an equal number of plus and minus point values. As there are four different tens, there are twenty cards counting as minus one. There are also twenty cards counting for plus one when the sevens are excluded.

In addition to the total point count, this strategy also requires to keep track of the unseen cards. The card for unseen cards is quite simple, the count starts at 52 multiplied by the number of decks. And each time a card is seen, just subtract 1 from the current total. The method of counting cards as soon as the cards are dealt, without waiting, is called the "running count".

The index ratio is used to determine the bet size and also in the strategy table. The ratio is determined as follows. The total point count divided by the total number of unseen cards and then multiplied by 100%. For example, in a one-deck game, if 3,4,8,A,5 fell, the point count would be +2, 47 cards remain, so the index becomes +2/47 or about 0.04, or in percentage 4%.

4.3.2 Occurrence of favourable situations

Like any winning strategy in blackjack the point count strategy also depends on favourable situations occurring in the deck and betting large when it happens. To obtain the frequency of these situations as well as the advantage of the player given a certain index, again a simulation was done. Edward Thorp using the calculations by Julian Braun, computed the strategy table for the point count strategy. He also estimated the player's advantage given a certain index. The result for the player's advantage given a certain index are given in figure 4.1.

Notice the gain that insurance gives the player. Another interesting fact is that the player gains more when the index is positive than he loses when it is negative. The reason for this is that the player can vary his bet. Therefore he is able to reduce the disadvantage when the deck is poor. And to exploit the advantage when the deck is good.

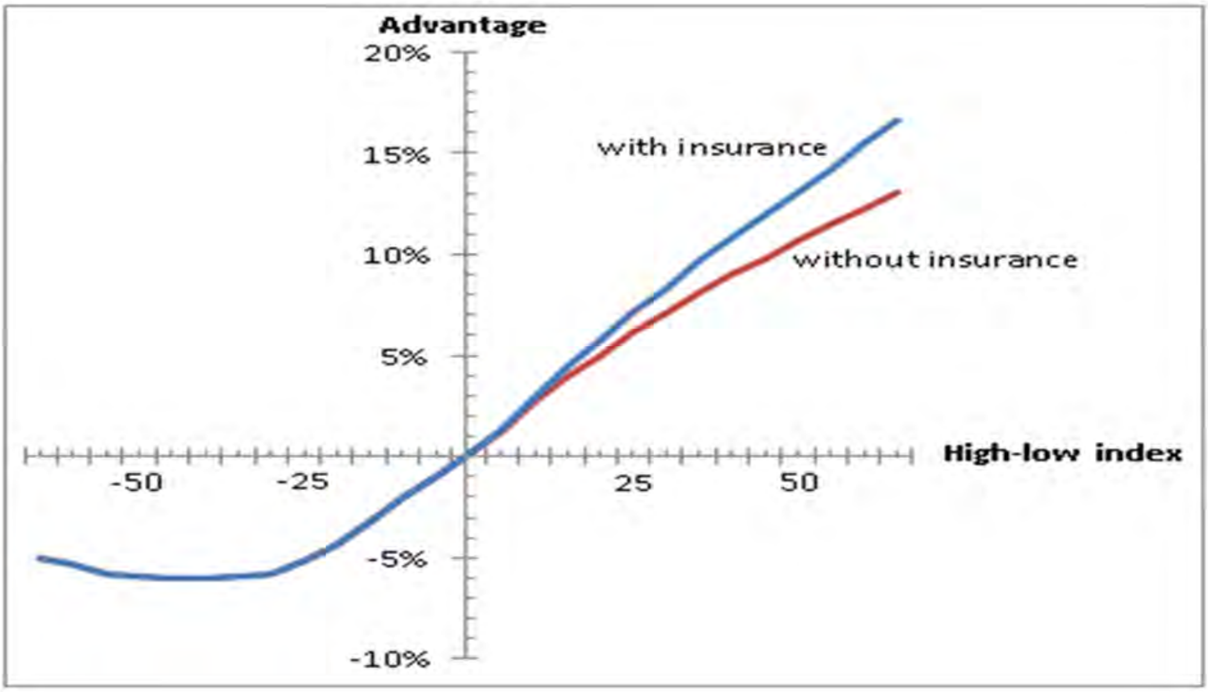


Figure 4.1 – Player's advantage using the point-count strategy

Figure 4.1 seems to give the impression with the left tail going slightly up again, that the player regains the advantage when the index becomes negative enough. This impression is actually correct but these situations appear very rarely. One of these situations comes from table 4.1, where the subset Q(10) = 0, giving an index of -16/36 or -44%, for an otherwise complete deck, the player has an advantage of 1.62%. However, the average situation with an index of -44 is disadvantageous to the player also indicated in figure 4.1. In another extreme case were the index is -100, thus there are only twos to nines remaining in the deck, the player has an average advantage of up to 50%, depending on the exact cards remaining. But this situation is almost theoretical as the probability of occurring is very small.

The results of the second simulation of 500.000 shuffled single decks to obtain the frequency of the favourable situation as well as the player's advantage, are displayed in table 4.11.

	Player's advantage with insurance												
	-5.7 above	-6.0 to -5.7	-6.0 to -5.9	-5.9 to -5.1	-5.1 to -3.5	-1.1 to -3.5	-1.1 to 1.4	1.4 to 4.3	4.3 to 7.2	7.2 to 9.7	9.7 to 12.7	12.7 to 14.6	14.6 above
	High-low index ranges												
	below -55	-55 to -45	-45 to -35	-35 to -25	-25 to -15	-15 to -5	-5 to 5	5 to 15	15 to 25	25 to 35	35 to 45	45 to 55	55 above
number of cards dealt													
0							100.0						
5						9.5	81.0	9.5					
10					0.4	15.8	67.7	15.8	0.4				
15					2.7	27.5	39.5	27.5	2.7				
20				0.5	6.8	24.2	37.0	24.2	6.8	0.5			
25			0.1	1.9	5.9	24.0	36.2	24.0	5.9	1.9	0.1		
30		0.1	0.7	3.2	9.4	18.3	36.6	18.3	9.4	3.2	0.7	0.1	
35		0.5	2.7	3.4	13.5	23.2	13.3	23.2	13.5	3.4	2.7	0.5	
40	0.7	1.3	2.6	5.0	19.3	13.8	14.6	13.8	19.3	5.0	2.6	1.3	0.7
45	5.3	0.0	7.3	12.1	0	16.3	18.0	16.3	0	12.1	7.3	0.0	5.3
The values in this table are given in percentage. The values on each row should therefore add up to 100%.													

Table 4.11 – frequency of favourable situations and the player's advantage for the point count strategy

Table 4.11 is read as follows. First of all the player's advantage and high-low index are one on one related. For example, for an index from 15 to 25 the corresponding player's advantage is equal to 4.3% to 7.2%. The lower part of the table gives the frequency of a certain high-low index after a number of cards are dealt. So when 30 cards are dealt, the probability that the high-low index is between -25 and -15 equals 9.4% and the probability that the high-low index is bigger than 5 equals 31.7% (=18.3%+9.4%+3.2%+0.7%+0.1%).

Notice from table 4.11 that the frequency part of the table is symmetric. This is due to the fact that this point count system is balanced, thus the probability of having an index of -35 after 20 cards are dealt, should be the same as for an index of 35 and 20 cards dealt. Also notice the pyramid shape of the table, meaning that the deck only becomes more favourable as the number of cards dealt increases.

4.3.3 Strategy table

The strategy table for the point count strategy was calculated by Julian Braun and Edward Thorp. Julian Braun worked for IBM Corporation at that time, which enabled him to do the calculations of one of the fastest computers at the time. The strategy table is displayed in table 4.12 and has the same format as the strategy table for the ten count strategy.

Hard standing numbers

x	D									
	2	3	4	5	6	7	8	9	10	A
18 or more	■	■	■	■	■	■	■	■	■	■
17	■	■	■	■	■	■	■	■	■	-15
16	-21	-25	-30	-34	-35	10	11	06	00	14
15	-12	-17	-21	-26	-28	13	15	12	08	16
14	-05	-08	-13	-17	-17	20	38			
13	01	-02	-05	-09	-08	50				
12	14	06	02	-01	00					

■ Standing numbers □ Drawing numbers

The indexes are in per cent. Stand if your index is larger than the appropriate entry in the table. And draw otherwise. This table assumes that the player has also taken the cards dealt in the current round into account.

Soft standing numbers

x	D									
	2	3	4	5	6	7	8	9	10	A
19 or more	■	■	■	■	■	■	■	■	■	■
18	■	■	■	■	■	■	■		12	-06
17						29				

■ Standing numbers □ Drawing numbers

Player should draw in case of soft 16 or less.

Soft doubling down

x	D				
	2	3	4	5	6
A,9		20	12	08	08
A,8		09	05	01	00
A,7		-02	-15	-18	-23
A,6	*	-06	-14	-28	-30
A,5		21	-06	-16	-32
A,4		19	-07	-16	-23
A,3		11	-03	-13	-19
A,2		10	02	-19	-13

Double down if the player's index is greater than the index in the squares. Otherwise do not double down. It is assumed that the cards dealt in the current round are included in the index.

* Double down with (A,6) versus a 2 only when the index is between 01 and 10.

Hard doubling down

x	D									
	2	3	4	5	6	7	8	9	10	A
11	-23	-26	-29	-33	-35	-26	-16	-10	-09	-03
10	-15	-17	-21	-24	-26	-17	-09	-03	07	06
9	03	00	-05	-10	-12	04	14			
8		22	11	05	05	22				
7		45	21	14	17					
6			27	18	24					
5				20	26					

Double down when the player's index is greater than the index in the square. Otherwise do not double down. The table assumes that the player has already counted the cards in the current round of play.

Pair splitting

x	D									
	2	3	4	5	6	7	8	9	10	A
A,A	■	■	■	■	■	-33	-24	-22	-20	-17
10,10	25	17	10	06	07	19				
9,9	-03	-08	-10	-15	-14	08	-16	-22		10
8,8	■	■	■	■	■	■	■	■	24*	-18
7,7	-22	-29	-35	■	■	■	■			
6,6	00	-03	-08	-13	-16	-08				
5,5										
4,4		18	08	00	05*					
3,3	-21	-34	■	■	■	■	06†			
2,2	-09	-15	-22	-30	■	■				

Split if the player's index is greater than the index in the squares or when the square is black. Otherwise do not split.

* Split (8,8) against 10 only if the index is below 24

* Split (4,4) against 6 when the index is greater than 05 only if doubling down is not permitted.

†Split (3,3) against 8 when the index is above 06 and also when the index is below -02.

Insurance

If the index is greater than 8, insurance should ben taken. Otherwise it should not be taken

Table 4.12 – Strategy table for the point count strategy

The table is read as follows. When a square has an index number in it, the player should stand, split or double down when the player's index is greater than the index number in the square. Except when the square has an asterisk (*) or cross in it, then follow the instructions under the table. As for insurance, the player should only insure when the high-low index is greater than 8. This high-low index implies that the deck is rich in tens and aces and therefore the probability of the dealer having a blackjack is greater than the dealer not having one.

4.3.4 Betting scheme

The betting scheme for this strategy again depends on the high-low index and thus the player's advantage. That is to bet large if the deck is favourable and small otherwise. Again the three different types of players are used; the conservative, moderate, and risky better. The betting scheme for the three players according to the ratio are displayed in table 4.13. Just like for the two previous strategies, when there are not enough cards left for another round of play, all three players will bet the minimum of 1 unit. The minimum amount of cards left for another round of play are assumed to be the same as for the five count strategy (table 4.4).

High-low index	Player's expectation (%)	Conservative better (units)	Moderate better (units)	Risky better (units)
≤ 2	< 0.5	1	1	1
2 - 4	≈ 1.0	2	2	2
4 - 6	≈ 1.7	3	3	3
6 - 8	≈ 2.5	4	4	5
8 - 10	≈ 2.8	5	5	7
10 – 15	≈ 3.2	5	7	10
≥ 15	> 3.5	5	10	20

Table 4.13 – betting scheme for the point count strategy

In chapter 5 the player's expectation and the winning probabilities for the three types of players using the point count strategy are simulated.

4.4 Zen count strategy

The zen count strategy is a used by professional blackjack players. And like all other professional blackjack strategies it is based on the point count system. It was published by an authority in the field of professional blackjack, Arnold Snyder, in his book "Blackbelt in Blackjack" [6]. Arnold Snyder is a professional blackjack gambler and a gambling author. For his record as a blackjack player and his innovations in blackjack techniques he was elected by other professional blackjack players as one of the seven original inductees into the Blackjack Hall of Fame (Edward Thorp was also an original inductee).

4.4.1 Counting the cards

The point count system only makes a difference between three types of cards: good cards for the player, bad cards for the player and unimportant cards. The good cards for the player are all tens and aces for which the player subtract minus one from the running count. For the bad cards (twos to sixes) the player adds plus one to the count. For the unimportant cards (sevens to nines) the count is unaffected. The point count system does not take into account that some cards influence the player's expectation more than other cards. For example, table 3.16 shows that the subset $Q(2) = 0$ gives the player an advantage of 1.75%, whereas the subset $Q(5) = 0$ give the player an advantage of 3.58%. Although the advantage of removing all the fives from the deck is almost twice as large as for removing all the twos, the point count system does not make a distinction between the two.

The zen count strategy, however, does make a distinction between the bad cards and the good cards. It also includes the sevens to the bad cards for the player. The point values for the zen count are given in table 3.14.

2	3	4	5	6	7	8	9	10	A
+1	+1	+2	+2	+2	+1	0	0	-2	-1

Table 3.14 – the point values for the zen count strategy

The zen count, like the point count strategy is a balanced point count system. This means that if a player would count down a complete deck, then the final count would be zero. The count again starts at zero and it requires the player to keep a running count for all the decision making. However unlike the point count strategy where the player keeps track of all the unseen cards the zen count strategy only demands the player to make an approximation of the number quarter-of-decks(13 cards) remaining. With this approximation the player is able to calculate the true edge over the house.

The way to adjust the running count to true edge is done as follows. First of all, any time the running count is negative, assume that either the house has the advantage, or the player's advantage is less than 0.5%. Now when the running count is positive, the true edge is calculated by dividing the running count by the remaining number of decks, or more precise the remainder of quarterdecks. For every quarter of a deck remaining accounts for plus one in the denominator. For example, if the running count is +7 and approximately half a deck is remaining, the true edge would be 7/2, or about an advantage of 3.5%.

4.4.2 Strategy table

The strategy table for this strategy was also given by Arnold Snyder and is shown in table 3.15.

Hard standing numbers

x	D									
	2	3	4	5	6	7	8	9	10	A
18 or more	■	■	■	■	■	■	■	■	■	■
17	■	■	■	■	■	■	■	■	■	-3
16	-4	■	■	■	■	5	4	2	0	3
15	-2	-3	-4	-4	■		5	4	1	4
14	-1	-2	-2	-3	-3				3	5
13	0	-1	-1	-2	-2					
12	1	1	0	-1	0					

■ Standing numbers □ Drawing numbers

Soft doubling down

x	D				
	2	3	4	5	6
A,9	4	3	3	2	2
A,8	3	2	1	0	0
A,7	0	-1	-2	-3	-3
A,6	0	-1	-2	-4	■
A,5	5	1	-1	-3	■
A,4		2	0	-2	-4
A,3		3	1	-1	-2
A,2		3	1	0	-1

■ Double down □ Do not double down

Soft standing numbers

x	D									
	2	3	4	5	6	7	8	9	10	A
19 or more	■	■	■	■	■	■	■	■	■	■
18	■	■	■	■	■	■	■			2

■ Standing numbers □ Drawing numbers

Pair splitting

x	D									
	2	3	4	5	6	7	8	9	10	A
A,A	■	■	■	■	■	■	■	■	■	-2
10,10	4	3	3	2	2					
9,9	-1	-2	-2	■	■	1	■	■		1
8,8	■	■	■	■	■	■	■	■	4	■
7,7	■	■	■	■	■	■	1			
6,6	-1	-2	■	■	■	■				
5,5										
4,4		3	1	0	-1					
3,3	-2	■	■	■	■	■	2			
2,2	-2	-2	■	■	■	■	3			

■ Split □ Do not split

Hard doubling down

x	D									
	2	3	4	5	6	7	8	9	10	A
11	■	■	■	■	■	-4	-3	-2	-2	0
10	-4	-4	■	■	■	-3	-2	-1	1	1
9	0	0	-1	-2	-3	2	4			
8	5	4	3	2	1					
7			5	5	5					

■ Double down □ Do not double down

Insurance: 1

Table 4.15 – Strategy table for the zen count strategy

The table is read as follows. When a square has a number in it, the player should stand, split or double down when the player's true edge is equal or greater than the number in the square. As for insurance, the player should only take insurance when the true edge is equal or greater than 1.

4.4.3 Betting scheme

The betting scheme for this strategy again depends on the player's true edge. That is bet large if the deck is favourable and small otherwise. Again the three different types of players are used; the conservative, moderate, and risky better. The betting scheme for the three players according to the ratio are displayed in table 4.16. Just like for the two previous strategies, when there are not enough cards left for another round of play, all three players will bet the minimum of 1 unit. The minimum amount of cards left for another round of play are assumed to be the same as for the five count strategy (table 4.4).

True edge	Conservative better (units)	Moderate better (units)	Risky better (units)
≤ 0	1	1	1
0 - 2	2	2	2
2 - 4	4	5	8
4 - 6	5	7	10
> 6	5	10	20

Table 4.16 – betting scheme for the zen count strategy

In chapter 5 the player’s expectation and the winning probabilities for the three types of players using the zen count strategy are simulated.

4.5 Is card counting cheating?

This is an obvious question the reader of this thesis might now have. The casino industry has done its best to change the public opinion that card counting is indeed cheating. Robbins Cahill, director of the Nevada Resort Association was quoted in the Las Vegas Review of August 4, 1976 as saying that most casinos “Don’t really like the card counters because they’re changing the natural odds of the game”. This is obvious complete nonsense, as if a card counter has an influence on the sequence in which the cards are drawn from the deck. And what are these “natural odds”? The casino can’t demand a blackjack players to go into the casino blindfolded.

It is certainly understandable that casinos do not like that people that can beat them at their own game are coming to their casino and gain large amounts. Since the publishing of the first winning blackjack strategies, there has been a reversal in the roles. Suddenly the casinos became the losers, and the card counters in their turn became the winners, earning quick money. Peter Griffin writes about the paradox in the casino’s attitude in his book “the theory of blackjack”. He writes that casinos make a living trying to encourage people to believe in systems, in luck, cultivating the notion that some players are better than others, that there is a savvy, macho personality that possesses the holy grail, which allows him to make a fortune in the casino. When in fact there exists such a system, the casino actually takes countermeasures to prevent this instead.

Blackjack dealers nowadays are trained to spot card counters. The casinos taught their dealers how to count cards in order to be able to spot card counters. The most common reason for a card counter to be spotted as one, is by his betting spread. If a player continuously makes large bets only when a lot of low cards come from the deck, the dealer will alarm the flour manager, who in his turn will ask the card counting player to leave the casino. This player will also be put on a blacklist, known as Griffin Book.

The Griffins Book or the Black book is the nickname frequently used to refer to persons that are unwelcome to the casino. The Griffin Book, is published by so-called Griffin Investigations and was originally introduced to block access to casinos to persons that are suspected of having, or known to have, ties with the organized crime. Casinos were obliged by regulation of the Gaming Board Control to deny access to these persons. But casinos soon also listed individuals suspected of being, or known to be card counters. The Griffin Book also keeps pictures of all the persons on the list. This combined with modern computer technology on face recognition makes it extremely complicated for a player on the list to enter a casino which has access to this list.

Officially card counting is not a form of cheating and it is not illegal, however, the casinos do have the right to deny anyone from playing at their casino. This means that once a player has been labelled as a card counter and therefore has been blacklisted, it will be near to impossible for that player to play blackjack in a casino with access to the blacklist.

Chapter 5

"Good grief. The poor blackjack deck is being
stripped naked of all her secrets."

Richard Epstein

Simulation

The strategies from the two previous chapters seem to work in theory for the classic rules as they were in beginning of the game. However, casinos have altered these rules slightly over the years to make card counting more difficult. Now the question arises whether these strategies also work in practice for the new casino rules. In order to investigate if this is true, a simulation program is the fastest way to find out.

The simulation program will test the strategies for six different kind of rules. These rules will be described later in this chapter. For these six different type of rules and for the five different strategies, first the program will simulate the player's expectation per combination of rules and strategy. Also the number of players at the table will be adjustable in this part. Secondly, the program simulates the probability that a player will end up in the money after a certain number of time units for the strategy that has the highest long term expectation under one of the casino rules used nowadays.

5.1 The rules

It was due to Edward Thorp's book "Beat the dealer" that The Las Vegas Resort Hotel Association on April 1, 1964, announced to change the rules of blackjack. This was the first time in history that the rules of a mayor casino gambling game were significantly changed. The spokesman of The Las Vegas Hotel Association, Gabriel Vogliatti explained the reason for this changes by "In the last 15 years there hasn't been one plane landed without at least one person in possession of a system. This guy [Edward Thorp] is the first in Las Vegas history to have a system that works."

The specific rules changes were that splitting aces was forbidden and that doubling down was restricted to hard totals of 11. Edward Thorpe calculated that the impact on the player's expectation by these countermeasures was about -1%.

However, these countermeasures didn't last that long. It took The Las Vegas Resort Hotel Association exactly three weeks to go back to the old rules. Why change it back? Players overnight, massively stopped playing blackjack at the casinos and also the play at the other casino games declined as the flow of tourists into the city diminished. Also the casino employees began to protest against the taken countermeasures, because their income largely depend on the number of tips they receive.

Over the years, the casinos all over the world have gradually made changes in the original rules. The rule changes include for instance increasing the number of decks and allowing doubling down on restricted totals only. Thus to make the simulation complete, the rules for five different places around the world were tested as well as the classic rules. These five different places include: Las Vegas strip, Las Vegas downtown, Atlantic City, Europe and the Holland casino in the Netherlands. An overview of the differences between the six types of rules are displayed in table 5.1.

	Classic blackjack	Las Vegas strip	Las Vegas downtown	Atlantic City	Europe	Holland casino
Number of decks	1	4	2	8	2	6
Shuffling between rounds						■
Dealer peeks for blackjack	■	■	■	■	■	■
Dealer stands on hard totals of 17	■	■	■	■	■	■
Dealer stands on soft totals of 17	■	■		■	■	■
Doubling down on these totals only	All	All	All	All	9, 11	9,10,11
Double down after split	■	■	■	■		■
Pair splitting on these pairs only	All	All	All	All	All	All
Unalike 10-valued cards can be split		■	■	■	■	
Split up to maximum this number of hands	2	4	4	3	2	2
Insurance	■	■	■	■	■	■
Maximum bet size	$100	$500	$300	$300	$200	$100
Minimum bet size	$5	$25	$15	$15	$10	$5
Pay-out for a normal win is	1 to 1	1 to 1	1 to 1	1 to 1	1 to 1	1 to 1
Pay-out for a blackjack is	3 to 2	3 to 2	3 to 2	3 to 2	1 to 1	3 to 2
Pay-out for insurance is	2 to 1	2 to 1	2 to 1	2 to 1	2 to 1	2 to 1
Pay-out for three 7's is	0	0	0	0	0	1 to 1

Table 5.1 – Overview of the different kind of rules

Table 5.1 is read as follows, a black square stands for a yes and a blank square for a no. The most striking difference from the classic rules is that the number of decks is at least two. Also the Holland casino shuffles the six decks after each round of play, which makes card counting impossible.

5.2 Player's long run expectation

In this part of the simulation the long run expectation of the player is simulated for each strategy per casino. Each simulation run was based on 500.000 of rounds of play. Also the number of players at the table were taken into account. When there are other players at the table an assumption was made, namely that the player always sits most left to the dealer and thus has to make his decisions last. This is favourable to the player because the player can then take into account the cards drawn by the other players. There will be two statistics to simulate, the player's mean expectation and a 95%-confidence interval. The 95%-confidence interval should be interpreted as an interval for which with 95% confidence can be said that the player's long run expectation lays in that interval.

5.2.1 Basic strategy

Rules	Statistic	Number of players		
		1	4	7
Classic rules	**Mean**	0.04765	-0.02170	-0.1116
	95%-Conf. Int.	[-0.2630, 0.3583]	[-0.4501, 0.4067]	[-0.6248, 0.4017]
Las Vegas Strip	**Mean**	-0.5108	-0.9322	-0.8564
	95%-Conf. Int.	[-0.8203, -0.2012]	[-1.3597, -0.5047]	[-1.3689, -0.3439]
Las Vegas downtown	**Mean**	-0.7663	-0.7637	-1.1748
	95%-Conf. Int.	[-1.0760, -0.4565]	[-1.1918, -0.3356]	[-1.6872, -0.6623]
Atlantic City	**Mean**	-0.3098	-0.9713	-1.3467
	95%-Conf. Int.	[-0.8192, 0.1997]	[-1.3992, -0.5434]	[-1.8587, -0.8347]
Europe	**Mean**	-0.8837	-1.19225	-0.9540

Europe	**95%-Conf. Int.**	[-1.1840, -0.5834]	[-1.8411, -1.0122]	[-1.2544, -0.6536]
Holland casino	**Mean**	-0.4972	-1.1957	-1.0144
	95%-Conf. Int.	[-0.8061, -0.1882]	[-1.6224, -0.4267]	[-1.5039, -0.5101]
All values in this table are displayed in percentage				

Table 5.2 – Player's long run expectation using the basic strategy under different casino rules and various number of players

It follows from table 5.2 that a player using the basic strategy under the current rules, the casino will always have the edge over the player. However, the edge is the smallest when playing in a casino in Atlantic City or Las Vegas downtown. Overall the European rules perform the worst for a player using the basic strategy.

Change in rules		**Player's expectation**
Increasing the number of decks	2 decks	-0.21
	4 decks	-0.44
	8 decks	-0.52
Shuffling between rounds		0.14
Dealer draws on soft totals of 17		-0.23
Pair splitting up to 3 hands		0.05
Pair splitting up to 4 hands		0.13
No doubling down after splitting		-0.21
Doubling down on 9,10 and 11 only		-0.51
The numbers in this table are given in percentage and were obtained by simulating 1 million hands.		

Table 5.3 – Effect of rule changes on the player's expectation

Under the classic rules the basic strategy performs as expected, ranging from -0.25% to 0.35% assuming that there are no other players at the table. Interesting is what rule changes influences the decrease in the player's expectation. Table 5.3 gives some of these rule changes and the effect on the player's expectation. From this table it follows that the effect of increasing the number of decks to 8 is the greatest. Also the restriction of doubling down on total of 9,10 and 11 only has a great impact on the player's expectation (dropping almost a half per cent). This restriction and the rule change on no doubling down after splitting make that the European rules are so bad for a player using the basic strategy.

Another interesting fact from table 5.3 is the positive effect of shuffling the deck between rounds. This is because the basic strategy was calculated assuming a full deck, therefore when the deck is more depleted the strategy becomes less precise, but shuffling between every round makes that the basic strategy is the most effective every new round.

5.2.2 Five count strategy

Rules	**Risk profile**	**Statistic**	**Number of players**		
			1	**4**	**7**
Classic rules	Conservative	**Mean**	0.8556	0.1919	-0.6764
		95%-Conf. Int.	[0.2291, 1.4821]	[-0.6660, 0.8579]	[-1.1672, 0.4908]
	Moderate	**Mean**	1.2998	0.7142	0.2723
		95%-Conf. Int.	[0.7804, 1.8192]	[-0.3293, 1.7577]	[-0.9419, 1.4865]
	Risky	**Mean**	3.0616	3.2781	3.0677
		95%-Conf. Int.	[0.7275, 5.3957]	[0.8184, 5.7377]	[0.6525, 5.4828]
Las Vegas strip	Conservative	**Mean**	0.3813	0.3646	-0.2101
		95%-Conf. Int.	[-1.3247, 2.0873]	[-1.4566, 2.1858]	[-2.3534, 2.1433]
	Moderate	**Mean**	-0.004	0.8542	0.7298
		95%-Conf. Int.	[-2.2885, 2.2805]	[-1.4990, 3.2073]	[-1.6475, 3.1071]
	Risky	**Mean**	2.1269	1.1761	0.2987
		95%-Conf. Int.	[-0.7792, 5.0329]	[-1.2836, 3.6358]	[-2.0838, 2.6812]
Las Vegas downtown	Conservative	**Mean**	-0.3230	-0.54255	-1.07925
		95%-Conf. Int.	[-0.9063, 0.2603]	[-1.2886, 0.2035]	[-1.9238,-0.2347]
	Moderate	**Mean**	-0.3451	-0.7324	-0.8583

Las Vegas downtown		**95%-Conf. Int.**	[-1.0387, 0.3486]	[-1.5433, 0.0785]	[-1.7686, 0.0521]
	Risky	**Mean**	0.4942	0.8978	0.7905
		95%-Conf. Int.	[-1.5487, 2.5371]	[-1.3043, 3.0999]	[-1.4353, 3.0162]
Atlantic City	Conservative	**Mean**	0.3801	0.3527	-0.0946
		95%-Conf. Int.	[-1.2154, 1.9756]	[-1.3694, 2.0748]	[-2.1034, 1.9143]
	Moderate	**Mean**	0.4633	0.5349	0.5597
		95%-Conf. Int.	[-1.2379, 2.1645]	[-1.5892, 2.6589]	[-1.6810, 2.8003]
	Risky	**Mean**	0.8646	0.8077	0.7509
		95%-Conf. Int.	[-1.3701, 3.0992]	[-1.8836, 3.4990]	[-2.0215, 3.5233]
Europe	Conservative	**Mean**	0.1614	0.2179	-0.2377
		95%-Conf. Int.	[-1.4796, 1.8023]	[-1.6010, 2.0367]	[-2.6541, 2.1787]
	Moderate	**Mean**	0.1513	0.2910	0.43485
		95%-Conf. Int.	[-1.7158, 2.0184]	[-1.9452, 2.5271]	[-2.5644, 3.4341]
	Risky	**Mean**	0.6875	1.0605	0.7532
		95%-Conf. Int.	[-1.7799, 3.1548]	[-1.3644, 3.4854]	[-1.9721, 3.4785]
Holland casino	Conservative	**Mean**	0.1751	0.0160	0.6770
		95%-Conf. Int.	[-0.4493, 0.7994]	[-0.7436, 0.7755]	[-1.5022, 0.1483]
	Moderate	**Mean**	0.2911	0.776	-0.5088
		95%-Conf. Int.	[-0.6474, 1.2296]	[-1.0535, 1.2087]	[-1.8975, 0.8799]
	Risky	**Mean**	0.2965	0.1989	0.2238
		95%-Conf. Int.	[-0.7292, 1.3221]	[-1.0232, 1.4209]	[-1.1116, 1.5592]
All values in this table are displayed in percentage					

Table 5.3 – Player's long run expectation using the five count strategy under different casino rules and various number of players

Under the classic rules, the five count strategy performs as expected, giving the player an edge over the house for almost every better profile and different number of players at the table. Only the conservative better playing at a full table has a negative expectation. The risky better gains a considerable edge over the house of around 3.0% under the classic rules.

But not only the classic rules give the player an advantage over the house, also some of the other rules perform well. In particularly the Las Vegas strip rules are quite good for the five count strategy as the player's overall expectation is positive. A risky player who plays at a table that is except for him empty, has an advantage of 2.1% over the house.

The Holland casino rules performs the worst for the five count strategy, because the player's advantage over the house is at most 0.3%. This due to reshuffling after every single round of play, which makes it impossible for the player to get into large bets for favourable situations (no fives).

Playing at the other casinos does not give the player very good results. The player's expectation in these casinos ranges from -1.0% to +1.0% depending on the betters profile of the player.

5.2.3 Ten count strategy

Rules	**Risk profile**	**Statistic**	**Number of players**		
			1	**4**	**7**
Classic rules	Conservative	**Mean**	2.3422	1.2793	0.6246
		95%-Conf. Int.	[1.8616, 2.8227]	[0.9547, 1.6040]	[0.4223, 0.8270]
	Moderate	**Mean**	3.9346	1.9539	1.1908
		95%-Conf. Int.	[3.2347, 4.6344]	[1.5446, 2.3633]	[0.8196, 1.5620]
	Risky	**Mean**	7.0098	2.8527	1.7317
		95%-Conf. Int.	[5.8104, 8.2091]	[2.1436, 3.5618]	[1.2019, 2.2615]
Las Vegas strip	Conservative	**Mean**	1.1267	0.6442	0.2514
		95%-Conf. Int.	[0.7212, 1.5314]	[0.1928, 1.0957]	[0.0468, 0.4560]

Las Vegas strip	Moderate	**Mean**	5.0799	2.0836	0.7850
		95%-Conf. Int.	4.2450, 5.9335	[1.5005, 2.6666]	[0.3659, 1.2040]
	Risky	**Mean**	5.1632	2.5222	1.2823
		95%-Conf. Int.	[4.1835, 6.1248]	[1.7399, 3.3045]	[0.7121, 1.8525]
Las Vegas downtown	Conservative	**Mean**	1.4601	0.8989	0.4152
		95%-Conf. Int.	[1.0173, 1.9109]	[0.5837, 1.2140]	[0.0233, 0.8070]
	Moderate	**Mean**	2.5209	2.1037	1.2582
		95%-Conf. Int.	[1.8911, 3.1507]	[1.4527, 2.7547]	[0.8330, 1.6833]
	Risky	**Mean**	5.0806	2.6185	1.4107
		95%-Conf. Int.	[4.0350, 6.1261]	[1.9077, 3.3292]	[0.7970, 2.0244]
Atlantic City	Conservative	**Mean**	0.9615	0.3666	-0.1891
		95%-Conf. Int.	[0.4472, 1.4758]	[-0.0442, 0.7775]	[-0.5072, 0.1290]
	Moderate	**Mean**	1.1939	0.7378	0.5030
		95%-Conf. Int.	[0.7258, 1.6620]	[0.1104, 1.3652]	[0.1124, 0.8936]
	Risky	**Mean**	1.7271	1.1839	0.9929
		95%-Conf. Int.	[1.0354, 2.4187]	[0.4910, 1.8768]	[0.5249, 1.4609]
Europe	Conservative	**Mean**	0.9932	0.4141	-0.0581
		95%-Conf. Int.	[0.5689, 1.4177]	[0.0722, 0.7560]	[-0.3537, 0.2375]
	Moderate	**Mean**	2.1705	1.1322	-0.0458
		95%-Conf. Int.	[1.5735, 2.7676]	[0.6624, 1.6020]	[-0.4111, 0.4019]
	Risky	**Mean**	4.1040	2.3432	1.6267
		95%-Conf. Int.	[3.1120, 5.0960]	[1.7165, 2.9698]	[1.1013, 2.1521]
Holland casino	Conservative	**Mean**	-0.1552	0.1666	0.4458
		95%-Conf. Int.	[-0.5395, 0.2290]	[-0.1582, 0.4914]	[-0.0685, 0.8232]
	Moderate	**Mean**	-0.1112	0.3301	1.1229
		95%-Conf. Int.	[-0.5129, 0.2905]	[-0.0757, 0.7360]	[0.6414, 1.6045]
	Risky	**Mean**	0.1575	0.6719	1.9874
		95%-Conf. Int.	[-0.1625, 0.4775]	[0.1519, 1.1918]	[1.2532, 2.7216]
All values in this table are displayed in percentage					

Table 5.4 – Player's long run expectation using the ten count strategy under different casino rules and various number of players

The results show that the player's expectation depends largely on the better profile of the player for the ten count strategy. The riskier the player, the higher his long run expectation is. However, remember that the riskier the player, also the higher chance of being spotted as a counter. For five out of the six simulated rules the player's expectation is positive for the ten count strategy. Except the rules at the Holland casino are not advantageous for the player, this is due to the card shuffling machines (like for all strategies for which counting is required).

Under the classic rules the player's expectation is always positive, thus a player using the ten count strategy will have the edge over the dealer on the long run. The minimum edge for the player on the long run is 0.6% for when there are 6 other players playing at the table and the player is using the conservative betting scheme. The player's maximum edge over the house is 7% for when the player is alone at the table and is using the risky betting scheme.

For the rules that are applied nowadays but do not use card shuffling machines (Las Vegas strip, Las Vegas downtown, Atlantic City and Europe) the expectation using the ten count strategy is positive for the player in most cases. Only for the worst conditions (six other players at the table and using the conservative betting scheme) the long run expectation ranges from -0.2% to 0.4%. The most positive situations (no other players at the table and risky betting scheme) the player's long run expectation ranges from 1.7% to 5.1%. On average the Las Vegas strip rules give the best results when using the ten count strategy.

5.2.4 Point count strategy

Rules	Risk profile	Statistic	Number of players		
			1	4	7
Classic rules	Conservative	**Mean**	1.3348	0.6982	0.2870
		95%-Conf. Int.	[0.5234, 2.1452]	[-0.1354, 1.5318]	[-0.2718, 0.8458]
	Moderate	**Mean**	4.2614	2.4680	1.8973
		95%-Conf. Int.	[3.0531, 5.4696]	[1.5353, 3.4007]	[0.9859, 2.8088]
	Risky	**Mean**	5.8206	3.6052	2.5973
		95%-Conf. Int.	[4.6839, 8.9572]	[2.3154, 4.8950]	[1.5715, 3.6231]
Las Vegas strip	Conservative	**Mean**	1.1769	0.6988	0.2701
		95%-Conf. Int.	[0.4651, 1.8887]	[0.0829, 1.3146]	[-0.2473, 0.7874]
	Moderate	**Mean**	3.2576	2.4769	1.5149
		95%-Conf. Int.	[2.0472, 4.4680]	[1.2122, 3.7416]	[0.6071, 2.4227]
	Risky	**Mean**	7.1610	4.6848	3.2728
		95%-Conf. Int.	[5.0248, 9.2973]	[3.3640, 6.0056]	[1.9970, 4.5487]
Las Vegas downtown	Conservative	**Mean**	1.2971	0.6169	0.0034
		95%-Conf. Int.	[0.6966, 1.8976]	[-0.0362, 1.2699]	[-0.5043, 0.5111]
	Moderate	**Mean**	3.6237	1.9091	1.3593
		95%-Conf. Int.	[2.4118, 4.8355]	[0.8574, 2.9607]	[0.4280, 2.2906]
	Risky	**Mean**	6.7038	4.0970	2.8973
		95%-Conf. Int.	[5.2360, 8.1716]	[2.8075, 5.3865]	[1.3441, 4.4504]
Atlantic City	Conservative	**Mean**	1.4786	0.9774	0.6689
		95%-Conf. Int.	[0.8114, 2.1459]	[0.4819, 1.4729]	[0.1222, 1.2157]
	Moderate	**Mean**	4.76743	3.0972	1.7300
		95%-Conf. Int.	[3.5589, 5.9760]	[2.0664, 4.1279]	[0.7434, 2.7167]
	Risky	**Mean**	7.4778	5.2686	3.1762
		95%-Conf. Int.	[6.2546, 8.7011]	[3.9939, 6.5433]	[1.8081, 4.5443]
Europe	Conservative	**Mean**	1.1795	0.6840	0.3446
		95%-Conf. Int.	[0.4502, 1.9089]	[0.0705, 1.2975]	[-0.2296, 0.9188]
	Moderate	**Mean**	3.3105	2.0923	1.1239
		95%-Conf. Int.	[2.2011, 4.4199]	[1.0485, 3.1360]	[0.2667, 1.9811]
	Risky	**Mean**	5.9585	2.8033	1.6777
		95%-Conf. Int.	[4.6304, 7.2867]	[1.5509, 4.0558]	[0.4521, 2.9032]
Holland casino	Conservative	**Mean**	-0.1468	0.1710	0.4854
		95%-Conf. Int.	[0.2375, -0.5310]	[-0.1539, 0.4958]	[-0.0289, 0.9997]
	Moderate	**Mean**	-0.1054	0.3039	1.0946
		95%-Conf. Int.	[0.2963, -0.5071]	[-0.1019, 0.7098]	[0.6131, 1.5761]
	Risky	**Mean**	0.1641	0.6751	1.9699
		95%-Conf. Int.	[0.4841, -0.1559]	[0.1552, 1.1951]	[1.2357, 2.7041]
All values in this table are displayed in percentage					

Table 5.5 – Player's long run expectation using the point count strategy under different casino rules and various number of players

The results for the point count strategy, like the ten count strategy, depend largely on the conditions under which the player plays the game. These conditions are again the number of players at the table and the betting profile used by the player. Under the Holland casino rules the results are not particularly good, only when there are six other players at the table the player has an edge over the house. The reason for this is that the player can take into account the cards dealt to the other players before playing his own hand.

Under the classic rules the point count strategy performs slightly better than the ten count strategy. Again the player will in all conditions have an advantage over the house, ranging from 0.3% to 5.8%.

For the current rules, except for the Holland casino rules, the point count strategy performs also very positive and much better than the ten count strategy. It is not that the maximum edge over the house is much higher for the point count strategy compared to the ten count strategy, but more that under worse conditions the point count strategy performs much better than the ten count strategy.

The player always has the advantage over the house on the long run, ranging from 0% to 7.5% depending on the conditions. Most advantageous is playing under the Atlantic City rules, with no other players and using the risky betting scheme. On average also the Atlantic City rules perform best for the point count strategy of all the rules applied nowadays.

5.2.5 Zen count strategy

Rules	Risk profile	Statistic	Number of players		
			1	4	7
Classic rules	Conservative	**Mean**	1.4031	0.7200	0.2966
		95%-Conf. Int.	[0.9049, 1.9012]	[-0.0415, 1.4815]	[-0.5447, 1.1380]
	Moderate	**Mean**	3.1356	2.9022	1.7100
		95%-Conf. Int.	[2.1818, 4.0895]	[1.9421, 3.8622]	[0.8609, 2.5592]
	Risky	**Mean**	8.4893	5.6210	3.6010
		95%-Conf. Int.	[7.0675, 9.9111]	[4.1837, 7.0582]	[2.2150, 4.9871]
Las Vegas strip	Conservative	**Mean**	1.2397	0.8091	0.3137
		95%-Conf. Int.	[0.6183, 1.8610]	[0.2066, 1.4115]	[-0.4480, 1.0754]
	Moderate	**Mean**	4.8353	2.3949	2.0523
		95%-Conf. Int.	[4.0138, 5.6569]	[1.4466, 3.3431]	[1.2147, 2.8900]
	Risky	**Mean**	6.1932	3.4503	2.3774
		95%-Conf. Int.	[5.1428, 7.2436]	[2.0033, 4.8974]	[1.1538, 3.6011]
Las Vegas downtown	Conservative	**Mean**	1.5449	0.6573	0.0039
		95%-Conf. Int.	[0.8040, 2.2859]	[0.1191, 1.1955]	[-0.4906, 0.4984]
	Moderate	**Mean**	4.0672	1.7829	1.4302
		95%-Conf. Int.	[2.9418, 5.1927]	[0.7689, 2.7970]	[0.3396, 2.5209]
	Risky	**Mean**	7.2520	4.8909	3.3844
		95%-Conf. Int.	[5.9144, 8.5897]	[3.7210, 6.0608]	[2.0569, 4.7118]
Atlantic City	Conservative	**Mean**	1.6169	1.0903	0.7137
		95%-Conf. Int.	[1.0691, 2.1646]	[0.3430, 1.8377]	[-0.0011, 1.4285]
	Moderate	**Mean**	4.5106	3.7045	1.6997
		95%-Conf. Int.	[3.7523, 5.2689]	[2.7001, 4.7089]	[0.8116, 2.5877]
	Risky	**Mean**	7.4459	5.0415	3.0548
		95%-Conf. Int.	[6.0480, 8.8439]	[3.6798, 6.4031]	[1.6956, 4.4139]
Europe	Conservative	**Mean**	1.1731	0.8115	0.3113
		95%-Conf. Int.	[0.6974, 1.6489]	[0.0817, 1.5414]	[-0.1517, 0.7743]
	Moderate	**Mean**	3.5346	2.4971	1.1723
		95%-Conf. Int.	[2.5785, 4.4907]	[1.5805, 3.4138]	[0.0444, 2.3001]
	Risky	**Mean**	6.4845	3.2839	1.7966
		95%-Conf. Int.	[5.1587, 7.8103]	[2.1559, 4.4119]	[0.8222, 2.7709]
Holland casino	Conservative	**Mean**	-0.1374	0.2017	0.4741
		95%-Conf. Int.	[-0.4928, 0.2179]	[-0.0379, 0.4413]	[0.1553, 0.7928]
	Moderate	**Mean**	-0.1129	0.2753	1.1786
		95%-Conf. Int.	[-0.3172, 0.0913]	[-0.0215, 0.5722]	[0.7352, 1.6221]

Holland casino	Risky	**Mean**	0.1701	0.6544	2.3274
		95%-Conf. Int.	[-0.2427, 0.5829]	[0.2709, 1.0379]	[1.9513, 2.7036]
All values in this table are displayed in percentage					

Table 5.6 – Player's long run expectation using the zen count strategy under different casino rules and various number of players

Again the results for the zen count strategy like the previous strategies, depend largely on the conditions under which the game is played. Applying a riskier a betting scheme influences the player's long run expectation greatly. In lesser extent increasing the number of players also negatively impacts the player's long run expectation. Except again for the Holland casino rules for the same reasons as the previous strategies.

The results for the classic rules show that the zen count is the best strategy for the player to use. Increasing the player's long run expectation slightly compared to the point count strategy. The player's advantage over the house ranges between 0.3% and 8.5%, depending on the conditions.

Also for the current rules, the zen count strategy performs slightly better than the point count system. The player again in all situations has the advantage over the house on the long run, ranging from 0% to 7.45% depending on the conditions. Also the Atlantic City are the most advantageous rules for playing using the zen cunt strategy nowadays.

5.3 Player's expectation per time units

The player's long run expectation says something about where the expectation will convert to if the player plays long enough. However, most players will not play long enough for his return to convert to this expectation. Therefore this part of the simulation chapter is to see what happens to the player's expectation if a player plays for a certain amount of time units.

For this part only the best situation (based on the player's long run expectation) of last simulation part will be used, the zen count strategy under the Atlantic City rules. Three different time units will be simulated, these will include a player playing one night (4 hours), one day (8 hours) and one week (56 hours).

As shown in the first part of the simulation, the number of players at the table has a negative influence on the player's expectation. In this second part of the simulation, however, it is assumed that the player will have the privilege of being able to play alone at a table for the whole time period.

Lastly, the player's expectation for certain time units also depends on the starting capital of the player. For simulating the long run expectation of the player it is assumed that the player has unlimited amount of money and can take a very bad loss streak without running out of money. However, in this part the player can actually run out of money. There will be three different type of starting capitals simulated; $1000, $2500 and $5000. Remember that the minimum bet size in the Atlantic City casino is $15 and the maximum bet is $300.

5.3.1 Speed of blackjack

Stanford Wong in his book "Professional Blackjack" [10] performed an empirical study to find out how fast blackjack is played. He used an stopwatch to measure the time it takes to shuffle the cards and the time it takes to deal cards to various number of players. The empirical study consisted of 42 shuffles and 167 rounds of play for a variety of dealers at a variety of casinos.

The first part of the study was the shuffle time which is the total elapsed time from the instant the dealer broke the deck to the instant the first card was dealt on the next round. The shuffle time thus includes burning the first card and also any interruptions on the dealer during the shuffle. This resulted in a mean time of 23 seconds.

Secondly Wong measured the dealing time, the time from the instant the first card was dealt to the instant the first card was dealt on the following round or to the instant the dealer broke the deck for a shuffle. The dealing time also includes interruptions like players buying chips. The study showed that the mean dealing time increased approximately linearly with the number of players. The combined mean time for the dealer and the first player was 17 seconds. Each additional player took 7 more seconds to play a hand.

5.3.2 One night of play

Using the shuffle time and dealing time calculated by Stanford Wong, playing for one night equals 14400 seconds (4*60*60) and comes down to playing around 200 hands of blackjack. To get an accurate result a simulation of 2500 nights was done.

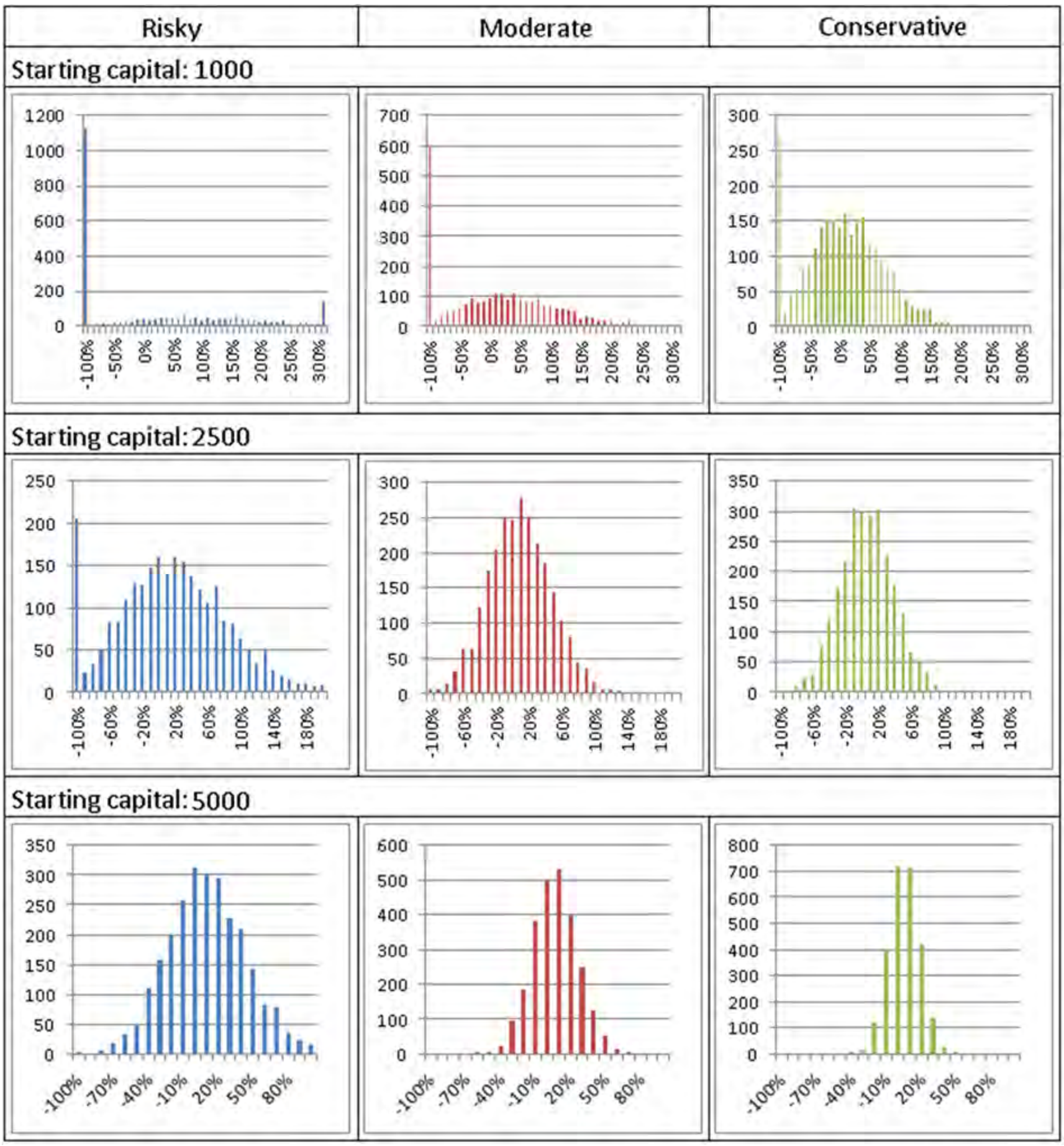


Figure 5.1 – Histogram of players using different betting schemes and with different starting capitals that play for one night (4 hours) under the Atlantic City rules.

The result show that any betting scheme a starting capital of $1000 is too little to play with, as the probability of running out of money is very high. Even for the conservative betting player the probability of running out of money is over 10%.

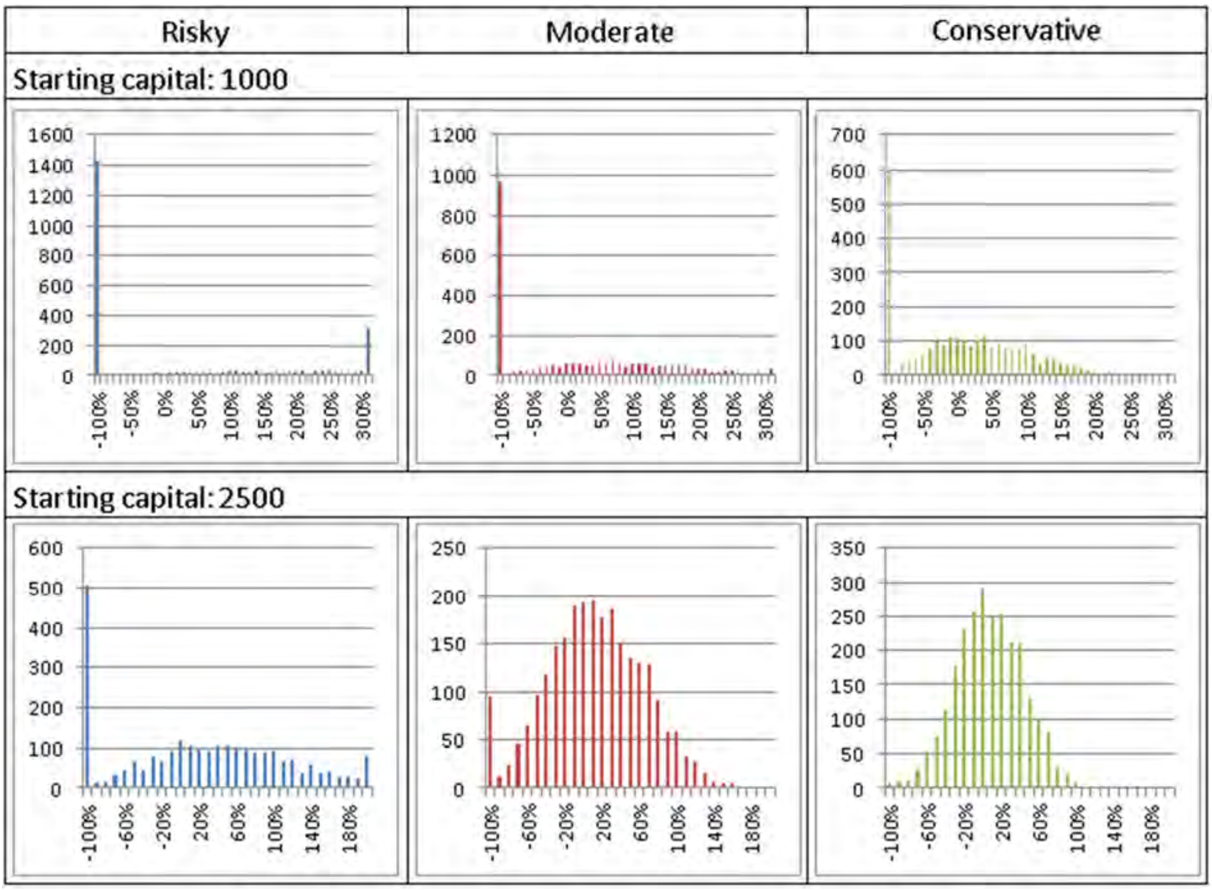
A risky betting player should not go into a casino using the Atlantic City rules with less than $5000. Even for a starting capital of $2500 the player has 8% change of losing all of his money. When the risky betting player does take a minimum of $5000 starting capital to the casino, the probability of ending up in the money is around 67% of the time. And making a profit of at least 50% about 15% of the time.

For the moderate betting player it also advisable to only enter a casino using the Atlantic City rules with at least $5000. As for only taking $2500 the probability of the player taking large losses is pretty high compared to taking $5000 as starting capital. The moderate betting player has a probability of 73% of ending up in the money when taking $5000 as starting capital.

Lastly, it is also advisable for the conservative better to take $5000 as starting capital, for the same reason as the moderate better. The probability of ending up in the money for the conservative player is about 78% when the starting capital is $5000. However the probability of gaining big profits of at least 20% or more is around 25% of the nights.

5.3.2 One day of play

One day of play equals 28800 seconds (8*60*60) and comes down to playing around 400 hands of blackjack. To get an accurate result a simulation of 2500 nights was done.



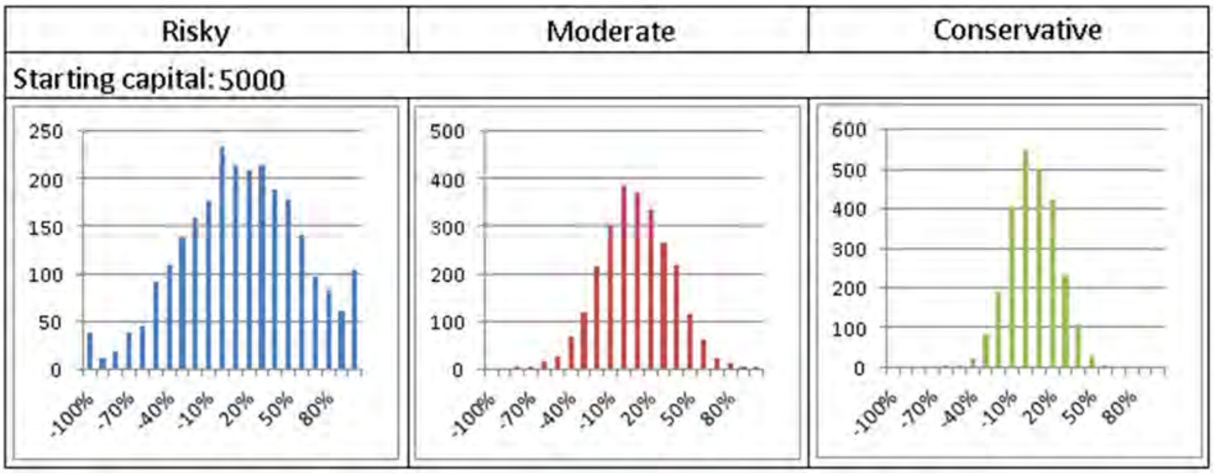


Figure 5.2 – Histogram of players using different betting schemes and with different starting capitals that play for one day (8 hours) under the Atlantic City rules.

If any player independent of his betting scheme intents to play at a casino using the Atlantic City rules and use the zen count strategy, he should at least take $5000 as starting capital. Taking less than that amount the probability of running out of money is way too high. Even over 50% for the risky betting player when taking $1000 as starting capital. Except for the conservative betting player it is not necessary to take $5000. For this player also $2500 as starting capital would suffice, although $5000 is preferable considering the high probability of large losses when only taking $2500.

For the risky player even a starting capital of $5000 does not suffice and it's advisable to take $10000 instead. The histogram of a risky player taking $10000 as starting capital for one night of play is displayed in figure 5.3. This figure shows that this player has a probability of ending up in the money equals 82.5%. And the probability of winning more than $2000 for this player is around 40%.

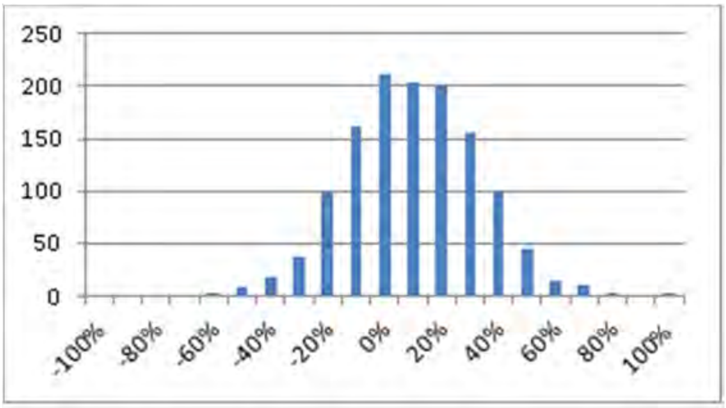


Figure 5.3 – Histogram of a simulation of 1000 nights for a risky player with a starting capital of $10000 under the Atlantic City rules.

The moderate betting player with a starting capital of $5000 will end up in the money around two thirds of the time. Also the probability of gaining big profits are also pretty high for the moderate better. A conservative player with a starting capital of $5000 will only lose money around 27% of the time when playing one night of blackjack.

5.3.2 One week of play

One week of play equals 201600 seconds (7*8*60*60) and comes down to playing around 2800 hands of blackjack. To get an accurate result a simulation of 1000 weeks was done. For this simulation also the starting capital of $10000 was included for all three types of betting schemes. The reason for this is that the minimum amount was already $5000 for one day of play for the moderate and risky betting scheme and it is expected to be higher for one week.

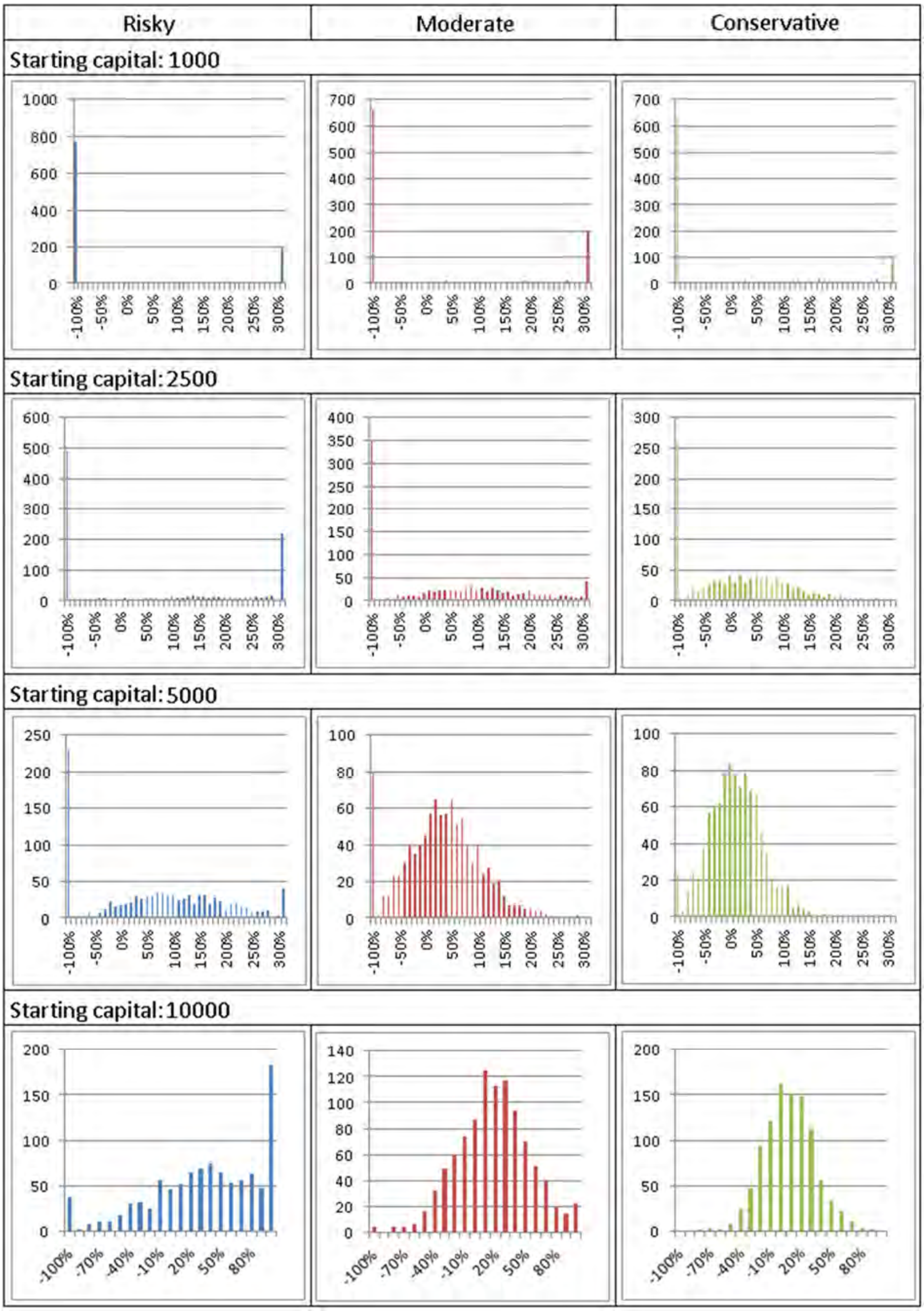


Figure 5.4 – Histogram of players using different betting schemes and with different starting capitals that play for one week (56 hours) under the Atlantic City rules.

The result from figure 5.4 show that for any player independent of their betting scheme and playing under the Atlantic City rules for one week a starting capital of at least $10000 is needed. For the conservative better $5000 could also suffice, but still the probability of running out of money or taking large losses is very high.

For the risky betting player the probability of running out of money when having a starting capital of $1000 is still around 4%. Therefore it is advisable for the risky player playing for one week in the Atlantic City casinos to take a starting capital of $20000. The result of a 1000 simulated weeks with a starting capital of $20000 for a risky betting player is shown in figure 5.5. This figure shows that a risky betting player playing in the Atlantic City casinos has a probability of ending up in the money of 81% when playing for one week. The mean expectation for this type of player is about 27.5% or $5000 profit. Remember that one week is 56 hours in this simulation, thus this comes down to a profit of around $90 per hour.

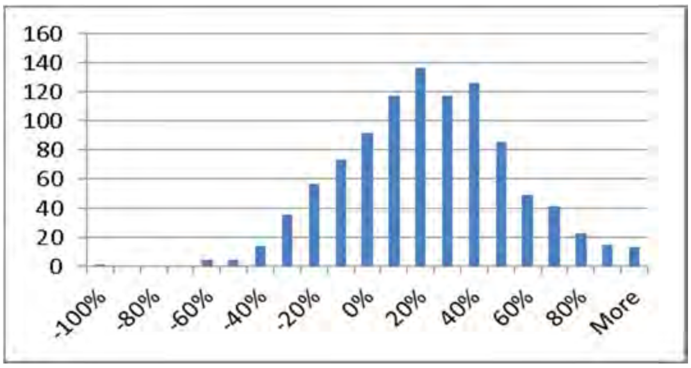


Figure 5.5 - Histogram of a simulation of 1000 weeks for a risky player with a starting capital of $20000 under the Atlantic City rules.

A moderate betting player using the starting capital of $10000 has a probability of 75% of ending up in the money. And a mean expectation of around 9% or in money terms, $900 per week which comes down to $16 per hour.

Finally, the conservative better playing a week at an Atlantic City casino using the zen count strategy has a probability of ending up in the money of about 70% with a starting capital of $10000. And a mean expectation of about 4%, which comes down to $7 per hour.

Chapter 6

"You have to be smart enough to understand the game
and dumb enough to think it matters."

Eugene McCarthy, on the similarity
Between politicians and football coaches.

Conclusion

The main question of this thesis was "can you still beat the dealer?". The answer to this question based solely on the results of the simulation should be yes for the rules in Las Vegas strip, Las Vegas downtown, Atlantic City and Europe. But the casinos using shuffling machines like the Holland casino can only be beaten under some conditions (high number of other players at the table) and the advantage for the player in the most favourable situation is too small to conclude that the casinos using shuffling machines can be beaten.

The results of the simulation of a certain amount of time units using the zen count strategy under the Atlantic City rules are also very positive. However, in this simulation the conditions were considered ideal, as it was assumed that there were no other players at the table. But even under these perfect conditions it is only really profitable to play blackjack when the player can use the risky betting scheme. However, the chance of being spotted as a card counter when applying this betting scheme in practice is very big.

To conclude, it is still very possible for a player to beat the dealer under the current rules. This is even possible for multiple strategies, with the remark that some strategies perform better than others. However, it is only possible to beat the dealer if a player only plays occasionally and keeps a low profile. To beat the dealer on a regular basis it is practically impossible without being spotted as a card counter. And once spotted as a card counter and therefore with your name and picture on a blacklist, it becomes nearly impossible to beat the dealer.

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