

Continuously Changing Correlations

How to model a correlation between risk factors?

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1 | Introduction

In this paper the behaviour of correlations between risk factors will be studied over time. In this chapter one will first discuss the background of the research and then pose the problem that we're looking to solve including. One will give a brief overview of the approach and the data used in this paper and how this fits into existing research in this field.

1.1 | Background

Insurance companies deal with many risk factors which have to be taken into account in their models. These risk factors may be correlated, which means that the risk factors may move in the same direction or in an opposite direction. Nowadays insurance companies that apply an internal model to calculate their Solvency Capital Requirement¹ tend to use stochastic simulation models where they use static (deterministic) correlations between risk factors. The number of risk factors an insurance company takes into account strongly depends on the company, however it may easily be over 100 risk factors. Unstable or unpredictable correlations make it very hard if not impossible to hedge the exposure of risk factors.

1.2 | Problem Statement

In the current situation insurance companies calculate the correlations between the different risk factors deterministically and keep them constant in their models for an entire year. The aim of this research is to determine if it is possible and more importantly useful to change the deterministic approach and make the correlations between risk factors dependent on time. The aim is to know what happens to correlations in times of economic stress. The following two sub questions should be answered in this paper:

- Can the correlations between certain market risk factors be linked to a probability density function that can describe the correlations well?
- Can the correlations between certain market risk factors be incorporated in insurance models in a non-deterministic way, such that they change over time? More specifically, what is the impact of incorporating this for the insurance company?

The main goal of this research is to investigate if there exists a non-deterministic way to calculate and incorporate correlations between risk factors in insurance models. This is done by first examining if correlations between risk factors can be linked to a probability density function, and second investigating non-deterministic methods to incorporate these correlations between risk factors in insurance models. Hence, the main components of this paper are:

- Evidence that the correlations between different risk factors can or can't be linked to a probability density function (based on the data that will be used in the research). This evidence can later be developed into a tool so that it can be used for insurance models.
- An indication of the effects of incorporating a non-deterministic approach in insurance models.

¹The required amount of funding that an insurance company in the European Union has to hold[10].

1.3 | Approach

For this research we are focussing on market risks. The data on selected market risk factors is obtained from Bloomberg². The downloaded data is the historical prices of the AEX index, the 6 month Euribor rate, the S&P 500 index and the exchange rate between Euros and US Dollars. The period in scope is January 4th, 1999 until August 1st, 2017. In this way one can investigate if the correlations between risk factors can either be linked to a probability density function or if they can be simulated by using for example stochastic modelling.

1.4 | Related work

In current literature there hasn't been written a lot about the correlations between risk factors. However in 1999 Boyer, Gibson and Loretan[4] wrote an article together on the pitfalls in tests for changing correlations. They argue that data should not be divided for normal and economically stressed situations but should be investigated as a whole³. Robert F. Engle[5] wrote a book on the importance of correlations in risk management, which does give a nice view on why this research is important, but doesn't analyse what happens to the correlation between a pair of commodities over time. Statistical analysis that will be the basis of this paper has rarely been performed on the correlation between the logarithmic returns of risk factors.

²Bloomberg is a company that provides historical financial data

³This separation between normal and economically stressed situations is called "correlation breakdowns"

2 | Methodology

For this research we are focussing on market risks. The downloaded data is the historical prices of the AEX index, the 6 month Euribor rate, the S&P 500 index and the exchange rate between Euros and US Dollars. The period in scope is January 4th, 1999 until August 1st, 2017. Note that in this paper we will only investigate and discuss the pairwise correlations between two risk factors and not more. The main reason for this is to keep the research a bit simplistic to perform small and exploratory research on the behaviour of correlations.

2.1 | Data Preprocessing

To avoid big data gaps in the data, weekends are skipped, and if there is data missing for one of the commodities, the value of the previous day will be taken.

A few demographics are being deduced from the date, such as the day of the week, the week-number, the year, etcetera. These demographics are used later to easier select subsets of the data and compute the correlation. Furthermore, a sequential number (or index) has been created for each complete week of data; this is to make sure that partial weeks at the beginning and the end of each year are taken together as one week.

Before computing any correlations one first needs to compute the daily logarithmic return[1], this can be computed using the following formula:

$$R = \ln \left(\frac{P_t}{P_{t-1}} \right)$$

where R represents the logarithmic return, P_t the price of the commodity at time t and P_{t-1} the price of the commodity at time $t - 1$. Note that in this case the logarithmic return is the same as the logarithmic rate of return as we compute it on a daily basis.

Correlation formula[3]: The correlation between two variables X and Y is computed using the following equation:

$$\rho(X, Y) = \frac{\text{Cov}(X, Y)}{\sigma(X) \cdot \sigma(Y)} = \frac{\mathbb{E}(X \cdot Y) - \mathbb{E}(X) \cdot \mathbb{E}(Y)}{\sigma(X) \cdot \sigma(Y)}$$

where $\rho(X, Y)$ depicts the correlation between variables X and Y , $\mathbb{E}(\cdot)$ represents the expectation and $\sigma(\cdot)$ the standard deviation.

The everyday correlation between the logarithmic rate of return of two commodities can be computed using the formula above. However the amount of data used to compute the correlation can be chosen. For example one could compute a weekly, biweekly or monthly correlation, using only the data within the given timeframe. Furthermore, one could use the data of two periods the current and the one preceding the current period. This will partially smooth the curve, and possibly make it better predictable.

After computing the correlations the investigation on distribution identification and other data patterns can take place. One can investigate whether the correlations fit a certain distribution on a particular scale and furthermore investigate whether other patterns are present in the data.

2.2 | Distribution identification

Now that the correlations between the risk factors have been calculated one can investigate whether the correlation data fits a particular distribution. One can use several methods of distribution investigation. First, a plot of the correlations over time will be given. After which histograms for each pair of risk factors and the different computation methods¹ will be created. Then we will investigate whether the distribution of the data is remotely symmetrical, and we will use QQ-plots to investigate whether the data fits a known distribution. In addition, boxplots will be created to give more insight as well.

If there will be a candidate distribution for a particular combination of risk factors, one needs to test that more thoroughly by using statistical tests pending on the suspected distribution. If a normal distribution is suspected one will use the Shapiro-Wilk normality test and for any other distribution the Kolmogorov-Smirnov test can be applied.

2.2.1 Goodness of fit tests

The Shapiro-Wilk normality test[2]

With the Shapiro-Wilk normality test one can test the composite null hypothesis, H_0 , that the sample belongs to a normal distribution with a real mean and a variance above 0 against the alternative hypothesis, H_1 , that the sample is not normally distributed. That is:

$$\begin{aligned} H_0 : F &\in \{\mathcal{N}(\mu, \sigma^2); \mu \in \mathbb{R}, \sigma^2 > 0\} \\ H_1 : F &\notin \{\mathcal{N}(\mu, \sigma^2); \mu \in \mathbb{R}, \sigma^2 > 0\} \end{aligned}$$

where F is the sample distribution, and $\mathcal{N}(\mu, \sigma^2)$ represents a normal distribution with mean μ and variation σ^2 .

The test statistic, W for the Shapiro-Wilk normality test for a random variable X is given by:

$$W = \frac{(\sum_{i=1}^n a_i X_{(i)})^2}{\sum_{i=1}^n (X_i - \bar{X})^2} \in (0, 1]$$

where a_1, \dots, a_n are certain constants and $X_{(1)}, \dots, X_{(n)}$ are the order statistics. Note that X_i is the i -th value of variable X and that \bar{X} is the average value of the random variable X , that is:

$$\bar{X} = \frac{1}{n} \sum_{i=1}^n X_i$$

The null hypothesis, H_0 , is rejected for “small” values of W and p -value $< 1 - \alpha$, where α is the desired confidence level. Small is in this context a slightly vague expression, since a value of 0.9 for W could be considered small.

¹biweekly, monthly, biweekly and monthly based on two periods and daily based on a specific number of days back

The Kolmogorov-Smirnov test[2]

Using a one-sample Kolmogorov-Smirnov test to test whether the sample originates from the suspected distribution, tests the following:

$$\begin{aligned}H_0 : F &= F_0 \\H_1 : F &\neq F_0\end{aligned}$$

where F represents the sample distribution and F_0 the suspected distribution.

The test statistic, D_n , for the Kolmogorov-Smirnov test is given by:

$$D_n = \sup_{-\infty < x < \infty} |\hat{F}_n(x) - F_0| \quad \text{with } \hat{F}_n(x) = \frac{1}{n} \sum_{j=1}^n 1_{(X_j \leq x)}$$

where x_1, \dots, x_n are the realizations from random variables X_1, \dots, X_n that are independent and identically distributed. The indicator $1_{(X_j \leq x)}$ equals 0 or 1 when $X_j > x$ or $X_j \leq x$, respectively. Hence, the random variable $n\hat{F}_n(x)$ equals the number $\#(X_j \leq x)$.

The null hypothesis is rejected for large values of D_n .

2.3 | Impact of incorporating non fixed correlations

To investigate what the impact of the proposed method on an insurance company will be, simulation will be used to give an idea of how much extra time is needed to compute the necessary correlations several times per year. The simulation needs to estimate the additional computation time on a yearly basis that is needed to compute the correlations.

In order to so a function has been created that will compute a new correlation based on the number of risk factors involved, the number of times per year the correlations should be updated and a number of simulations that should be performed.

The function should first compute the logarithmic rate of return for each risk factor. Then for each combination of risk factors it should compute the new correlation based on the previous data and the newly added logarithmic rate of return.

Pending on how many times a year one would like to update the correlations in the model, the computation of the correlations should be performed multiple times, in the pseudo code this amount is named `numberOfComputationsPY`.

The entire procedure should also be repeated multiple times, to find a reliable estimation of the time necessary to perform the computations. The number of repetitions of the procedure is given by `numberOfTests` in the pseudo code.

In the box below one can see the pseudo code of such a simulation function:

```
Simulation pseudo code:
simulationComptime <- function(numberOfComputationsPY, numberOfRiskFactor, numberOfTests) {
  numberOfRiskFactorCombinations <- choose(numberOfRiskFactor, 2)
  starttime <- Sys.time()

  #compute the latest rate of returns
  for(index in 1:numberOfRiskFactor) {
    new_RR1_riskfactor <- log>LastPrice[today]/LastPrice[yesterday]
  }
  for(i in 1:numberOfTests) {
```

```

for(j in 1:numberOfComputationsPY){
  for(k in 1:numberOfRiskFactorCombinations){
    #combine with a range of previous rate of returns from the same commodities
    CORdata1 <- rbind(cbind(previous_RRX, previous_RRY),
                     c(new_RR1_riskfactorX,new_RR1_riskfactorY))
    #and compute the correlation based on the latest data
    cor(CORdata1,use = "pairwise.complete.obs")[1,2]
  }
}

endtime <- Sys.time()
totalTime <- as.numeric(endtime-starttime, units="secs")
averageTime <- totalTime/numberOfTests

implications1 <- as.data.frame(cbind(numberOfComputationsPY
                                     ,numberOfRiskFactor
                                     ,numberOfRiskFactorCombinations
                                     ,numberOfTests
                                     ,starttime
                                     ,endtime
                                     ,totalTime
                                     ,averageTime))

return(implications1)
}

```

Note that the function `choose(n,m)` returns the number of possible combination given the number of options, n and the amount that you want to choose, m ²

²Compare the choose function from R to the nCr-button on a calculator.

3 | Initial Data Analysis

Within the data analysis one has differentiated between the following methods of computing the correlations between the risk factors:

- biweekly using only data from the two weeks in scope
- biweekly using data from the two weeks in scope and the data from the two weeks prior to the current period
- monthly using only the data from the month at hand
- monthly using the data from the month at hand and the data from the month prior to the current month
- daily using the data from an n number of days back, in this paper we will use $n \in \{15, 20\}$.

3.1 | A first glance

To get some basic idea of how the correlations between each pair of risk factors compares to one another one has created a set boxplots for the biweekly, the monthly and the daily (20 days based) computation of the correlation:

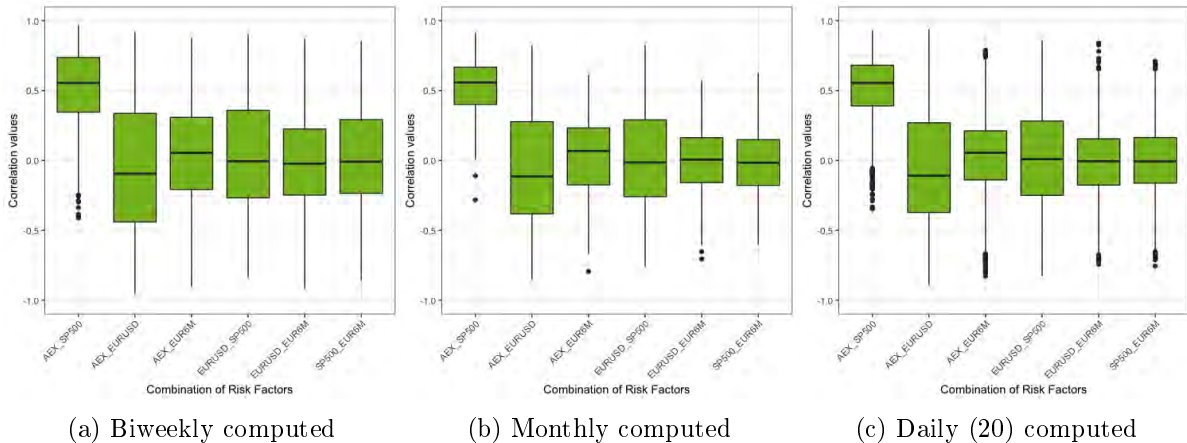


Figure 3.1: Boxplots of the correlations between the different risk factor combinations.

As depicted in Figure 3.1 the correlation between the AEX index and the S&P500 behaves considerably different than the other ones. Furthermore note that the boxes and whiskers are smaller when using the monthly computation for all combinations of risk factors. When looking at the daily computation this seems less clear but keep in mind that these boxplots contain almost 22 times as many entries as the monthly computation and nearly 10 times as many as the biweekly computation. Therefore it is more likely that certain data points are considered outliers and that the spread might be bigger, but is probably more stable over time.

Note furthermore that the correlations “EURUSD exchange rate vs. 6 month Euribor” and “S&P500 vs. 6 month Euribor” seem to be highly comparable to one another.

Next to the created boxplots, plots of the correlations over time, also known as timeseries[6], have been made. In Figure 3.2 one can see how the correlation between AEX and S&P500 fluctuates over time. In Figure 3.3 one can see how the correlation between the EURUSD exchange rate

and the 6 month Euribor moves. As can be seen in both figures the correlation shows very volatile behaviour.¹

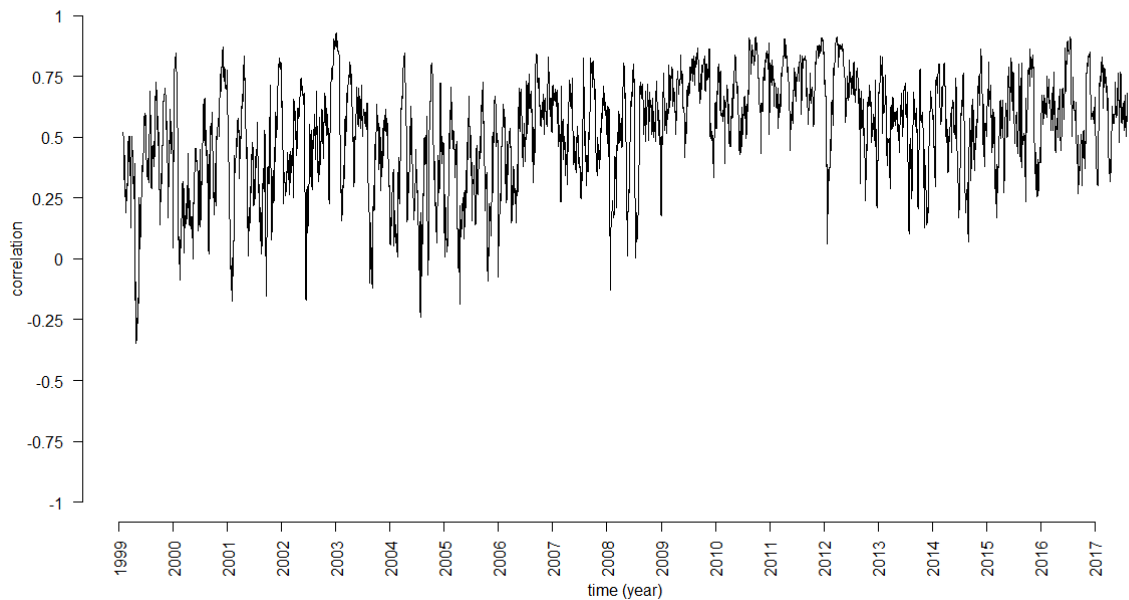


Figure 3.2: The timeline of daily (20 days based) computed correlations between the AEX and the S&P500.

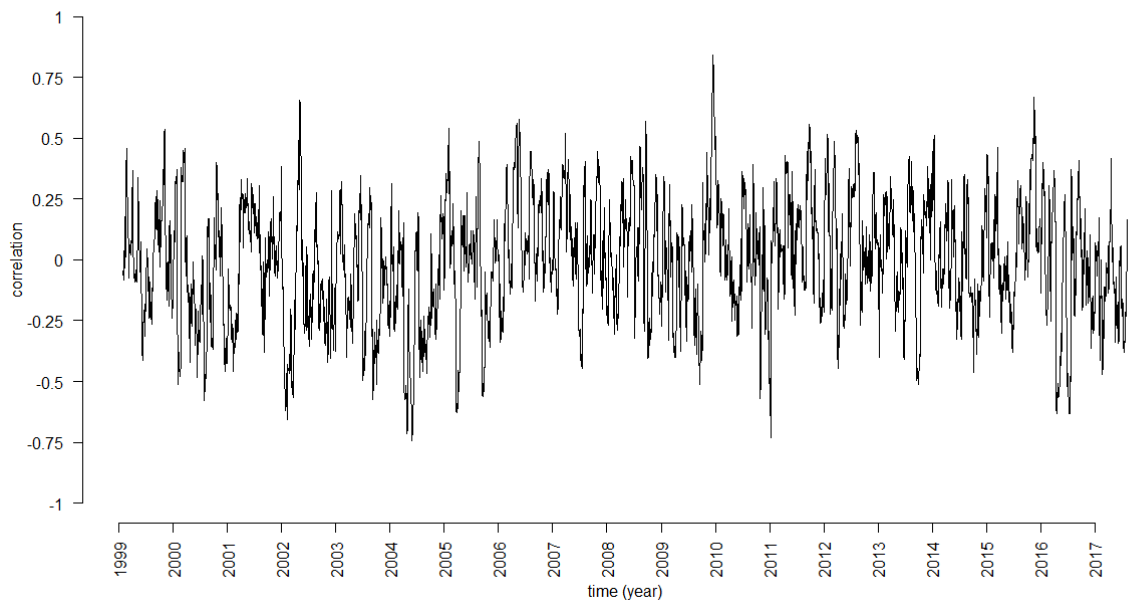


Figure 3.3: The timeline of daily (20 days based) computed correlations between the EURUSD exchange rate and the 6 month Euribor.

¹The other correlations between risk factors show the same kind of volatility. The corresponding timeline figures (Figures B.4 to B.7) have been placed in Appendix B.

The volatility of the correlations even when using an rolling horizon approach means that a predictive model will most likely not improve the current method of fixing the correlation between two risk factors for an entire year.

3.2 | The influence of the data taken into account per correlation

Now that one has some basic idea of how the correlations compare to each other one moves to investigate the individual correlations. The first investigation into the individual correlations is on the histograms of the correlations between the AEX index and the S&P500. One will show all the created histograms to investigate not only the distribution of the correlation but also investigate the influence of the computation technique.

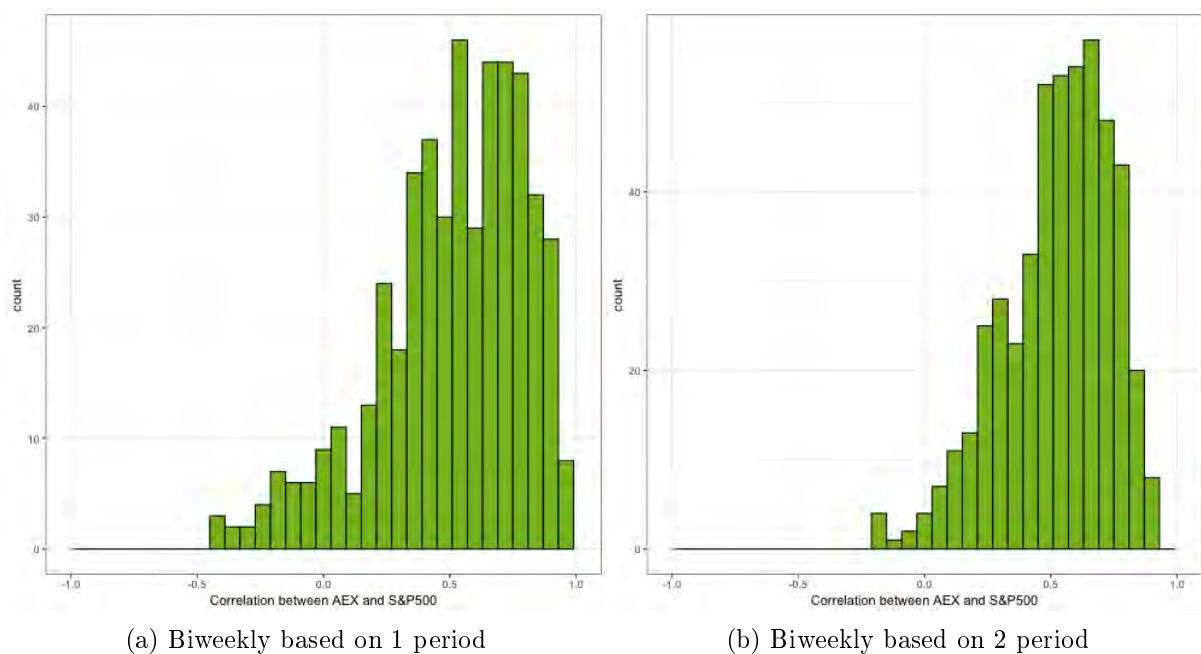


Figure 3.4: Histograms of the biweekly correlations between the AEX index and the S&P500 index.

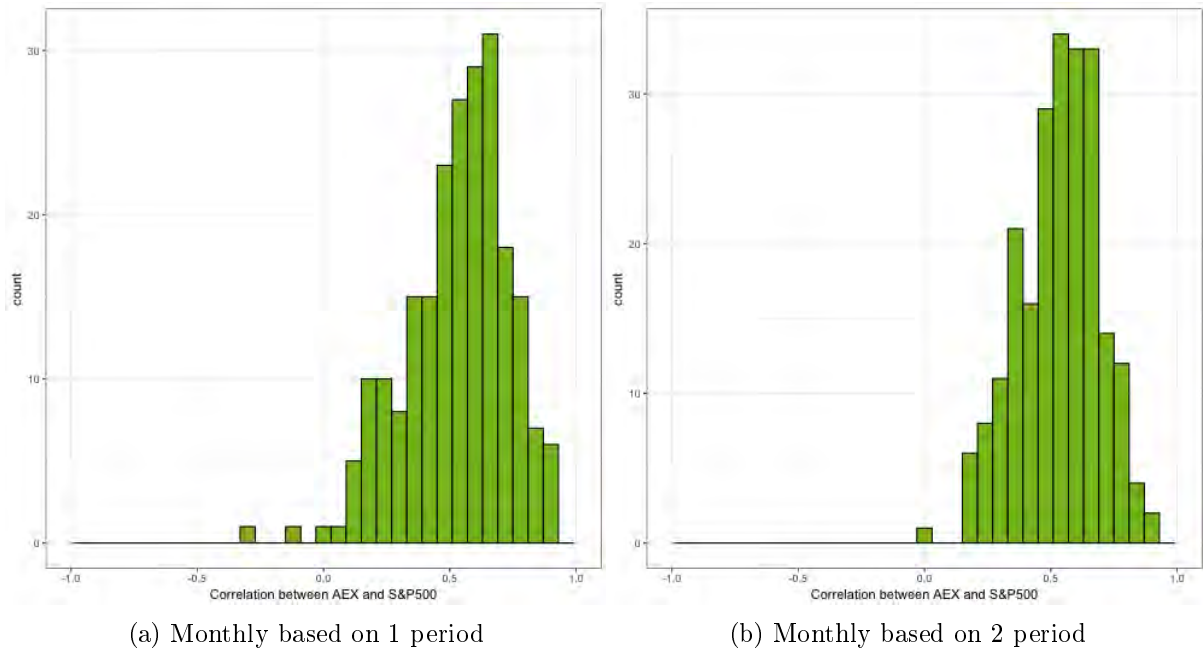


Figure 3.5: Histograms of the monthly correlations between the AEX index and the S&P500 index.

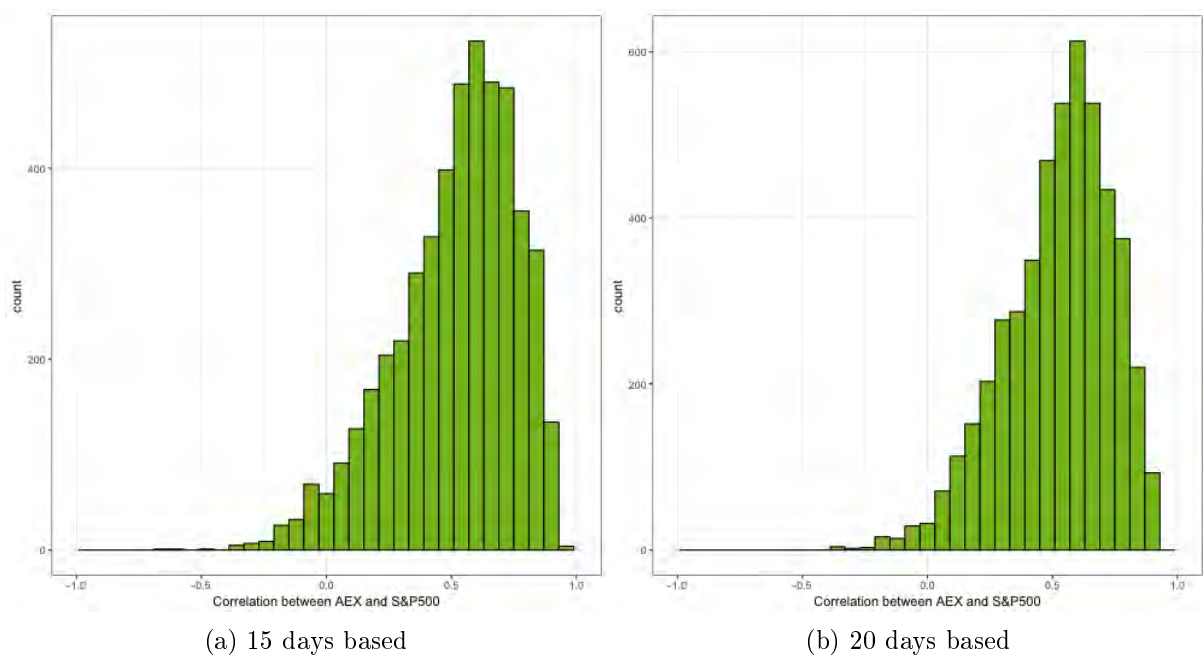


Figure 3.6: Histograms of the daily correlations between the AEX index and the S&P500 index.

The 20 day based daily computation evidently creates the most distinctive picture. This has a couple of underlying causes, such as the amount of data that goes into each computation of one correlation value, the daily computation which results in smaller differences between every two computations and furthermore generates daily data-points instead of one every two weeks or one a month.

3.3 | Distribution identification

In this section we will investigate whether the correlations can be linked to a probability distribution function[2], which if found may be a suitable starting point to model the correlation properly over time.

3.3.1 Histograms

For investigating the distribution of the correlations for each of the other combinations of risk factors one will keep on using the daily computation as it helps to give a more distinctive picture.

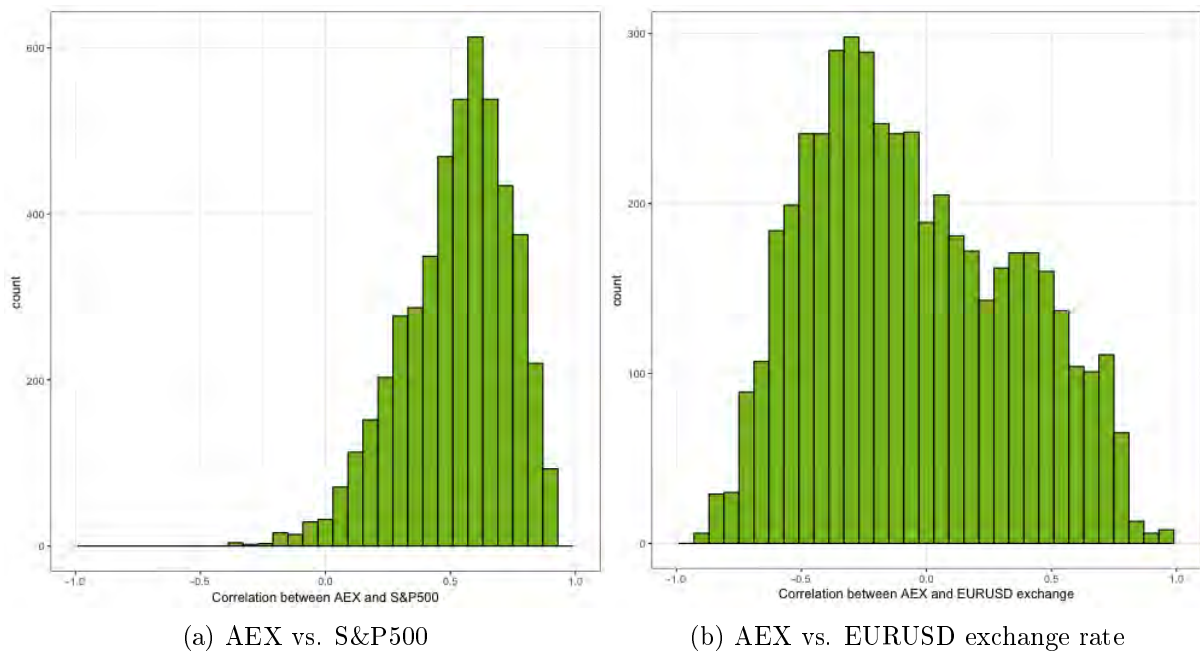


Figure 3.7: Histograms of the daily correlations based on 20 days of data.

In Figure 3.6b one can see that the correlation between AEX and S&P500 is mostly positive and that it seems to have a parabolic shape. In Figure 3.7b one can see that the correlations have been positive and negative and not evenly distributed.

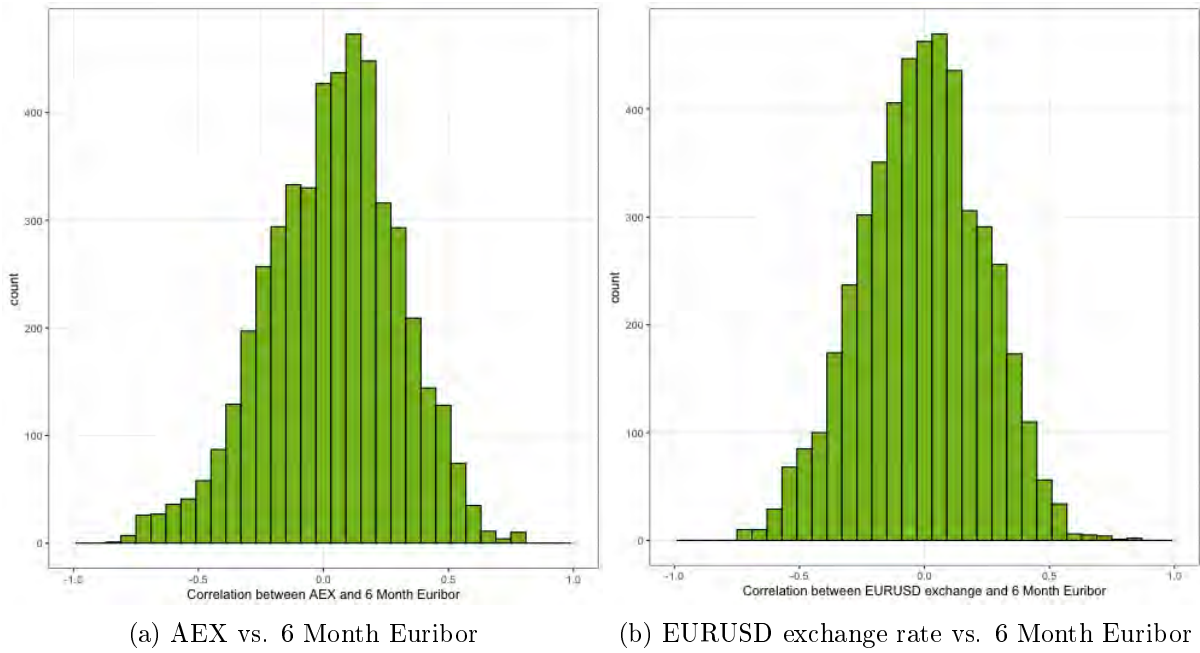


Figure 3.8: Histograms of the daily correlations based on 20 days of data.

Given the histogram in Figure 3.8a one could suspect that the correlation between AEX and 6 month Euribor follows a normal distribution and that it peaks a little over zero. In Figure 3.8b one can see that the correlation between the EURUSD exchange rate and the 6 month Euribor shows even stronger indications of a normal distribution.

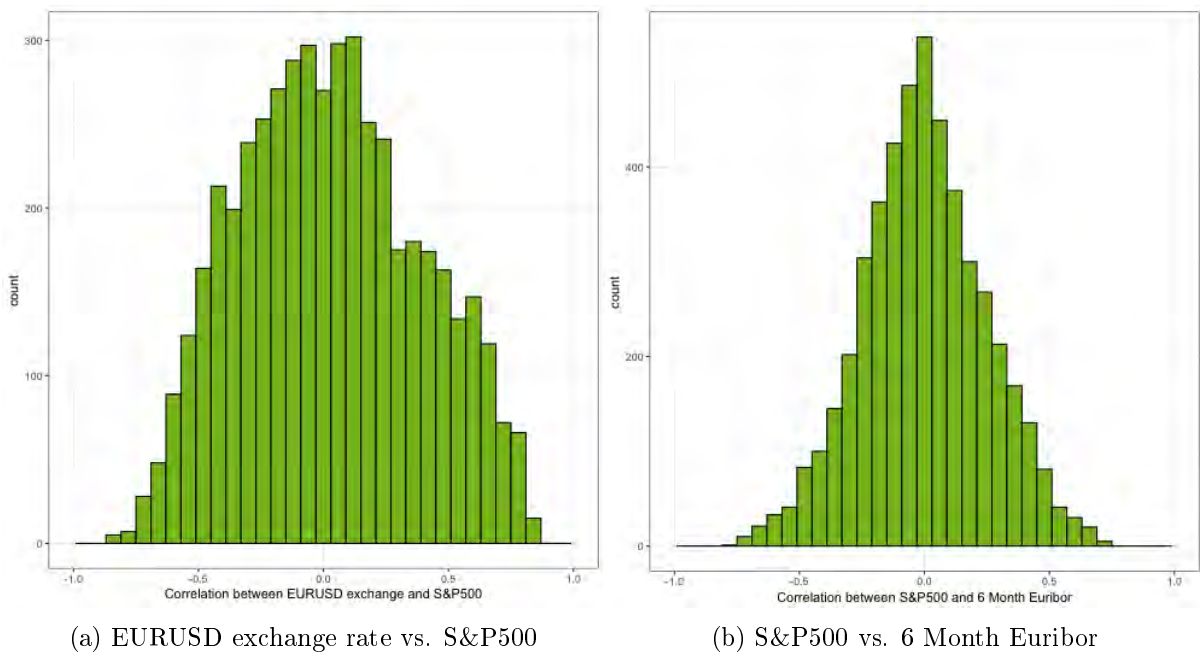


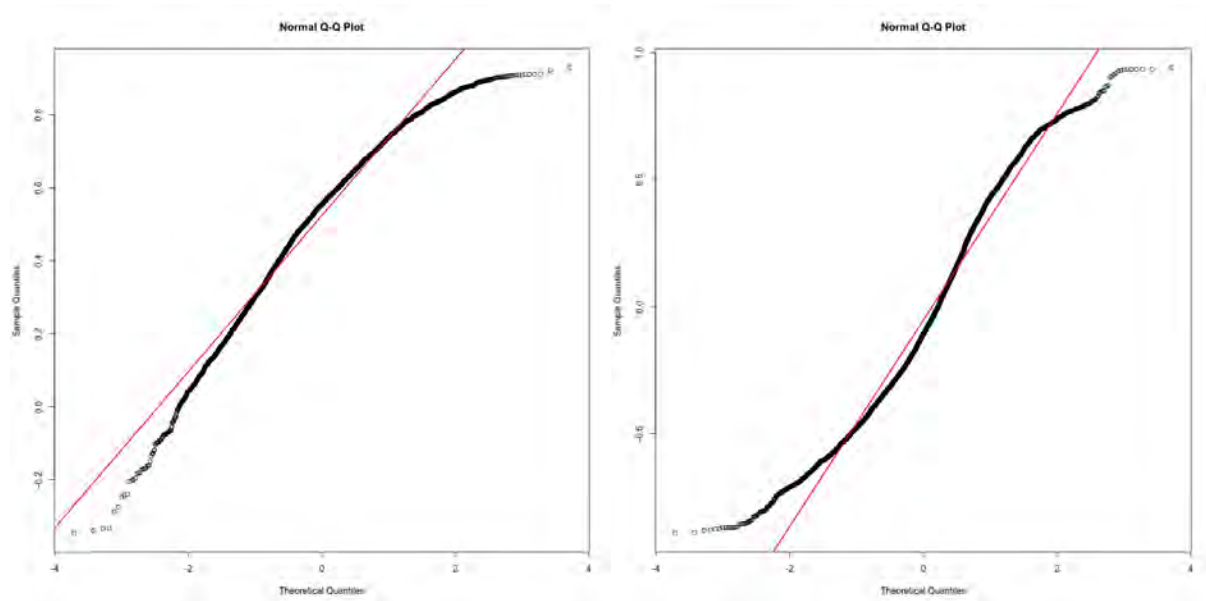
Figure 3.9: Histograms of the daily correlations based on 20 days of data.

In Figure 3.9a one can see that the correlation between EURUSD exchange rate and S&P500 seems to have a some sort of an uneven parabolic shape. In Figure 3.9b one can see that the correlation between S&P500 and 6 month Euribor has an eminent peak around zero and has a steep slope directly surrounding zero which becomes less steep as the correlation moves further

away from zero.

3.3.2 QQ-plots

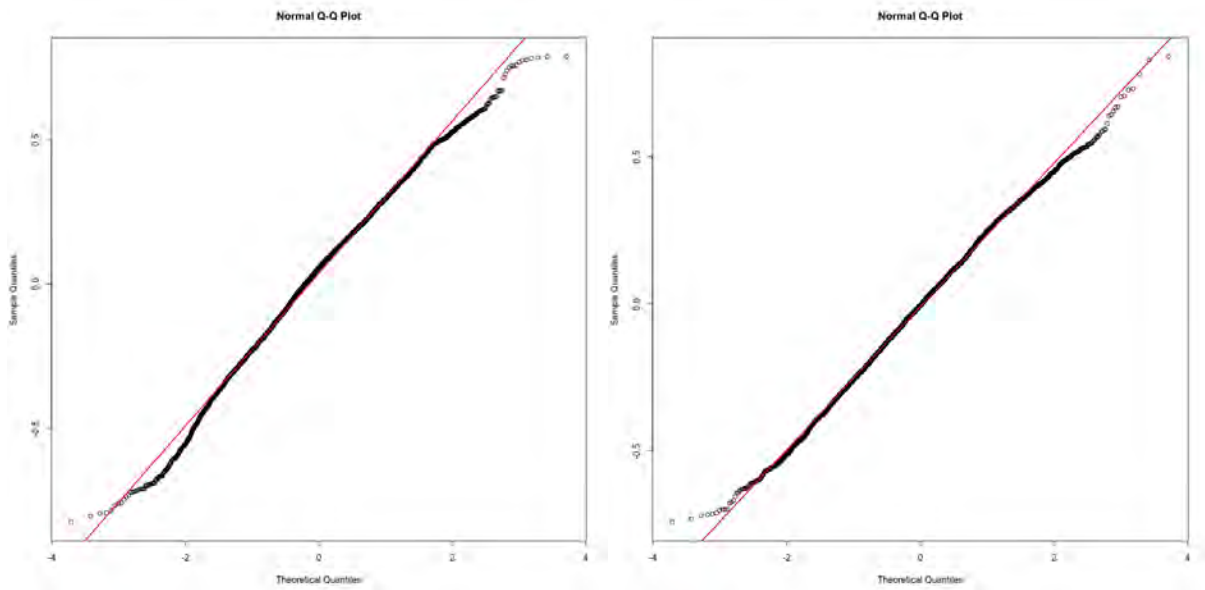
Investigating these distributions more thoroughly one has created QQ-plots for different distributions with different degrees of freedom if applicable. The majority of the QQ-plots are shown in Figures B.8 to B.13 in Appendix B. As one could previously see in the histograms the most interesting QQ-plot will be the QQ-plots considering a normal distribution, therefore the QQ-plots for the normal distribution and the daily (20 days based) correlations will be shown here.



(a) AEX vs. S&P500

(b) AEX vs. EURUSD exchange rate

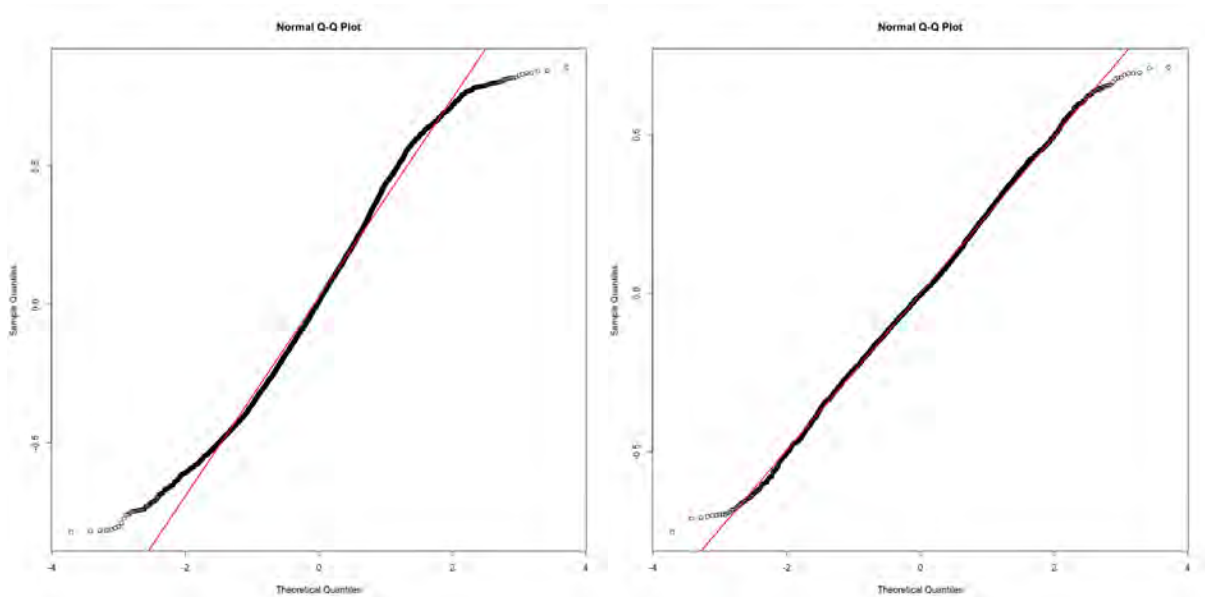
Figure 3.10: QQ plots of the daily correlations based on 20 days of data.



(a) AEX vs. 6 month Euribor

(b) EURUSD exchange rate vs. 6 month Euribor

Figure 3.11: QQ plots of the daily correlations based on 20 days of data.



(a) EURUSD exchange rate vs. S&P500

(b) S&P500 vs. 6 month Euribor

Figure 3.12: QQ plots of the daily correlations based on 20 days of data.

As one can easily see in Figures 3.12 and 3.12b, the correlations between the 6 month Euribor and the other risk factors seem to follow some sort of the normal distribution². The remaining combinations of risk factors (those where the 6 month Euribor is not involved) do evidently not follow a normal distribution.

²They show a more or less straight diagonal line from the left bottom corner to the right upper corner of the QQ-plot[2].

4 | Results

4.1 | Goodness of fit tests

To test the hypotheses that the 6 month Euribor and the AEX, S&P500 and EURUSD exchange rate follow indeed a normal distribution one can use the Shapiro-Wilk test. Recall that the composite null hypothesis and the alternative hypothesis are given by:

$$H_0 : F \in \{\mathcal{N}(\mu, \sigma^2); \mu \in \mathbb{R}, \sigma^2 > 0\}$$

$$H_1 : F \notin \{\mathcal{N}(\mu, \sigma^2); \mu \in \mathbb{R}, \sigma^2 > 0\}$$

The test statistic, W , for the Shapiro-Wilk normality test is given by:

$$W = \frac{(\sum_{i=1}^n a_i X_{(i)})^2}{\sum_{i=1}^n (X_i - \bar{X})^2} \in (0, 1]$$

In Table 4.1 one can see the results of the Shapiro-Wilk normality tests that have been performed. Note that in the last column one can see whether the (composite) null hypothesis, H_0 , is rejected or not.

Risk factor	Test-statistic (W)	p-value	Testresult
AEX	0.99492	5.56×10^{-12}	H_0 is rejected
EUR-USD exchange rate	0.99809	1.17×10^{-5}	H_0 is rejected
S&P500	0.99823	2.88×10^{-5}	H_0 is rejected

Table 4.1: The results of the Shapiro-Wilk normality test of the correlation between the 6 month Euribor and the names risk factor.

The composite null hypothesis is rejected since all p -values are less than $1 - \alpha$ even if α would have been set to 0.995 which corresponds with a confidence level of 99.5%. Furthermore, W is not small as it is almost as big as it can be. The results of the Shapiro-Wilk test suggest that the correlations between the 6 month Euribor and the other commodities are not normally distributed. This is not a surprise as the normal distribution could also get a value below -1 or above 1 .

4.2 | Simulation

The average time that is needed to implement the frequently updating correlations is evidently dependent on the number of risk factors. The number of correlations that should be computed is given by the number of possible combinations of two different risk factors, that is, $\frac{n \times (n - 1)}{2 \times 1}$. In Table 4.2 an overview is given of the amount of risk factors involved and the update frequency in number of times per year. In the column ‘‘Average Time (s)’’ the average computational time is given on a yearly basis. The results have been obtained by finding the average time to perform

the necessary computations of correlations between each pair of risk factors 100 times. Note that the correlations computed in this simulation are pairwise en based on 20 datapoints each.

Risk factors	Computations per year	Starttime	Endtime	Total Time (hh:mm:ss)	Average Time (s)
50	365	22/04/2018	22/04/2018	0:30:45	18.4589
		13:37:20	14:08:06		
	52	22/04/2018	22/04/2018	0:04:21	2.6102
		14:08:06	14:12:27		
26	22/04/2018	22/04/2018	0:02:20	1.4002	
	14:12:27	14:14:47			
12	22/04/2018	22/04/2018	0:01:05	0.6516	
	14:14:47	14:15:52			
100	365	22/04/2018	22/04/2018	2:04:57	74.9722
		14:15:52	16:20:49		
	52	22/04/2018	22/04/2018	0:17:31	10.5124
		16:20:49	16:38:20		
26	22/04/2018	22/04/2018	0:08:47	5.2748	
	16:38:20	16:47:08			
12	22/04/2018	22/04/2018	0:04:03	2.4343	
	16:47:08	16:51:11			
200	365	22/04/2018	23/04/2018	8:19:56	299.9648
		16:51:11	01:11:08		
	52	23/04/2018	23/04/2018	1:10:27	42.2773
		01:11:08	02:21:35		
26	23/04/2018	23/04/2018	0:35:10	21.1072	
	02:21:35	02:56:46			
12	23/04/2018	23/04/2018	0:16:13	9.7396	
	02:56:46	03:13:00			

Table 4.2: The simulation results

From Table 4.2 it becomes clear that even when considering 200 risk factors to be in scope on average it takes a mere 5 minutes per year to compute the correlation daily when basing the correlation on 20 datapoints.

5 | Conclusion

In this chapter one will answer the questions posed in the introduction:

- Can the correlations between certain market risk factors be linked to a probability density function that can describe the correlations well?
- Can the correlations between certain market risk factors be incorporated in insurance models in a non-deterministic way, such that they change over time? More specifically what is the impact of incorporating this for the insurance company?

Furthermore the some suggestions for further research will be discussed and what could be a simple intermediate solution for updating the correlation within the bigger model more frequently.

5.1 | Findings

During this small research it became clear that the correlation between two risk factors doesn't always show the same behavioural pattern. Therefore a simple solution as to finding a non-deterministic approach to find or predict the correlation is ambiguous. Given the Shapiro-Wilk test results one can conclude that there is no evidence that the correlation between any of the risk factors in this behaviour follows a normal distribution. This result is expected as a normal distribution will not have a clear cut off at -1 and 1 which does happen with correlations. Furthermore one doesn't seem to have another candidate distribution that might fit better.

One has looked at the influence of computation methods, that is, the amount of data points that should be taken into account for the computation of one single correlation. From the tested computational methods the daily computed correlation based on 20 days of data seems to be most suitable for research. However one could increase this number to 25 or 30, which may lead to higher stability in the timeseries. In this paper the number 20 has been chosen as it is the closest to the average number of trading days in a month and doesn't take partial weeks. This is in agreement with the methods posed by McNeil, Frey and Embrechts (2015)[8].

In addition, one has looked into how frequently such a computation should be performed. Frequently updated computations should be preferred to less frequently updated. However the computational time is possibly not available and therefore an insurance company might opt for updating only once a week, or even once a month. The number of risk factors in scope is leading in this, as it influences the number of necessary computations the most. Note that the computational time is highly dependable on the machines specifications and whether the machine is performing more tasks simultaneously.¹

5.2 | Further Research

The found results definitely ask for more research on correlations between market risk factors, as they do not all behave in the same way, there is not just one simple solution that can directly substitute current methods. In this paper we have looked into pairwise correlations, but we

¹The computations in this paper have been performed on a HP EliteBook, with a Intel(R) Core(TM) i5-6300U CPU @ 2.40GHz 2.50 GHz processor and 16 GB (15.9 GB usable) memory.

haven't looked at the increasing dimensionality, that is, what happens between has not been investigated. Furthermore one could also look into the possibility of applying copulas.

Clearly in this paper we've strictly looked at the correlations between the logarithmic returns of two commodities, and not at the underlying commodities. In further research one could also look at more than two commodities simultaneously and investigate the underlying commodities as well.

In the meantime one could implement a simple "scraper" that pulls the necessary closing prices² on a daily bases and then computes the rate of return and subsequential computes the correlations. This will at least update the used information on a daily basis.

Pending on the existing model, computation time and associated costs of updating (running) the model on a daily basis, this would be a feasible and satisfying solution. Furthermore the solution doesn't involve a highly complicated model that needs to be updated itself³.

²The term necessary is used as it differs per company which closing prices are relevant.

³Cathy O'Neil wrote a book called *Weapons of Math Destruction* on the problems with complicated mathematical systems and their use of such systems in our daily lives.

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A | Tables

Correlation	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
AEX vs. S&P500	-0.41	0.35	0.56	0.51	0.74	0.97
AEX vs. EURUSD	-0.96	-0.44	-0.09	-0.06	0.34	0.92
AEX vs. EUR6M	-0.90	-0.21	0.05	0.05	0.31	0.88
EURUSD vs. S&P500	-0.84	-0.27	-0.01	0.02	0.36	0.90
EURUSD vs. EUR6M	-0.92	-0.25	-0.02	-0.01	0.23	0.87
S&P500 vs. EUR6M	-0.86	-0.23	-0.01	-0.00	0.29	0.85

Table A.1: Statistics of the biweekly computed correlations.

Correlation	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
AEX vs. S&P500	-0.20	0.41	0.56	0.53	0.69	0.91
AEX vs. EURUSD	-0.87	-0.38	-0.10	-0.06	0.26	0.92
AEX vs. EUR6M	-0.71	-0.13	0.07	0.04	0.21	0.78
EURUSD vs. S&P500	-0.82	-0.24	-0.01	0.02	0.27	0.81
EURUSD vs. EUR6M	-0.70	-0.18	-0.00	-0.01	0.15	0.78
S&P500 vs. EUR6M	-0.76	-0.17	0.01	-0.00	0.14	0.60

Table A.2: Statistics of the 2 period based biweekly computed correlations.

Correlation	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
AEX vs. S&P500	-0.28	0.40	0.56	0.53	0.67	0.91
AEX vs. EURUSD	-0.85	-0.38	-0.11	-0.05	0.28	0.82
AEX vs. EUR6M	-0.79	-0.17	0.07	0.04	0.23	0.61
EURUSD vs. S&P500	-0.76	-0.26	-0.01	0.02	0.29	0.82
EURUSD vs. EUR6M	-0.70	-0.16	0.01	-0.00	0.16	0.57
S&P500 vs. EUR6M	-0.60	-0.18	-0.02	-0.00	0.15	0.63

Table A.3: Statistics of the monthly computed correlations.

Correlation	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
AEX vs. S&P500	-0.02	0.43	0.55	0.53	0.65	0.88
AEX vs. EURUSD	-0.75	-0.35	-0.11	-0.05	0.24	0.78
AEX vs. EUR6M	-0.62	-0.11	0.05	0.03	0.16	0.46
EURUSD vs. S&P500	-0.64	-0.22	-0.01	0.02	0.25	0.80
EURUSD vs. EUR6M	-0.63	-0.12	-0.00	-0.01	0.11	0.42
S&P500 vs. EUR6M	-0.49	-0.12	-0.01	-0.01	0.09	0.45

Table A.4: Statistics of the 2 period based monthly computed correlations.

Correlation	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
AEX vs. S&P500	-0.67	0.37	0.56	0.52	0.70	0.97
AEX vs. EURUSD	-0.91	-0.39	-0.11	-0.05	0.28	0.95
AEX vs. EUR6M	-0.84	-0.15	0.05	0.04	0.25	0.88
EURUSD vs. S&P500	-0.88	-0.26	0.02	0.03	0.31	0.89
EURUSD vs. EUR6M	-0.79	-0.20	-0.01	-0.01	0.18	0.86
S&P500 vs. EUR6M	-0.78	-0.20	-0.01	-0.00	0.20	0.83

Table A.5: Statistics of the daily (15 days based) computed correlations.

Correlation	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
AEX vs. S&P500	-0.35	0.39	0.55	0.52	0.68	0.93
AEX vs. EURUSD	-0.89	-0.37	-0.11	-0.05	0.27	0.94
AEX vs. EUR6M	-0.83	-0.14	0.06	0.04	0.21	0.79
EURUSD vs. S&P500	-0.83	-0.25	0.01	0.03	0.28	0.86
EURUSD vs. EUR6M	-0.74	-0.18	-0.01	-0.01	0.16	0.84
S&P500 vs. EUR6M	-0.76	-0.16	-0.01	-0.00	0.16	0.71

Table A.6: Statistics of the daily (20 days based) computed correlations.

B | Remaining Figures

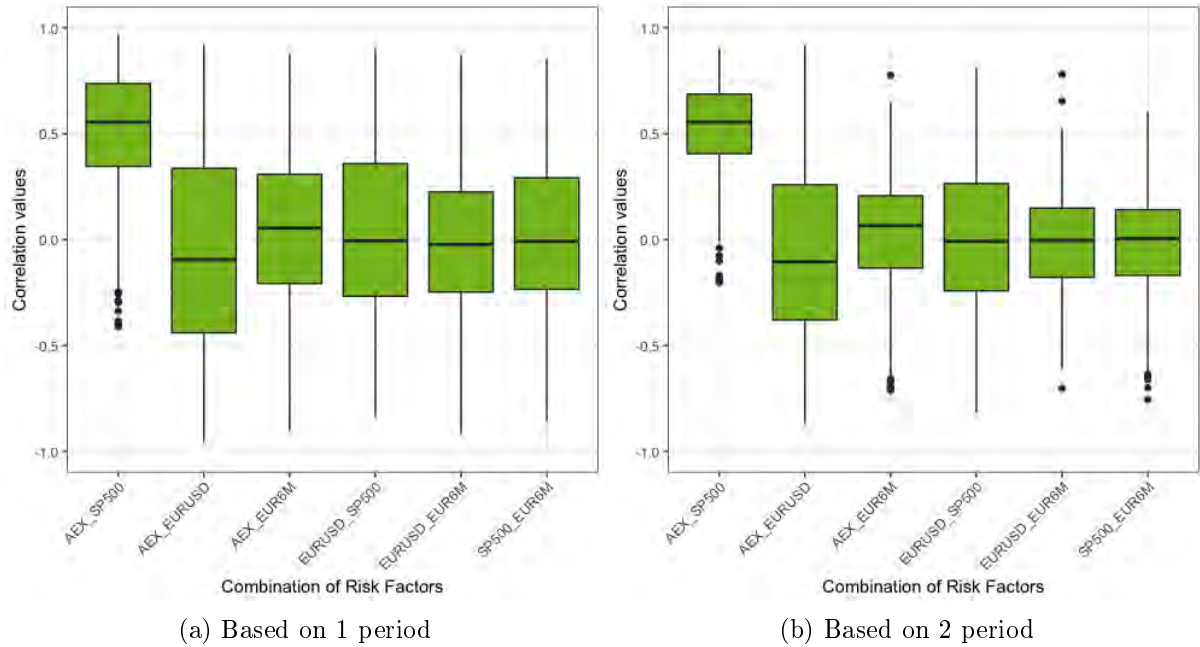


Figure B.1: Boxplots of the biweekly correlations between the different risk factor combinations.

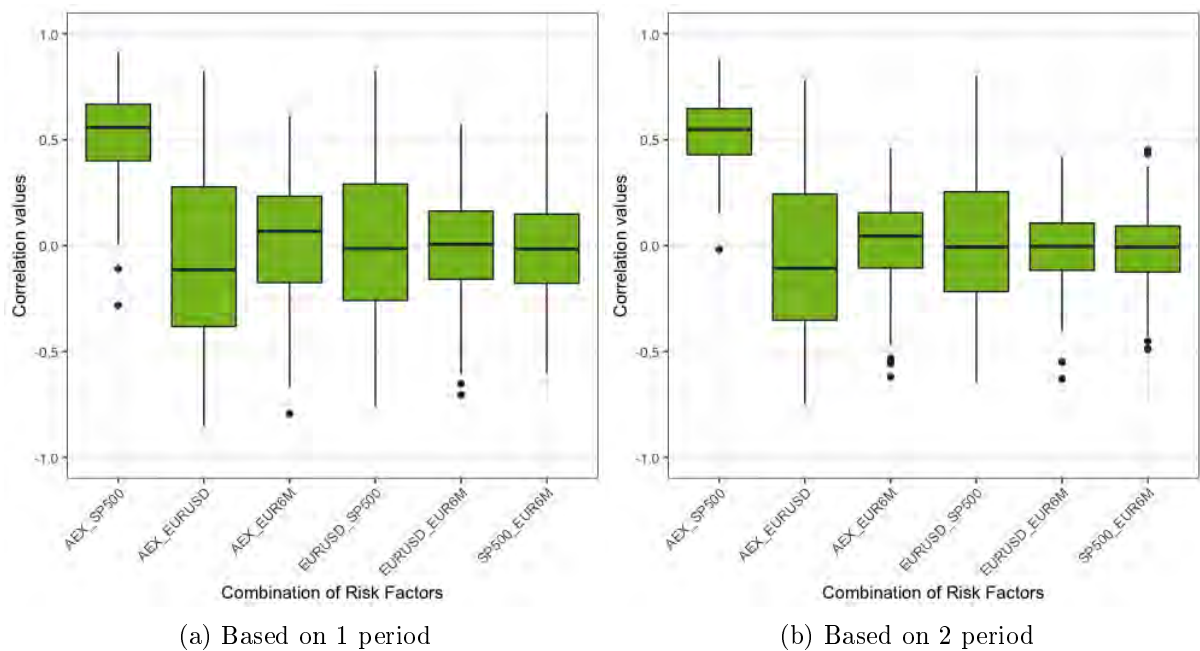


Figure B.2: Boxplots of the monthly correlations between the different risk factor combinations.

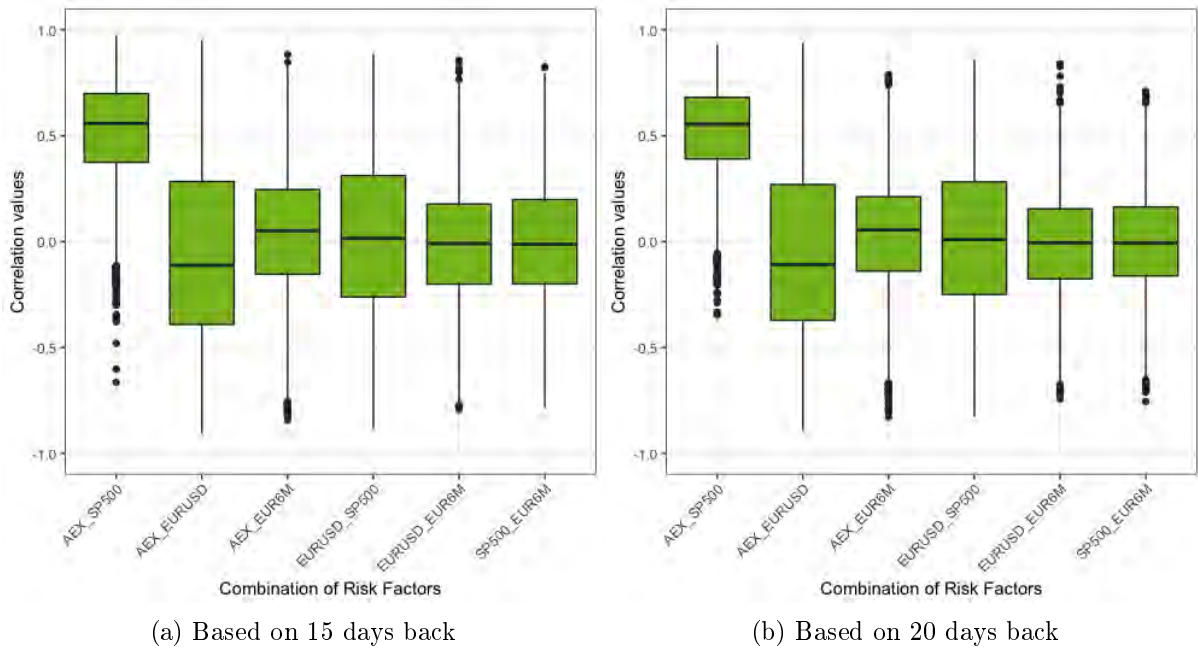


Figure B.3: Boxplots of the daily correlations between the different risk factor combinations.

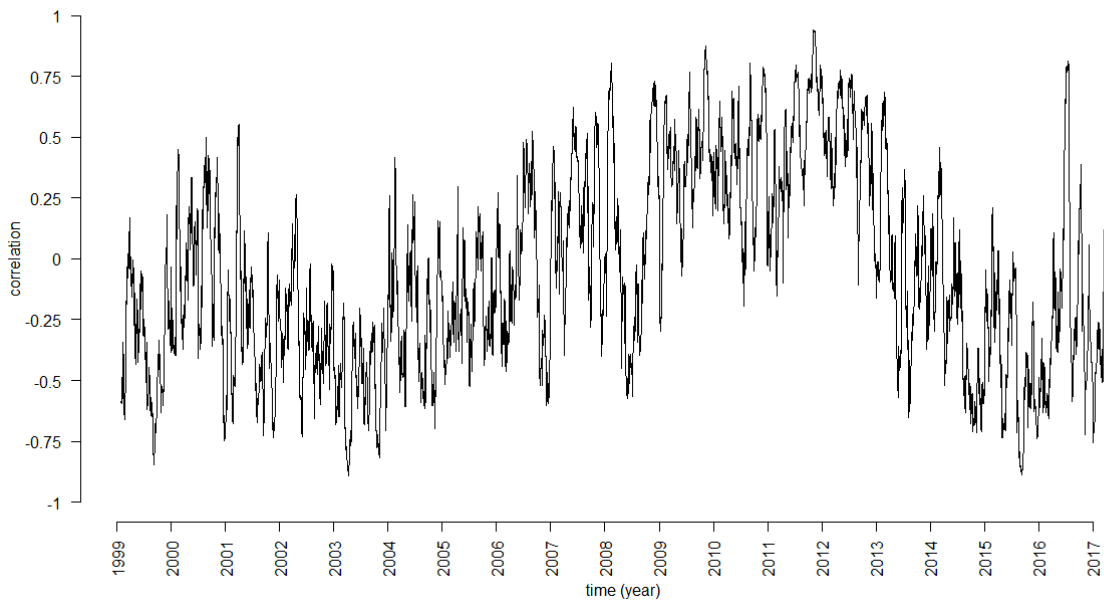


Figure B.4: The timeline of daily (20 days based) computed correlations between the AEX and the EURUSD exchange rate.

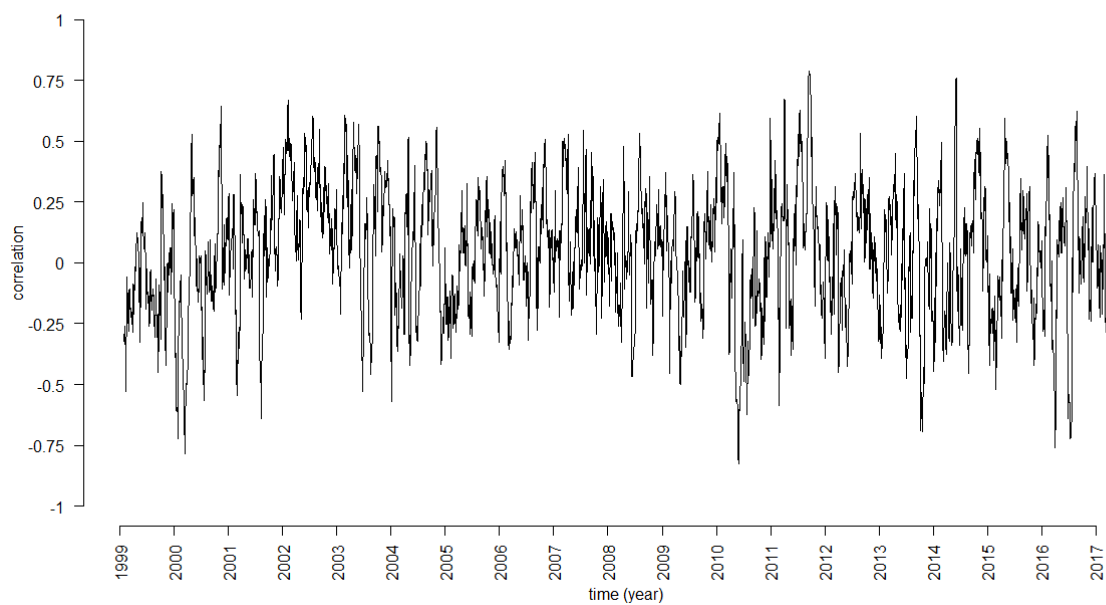


Figure B.5: The timeline of daily (20 days based) computed correlations between the AEX and the 6 month Euribor.

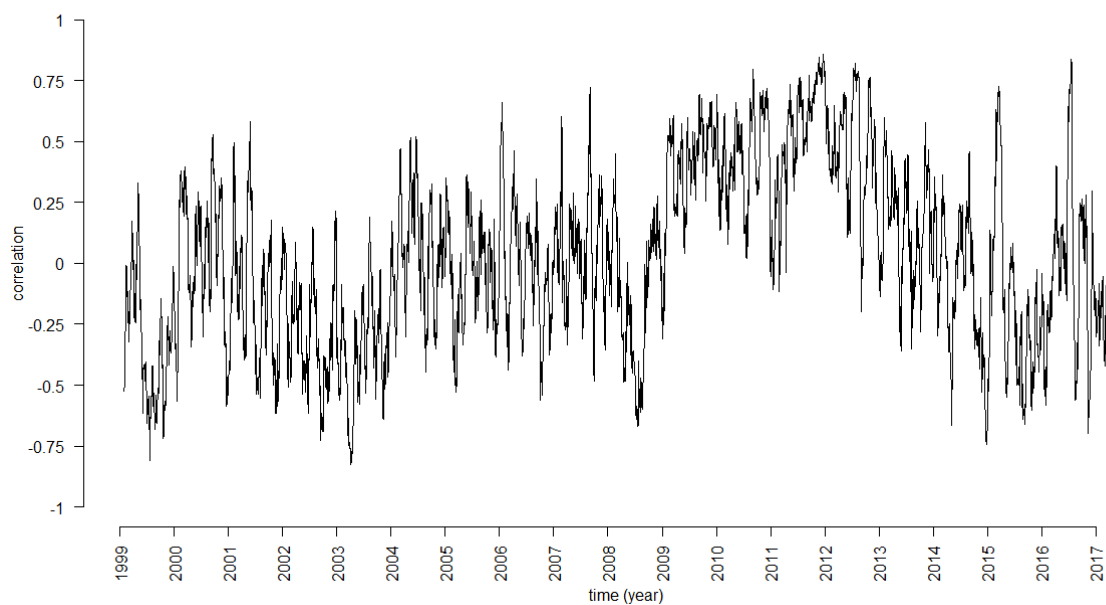


Figure B.6: The timeline of daily (20 days based) computed correlations between the EURUSD exchange rate and the S&P500.

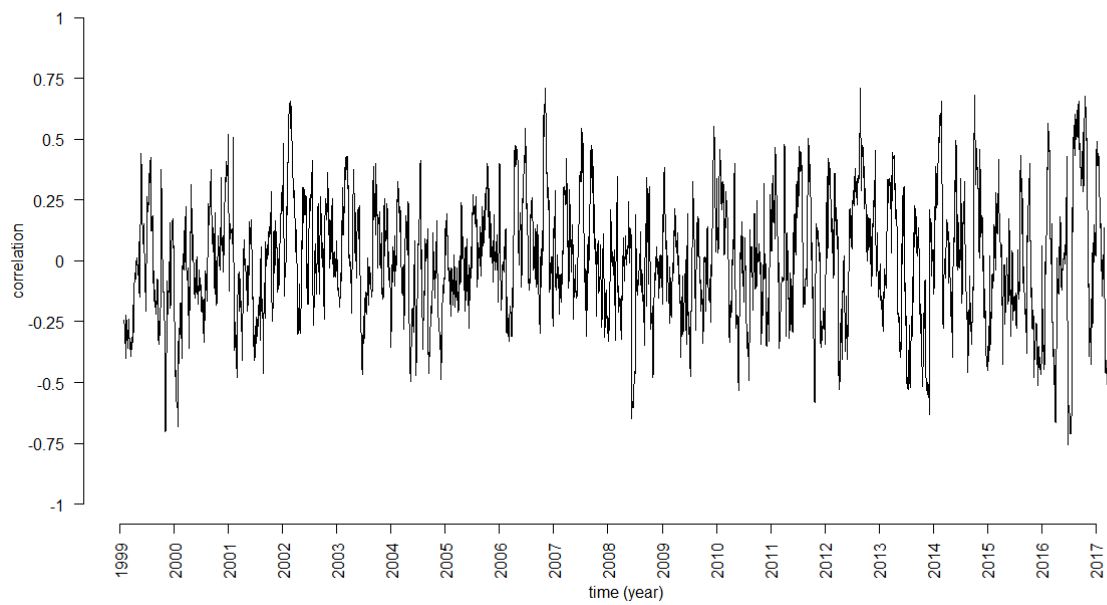


Figure B.7: The timeline of daily (20 days based) computed correlations between the S&P500 and the 6 month Euribor.

QQplots for daily (20) correlations between AEX & SP500

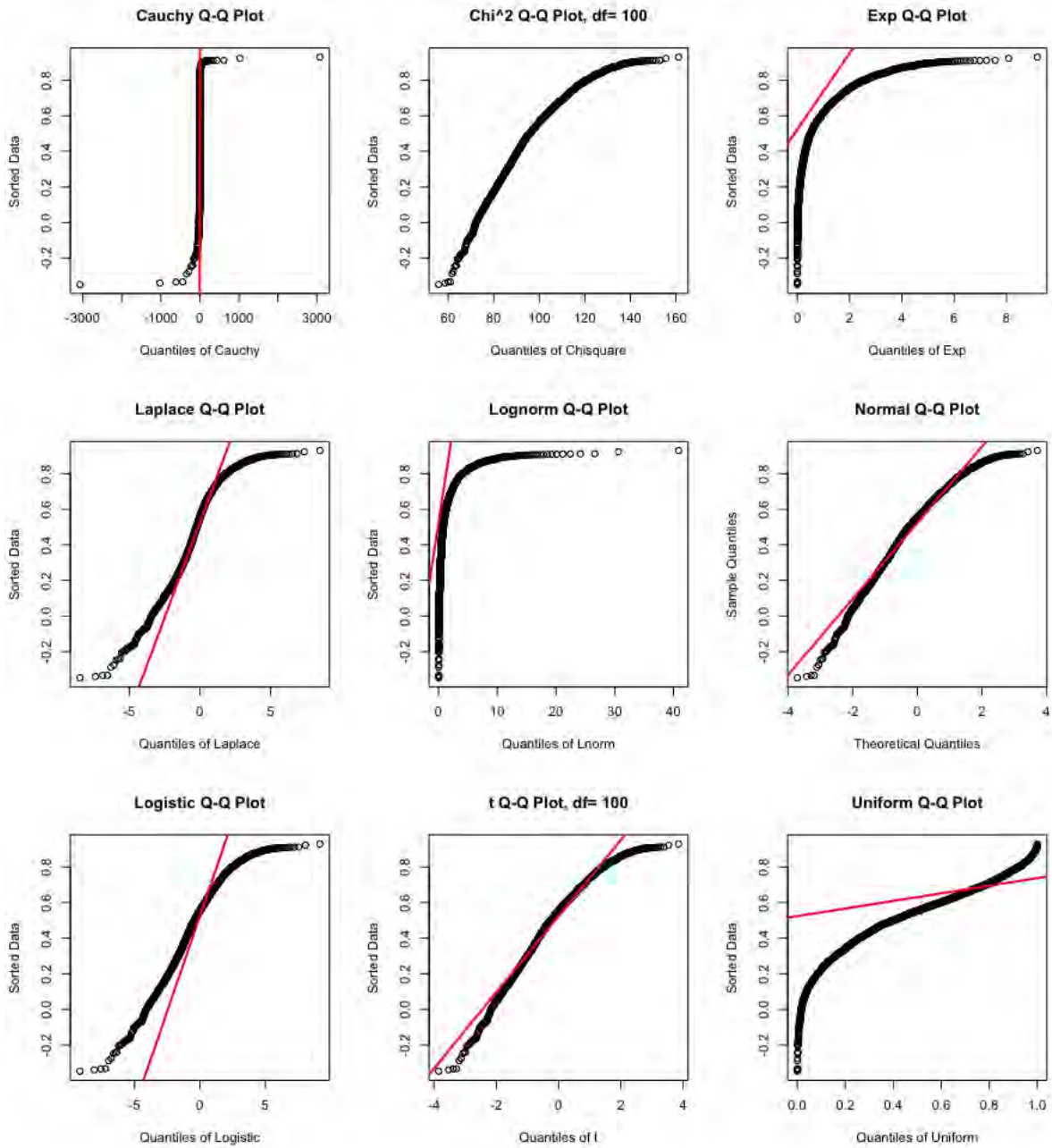


Figure B.8: Nine QQ-plots of daily (20 days based) computed correlations between the AEX and the S&P500 for different density functions.

QQplots for daily (20) correlations between AEX & EURUSD exchange

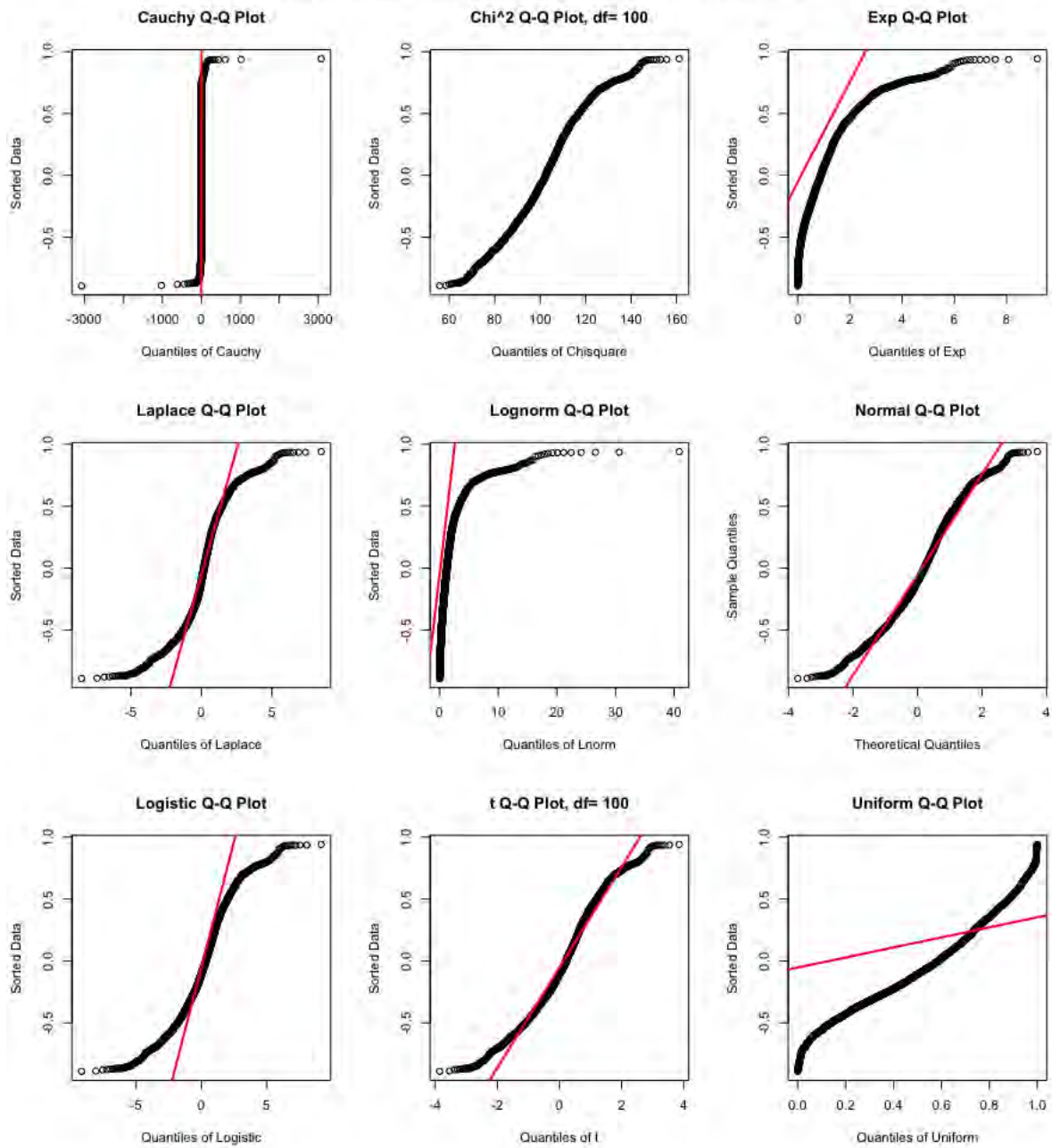


Figure B.9: Nine QQ-plots of daily (20 days based) computed correlations between the AEX and the EURUSD exchange rate for different density functions.

QQplots for daily (20) correlations between AEX & EUR6M

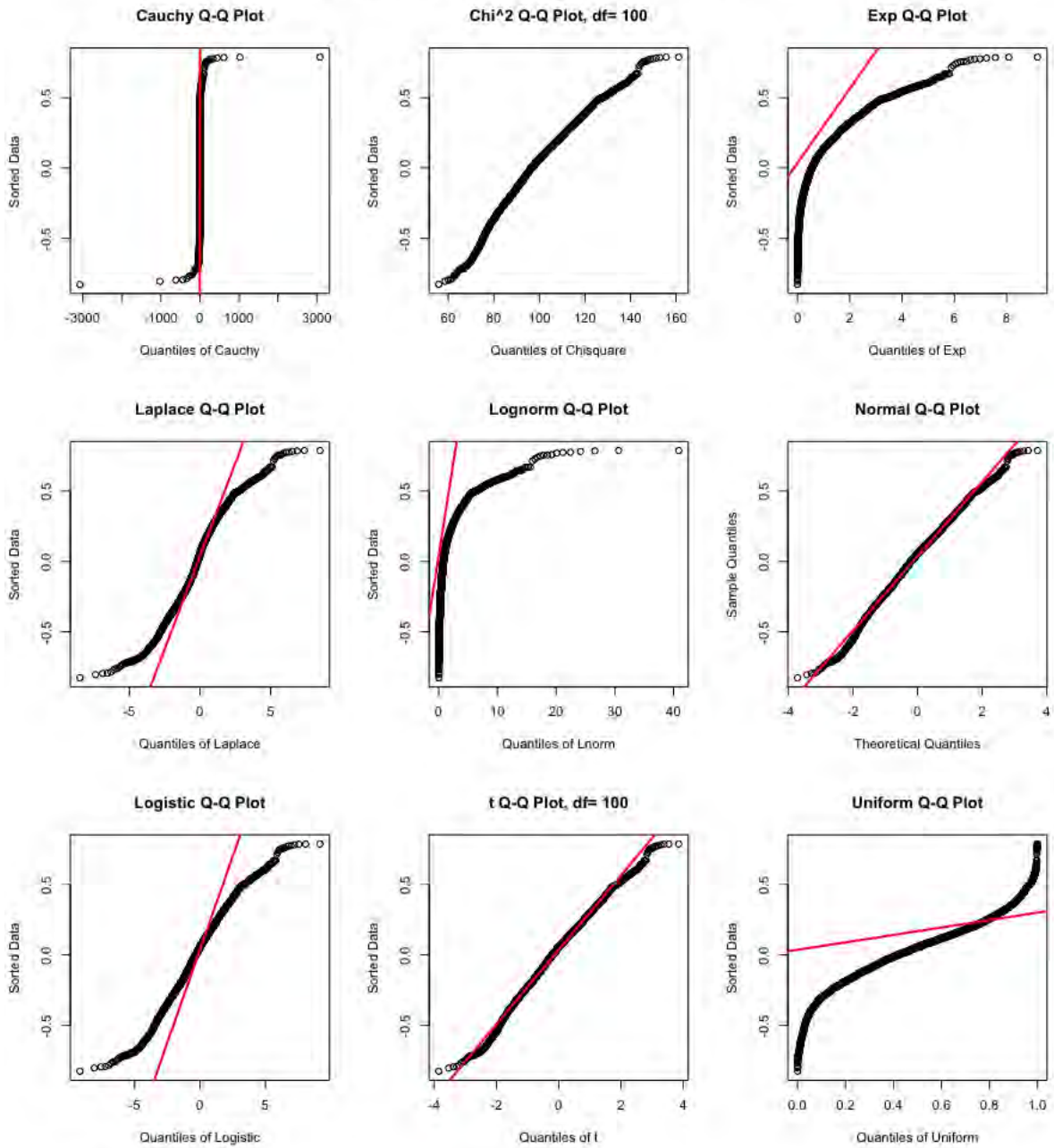


Figure B.10: Nine QQ-plots of daily (20 days based) computed correlations between the AEX and the 6 month Euribor for different density functions.

QQplots for daily (20) correlations between EURUSD exchange & SP500

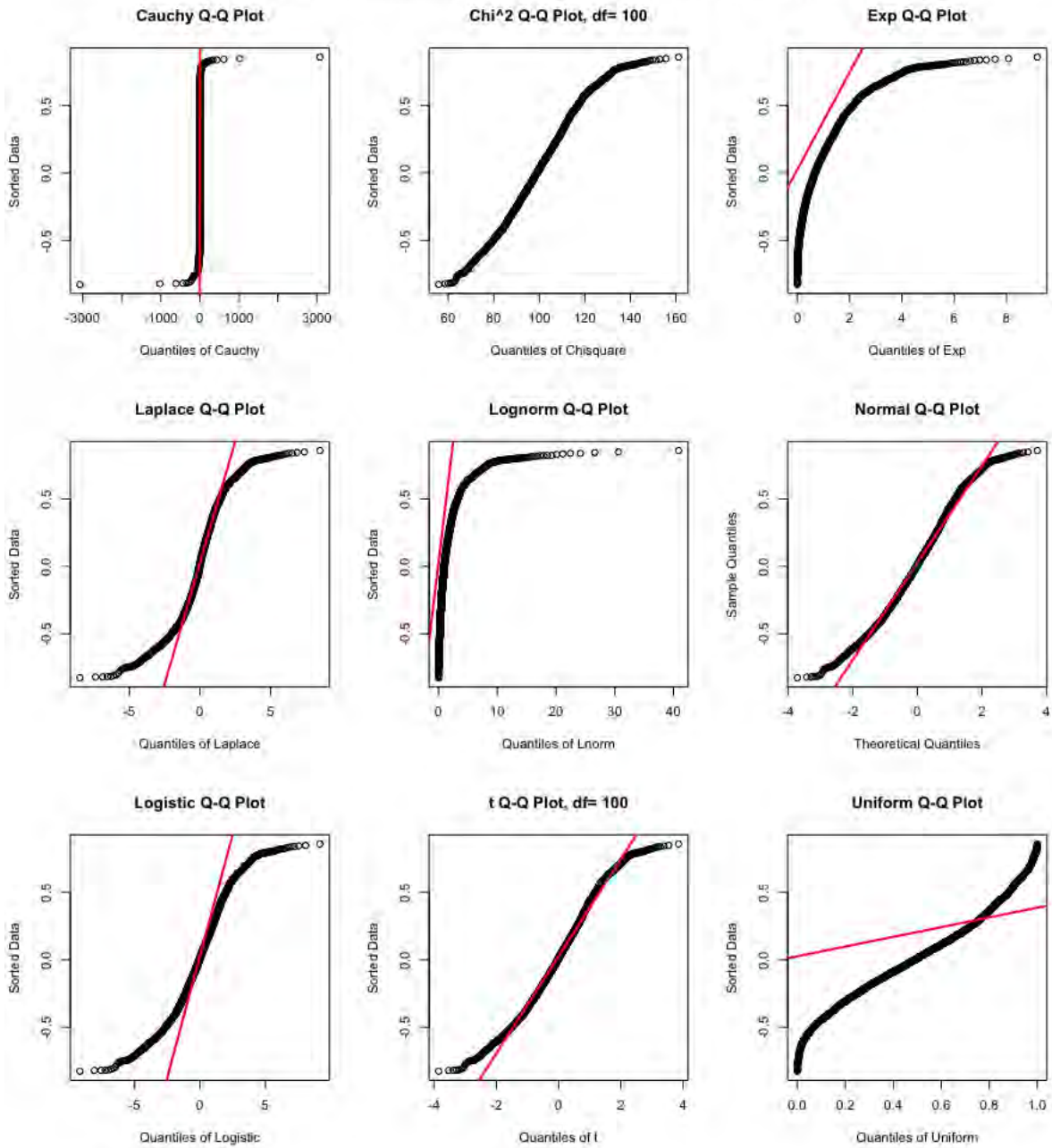


Figure B.11: Nine QQ-plots of daily (20 days based) computed correlations between the EU-RUSD exchange rate and the S&P500 for different density functions.

QQplots for daily (20) correlations between EURUSD exchange & EUR6M

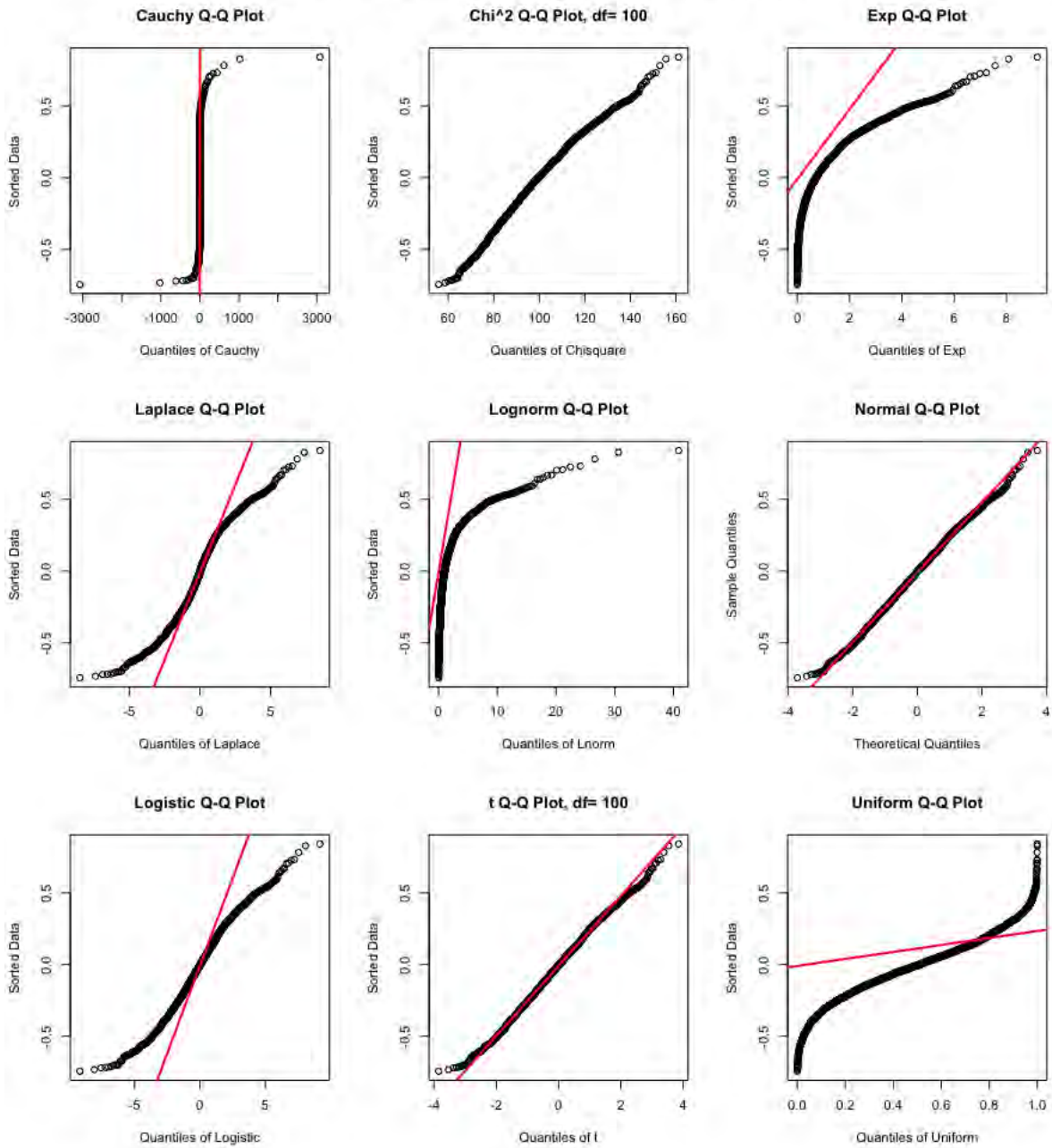


Figure B.12: Nine QQ-plots of daily (20 days based) computed correlations between the EU-RUSD exchange rate and the 6 month Euribor for different density functions.

QQplots for daily (20) correlations between SP500 & EUR6M

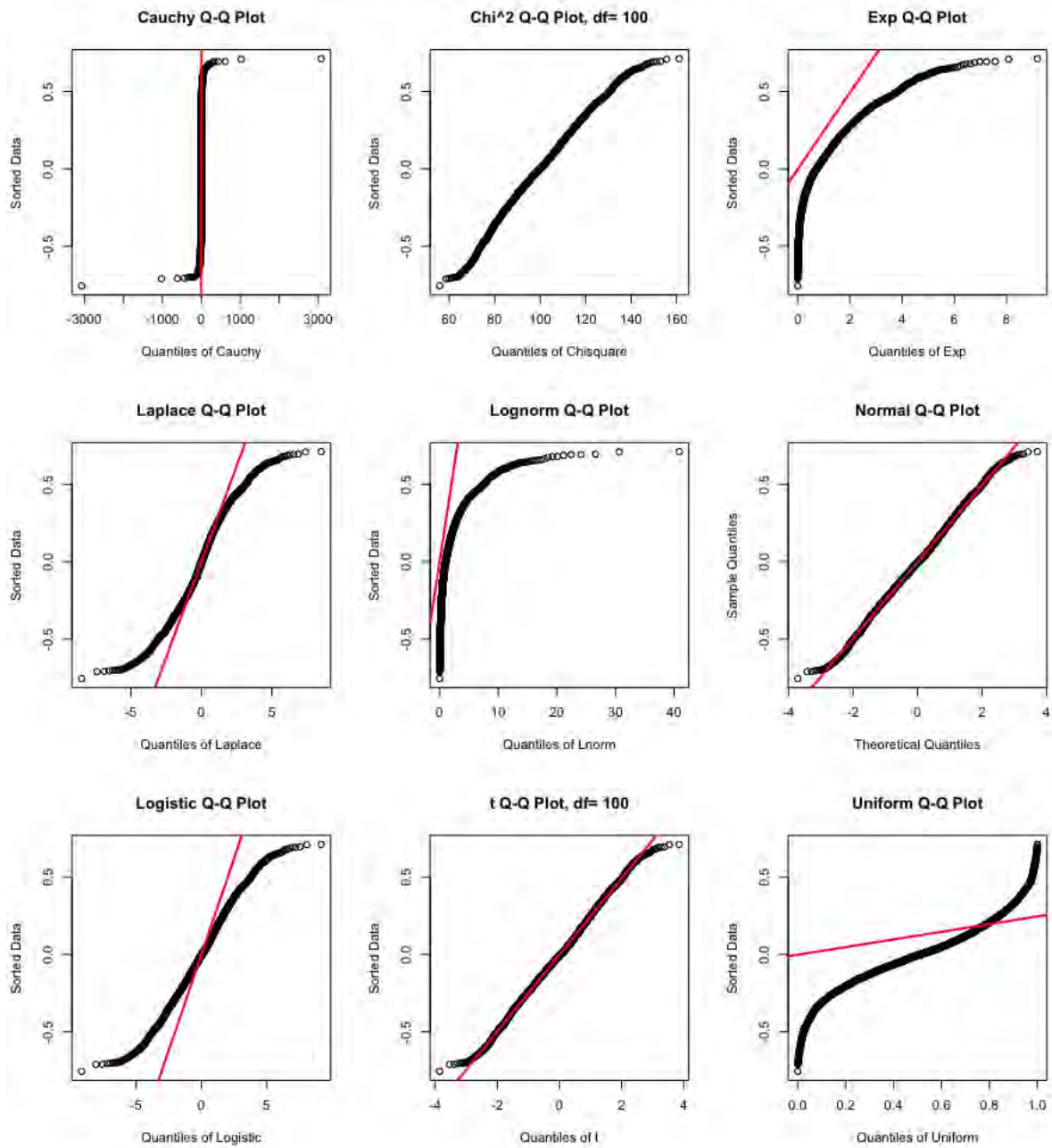


Figure B.13: Nine QQ-plots of daily (20 days based) computed correlations between the S&P500 and the 6 month Euribor for different density functions.

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