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Equity and Foreign Exchange Dependencies in Central and Eastern Europe

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Toma Donchev

VU Amsterdam

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Supervisor: dr. S.A. Borovkova **VU Amsterdam**

Abstract

Studying the dependences across financial markets is an important issue for risk management and portfolio management. This paper investigates the dependence structures of several emerging financial markets in Central and Eastern Europe – Bulgaria; Romania; Poland; Check Republic; Hungary; Slovakia and Russia. Focus falls on two kinds of dependence – first the correlation between equity prices and foreign exchange rates in each country and second - the co-movements of neighboring equity markets.

To model the dependences, we use the copula approach and we discuss its advantages over the standard correlation-based approach. The questions we intend to answer are: what is the dependence structure between equity and FX rates in Central and Eastern European financial markets? Is there any extreme (upper or lower) tail dependence? Is it symmetric or asymmetric? By answering these questions, we hope to better understand the co-movements across these financial markets and the risks associated with them.

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1. Introduction

Studying the dependences across financial markets is an important issue for the risk management and portfolio management. A great deal of research exists today, focusing on the co-movements of international equity markets. *Roll and Chakrabarti (2002)* compare the Asian stock markets with the European stock market before and during the Asian crisis. They find out that correlations, and volatilities increased from the pre-crisis to the crisis period in both regions, but the percentage increases were much higher in Asia. They also find out that diversification potential was better in Asia than in Europe before the crisis but this was reversed during the crisis. Other examples of research on the co-movements of equity markets can be found in *Karolyi and Stulz (1996), Longin and Solnik (2001), Forbes and Rigobon (2002)*. They all used methodology based on correlations and conditional correlations.

After the limitations of correlation based models were identified in *Embrechts et al. (2002)*, researchers have started to use copulas to directly model the dependences and co-movements across financial markets. There are a number of works that go along this line - *Mashal and Zeevi (2002), Hu (2003)* and *Chollete, Pena and Lu (2005).* They all report asymmetric extreme dependence between equity returns, in other words, stock markets tend to crash together but do not boom together.

All of the above literature focuses on the dependence structure and co-movements in equity markets via copulas. *Patton (2005)* also uses copulas to model the asymmetric exchange rate dependence. He finds that the German Mark-Dollar and Yen-Dollar exchange rates are more correlated when they are depreciating against the U.S. Dollar than when they are appreciating.

While there are a number of papers studying the co-movements of international equity markets, there is little literature on studying dependences between equity and exchange rates. *Ning (2006)* uses a Symmetrized Joe-Clayton (SJC) copula to directly model the underlying dependence structure between the equity market and the foreign exchange market in the G5 countries (US, UK, Germany, Japan and France) for the period 1/1/1991 to 31/12/1998. She finds that there exists significant upper and lower tail dependence between equity market and foreign exchange market, and the dependence is symmetric.

On the other hand, there is little or almost no research that studies dependences between equity and exchange rates in Central and Eastern Europe emerging markets. A possible reason for this is the fact that these financial markets are relatively young. Nevertheless, the results of such research will have important implications for both global investment management and asset pricing modeling. Central and Eastern European markets can become a very attractive option for global investors who want to diversify their portfolios internationally.

In this paper, we will investigate the dependence between the equity returns and the exchange rate returns, by the use of the copula approach. The rest of the paper is organized in the following: first we present the theoretical reasoning behind the research and give the definitions of the main concepts to be used; second the dependences between the main stock exchange index and the FX rate of a country's currency with the US Dollar are investigated; third, we model the co-movements of main stock indices for pairs of neighboring countries and last but not least, we will build a number of different portfolios that follow those indices and determine their Value at Risk, expressed in US Dollar.

In the next section we will describe the main theory behind the research. We assume that the reader is familiar with the concept of normal distribution, correlation and covariance. In order to better understand the ideas discussed in the paper we will give short definitions of multivariate normal distribution, dependence measures, Copulas and Value at Risk. Most of the theoretical explanations are based on the book - *"Quantitative Risk Management – Concepts, Techniques and Tools" by McNeil, Frey & Embrechts*

2. Theoretical Foundation

2.1 Risk Factors and Loss Distributions

Let us consider a financial portfolio as a collection of risky assets (e.g. stocks, bonds, derivatives, risky loans etc.). We denote the value of this portfolio at time t by V_t and we assume that V_t is observable at time t. The *loss* of the portfolio over a given time horizon Δ is defines as

$$L_{[t,t+\Delta]} := -(V_{t+\Delta} - V_t) \tag{2.1}$$

The distribution of $L_{[t,t+\Delta]}$ is called the *loss distribution*.

Following standard risk management practice the value V_t is modeled as a function of time t and a d-dimensional random vector $\mathbf{Z}_t = (Z_{t,1},....,Z_{t,d})'$ of *risk factors*, i.e. we get the following representation of V_t:

$$V_t = f(t, \mathbf{Z}_t) \tag{2.2}$$

with a measurable function $f : \mathbf{R}_+ \times \mathbf{R}^d \to \mathbf{R}$. The risk factors $\mathbf{Z}_t$ are observable (known) at time t. Frequently used risk factors in practice are logarithmic prices of financial assets, yields and logarithmic exchange rates.

Now let us define the series of *risk factor changes* $(\mathbf{X}_t)_{t\in\mathbf{N}}$ by $\mathbf{X}_t := \mathbf{Z}_t - \mathbf{Z}_{t-1}$ since they are the objects of interest in most statistical studies of financial time series. Note that if $\mathbf{Z}_t$ are logarithmic prices, then $\mathbf{X}_t$ are simply the *logarithmic returns* of the prices in consideration.

Using (2.2) the loss of the portfolio can be written as

$$L_{t+1} = -\big(f(t+1, \mathbf{Z}_t + \mathbf{X}_{t+1}) - f(t, \mathbf{Z}_t)\big) \tag{2.3}$$

Since $\mathbf{Z}_t$ is known at time t, the loss distribution is completely determined by the distribution of the risk factor change $\mathbf{X}_{t+1}$. We can therefore introduce the *loss operator* $l_{[t]} : \mathbf{R}^d \to \mathbf{R}$, which maps risk factor changes into losses. It is defined as

$$l_{[t]}(x) := -\left(f(t+1, \mathbf{Z}_t + x) - f(t, \mathbf{Z}_t)\right) \qquad x \in \mathbf{R}^d \tag{2.4}$$

and we obviously have $L_{t+1} = l_{[t]}(\mathbf{X}_{t+1})$

If f is differentiable we can consider the first-order approximation L_{t+1}^{Δ} of the loss (2.3), which is given by

$$L_{t+1}^{\Delta} := -\left(f_t(t, \mathbf{Z}_t) + \sum_{i=1}^{d} f_{Z_i}(t, \mathbf{Z}_t) * X_{t+1,i} \right) \tag{2.5}$$

where f_t and f_{Z_i} are the partial derivatives of f with respect to time t and the risk-factors Z_i.

Using this we can define the linearized loss operator, corresponding to (2.5) as

$$l_{[t]}^{\Delta}(x) := -\left(f_t(t, \mathbf{Z}_t) + \sum_{i=1}^{d} f_{Z_i}(t, \mathbf{Z}_t) * x_i \right) \tag{2.6}$$

The first order approximation is very useful as it allows us to express the loss as a *linear* function of the risk factor changes. The quality of the approximation is obviously best when the risk factor changes are likely to be small (i.e. if we are measuring risk over a short time horizon) and if the portfolio value is almost linear in the risk factors (i.e. if the function f has small second derivatives).

2.2 Multivariate Normal Distribution

A random variable $\mathbf{X} = (X_1, ..., X_d)$ is multivariate normally distributed if

$$\mathbf{X} \overset{d}{=} \mu + A\mathbf{Z}$$

where $\mathbf{Z} = (Z_1, ..., Z_k)'$ is a vector of independent identically distributed *standard normal* random variables (mean 0 and variance 1), and $A \in \mathbf{R}^{d \times k}$ and $\mu \in \mathbf{R}^d$ are a matrix and a vector of constants, respectively.

It is easily verified that the mean vector of this distribution is $\mathrm{E}(\mathbf{X}) = \mu$ and the covariance matrix is $\mathrm{cov}(\mathbf{X}) = \Sigma$, with $\Sigma = AA'$ and Σ is a positive semi-definite matrix. It is obvious that the distribution is characterized by its mean vector and covariance matrix, therefore a standard notation is $\mathbf{X} \sim N_d(\mu, \Sigma)$

2.3 Standard Estimators of Covariance and Correlation

Let $\mathbf{X}_1,.....,\mathbf{X}_n$ be n observations of a d-dimensional risk-factor return vector and assume that the observations come from a distribution with mean vector μ, finite covariance matrix Σ and correlation matrix P.

Standard estimators of μ and Σ are given by the *sample mean vector* $\overline{\mathbf{X}}$ and *sample covariance matrix* S, defined by

$$\overline{\mathbf{X}} := \frac{1}{n}\sum_{i=1}^{n}\mathbf{X}_i \,, \qquad S := \frac{1}{n}\sum_{i=1}^{n}(\mathbf{X}_i - \overline{\mathbf{X}})(\mathbf{X}_i - \overline{\mathbf{X}})' \tag{2.7}$$

where arithmetic operations on vectors and matrices are performed componentwise. $\overline{\mathbf{X}}$ is unbiased estimator but S is biased. An unbiased version can be obtained by setting $S_u := nS/(n-1)$.

The *sample correlation matrix* R may be easily calculated from S; the (j,k)-th element is $r_{jk} = s_{jk}/\sqrt{s_{jj}s_{kk}}$, where s_{jk} is the (j,k)-th element of S.

The properties of the estimators $\overline{\mathbf{X}}$, S and R will depend very much on the true multivariate distribution of the observations. These quantities are not necessarily the best estimators of the corresponding theoretical quantities in all situations. This is often forgotten in financial risk management, where sample covariance and correlation matrices are routinely calculated and interpreted with little crucial consideration of underlying models.

When our data $\mathbf{X}_1,.....,\mathbf{X}_n$ are IID multivariate normal, then $\overline{\mathbf{X}}$ and S are *maximum likelihood estimators* of the mean vector μ and covariance matrix Σ. Their behavior is well understood and statistical inference for the model can be made.

However, over short time intervals such as daily data, the multivariate normal distribution is certainly not a good description of financial risk factors, and it is often not very good for longer time intervals either. Under these circumstances the behavior of the standard estimators (2.7) is less well understood and statistical inferences based on the estimators might not produce an accurate model.

2.4 Copulas

One method of modeling dependencies which has become very popular recently is the *copula*. The word copula is a Latin noun which means 'a link, tie or bond', and was first employed in a mathematical or statistical sense by *Abe Sklar*. Mathematically, a copula is a function which allows us to combine univariate distributions to obtain a joint distribution with a particular dependence structure.

Every joint distribution function for a random vector of risk factors contains both a description of marginal behavior of individual risk factors and a description of their *dependence structure*. The copula approach gives us a way to separate the description of the dependence structure. Copulas help us understand the potential pitfalls of approaches that focus only on linear correlation and show us how to define a number of alternative dependence measures. Copulas express dependence on a *quantile scale*, which is useful for describing the dependences of extreme outcomes.

The copula approach makes it possible to combine more developed marginal models with a variety of dependence models and to investigate the sensitivity of risk to the dependence specification.

Definition 2.4.1 A d-dimensional copula is a distribution function on $[0,1]^d$ with standard uniform marginal distributions.

In other words, a copula $C(\mathbf{u}) = C(u_1,.....,u_d)$ is a multivariate distribution function of the unit hypercube to the unit interval. A mapping of the form $C:[0,1]^d \rightarrow [0,1]$ is a copula if the following three properties hold:

a) $C(u_1,.....,u_d)$ is increasing in each component u_i
b) $C(1,....,1,u_i,1,...,1) = u_i$ for all $i \in \{1,....,d\}$, $u_i \in [0,1]$
c) For all $(a_1,.....,a_d),(b_1,.....,b_d) \in [0,1]^d$ with $a_i \le b_i$ we have

$$\sum_{i_1}^{2} \cdots \sum_{i_d}^{2} (-1)^{i_1+....+i_d} C(u_{1i_1},.......u_{di_d}) \ge 0 \quad (2.8)$$

where $u_{j1} = a_j$ and $u_{j2} = b_j$ for all $j \in \{1,....,d\}$.

The first property is a requirement for any multivariate distribution function and the second property is a requirement of uniform marginal distributions. The last one is less obvious, but it ensures that is the random vector $(U_1,.....,U_d)'$ has a distribution function C, then $P(a_1 \le U_1 \le b_1,.....,a_d \le U_d \le b_d)$ is non-negative.

The following theorem summarizes the importance of copulas in the study of multivariate distribution functions:

Theorem 2.4.2 (Sklar 1959) *Let F be a joint distribution function with margins $F_1,....,F_d$. Then there exist a copula $C:[0,1]^d \to [0,1]$ such that, for all $x_1,....,x_d$ in $[-\infty,\infty]$,*

$$F(x_1,....,x_d) = C(F_1(x_1),....,F_d(x_d)) \tag{2.9}$$

If the margins are continuous, than C is unique.

Conversely, if C is a copula and $F_1,....,F_d$ are univariate distribution functions, then the function F defined by (2.9) is a joint distribution function with margins $F_1,....,F_d$.

Proof: see McNeil, Frey & Embrechts - "Quantitative Risk Management – Concepts, Techniques and Tools"

Another essential property of copulas is given in the following proposition:

Proposition 2.4.3 *Let $(X_1,.....,X_d)$ be a random vector with continuous margins and a copula C and let $T_1,.....,T_d$ be strictly increasing functions. Then $(T_1(X_1),.....,T_d(X_d))$ also has copula C.*

2.4.4 Examples of copulas

There are three main categories of copulas: *fundamental copulas* represent a number of important special dependence structures; *implicit copulas* are extracted from well known multivariate distributions using Sklar's Theorem, but do not necessary have simple closed-form expressions and *explicit copulas* – copulas that have simple closed-form expressions and follow general mathematical constructions.

A. Fundamental copulas

I. The *independence* copula is

$$\Pi(u_1,....,u_d) = \prod_{i=1}^{d} u_i \quad (2.10)$$

It is clear from Sklar's Theorem and equation (2.9), that random variables with continuous distributions are independent if and only if their dependence structure is given by (2.10)

II. The *comonotonicity* copula is given by

$$M(u_1,....,u_d) = \min\{u_1,....,u_d\} \quad (2.11)$$

The comonotonicity copula represents *perfect positive dependence*. If $X_1,.....,X_d$ are perfectly positive dependent in the sense that they are almost surely strictly increasing functions of each other, then (2.11) is their copula.

III. The *countermonotonicity* copula is given by

$$W(u_1,u_2) = \max\{u_1 + u_2 - 1,0\} \quad (2.12)$$

This copula is the joint distribution function of the random vector $(U,1-U)$, where $U \sim U(0,1)$. If X_1 and X_2 have continuous distribution functions and are *perfectly negatively dependent,* in the sense that X_2 is almost surely a strictly decreasing function of X_1, then (2.12) is their copula. Note that the concept of countermonotonicity is defined only for two random variables.

B. Implicit copulas

I. *Gauss Copula*.

If $\mathbf{Y} \sim N_d(\mu, \Sigma)$ is a Gaussian (Normal) random vector, then its copula is the Gauss Copula. By Proposition 2.4.3 the copula of $\mathbf{Y}$ is exactly the same as the copula of $\mathbf{X} \sim N_d(\mathbf{0}, \mathrm{P})$ where P is the correlation matrix of $\mathbf{Y}$. Then this copula is

$$\begin{aligned} C_{\mathrm{P}}^{Ga}(\mathbf{u}) &= P(\Phi(X_1) \le u_1, \ldots.., \Phi(X_d) \le u_d \\ &= \Phi_{\mathrm{P}}(\Phi^{-1}(u_1), \ldots.., \Phi^{-1}(u_d)) \end{aligned} \tag{2.13}$$

where Φ denotes the standard univariate normal distribution function and Φ_{P} denotes the joint distribution function of $\mathbf{X}$. When there are only two random variables X_1 and X_2 in consideration, we write C_{ρ}^{Ga}, with $\rho = \rho(X_1, X_2)$ – the correlation coefficient.

II. *t-Copula*

We can use the same reasoning to derive an implicit copula from any other multivariate with continuous margins. For example the d-dimensional t-copula is the following:

$$C_{\nu,\mathrm{P}}^{t}(\mathbf{u}) = \mathbf{t}_{\nu,\mathrm{P}}(t_{\nu}^{-1}(u_1), \ldots.., t_{\nu}^{-1}(u_d)) \tag{2.14}$$

where t_{ν} is the distribution function of a standard univariate t-distribution, $\mathbf{t}_{\nu,\mathrm{P}}$ is the joint distribution function of the vector $\mathbf{X} \sim t_d(\nu, \mathbf{0}, \mathrm{P})$ and P is the correlation matrix.

C. Explicit copulas

The Gauss and t-copula are copulas implied by well-know multivariate distribution functions and do not have a simple closed forms. Nevertheless, we can derive a number of copulas which do have simple closed forms:

I. *Gumbel copula*

$$C_{\theta}^{Gu}(u_1, u_2) = \exp\left(-\left((-\ln u_1)^{\theta} + (-\ln u_2)^{\theta}\right)^{1/\theta}\right) \qquad 1 \le \theta < \infty \tag{2.15}$$

If the parameter $\theta = 1$ we obtain the independence copula as a special case, and the limit of C_{θ}^{Gu} as $\theta \to \infty$ is the two-dimensional comonotonicity copula. Therefore the Gumbel copula interpolates between independence and perfect dependence and the parameter θ represents the strength of the dependence.

II. *Clayton copula*

$$C_{\theta}^{Cl}(u_1,u_2) = \left(u_1^{-\theta} + u_2^{-\theta} - 1\right)^{1/\theta} \qquad 0 < \theta < \infty \qquad (2.16)$$

The Clayton copula approaches the independence copula as $\theta \to 0$, and the 2-dimensional comonotonicity copula as $\theta \to \infty$.

III. *Frank copula*

$$C_{\theta}^{Fr}(u_1,u_2) = -\frac{1}{\theta}\ln\left(1 + \frac{(\exp(-\theta u_1)-1)(\exp(-\theta u_2)-1)}{\exp(-\theta)-1}\right) \qquad \theta \in R \qquad (2.17)$$

IV. Symmetrized Joe-Clayton Copula (SJC)

The *SJC* copula is a modification of the so-called *BB7* copula of Joe (1997). It is defined as

$$C_{SJC}(u,v \mid \lambda_r,\lambda_l) = 0.5 \times \left(C_{JC}(u,v \mid \lambda_r,\lambda_l) + C_{JC}(1-u,1-v \mid \lambda_r,\lambda_l) + u + v - 1\right) \qquad (2.18)$$

where $C_{JC}(u,v \mid \lambda_r,\lambda_l)$ is the *BB7* copula (also known as *Joe-Clayton* copula), which is defined as

$$C_{JC}(u,v \mid \lambda_r,\lambda_l) = 1 - \left(1 - \left[\left(1-(1-u)^k\right)^{-r} + \left(1-(1-v)^k\right)^{-r} - 1\right]^{-1/r}\right)^{1/k} \qquad (2.19)$$

with $k = 1/\log_2(2-\lambda_r)$, $r = -1/\log_2(\lambda_l)$ and $\lambda_r \in (0,1)$, $\lambda_l \in (0,1)$. λ_r and λ_l are called the (upper and lower) coefficients of tail dependence and are explained in the next section. The SCJ copula is symmetric when $\lambda_r = \lambda_l$.

Gumbel, Clayton, Frank and Symmetrized Joe-Clayton Copula copulas belong to the so-called *Archimedean* copula family.

2.5 Dependence Measures

In this section we will discuss the differences between three kinds of dependence measures – the usual Pearson *linear correlation*, the *rank correlations* and the *coefficients of tail dependence*. All these measures yield a scalar measurement for a pair of random variables (X_1, X_2), but they differ in nature and properties.

2.5.1 Linear Correlation.

The standard linear correlation is a measure of linear dependence that takes values in $[-1, 1]$. If X_1 and X_2 are random vectors than their linear correlations is defined by

$$\rho(X_1, X_2) = \frac{Cov(X_1, X_2)}{\sqrt{Var(X_1)} * \sqrt{Var(X_1)}}$$

If X_1 and X_2 are independent, then $\rho = 0$, but the converse is false: if X_1 and X_2 are uncorrelated, this does not imply that they are independent.

A disadvantage of linear correlation is that it is **not** invariant under **non-linear** strictly increasing transformations. It is invariant only under strictly increasing **linear** transformations. If $T : \mathbf{R} \to \mathbf{R}$ is a strictly increasing function then, in general, $\rho(T(X_1), T(X_2)) \neq \rho(X_1, X_2)$. Another obvious, but important, remark is that correlation is only defined when the variances of X_1 and X_2 finite. The restriction to finite-variance models is not ideal for a dependence measure and can cause problems when we work with heavy tailed distributions.

Next we present two incorrect assumptions usually made in financial risk management when modeling a multivariate risk-return factors. Both these statements are true only if we restrict ourselves to elliptically distributed risk factors (e.g. normal distribution), but are false in general.

Assumption 1. The marginal distributions and pairwise linear correlations of a random vector determine its joint distribution.

Assumption 2. For given univariate distributions F_1 and F_2 and any linear correlation value ρ in $[-1, 1]$ it is always possible to construct a joint distribution function F with margins F_1 and F_2 and correlation ρ.

2.5.2 Rank Correlations.

Rank correlations, on the other hand, are simple scalar measures of dependence that depend only on the copula of a bivariate distribution and not on the marginal distributions, unlike linear correlation which depends on both. The standard empirical estimators of rank correlations may be calculated by looking at the *ranks* of the data alone, hence the name. In other words, we only need to know the ordering of the sample for each variable of interest and not the actual numerical values.

Kendall's Tau

Kendall's rank correlation is a measure of concordance for bivariate random vectors. Two points in the real plane, denoted by (x_1, x_2), and $(\tilde{x}_1, \tilde{x}_2)$ are said to be *concordant* if $(x_1 - \tilde{x}_1)(x_2 - \tilde{x}_2) > 0$ and to be *discordant* if $(x_1 - \tilde{x}_1)(x_2 - \tilde{x}_2) < 0$. Let us denote Kendall's tau for a random vector (X_1, X_2) by $\rho_\tau(X_1, X_2)$ and let $(\tilde{X}_1, \tilde{X}_2)$ be an independent copy of (X_1, X_2) i.e. a vector with the same distribution but independent of (X_1, X_2). Kendall's rank correlation is then simply the probability of concordance minus the probability of discordance:

$$\rho_\tau(X_1, X_2) = P((X_1 - \tilde{X}_1)(X_2 - \tilde{X}_2) > 0) - P((X_1 - \tilde{X}_1)(X_2 - \tilde{X}_2) < 0) \tag{2.20}$$

We can also define Kendall's tau as an expectation:

$$\rho_\tau(X_1, X_2) = \mathrm{E}\left(sign\left((X_1 - \tilde{X}_1)(X_2 - \tilde{X}_2)\right)\right) \tag{2.21}$$

where $(\tilde{X}_1, \tilde{X}_2)$ is an independent copy of (X_1, X_2)

Spearman's Rho

Let X_1 and X_2 be two random variables with marginal distribution functions F_1 and F_2. Then Spearman's rank correlation is given by:

$$\rho_S(X_1, X_2) = \rho(F_1(X_1), F_1(X_1)) \tag{2.22}$$

In other words, Spearman's rho is simply the linear correlation of probability transformed random variables, which for continuous random variables is the linear correlation of their unique copula.

Properties

Kendall's tau and Spearman's rho have many common properties. They both take values in $[-1,1]$ and both are symmetric dependence measures. For independent random variables both ρ_τ and ρ_S give the value 0, although, like linear correlation, a rank correlation of 0 does not imply independence. For comonotonic (perfectly positive dependent) random variables both ρ_τ and ρ_S give the value 1 and for countermonotonic (perfectly negative dependent) they give the value -1.

The most important property of rank correlations however, is the fact that both Kendall's tau and Spearman's rho are invariant under non-linear strictly increasing transformations. In contrast, linear correlation is only invariant under strictly increasing *linear* transformations.

Now for two random variables X_1 and X_2 with continuous marginal distribution functions F_1 and F_2 and unique copula C, we can express ρ_τ and ρ_S only in terms of C:

$$\rho_\tau(X_1, X_2) = 4\int_0^1\int_0^1 C(u_1, u_2)\, d\,C(u_1, u_2) \;\; -1 \tag{2.23}$$

$$\rho_S(X_1, X_2) = 12\int_0^1\int_0^1 \left(C(u_1, u_2) - u_1 u_2\right) d\,u_1\, du_2 \tag{2.24}$$

2.5.3 Coefficients of Tail Dependence.

Let X_1 and X_2 be two random variables with continuous marginal distribution functions F_1 and F_2 and unique copula C. Similar to the rank correlations, the *coefficients of tail dependence* are measures of tail dependence that depend only on the copula of X_1 and X_2. The importance of those coefficients is that they provide measures of *extremal dependence*. In other words, they measure the strength of dependence in the tails of bivariate distribution. The coefficients we describe here are defined in terms of limiting conditional probabilities of *quantile exceedances*.

The coefficients of tail dependence are of two kinds – upper and lower tail dependence. In the case of upper tail dependence we look at the probability that X_2 exceeds its q-quantile, given that X_1 exceeds its q-quantile and then consider the limit as $q \to \infty$. Obviously X_1 and X_2 are interchangeable. The formal definition is the following:

Let X_1 and X_2 be random variables with distribution functions F_1 and F_2. The coefficient of upper tail dependence of X_1 and X_2 is

$$\lambda_u := \lambda_u(X_1, X_2) = \lim_{q \to 1^-} P(X_2 > F_2^{\leftarrow}(q) \mid X_1 > F_1^{\leftarrow}(q)), \tag{2.25}$$

provided a limit $\lambda_u \in [0,1]$ exists. If $\lambda_u \in (0,1]$, then X_1 and X_2 are said to show upper tail dependence or extremal dependence in the upper tail; if $\lambda_u = 0$ they are asymptotically independent in the upper tail.

Analogously, the coefficient of lower tail dependence is

$$\lambda_l := \lambda_l(X_1, X_2) = \lim_{q \to 0^+} P(X_2 \le F_2^{\leftarrow}(q) \mid X_1 \le F_1^{\leftarrow}(q)), \tag{2.26}$$

provided a limit $\lambda_u \in [0,1]$ exists.

Now if F_1 and F_2 are continuous distribution functions and F is the joint distribution function of X_1 and X_2, we can derive simple expressions for λ_u and λ_l in terms of the unique copula C of the bivariate distribution function. Using that

$$C(u_1,....,u_d) = F(F_1^{\leftarrow}(u_1),......,F_d^{\leftarrow}(u_d)) \tag{2.27}$$

we get:

$$\begin{aligned}\lambda_l &= \lim_{q\to 0^+} \frac{P(X_2 \le F_2^{\leftarrow}(q) \mid X_1 \le F_1^{\leftarrow}(q))}{P(X_2 \le F_2^{\leftarrow}(q))} \\ &= \lim_{q\to 0^+} \frac{C(q,q)}{q}\end{aligned} \tag{2.28}$$

In the same way, for upper tail dependence we get:

$$\lambda_u = \lim_{q\to 1^-} \frac{\hat{C}(1-q,1-q)}{1-q} = \lim_{q\to 0^+} \frac{\hat{C}(q,q)}{q} \tag{2.29}$$

where $\hat{C}$ is the survival copula of C, i.e. $\hat{C}(u_1,u_2) := 1 - C(u_1,u_2)$

Examples

Let $\hat{C}_\theta^{Gu}$ denote the Gumbel survival copula, then

$$\lambda_u = \lim_{q\to 1^-} \frac{\hat{C}_\theta^{Gu}(1-q,1-q)}{1-q} = 2 - \lim_{q\to 1^-} \frac{C_\theta^{Gu}(q,q)-1}{1-q} \tag{2.30}$$

and using the fact that $C_\theta^{Gu}(u,u) = u^{2^{1/\theta}}$, we get:

$$\lambda_u = 2 - 2^{1/\theta} \tag{2.31}$$

for the upper tail dependence of the Gumbel copula. In other words, provided that $\theta > 1$, the Gumbel copula has upper tail dependence.

In a similar way we can show that the coefficient of lower tail dependence for the Clayton copula is:

$$\lambda_l = 2^{-1/\theta} \tag{2.32}$$

Proof: see McNeil, Frey & Embrechts - "Quantitative Risk Management – Concepts, Techniques and Tools"

2.6 Value at Risk

Today, the most widely used risk measure in financial institutions is *Value-at-Risk* (VaR). It also plays a significant role in the *Basel II Capital-adequacy framework*.

Let us consider a portfolio of risky assets and a fixed time horizon Δ, and let us denote the distribution function of the corresponding loss distribution by $F_L(l) = P(L \leq l)$. We want to define a statistic based on F_L, which measures the severity of the risk of holding our portfolio over this time period. An obvious candidate is the maximal possible loss; however, in most cases the possible loss is unbounded so that the maximal loss is simply infinity. Moreover, if we use the maximal loss we neglect any probability information contained in the specific distribution function F_L. Value-at-Risk takes these disadvantages into account. It is a straightforward extension of the maximal loss and stands for "maximal loss which is not exceeded with a given high probability". This probability is predetermined and is called confidence level.

Definition 2.5.1 (Value-at-Risk): The Value-at-Risk of a portfolio at a certain confidence level α is given by the smallest number l such that the probability that the loss L exceeds l is not larger than $(1-\alpha)$. Mathematically:

$$VaR_\alpha = \inf\left\{l \in \mathbf{R} : P(L > l) \leq 1-\alpha\right\} = \inf\left\{l \in \mathbf{R} : F_L(l) \geq \alpha\right\} \quad (2.33)$$

In statistical terminology, VaR is simply a quantile of the loss distribution. Typical values of the confidence level α are $\alpha = 0.95$, $\alpha = 0.99$ or $\alpha = 0.999$. The time horizon Δ is different depending on the particular portfolio and types of risk factors in consideration. Usually in market risk management the VaR is calculated for short time horizons – 1 or at most 10 days, and in credit and operational risk management the time horizon Δ is usually 1 year.

2.7 Standard methods for measuring Market Risk

There are three standard methods used in the financial industry for measuring market risk over short time intervals (i.e. methods for calculating VaR of a portfolio of risky assets): *Variance-Covariance Method, Historical Simulation Method* and *Monte Carlo Simulation Method*.

In market risk managements, we are interested in estimating VaR for the distribution of a loss $L_{t+1} = l_{[t]}(\mathbf{X}_{t+1})$, where $\mathbf{X}_{t+1}$ is the vector of risk-factor changes from time t to time $t+1$ and $l_{[t]}$ is the loss operator based on the portfolio at time t.

2.7.1 Variance-Covariance Method

The main characteristic of the method is the assumption that the risk-factor changes $\mathbf{X}_{t+1}$ have a multivariate normal distribution. The notation is $\mathbf{X} \sim N_d(\mu, \Sigma)$, where μ is the mean vector and Σ is variance-covariance matrix of the distribution. The method also assumes that the linearized loss is a sufficiently accurate approximation of the actual loss. In this way the problem is simplified to considering the distribution of $L_{t+1}^{\Delta} = l_{[t]}^{\Delta}(\mathbf{X}_{t+1)})$, with $l_{[t]}^{\Delta}$ defined in (2.6).

Then the linearized loss operator will be a function with the following structure

$$l_{[t]}^{\Delta}(x) = -(c_t + \mathbf{b}_t' x) \tag{2.34}$$

with some constant c_t and a constant vector $\mathbf{b}_t$, which are known at time t.

The idea is to use the fact that a linear function (2.34) of a multivariate normal distribution has a univariate normal distribution. Using the rules of mean and variance of linear combinations of random vectors we get that

$$L_{t+1}^{\Delta} = l_{[t]}^{\Delta}(\mathbf{X}_{t+1)}) \sim N(-c_t - \mathbf{b}_t'\mu,\ \mathbf{b}_t'\Sigma\mathbf{b}_t) \tag{2.35}$$

Value-at-risk can then be easily calculated for this loss distribution by

$$\mathbf{VaR}_{\alpha} = \mu_p + \sigma_p \Phi^{-1}(\alpha) \tag{2.36}$$

where μ_p and σ_p are mean and variance of the linearized the loss distribution, Φ denotes the standard normal cumulative distribution function and $\Phi^{-1}(\alpha)$ is the α -quantile of Φ.

2.7.2 Historical Simulation Method

The method tries to estimate the distribution of the loss operator under the *empirical distribution* of the data $X_{t-n+1},...,X_t$. We construct a univariate dataset by applying the loss operator to each of our historical observations of the risk-factor changes and we get a set of historically simulated loses:

$$\{\tilde{L}_s = l_{[t]}(X_s) : s = t-n+1,....t\} \tag{2.37}$$

The values $\tilde{L}_s$ show what would happen to the current portfolio if the risk-factor change on day s was to reoccur. We can now use these historically simulated losses to make inference about the loss distribution and the VaR.

There are a number of ways we can use the historically simulated loss data. In practice it is common to estimate VaR using the method of *empirical quantile estimation,* where theoretical quantiles of the loss distribution are estimated by sample quantiles of the data. Let us denote the ordered values of the dataset (2.37) by $\tilde{L}_{n,n} \leq ..... \leq \tilde{L}_{1,n}$. Then a possible estimator of VaR_α is the $n \times (1-\alpha)$-th largest value in (2.37). For example if $n = 1000$ and $\alpha = 0.95$ we would estimate VaR by the 50-th largest historically simulated loss.

2.7.3 Monte Carlo Simulation Method

The term Monte Carlo Method is a rather general name for any approach that uses a number of simulations under an explicit parametric model. In risk management the idea is to simulate a large number of risk-factor changes and apply the loss operator on them.

The first step is to choose of the model and to calibrate this model to fit the historical risk factor data $X_{t-n+1},...,X_t$. It should be a model from which we can easily simulate, because in the second stage we generate m independent realizations of risk-factor changes for the next time period. Let us denote those realizations by $\widetilde{X}_{t+1}^{(1)},.....\widetilde{X}_{t+1}^{(m)}$.

Just like in the case of historical simulation method, we can now apply the loss operator to these simulated risk-factor changes and we obtain a number of simulated realizations $\{\widetilde{L}_{t+1}^{(i)} = l_{[t]}(\widetilde{X}_{t+1}^{(i)}) : i = 1,....,m\}$ from the loss distribution. Again we can estimate VaR by simple empirical quantile estimation. An important feature of Monte Carlo method is that we are free to chose the model and the number of replications m. Generally, m can be chosen to be much larger than n, so that we will obtain more accurate estimate of VaR than is possible in the case of historical simulation.

Since all three methods have their advantages and disadvantages, we cannot say which VaR estimate is the most accurate one. None of the methods is generally considered better than the others.

The model we use in this paper to estimate VaR is the following: we are considering a pair of risk factor changes together. The marginal distribution of each single risk-factor is approximated by its true empirical distribution and the joint distribution of the two risk-factors is modeled by the 'best fit' bivariate copula. Our goal is not to fit the best model to the marginal distributions; we focus only on the dependence structure between the risk factors. That is why we take the historical empirical distribution for the marginals. In the next section we present the data and we discuss the results of our research.

3. Data and the Discussion of Results

3.1 Data

We use daily data from DataStream for the period 20.10.2000 to 17.11.2008. We are considering seven countries in Central and Eastern Europe – Bulgaria, Romania, Poland, Czech

Republic, Hungary, Slovakia and Russia. As a representation of the stock market in each country, we take the main stock exchange index of that country. The foreign exchange (FX) rates are expressed in US dollars per local currency e.g. if BGN/USD = 0.647 this means that 1 Bulgarian Lev is traded for 0.647 US Dollars (i.e. the 'price' of 1 BGN in $)

The following table summarizes the data we use:

Country	Observations	Main Stock Exchange Index	FX rate in $
Bulgaria	2107	SOFIX – Bulgarian Stock Exchange, Sofia	BGN/USD
Romania	2107	BET - Bucharest Exchange Trading Index	RML/USD
Poland	2107	WIG - Warsaw Stock Exchange Index	PLZ/USD
Czech Republic	2107	PX - Prague Stock Exchange Index	CZK/USD
Hungary	2107	BUX - Budapest Stock Exchange Index	HUF/USD
Slovakia	2107	SAX – Bratislava Stock Exchange Index	SKK/USD
Russia	2107	RTS - Russian Trading System Index	RUR/USD

For both stock index and exchange rate we are considering the daily logarithmic returns of the raw data. The return series are labeled: R_SOFIX; R_BET; R_WIG; R_PX; R_BUX; R_SAX; R_RTS, for each index respectively. The FX rate returns are labeled: R_FX_BGN; R_FX_RML; R_FX_PLZ; R_FX_CZK; R_FX_HUF; R_FX_SKK; R_FX_RUR, respectively.

Tables 1 and 2 in the Appendix give an overview of the return series statistics. The tables show that all the means of the returns are small relative to their standard deviations. The standard deviations of stock returns are higher than the standard deviation of FX returns, which indicates that stock markets are more volatile that FX markets. All the return series have skewness different from 0 and all, except Bulgarian and Slovakian FX rate returns, are skewed to the left. Skewness, Kurtosis and Jarque-Bera Statistic clearly show that the returns are not normally distributed. Jarque-Bera test rejects the null hypothesis (that data is normally distributed) at 100% level for all the return series.

3.2 Dependence structure of Equity and FX rate

Table 3 shows the linear correlations between each country's stock return and foreign exchange return. We can see that the correlations are all positive, but relatively small. This shows a very small positive dependence between stock and FX rate in each country. The linear correlation is the largest for Poland (0.121), indicating that the increase (decrease) of the local stock market is associated with (around 12% on average) appreciation (depreciation) of the local currency. Romania and Russia also show a bit larger linear correlations, which is a sign of again small (8% and 9% on average) positive dependence between stock market and FX market. Slovakia has the smallest (and insignificant at 10% level – p-value > 0.1) linear correlation. This indicates that Slovakian stock market and foreign exchange market are not dependent from each other.

Tables 4 and 5 represent Kendall's Tau and Spearman's Rho correlations for each county's stock return and FX return. Kendall's tau measures the difference between the probability of the concordance and the probability of the discordance and Spearman's Rho measures the rank correlation between variables. We can see that both rank correlations are consistent with each other and with the linear correlation (except for Czech Republic and Hungary, where the very small negative rank correlations are insignificant at 10% level – p-value is much larger than 0.1). Just like with linear correlation – rank correlations are a bit higher for Poland $(\rho_\tau = 0.059$ and $\rho_S = 0.088)$, followed by Russia and Romania. Both linear correlation and rank correlations show no indication of significantly large dependence between equity market and foreign exchange market, for all seven countries in consideration.

Next we will take a look at the tail dependence of stock market and FX market. To see the dependence structure from the data we build a frequency table. To do this, we first rank the pair of return series in ascending order and then we divide each series evenly into 20 bins. Bin 1 includes the observations with the lowest values and bin 20 includes observations with the largest values. In other words bin 1 contains all the values that are smaller than or equal to the 5%-quantile (of the Empirical Distribution) of the return data and bin 20 contains the values that are larger than the 95%-quantile. We want to know how the values of one return series are associated with the values of the other return series, especially whether lower returns in stock market is associated with lower returns in FX market. In other words, we are interested in the empirical probability that one return series is in its i-th bin, given that the other return series is in its j-th bin. Let the two return series in consideration be X_1 and X_2. The cell (i, j) of the frequency table shows the number of times series X_1 is in its i-th bin, given that series X_2 is in its j-th bin.

Obviously we will concentrate our attention to cell $(1,1)$ and cell $(20,20)$, because they will represent the extreme (lower and upper) tail dependence. The information we can obtain from the frequency table is as follows: if the X_1 and X_2 are perfectly positively correlated, we would see most observations lie on the main diagonal; if they are independent, then we would expect that the numbers in each cell are about the same; If the series are perfectly negatively correlated, most observations should lie on the diagonal connecting the upper-right corner and the lower left corner; If there is positive lower tail dependence between the two series, we would expect more observations in cell $(1,1)$; If there exists positive upper tail dependence, we would expect a large number in cell $(20,20)$.

Tables 6 through 12 in the Appendix show the frequency tables for each country's stock return and FX return. As we can see there is no indication of upper or lower tail dependence for Bulgarian Index and FX rate. This can be partly explained by the fact that Bulgaria has been in a situation of Monetary Board for the last 10 years and its currency is bound to the Euro. According to our results there is some small lower tail dependence for Romanian stock index and FX rate. This asymmetric dependence indicates that Romanian equity and currency are more likely to depreciate together then to appreciate together. The frequency tables for Czech Republic, Hungary and Russia show similar dependence structures as the one for Romania. There is no indication of upper or lower tail dependence for Slovakian stock index and currency. Only the frequency table for Poland shows signs of upper tail dependence, although it is still asymmetric because the lower tail dependence is stronger than the upper tail dependence.

The frequency tables' analysis shows consistent results to linear and rank correlation analysis: there is no indication of dependence between stock return and FX rate return for Bulgaria and Slovakia; there is some lower tail dependence between Romanian, Czech, Hungarian and Russian stock markets and local FX rates and there is some lower and upper tail dependence for Polish stock return and FX rate.

3.3 Dependence structure between Equity markets

We are now going to perform the same analysis but this time for all the pairs of stock exchange returns of the countries in consideration. We want to analyze the co-movements of equity markets. We are not going to consider the FX rate of each country but only the return on its main stock exchange index. The goal is to see if Central and Eastern European Equity markets are dependent on each other.

Again we start our analysis with linear correlation. Table 13 shows the linear correlations between the stock returns of each country. As we see all the pairs show significant positive correlation, except for Slovakian stock return which seems to be uncorrelated with the others. The correlations are highest for the pairs – Poland-Czech Republic (0.5743); Poland-Hungary (0.5634) and Czech Republic-Hungary (0.5699), which indicates strong dependence between those three equity markets. An increase (decrease) of one of those three stock markets is associated with (more than 56% on average) increase (decrease) of the other two. Russian stock returns are also highly correlated with Polish, Czech and Hungarian stock returns. Such high linear correlation indicates that Polish, Czech and Hungarian equity markets are highly dependent on the Russian market (as the largest financial market in consideration). Bulgarian equity market is also positively dependent on the others, although the correlations are relatively lower. The highest correlations are with Romanian (0.1318), Czech (0.1478) and Russian (0.1473) markets. Romanian stock returns are also positively correlated with the others – the highest is the correlation with Czech stock returns - 0.3415.

Tables 14 and 15 represent the rank correlations for each pair of stock returns. We can see that the result is consistent with linear correlation (although Kendall’s Tau is relatively smaller). Again Polish, Czech and Hungarian stock returns are highly correlated with each other and with Russian stock returns. Equity returns in Slovakia seem completely uncorrelated with the rest of the equity markets in consideration. Bulgarian stock returns do not show high rank correlations with any other stock returns. Romanian stock market indicates relatively high rank correlations with Czech and Polish stock markets.

Let us now examine the frequency tables of all the pairs of stock markets. Tables 16 to 37 represent the frequency tables. Again we find that Slovakian stock return shows no tail correlation with any other stock return in consideration. Bulgarian stock returns show lower tail dependence with all the other returns (except Slovakia). This indicates that Bulgarian stock market tends to fall together with Central European stock market and Russian stock market. Again, Poland, Czech Republic and Hungary show high lower and upper tail dependence between each other. This finding indicates that those three stock markets crush and boom together. Nevertheless, the dependence is asymmetric because the lower tail dependence is much higher than the upper tail dependence. Our analysis shows again that these three stock markets are also highly dependent on the Russian stock market. We find evidence of lower and upper tail dependence between Russian stock returns and all the three Polish, Czech and Hungarian stock returns. Here again the lower tail dependence is higher than the upper tail dependence. Romanian stock returns show relatively weaker but almost symmetric tail dependence (upper and lower almost the same) with Polish, Czech, Hungarian and Russian stock returns.

4. Model Estimation and Value-at-Risk

4.1 Model and Optimal Copula

As mentioned above, in this paper we are not trying to create a model for the univariate return series, we are only interested in their dependence structure. Therefore when modeling the behavior of a specific pair of return series we will use the empirical distribution for the marginals and we will try to find the copula that best describes the co-movements (dependence) of the series in consideration.

Let us look again at the dependence structure of stock returns and FX returns in each of the seven countries. With the help of Andrew Patton's *Copula toolbox* for Matlab® [10] we are able to find the best fitting copula for all the pairs of return series. The toolbox uses *Maximum likelihood Estimation* to find the optimal copula (the one that best fits the empirical data). The results are presented in table 38. We can see that for all countries, except Poland, student-t is the optimal copula. Student-t Copula is a symmetric copula (shows both upper and lower tail dependence), but the estimated coefficients of tail dependence we get are very small (weak or no tail dependence) which is consistent with our preliminary findings. Only for Poland the optimal copula is the so called *Rotated Gumbel Copula*, which is derived from the Gumbel copula in the following way:

$$C_{\theta}^{Gu-Rotated}(u,v) = C_{\theta}^{Gu}(1-u,1-v)$$

The Gumbel Copula has upper tail dependence and no lower tail dependence. On the other hand the Rotated Gumbel Copula is exactly the opposite – it has lower tail dependence and no upper tail dependence. This again is consistent with the result of the frequency table where Polish stock and FX return showed stronger lower tail dependence.

In the same way in table 39, we present the optimal copula and the estimated coefficients of tail dependence for all the pairs of stock returns of the countries in consideration. We can see that the coefficients of tail dependence are consistent with the frequency tables, previously discussed. The highest coefficients of tail dependence are between Poland, Czech Republic and Hungary, also relatively high are the dependences between these three stock markets and the Russian stock market. Student-T copula and Symmetrized Joe Clayton copula turn out to be very good descriptions of the dependence structure for most of the pairs.

We can now use this optimal copula to simulate 5000 random risk-factor changes and use those in the Monte Carlo Simulation method to calculate Value-at-Risk.

4.2 Value-at-Risk

Now we are going measure the market risk associated with each financial market in consideration. First we will focus on the pairs of stock return and FX rate return for each country. We will compute Value-at-Risk of a hypothetical portfolio of 100 million US Dollars invested 100% in the given county's stock market and we will consider two risk-factors changes – the stock returns and the FX rate returns of the particular country.

We are considering the following situation: today (17/11/2008) we invest the $100M in the market portfolio of each country (represented by the main stock exchange index) and we are interested for the VaR (in $) for the next day i.e. the 1-day VaR at 95%, 99% and 99.9% confidence levels. We use the three methods discussed above – Variance-Covariance, Historical Simulation and Monte Carlo Simulation (with copulas and the model we gave in the previous section). The results are presented in tables 40, 41 and 42 for the 95%, 99% and 99.9% VaR, respectively. All values are in $.

We can see that in the case of Slovakia the Value-at-Risk is the lowest, indicating that this financial market is less risky than the others in consideration. Russian stock market appears to be the most risky one. The other countries show similar values of VaR to each other.

The results also show that, in the case of 95% VaR, Variance-Covariance method gives higher estimates than Historical Simulation and Monte Carlo Simulation. In the case of 95% confidence level the Variance-Covariance method overestimates the tail of the loss distribution. On the other hand when we consider higher quantiles – 99% and 99.9% we see that Variance-Covariance method underestimates the true loss distribution and gives VaR much lower than Historical Simulation and Monte Carlo simulation, especially in the case of 99.9% confidence level where the difference is really significant – Variance-Covariance VaR is sometimes almost twice lower than Historical Simulation. This difference can be explained by the non-normality of the data. We saw already that none of the return series in consideration is close to being normally distributed and moreover - the joint distribution of the risk factors is not the multivariate normal distribution – which is the main and crucial assumption of Variance-Covariance Method.

We also note that Monte Carlo method gives VaR estimates which are very close to Historical simulation. This makes sense, because the model for the Monte Carlo simulations uses empirical distribution of the univariate risk-factors and describes their dependence with the optimal copula, which turns out to be a very good approximation of the true multivariate distribution of the risk-factors.

Now let us look at pairs of countries together. We will again compute Value-at-Risk of a hypothetical portfolio of $100M, but this time it is equally invested in the markets of the two countries in consideration - $50M in one stock market and $50M in the other. Here we will consider only the returns of the stock markets as risk-factors and not the FX rates of the two countries. Situation is as follows: today (17/11/2008) we invest $50M in one stock market and $50M in the other. We assume that the FX rates do not change for 1 day and we do not consider them as risk factors. Again, we are interested in the 1-day VaR at 95%, 99% and 99.9% confidence levels. We expect that VaR is lower than if we invest $100M in one market only, because of the diversification effect. The actual results are summarized in tables 43, 44 and 45.

Results are consistent with the VaR for a portfolio invested in one market only – again for 95% confidence level Variance-Covariance Method gives VaR higher than the other two methods. But when we consider higher quantiles Variance-Covariance method underestimates the tail of the loss distribution. The Value-at-Risk calculated with Monte Carlo Simulation Method is very close to the one calculated with Historical Simulation. This again indicates that the copula approach gives a very good approximation of the true multivariate dependence structure of the risk-factors. We have to note that VaR is lower for the pairs of countries that include Slovakia. As we already found out – Slovakian market has very low correlation with the others which makes it a very good diversifier in a given portfolio.

One more thing worth mentioning is that none of the methods for estimating VaR is generally considered ‘better’ than the others. In practice, risk managers take all of them into account. But the most important result of our research is that the ‘best fit’ copula is a very good description of the true dependence structure of the risk-factors.

5. Conclusion

The goal of this paper was to analyze the dependences across financial markets in Central and Eastern Europe. We focused our attention on two kinds of dependences – the relation between stock returns and FX rate returns for each country and the co-movements of stock markets. We considered the following seven countries: Bulgaria, Romania, Poland, Czech Republic, Hungary, Slovakia and Russia.

Our results showed no indication of any dependence between stock returns and FX rate returns for Bulgaria and Slovakia. We also found some lower tail dependence between the stock markets and FX rates of Romania, Czech Republic, Hungary and Russia and some lower and upper tail dependence for Polish stock returns and FX rate returns, but all those dependences were relatively small. The conclusion is: there is no evidence of significantly large dependence between equity market and foreign exchange market, for all the 7 countries in consideration.

On the other hand we found significant correlation between stock returns of Poland, Czech Republic and Hungary. Our results show high lower and upper tail dependence between each of these countries, indicating that those three stock markets tent to crush and boom together. There is also evidence that Polish, Czech and Hungarian stock markets are strongly correlated with the Russian stock market. Russian stock market has significant tail dependence with all of the others stock markets in consideration (except Slovakia). This dependence is again stronger in the lower tail than in the upper tail – meaning that a crush in Russian stock returns is likely to cause crushes in the other Central and Eastern European markets. Slovakian stock returns show no correlation with any other stock return in consideration, a property that makes Slovakian stock market attractive for investors because of the good diversification effect. Bulgarian stock returns show lower tail dependence (although relatively weaker) with all the other returns (except Slovakia), indicating that Bulgarian stock market tends to fall together with Central European stock market and the Russian stock market.

We then turned our attention to modeling these relations and co-movements across financial markets. We focused on the copulas of the risk-factors. For each pair of risk-factors in consideration we fitted the optimal copula and we calculated Value-at-Risk based on the three most common methods – Variance-Covariance, Historical Simulation and Monte Carlo Simulation. Our results show that for the case of 95% confidence level Variance-Covariance method actually overestimates the tail of the loss distribution and produces higher values of VaR than the other two methods. But if we consider higher confidence levels – 99% and 99.9% we find that Variance-Covariance method gives much lower estimates of VaR than the other two methods – an indication that, in this case, the assumption of normally distributed risk-factors is inadequate. An important result of our research is that copula approach gives a very good description of the true dependence structure of risk-factors.

References

[1] McNeil, A. J., Frey, R., & Embrechts, P. - "Quantitative Risk Management – Concepts, Techniques and Tools" – Princeton series in Finance (2005)

[2] Chakrabarti, R. and Roll R. (2002), "East Asia and Europe during the 1997 Asian Collapse: a Clinical Study of a Financial Crisis", Journal of Financial Markets 5 (2002) 1-30.

[3] Karolyi, G. A. and Stulz, R. M., 1996, "Why do Markets Move Together? An Investigation of U.S.-Japan Stock Return Comovements", Journal of Finance, 51(3):951-986.

[4] Longin, F. and B. Solnik (2002) "Extreme Correlation of International Equity Markets", Journal of Finance, 56(2):649-679.

[5] Forbes K. and R. Rigobon (2002) "No Contagion, Only Interdependence: Measuring Stock Market Co-movements", Journal of Finance, 57(5): 2223-2261

[6] Hu, L (2005), "Dependence Patterns across Financial Markets: a Mixed Copula Approach", forthcoming in the Applied Financial Economics.

[7] Chollete, Victor de la Pena, and Ching-Chih Lu (2005), "Comovement of International Financial Markets", working paper.

[8] Marshal R. and A. Zeevi (2002) "Beyond Correlation: Extreme Co-movements between Financial Assets", Working Paper, Columbia Business School

[9] Patton, A.L. (2006), "Modeling Asymmetric Exchange Rate Dependence", International Economic Review, 47(2), 527-556.

[10] Andrew Patton's Copula toolbox for Matlab® - http://www.financialmathematics.com/wiki/Code:patton_copula_toolbox

[11] Ning, Cathy (2006), " Dependence Structure between the Equity Market and the Foreign Exchange Market—A Copula Approach", Department of Economics, Ryerson University, Toronto, Ontario, M5B 2K3, Canada

Appendix

Table 1. Descriptive statistics of the daily logarithmic returns of stock exchange indices.

	R_SOFIX	R_BET	R_WIG	R_PX	R_BUX	R_SAX	R_RTS
Mean	0.00061	0.000829	0.000234	0.000222	0.000177	0.000624	0.000528
Std. Dev.	0.018579	0.016365	0.012933	0.015077	0.015319	0.011098	0.022245
Skewness	-0.609329	-0.27997	-0.32598	-0.70406	-0.322879	-0.042637	-0.736228
Kurtosis	31.57748	10.11759	5.946227	20.39308	12.01723	9.47241	16.3606
Jarque-Bera	71827.43	4475.061	799.3698	26732.73	7174.993	3678.415	15861.65
Probability of Normality	0	0	0	0	0	0	0
Observations	2107	2107	2107	2107	2107	2107	2107

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Table 2. Descriptive statistics of the daily logarithmic returns of foreign exchange rates.

	R_FX_BGN	R_FX_RML	R_FX_PLZ	R_FX_CZK	R_FX_HUF	R_FX_SKK	R_FX_RUR
Mean	0.000192	-8.63E-05	0.000218	0.000345	0.000184	0.000367	1.02E-05
Std. Dev.	0.006116	0.006531	0.007967	0.007317	0.008549	0.006955	0.002658
Skewness	0.103095	-0.184091	-0.703651	-0.094485	-0.539868	0.002654	-1.123123
Kurtosis	4.771448	11.31232	10.52722	8.434286	9.074085	4.165953	27.03313
Jarque-Bera	279.225	6077.841	5148.058	2595.751	3341.381	119.3506	51150.67
Probability of Normality	0	0	0	0	0	0	0
Observations	2107	2107	2107	2107	2107	2107	2107

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Table 3. Linear correlation between each country's stock index return and FX rate return.

Linear Correlation Index vs. FX	**Bulgaria**	**Romania**	**Poland**	**Czech Republic**	**Hungary**	**Slovakia**	**Russia**
	0.04441	0.080073	0.121418	0.039181	0.058398	0.018678	0.093028
P-value	(0.0416)	(<0.001)	(<0.001)	(0.0155)	(<0.001)	(0.24)	(<0.001)

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Table 4. Kendall's Tau rank correlation between each country's stock index return and FX rate return.

Kendall's Tau Index vs. FX	**Bulgaria**	**Romania**	**Poland**	**Czech Republic**	**Hungary**	**Slovakia**	**Russia**
	0.01701	0.023215	0.05961	-0.003867	-0.00118	0.018527	0.02598
P-value	(0.2438)	(0.0643)	(<0.001)	(0.7212)	(0.9065)	(0.0844)	(0.0243)

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Table 5. Spearman's Rho rank correlation between each country's stock index return and FX rate return.

Spearman's Rho	Bulgaria	Romania	Poland	Czech Republic	Hungary	Slovakia	Russia
Index vs. FX	0.02405	0.034381	0.088743	-0.005396	-0.00181	0.027054	0.036978
P-value	(0.27)	(0.0672)	(<0.001)	(0.739)	(0.9042)	(0.0888)	(0.03)

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Table 6. Frequency table for Bulgaria
(R_SOFIX vs. R_FX_BGN)

13	7	4	6	5	6	2	3	6	1	6	4	7	5	5	4	4	5	5	8
11	3	8	7	1	6	4	3	7	1	5	4	4	6	4	8	4	6	10	3
8	0	8	3	6	3	7	4	7	2	8	4	1	6	5	11	7	7	4	5
5	6	9	8	4	6	4	8	7	2	9	6	5	5	6	2	4	1	3	5
6	7	3	5	6	10	5	6	5	0	8	3	5	6	3	6	4	4	7	6
6	3	8	3	4	4	4	8	4	3	3	9	4	1	7	10	5	6	7	7
2	6	3	8	4	7	10	4	6	0	6	6	5	5	7	7	8	3	3	5
2	2	6	7	7	6	12	4	4	2	4	7	7	5	4	9	6	3	5	3
7	4	4	5	3	5	4	7	11	4	9	4	5	2	9	3	5	4	5	6
1	0	1	4	1	3	1	4	15	57	0	0	3	1	3	2	0	3	4	2
2	10	7	4	4	2	3	4	2	18	3	10	11	4	6	2	6	4	1	2
6	11	6	4	7	4	7	5	3	2	3	7	6	7	4	3	4	8	4	5
2	4	3	3	7	7	7	7	4	1	5	4	5	7	4	9	6	9	4	7
3	9	5	6	6	4	5	3	4	0	8	7	6	3	7	6	4	8	6	5
8	3	4	6	6	6	6	3	0	0	2	8	8	7	7	6	8	5	7	6
2	6	8	3	7	8	4	7	5	1	1	3	6	12	4	4	6	7	5	6
2	11	3	4	11	6	5	6	4	2	4	7	3	11	5	3	5	4	8	1
8	4	6	6	3	6	2	5	4	1	8	3	5	6	6	5	7	9	3	9
7	3	8	10	4	4	3	7	2	4	7	8	3	5	4	2	3	5	8	8
5	6	2	3	9	3	10	7	6	4	6	2	6	1	6	3	9	5	6	**7**

Table 7. Frequency table for Romania
(R_BET vs. R_ FX_RML)

20	9	1	3	4	4	4	5	3	1	4	4	6	7	5	6	5	6	3	6
10	7	7	6	5	7	4	1	3	9	6	5	4	8	2	7	2	5	6	1
3	1	8	3	5	7	5	5	5	2	5	7	4	6	5	7	6	8	9	5
10	4	3	7	2	9	4	4	2	6	4	7	7	8	6	8	3	8	2	1
4	6	6	5	2	4	8	7	10	2	9	5	5	4	7	4	3	4	5	5
5	4	7	4	8	1	2	5	4	4	8	5	10	6	5	11	4	5	5	3
5	7	6	6	6	6	8	2	9	1	6	6	2	3	6	6	7	6	3	4
7	9	6	4	6	6	5	6	2	3	5	4	6	5	4	4	3	3	9	8
7	6	7	5	8	4	7	4	4	2	4	2	4	3	8	4	7	6	8	6
2	3	8	4	7	8	1	8	4	2	3	5	6	4	7	4	10	4	10	5
3	3	4	4	1	4	5	3	13	39	3	1	5	3	3	3	1	4	1	2
7	6	4	8	6	3	7	5	7	8	4	4	4	3	5	9	4	5	2	5
3	7	10	5	6	7	4	4	7	1	4	10	8	7	1	5	1	4	6	5
0	5	3	5	9	2	8	8	8	4	8	7	5	4	5	5	3	5	4	7
1	3	4	5	4	7	1	5	3	2	7	11	6	4	11	3	10	6	2	11
3	3	3	4	3	8	6	8	5	4	5	8	5	7	6	4	9	5	6	3
3	4	8	8	8	4	6	6	3	2	5	3	6	4	5	2	6	9	6	7
3	5	5	8	6	6	5	10	2	6	6	4	4	5	5	4	6	3	7	6
4	3	3	9	5	4	8	3	9	5	6	6	6	3	6	4	5	5	6	5
6	10	3	2	4	5	7	6	3	2	3	2	2	11	4	5	10	5	5	**11**

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Table 8. Frequency table for Poland
(R_WIG vs. R_ FX_PLZ)

20	9	8	8	3	4	3	3	6	0	7	2	2	4	8	7	2	6	2	2
8	6	10	4	5	9	8	4	4	0	3	5	6	4	6	3	2	1	9	8
11	6	10	11	6	5	5	3	4	1	4	2	6	5	4	6	7	5	3	2
6	6	8	5	7	5	3	3	4	1	6	4	4	11	3	10	5	3	5	6
3	5	5	12	2	5	4	8	10	3	5	5	8	3	1	6	4	6	4	6
2	5	4	3	6	6	7	6	7	5	6	6	6	5	5	5	4	7	5	6
6	8	1	5	7	7	4	4	5	3	3	10	4	4	8	5	4	8	7	2
5	6	3	7	6	3	3	5	5	5	8	6	8	7	7	3	4	5	5	4
4	9	6	3	7	2	7	7	6	1	9	7	5	6	7	6	7	3	3	1
1	0	1	0	2	3	2	4	3	67	1	2	0	5	5	3	3	2	1	0
5	6	4	3	8	4	4	5	7	3	4	6	3	5	6	6	7	7	5	7
4	4	5	10	3	7	8	9	8	2	1	7	5	5	1	4	7	6	5	5
4	4	5	4	8	5	8	5	8	2	6	5	5	6	6	5	4	6	3	6
3	0	6	5	6	8	5	8	1	0	3	9	5	4	5	6	6	5	12	8
2	2	7	7	6	7	5	4	7	2	2	5	8	5	5	4	10	5	9	4
2	5	4	1	6	3	4	6	2	2	9	6	7	7	4	6	5	5	11	10
4	8	6	4	4	4	8	4	3	3	9	3	4	3	6	5	9	9	2	7
7	9	5	5	6	6	4	3	7	1	5	5	4	5	7	8	5	7	1	6
3	2	5	6	6	5	7	7	4	2	7	5	8	4	7	5	6	6	7	3
6	5	3	2	1	8	6	7	5	2	7	6	7	7	5	2	4	4	6	**13**

Table 9. Frequency table for Czech Republic
(R_PX vs. R_ FX_CZK)

17	7	7	5	1	3	0	5	4	1	4	8	3	7	1	8	6	5	7	7
6	4	5	5	6	6	5	3	5	2	3	11	8	5	1	1	3	6	11	9
5	11	7	3	3	7	6	0	3	2	7	4	5	8	6	8	3	6	7	5
3	4	7	11	7	5	7	5	7	0	4	5	2	5	5	5	4	7	9	3
2	6	9	3	5	5	6	6	4	1	5	7	6	7	9	3	8	2	3	8
1	1	4	6	10	4	5	4	10	2	7	6	8	9	5	9	4	4	5	2
7	6	4	2	10	7	4	3	5	1	5	2	8	7	4	8	4	7	7	4
4	3	8	3	5	4	4	12	3	6	5	6	6	5	7	4	9	8	1	2
3	6	4	14	13	11	2	5	7	0	6	4	3	2	4	1	7	5	5	4
0	1	0	0	0	0	0	3	6	68	3	2	0	1	3	6	2	2	2	6
2	3	6	6	4	5	3	9	11	2	4	8	2	3	10	4	5	8	6	4
2	2	5	7	6	8	10	7	6	2	5	7	10	2	5	7	3	6	3	3
7	5	5	4	6	4	3	3	6	3	6	5	6	9	6	8	5	4	4	6
6	3	6	3	8	9	2	5	7	0	6	7	2	5	4	5	10	8	5	4
5	7	3	2	4	3	10	5	3	3	4	4	7	7	13	6	7	4	5	4
8	5	6	5	6	3	7	3	7	1	5	2	9	5	6	8	6	3	4	6
5	6	5	2	3	4	12	7	3	3	7	6	7	7	4	4	6	3	5	6
7	8	6	7	1	5	8	6	2	5	8	1	4	4	7	6	4	5	9	3
7	9	3	8	3	4	4	8	2	2	5	6	6	6	4	2	7	7	3	9
9	8	6	9	4	9	7	6	5	1	6	5	3	1	2	2	2	6	4	**11**

Table 10. Frequency table for Hungary
(R_BUX vs. R_FX_HUF)

19	10	8	8	5	3	5	7	0	0	3	8	3	5	2	1	7	4	4	4
12	6	5	6	4	5	2	7	9	1	3	2	6	3	10	6	5	5	4	4
10	3	5	7	5	6	3	5	5	3	2	2	6	7	6	6	6	8	5	6
8	8	9	5	4	5	4	4	4	1	4	3	6	5	5	9	8	3	6	4
4	5	3	5	6	10	9	7	8	0	4	3	2	2	6	4	8	3	7	9
3	3	4	5	8	7	5	8	2	1	9	5	5	3	6	2	5	9	9	7
4	7	6	5	2	6	8	4	7	3	3	4	4	4	6	7	9	2	6	8
4	2	6	12	6	5	5	7	5	3	6	7	7	7	6	4	4	3	4	2
4	10	1	6	5	8	2	7	9	1	6	7	2	9	5	10	5	3	3	3
1	0	3	1	1	0	1	0	2	69	18	0	0	2	1	1	0	2	1	2
1	7	6	2	5	4	5	6	6	0	4	3	10	4	5	9	8	7	6	7
6	4	6	4	4	8	11	3	0	4	2	7	6	5	3	4	4	6	11	8
3	3	4	4	6	4	7	8	7	2	6	4	7	7	4	6	8	5	3	7
2	4	10	7	5	5	4	3	5	2	7	6	9	6	5	4	5	7	3	6
4	5	6	4	7	5	5	6	4	1	7	5	8	7	6	5	6	5	7	3
4	6	9	3	5	7	7	3	4	4	2	8	7	7	6	5	4	4	4	6
3	6	4	4	6	6	5	7	5	0	3	6	6	6	7	8	2	11	3	7
5	8	2	8	8	2	4	4	9	4	5	10	4	6	5	6	4	4	7	1
3	4	3	3	10	8	7	5	8	2	7	9	3	4	8	3	2	10	2	4
6	4	6	6	3	2	6	4	7	4	4	7	4	6	4	5	5	5	10	**8**

Table 11. Frequency table for Slovakia
(R_SAX vs. R_ FX_SKK)

7	5	7	5	6	5	7	6	4	4	12	4	8	1	6	6	4	2	2	5
7	5	9	7	1	7	4	1	4	3	3	6	6	6	6	9	4	7	5	5
5	5	10	4	6	4	4	3	5	4	3	10	5	7	6	7	5	4	6	3
6	6	5	3	8	8	2	5	3	5	4	7	7	4	2	4	9	2	5	10
8	5	6	4	5	5	8	2	3	5	3	3	4	7	6	8	4	8	5	6
7	11	8	6	4	3	7	3	5	1	6	2	4	7	4	6	6	5	5	6
6	6	7	7	5	5	2	3	2	5	4	5	4	6	6	9	3	12	1	7
4	3	4	7	8	6	5	2	6	7	4	6	5	7	4	7	8	3	6	3
7	6	7	6	5	6	7	11	6	3	4	5	4	4	3	5	5	7	3	2
0	0	1	1	1	0	1	30	27	16	9	3	1	2	3	1	3	2	3	1
4	5	6	3	7	7	6	4	5	8	7	5	4	2	4	2	6	4	10	6
8	3	4	8	6	4	9	5	4	5	5	4	2	7	8	2	3	5	9	5
3	7	4	3	4	10	7	4	3	6	7	9	7	7	6	2	4	7	4	1
3	6	5	2	4	6	4	2	4	8	4	5	5	9	6	8	4	6	6	8
4	4	7	6	5	3	5	4	4	3	6	4	8	4	8	11	5	6	5	4
4	5	2	8	8	7	3	4	4	5	5	3	3	5	8	6	9	3	6	7
4	6	2	9	8	4	7	2	4	3	5	8	8	6	5	3	5	5	6	5
2	5	4	6	6	6	6	4	5	3	1	5	9	8	6	4	5	8	5	8
9	3	3	6	5	5	6	3	4	6	7	5	6	3	5	3	4	8	4	10
8	9	5	4	3	5	5	7	4	5	6	7	5	3	4	2	9	2	9	**4**

Table 12. Frequency table for Russia
(R_RTS vs. R_FX_RUR)

18	6	4	5	4	6	6	6	2	4	3	1	5	6	5	5	5	3	2	10
7	5	4	4	5	5	4	3	4	6	5	6	7	7	5	6	3	5	4	10
4	9	4	6	9	5	7	6	4	2	4	7	7	1	8	5	5	6	5	2
3	7	7	4	10	5	8	4	3	2	8	5	4	3	6	8	6	4	4	4
3	3	7	5	6	4	10	7	3	5	3	6	6	5	4	4	7	5	4	8
6	8	4	6	4	8	4	6	2	4	5	8	5	5	1	5	5	9	8	3
3	9	4	8	6	7	3	3	4	2	2	7	3	7	11	6	3	6	6	5
3	4	9	3	4	10	4	5	1	5	7	6	3	7	7	6	4	2	9	6
18	5	6	3	5	6	3	3	6	3	3	3	8	3	5	3	9	4	6	4
0	3	7	4	8	8	8	8	51	8	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	20	10	4	7	9	7	6	5	14	8	15
1	4	8	4	2	2	7	7	3	8	6	5	8	5	6	5	6	3	5	11
4	8	6	3	4	6	6	9	3	4	9	5	2	7	3	1	3	8	10	4
3	6	1	7	3	4	5	8	6	4	6	9	8	2	6	5	6	6	8	2
6	4	9	5	3	3	3	6	2	3	4	5	7	15	1	6	4	8	6	6
3	3	7	10	7	4	4	5	2	4	7	7	6	6	4	7	7	6	5	1
4	5	5	8	8	7	5	5	2	5	5	6	7	4	7	5	7	2	6	2
5	5	5	7	2	5	6	8	0	2	8	6	6	9	7	8	5	4	5	3
6	3	3	5	11	8	9	5	4	5	5	3	2	3	6	8	9	7	1	2
9	8	6	8	4	3	3	1	4	9	5	7	4	1	7	6	6	4	3	**8**

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Table 13. Linear Correlations between Equity market returns

Linear Correlation	R_SOFIX	R_BET	R_WIG	R_PX	R_BUX	R_SAX	R_RTS
R_SOFIX	1	0.1318	0.075	0.1478	0.0861	0.0263*	0.1473
R_BET	0.1318	1	0.2395	0.3415	0.2462	0.01*	0.2269
R_WIG	0.0750	0.2395	1	0.5743	0.5634	0.0153*	0.4598
R_PX	0.1478	0.3415	0.5743	1	0.5699	0.0153*	0.5241
R_BUX	0.0861	0.2462	0.5634	0.5699	1	-0.0041*	0.4440
R_SAX	0.0263*	0.01*	0.0153*	0.0153*	-0.0041*	1	-0.0101*
R_RTS	0.1473	0.2269	0.4598	0.5241	0.444	-0.0101*	1

*-insignificant at 10% level (p-value much larger than 0.1)

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Table 14. Kendall’s Tau Rank Correlations between Equity market returns

Kendall's Tau	R_SOFIX	R_BET	R_WIG	R_PX	R_BUX	R_SAX	R_RTS
R_SOFIX	1	0.0467	0.0301	0.051	0.0219	0.041	0.0572
R_BET	0.0467	1	0.1054	0.1341	0.0917	0.0032*	0.0758
R_WIG	0.0301	0.1054	1	0.3533	0.3762	0.0006*	0.2772
R_PX	0.051	0.1341	0.3533	1	0.3533	0.0266	0.2889
R_BUX	0.0219	0.0917	0.3762	0.3533	1	0.0108*	0.238
R_SAX	0.042	0.0032*	0.0006*	0.0266	0.0108*	1	0.0097*
R_RTS	0.0572	0.0758	0.2772	0.2889	0.238	0.0097*	1

*-insignificant at 10% level (p-value much larger than 0.1)

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Table 15. Spearman’s Rho Rank Correlations between Equity market returns

Spearman's Rho	R_SOFIX	R_BET	R_WIG	R_PX	R_BUX	R_SAX	R_RTS
R_SOFIX	1	0.0687	0.0443	0.0758	0.0326	0.0598	0.0854
R_BET	0.0687	1	0.1549	0.1949	0.1353	0.0052*	0.1116
R_WIG	0.0443	0.1549	1	0.5011	0.5298	0.0005*	0.398
R_PX	0.0758	0.1949	0.5011	1	0.4981	0.0384	0.4098
R_BUX	0.0326	0.1353	0.5298	0.4981	1	0.015*	0.3437
R_SAX	0.0598	0.0052*	0.0005*	0.0384	0.015*	1	0.0142*
R_RTS	0.0854	0.1116	0.398	0.4098	0.3437	0.0142	1

*-insignificant at 10% level (p-value much larger than 0.1)

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Table 16. Frequency table Bulgaria – Romania
(R_SOFIX vs. R_BET)

23	9	8	4	4	7	5	5	4	1	1	3	4	3	3	6	2	4	5	5
7	4	4	7	7	6	7	6	6	2	5	4	6	9	5	2	8	6	2	2
6	6	8	2	6	8	6	6	3	2	4	8	3	5	6	5	7	3	4	8
2	6	8	9	9	1	5	4	7	2	5	6	11	5	5	1	5	5	3	6
5	4	10	11	4	5	3	6	6	4	7	1	6	3	6	7	5	3	4	5
4	5	6	5	5	6	3	5	6	1	8	8	1	10	6	7	8	2	7	3
4	4	6	6	4	3	4	7	5	4	5	9	4	6	6	9	4	2	7	6
2	4	2	5	8	4	11	4	8	4	3	3	8	3	4	11	1	8	10	2
9	5	2	6	7	9	7	5	11	3	2	7	5	6	3	3	5	4	1	6
0	0	0	1	1	2	0	2	10	57	4	3	2	4	2	3	3	4	3	4
3	4	7	5	7	3	4	10	4	1	5	4	3	2	8	8	7	6	5	9
1	4	9	5	6	7	3	5	5	4	4	5	6	4	6	6	5	8	5	8
1	5	8	4	6	8	4	3	4	3	10	7	6	6	4	6	3	10	2	5
5	9	2	2	5	5	6	7	2	2	8	6	7	8	7	1	8	4	8	3
6	3	1	6	2	6	9	7	4	4	4	3	11	5	5	6	5	5	8	6
3	8	4	3	6	4	3	4	5	2	8	5	7	9	7	7	5	1	9	5
8	6	4	6	5	5	9	9	3	2	7	7	2	5	5	6	8	4	0	4
6	10	4	6	5	5	8	5	5	2	1	6	7	4	6	7	3	6	6	4
7	4	5	6	3	4	4	2	6	3	5	6	2	5	7	2	6	11	9	8
4	5	8	6	5	8	4	3	2	2	9	5	4	3	5	2	7	10	7	**7**

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Table 17. Frequency table Bulgaria – Poland
(R_SOFIX vs. R_WIG)

18	2	12	7	4	7	11	5	2	4	3	4	2	3	5	1	6	3	2	5
9	10	11	7	5	9	6	5	3	1	2	0	3	3	8	3	5	4	8	3
4	5	5	3	7	6	9	4	3	1	5	10	8	7	4	2	3	7	5	8
6	4	4	4	6	4	5	6	8	5	2	3	10	5	8	2	4	7	3	9
2	4	9	6	9	2	5	10	2	5	7	7	8	2	5	3	7	2	8	2
6	3	3	0	5	2	6	8	6	6	4	11	2	12	2	6	4	4	7	9
2	3	0	3	6	4	7	6	8	5	8	8	5	5	4	5	8	5	7	6
5	4	5	6	5	6	9	4	2	5	5	9	4	7	4	8	4	1	7	5
4	5	7	9	6	4	5	3	4	5	3	5	5	3	8	9	2	7	2	10
4	6	3	2	3	2	0	5	14	28	2	3	5	6	2	4	3	4	7	2
6	7	5	8	5	3	2	4	5	3	7	5	6	5	4	6	7	5	8	4
7	5	5	4	3	7	4	3	8	4	6	4	3	7	7	6	6	8	6	3
2	7	4	6	5	9	4	5	3	8	11	4	4	7	5	2	6	4	4	5
4	6	5	5	8	4	6	4	1	4	7	3	9	4	6	10	3	8	5	3
2	6	5	3	7	5	3	12	8	5	8	2	3	5	5	7	5	4	5	6
2	2	4	6	3	8	4	4	4	1	5	9	13	7	7	8	8	4	2	4
5	2	4	5	5	7	0	5	7	5	4	5	3	9	10	8	4	9	5	3
4	7	5	7	5	3	4	4	6	4	8	7	4	3	5	5	6	6	5	8
7	5	6	7	3	8	8	3	8	5	5	3	5	0	6	3	5	9	2	7
7	12	4	7	5	6	7	5	4	1	3	4	3	5	1	7	9	5	7	**4**

Table 18. Frequency table Bulgaria – Czech Republic
(R_SOFIX vs. R_PX)

22	6	7	7	6	5	7	5	5	5	3	5	5	3	4	2	3	0	3	3
11	7	9	7	4	4	6	6	5	3	5	5	5	3	5	3	2	5	5	5
5	6	6	8	6	5	4	4	5	1	5	7	6	4	7	4	5	4	7	7
6	6	9	5	6	5	6	7	5	5	3	8	2	9	2	5	3	2	6	5
4	3	4	4	5	5	5	9	3	5	3	5	9	5	5	5	10	8	5	3
2	7	5	4	8	9	5	1	7	5	3	5	2	6	6	4	7	5	11	4
4	8	6	3	5	3	4	9	3	2	9	7	5	7	6	8	6	2	6	2
2	6	5	8	5	3	7	6	0	5	7	3	6	4	7	7	4	7	5	8
10	5	6	4	8	3	4	5	9	3	4	8	4	1	4	11	4	5	3	5
0	2	1	3	4	2	7	3	10	31	1	6	6	6	5	1	5	5	3	4
3	2	5	2	3	7	8	7	7	6	2	2	5	6	7	6	8	6	10	3
2	6	5	2	6	10	5	6	6	2	2	5	6	6	10	7	1	8	1	10
4	7	3	11	2	4	4	6	5	3	3	6	6	6	4	7	6	9	5	4
4	4	1	5	4	5	4	3	8	6	10	8	4	4	4	5	7	7	8	4
2	6	6	7	5	5	7	9	5	1	9	3	8	7	4	4	7	5	3	3
3	4	6	2	4	6	7	4	3	5	11	5	5	6	5	5	5	8	4	7
4	5	4	6	5	10	5	4	6	3	6	8	7	5	6	9	1	2	6	3
5	1	2	2	9	3	5	2	7	6	7	4	9	7	7	6	5	11	2	6
4	6	9	7	6	4	2	4	3	4	7	4	4	5	2	1	10	5	5	13
9	8	7	8	4	8	3	5	4	4	5	2	1	5	6	5	6	2	7	7

Table 19. Frequency table Bulgaria – Hungary
(R_SOFIX vs. R_BUX)

20	4	3	3	7	6	10	6	4	3	2	6	5	4	4	4	4	3	6	2
9	8	4	6	3	6	7	3	3	6	3	6	7	11	4	3	4	6	5	1
5	8	6	7	4	4	8	5	5	2	4	7	6	5	6	4	4	4	6	6
3	2	9	2	6	7	5	6	5	5	6	6	4	3	5	5	6	4	6	10
5	7	3	5	10	1	9	6	2	1	7	5	5	3	3	6	10	7	3	7
2	7	10	4	8	4	5	5	0	3	8	4	4	5	9	4	6	6	8	4
6	4	4	1	8	9	5	6	3	3	5	1	8	6	9	6	3	5	4	9
3	8	7	2	2	7	4	8	6	7	5	5	8	1	3	5	5	8	4	7
1	6	5	9	8	7	6	4	5	4	7	3	7	5	5	2	7	3	8	4
3	4	3	3	4	4	3	3	15	25	1	1	7	6	2	3	4	3	7	4
3	1	9	7	7	4	5	3	3	10	3	8	3	8	1	8	8	6	6	2
6	7	4	3	7	6	3	3	6	5	3	5	6	8	6	7	8	6	2	5
4	2	4	9	5	5	4	8	2	1	5	11	3	6	4	6	7	11	4	4
6	8	6	4	3	5	4	8	8	3	9	4	3	4	7	3	3	4	7	6
1	6	3	3	2	3	3	7	11	6	10	6	5	5	9	7	3	6	4	6
2	3	4	6	9	6	5	6	4	7	6	7	4	7	5	8	2	4	3	7
6	3	5	3	2	4	2	3	8	4	5	4	7	6	4	13	8	5	5	8
10	3	10	7	4	3	5	5	4	5	4	5	6	4	5	4	8	8	2	4
3	7	4	8	4	7	6	6	5	4	8	8	5	5	7	5	1	2	6	4
8	7	3	13	2	8	6	4	7	1	4	4	2	3	8	2	4	5	9	**6**

Table 20. Frequency table Bulgaria – Slovakia (R_SOFIX vs. R_SAX)

5	5	6	4	10	9	6	4	3	5	3	8	7	5	2	4	1	3	10	6
3	7	8	5	5	6	3	3	4	5	5	7	6	7	6	4	6	4	5	6
12	10	2	9	5	4	4	3	4	2	9	4	5	4	4	8	6	2	6	3
5	4	6	4	6	11	6	7	10	3	7	5	4	4	4	3	3	3	4	6
5	11	11	4	6	2	7	1	6	1	11	5	5	4	2	4	8	3	2	7
4	10	3	3	4	9	8	1	3	1	4	5	6	5	11	2	7	5	8	7
4	9	7	7	6	2	3	7	4	1	4	4	7	2	5	5	3	10	7	8
2	6	5	8	4	3	4	6	14	24	3	3	1	3	2	2	1	6	6	2
0	3	1	2	6	8	7	7	5	12	5	5	6	9	8	4	8	8	2	0
3	3	7	2	2	4	6	4	3	17	4	4	7	9	3	9	7	6	4	1
17	6	10	10	2	6	4	3	5	6	2	3	1	3	3	9	3	4	4	4
5	3	2	1	6	6	4	8	4	4	2	7	6	8	9	8	8	3	6	6
8	6	6	3	3	6	8	7	6	1	3	5	8	6	6	1	5	6	3	8
3	1	8	7	5	4	2	7	6	2	8	5	6	7	10	6	2	8	4	4
4	3	5	7	4	3	9	8	3	5	6	3	2	7	8	7	5	9	4	4
3	4	3	4	4	3	6	6	8	6	6	8	7	10	4	6	8	1	4	4
1	5	3	9	1	3	4	8	4	4	6	6	5	3	3	7	6	10	10	7
9	3	3	5	8	5	5	5	5	1	6	7	7	3	4	6	5	7	5	7
5	4	7	5	8	9	5	8	6	1	5	5	7	1	5	6	3	4	5	6
8	2	3	6	10	3	4	2	3	4	6	7	2	5	7	4	10	4	6	**10**

Table 21. Frequency table Bulgaria – Russia (R_SOFIX vs. R_RTS)

22	4	6	4	5	8	11	3	7	1	6	5	3	4	2	4	1	4	3	3
9	8	8	6	9	7	4	6	4	4	1	5	7	4	3	7	4	5	3	1
7	7	10	1	7	5	3	2	5	2	6	4	4	4	8	8	5	7	7	4
3	7	7	6	6	3	5	5	7	4	4	5	7	4	3	6	7	4	6	6
5	4	11	5	0	5	5	8	10	0	3	5	10	2	5	5	6	7	4	5
4	2	7	7	7	5	5	8	6	3	8	2	8	10	5	4	4	2	5	4
7	7	4	6	3	4	5	4	3	8	4	5	3	4	6	5	6	9	4	8
4	8	5	8	3	4	6	8	5	2	3	7	2	6	9	5	6	6	4	4
4	4	5	7	3	4	5	4	5	15	9	4	5	7	6	2	4	4	4	5
8	4	3	9	6	3	3	3	3	17	6	2	3	8	5	1	7	5	4	5
1	4	0	4	4	4	3	6	7	6	8	4	7	8	6	8	7	7	7	4
2	2	5	6	5	5	4	8	7	8	7	3	8	5	9	4	6	3	5	4
3	5	4	7	1	6	4	5	4	7	5	9	3	5	1	7	4	3	11	11
8	3	4	3	9	6	3	5	6	4	3	6	3	7	6	6	6	6	7	4
3	7	2	2	4	7	6	6	3	9	7	12	6	3	4	5	3	4	6	7
3	5	4	3	9	7	6	4	4	8	4	9	6	3	6	3	7	6	3	5
2	5	7	2	6	6	8	9	3	2	6	6	3	6	5	9	3	5	5	7
2	6	4	3	8	7	8	4	7	3	7	6	4	6	3	8	4	8	3	5
2	7	5	4	5	5	6	4	5	0	4	4	8	5	8	5	8	5	7	8
7	6	5	12	5	5	5	3	5	2	4	3	5	4	6	3	7	6	7	**6**

Table 22. Frequency table Romania – Poland (R_BET vs. R_WIG)

25	10	5	4	5	2	2	7	8	1	4	3	4	3	5	4	3	2	5	4
14	5	7	12	5	7	6	6	2	2	3	3	3	5	6	3	2	5	4	5
5	10	13	6	5	6	4	6	7	1	5	8	1	3	5	4	5	4	6	2
5	8	7	5	9	6	5	4	4	7	3	5	1	3	5	7	7	4	5	5
4	9	2	8	8	6	3	3	6	2	4	6	8	6	6	5	8	2	6	3
4	7	8	3	5	4	10	5	5	4	7	2	9	5	6	6	4	6	2	4
3	7	6	5	4	9	7	6	7	0	6	4	13	4	5	2	4	7	2	4
1	3	9	5	3	5	1	5	8	5	3	9	8	8	4	8	4	4	7	5
5	5	2	3	3	8	4	4	8	7	6	6	3	8	6	7	7	7	3	4
5	3	3	3	1	4	8	5	10	29	3	3	2	1	4	6	4	4	4	3
6	2	3	4	10	8	6	5	3	3	7	5	6	6	4	6	4	6	8	3
3	4	2	7	4	4	11	5	7	5	5	5	5	8	6	4	5	4	7	5
4	3	5	7	5	5	3	9	6	4	7	4	2	13	4	2	5	4	7	6
4	3	5	6	7	4	0	7	2	3	7	6	9	7	8	4	6	7	3	7
2	6	4	1	7	4	6	6	3	5	9	6	6	5	4	11	6	5	6	4
6	8	2	7	6	5	5	6	3	6	4	2	6	9	6	3	7	4	4	6
2	3	7	4	5	4	4	2	5	7	3	5	4	4	8	10	7	10	4	7
5	1	4	5	6	8	7	3	5	4	5	11	6	3	6	5	6	5	6	5
2	4	6	8	3	4	5	5	3	4	7	6	6	2	2	4	8	7	10	9
1	4	6	2	4	3	8	6	4	6	7	7	3	2	6	4	3	9	6	**15**

Table 23. Frequency table Romania – Czech Republic (R_BET vs. R_PX)

32	14	5	10	2	0	4	4	3	1	3	2	7	0	2	2	3	4	6	2
10	12	9	4	9	6	1	9	5	3	6	6	3	2	3	6	3	1	1	6
6	6	4	7	10	9	9	3	4	1	4	5	5	3	10	5	2	5	4	4
9	5	6	5	5	7	2	4	3	1	4	8	5	6	10	5	4	6	5	5
3	7	8	4	8	8	6	2	8	3	5	7	8	4	4	6	3	3	5	3
5	6	6	6	5	3	8	7	7	1	4	9	3	10	5	2	6	3	7	3
6	1	1	6	5	10	7	9	5	6	6	3	5	9	7	7	2	3	3	4
2	7	4	5	9	4	5	9	3	5	9	5	4	3	4	6	5	5	4	7
4	5	1	5	7	7	12	4	6	5	9	2	6	5	5	8	5	5	3	2
3	3	6	2	2	4	7	5	7	30	3	1	4	4	7	3	2	6	3	3
2	4	10	2	6	12	1	5	6	3	9	6	1	7	3	4	7	3	9	5
4	6	5	6	4	3	7	6	2	5	8	6	6	9	7	2	5	3	8	4
0	4	3	8	3	9	5	5	5	7	9	5	7	5	1	8	3	10	4	4
4	2	7	5	3	6	5	6	10	4	1	5	5	6	4	7	9	6	6	4
3	4	6	6	5	3	5	4	5	7	7	4	5	11	4	8	9	4	4	2
2	4	5	5	5	2	1	2	8	7	7	6	5	4	8	4	9	7	8	6
1	1	8	4	5	4	7	7	4	4	2	5	9	6	3	8	11	6	5	5
4	0	2	4	4	1	2	9	6	4	6	12	6	4	3	5	6	11	10	7
2	7	5	9	5	6	6	4	1	3	1	5	5	4	6	6	7	8	5	10
4	7	5	2	3	2	5	1	8	5	2	4	6	3	10	3	4	7	5	**20**

Table 24. Frequency table Romania – Hungary
(R_BET vs. R_BUX)

24	14	9	3	4	5	2	3	5	1	3	4	5	3	6	3	1	3	6	2
7	9	8	1	11	4	8	7	3	0	5	3	3	9	6	6	4	6	4	1
8	6	7	8	4	7	6	2	4	2	5	9	3	5	7	3	7	4	5	4
7	4	3	10	9	8	9	4	4	4	3	5	6	3	6	3	7	2	5	3
6	6	4	7	5	5	7	3	3	2	7	5	6	7	2	8	6	7	4	5
4	7	6	5	6	5	2	7	5	1	4	6	6	4	6	8	7	5	7	5
6	4	7	7	6	5	2	6	7	2	8	7	4	8	8	4	4	2	4	4
4	4	1	4	4	7	6	8	8	5	4	7	6	4	4	6	9	5	3	6
4	6	7	10	10	5	1	4	7	5	5	2	5	2	4	8	4	7	4	6
2	4	1	4	4	2	6	4	14	23	2	5	3	6	6	4	2	4	4	5
4	6	6	6	3	5	5	6	6	9	8	4	6	4	6	4	3	4	5	5
4	4	5	4	6	8	4	3	4	5	4	7	5	5	5	9	3	10	6	5
2	4	5	7	3	4	4	6	2	9	10	7	8	5	4	4	4	4	8	5
6	3	5	2	5	5	8	3	7	6	10	4	6	4	5	4	5	7	5	5
2	6	3	8	6	5	3	8	7	7	4	5	5	11	8	3	4	4	5	2
6	3	7	2	6	4	7	7	5	7	4	3	4	1	6	8	6	9	7	3
0	2	8	2	2	9	1	11	3	3	5	4	7	9	6	10	6	7	6	4
3	3	2	6	8	7	11	1	5	8	3	4	5	4	3	5	11	7	2	8
3	3	6	5	3	3	7	6	4	4	5	10	6	9	3	2	7	7	5	7
4	7	6	4	0	3	6	6	3	2	6	5	6	2	5	3	5	2	10	**21**

Table 26. Frequency table Romania – Slovakia
(R_BET vs. R_SAX)

11	6	5	8	6	4	4	4	6	4	3	5	5	8	7	3	3	5	4	5
7	2	7	3	5	7	5	6	3	4	6	9	6	7	3	9	3	4	4	5
7	5	6	9	5	5	3	3	7	1	5	6	3	7	3	4	10	3	4	10
3	6	7	4	8	5	5	6	4	1	3	2	9	8	6	3	7	3	4	11
7	8	8	4	3	2	5	2	6	3	8	10	4	4	9	9	1	4	3	5
1	10	3	6	4	8	8	9	4	1	6	4	2	6	8	6	3	7	5	5
3	6	6	4	4	4	2	7	6	1	9	9	7	5	5	6	6	4	7	4
2	2	3	4	2	2	6	5	18	25	4	3	3	0	2	6	8	2	4	4
3	3	4	4	8	5	6	4	4	15	4	4	3	4	9	4	3	6	6	7
4	2	5	5	4	10	5	3	1	17	4	4	7	3	5	3	5	9	8	1
22	9	4	4	5	3	2	3	0	2	3	3	4	5	4	3	4	8	5	12
2	4	9	7	6	3	5	7	5	3	4	5	2	4	5	6	5	14	8	2
5	5	4	4	5	7	6	7	5	2	7	3	6	6	7	5	7	4	6	4
3	6	6	3	7	10	5	4	5	2	3	11	5	7	5	5	8	1	6	3
5	8	5	2	6	6	3	4	7	3	8	5	6	6	6	5	7	2	7	5
3	6	4	8	5	2	5	4	9	6	1	3	11	5	3	5	7	5	9	4
2	3	7	6	3	6	8	8	3	6	4	9	4	4	10	8	2	5	2	5
8	5	3	7	8	5	6	5	6	2	11	3	2	6	4	5	4	6	5	5
2	4	4	7	7	6	8	8	2	2	7	6	7	4	3	7	5	9	3	4
6	5	6	6	4	6	8	6	5	5	5	2	9	6	2	3	7	5	5	**5**

Table 27. Frequency table Romania – Russia
(R_BET vs. R_RTS)

20	10	5	3	6	4	3	7	3	1	3	4	5	2	6	6	8	3	4	3
15	5	5	5	7	0	2	7	5	3	8	5	3	7	6	6	2	7	5	2
5	10	7	6	3	5	3	9	4	2	1	6	5	6	9	4	5	8	6	2
4	5	6	3	5	9	4	8	10	1	5	6	7	6	2	5	4	8	4	3
7	8	6	8	7	5	4	6	4	2	3	7	5	3	5	4	6	6	4	5
5	5	7	10	4	7	8	3	6	6	8	4	5	7	6	5	3	0	3	4
4	6	4	4	9	8	5	2	3	7	12	4	6	4	8	2	4	5	5	3
5	3	4	6	7	1	2	1	4	6	5	10	8	4	10	6	6	5	8	4
3	2	4	2	5	8	6	5	7	15	1	9	2	2	4	8	3	5	5	10
7	8	8	6	6	6	6	1	5	16	2	3	3	5	5	2	2	3	4	7
5	7	3	4	5	9	1	5	5	6	2	4	8	5	8	6	7	4	7	4
6	6	6	9	1	5	8	2	4	7	5	5	5	8	4	4	5	7	7	2
3	4	4	5	4	4	7	7	5	4	7	5	7	8	4	6	10	5	1	5
1	3	6	7	6	5	6	4	8	3	6	7	5	4	3	5	6	9	8	3
3	3	4	7	9	5	7	7	4	8	5	5	6	7	2	5	5	7	3	4
3	5	7	2	6	5	3	3	4	7	4	4	7	7	7	7	9	2	6	7
4	2	4	5	3	7	6	5	9	4	8	7	4	6	4	6	6	4	6	5
2	3	7	8	6	5	10	10	7	2	6	2	4	7	1	3	2	5	7	9
1	5	3	1	2	4	6	6	5	3	10	6	6	4	7	13	3	8	3	9
3	5	6	4	4	4	8	7	4	2	4	3	4	3	5	2	9	5	9	**15**

Table 28. Frequency table Poland – Czech Republic
(R_WIG vs. R_PX)

47	15	8	6	6	4	2	2	2	2	5	0	0	1	2	1	1	1	1	0
16	16	17	14	8	1	0	5	4	3	2	4	4	4	2	0	1	1	1	2
9	13	12	11	9	9	7	4	8	2	4	4	5	5	1	2	0	0	1	0
4	7	8	7	13	8	9	8	5	3	1	3	6	5	4	2	6	3	3	0
10	5	9	10	7	7	4	5	8	3	5	7	5	6	3	3	3	2	0	3
3	11	13	12	5	3	7	2	2	3	8	7	6	3	5	6	4	4	1	1
3	9	5	4	4	10	8	9	9	3	8	4	6	5	7	2	2	4	2	1
3	7	3	4	7	6	9	6	9	6	5	8	7	3	2	7	6	3	3	1
3	5	4	10	3	9	9	8	8	2	8	4	3	6	1	7	4	6	4	2
0	0	1	4	5	2	4	5	2	40	5	5	3	3	6	4	4	3	3	6
3	5	4	8	5	5	6	6	6	3	3	5	3	11	7	5	7	3	6	4
0	1	6	2	2	7	5	5	7	6	7	8	9	7	8	6	5	6	1	8
3	1	3	2	6	4	3	7	4	5	7	8	6	8	9	7	8	6	2	6
1	1	2	0	5	7	6	6	1	6	8	9	9	9	9	4	5	5	8	4
0	4	2	3	8	3	6	6	8	2	4	7	6	3	8	6	7	9	8	6
0	1	3	3	3	6	3	5	6	4	3	3	6	8	10	3	11	11	9	7
0	0	0	3	3	5	3	5	7	4	10	7	3	5	5	9	10	13	5	8
1	1	3	0	3	5	8	6	3	4	3	5	6	3	9	10	10	7	11	8
0	3	0	0	1	4	4	4	4	1	5	4	9	7	4	13	4	8	15	15
0	0	3	2	2	1	2	1	3	3	4	4	3	3	4	8	7	11	21	**24**

Table 29. Frequency table Poland – Hungary
(R_WIG vs. R_BUX)

43	18	11	6	7	4	5	1	1	2	0	2	1	2	0	0	1	1	0	1
14	12	15	14	6	8	4	6	4	2	4	5	0	2	2	5	1	1	0	0
13	8	12	10	6	8	9	6	5	2	2	4	3	6	2	2	1	3	2	2
4	13	16	11	6	7	9	6	7	1	5	1	2	4	5	2	2	2	1	1
7	12	5	5	14	5	5	2	10	2	6	5	6	2	5	5	2	3	1	3
5	7	3	9	11	9	8	8	6	3	4	8	8	3	2	0	5	2	3	2
4	5	3	9	6	5	8	7	10	1	12	6	5	10	4	4	1	2	2	1
1	5	6	5	9	9	8	4	8	4	8	4	10	5	5	6	1	2	4	1
2	5	7	6	12	10	3	6	3	4	9	3	7	7	4	3	5	4	4	2
1	3	2	3	3	4	1	5	4	43	3	4	4	5	3	4	3	3	3	4
5	3	3	5	2	5	9	4	3	16	7	7	5	4	4	5	5	2	9	2
1	5	5	3	5	6	4	7	6	3	5	5	9	8	7	9	8	5	2	3
2	3	1	2	3	7	5	8	6	3	5	10	9	9	7	2	5	9	6	3
1	2	4	4	2	3	5	6	7	3	6	11	9	8	9	2	6	9	4	4
0	2	3	3	6	4	2	5	9	2	7	5	8	6	10	9	8	8	5	4
2	0	2	3	2	4	5	4	3	3	3	11	6	6	9	11	12	8	5	6
0	1	2	3	4	0	7	5	5	2	5	2	5	5	7	9	11	14	9	9
0	1	3	3	1	2	3	6	2	7	9	5	2	3	10	7	10	9	13	10
1	0	3	1	0	3	4	4	4	1	1	4	4	5	7	14	8	11	9	21
0	0	0	0	0	3	1	5	3	1	4	4	2	5	4	6	10	8	23	**27**

Table 30. Frequency table Poland – Slovakia
(R_WIG vs. R_SAX)

5	8	4	6	6	4	10	7	5	2	9	2	5	4	5	8	6	3	1	6
9	7	9	5	5	2	3	5	3	2	5	7	7	9	6	1	6	3	5	6
7	4	5	6	5	6	6	1	3	3	5	3	9	3	4	6	10	7	7	6
3	7	5	3	8	6	6	7	6	3	6	6	4	6	1	9	3	7	5	4
4	2	7	6	7	6	8	5	8	2	7	4	7	7	5	1	4	7	4	4
5	1	4	6	8	8	3	4	9	4	6	5	4	5	5	3	5	6	8	7
1	3	9	10	9	1	6	7	8	0	5	5	6	4	7	4	3	8	6	3
5	3	3	4	0	4	4	5	5	30	2	4	8	2	2	3	2	3	4	12
3	3	0	4	4	8	4	1	4	14	7	6	6	4	6	7	7	4	6	8
4	9	3	3	2	5	5	6	7	14	5	6	1	2	4	8	5	4	8	4
15	6	6	4	2	4	2	5	5	3	2	8	5	7	6	6	3	6	3	7
2	7	5	10	6	8	4	3	4	3	6	3	5	5	5	5	8	12	3	2
7	6	2	5	3	7	5	4	5	2	6	7	5	5	11	6	9	2	5	3
10	7	4	1	6	2	8	8	4	1	6	5	6	2	5	4	8	9	5	4
4	4	10	6	4	5	6	4	3	4	6	6	5	8	7	6	8	2	4	4
8	2	2	7	8	5	7	12	6	1	5	5	7	4	7	7	0	3	4	5
3	6	7	6	7	5	2	6	2	2	6	11	5	8	4	3	5	3	12	2
2	6	6	3	5	7	4	6	8	6	4	7	1	9	3	10	2	8	2	7
5	8	8	7	4	5	4	3	5	6	4	3	4	10	8	3	4	4	6	4
4	6	7	3	6	8	8	6	6	3	3	3	5	1	5	5	7	5	7	**8**

Table 31. Frequency table Poland – Russia
(R_WIG vs. R_RTS)

39	10	8	10	4	8	4	1	1	2	1	0	3	5	2	2	3	1	1	1
18	13	11	7	6	3	8	4	2	3	3	3	2	6	5	3	1	2	4	1
8	14	7	12	10	5	2	6	5	4	3	11	2	3	1	4	2	6	1	0
8	8	11	10	6	5	6	7	6	3	5	4	4	1	9	4	4	2	1	1
4	11	12	6	5	5	3	5	7	5	8	6	3	4	6	5	4	2	2	2
4	7	6	8	11	7	4	5	9	6	7	4	6	7	1	2	3	4	3	2
3	3	7	4	5	10	3	6	8	9	7	4	7	5	5	5	6	4	2	2
2	3	5	4	9	5	8	9	9	7	4	5	7	5	4	6	6	5	1	1
4	3	3	9	2	6	4	4	8	11	7	7	6	6	6	5	4	4	2	5
3	6	4	1	4	4	5	10	6	12	9	5	6	4	3	1	5	7	4	6
2	4	7	3	3	6	10	6	8	7	4	7	6	7	5	5	2	7	3	3
1	3	5	9	5	1	8	1	2	6	7	8	9	4	8	8	9	4	5	3
3	2	6	1	8	8	2	5	6	3	6	6	6	5	6	9	10	2	5	6
1	3	4	3	8	8	7	4	1	5	7	5	6	7	5	7	10	4	4	6
1	5	0	4	1	2	7	11	11	6	6	2	2	3	9	2	7	7	13	7
0	3	2	1	6	6	5	3	4	5	4	6	6	9	8	10	9	5	8	5
2	1	4	2	7	5	5	6	5	2	5	5	5	6	7	3	6	13	5	11
1	3	1	5	3	8	6	4	4	3	3	4	11	8	6	9	4	8	5	10
2	2	0	3	1	3	3	6	2	4	5	6	4	8	8	7	5	7	18	11
0	1	3	3	1	1	5	2	2	2	4	8	4	2	2	8	5	12	18	**23**

Table 32. Frequency table Czech Republic – Hungary
(R_PX vs. R_BUX)

52	19	6	5	7	2	2	1	1	0	1	1	0	3	0	1	1	0	1	3
10	16	15	6	7	9	9	6	4	4	4	3	5	2	2	0	0	1	0	2
8	10	11	12	7	10	8	7	3	0	5	5	3	1	4	1	2	1	6	2
6	8	12	9	8	11	10	7	6	1	1	6	2	5	2	4	3	1	2	1
5	5	7	10	9	7	8	10	7	7	2	3	1	4	3	8	2	3	3	1
4	6	5	9	8	5	10	3	4	2	12	5	7	4	7	2	6	3	4	0
4	7	3	5	17	6	9	6	6	5	3	8	3	5	2	3	5	3	3	2
3	3	4	8	6	10	4	11	4	5	8	11	4	7	1	4	1	8	2	1
4	3	8	7	4	6	6	11	6	3	6	4	7	5	7	4	4	4	4	3
3	4	5	4	1	7	3	1	9	39	4	6	5	3	1	1	0	4	4	1
0	3	2	7	7	4	5	2	6	5	6	5	11	2	8	7	7	7	7	4
3	4	9	5	4	5	4	5	7	4	8	3	7	3	3	8	8	6	4	6
1	6	4	1	11	3	2	6	6	4	6	4	15	6	7	4	8	6	4	1
2	3	6	2	1	5	8	3	6	4	6	6	6	8	10	7	7	7	3	5
0	0	3	4	2	6	6	3	7	6	7	9	7	9	11	10	8	4	1	3
0	2	1	3	2	3	3	8	7	3	4	7	6	9	9	10	5	7	10	6
1	0	1	4	2	2	0	2	1	1	7	7	4	9	14	8	10	13	9	10
0	2	1	2	1	3	3	4	8	3	8	3	2	8	3	8	15	9	10	13
0	2	1	1	0	0	4	8	4	7	4	5	8	6	8	7	4	11	15	10
0	2	2	1	1	2	1	1	4	2	3	5	2	6	4	8	9	8	13	**32**

Table 33. Frequency table Czech Republic – Slovakia (R_PX vs. R_SAX)

9	5	5	7	9	6	6	5	6	0	4	7	6	3	6	3	1	5	4	9
7	9	10	6	5	1	3	4	3	3	5	4	3	5	3	7	3	8	4	12
6	10	8	6	9	5	0	4	8	0	6	4	1	7	4	6	7	4	3	8
7	6	11	7	4	6	8	1	6	3	4	5	1	2	5	5	5	2	7	10
4	2	8	3	4	6	8	6	4	5	6	4	8	8	5	9	6	4	4	1
7	3	3	10	4	6	2	4	6	3	9	7	6	4	4	6	8	5	5	4
7	5	5	7	4	9	6	6	8	2	7	2	6	3	8	6	2	5	1	6
3	2	2	4	4	2	3	1	4	29	6	4	5	3	2	5	8	4	7	7
2	0	3	6	2	3	8	2	3	16	9	6	7	9	5	8	7	5	4	1
2	5	1	6	6	4	4	8	8	13	2	3	5	5	7	5	7	6	6	2
15	8	5	7	7	7	2	2	4	4	4	4	2	2	3	3	5	5	4	12
3	11	3	10	5	6	4	8	4	4	4	4	7	8	5	4	6	2	4	4
2	4	3	5	8	9	6	8	3	4	6	9	4	3	7	3	7	7	7	0
6	2	3	4	2	6	4	7	7	1	7	8	10	9	9	3	4	6	4	3
2	5	5	2	5	5	7	9	5	4	4	5	3	9	5	8	5	7	5	6
4	6	4	3	7	4	7	5	3	2	5	8	6	5	8	4	10	5	7	2
3	5	8	6	4	3	2	10	5	3	3	7	7	11	4	1	1	9	9	4
3	6	7	3	6	10	6	6	4	2	4	7	7	3	8	3	4	6	7	4
6	5	6	1	6	4	10	5	10	3	6	5	7	2	4	7	3	3	7	5
8	6	6	2	4	4	9	4	5	4	4	3	4	4	4	9	6	8	6	**6**

Table 34. Frequency table Czech Republic – Russia (R_PX vs. R_RTS)

43	11	9	5	4	7	3	1	1	2	4	2	1	1	1	2	2	3	1	3
21	12	9	9	7	8	3	5	3	3	5	1	3	5	2	3	1	1	2	2
8	15	8	6	6	5	8	5	7	6	4	4	3	6	4	4	1	1	1	4
6	10	7	10	13	10	5	5	4	6	3	2	1	3	3	5	5	3	1	3
2	11	11	8	13	7	8	4	3	4	6	5	1	5	4	3	3	4	2	1
2	8	13	4	8	9	10	3	6	3	8	3	6	4	3	6	2	4	2	2
2	3	3	5	7	4	10	11	8	7	6	7	6	6	8	2	4	3	1	2
3	3	4	5	6	7	5	6	8	3	3	6	12	9	6	4	7	5	3	0
1	6	10	4	5	2	2	6	7	10	9	5	2	5	3	5	4	6	9	5
3	4	3	7	4	4	5	8	3	14	7	5	2	5	2	6	9	4	6	4
3	2	5	3	6	8	3	3	11	3	5	9	9	8	8	5	3	5	4	2
2	0	1	6	2	7	6	9	7	5	8	7	3	11	9	3	8	4	5	3
1	6	7	6	4	4	8	6	6	1	4	6	7	8	8	9	5	3	2	4
3	4	1	6	2	5	2	7	12	4	4	7	5	8	9	6	6	8	4	2
0	2	2	4	5	6	10	6	3	4	5	8	6	6	8	4	6	11	5	5
0	2	1	3	4	2	6	4	3	4	6	7	6	5	9	12	9	3	9	10
2	1	0	3	0	3	6	3	3	10	5	4	10	3	6	11	6	12	12	5
1	0	4	5	3	5	4	4	3	4	6	7	14	1	6	4	10	8	10	7
1	3	6	4	1	3	1	6	3	6	5	6	5	3	3	7	7	10	11	14
2	2	2	2	5	0	0	3	5	6	2	5	3	3	4	4	7	8	15	**28**

Table 35. Frequency table Hungary – Slovakia (R_BUX vs. R_SAX)

9	3	5	9	6	4	6	6	5	5	7	3	3	1	3	4	8	7	6	6
10	4	5	7	3	4	8	4	7	1	4	3	3	11	4	9	2	5	5	6
7	4	5	6	5	4	3	4	4	6	3	5	5	7	6	5	4	9	5	9
4	5	4	5	5	7	9	4	6	6	2	1	9	8	3	6	8	2	6	5
5	5	6	2	8	5	7	3	10	0	3	8	3	9	7	3	7	8	3	3
5	6	6	6	3	5	9	5	6	4	7	2	1	3	5	7	5	6	4	11
7	8	7	4	10	5	8	8	2	2	5	7	4	1	9	1	3	5	7	2
20	18	17	20	21	0	1	1	1	0	0	0	0	0	1	0	0	1	2	2
0	0	0	0	0	24	10	20	9	43	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	5	28	18	18	20	16	0	0	0	0	0
0	0	1	0	1	1	0	1	1	1	0	2	3	0	5	20	19	14	16	20
4	8	2	6	3	9	3	4	10	7	7	3	4	1	8	5	7	5	4	6
2	8	4	4	4	4	9	2	10	4	7	6	12	8	4	3	3	3	6	2
6	6	11	3	4	7	0	3	4	4	6	6	5	5	7	8	5	7	7	1
3	5	6	4	5	2	7	9	7	4	4	8	7	6	3	5	2	7	5	7
5	5	6	5	3	4	8	5	4	2	3	9	5	3	7	7	7	9	5	3
7	4	9	3	6	6	2	8	5	4	3	5	11	6	2	4	7	2	4	7
2	5	5	4	7	5	9	6	3	1	8	4	7	5	8	5	8	6	6	2
2	7	4	10	6	3	1	6	8	5	6	12	3	5	4	4	6	2	5	6
8	4	3	7	5	7	5	6	4	1	2	4	2	6	4	9	4	8	9	**8**

Table 36. Frequency table Hungary – Russia (R_BUX vs. R_RTS)

41	10	4	7	5	7	0	5	2	0	4	5	2	5	1	0	3	1	2	2
19	14	8	9	7	1	5	8	8	2	3	2	2	6	0	3	3	1	0	4
8	10	11	6	5	9	6	5	1	1	11	7	5	2	4	3	7	2	2	1
8	8	3	8	6	6	10	6	3	2	8	3	9	5	5	2	1	4	5	3
7	10	5	5	16	8	5	6	4	5	4	4	4	4	3	4	3	4	3	1
4	10	8	5	7	7	7	9	3	3	3	4	6	9	3	3	7	2	1	5
2	4	5	4	5	1	12	5	9	9	9	3	6	8	3	4	4	4	5	3
3	2	7	9	5	4	5	2	7	4	5	8	6	7	11	7	4	4	3	2
4	2	5	6	5	6	6	8	5	14	6	5	3	6	5	6	1	9	2	2
5	4	4	6	4	6	4	4	5	7	9	5	5	5	9	3	4	6	6	4
2	5	10	8	2	6	7	4	6	7	6	5	9	5	3	7	4	1	4	4
0	3	3	4	6	4	6	5	11	4	5	3	4	8	8	9	6	9	5	3
0	4	5	2	4	7	8	5	3	5	1	3	6	8	4	8	8	5	12	7
0	2	2	5	3	9	5	4	7	10	5	5	7	5	4	4	9	12	2	5
0	2	7	8	7	2	5	6	5	4	5	7	6	2	8	6	8	3	9	6
0	1	3	7	4	6	3	3	5	8	3	6	7	4	6	8	4	10	10	7
0	2	6	2	4	5	1	4	8	4	7	10	3	5	7	7	6	8	8	8
0	5	4	2	4	4	4	6	4	5	9	10	8	4	9	7	7	4	6	4
1	3	1	0	4	8	1	7	2	7	1	4	7	3	10	9	11	9	6	11
2	4	5	2	2	0	5	3	8	4	1	7	0	4	3	5	5	8	14	**24**

Table 37. Frequency table Slovakia - Russia
(R_SAX vs. R_RTS)

5	8	7	7	4	7	6	5	1	1	18	3	3	8	4	4	1	1	6	7
10	7	6	5	8	7	7	3	3	2	8	5	3	2	5	5	5	6	3	5
4	4	7	2	2	5	7	2	3	2	6	10	9	5	3	9	9	7	6	4
3	9	6	3	5	6	12	5	1	7	6	3	7	4	2	8	1	3	4	10
6	2	6	4	9	1	5	8	6	6	7	4	6	4	4	6	6	6	6	3
4	2	6	6	5	4	7	1	4	4	3	5	7	11	5	5	11	5	6	5
5	6	3	3	5	8	5	5	10	5	3	8	6	4	3	7	5	5	1	8
6	3	4	7	9	5	7	3	5	6	2	3	6	1	5	6	7	8	7	5
13	9	7	4	6	10	5	16	16	5	1	1	3	0	0	2	5	0	2	1
1	4	5	3	3	1	2	6	2	9	9	12	3	8	10	3	3	7	5	9
2	4	5	8	7	1	3	4	3	7	6	3	7	10	4	7	3	7	8	6
8	3	6	3	8	4	1	6	8	8	3	5	5	5	7	3	8	7	3	5
4	9	5	5	1	5	3	7	3	9	2	5	3	6	9	6	6	3	2	12
3	2	3	7	4	8	5	3	6	10	4	9	5	4	1	5	7	4	12	3
8	4	5	5	3	4	3	4	3	5	4	5	12	6	8	6	2	8	6	5
4	1	0	13	6	3	6	4	11	4	4	7	6	3	9	4	7	4	7	2
6	7	4	4	7	6	5	5	8	6	2	5	3	5	9	5	4	7	5	2
4	5	4	4	7	8	7	3	5	5	4	6	4	5	7	6	5	8	5	4
5	10	10	3	2	5	6	5	6	3	5	4	5	8	4	4	6	5	6	3
5	6	7	9	4	8	3	10	2	1	8	3	2	6	7	4	4	5	5	**7**

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Table 38. Best fitting copula for stock returns and FX returns in each country

Country	Optimal Copula	Estimated Parameters	Estimated Coefficients of tail dependence	
			Lower	Upper
Bulgaria R_SOFIX vs. R_FX_BGN	Student-t	ρ =0.0279 DOF[1] = 10	0.0126	0.0126
Romania R_BET vs. R_FX_RML	Student-t	ρ =0.0682 DOF = 12	0.0088	0.0088
Poland R_WIG vs. R_FX_PLZ	Rotated Gumbel[2]	θ=1.099433	0.1267	0
Czech Republic R_PX vs. R_FX_CZK	Student-t	ρ =-0.0143 DOF = 4	0.0593	0.0593
Hungary R_BUX vs. R_FX_HUF	Student-t	ρ =0.0665 DOF = 7	0.0383	0.0383
Slovakia R_SAX vs. R_FX_SKK	Student-t	ρ =0.034 DOF = 22	0.0004	0.0004
Russia R_RTS vs. R_FX_RUR	Student-t	ρ =0.0169 DOF = 9	0.0128	0.0128

[1]DOF = degrees of freedom parameter of the t-distribution

[2]Rotated Gumbel Copula is given by: $C_{\theta}^{Gu-Rotated}(u,v) = C_{\theta}^{Gu}(1-u,1-v)$

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Table 39. Best fitting copula between stock returns of each country

Copula	R_SOFIX	R_BET	R_WIG	R_PX	R_BUX	R_SAX	R_RTS
R_SOFIX	-	**Student T** τ_l =0.0386 τ_u =0.0386	**Student T** τ_l = 0.0243 τ_u = 0.0243	**Student T** τ_l = 0.0419 τ_u = 0.0419	**Clayton** τ_l = 0.0015 τ_u = 0	**Student T** τ_l = 0.0004 τ_u = 0.0004	**SJC** τ_l = 0.0721 τ_u = 0
R_BET	**Student T** τ_l =0.0386 τ_u =0.0386	-	**SJC** τ_l = 0.1308 τ_u = 0.0111	**Student T** τ_l = 0.1262 τ_u = 0.1262	**Student T** τ_l = 0.0656 τ_u = 0.0656	**Clayton** τ_l = 0 τ_u = 0	**SJC** τ_l = 0.0665 τ_u = 0.0171
R_WIG	**Student T** τ_l = 0.0243 τ_u = 0.0243	**SJC** τ_l = 0.1308 τ_u = 0.0111	-	**SJC** τ_l = 0.4465 τ_u = 0.2262	**Student T** τ_l = 0.1706 τ_u = 0.1706	**Student T** τ_l = 0 τ_u = 0	**SJC** τ_l = 0.3183 τ_u = 0.1852
R_PX	**Student T** τ_l = 0.0419 τ_u = 0.0419	Student T τ_l = 0.1262 τ_u = 0.1262	**SJC** τ_l = 0.4465 τ_u = 0.2262	-	**SJC** τ_l = 0.4215 τ_u = 0.2734	**Student T** τ_l = 0.0088 τ_u = 0.0088	**Student T** τ_l = 0.2393 τ_u = 0.2393
R_BUX	**Clayton** τ_l = 0.0015 τ_u = 0	**Student T** τ_l = 0.0656 τ_u = 0.0656	**Student T** τ_l = 0.1706 τ_u = 0.1706	**SJC** τ_l = 0.4215 τ_u = 0.2734	-	**Student T** τ_l = 0.001 τ_u = 0.001	**SCJ** τ_l = 0.3015 τ_u = 0.1264
R_SAX	**Student T** τ_l = 0.0004 τ_u = 0.0004	**Clayton** τ_l = 0 τ_u = 0	**Student T** τ_l = 0 τ_u = 0	**Student T** τ_l = 0.0088 τ_u = 0.0088	**Student T** τ_l = 0.001 τ_u = 0.001	-	**Student T** τ_l = 0 τ_u = 0
R_RTS	**SJC** τ_l = 0.0721 τ_u = 0	**SJC** τ_l = 0.0665 τ_u = 0.0171	**SJC** τ_l = 0.3183 τ_u = 0.1852	**Student T** τ_l = 0.2393 τ_u = 0.2393	**SJC** τ_l = 0.3015 τ_u = 0.1264	**Student T** τ_l = 0 τ_u = 0	-

Note: SCJ stands for Symmetrized Joe-Clayton Copula, R-Gumbel for Rotated Gumbel Copula, τ_l and τ_u are the estimated coefficients of lower and upper tail dependence, respectively.

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Table 40. 1-day VaR 95% of a $100M portfolio invested in each country

VaR 95%	Bulgaria	Romania	Poland	Czech Republic	Hungary	Slovakia	Russia
Variance-Covariance	3 339 566	3 089 122	2 761 703	2 872 916	3 120 360	2 276 290	3 753 371
Historical Simulation	2 384 226	2 785 286	2 466 410	2 312 066	2 825 292	1 970 513	3 325 392
Monte Carlo	2 490 821	2 745 866	2 630 856	2 513 852	2 703 344	2 044 879	3 308 839

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Table 41. 1-day VaR 99% of a $100M portfolio invested in each country

VaR 99%	Bulgaria	Romania	Poland	Czech Republic	Hungary	Slovakia	Russia
Variance-Covariance	4 690 001	4 338 232	3 887 201	4 039 724	4 398 256	3 178 352	5 286 148
Historical Simulation	6 242 627	5 193 867	4 545 001	4 697 228	5 157 911	3 536 185	7 285 201
Monte Carlo	6 103 115	4 854 079	4 676 720	4 829 319	5 428 017	3 582 317	6 852 811

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Table 42. 1-day VaR 99.9% of a $100M portfolio invested in each country

VaR 99.9%	Bulgaria	Romania	Poland	Czech Republic	Hungary	Slovakia	Russia
Variance-Covariance	6 203 700	5 738 356	5 148 767	5 347 595	5 830 646	4 189 470	7 004 232
Historical Simulation	11 174 188	9 821 534	9 636 634	10 369 590	10 356 865	5 174 726	13 469 366
Monte Carlo	18 153 204	8 891 851	8 021 753	11 701 903	11 216 860	7 685 648	18 978 423

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Table 43. 1-day VaR 95% of a $100M portfolio invested equally in 2 countries

Var-Cov **Hist.Sim** **MC Sim.**	**R_SOFIX**	**R_BET**	**R_WIG**	**R_PX**	**R_BUX**	**R_SAX**	**R_RTS**
R_SOFIX	-	2 309 132 1 669 939 1 851 184	2 010 470 1 604 025 1 678 283	2 188 493 1 702 992 1 713 414	2 141 058 1 636 947 1 700 754	1 923 652 1 481 499 1 598 713	2 664 340 2 192 224 2 248 805
R_BET	2 309 132 1 669 939 1 851 184	-	2 010 980 1 645 129 1 719 060	2 223 763 1 812 121 1 824 863	2 158 118 1 816 610 1 701 961	1 779 049 1 505 458 1 559 356	2 640 806 2 133 969 2 274 075
R_WIG	2 010 470 1 604 025 1 678 283	2 010 980 1 645 129 1 719 060	-	2 090 734 1 846 515 1 823 242	2 097 103 1 893 331 1 834 010	1 497 802 1 372 154 1 314 753	2 579 475 2 297 705 2 337 052
R_PX	2 188 493 1 702 992 1 713 414	2 223 763 1 812 121 1 824 863	2 090 734 1 846 515 1 823 242	-	2 254 517 1 934 113 2 048 030	1 635 492 1 449 201 1 422 138	2 769 753 2 348 289 2 384 120
R_BUX	2 141 058 1 636 947 1 700 754	2 158 118 1 816 610 1 701 961	2 097 103 1 893 331 1 834 010	2 254 517 1 934 113 2 048 030	-	1 632 659 1 460 854 1 413 253	2 712 502 2 326 237 2 343 624
R_SAX	1 923 652 1 481 499 1 598 713	1 779 049 1 505 458 1 559 356	1 497 802 1 372 154 1 314 753	1 635 492 1 449 201 1 422 138	1 632 659 1 460 854 1 413 253	-	2 151 460 1 908 437 1 802 729
R_RTS	2 664 340 2 192 224 2 248 805	2 640 806 2 133 969 2 274 075	2 579 475 2 297 705 2 337 052	2 769 753 2 348 289 2 384 120	2 712 502 2 326 237 2 343 624	2 151 460 1 908 437 1 802 729	-

The first value in each box represents the 1-day VaR 95% calculated by Variance-Covariance Method and the second and third - by Historical Simulation and Monte Carlo Simulation, respectively.

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Table 44. 1-day VaR 99% of a $100M portfolio invested equally in 2 countries

Var-Cov Hist.Sim MC Sim.	**R_SOFIX**	**R_BET**	**R_WIG**	**R_PX**	**R_BUX**	**R_SAX**	**R_RTS**
R_SOFIX	-	3 206 223 4 451 470 4 046 985	2 808 316 3 739 438 3 353 854	3 060 742 4 142 946 3 748 734	2 995 534 4 064 027 3 627 069	2 669 543 3 142 647 3 459 448	3 721 067 4 784 668 4 504 466
R_BET	3 206 223 4 451 470 4 046 985	-	2 800 134 3 290 773 3 457 440	3 101 549 3 800 831 3 599 666	3 010 603 3 434 773 3 128 637	2 455 950 2 501 687 2 809 109	3 678 701 4 540 137 4 478 208
R_WIG	2 808 316 3 739 438 3 353 854	2 800 134 3 290 773 3 457 440	-	2 938 064 3 473 311 3 567 670	2 948 969 3 416 301 3 114 317	2 082 839 2 198 526 2 278 708	3 616 619 4 598 798 4 719 290
R_PX	3 060 742 4 142 946 3 748 734	3 101 549 3 800 831 3 599 666	2 938 064 3 473 311 3 567 670	-	3 172 076 3 838 687 3 880 622	2 278 049 2 391 259 2 713 687	3 886 205 5 103 499 4 882 819
R_BUX	2 995 534 4 064 027 3 627 069	3 010 603 3 434 773 3 128 637	2 948 969 3 416 301 3 114 317	3 172 076 3 838 687 3 880 622	-	2 275 939 2 414 149 2 543 685	3 807 131 4 875 629 5 184 104
R_SAX	2 669 543 3 142 647 3 459 448	2 455 950 2 501 687 2 809 109	2 082 839 2 198 526 2 278 708	2 278 049 2 391 259 2 713 687	2 275 939 2 414 149 2 543 685	-	2 995 111 3 676 680 3 998 030
R_RTS	3 721 067 4 784 668 4 504 466	3 678 701 4 540 137 4 478 208	3 616 619 4 598 798 4 719 290	3 886 205 5 103 499 4 882 819	3 807 131 4 875 629 5 184 104	2 995 111 3 676 680 3 998 030	-

The first value in each box represents the 1-day VaR 99% calculated by Variance-Covariance Method and the second and third - by Historical Simulation and Monte Carlo Simulation, respectively.

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Table 45. 1-day VaR 99.9% of a $100M portfolio invested equally in 2 countries

Var-Cov **Hist.Sim** **MC Sim.**	**R_SOFIX**	**R_BET**	**R_WIG**	**R_PX**	**R_BUX**	**R_SAX**	**R_RTS**
R_SOFIX	-	4 211 781 8 505 447 8 717 601	3 702 761 7 745 184 8 762 875	4 038 448 9 891 520 8 590 670	3 953 352 7 356 687 9 985 620	3 505 614 6 565 629 6 170 463	4 905 549 10 409 479 9 506 901
R_BET	4 211 781 8 505 447 8 717 601	-	3 684 694 5 588 450 6 776 659	4 085 456 7 305 200 7 674 009	3 966 150 8 090 401 6 400 540	3 214 687 5 559 241 5 423 847	4 842 073 10 148 789 9 375 731
R_WIG	3 702 761 7 745 184 8 762 875	3 684 694 5 588 450 6 776 659	-	3 887 833 7 409 842 11 448 569	3 903 822 5 799 496 6 779 592	2 738 605 3 117 238 3 977 014	4 779 151 8 282 772 9 960 172
R_PX	4 038 448 9 891 520 8 590 670	4 085 456 7 305 200 7 674 009	3 887 833 7 409 842 11 448 569	-	4 200 564 8 208 974 8 760 228	2 998 288 5 085 746 6 188 815	5 137 633 8 628 102 10 493 714
R_BUX	3 953 352 7 356 687 9 985 620	3 966 150 8 090 401 6 400 540	3 903 822 5 799 496 6 779 592	4 200 564 8 208 974 8 760 228	-	2 996 990 4 978 258 5 166 175	5 034 098 9 261 246 11 517 724
R_SAX	3 505 614 6 565 629 6 170 463	3 214 687 5 559 241 5 423 847	2 738 605 3 117 238 3 977 014	2 998 288 5 085 746 6 188 815	2 996 990 4 978 258 5 166 175	-	3 940 757 6 597 707 6 265 081
R_RTS	4 905 549 10 409 479 9 506 901	4 842 073 10 148 789 9 375 731	4 779 151 8 282 772 9 960 172	5 137 633 8 628 102 10 493 714	5 034 098 9 261 246 11 517 724	3 940 757 6 597 707 6 265 081	-

The first value in each box represents the 1-day VaR 99% calculated by Variance-Covariance Method and the second and third - by Historical Simulation and Monte Carlo Simulation, respectively.

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