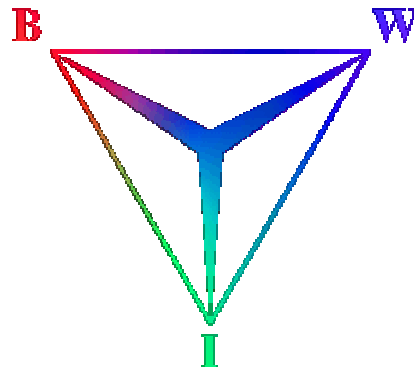# Stock price modelling:
Theory and Practice

Abdelmoula Dmouj

Supervisor: drs. A.M. Dobber

Vrije Universiteit
Faculty of sciences
Amsterdam, The Netherlands

Preface

This dissertation (BWI-werkstuk) forms a compulsory part of my Business Mathematics and Informatics (BMI) Masters degree at the *Vrije universiteit* in Amsterdam. It is about the geometric Brownian motion model. This model is one of the most mathematical models used in asset price modelling. According to the geometric Brownian motion model the future price of financial stocks has a lognormal probability distribution and their future value therefore can be estimated with a certain level of confidence.
The goal of this paper is to study the modelling of future stock prices. The discussion will be focused on the most used financial stock model; the just mentioned geometric Brownian motion. Naturally everyone who is interested in the topic cab read this document, but I especially hope that this study will, help people with mathematical background to get more understanding about stock price modelling.
I would not have been able to write this dissertation without the help of my mentor Menno. He provided me with ideas and kept me in the right track. Hereby I express my special thank to him.

Amsterdam, November 2006

Contents

Abstract

During the twentieth century major financial crisis have been strong motives to lots of studies and research in financial modelling in order to minimize such risks for the future. Through the years a lot of work has been done in this area. Mathematicians and financial engineers developed many mathematical models and the geometric Brownian motion is now widely used in stock price modelling. The assumptions on which this model is based meet the financial market laws and rules imposed by the Market Efficiency Hypothesis. These rules and laws suppose that only present information about a stock efficient to determine the future price of this that stock. According to the geometric Brownian motion model, the returns on a certain stock in successive, equal periods of time are independents and normally distributed. Thus they form a Markov process.

So theoretically the geometric Brownian motion seems to be a good way to model future stock. In practice however it shows some shortcomings, especially when it is used to model the price over short periods of time. Whereas it is more accurate when used for modelling stock prices over longer periods of time. This is due to the fact that the expected rate of return and volatility of a stock are assumed to be constant. To assess the accuracy of the model, it is preferable to model these parameters as stochastic function of time and not as constants.

The first part of this work covers the mathematical construction of the geometric Brownian motion. Concepts like 'random walk' and 'Ito's process 'will be used n the construction and understanding of the geometric Brownian motion. Statistics of the model will also be derived.

In the second part of this work, real data will be used to estimate model parameters, test assumptions and study the accuracy of the model. Given a real stock value another value of the same stock will be simulated with a 95% confidence interval.

Chapter 1   Introduction

Towards the end of the 16th century, Dutch traders from various towns decided to take charge of the import of spices from Asia. In order to finance the ships and equipment, companies were formed. On March 20th, 1602, the prime companies of Holland merged to form one company called the "Vereenigde Oostindische Compagnie "(VOC). This merger was the breakthrough of the first, and soon largest, worldwide dominating trading company of its time, and as the 'first modern joint-stock company' the VOC would write economic and financial history.

At the beginning the company was run by the chambers of commerce of Amsterdam and other cities. The original paid up share capital of the company was 6,424,588 Guilders, which was a huge amount at that time. A few years later the company's shares became publicly sold. As result the shares were sold rapidly, mostly at a nominal value of 3000 Guilders, and they were tradable, as any Dutchman could buy and sell them. The share price was not set by the Dutch government, but by an independent joint-stock corporation interested in profit. The "Amsterdam Kontor" of the VOC became the "first stock exchange in the world" by trading in its own shares and the share became the first stock in the world.

Centuries later, trading in stock markets would become much more sophisticated and complicated, but the principle remained the same: the forces of supply and demand determine the value of a stock. Investors are attracted to stocks of companies they expect will earn substantial profits in the future. Consequently if people are not interested in company, which faces bleak profits, the stock of that company will go down.
Certainly people want to make profit by investing in stock and companies want to minimize or control risk caused by such decreasing in their stock value. Is there any way to achieve this? Or is there any model, which can give an indication about future behaviour of stock prices?
There are of course market laws, such as the Market Efficiency Hypothesis[1], that guarantee market transparency. This transparency is very important in the scene that it gives everyone the same information about a certain stock. Otherwise everyone would try to realise the risky profit instantaneously. According to this hypothesis, the only relevant information about the stock is its current value. Past price values do not say anything about the future behaviour future values. Thus modelling stock prices is about modelling new information about stocks.

Based on market restrictions and laws, geometric Brownian motion is a mathematical approach for stock price modelling. It is a stochastic process, which assumes that the returns, profits or losses, on the stock are independent and normally distributed.
In this dissertation I will discuss the Geometric Brownian motion process as a stochastic Markov [2] process and study its accuracy when used to model future stock prices.
In the first part I will explain the geometric Brownian motion as a mathematical model. Also all its statistics will be derived.
 In the second part I will investigate the assumptions on which the model is based, using real and simulated data to test the model's accuracy.

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[1] Appendix 1: Efficient Market
[2] Appendix 2: Markov propretés

In the first section of Chapter 2, I will give an overview of stock and the Market Efficiency Hypothesis. The next sections deal with concepts such as random walk and Brownian motion. Both processes are conditional to understanding the geometric Brownian motion. They are exposed heuristically.

Chapter 3 is an expansion of the mathematical results of Chapter 2. It deals with the stock return as a generalized random walk. The chapter ends with the solution of the stochastic differential equation. The solution is the mathematical modelling of the future stock price.

Chapter 4 introduces the distribution of the geometric Brownian motion and other statistics such as expected value of the stock price and confidence interval.

In Chapter 5 results developed in Chapter 4 will be tested. Assumption on which the geometric Brownian motion is based will be investigated. In the next section parameters of the stock, like the volatility and drift, will be estimated according to their biased estimators.

Using the geometric Brownian motion model a series of stock price paths will be simulated. The estimated future stock value will then be compared to the real value of the same stock to see if the geometric Brownian motion does really model the future stock price.

Chapter 2   Stock price behavior

In this chapter I will first give an overview of the stock and the strong market law that governs the behavior of the stock price. Further explanation of the stock price modelling will be also given.

2.1   Stock definition

In finance, a stock represents a share in the ownership of an incorporated company. Stocks are evidences of ownership, or equity. Investors buy stocks in the hope that it will yield income from dividends and appreciate, or grow, in value. Shares of widely held companies are traded on stocks markets. Stockholding is popular because stocks represent ownership of capital that can be easily transferred by means of organized trading in the stock markets.

2.2   Efficient market hypothesis

In financial markets the dynamics of stock prices are reflected by uncertain movements of their value over time. One possible reason for the random behavior of the asset price is the efficient market hypothesis (EMH).The EMH basically states two things:
1- The past history of a stock price is fully reflected in present prices.
2- The markets respond immediately to any new information about the stock.

These two assumptions imply that changes in the stock price are a Markov process. This means that the expected future value of a stock depends only on its current price. Predictions remain uncertain and may be only expressed in term of probability distribution.
In this context, modelling the stock price is concerned with modelling the arrival of new information, which affects the price. Two important things to retain are:
1- Probability distribution
2- Information

These play a major role in the modelling of future stock prices. In other words, the future price of a stock can be predicted within a certain level of exactitude, if one can anticipate new information about the stock.

2.3   Stock price process

Figure 2.1 shows how the price of a financial stock varies over time.
The horizontal axis represents the time in years. The vertical axis is the daily price closing price of a "Google" stock during one year.
The price behaviour shows the same behaviour as a stochastic process called "Brownian motion ". Thus some properties of the stock price process can be derived from those of the Brownian motion process.
In the next paragraph I will give a more detailed explanation of the Brownian motion.
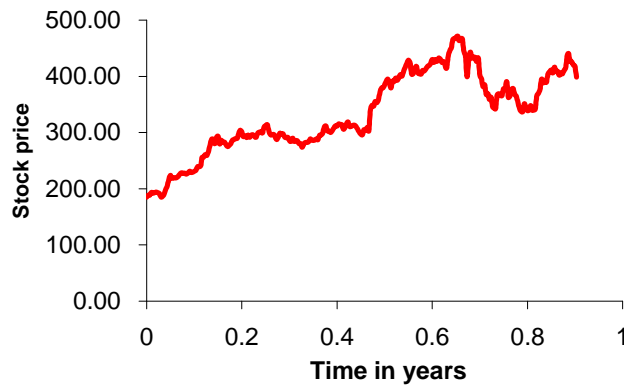
Example of Stcok price process

Figure 2.1: Closing price of Google stocks over a one-year period.

2.4   Random walk

A random walk, sometimes also called a "drunkard's walk", is the first step in understanding the Brownian motion. A random walk is a formalization of the intuitive idea of taking successive steps. The simplest random walk is a path constructed according to the following rules:

For an integer n, $n > 0$, we define the Random Walk process at the time t $\{W_n(t), t>0\}$ as follows:

  1. The initial value of the process is: $W_n(0) = 0$

  2. The layer spacing between tow successive jumps is equal to 1/n

  3. The "Up" and "down" jumps are equal and of size $1/\sqrt{n}$, with equal probability.

In other words, if we consider a sequence of independent binomial variable X_i taking values +1 or –1 with equal probability ½, then the value of the random walk at the i-th step is defined recursively as follows:

$$W_n(\frac{i}{n}) = W_n(\frac{i-1}{n}) + \frac{X_i}{\sqrt{n}} \qquad \text{for all i} \geq 1 \tag{2.1}$$

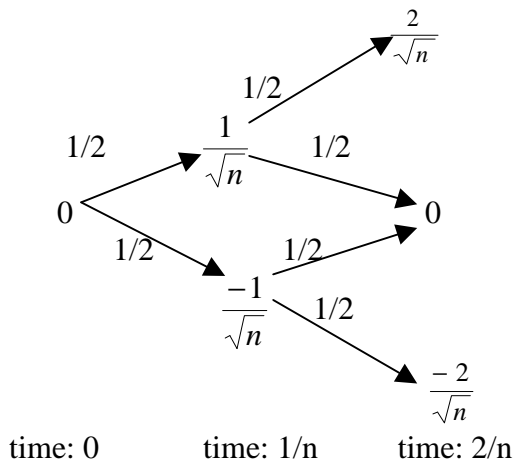time: 0   time: 1/n   time: 2/n

Figure 2.2: The first two steps of the random walk W_n

Figure 2.2 shows the first two steps of W_n according to the definition above. At time 1 the process has only two values with the same probability:
It moves "up" to the value $(1/n)^{1/2}$ with probability 0.5 or moves "down " to the value $-(1/n)^{-1/2}$ with the same probability 0.5. In this case the mean and the variance of the process are:

$$E\ [W_n(1)] = 0.5*\ (\frac{1}{\sqrt{n}})+ 0.5*(\frac{-1}{\sqrt{n}})=0$$

$$Var\ [W_n(1)]=E[W_n(1))- E(W_n(1)]^2 = 0.5*(\frac{1}{\sqrt{n}}-0)^2 + 0.5*(\frac{-1}{\sqrt{n}}-0)^2 = \frac{1}{n}$$

Using the same procedure we obtain the value of the random walk at time 2, which equal to 2/n. According to (2.1) the random walk at time 2 is given by:

$$W_n(\frac{2}{n}) = W_n(\frac{1}{n}) + \frac{X_1}{\sqrt{n}}$$

As the Figure 2.2 shows $W_n(2/n)$ can have 3 different states:
The state $2*(1/n)^{1/2}$ with probability 0.25 if $W_n(1/n)=(1/n)^{1/2}$ and $X_1=+1$.
The state "0" with probability $2*0.25=0.5$ if from the state $(1/n)^{1/2}$ the random walk goes "down" i.e. $X_1= -1$, or jumps "up" from the state "$-(1/n)^{1/2}$ " i.e. $X_1= +1$.
In the state "$–2/(n^{1/2})$" the process goes "down" from the state "$-1/(n^{1/2})$" with probability 0.25. In this case $X_1= -1$.
The mean and the variance of the random walk process after the second step can be calculated in the same manner as before. Thus we have:

$$E\left(W_n(\frac{2}{n})\right) = \frac{2}{\sqrt{n}}\ 0.25+ 0*0.5+ \frac{-2}{\sqrt{n}}*0.25 =0 \text{ and hence the variance}$$

$$Var\left(W_n(\frac{2}{n})\right) = E\left(W_n(\frac{2}{n}) - E\left(W_n(\frac{2}{n})\right)\right)^2$$

$$= 0.25 * \left(\frac{-2}{\sqrt{n}} - 0 \right)^2 + 0.25 * \left(0 - 0 \right)^2 + 0.25 * \left(\frac{2}{\sqrt{n}} - 0 \right)^2$$

$$= 0.258 * \frac{4}{n} + 0 + 0.25 * \frac{4}{n}$$

$$= \frac{2}{n}$$

As an example, the plot below in figure 2.2 shows a random process of n steps.

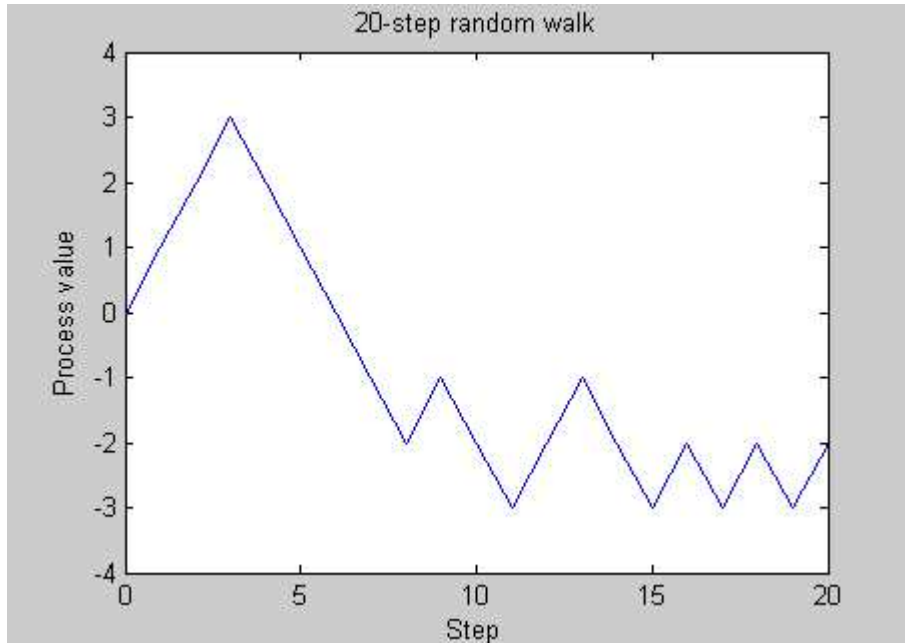**Figure 2.3**: Example of random walk with 20 steps in the time.

The process starts from the value "0" and randomly makes a series of 3 successive "up" jumps followed by 5 successive "down" jumps. At the end the random walk seems to show a different behavior like an "up" and "down" process.
But what does the random walk look like if n gets large i.e. the intervals of time become smaller?
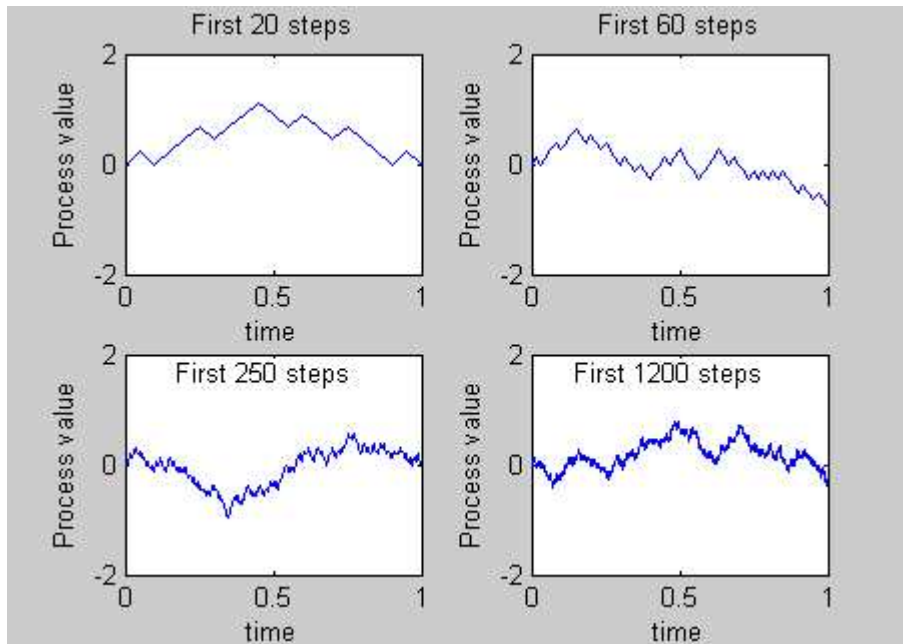
Figure 2.4: Scaled random walks of 20, 60,250 and 1200 steps respectively

The Figure 2.4 shows that family portraits appear to be settling down towards something as the number of steps n increases. The moves of size $1/(n^{1/2})$ seem to force some kind of convergence. According to the probability theory the process $W_n(t)$ has a normal distribution with mean zero and variance equal to t. Of course this is statement of course have to be proved.

Proof:
If we define $X_1, X_2\ldots$ a series of n independent and identically distributed binomial random variables taking the values -1 and +1 with probability P=0.5
Starting from $W_n(0) = 0$, we have:

$$W_n(\frac{1}{n}) = W_n(0) + \frac{X_1}{\sqrt{n}} = \frac{X_1}{\sqrt{n}}$$

$$W_n(\frac{2}{n}) = W_n(\frac{1}{n}) + \frac{X_2}{\sqrt{n}} = \frac{X_1}{\sqrt{n}} + \frac{X_2}{\sqrt{n}} = \frac{1}{\sqrt{n}}(X_1 + X_2)$$

.
.

$$W_n(\frac{i-1}{n}) = W_n(\frac{i-2}{n}) + \frac{X_{i-1}}{\sqrt{n}} = \frac{1}{\sqrt{n}}(X_1 + X_2 + \ldots + X_{i-1})$$

$$W_n(\frac{i}{n}) = W_n(\frac{i-1}{n}) + \frac{X_i}{\sqrt{n}} = \frac{1}{\sqrt{n}}(X_1 + X_2 + \ldots + X_i), \forall i \geq 1$$

.
.

And so recursively we get the value of the Random Walk at time after n steps starting from the initial value at time zero $W_n(0)$ that is:

$$W_n(t) = \frac{1}{\sqrt{n}} \sum_{i=1}^{nt} X_i \qquad\qquad (2.2)$$

We multiply the numerator and denominator in (2.2) by $\sqrt{t}$, we have:

$$W_n(t) = \sqrt{t} \left(\frac{\sum_{i=1}^{nt} X_i}{\sqrt{nt}} \right). \qquad\qquad (2.3)$$

According to Central Limit Theorem, tends the quantity between brackets in [2.3] in distribution, to the standard normal distribution with mean zero and variance one N(0,1). Hence the random walk process $W_n(t) \rightarrow$ N(0,t) in distribution when n gets large.

2.5   Brownian motion

It was the botanist Robert Brown who first realized the Brownian motion (also called Wiener process) when trying to describe the motion exhibited by particles immersed in a gas or liquid. The particles were essentially being bombarded by molecules present within the matter causing displacement or movement.

2.5.1   Definition:

The Brownian motion is a fundamental process that serves as a part of many different processes. It refers either to:
  1-The physical phenomenon that small particles immersed in a fluid , move randomly or
  2-The mathematical models used to describe those random movements.
  It is the scaling limit of random walk in one dimension as the time steps go to zero i.e. the number of steps becomes large. The Figure 2.4 shows that the scaled random walk tends to a Brownian motion when the number of steps in the random in figure Figure2.3 increases.

2.5.2   Properties:

We refer to the Brownian motion as a process B_t. The Brownian motion has the following properties:
  1. Continuity: B_t has a continuous path and B (t=0) = 0
  2. Normality: The increment of the Brownian process in the interval of time of length t between the tow moments s and s+t is B_{s+t} -B_s. This increment is normally distributed with mean zero and variance equal to the time increment t.
      B_{s+t}-B_s ~N (0, t).
  3. Markov property: The conditional distribution B(t) given information up to time s<t depends only on the B(s).

2.5.3   Generalized random walk

The generalized random walk, called also 'Brownian motion with drift', is a stochastic process B_t. For given constants μ and σ the process has the following form:
$$B_t = \mu t + \sigma W_t \qquad\qquad (2.3)$$
Where t represents time and W_t is a random walk process as described in the previous Section (2.4), W_t can be written as:

$$W_t = \varepsilon\sqrt{t} \tag{2.4}$$

ε is a random number drawn from a standard normal distribution.

The process described by [2.3] is normally distributed with mean $\mu*t$ and variance σ^2*t:

If we define the mean of the Brownian motion process as $E(B_t)$ and it's variance as $Var(X)$ we have:

$$E(B_t) = E(\mu t + \sigma W_t) = E(\mu t) + E(\sigma W_t)$$

$$= \mu t + \sigma E(W_t)$$

This is because the function mean is linear. Using the fact that the mean of the random walk is zero i.e. $E(W_t) = 0$ we finally have

$$E(B_t) = \mu t \tag{2.5}$$

For the variance of the process B_t we have , according to the function variance we have:

$$Var(B_t) = Var(\mu t + \sigma W_t)$$

$$= E[(\mu t + \sigma W_t)^2] - [E(\mu t + \sigma W_t)]^2$$

Using again the linearity property of the mean function, the first term in the right side of this equality is equal to:

$$E[(\mu t)^2] + E[(\sigma W_t)^2] + 2E[\mu\sigma t W_t] = (\mu t)^2 + \sigma^2 E[(W_t^2] + 2\mu t\sigma E(W_t)$$

The second term of the equality is equal to the mean of the process $E(B_t)$

Because of $E(W_t)=0$ we have:

$$E[(\mu t + \sigma W_t)^2] = (\mu t)^2 + \sigma^2 E[W_t^2] + 0$$

According to the definition of the variance we have:

$$E(W_t^2) = Var(W_t) + [E(W_t)]^2 = t + 0 = t \text{ and thus}$$

$$E[(\mu t + \sigma W_t)^2] = (\mu t)^2 + \sigma^2 E[W_t^2] + 0$$

$$= (\mu t)^2 + t$$

Finally $Var(B_t) = (\mu t)^2 + t - (\mu t)^2 = t \tag{2.6}$
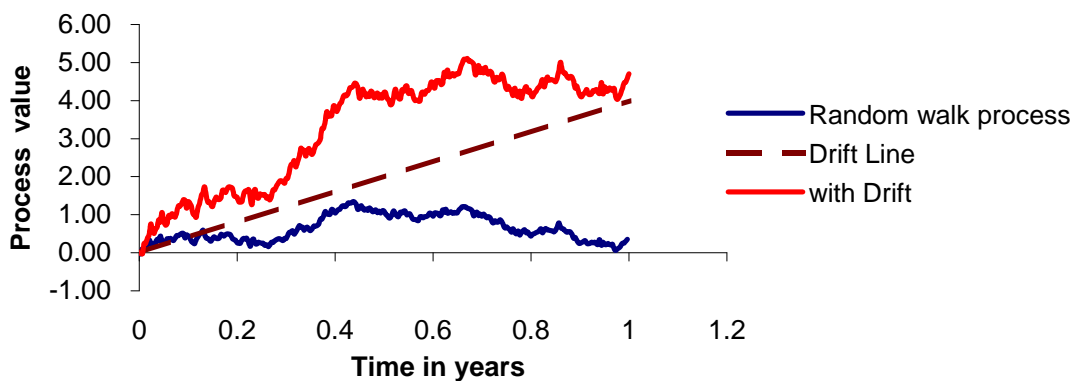
Figure 2.4: Generalized Brownian motion process with positive drift

The Figure [2.4] shows the generalized Brownian process given by [2.3] with its components. It consists of a random walk process (below) with a drift (line). The result is an increasing process if the constant μ is positive or a decreasing one if the constant is negative.

If we compare the closing price process plot in Figure 4.1 of and with the generalized Brownian motion in Figure 2.4, we can conclude that the two processes show the same behavior in time. This statement is the first step towards the mathematical modelling of the stock closing price. This will be the subject of the next chapter.

Chapter 3   Stock price modelling

In the previous chapter similarities have been shown between the Brownian motion and the stock price process. However there is a misgiving about Brownian motion as a global model for stock behavior. This is because of the property 2 of the Brownian motion, which states that the process is normally distributed with mean equal zero and hence negative with probability 1/2 whereas the price of a stock normally grows at some rate. One way to solve this problem is to model the stock price as a sum of a positive, deterministic function of time and a Brownian motion.

In the real world of financial markets, investors and financial analysts are generally more interested in the profit or loss of the stock over a period of time i.e. the increase or decrease in the price, than in the price self. Therefore modelling a behavior of a stock price can be made through its relative change in the time. In this chapter I will give a modelling of the stock return through a stochastic differential equation. The solution of the differential equation will be used to find a mathematical model of the stock price.

3.1 Stock return

Like the particle being bombarded in the Brownian motion, stock prices deviate from a steady state as a result of being jolted by trades i.e. ask and bid in financial markets.

If we consider a with price S_t at time t and an expected rate of return μ, then the return or relative change in its price during the next period of time dt can be decomposed in two parts:

1. A ppredictable, deterministic and anticipated part that is the expected return form the stock hold during a period of time dt. This return is the equal to $\mu S dt$.

2. A stochastic and unexpected part, which reflects the random changes in stock price during the interval of time dt, as response to external effects such as unexpected news on the stock.

A reasonable assumption is to take this contribution, as proportional to the stock. For some constant σ and a random walk process dB_t. This unexpected part of the return is equal to $\sigma S_t dB_t$.

This definition of the daily return leads to the stochastic differential equation followed by the stock price:

$$dS_t = \mu S_t dt + \sigma S_t dB_t \tag{3.1}$$

or

$$\frac{dS_t}{S_t} = \mu dt + \sigma dB_t \tag{3.2}$$

The stochastic differential Equation [3.1] is the Brownian motion with drift followed by the stock price S_t. The Equation [3.2] is the instantaneous rate of return on S_t. The return on S_t in the period of time dt follows an Ito's process. It can be also written for every interval of time of length dt between two consecutive instants, as:

$$\frac{dS_t}{S_t} = d(\ln S_t) = \ln(S_t) - \ln(S_{t-1}) = \ln\left(\frac{S_t}{S_{t-1}}\right)$$

$$\ln\left(\frac{S_t}{S_{t-1}}\right) = \mu dt + \sigma dB_t \tag{3.3}$$

3.2   Solution of the SDE

The solution S_t the Formula [3.2] can be found by applying the famous Itô's[3] formula. Before we apply the Formula to [3.2] I want first to remind it general form and. More explanation about the formula and the Itô's are given in the Appendix:

For any function G(X, t) of two variables X and t where X satisfies the following stochastic differential equation:

dX=adt +bdB$_t$, for some constants a and b.

dB$_t$ is a Brownian motion

The general form of the Itô's formula is:

$$dG = \left(\frac{\partial G}{\partial X} a + \frac{\partial G}{\partial t} + \frac{1}{2} \frac{\partial^2 G}{\partial X^2} b^2 \right) dt + \frac{\partial G}{\partial X} dB \qquad (3.4)$$

If we now consider the function $G=\ln(S_t)$ where S_t satisfies the Formula[3.1], the Formula [3.4] is applied to we obtain:

$$\frac{\partial G}{\partial S} = \frac{1}{S_t} \qquad (3.5)$$

$$\frac{\partial G}{\partial S} = -\frac{1}{S_t^2} \qquad (3.6)$$

$$\frac{\partial G}{\partial S} = 0 \qquad (3.7)$$

By inserting these into the Ito's formula we obtain:

$$d(\ln S_t) = \left[\mu S_t \frac{1}{S_t} + \frac{1}{2} \sigma^2 S_t^2 (-\frac{1}{S_t^2}) \right] dt + \sigma S \frac{1}{S_t} dB_t$$

$$d(\ln S_t) = \ln(S_t) - \ln(S_{t-1}) = \ln(\frac{S_t}{S_{t-1}}) = \left(\mu - \frac{1}{2} \sigma^2 \right) dt + \sigma \varepsilon \sqrt{dt} \qquad (3.8)$$

It follows from [3.9] that dlnS$_t$ has a Brownian motion with drift. It's has a normal distribution with mean $(\mu-\sigma^{1/2}/2)*dt$ and variance $\sigma^2 *dt$. The solution of S_t can be easily drawn as follows:

$$\ln S_t = \ln S_{t-1} + \left(\mu - \frac{1}{2} \sigma^2 \right) dt + \sigma \varepsilon \sqrt{dt} \qquad (3.9)$$

$$S_t = S_{t-1} e^{\left(\mu - \frac{1}{2}\sigma^2 \right) dt + \sigma \varepsilon \sqrt{dt}} \qquad (3.10)$$

The Solution [3.9] is the geometric Brownian motion model of the future stock price. It can be extended to any period of time t. In this case the future stock price S_t can be derived from the initial value S_0 just by applying [3.9] on the period of time t when the time dt =t

---

[3]Appendix 3: Ito's process

$$S_t = S_0 e^{\left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma\varepsilon\sqrt{t}}$$

(3.11)

3.3   Parameters estimation

In the form of stock price S_t in the previous section there are two important parameters. These parameters are supposed to be known. If for some reason there is no available information about the drift and the volatility of the stock, they need to be estimated form the historical data. That is what I will do in this section. Using the data of the daily returns on results of the Section 3.2, I will give an estimation of the two parameters.

3.3.1   Volatility

The volatility is a constant characteristic of a stock, expressed as an annual percentage. It gives an idea about the stability of stock price. Relatively high volatility means that the stock price varies continuously within relatively large interval. The value most usual method of measuring the stock volatility is the standard deviation of the price returns. This procedure is fairly standard. The practical way to estimate empirically the volatility of a stock is to observe the historical data at fixed intervals of time, for example the daily closing price.
If we denote S_i as the stock closing price at the end of i-th trading period and τ is the length of time interval between two consecutive trading periods expressed in years,
$\tau = t_i - t_{i-1}$ for i positive integer.
If u_i is the logarithm of the daily return on the stock over the short time interval τ i.e.

$$u_i = \ln\left(\frac{S_i}{S_{i-1}}\right) \qquad\qquad \text{for i=1........n} \qquad\qquad (3.12)$$

Then the unbiased estimator $\bar{u}$ of the logarithm of the returns is given by:

$$\bar{u} = \frac{1}{n}\sum_{i=1}^{n} u_i$$

The estimator of the of the standard deviation of the u_i's is given by

$$v = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(u_i - \bar{u}\right)^2}$$

or

$$v = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n} u_i^2 - \frac{1}{n(n-1)}\left(\sum_{i=1}^{n} u_i\right)^2}$$

Where $\bar{u}$ is the unbiased estimator of the log return u_i.
From the Formula[3.8] the estimator of the standard deviation of the daily return on the stock is equal to $\sigma^*\tau^{1/2}$. It follows that σ can simply estimated by:

$$\sigma^* = \frac{v}{\sqrt{\tau}}$$

(3.13)

With a standard error equal: $\sigma^*/(2n)^{1/2}$

3.3.2   Drift

Using the result given in the Formula [3.9] and the unbiased estimator of the daily return given by [4.8] we obtain the expected annual rate of return or the drift μ

$$\bar{u} = \left(\mu - \frac{1}{2}\sigma^2 \right)\tau \text{ Thus:}$$

$$\mu = \frac{\bar{u}}{\tau} + \frac{1}{2}\sigma^2 \qquad\qquad (3.14)$$

Chapter 4   Stock price distribution

Up to now, only the mathematical form of Geometric Brownian motion has been derived. To understand more the model we need also to derive other features like probability density function, mean and confidence interval.
First I will introduce the lognormal distribution and its properties. Then the distribution of the stock price at a future time will be derived. Other statistics like the expected value and the confidence of the stock at future time will also be derived.

4.1   Lognormal density function

A random variable Q has a lognormal distribution if V=ln(Q) had a normal distribution. Suppose that V is φ(m,s); that is it has a normal distribution with a mean m and a standard deviation s. The probability density function for V is given

by $f(V) = \dfrac{1}{\sqrt{2\pi}\,s}\exp\left(-\dfrac{1}{2}\left(\dfrac{V-m}{s}\right)^2\right)$ (4.1)

The lognormal probability density function of the random variable Q can be obtained, realizing that for equal probabilities under the normal and lognormal probability density functions, incremental areas should also be equal i.e.: f(V)dV=h(Q)dQ
By dividing both terms by dQ we obtain:

$h(Q) = \dfrac{f(V)dV}{dQ}$.

Using the derivative of the formula V=ln(Q) we get:
dV=d(ln(Q)); that is dV=dQ/Q. By substitution of dV in h(Q) we obtain:

Hence the probability density function of the lognormal distribution of the variable Q is given by:

$h(Q) = \dfrac{1}{\sqrt{2\pi}\,sQ}\exp\left(-\dfrac{1}{2}\left(\dfrac{\ln(Q)-m}{s}\right)^2\right)$ (4.2)

4.2   Mean lognormal distribution

Using the probability density function, the mean of the variable Q is given by the following integral:

$E(Q) = \int_0^\infty Q h(Q)dQ$

by substitution of Q=exp(V) and dQ=exp(V)dV in the integral , we have:

$$E(Q) = \int_0^\infty \frac{Q}{sQ\sqrt{2\pi}}\exp\left(-\frac{1}{2}\left(\frac{\ln Q - m}{s}\right)^2\right)dQ = \int_{-\infty}^\infty \frac{1}{s\sqrt{2\pi}}\exp(V)*\exp\left(-\frac{1}{2}\left(\frac{V-m}{s}\right)^2\right)dV$$

$$= \int_{-\infty}^\infty \frac{1}{s\sqrt{2\pi}}\exp\left(-\frac{(V-m-s^2)^2}{2s^2}\right)*\exp\left(\frac{ms^2+s^4}{2s^2}\right)dV$$

$$= \exp(m+\frac{s^2}{2})*\int_{-\infty}^\infty \frac{1}{s\sqrt{2\pi}}\exp\left(-\frac{(V-m-s^2)^2}{2s^2}\right)dV$$

The integral in this expression is the integral of a normal density function with mean $m+s^2$ and standard deviation s and is therefore equal to 1. it follows that the expectation of the lognormal variable Q is:

$$E(Q) = e^{m + \frac{s^2}{2}}$$
(4.3)

.

4.3 Stock expected value

According to the results of the previous section of Chapter 4, the variable $Y = \ln S_t$ is normally distributed with mean $\ln S_0 + (\mu - \sigma^2/2)*t$ and variance σ^2*t.
The expected value $E(S_t)$ of the stock price at the future time t is given by:

$$E(S_t) = \exp\left(\ln S_0 + \left(\mu - \frac{\sigma^2}{2}\right)*t + \sigma^2 *t\right)$$

$$= \exp(\ln S_0)*\exp\left(\left(\mu + \frac{\sigma^2}{2}\right)*t\right)$$

$$= S_0 *\exp\left(\left(\mu + \frac{\sigma^2}{2}\right)*t\right)$$
(4.4)

4.4 Confidence interval

The variable $\ln S_t$ follows a normal distribution with mean $\ln S0 + (\mu - \sigma^2/2)*t$ and variance σ^2*t. A confidence interval with 95% confidence level of the variable $\ln S_t$ is given by:

$$\ln(S_0) + \left(\mu - \frac{1}{2}\sigma^2\right)t - 1.96*\sigma\sqrt{t} \le \ln(S_t) \le \ln(S_0) + \left(\mu - \frac{1}{2}\sigma^2\right)t + 1.96*\sigma\sqrt{t}$$
(4.4)

Hence, By taking the exponentials in [4.6] we obtain the 95% confidence interval of the lognormal variable distributed variable S_t. It has the following form:

$$e^{\ln(S_0) + \left(\mu - \frac{1}{2}\sigma^2\right)t - 1.96*\sigma\sqrt{t}} \le S_t \le e^{\ln(S_0) + \left(\mu - \frac{1}{2}\sigma^2\right)t + 1.96*\sigma\sqrt{t}}$$
(4.5)

Chapter 5   Results

In this chapter, I will illustrate the geometric Brownian motion model that has been developed by applying the obtained results in the previous chapters to real data. First, I will analyze the used data by testing the normality assumption on the daily return. Secondly, I will use a part of the data to estimate the parameters drift and volatility of the stock. The remained data will be used to test the results. The model will be used to estimate future prices and other statistical quantities like the mean and confidence interval.

5.1 Data

The data that are used for further work in table 1 have been obtained from Internet. They are an historical set of data of 2627 closing prices of Hewlett-Packard stocks, traded at the New York Stock Exchange NYSE from 02 January 1996 until 15 June 2006. To understand more about the historical behavior of the stock, instead of dealing with the period as a whole I will subdivide the data into ten smaller data samples. Each sample consists of one-year daily closing price of the same stock. One trading year is on average 252 days.
Hewlett-Packard commonly known as HP is one of the world's largest information technology corporations. The company operates worldwide in the fields of computing, printing, and digital imaging, sells software and offers related services.

Date	Open	High	Low	Close	Volume	Adj. Close*	Return
2-Jan-96	28.57	28.69	28.34	28.34	104,600	9.33	0
3-Jan-96	28.45	28.45	27.86	28.34	57,600	9.33	0
4-Jan-96	28.34	28.34	27.5	27.62	163,400	9.09	-0.02573
10-Jan-96	27.26	27.26	26.55	26.55	60,600	8.74	-0.03516
11-Jan-96	26.55	26.67	26.19	26.31	84,400	8.66	-0.00908
12-Jan-96	26.43	26.43	25.84	25.84	78,000	8.5	-0.01803
15-Jan-96	25.84	26.07	25.84	25.95	15,600	8.54	0.004248
16-Jan-96	26.07	26.19	25.95	26.07	48,800	8.58	0.004614
………… ………… …………..							
2-Jun-06	66.5	67.28	65.65	67.11	687,400	67.11	0.026882
5-Jun-06	68.3	68.39	63.27	63.42	1,213,300	63.42	-0.05655
6-Jun-06	63.33	64.84	62.31	63.4	789,400	63.4	-0.00032
12-Jun-06	58.68	58.72	54.89	54.99	1,010,400	54.99	-0.06222
13-Jun-06	54.07	55.44	52.05	52.75	1,084,300	52.75	-0.04159
14-Jun-06	53.15	55.29	53.15	55.23	1,213,300	55.23	0.045943
15-Jun-06	56	58.68	56	58.42	851,800	58.42	0.056152

Table1: Data of HP stock form 02 January 1996 until 15 June 2006
Source: http://chart.yahoo.com/

The first column of the table 1 shows the dates, from 2 January 1996 until 15 June 2006. The 2nd, 3d and 4th columns consist respectively of the openings and the highest and

lowest trading prices of the stock on the same trading day. The daily closing price, that is the price of the stock at the end of the trading day and the adjusted closing price[4] are presented respectively in the 5th and 7th columns of the table. The 8th column was initially not included in the table above. It has been inserted into the original data table for calculation purposes. It consists of the logarithm of the daily return of the stock calculated according to the Formula [4.8].
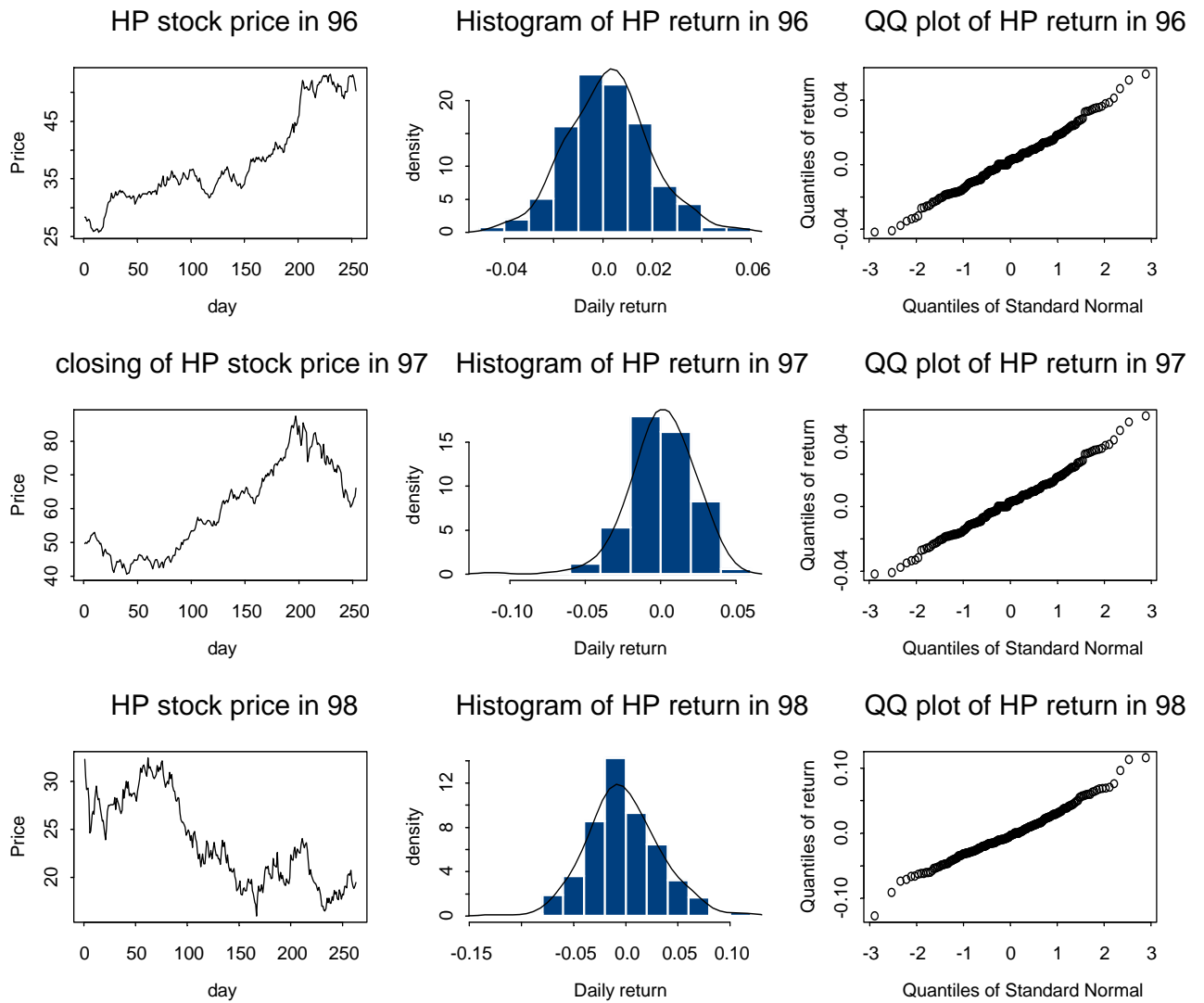
5.2   Normality test

Figure 5a: Plot of closing price, histogram and QQ-plot of HP stock price in the periods 1996, 1997 and 1998
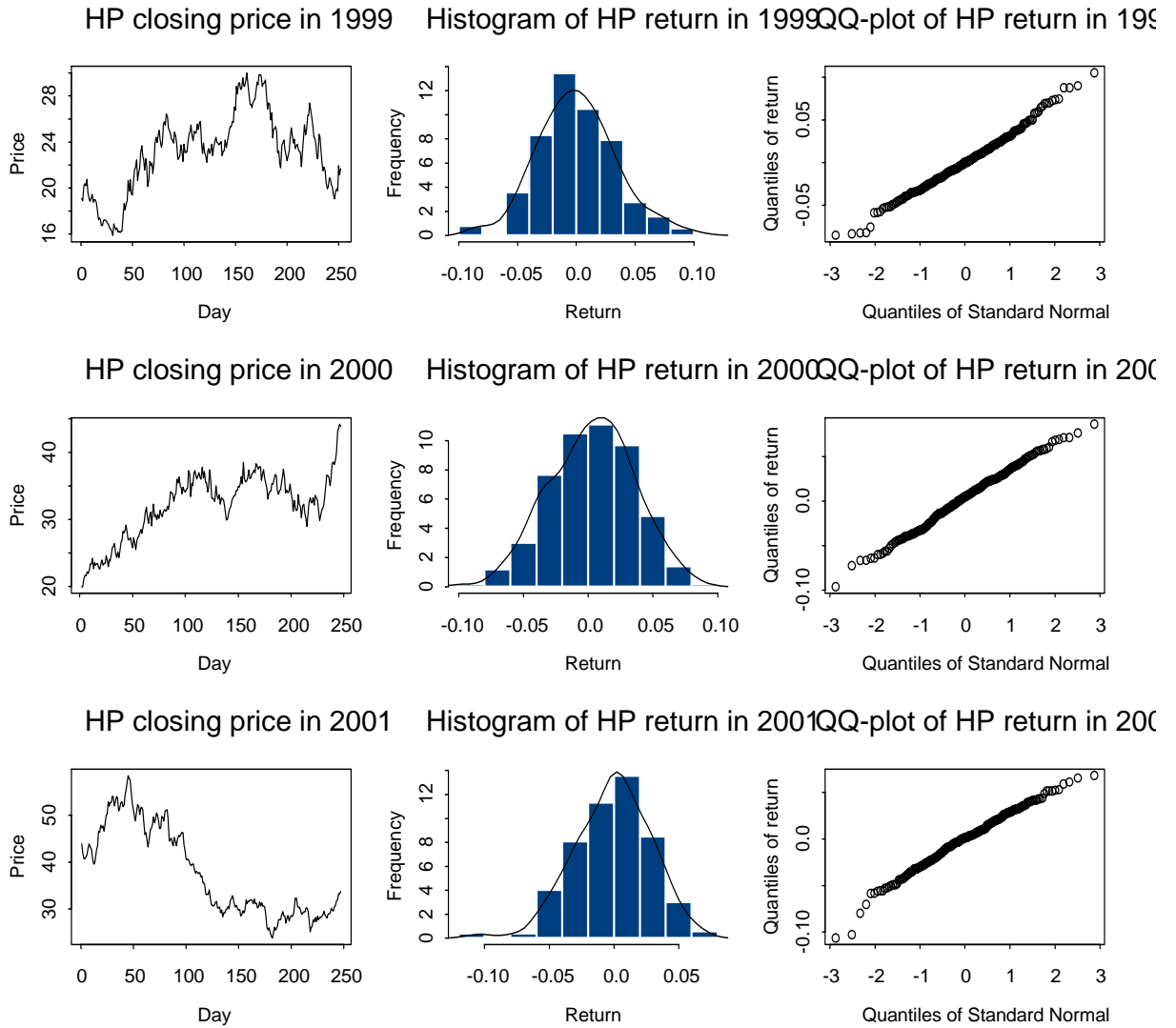
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[4] Appendix 4: Adjusted price

Figure 5b: Plot of closing price, histogram and QQ-plot of HP stock price in the periods
1999, 2000 and 2001

closing of HP stock price in 2002 Histogram of HP return in 2002 QQ-plot of HP return in 2002

closing of HP stock price in 2003 Histogram of HP return in 2003 QQ-plot of HP return in 2003

closing of HP stock price in 2004 Histogram of HP return in 2004 QQ-plot of HP return in 2004
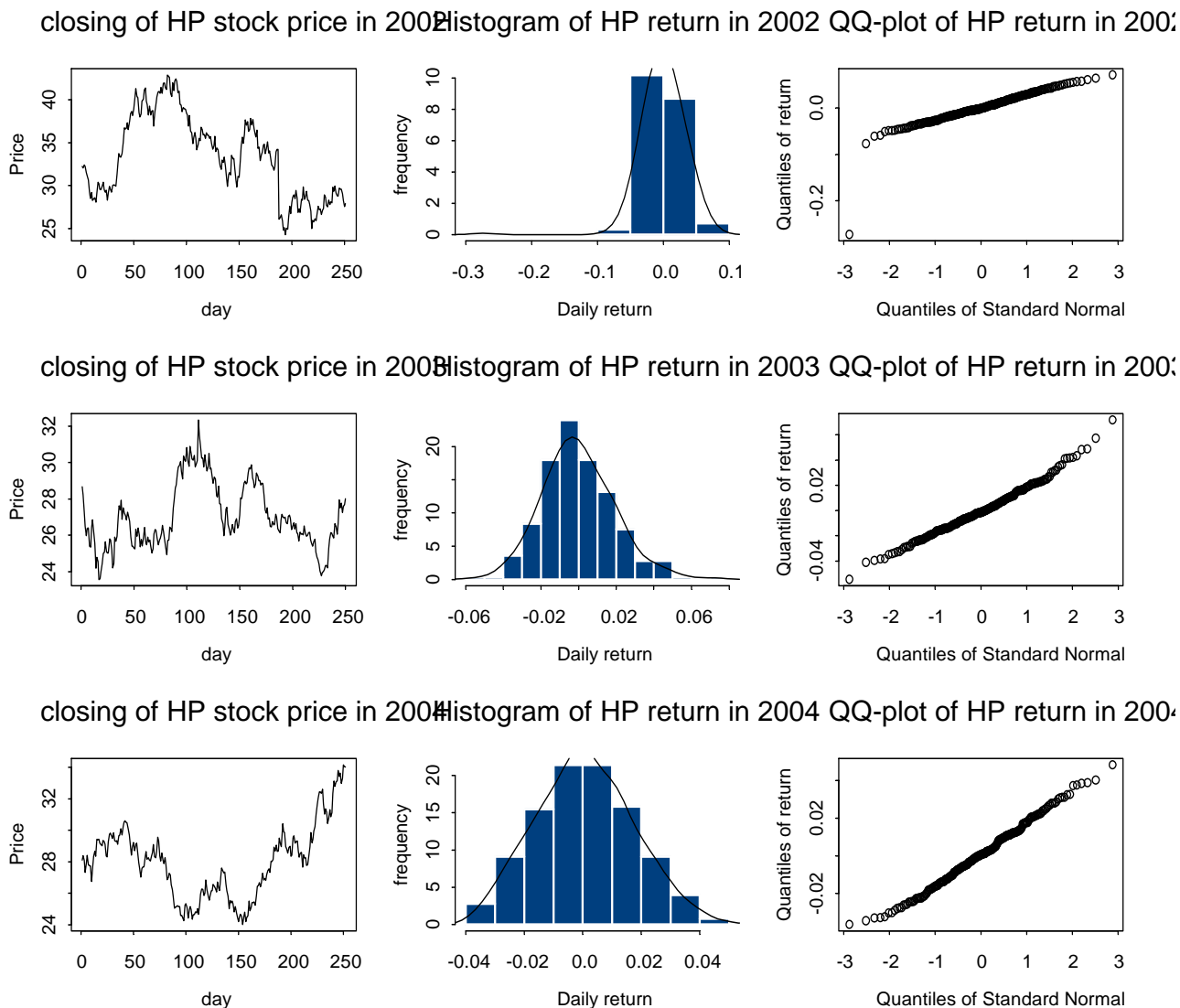
Figure 5c: Plot of closing price, histogram and QQ-plot of HP stock price in the periods 2002, 2003 and 2004

According to the Property 2 of the Brownian motion the daily return on the stock during a period of time is normally distributed.

In the Tables 2a, 2b and 2c below is overview of the data. It consists of respectively the plot of closing prices of the HP stock, the histogram of the daily return and the QQ-plot of the daily return on the same stock. This has been done for the periods 1996, 1997, 1998, 1999, 2000, 2001, 2002, 2003, 2004, 2005.

Because the data set over a one-year period of time is relatively large, the histogram is most effective for this purpose. The histogram is used to determine whether or not the data are approximately normally distributed. A bell shaped histogram gives an indication that the data are normally distributed. A graphical alternative to the histogram is the Q-Q plot. This plot displays the data values versus spaced percentiles from a normal distribution. If the Q-Q plot shows a line that is if there is a good correspondence between the data. A normal distribution is often a reasonable model for the data.

Back to the Figures 5a, 5b and 5c. The histograms of the daily return in all the periods have a unimodal form. They also have a bell shaped form. Some of them show a perfect symmetry around the mean, for example in the periods 1996, 1998, 2000 and 2004. Others histograms show less symmetry in their form and have skewed right or left sides like in the periods 1997, 2001 and 2002. The form of the histograms in all the periods advances an assumption that the logarithm of the daily return is normally distributed. Is this assumption reasonable? An analysis of the normal probability QQ-plot will help to confirm or reject this assumption. The QQ-plots relative to all the periods are almost linear. The skewness in some of the histograms can now be explained by the presence of outliers in the data. This can be easily seen in the few points, which do not belong to the line in the QQ-plots. The QQ-plot lines in all the periods confirm that the normality assumption of the daily return may be reasonable. However a parametric statistical test needs to be performed in order to be able to state with confidence that the hypothesis of normality can be accepted. The Kolmogorov-Smirnov test is one the strongest statistical test used for this purpose.

Using the statistical tools S-Plus. I have performed the test at 5% significance level. The result is shown in table2.

Period	Goodness-of-fit test	
	ks	p-value
1996	0.0526	0.4836
1997	0.0507	0.5332
1998	0.0514	0.5181
1999	0.033	0.9345
2000	0.0354	0.9161
2001	0.0457	0.6831
2002	0.0552	0.4288
2003	0.0487	0.594
2004	0.0366	0.8884

Table 2: Results of the Kolmogorov-Smirnov test of normality

The 2nd column in the table shows the value of the performed Kolmogorov-smirnov test whereas the 3d column displays the p-value relative to the same test value.
The Kolmogorov-Smirnov test is a very useful test of normality.
The p-value of the test in the 3d column of Table 2 is greater than significance level of 5%. Therefore the normality assumption of the HP stocks daily return may be accepted.

5.3   Parameters estimation

The unit period of time, which I use in this simulation, is of one-day length (1 day $=1/252 \approx 0.004$ year). On average there are 252 trading days in the year. The value of the stock volatility and its drift can be respectively estimated according to the formulas [3.13] and [3.14] in Chapter 3. Using the data in table1 an estimation of the two parameters of the stock HP has been made. The results are shown in table 3.

Period	$\mu_{daily}(\%)$	$\mu_{annualized}(\%)$	$\sigma_{daily}(\%)$	$\sigma_{annualized}(\%)$
1996	0.23	62.52	0.0167	26.5
1997	0.1	30.34	0.020	33.14
1998	-0.23	-42.7	0.035	55
1999	0.041	24.39	0.033	52.82
2000	0.31	91.77	0.032	51.85
2001	-0.11	-17.94	0.03	46.52
2002	-0.06	-1.67	0.031	50.38
2003	-0.014	-1.03	0.0189	30
2004	0.07	22.7	0.0165	26.2
2005	0.025	69.32	0.02	31.73

Table 3: Estimated values of the drift and volatility.

The Table 3 exhibits for each period the standard deviation of the daily return in column 1. The annualized standard deviation or the annual volatility of the stock obtained by applying the Formula (4.11) is displayed in the 4th column of table 3. The 1st and the 2nd respectively show the daily return and the annual expected rate of return on the stock HP given by [4.8] in every period from 1996 until 2005.

The table gives very important information about the behavior of the stock: the drift or assumed annual expected return of the stock differs from one period to another. For example in 1996, 2000 and 2005 the stock was very promising with a very high return rate expectancy of respectively 62.52%, 91% and 69.32%. In other periods though, it shows a considerable decrease like - 42.7 % in 1998. So the expected return is not equally distributed over the different periods.

The volatility of the stock also shows a variation from one period to another. In 1998, 1999 and 2000 the stock shows a very high volatility of 55%, 52.82% and 51% while it is relatively normal between 20% and 40%.

5.4 Testing the model

Up to now, the geometric Brownian motion that has been closely studied and the distribution of the future stock price have been given. The properties and hypothesis of the geometric Brownian motion have also been successfully tested. The major question that remains unanswered is: does the theoretical model, with assumptions as stated before, reproduce a correct image of the reality of stock price.

To answer this question I will use the model to generate a series of simulated stock values and compare the results to the real data. Using the formulas of the expected value and the confidence interval, the value of the stock at a future time will be estimated and also compared to the real stock value at the same future time.

5.4.1 Simulation

In this simulation I will use the formulas developed in section 3.2 to generate a series of stock price for one-year periods of time. This process will be repeated for all the nine periods 1997… 2005. There are two ways to do it:

The first method consists of first simulating a sample sequence for the log price and then to make exponentials of the numbers obtained. If we want to generate X_0, X1… ..X_n of

log price $\{X_t = \ln S_t\}$ separated in time by time interval Δt, we set $X_0=0$ and we generate a series of random numbers ε from of $N(0,1)$ by using the recursive formula:

$$X_{i+1} = X_i + \left(\mu - \frac{\sigma^2}{2} \right) \Delta t + \sigma \sqrt{\Delta t} \varepsilon_{i+1}.$$

The second method, used in this work, is simply based on the form of S(t) as given by (3.10). The algorithm of this method is to be found in Appendix 4. This simulation consists of 1000 random paths of the stock price in nine successive periods from 1998 until 2005. I used a data set to estimate the parameters drift and volatility. I use these estimated parameters and the form of the stock price given by the Formula [3.10] to simulate the price in the following period. For example I used the estimated drift and volatility in 1996 to generate a time series of price values for 1997. I used the same method for every two consecutive periods. The results of the simulation are shown in Figure 6.
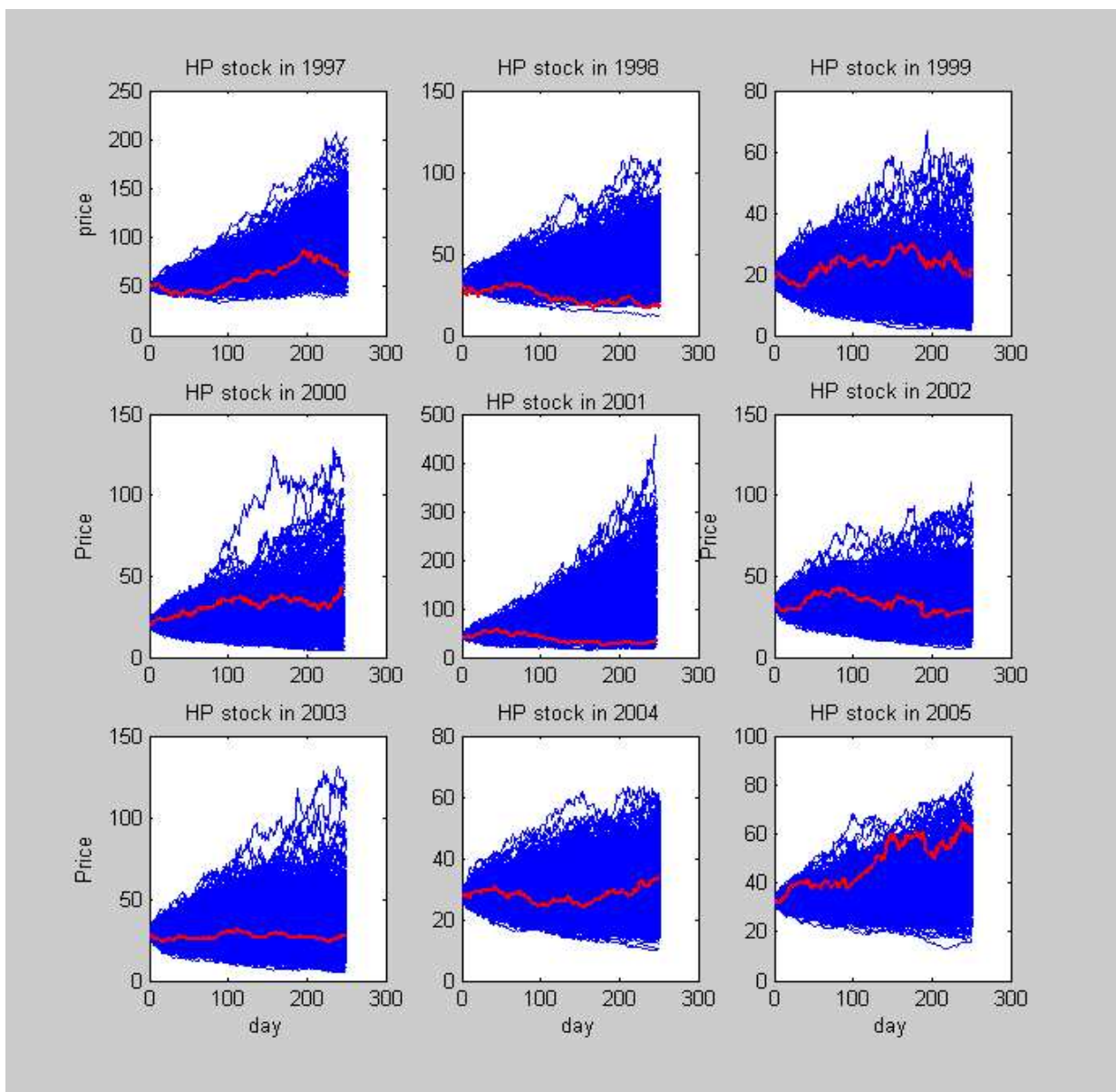

Figure 6: One year actual and 1000 simulated paths of HP closing price .

5.4.2   Results

Figure 6 exhibits the actual value and 1000 simulated paths of HP closing prices for 252 days. On the horizontal axis are the day numbers. The simulated prices are shown on the vertical axis. This figure gives a first idea about the distribution of simulated prices around the actual value. To better understand the graphics I performed more numerical calculations. The results are shown in table 3 and table 4.

Period	95 % confidence interval			
	Simulated S_T	LCL	UCL	Actual S_T
1997	67.60	66.23	69.00	66.7
1998	21.43	20.61	22.26	20.08
1999	23.20	22.38	24.00	22.69
2000	51.30	49.60	53.00	43.88
2001	36.56	35.43	37.70	33.69
2002	31.20	30.20	32.21	27.91
2003	29.05	28.47	29.63	28.01
2004	34.95	34.35	35.55	34.04
2005	64.82	63.51	66.13	61.91

Table 3 Simulated value of one-day stock price

For each period in the 1st column, corresponds an average of 1000 simulated values after only one trading day. I choose the last day 31 December of each year. This expected value and its lower (respectively upper) value are shown respectively in the 2nd and 3d (respectively 4th) column. The actual value at fixed time T is given in the 5th column. Table 3 shows that in the most of cases real stock prices are not in the 95% confidence interval of the simaulated stock price. The confidence intervals are relatively small for one day stock price simulation.

Period	95 % confidence interval			
	Simulated S_T	LCL	UCL	Actual S_T
1997	92.83	28,97	222,46	66.07
1998	43.20	21,63	78,86	19.08
1999	12.10	3,65	31,54	21.69
2000	26.15	8,69	66,74	43.88
2001	110.23	35,23	260,09	33.69
2002	27.29	9,82	59,61	27.91
2003	27.69	9,34	66,27	28.01
2004	20.33	14,77	47,88	34.04
2005	50.65	23,35	64,71	61.9

Table 4: Simulated value of the stock after one year.

Table 4 presents the results of simulating the expected stock S_T value after one year i.e. at 31 December of each year from the present time with its 95% confidence limits. The parameters drift and volatility as the value S_0 of the stock value are known at the beginning of each year.

Table 4 clearly shows us that in the most of cases the true value of the stock is in the 95% confidence interval of the simulated value. In the periods 1998 and 2001 the real stock value is out of the confidence interval. In this case the confidence interval are relatively large.

5   Conclusion

As stated in the introduction, modelling of stock prices is about modelling new information about stocks. In this dissertation modelling has been realized through the quantification of a stochastic part in the general expression of the model.
The results of the simulation performed in this work do not always match those of the theoretical model even if the assumptions on which the model is based meet the financial market rules.

The analysis of simulation results provides two important things:
1. In one-day simulation almost 80% of the real price values of the stock are not in the confidence interval of the simulated value and the confidence intervals are small.
2. In the long term simulation almost all real price values are in the confidence interval of the simulated stock price. The confidence intervals are large.

The geometric Brownian motion shows less accuracy in short time modelling. This is due to the assumption that the parameters drift and volatility are constant. However its accuracy may be improved by modelling the parameters as stochastic function of time and other macroeconomic factors, which may influence the course the stock price

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APPENDIX

Appendix 1: Efficient Market
An 'efficient' market is defined as a market where there are large numbers of rational, profit-maximizers actively competing, with each trying to predict future market values of individual securities, and where important current information is almost freely available to all participants. In an efficient market, competition among the many intelligent participants leads to a situation where, at any point in time, actual prices of individual securities already reflect the effects of information based both on events that have already occurred and on events which, as of now, the market expects to take place in the future. In other words, in an efficient market at any point in time the actual price of a security will be a good estimate of its intrinsic value.

Appendix 2: Markov Property
The Markov property states that the expected value of a random variable S_i conditional on all past events only depends on the immediate previous value of the random variable S_{i-1}. This property can be expressed as:
$E(S_t | I_{t-1}) = E(S_t | S_{t-1})$, where I_{i-1} is the information available up to t_{i-1}

Appendix 3: Ito's Lemma
Ito's lemma is the most important result about the manipulation of random variables that is required to Ito's processes. It is to function of random variable what Taylor 's theorem to function to deterministic variable.

Consider a function of two variables G (x, t). Its Taylor expansion to the second order is:

$$dG \approx \frac{\partial G}{\partial X} dX + \frac{\partial G}{\partial t} dt + \frac{1}{2} \left(\frac{\partial^2 G}{\partial X^2} (dX)^2 + \frac{\partial^2 G}{\partial t^2} (dt)^2 \right)$$

Where dX= adt+bdB

$$dG = \frac{\partial G}{\partial X} (adt + bdB) + \frac{\partial G}{\partial t} dt + \frac{1}{2} \left(\frac{\partial^2 G}{\partial X^2} b^2 dt \right)$$

$$dG = \left(\frac{\partial G}{\partial X} a + \frac{\partial G}{\partial t} + \frac{1}{2} \frac{\partial^2 G}{\partial X^2} b^2 \right) dt + \frac{\partial G}{\partial X} dB$$

This is the Ito's Lemma that can be applied to any stochastic variable that follows an Ito process.

Appendix 4: Price adjustment Rule
If there is a market surplus or glut (excess supply), prices fall, ending the glut, while a shortage (excess demand) causes price rises. However, instead of price adjustment --or, more likely, simultaneously with price adjustment -- quantities may adjust: a market surplus leads to a cut-back in the quantity supplied, while a shortage causes a cut-back in the quantity demanded. The "short side" of the market dominates, with limited quantity demanded constraining supply in the first case and limited quantity supplied constraining demand in the second."

Matlab codes:

```
%Random walk:

%walk(N) displays outcomes of the N-step random walk
function [S] = walk(N)
clf;                            % clear the entire figure

t = (0:1:N)';                   % t is the column vector [0 1 2 3 ... N]

S = [0; cumsum(2*(rand(N,1)>0.5)-1)];  % S is the running sum of +/-1 variables
M = max(abs(S))+1;
plot(t,S);          % plot the path
axis([0 N -M M])
title([int2str(N) '-step random walk'])
%Brownian motion:
%function Brownian motion displays outcomes of the N-steps Brownian motion with
drift
function [B] = Brownian(N, b, sigma ,T)
clf;                            % clear the entire figure
t = (0:1:N)'/N;                 % t is the column vector [0 1/N 2/N ... 1]
W = [0; cumsum (randn(N,1))]/sqrt(N); % S is running sum of N(0,1/N) variables
t = t*T;
%W = W*sqrt(T);
B = b*t + sigma*W;
plot( t, B );                   % plot the path
hold on
plot(t, b*t,':');
axis([0 T min(-sigma,(b-2*sigma)*T) max (sigma,(b+2*sigma)*T)])
title([int2str(N) '-step version of Brownian motion and its mean'])
xlabel(['Drift ' num2str(b) ', diffusion coefficient ' num2str(sigma)])
hold off% GBM(N, mu ,sigma, T) simulates a geometric Brownian motion
% on [0,T] using N normally distributed steps and parameters mu and sigma.

%Geometric Brownian:

% this displays outcomes of the N-steps of Geometric Brownian motion
function [X] = GBM(N ,mu, sigma, T)
t = (0:1:N)'/N;                         % t is the column vector [0 1/N 2/N ... 1]
W = [0; cumsum(randn(N,1))]/sqrt(N); % W is running sum of N(0,1/N) variables
t = t*T;
B = W*sqrt(T);
Y = (mu-(sigma^2)/2)*t + sigma * B;
X = exp(Y);
hold on;
plot(t,X);          % plot the path
plot(t,exp(mu*t),'r')
axis([0 T 0 max(1,exp((mu-(sigma^2)/2)*T+2*sigma))])
title([int2str(N) '-step geometric Brownian motion '])
xlabel(['mu = ' num2str(mu) ' and sigma= ' num2str(sigma)])
```

hold off;

%Simulation of stock price paths
function [S, t]=stockPrice(s0, mu, sig, n); %displays n random paths of future price up t
% s0 is the initial price,
%mu the annual growth rate,
%sig the volatility
% This simulation is going in steps of 1 day at a time and n is the number
% of simulated paths of the stock price

x=load('hp05.m');   %load the actual stock price
h=252;
dt=1/h;
S(1)=s0;
t(1)=0;

for i=1:1000
for j=1:1:n   % The following value of the stock is evaluated
    S(j+1)=S(j)*exp((mu-sig^2/2)*dt+sig*sqrt(dt)*randn);
    t(j+1)=j;
    end;
    plot(t,S);
    hold on;
end
plot(x,'-r');
xlabel('day')
ylabel('price')
title(' HP stock price in 2005')
hold off;

S-plus codes: Estimated future price using S-plus
N<-10000
S0= -6.9
mu=0.0927
sig=0.30
TT=1
muT=log(S0)+(mu-sig^2/2)*TT
sigT=sig*sqrt(TT)
SAMPLE<-rlnorm(N,meanlog=muT,sdlog=sigT)
Summary (SAMPLE);