Determining Ruin Probabilities with Monte Carlo Simulation

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June 15, 2009

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Acknowledgements

This paper was written as part of the master Business Mathematics and Informatics at the Vrije University, Amsterdam. The main goal of this assignment is to write a clear and concise paper on a scientific problem, with a knowledgeable manager as target audience.

I want to thank dr. Erik Winands for helping me define a good subject for this paper and for his comments and advice during the writing-process.

Abdil Ciblak
Amsterdam, June 11, 2009

Management samenvatting

Dit werkstuk behandelt de kans dat de uitgaven van een verzekeringsmaatschappij groter zijn dan de inkomsten. Veel gebruikte termen in levensverzekering en oplossingsmethoden voor het schatten van inkomsten en uitgaven van een verzekeringsmaatschappij worden in de inleiding behandeld.

Het eerste gedeelte van het werkstuk richt zich op verscheidene analytische modellen die gebruikt worden voor het modelleren van de risico's van verzekeringsmaatschappijen, zogenaamde risico modellen. Het meest bestudeerde risico model (de standaard risico model) en een risicomodel gebaseerd op polissen met een exponentiële levensduur worden in detail behandeld. Afhankelijk van het risicomodel dat gebruikt wordt, kunnen analytische oplossingstechnieken worden toegepast om de exacte kans op ruin te berekenen. Het komt regelmatig voor dat voor een risico model alleen een benadering van de ruin kans kan worden gegeven met behulp van analytische methoden. Een andere optie voor het schatten van de ruin kans is het toepassen van simulatie. Een hoofdstuk is toegewijd aan processen dat gebruikt worden voor het simuleren van de claim aankomsten.

Het tweede gedeelte van het werkstuk behandelt een simulatie studie van enkele risico modellen. De simulatie studie behandelt simulatie algoritmes die gebruikt worden voor het simuleren van risicomodellen. Microsoft Excel is gebruikt om de verschillende risicomodellen te simuleren.

De uitkomsten van de simulatiemodellen zijn vergeleken met exacte oplossingen die voortvloeien uit analytische methoden. Op deze manier hebben we de simulatie modellen gevalideerd. De simulatie studie heeft aangetoond dat risicomodellen met een gamma verdeelde levensduur van polissen en normaal verdeelde levensduur van polissen niet dezelfde ruin kans opleveren als de standaard risicomodel.

Executive summary

This paper focuses on the estimation of the probability that the expenses of an insurance company exceed the income. The paper starts with the introduction of some definitions used in life insurance and solution methods for estimating the income and costs of an insurance company.

The first part of the paper concentrates on several analytical models used for modelling the risk of an insurance company, risk models. The most studied risk model (standard risk model) and a risk model based on policies with an exponential distributed lifetime are discussed in detail. Depending on the risk model, analytical solution techniques can be used to calculate the exact ruin probability. There are also a lot of risk models for which only an approximation of the ruin probability can be given. Next to analytical solution techniques simulation can be considered to determine the ruin probability. We have dedicated a section to some processes that are used to model the arrival of claims.

The second part of the paper consists of a simulation study on some risk models. The simulation study examines simulation algorithms to perform simulations of risk models. A Microsoft Excel environment is used to carry out simulations of different risk models.

For the validation of our simulation models the outcomes were compared with the solutions based on analytical methods. The simulation study showed that risk models with gamma distributed policy lifetimes and normal distributed policy lifetimes is not identical to the ruin probability of the standard risk model.

Contents

1. Introduction

Losing someone close is one of the toughest things we will all have to deal with. If that person has not properly planned ahead to cover the expenses they have left behind then they leave a tremendous burden on their loved ones. The last thing someone wants to be remembered for is not properly planning ahead. Life insurance is a cost effective way to protect the ones you care about from having to clean up a financial mess after you have passed on.
A life insurance policy is a contract in which an insurance company agrees to pay a sum of money to a designated beneficiary upon the death or other event, such as terminal illness or critical illness of the policyholder. In exchange, the policyholder pays a stipulated amount called a premium at regular intervals [12]. As with most insurance policies, life insurance is a contract between the insurer and the policyholder whereby a benefit is paid to the designated beneficiaries if an insured event occurs which is covered by the policy.

Like any other company, an insurer has to show that its capital and expected income also referred to as assets exceed its costs also referred to as liabilities to be solvent. In the insurance industry, however, assets and liabilities are not known entities. They depend on how many policies result in claims, inflation from now until the claim, investment returns during that period, and so on.
So the valuation of an insurer involves a set of projections, looking at what is expected to happen, and thus coming up with the best estimate for assets and liabilities, and therefore for the company's level of solvency.

Different approaches can be taken to estimate the assets and liabilities of an insurance company. We discern two different approaches.

Deterministic approach

The deterministic approach is an approach where no randomness is involved in the estimation of values. The simplest way of estimating the assets and liabilities of an insurance company is to look at best estimates [12].
The projections in financial analysis usually use the most likely rate of claim, the most likely investment return, the most likely rate of inflation, and so on. The result provides a point estimate, the best single estimate of what the company's current solvency position is or multiple points of estimate. The downside of the deterministic approach is that it does not consider randomness and that there is a whole range of possible outcomes. Some of these outcomes are more probable and some are less.

Stochastic modeling

A stochastic model is the counterpart of the deterministic model and involves probability or randomness. A stochastic model is a tool for estimating probability distributions of potential outcomes by allowing for random variation in one or more inputs over time [10]. The random variation is usually based on fluctuations observed in historical data for a selected period using standard time-series techniques.

A stochastic model for an insurance company would be to set up a simulation model or an analytical model which looks at a single policy, an entire portfolio or an entire company. But rather than setting investment returns according to their most likely estimate, for example, the model uses random variations to look at what investment conditions might be like.
Based on a set of random outcomes, the outcome is noted. Then this is done again with a new set of random variables. In fact, this process is repeated thousands of times. At the end, a distribution of

outcomes is available which shows not only what the most likely estimate is, but what ranges are reasonable too.

A deterministic simulation, with varying scenarios for future investment return, does not provide a good way of estimating the cost of providing this guarantee. This is because it does not allow for the volatility of investment returns in each future time period or the probability that an extreme event in a particular time period leads to an investment return less than the guarantee. Stochastic modeling builds volatility and randomness into the simulation model and therefore provides a better representation of real life from more angles.

Stochastic models help to assess the interactions between variables, and are useful tools to numerically evaluate quantities, as they are usually implemented using Monte Carlo simulation techniques (see Section 1.2). While there is an advantage here, in estimating quantities that would otherwise be difficult to obtain using analytical methods, a disadvantage is that such methods are limited by computing resources as well as simulation error.

1.1 Simulation

In finance, computer simulations are often used for scenario planning. The arrival of claims, for example, is computed from not always known inputs. Monte Carlo simulation is often used to calculate the value of companies or to evaluate financial derivatives.

1.2 Monte Carlo Simulation

Monte Carlo Simulation (MCS) is a technique that involves using random numbers and probability to solve problems. By combining the distributions and randomly selecting values from them, it recalculates the simulated model many times and brings out the probability of the output. Some basic characteristics for MCS are defined [2]:

- The use of random numbers characterizes MCS as a stochastic method. The random numbers have to be independent; no correlation should exist between them.
- MCS allows several inputs to be used at the same time to create the probability distribution of one or more outputs.
- Different types of probability distributions can be assigned to the inputs of the model. When the distribution is unknown, the one that represents the best fit to the data could be chosen.
- MCS generates a range of values as output and shows how likely the output value is to occur

The MCS is performed with Excel functions and an Excel add-in for simulation and optimization: Crystal Ball. With the help of Crystal Ball a deterministic spreadsheet can be turned into a probabilistic one. To use the Crystal Ball tool correctly we need to make sure that the probabilistic model structure reflects the underlying problem.

This paper describes a simulation approach for the estimation of the probability that the expenses of an insurance company exceed the income within a predefined time period. This probability, called the ruin probability, is discussed in Chapter 2. A standard model used for the determination of the ruin probability is described in Chapter 2. Next to that some variations on the standard model are discussed in Chapter 2. A simulation study is conducted on these models and results are discussed in Chapter 3. Based on the observations and analyses a conclusion is drawn in Chapter 4.

2. The ruin probability

In examining the nature of the risk associated with the solvency of an insurance company, it is often of interest to assess how the insurance company may be expected to perform over an extended period of time. One measure of risk is the probability that a ruin occurs. Ruin theory is concerned with the excess of the income over the expenses. This quantity, called the surplus, varies in time. A simple representation of the surplus is given by

surplus = *income* – *expenses*.

Ruin is said to occur if the surplus reaches a specified lower bound [4]. To be able to determine the surplus we first have to determine the income and the expenses. There are many variables and uncertainties involved with the income as well as the expenses of an insurance company. Risk models are used to model these uncertainties and variables to determine the surplus. This chapter describes some models used in risk theory for the determination of the surplus, mostly following to the work of [1], [4],[6] and [7].

2.1 Standard risk model

The most studied literature in risk process is the standard mathematical model for the insurance of risk [5][6]. The standard model assumes that the insurance company only receives income from premiums paid by the policyholder. The income consists of premiums received at a constant rate r per unit time t. Let's denote the total income for the insurance company by TI. The total income TI for the insurance company is

$$TI = rt. \qquad (2.1)$$

Next to the total income the surplus depends on the total expenses denoted by TE. The total expenses for the standard model are only based on the occurrences of claims. Every time a policy expires a claim occurs. The number of claims within a time interval (0,t] is described by a Poisson process N_t with intensity rate μ. The properties of the Poisson process are:

- The arrival time between two claims is independent and exponential distributed with expected value of 1/ μ.
- And the expiration of a policy leads to only one claim at a given time.

Based on the insurance type and contract agreement a claim has a size. The size of the claim, also referred to as claim severity, is the amount of money the insurance company needs to pay the beneficiary. In the standard model the claim severities X_k are random variables with mean value ϴ, independent of the arrival of the claims N_t. The total expenses for a time interval (0,t] are the aggregated claim severities within the defined time interval

$$TE = \sum_{i=1}^{N(t)} X_k \qquad (2.2)$$

Using the equation for the total income (2.1) and the equation for the total expenses (2.2) the surplus of the insurance company can be determined. Let's denote the surplus of the insurance company at time t by S_t. The surplus S_t of the insurance company at a time is the excess of the total income over the total expenses

$$S_t = rt - \sum_{i=1}^{N(t)} X_k \qquad (2.3)$$

It is reasonable to assume that the insurance company has extra capital that is not part of the income. This capital is the initial capital of the insurance company and can be used as buffer against fluctuation in claim occurrences and claim severities. Let's denote the initial capital by *U,* where $S_0=U$. The surplus $\{S_t\}_{t\geq 0}$ of the insurance company for the standard risk model with initial capital *U* is given by

$$S_t = U + rt - \sum_{k=1}^{N(t)} X_k \qquad (2.4)$$

Now we have the equation to determine the surplus of the insurance company at a time.

The sample path of the surplus *S* in time *t* is shown in Figure 2.1. This figure clearly illustrates how the surplus drops according to a claim severity every time a claim occurs.

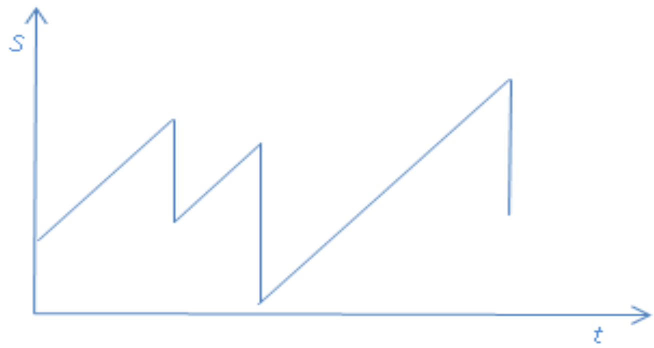

Figure 2.1: Sample path of the surplus process *S*

Ruin occurs if the surplus drops below a certain point. More specifically in this paper ruin is defined as the occurrence that the surplus of an insurance company has a negative value. Therefore, the probability of ruin is the probability that the surplus of an insurance company has a negative value within a predefined time horizon. According to this definition, the ruin probability ψ(U) for an insurance company with initial capital *U* is

$$\psi(U) = P(S_t<0 \ \forall \ t > 0 | \ S_0=U). \qquad (2.5)$$

We mentioned already that income generated by premiums and the expenses from claims are the variables that influence the ruin probability. The premium of a policy should be chosen such that they cover the expenses from claims. However a higher premium rate leads to more costs to the policyholder. Therefore, to somewhat "guarantee" the survival of the insurance company the premium rate should meet the expected expenses without burdening the policyholder with too high of a premium rate. The expected expenses of the insurance company are the expected number of arrivals multiplied with the expected claim severity: $\mu \ \Theta$. The premium rate *r* should be chosen such that $r \geq \mu \ \Theta$. More specifically $r=(1+\varphi) \ \mu \ \Theta$ and $\varphi > 0$ is called the relative safety loading [1],[4].

The standard risk model with constant interest rate

It is possible to extend the standard risk model with interest earned on the surplus. It is likely that the insurance company earns interest over the capital. Let $v_1, v_2, \ldots, v_T$ be a sequence of random variables representing the rates of interest earned by the insurer in successive years. Dickson et al.[3] present the standard model with the use of interest *v*. Adding this extension to our formula for the surplus of an insurance company results in

$$S_t = S_{(t-1)}\,(1+v_t) + rt - \sum_{k=1}^{N(t)} X_k$$

This expression is based on a constant value of the interest.

2.2 Analytical solutions of the standard risk model

We have discussed the standard risk model where the Poisson process is used to describe the arrival of the claims. In the standard model the claim severities X_k are random variables with mean value Θ, independent of the arrival of the claims N_t. Depending on the probability distribution of the claim severities exact solutions for the ruin probability can be calculated analytically. There are two types of claim severity distributions known for which the ruin probability can be calculated easily [13]. These are the exponential distributions and mixtures and combinations of these distributions, as well as distributions with only a finite number of values. For other distributions good approximations of the ruin probability can be found. An extended overview of exact solutions and methods for a good approximation of the ruin probability is given by [4].

Exact solution to the standard risk model

In this section we will present two equations to calculate the exact ruin probability for the standard risk model. Both of these equations are only valid if the time horizon is infinite. The first equation we present to calculate the ruin probability of the standard risk model assumes that there is no initial capital, i.e. when U=0, the ruin probability of the standard risk model can be calculated by [4]

$$\psi(U)=\frac{1}{1+\varphi}, \qquad (2.6)$$

The equation is valid for the standard risk model regardless of the claim arrival rate μ or the claim severity distribution. The ruin probability only depends on the safety loading φ.

The second equation we present gives an exact solution for the ruin probability of the standard risk model with exponentially distributed claim severities. This equation holds for all values of the initial capital and like Equation (2.6) does not depend on the claim arrival rate .

$$\psi(U) = \frac{1}{1+\varphi} e^{\left(-\theta\left(1-\frac{1}{1+\varphi}\right)U\right)}. \qquad (2.7)$$

2.3 Risk models based on policies with lifetimes

The standard model described in Section 2.1 makes the assumption that the time between the arrivals of two consecutive claims is exponential distributed. In this section we consider risk models where a claim occurs when the lifetime of a policy expires. The risk models we discuss in this section do also take the arrival of policies into consideration.
The risk models we discuss in this section are based on the assumption that policies arrive independently and at random point in time. This corresponds with the Poisson process. The arrival rate of the policies is described with the parameter λ. The time between the arrival of two customers is exponential distributed with expected value 1/λ and every customer purchases one policy. The second assumption we make for these risk models is that every active policy has a certain lifetime. The length of the lifetimes of these policies is modeled according to some probability distribution. The insurance company will receive premium as long as the policy is active. When the lifetime of a policy expires not only a claim occurs, the insurance company will also miss further premium income for the expired policy.

The first risk model we discuss is a risk model based on exponential lifetime of the policies. The reason for this is that the exponential distribution is both relatively easy to work with and is often a good approximation to the actual distribution. The property of the exponential distribution that makes it easy to analyze is that it does not deteriorate with time. By this we mean that if the lifetime of a policy is exponentially distributed, then a policy that has been active for any number of years is as good as a new arrived policy in regards to the amount of time remaining until the policy expires. The exponential distribution is the only continuous distribution which possesses this property [10].

A random variable is said to be without memory, or memoryless, if

$P\{X>s+t|X>t\}=P\{X>s\}$ for all s,t>=0.

If we think of X as being the lifetime of some policy, then this equation states that the probability that the policy stays active for at least $s+t$ years given that it has been active for the past t years is the same as the initial probability that it stays active for at least s years. In other words, if the policy is active at time t, then the distribution of the remaining amount of time that it survives is the same as the original lifetime distribution; that is, the policy does not remember that it has already been active for a time t [13].

The surplus process of the risk model with exponential lifetimes of policies has the same structure as the standard risk model and is based on the total income *TI* and the total expenses *TE*.
The total income during a period is the number of active policies during the period multiplied by the rate. Let's denote the number of active policies at time t by I_t. Initially, I_0 policies are active, which were issued before or at time 0. We further assume that each active policy pays premiums continuously at constant rate r to the insurance company [1]. The instantaneous total rate at which premiums are paid to the company at time t is thus equal to rI_t. When a policy expires the insurance company pays out a claim, the mean size of which is denoted by ϴ, and the number of active policies decreases by one. The rate of the income will fluctuate every time a policy arrives or a claim occurs.

Let's denote the time an event takes place by T_z. Every time an event takes place: a policy has arrived or a claim has occurred. If a policy arrives, the number of active policies I increases. If a claim occurs, the number of active policies decreases with one and the surplus decreases with the amount of a claim. The income between two events is the number of active policies in the time interval multiplied with the premium rate r during time $(T_{z-1}, T_z]$. The income between two events denoted by ΔTI can be calculated using

$$\Delta TI = IT_{z-1} * (T_z - T_{z-1}] r \qquad (2.8)$$

The surplus at time of event S_{Tz} is given by

$$S_{Tz} = S_{Tz-1} + I_{Tz-1} * (T_z - T_{z-1})\ r - \delta X \qquad (2.9)$$

where, δ =1 if a claim occurs at the time interval $(T_{z-1},T_z]$ and δ =0 if there is no occurrence of a claim at the interval $(T_{z-1},T_z]$. Note that $S_{T0} = U$.

Exact solution to the risk model with exponential distributed lifetimes of policies

Adan et al.[1] present a formula to determine the exact ruin probability in case the arrival of policies is Poisson distributed and they have an exponential distributed lifetime and the claim size distribution is exponential. In this paper we restrict by notifying the reader with this equation only, for the mathematical foundation of this equation the reader is referred to Adan et al. [1]. According to Adan et al. [1] the ruin probability for an insurance company with initial capital *U* and initial active policies I_0 is defined by

$$\psi(U, I_0) = \frac{1}{1+\varphi} e^{\left(-\theta\left(1-\frac{1}{1+\varphi}\right)U\right)} \qquad (2.10)$$

Note that the ruin probability is independent of the number of initially active policies, the arrival of the policies nor the lifetime of the policies.

It is possible to model the lifetime of a policy using other probability distributions. Note that this probability distribution may not be memoryless, and the residual lifetime of initially active policies will depend on the time they have already been active.

2.4 Risk processes and simulation

In this section simulation as a tool to estimate the ruin probability is discussed in more detail. There are risk models where calculation or an analytical approach results in no exclusive solution. In those situations simulation can be a valuable option. Simulation is a good way to present different scenarios for risk models. This way the impact of the uncertainties involved with risk theory can be shown. Setting up a correctly working simulation model can be a time consuming and challenging thing to do. Often good knowledge of statistics is required to draw a reliable conclusion from a simulation model that is based on stochastic variables.

Advantages of simulation of risk models

- Simulation models are useful to make estimations of ruin probability in situations where analytical models do not lead to an exclusive solution
- Compared to analytical models simulation models are often more convincing and easier to understand for management

Disadvantages of simulation of risk models

- The calculation time of simulations increase when a more accurate and reliable estimation of the ruin probability is required [3].
    - The accuracy of the estimation of the ruin probability is the relative difference between the estimated ruin probability and the true ruin probability.

    - The reliability of the estimation is the risk one wants to take that an estimation of the ruin probability exceeds the accepted error margin.
- Mismodeling. It is common to make mistakes in setting up a simulation model. Like any model verification and validation of the simulation model is necessary. Cao et al. [8] discuss one of the most common mistakes made in spreadsheet simulation applications.

Key factor for simulation of risk processes for an insurance company is simulating the arrival of claims. The generation of aggregate claims is vital for calculation of the amount of loss that may occur. Claims of random size X_k arrive at random times T_i. The number of claims up to time t is described by the stochastic process N_t.

2.4.1 Homogenous Poisson Process

Homogenous Poisson Process is one of the most common claim arrival point processes which have stationary and independent increments and Poisson distributed number of claims in a given time interval [7]. This process is normally appropriate for life insurance modeling. The choice of a homogenous Poisson process implies that the size of the portfolio cannot increase or decrease.
Formally, a continuous-time stochastic process $\{N_t : t \geq 0\}$ is a (homogenous) Poisson process with intensity (or rate) l > 0 which have the following properties

- N_t is a point process,
- the times between two arrivals W_i are independent and identically distributed and follow an exponential distribution with intensity λ, i.e. with mean 1/λ.

The expected value $E(N_t)=\lambda t$ for the homogenous Poisson process. Therefore, it is natural to define the income from premium in this case as $r(t)=rt$, where $r=(1+\varphi)\Theta\lambda$, $\Theta=EX_k$ and $\varphi>0$ is the relative safety loading. The time of the arrival of a (claim) process is generated with the use of an algorithm. This algorithm is described in Burnecki et al. [7].

Algorithm for the Homogenous Poisson Process

Step 1: set $T_0 = 0$

Step 2: for i= 1,2,…,n do
Step 2a: generate an Exponential random variable E with intensity λ
step 2b: set $T_i = T_{i-1} + E$

This algorithm can be used to generate the time of the arrival of (claim) process. When the arrival times of the claims are generated the surplus process at the time of arrival can be determined using the corresponding risk process formula.

2.4.2 Non-homogenous Poisson Process

The non-homogenous Poisson process is a Poisson process with a variable intensity defined by the deterministic intensity (rate) function λ(t). The characteristics of non-homogenous Poisson process is that the increment is not necessarily stationary and when λ(t) takes the constant value l , the non-homogenous Poisson process reduces to the homogenous Poisson process. The non-homogenous Poisson process can be used for example when a claim occurrence depends on seasonal factors.

2.4.3 Mixed Poisson Process

Often insurance companies split their portfolio into different risk associated groups. We assume in this situation that the claims come from a heterogeneous group of clients, each one of them generating claims according to Poisson distribution with the intensity varying from one group to another. The homogenous Poisson process and the non-homogenous Poisson process do not take the variability in intensity of the heterogeneous groups into account. The mixed Poisson process can be used to model the arrival of claims in this situation. The distribution of the point process is given by a mixture of Poisson process in the mixed Poisson process [9].

2.4.4 Cox Process

The Cox process provides flexibility by letting the intensity not only depend on time but also allowing it to be a stochastic process. The Cox process is a two-step process which contains an intensity process Λ(t) that is used to generate another process N_t is a Poisson process conditional on Λ (t) which itself is a stochastic process. The simulation algorithm of the Cox process can be explained that a non-negative stochastic process Λ (t) is generated and, conditioned upon its realization, N_t as a non-homogenous Poisson process with that realization as its intensity is constructed [7].

2.4.5 The renewal Process

A renewal process is a stochastic model for events that occur randomly in time (generically called renewals or arrivals). The basic mathematical assumption is that the times between the successive arrivals are independent and identically distributed. In the renewal process we assume that there can be different distributions on the sequence of inter-arrival times $\{W_1, W_2, \ldots\}$ of the claim arrival point process N_t . Note that the homogenous Poisson process is a renewal process with exponentially distributed inter-arrival times.

3. Simulation of the risk models

In this chapter a simulation study is performed to compare the ruin probability for the standard risk model with the ruin probability for other risk models. The approach we have taken to conduct the simulation study is discussed in Section 3.1. Parameters influencing the accuracy and the reliability of the simulation are also discussed. A simulation model representing the standard risk model is set up in Section 3.2. The ruin probability for the standard risk model is estimated under different conditions. The simulation model and the results for the risk model with exponential lifetimes of policies are discussed in Section 3.3. A similar simulation analysis is performed in Section 3.4 for risk models with policies having a lifetime according to different probability distributions. The results for the different simulation models are compared and a conclusion is drawn.

3.1 Approach for simulation study

The direct cause for our simulation study is a paper by Adan et al.[1]. Adan et al.[1] discuss in their paper the insurance risk with variable number of policies. They establish a result that, if the lifetimes of policies are independent and identically distributed (i.i.d.) exponential random variables with rate μ, then the ruin probability is identical to the one in the standard risk (compound Poisson) model where the capital increases at constant rate *r* and claims occur according to a Poisson process with rate μ. The objective of the simulation study is to estimate the ruin probabilities for the standard risk model and for the risk model with varying number of policies with exponentially distributed lifetimes. Next to that the simulation study is used to perform a sensitivity analysis on the results for the risk model with varying number of policies with exponentially distributed lifetimes. Finally, some other risk models having policies with lifetimes with different probability distributions are simulated and compared to the policies with exponentially distributed lifetimes.

In this section a general approach is described for setting up a simulation model for risk models. A subsection discusses the reliability and accuracy of the simulation model.

The first three steps of our approach describes the realization of the surplus process, this corresponds with one simulation run. One run of the simulation enables us to determine whether a ruin occurs within a time horizon or not.

Step 1: Initial situation

The first step of our approach is to define the initial situation and the initial values of the simulation. The input for the simulation model is defined in the initialization phase. The input consists of the variables that are used to estimate the ruin probability.

Step 2: Determine the events of the simulation

The second step of our approach is to determine the events in the simulation. The events of our simulation model are the events that influence the surplus process. We consider three possible events for our simulation model: the arrival of a policy, the occurrence of a claim and the claim severity. The type of event for our simulation model will depend on the risk model that is simulated.

Step 3: Set up a simulation algorithm

The simulation algorithm is an algorithm used to generate the time an event takes place. Using a simulation algorithm the time of the arrival of the next policy or the occurrence of the next claim can be generated.
Every time an event takes place the surplus process is calculated. The surplus of an insurance company increases when a policy arrives and will decrease when a claim arrives. The surplus is calculated until the time is equal to the time horizon: $t=T$.

Step 4: Estimate ruin probability

For the estimation of the probability of ruin we simulate a large number of realizations of the model. We count the number of realizations that result in ruin. Let's denote the number of realizations that we simulate by m. Let's denote the number of occurrences of ruin within the specified time horizon by l. Our estimation of the ruin probability within a time horizon T is $\psi(T) = l/m$. This procedure of estimating the probability of ruin corresponds to the procedure described by Dickson et al.[3]. The number of realizations to choose depends on the level of accuracy and reliability of the result that is aimed for.

Reliability and the number of simulation runs

The simulation gives only an estimation of the ruin probability for a risk model. It is important to assess how valid the estimation from our simulation is. The central limit theorem states that the sampling distribution of any estimated statistic will be normal distributed, if the sample size is large enough. Two criteria are used to determine the number of realizations for the simulation to choose: accuracy and reliability [16].

The accuracy of the estimation of the ruin probability is the relative difference between the estimated ruin probability $\tilde{\Psi}$ and the true ruin probability ψ. This difference is also known as the error margin denoted by e.

The reliability of the estimation is the risk one wants to take that an estimation of the ruin probability exceeds the accepted error margin e. An indication of the reliability of the estimation is given by a confidence interval. For example a 90% confidence interval means that if many simulation runs are conducted and the confidence interval computed, in the long run about 90% of these intervals would contain the true ruin probability [16].

The relationship between the accuracy and the reliability of the estimate is used to derive a formula for the number of simulation runs to perform [16]. We are interested in the probability of ruin, this corresponds to the ratio ruin to non-ruin.
The sample size formula for categorical data discussed in Bartlett et al. [16] is used to estimate the proportion of two different ratios. This formula uses the estimation of the variance. The variance is estimated from the probability for an element to be in one category and the probability to be in the other category. In our model the two categories are ruin and non-ruin, so we use the probability of ruin ψ and the probability of non ruin $(1-\psi)$ to estimate the variance: $\psi(1-\psi)$. Next to the estimation of the variance an acceptable margin of error for the probability being estimated (e) and a reliability using a t-value (t) are used for the determination of the number of realizations to use. To estimate the proportion of ruin we use a 95% confidence interval for the reliability of the estimation. This corresponds with a t-value of 1.96 (see also Appendix A).

$$m = \frac{(t)^2 \times \psi(1-\psi)}{(e\psi)^2}$$

Thus an increase in confidence in our estimate is achieved at the expense of conducting more simulation runs.

The number of realizations *m* to have a 95% confidence (*t=1,96)* that the estimated ruin probability is within 5% (e = 0,05) of the true ruin probability are given in Table 3.1.

$\tilde{\psi}$	0,80	0,75	0,50	0,40	0,30	0,20	0,10	0,05	0,03	0,01	0,005
m	385	513	1537	2306	3587	6148	13833	29203	~50000	~150000	~300000

Table 3.1: The number of realizations *m* to have a 95% confident estimate of the ruin probability with an accepted error margin of 5%.

The relative error margin for a given number of simulations *m* and a confidence level of 95% is shown in Table 3.2. Evidently, the relative error margin decreases if the number of simulations increases.

$\tilde{\psi}$	*m=1000*	*m=2000*	*m=5000*	*m=10000*
0,80	3%	2,2%	1,4%	1,0%
0,75	3,6%	2,5%	1,6%	1,1%
0,50	6,2%	4,4%	2,8%	2,0%
0,40	7,6%	5,4%	3,4%	2,4%
0,30	9,5%	6,7%	4,2%	3,0%
0,20	12,4%	8,8%	5,5%	3,9%
0,10	18,6%	13,1%	8,3%	5,9%
0,05	27%	19,1%	12,1%	8,5%
0,03	35%	25%	15,8%	11,1%
0,01	61,7%	43,6%	27,6%	19,5%
0,005	87,4%	61,8%	39,1%	27,6%

Table 3.2: The relative error margin for the ruin probabilities corresponding to a given number of realizations *m*

The objective of this simulation study is to compare as many results as possible with different risk models and different parameter values. Initially we choose to simulate 1000 realizations of the process, as we considered this number to be sufficiently large to give estimates of the ruin probability. This is the same number of realizations Dickson et al.[3] use for comparison purposes of ruin probabilities. In case the estimated ruin probability is very low larger number of realizations of the process are simulated to have a more accurate estimation.

3.2 Standard risk model

The most studied literature in risk process is the standard risk model described in Chapter 2.1. This is also referred to as the compound Poisson model [1]. The simulation model of the compound Poisson model is based on the formula of the standard risk model, as in Equation (2.4)

$$S_t = U + rt - \sum_{k=1}^{N(t)} X_k$$

The simulation of the compound Poisson model is performed using the general approach discussed in Section 3.1.

Step 1: Initial situation

In this step the initial capital U and the number of active policies at the beginning of the simulation I_0 are determined. We will perform a simulation for different values for the initial capital. There are no arrival of policies in the standard risk model. Therefore, we don't have to model the number of initially active policies. The premium rate r, the arrival rate of the claims μ and the claim severity mean ϴ are chosen such that r=(1+φ)μϴ .

Step 2: The events of the simulation

The only event in our simulation we have to take into consideration is the arrival of a claim. The time between the arrivals (inter-arrival time) of two claims is 1/ μ. Every time segment of 1/ μ a new event in the form of claim arrival occurs.

Step 3: Set up simulation algorithm

We have to set up an algorithm that generates the arrival of claim processes. The claim processes arrive according to a Poisson process. Therefore, the use of the algorithm described in Section 2.3 for the homogenous Poisson process is justified.

Step 1: set $T_0 = 0$

Step 2: for k= 1,2,…,n do
Step 2a: generate an exponential random variable E with intensity μ
step 2b: set $T_k = T_{k-1} + E$,
where T_k is the time claim k occurs with a random variable X with mean size ϴ .

This algorithm is used to generate the next arrival of a claim. Every time a claim occurs the surplus is calculated using Equation (2.4):

$$\begin{cases} S_{T_0} = U, \\ S_{T_k} = S_{T_{k-1}} + r(T_k - T_{k-1}) - X_k \end{cases}$$

Excel is used to generate the exponential random variables E and X. The following function is used to have an exponential random variable with expected mean value [15]

```
Exponential =-LN(RAND())*mean
```

3.2.1 Validation of our simulation model

Validation of the simulation model is used to make sure that the simulation model reflects the standard risk model and estimates the ruin probability within a pre-determined reliability and accuracy margin.

For the validation of the model a comparison is made with the outcome of our simulation model and the exact ruin probabilities for the standard risk model. In Section 2.1 we introduced an equation for the calculation of the ruin probability of the standard risk model with exponential claim size distribution (Equation 2.6 and 2.7) for the infinite time horizon.
Before we can compare the outcome of the simulation model and the exact ruin probabilities calculated by Equation (2.7) we need to determine the time horizon of our simulation. The simulation model is appropriate for the finite time horizon, however taking a large time horizon will approximate the infinite time case. In the next subsection the length of the time horizon is analyzed.

Time horizon and the length of a simulation run

One simulation run represents one realization of the surplus process for a length of time. In this paper we have presented exact solutions to some risk models for the infinite time horizon. In order to compare the results of our simulation model with the results of risk models with known exact solutions we have to simulate a realization for an infinite time horizon. Monte Carlo Simulation is useful for the finite time horizon [4], however the ruin probability for the finite time will converge to those calculated in infinite case as the time horizon is getting larger (a time horizon of T=20 years gives a similar ruin probability as the infinite time case [4]).
In this section we research the "time horizon" we have to take for our simulation model to approximate the ruin probability in infinite time. The time in our simulation model depends on the arrival of policies or the arrival of claims. So instead of simulating for a time horizon, we simulate up to a number of claim arrivals. The more arrivals of a claim we simulate the more our model will reflect the situation of the infinite time horizon. The ruin probability is calculated for different number of claim arrivals denoted by *K*. To have a reliable point estimate of the ruin probability we kept the number of simulation runs high (*m*=20.000). The results of our simulation are presented in Table 3.3.

Number of claims K	Average ruin probability (U=1)	Average ruin probability (U=10)	Average ruin probability (U=20)
K=20	0,5613	0,0275	0,00037
K=100	0,60	0,064	0,0060
K=200	0,6098	0,071	0,0069
K=500	0,611	0,075	0,0072
K=1000	0,608	0,076	0,0078
K=3000	0,610	0,077	0,0075
Exact	0,610	0,077	0,0076

Table 3.3: The number of claim arrivals and the corresponding average ruin probability for 10000 simulation runs and safety loading of φ =0.3.

From these results we see that the speed of convergence decreases as the initial capital U grows (i.e. as the ruin probability decreases). For our simulation model we will use K=3000. The results have shown that simulating this number of claim arrivals will approximate the infinite time horizon well, even when ruin probability is very small.

Comparison of results

We calculate the exact ruin probabilities for the standard risk model with exponential claim size distribution using Equation 2.7 and compare the simulation results with them. The results of the simulation and the exact solution based on Equation 2.7 are shown for a safety loading of in Table 3.4 φ=0.3 and a safety loading of φ=0.2 in Table 3.5. The results are compared under different values of the initial capital and different number of simulation runs.

U	*m=1000*	*m=5000*	Exact
0	0,778	0,77	0,769
1	0,606	0,612	0,61
2	0,474	0,485	0,485
5	0,265	0,245	0,243
7	0,148	0,152	0,153
10	0,74	0,083	0,077
15	0,027	0,023	0,024

20	0,0085	0,0079	0,0076

Table 3.4: The ruin probability of our simulation model and the exact ruin probability for exponential claim sizes when safety loading φ=0.3.

U	*m=1000*	*m=5000*	Exact
0	0,833	0,821	0,833
1	0,712	0,710	0,705
2	0,589	0,592	0,597
5	0,352	0,354	0,362
7	0,267	0,269	0,260
10	0,158	0,155	0,157
15	0,066	0,066	0,068
20	0,036	0,034	0,03

Table 3.5: The ruin probability of our simulation model and the exact ruin probability for exponential claim sizes when safety loading φ=0.2.

The results in Table 3.4 as well as in Table 3.5 show that our simulation model produce similar results to the exact solutions calculated with analytical methods. Note that an increase in the number of simulation runs leads to an increase in accuracy of our estimation.

The ruin probability of the standard risk model does not depend on the arrival rate of claims [4]. We have simulated the standard risk model for different values of the claim arrival rate μ. The results are shown in Table 3.6 for a safety loading of φ=0.3.

	U=0	*U*=1	*U*=2	*U*=5	*U*=7	*U*=10	*U*=15	*U*=20
μ=10	0,760	0,621	0,478	0,226	0,154	0,069	0,029	0,0065
μ=20	0,779	0,616	0,493	0,254	0,145	0,081	0,028	0,0072
μ=50	0,773	0,608	0,481	0,249	0,155	0,072	0,019	0,0085
μ=100	0,764	0,615	0,498	0,251	0,149	0,071	0,020	0,0071
Exact	0,769	0,610	0,485	0,243	0,153	0,077	0,024	0,0076

Table 3.6: The ruin probability of our simulation model for different claim arrival rates compared with the exact ruin probabilities

The results in Table 3.6 show that in our simulation model the arrival rate of claims do not influence the ruin probability. This corresponds to the analytical outcomes. From these results we conclude that the simulation model we have proposed is a good reflection of the standard risk model. The ruin probability estimations of our simulation model are within the accepted error margin. The proposed simulation model can also be used for the estimation of the ruin probabilities of the standard risk model with other claim size distributions. The same simulation algorithm can be used to generate the arrival times of claims. Every time a claim occurs a stochastic variable *F* is generated that represents the claim size according to a given claim size distribution.

In the next section we will perform a simulation study on risk models that consider income from premiums based on the number of active policies and the occurrences of claims on expiry of the lifetime for policies.

3.3 Risk model with exponential lifetimes for policies

The risk model with exponential lifetimes for policies are already discussed in Section 2.2. In this section we will set up a simulation model to estimate the ruin probability for this risk model.

A similar approach to the standard risk model is taken to set up a simulation model for the risk model with exponential lifetimes for policies.

Step 1: Initial situation

In this step the initial capital U and the number of active policies at the beginning of the simulation I_0 are determined. We will perform a simulation for different values for the initial capital. In this model we take the number of initially active policies into consideration. We will perform simulations with varying number of initially active policies.

Step 2: The events of the simulation

The policies in this model arrive according to a Poisson process with arrival rate λ. And the policies have an exponential distributed lifetime with rate μ and expected lifetime of 1/ μ [13]. We have to take three events into consideration; one event concerning the arrival of policies, one event concerning the occurrence of claims and the severity of a claim when a claim occurs. Every moment a policy arrives the number of policies will increase, therefore the income from premiums will increase. Every moment a claim arrives the capital of the company decreases with a claim size and the number of policies decrement by one.

Step 3: Set up simulation algorithm

We update the value of the surplus after every event (arrival of a policy or arrival of a claim). Therefore, we have to generate the arrival times of the policies and the time of occurrence of the claims. The simulation algorithm of the homogenous Poisson process can be used to generate the arrival of the policies. We have to adapt this algorithm to generate the time a claim occurs.

Step 1: set $T_0 = 0$

Step 2: for i= 1,2,…,m do
Step 2a: generate an exponential random variable E with intensity λ
step 2b: set $T_i = T_{i-1} + E$,
where T_i is the time policy i arrives.

Step 3: k= 1,2,…,n do
Step 3a: generate a random exponential variable F with mean value μ
Step 3b: set $T_k = T_i + F$
where T_k is the time a claim occurs for policy i, T_k is the lifetime of a policy added to the arrival time of the policy

Step 4: sort the arrival of events (policies and claims) in ascending order, this way all events are sorted in a chronological way and the surplus for every event can be calculated.

Let's denote the time an event takes place by T_z. Note that T_z is just the time of arrival of policy i, T_i , and the time an occurrence of claim k takes place, T_k , assorted.

Every time an event takes place determine the type of event: arrival of policy or arrival of claim. If a policy arrives, increase the number of active policies *I.* If a claim arrives, decrease the number of active policies with one. The income generated between two events is the number of active policies in the time interval multiplied with the premium rate *r* during time $(T_{z-1}, T_z]$. The income between two events denoted by ΔTI can be calculated using

$$\Delta TI = I_{T_{z-1}} * (T_z - T_{z-1}] r \qquad (3.1)$$

When a claim arrives the surplus process is decreased by the amount of the claim severity. A graphical representation of this simulation algorithm for the realization of the arrival of policies and occurrence of claims is given in the flowchart in Figure 3.1.

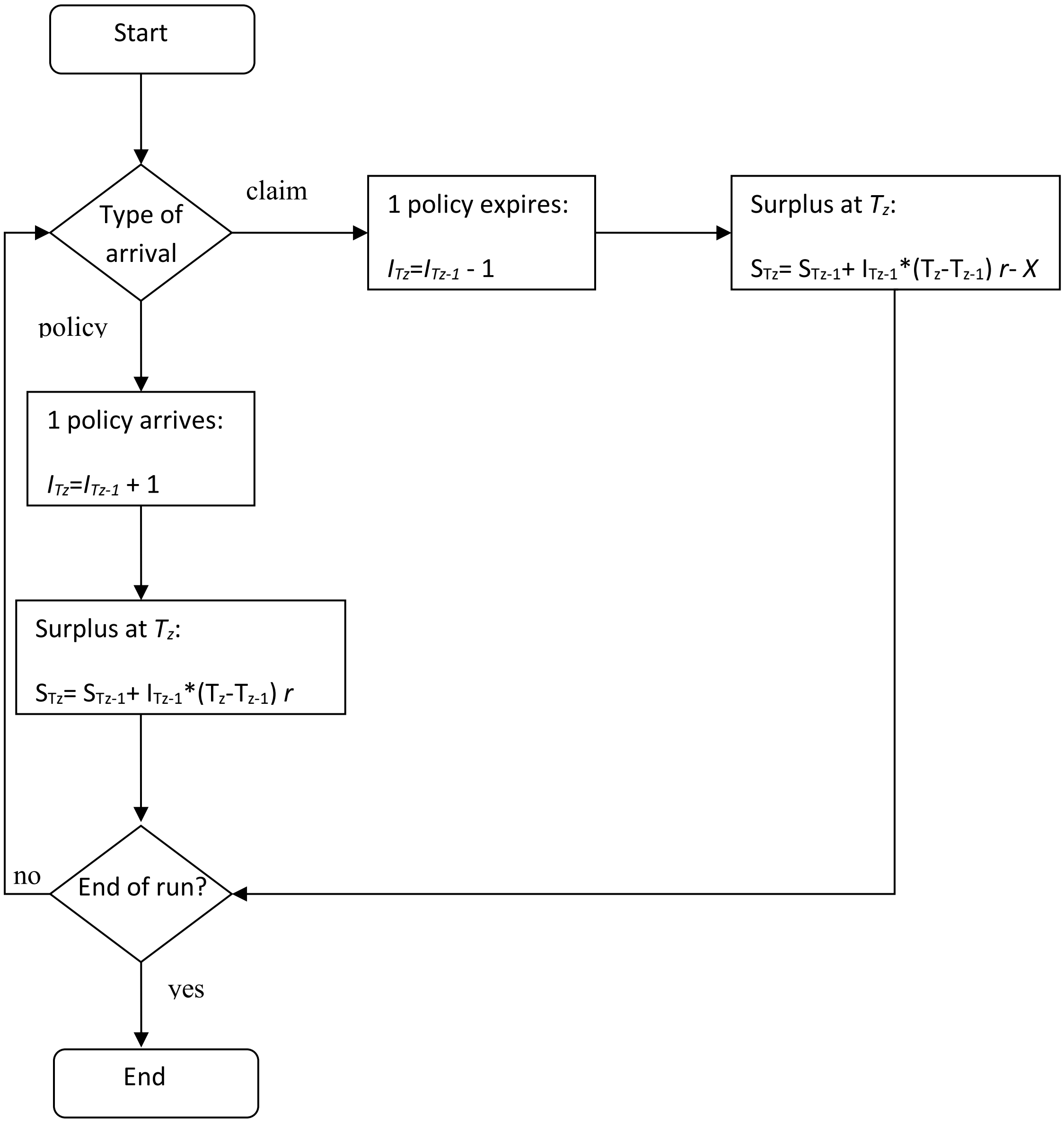

Figure 3.1: Flowchart of the simulation of the arrival of policies and occurrences of claims according to the risk model with exponentially distributed lifetimes for policies

This flowchart can be summarized with the formula for the determination of the surplus at a time of an event T_z (Equation (2.9)).

The realization of one simulation run is further illustrated in Figure 3.4, where the level of the surplus in time is shown. The influence of the arrival of policies and arrival of claims on the surplus level is made clear in Figure 3.2. Note that the surplus increases with a higher rate if an extra policy arrives (see (1) in the figure) and the rate is less when a claim occurs (see (2) in the figure).

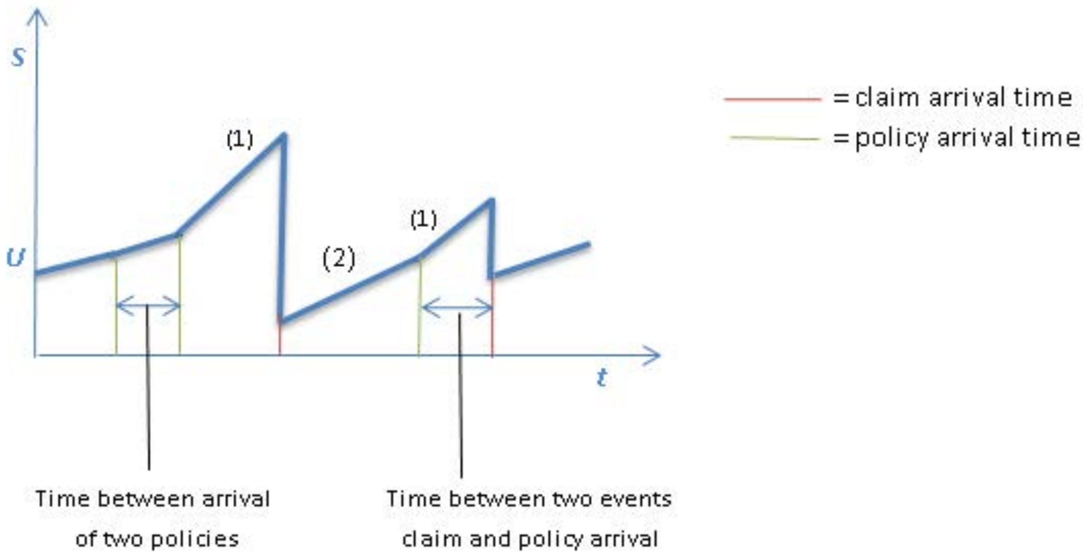

Figure 3.2: The realization of one simulation run, where the surplus *S* fluctuates according to the number of active policies *I* and the occurrences of claims

Validation of our simulation model

Validation of the simulation model is used to make sure that the simulation model reflects the standard risk model and estimates the ruin probability within a pre-determined reliability and accuracy margin.

For the validation of the model a comparison is made with the outcome of our simulation model and the exact ruin probabilities for the risk model with exponential lifetimes of policies. In Section 2.3 we introduced an equation for the calculation of the ruin probability of the risk model with exponentially distributed lifetimes of the policies and exponential claim size distribution (Equation 2.10). We calculate the exact ruin probabilities using Equation (2.10) and compared the simulation results with them. We have used the same set of initial capital and safety loading as for the standard risk model in Section 3.2. The results are shown in Table 3.7 for safety loading φ=0.3 and different values of the initial capital U and the initial number of active policies I_0. We have used the same set of initial capital U as in the standard risk model in Section 3.2. This enables us to compare the outcome of the risk model also with exponentially distributed lifetimes of policies with the standard risk model.

	I_0=0	I_0=10	I_0=20	I_0=50	I_0=100	Exact
U=0	0,774	0,773	0,761	0,758	0,77	0,769
U=1	0,63	0,628	0,61	0,595	0,6	0,61
U=2	0,481	0,480	0,477	0,467	0,466	0,485
U=5	0,256	0,255	0,249	0,251	0,234	0,243
U=7	0,148	0,151	0,156	0,147	0,148	0,153
U=10	0,074	0,069	0,066	0,083	0,069	0,077
U=15	0,027	0,026	0,028	0,023	0,023	0,024
U=20	0,0088	0,0081	0,007	0,0069	0,007	0,0076

Table 3.7: The ruin probability of our simulation model and the exact ruin probability for exponential claim sizes when safety loading φ=0.3 for m=1000.

Although there is some fluctuation in the outcome of the ruin probabilities compared to the exact solution, the results are within the relative accepted error margin of 5%. The results would be more accurate if a larger number of simulations was used.

Adan et al. [1] suggested that the ruin probability of the risk model with exponential policy lifetimes does not depend on the arrival of policies. We performed a second validation of our simulation model by comparing outcomes of the simulation model for different values of the arrival of policies λ. The ruin probability is estimated for different values of the initial capital with initial number of active policies I_0=0 and I_0=050. The results are shown in Table 3.8 and are within the accepted error margin of the exact values calculated by Equation (2.10).

		λ=10	λ=20	λ=40	λ=50	λ=100	Exact
I_0=0							
	U=0	0,771	0,774	0,759	0,767	0,785	0,769
	U=1	0,617	0,628	0,61	0,595	0,6	0,61
	U=2	0,481	0,480	0,477	0,467	0,466	0,485
	U=5	0,256	0,255	0,249	0,251	0,234	0,243
I_0=50							
	U=0	0,779	0,758	0,768	0,777	0,757	0,769
	U=1	0,607	0,598	0,618	0,621	0,603	0,61
	U=2	0,475	0,472	0,493	0,476	0,489	0,485
	U=5	0,248	0,253	0,259	0,240	0,235	0,243

Table 3.8: The ruin probability of our simulation model and the exact ruin probability for exponential claim sizes when safety loading φ=0.3 for m=1000.

From the results in this section we conclude that the simulation model we have presented for the risk model with exponentially distributed policy lifetimes is valid. For a given safety loading and initial capital the standard risk model produces similar results for the ruin probability as the risk model with exponentially distributed policy lifetimes. Also the number of initially active policies and the arrival rate of policies has no influence on the ruin probability of the insurance company when the lifetime of a policy is exponential distributed.

In the next section we will study the influence of the distribution of the lifetime of policies.

3.4 Risk models with lifetimes for policies

In the previous section we have discussed a simulation model for policies with an exponential lifetime. Due to its memoryless property the model with exponential lifetime for policies assumes that the

expected remaining lifetime of initially active policies is similar to the expected lifetime of a newly arrived policy regardless of the time it has already been active. The objective of the simulation study in this section is to examine whether the same conclusions can be drawn for risk models with different policy lifetime distributions as for the risk model with exponentially distributed policy lifetimes.
In this section two risk models with policy lifetimes are examined: a first risk model with normal distributed policy lifetimes and a second one with gamma distributed lifetimes. For both of these risk models we performed an analysis similar to the risk model with exponentially distributed policy lifetimes:

1. We examined the impact of the number of initially active policies on the ruin probability, and
2. the impact of the arrival rate of policies on the ruin probability

In the previous section we described a simulation algorithm to model the arrival of claims according to a homogenous Poisson process and the exponential lifetime of policies. We make use of the same simulation algorithm to model the arrival of policies. For the lifetime of policies a random variable G is generated with a mean from the corresponding distribution for the lifetime of a policy.

Unlike the exponential distribution these distributions assume that the expected lifetime of newly arrived policies is longer than the expected remaining lifetime of initially active policies. Therefore, we have to take the remaining lifetimes of the initially active policies into consideration in the simulation.
The following approach is taken to simulate the remaining lifetime of initially active policies is:

1. Generate a uniform (0,1) random number H
2. Generate a random variable G from the corresponding distribution with mean lifetime μ
3. expected remaining lifetime: $H\mu \leq \mu$

Since we do not know how long the policy is already active we multiply the generated random variable with a value between (0,1) to correct for this. Note that this approach is just an approximation where the remaining lifetime of an already active policy is a fraction of the lifetime of a newly arrived policy.

3.4.1 Risk model with normal distributed policy lifetimes

The first risk model we examine is the risk model with normal distributed policy lifetimes. To indicate that a real-valued random variable G is normally distributed with mean μ and variance $\sigma^2 \geq 0$, we write

$G \sim \text{normal}(\mu,\sigma^2)$

For our simulation we have set the mean lifetime to μ=20 with a standard deviation of 5. The Excel function we have used to generate a random normal variable with a mean and standard deviation is [15]

= Norminv (rand(), mean, standard deviation)

The influence of the arrival rate on the ruin probability is examined for this risk model. In the case of exponentially distributed policy lifetimes the arrival rate had no influence on the ruin probability. In Figure 3.3 the ruin probability for different values of the initial capital U and policy arrival rate λ are shown. The mean claim size for each case is Θ=1 and exponentially distributed and for the safety loading we have used φ=0.3. The number of initially active policies I_0=0.

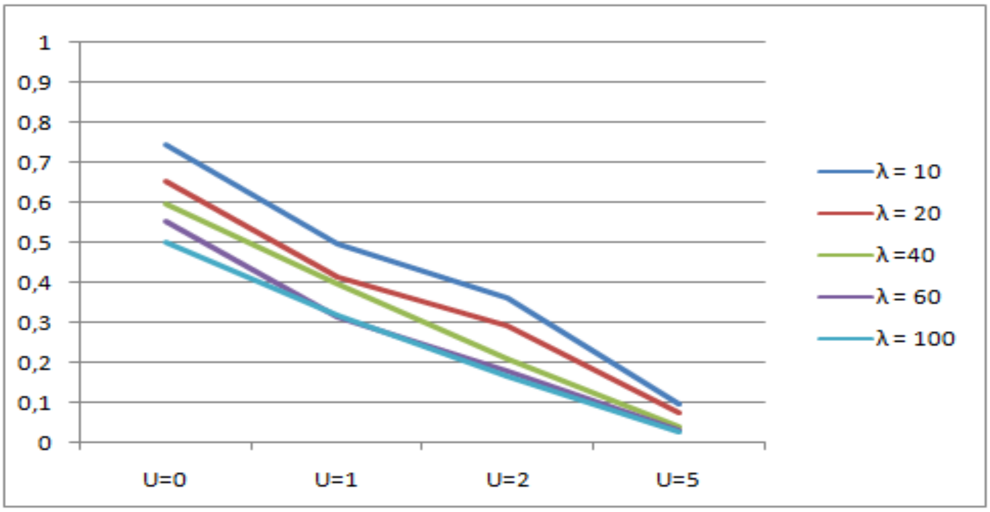

Figure 3.3: Ruin probability for the risk model with normal (20,5) distributed policy lifetimes for different values *λ* and *U*.

The results in Figure 3.3 tell us that the ruin probability decreases if the arrival rate increases. This is the case for all examined values of initial capital. Note that the differences are getting smaller when the arrival rate increases, e.g. the difference between the values for λ=60 and λ=100 are smaller than the differences between λ=40 and λ=60.
Next to the influence of the arrival rate on the ruin probability the influence of the number of initially active policies I_0 on the ruin probability is examined. In Figure 3.4 the ruin probability for different values of initially active policies corresponding to some initial capital are shown. The mean claim size is ϴ=1 and the safety loading φ=0.3. The arrival rate of policies is λ=20 for all cases.

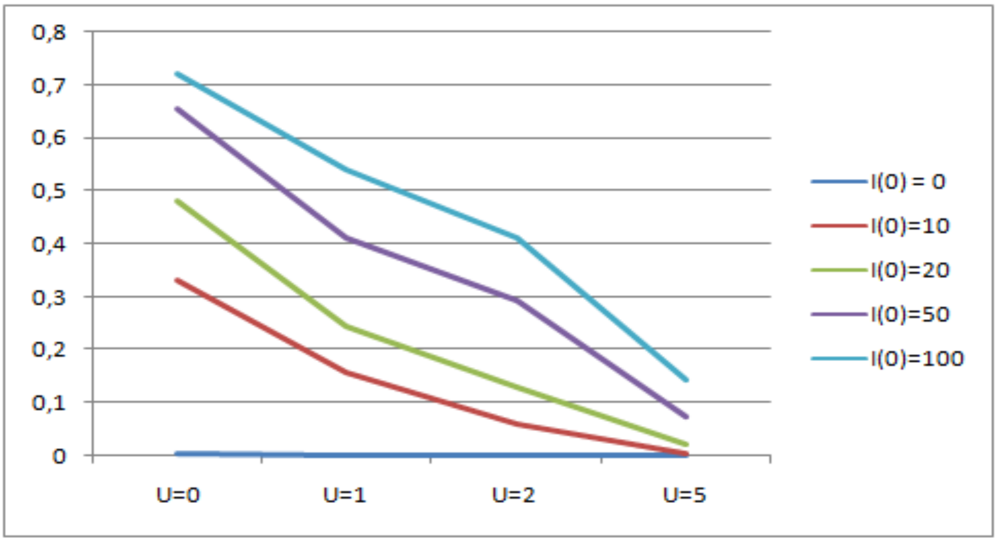

Figure 3.4: Ruin probability for the risk model with normal (20,5) distributed policy lifetimes for different values I_0 and *U*.

The results in Figure 3.4 show that the ruin probability increases if the number of initially active policies increases. This is the case for all values of the initial capital. We can draw a conclusion from these observations that the number of initially active policies has a great influence on the ruin probability.
The analysis for the risk model with normal distributed policy lifetimes indicate that the ruin probability for this model depends on the arrival rate of policies as well as the number of initially active policies.

3.4.2 Risk model with gamma distributed policy lifetimes

The second risk model we examine is the risk model with gamma distributed policy lifetimes. A random variable X that is gamma-distributed with scale β and shape *a* is denoted

$G \sim \text{gamma}(a, \beta)$

For our simulation model we chose a gamma(a,β) distribution with scale parameter β=3 and shape parameter a=5. The mean of the gamma distribution is a β=15.
The following excel function is used to generate a random gamma variable G with expected mean

=Gammainv(rand() , shape parameter, scale parameter)

The influence of the arrival rate on the ruin probability is examined for this risk model. In the case of exponentially distributed policy lifetimes the arrival rate had no influence on the ruin probability. In Figure 3.5 the ruin probability for different values of the initial capital U and policy arrival rate λ are shown. The mean claim size for each case is ϴ=1 and exponentially distributed and for the safety loading we have used φ=0.3. The number of initially active policies I_0=0.

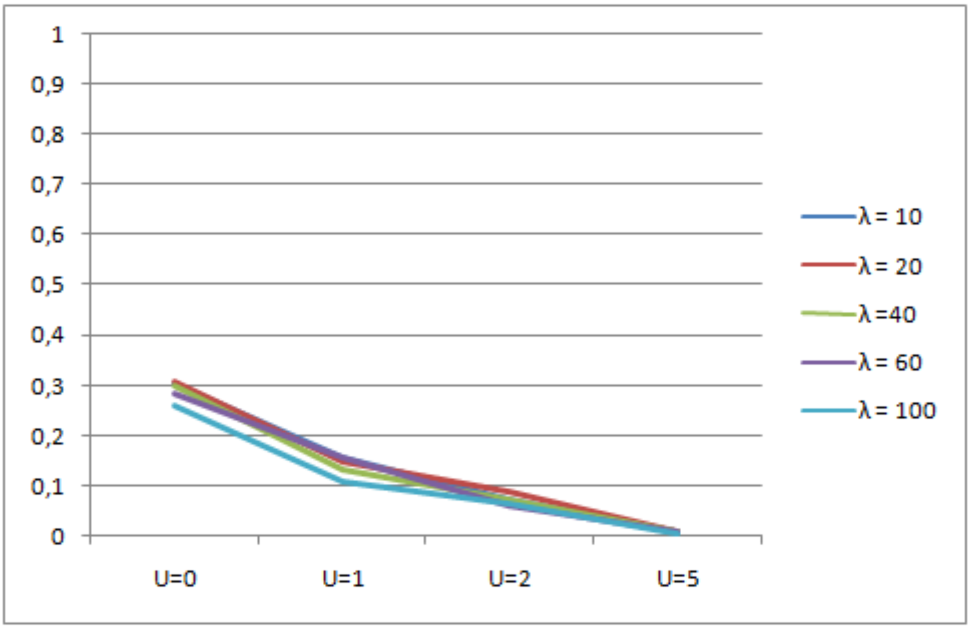

Figure 3.5: Ruin probability for the risk model with gamma (5,3) distributed policy lifetimes for different values *λ* and *U*.

Although not as clear as the risk model with normal distributed policy lifetimes the results in Figure 3.5 tell us that the ruin probability decreases if the arrival rate increases. The arrival rate has an impact on the ruin probability.
Next to the influence of the arrival rate on the ruin probability the influence of the number of initially active policies I_0 on the ruin probability is examined. In Figure 3.6 the ruin probability for different values of initially active policies corresponding to some initial capital are shown. The mean claim size is ϴ=1 and the safety loading φ=0.3. The arrival rate of policies is λ=20 for all cases.

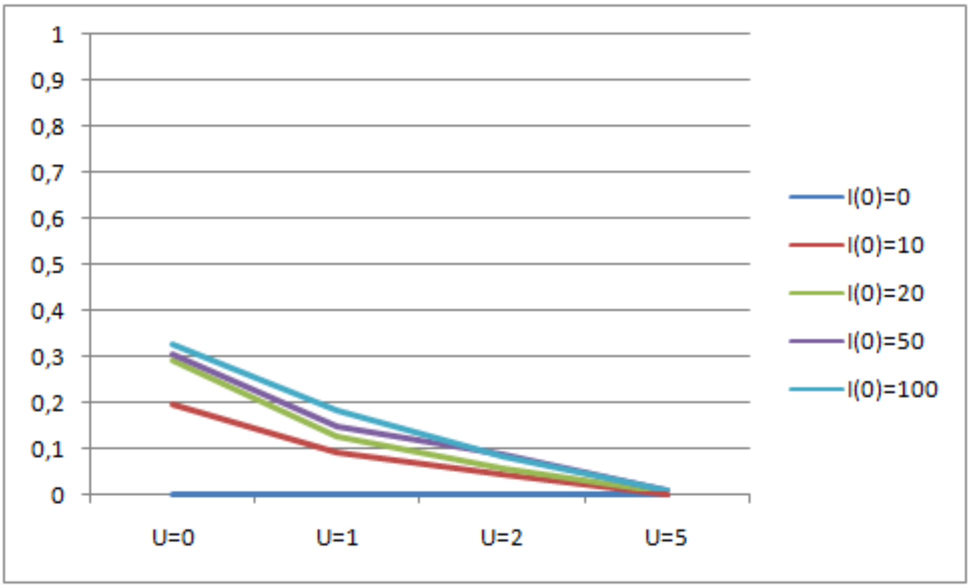

Figure 3.6: Ruin probability for the risk model with gamma (5,3) distributed policy lifetimes for different values *I_0* and *U*

The results in Figure 3.8 show that the ruin probability is higher for higher number of initially active policies. This corresponds with the results for the risk model with normal distributed policy lifetimes.

In summary, for the risk models examined the results clearly illustrate that the ruin probability for the risk models with gamma and normal distributed policy lifetimes depend on the arrival rate of policies as well as the number of initially active policies. This in contrast to the risk model with exponential policy lifetimes as discussed in Adan et al.[1].

4. Conclusion

For an insurance company, each contract of insurance brings a risk which occurs because of random frequency and non-negative random amount of claim. One of the main tasks of the insurance company is to determine premiums to prevent from ruin of the insurance company. To manage this risk, ruin probabilities are important tools which help insurance companies to control the risk of ruin. The probability of ruin is used for decision taking, for instance the premium calculation or the computation of reinsurance retention levels. For an actuary it is important to be able to take a good decision in reasonable time.

In this paper Monte Carlo simulation as a tool is used to model risk models. The simulation models used in this paper are modeled in a well-known environment: Microsoft Excel. The use of simulation algorithms to model the standard risk model and risk models with policy lifetimes are discussed and shown. Also a section is dedicated to factors that have effect on the reliability and accuracy of the simulation outcome. Therefore the paper can be a reference for the reader to simulate simple risk models in an accessible environment (Excel).

Secondly, we have studied the conclusions drawn in Adan et al. [1] using this simulation technique. Adan et al. [1] concluded that, if the lifetimes of policies are exponential random variables, then the ruin probability is identical to the one in the standard risk model where reserves increase at constant rate r and claims occur according to a Poisson process.
Two simulation models are presented in this paper. The first simulation model representing the standard risk model. The application of a simulation algorithm for the arrival of claims is described. This simulation model can be used as a framework to model standard risk models with all kind of claim size distributions. We have validated the simulation model for the standard risk model by comparing the outcome with known exact solutions to analytical methods.

The second simulation model we have presented is a simulation model based on the arrival of policies with an intensity described by the Poisson process and the occurrence of claims on expiry of the lifetime of policies. And is also validated by comparing the outcome with known exact solutions to analytical methods.

The simulation study confirmed the conclusion drawn by Adan et al. [1]. Further we have examined if the conclusions drawn by Adan et al. hold for risk models with different distributed policy lifetimes. A risk model based on normal distributed policy lifetimes and a risk model based on gamma distributed lifetimes is examined. The results for these two risk models showed that the number of initially active policies and the arrival rate of policies has an influence on the ruin probability.
We conclude that the standard risk model cannot be used to estimate the ruin probability for all risk models that consider the arrival of policies and a lifetime for policies. In contrary, it would be not a surprise if the results for the standard risk model only holds for a risk model with exponentially distributed policy lifetimes. Apparently, the memoryless property of the exponential distribution has a great influence on the ruin probability.

5 Bibliography

[1] Adan, I and Kulkarni, V. (2007). Insurance risk with variable number of policies.

[2] http://www.ualberta.ca/~cdeutsch/MCS-course.htm

[3] Dickson, D.C.M. and Waters, H. R. (1996). Ruin problems: simulation or calculation? B.A.J.2, Vol. III, p. 727-740.

[4] http://fedc.wiwi.hu-berlin.de/xplore/tutorials/

[5] Asmussen, S. and Binswanger, K. Simulation of ruin probabilities for sub exponential claims.

[6] Albrecher, H., Kainhofer, R. and Tichy, R. F. Simulation methods in ruin models with non-linear dividend barriers

[7] Burnecki, K., Hardle, W. and Weron, R. An introduction to simulation of Risk Processes.

[8] Cao, X. and Feigel, R. E. Compound process simulation on spreadsheet and application to insurance modeling

[9] Rolski, T., H. Schmidli, V. Schmidt and J. Teugels (1999). Stochastic processes for insurance and finance. *John Wiley & Sons*, Chichester.

[10] Tijms, H.C. (2002). Operationele analyse. Utrecht: Epsilon Uitgaven.

[11] Koole, G.M.(2006) Optimization of Business Processes. Lecture notes, Vrije Universiteit

[12] www.wikipedia.org

[13] Kaas, R., Goovaerts, M., Dhaene, J. and Denuit, M. (2001). Modern Actuarial Risk Theory. Dordrecht: Kluwer Academic Publishers.

[14] Journal of Applied Mathematics and Decision Sciences, 7(3), 133–146
Copyrightc 2003, Lawrence Erlbaum Associates, Inc.

[15] http://my.jce.ac.il/bani/StaticSimulation/MonteCarloTutorial.pdf

[16] Bartlett, J.E., II, Kotrlik, J.W.and Higgins, C. (2001). Organizational research: Determining appropriate sample size for survey research. Information Technology, Learning, and Performance Journal, 19(1) 43-50

Appendix A

						P						
df	0.25	0.2	0.15	0.1	0.05	0.025	0.02	0.01	0.005	0.0025	0.001	0.0005
1	1	1.376	1.963	3.078	6.314	12.706	15.895	31.821	63.657	127.321	318.309	636.619
2	0.817	1.061	1.386	1.886	2.92	4.303	4.849	6.965	9.925	14.089	22.327	31.599
3	0.765	0.979	1.25	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.215	12.924
4	0.741	0.941	1.19	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.61
5	0.727	0.92	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	0.718	0.906	1.134	1.44	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	0.711	0.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	0.706	0.889	1.108	1.397	1.86	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	0.703	0.883	1.1	1.383	1.833	2.262	2.398	2.821	3.25	3.69	4.297	4.781
10	0.7	0.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	0.697	0.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	0.696	0.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.93	4.318
13	0.694	0.87	1.079	1.35	1.771	2.16	2.282	2.65	3.012	3.372	3.852	4.221
14	0.692	0.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.14
15	0.691	0.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	0.69	0.865	1.071	1.337	1.746	2.12	2.235	2.583	2.921	3.252	3.686	4.015
17	0.689	0.863	1.069	1.333	1.74	2.11	2.224	2.567	2.898	3.222	3.646	3.965
18	0.688	0.862	1.067	1.33	1.734	2.101	2.214	2.552	2.878	3.197	3.61	3.922
19	0.688	0.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	0.687	0.86	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.85
21	0.686	0.859	1.063	1.323	1.721	2.08	2.189	2.518	2.831	3.135	3.527	3.819
22	0.686	0.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	0.685	0.858	1.06	1.319	1.714	2.069	2.177	2.5	2.807	3.104	3.485	3.768
24	0.685	0.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	0.684	0.856	1.058	1.316	1.708	2.06	2.167	2.485	2.787	3.078	3.45	3.725
28	0.683	0.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	0.683	0.854	1.055	1.311	1.699	2.045	2.15	2.462	2.756	3.038	3.396	3.659
30	0.683	0.854	1.055	1.31	1.697	2.042	2.147	2.457	2.75	3.03	3.385	3.646
40	0.681	0.851	1.05	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	0.679	0.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	0.679	0.848	1.045	1.296	1.671	2	2.099	2.39	2.66	2.915	3.232	3.46
80	0.678	0.846	1.043	1.292	1.664	1.99	2.088	2.374	2.639	2.887	3.195	3.416
100	0.677	0.845	1.042	1.29	1.66	1.984	2.081	2.364	2.626	2.871	3.174	3.39
1000	0.675	0.842	1.037	1.282	1.646	1.962	2.056	2.33	2.581	2.813	3.098	3.3
z*	0.674	0.841	1.036	1.282	1.645	1.96	2.054	2.326	2.576	2.807	3.09	3.291
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.50%	99.80%	99.90%
					Confidence level		C					