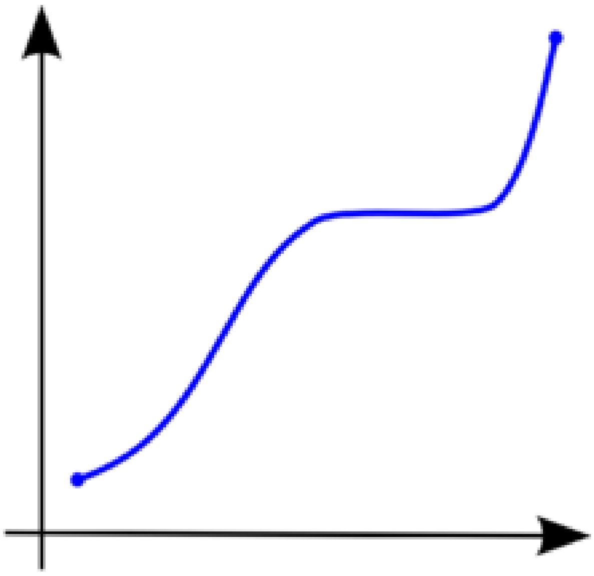

The Robustness Of Non-Decreasing Dynamic Pricing

Laura van der Bijl

Research Paper Business Analytics

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Laura van der Bijl
Laura.vanderbijl5@gmail.com

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Prof. dr. G.M. Koole
Ger.koole@vu.nl

VU University Amsterdam
De Boelelaan 1105,
1081 *HV Amsterdam*

Preface

This is a paper for the master Business Analytics. For two years I have worked at the department Revenue Management at Transavia, a lowcost airline in the Netherlands. This experience has increased my interest in new revenue management methods. Ger Koole, a professor at the VU Amsterdam, told me about the paper that he co-wrote with Daniel Hopman and Rob van der Mei. I was immediately enthusiastic about this subject.

I would like to thank my supervisor Ger Koole, who provided me with choices of subjects and made me excited about this subject. He has been a great coach, who always made time for me when I needed help and he was always ready for me.

1 Abstract

Nowadays, revenue management is very important in the airline business. There are multiple revenue management methods that can be used to steer the prices. The most simple method is the so-called "DPID method": there are different fares and the fare with the most demand is the fare that is used. This method is discussed. Another method we are going to study is the downselling method. Downselling happens when a customer pays a lower fare than he was willing to pay.

The ideal curve for selling tickets of a flight is first selling for a low price and as time goes on, the price gets higher. We call it non-decreasing control. But in reality, non-decreasing control is not what the airline business uses. If the demand is not correctly estimated and the price is already at a high point, not enough tickets will be sold. If the price decreases, the demand will attract again because customers do buy the ticket for a lower price.

In this paper, we test how robust the downselling method is in comparison with a derivative downselling UP method. The downselling UP method has a non-decreasing control, the price always goes up or stays the same.

By comparing different expected demand and real-time demand we can conclude that only at overforecasting the downselling UP method gives a higher revenue. With overforecasting there will be sold less tickets, but the tickets that are sold generates more revenue. The point of changeover differs. This depends on demand, capacity, and price per fare class.

Contents

2 Introduction

In the early days of air travel, a flight ticket to a destination had a fixed price. BOAC was the first company that started with ”early bird” tickets. These were discounted fares to sell seats that would not otherwise have been sold [7]. Nowadays the price of a ticket is not fixed anymore. All airlines use revenue management techniques for selling their tickets. The goal of revenue management is ”selling the right product to the right customer at the right time for the right price” [4]. The products are the flight tickets and the ancillaries. Airlines are using mathematical models to optimize the price. Zaki [6] writes about this subject: ”Revenue management techniques are essentially a set of balancing acts, each act or technique adds a small fraction to the airline revenue, and collectively they provide respectable increases, between 1% and 10%.”

In this paper, we focus only on the price of the flight tickets. The ideal curve for selling tickets of a flight is first selling for a low price and as time goes on, the price gets higher. We call it non-decreasing control. But in reality, non-decreasing control is not what the airline business uses. If the demand is not correctly estimated and the price is already at a high point, not enough tickets will be sold. If the price decreases, the demand will attract again because some customers do buy the ticket for a lower price. That gives higher revenue. Thus if the demand is not correctly estimated the airlines will send the price down causing the demand to increase.

Airlines use mathematics models to optimize the revenue. A famous revenue management technique is the dynamic programming formulation (DPID) [1]. It is widely used. The DPID looks which fare class generates the highest revenue and indicates to use this class. It does not take into account non-decreasing control. This method assumes that a customer buys a ticket for only the maximum price the customer accepts. But in reality, if the price is lower than the maximum spending price of the customer, he will buy it also if the fare conditions are the same. This is important, if the conditions are different at a lower price, we cannot assume that a customer buys that ticket because maybe he wants luxury for example. Therefore to prevent confusion, we assume that the fare conditions are the same.

When a customer pays less than he is willing to pay, we call it downselling. Hopman et al. [1] programmed a downselling model (DPDS) and compared this model to the regular dynamic programming method (DPID). They take into account three different demands: high, medium and low demand. The results were clear, the downselling model always gives a higher revenue.

As we mention, revenue management departments are always trying to send the price from a lower price long before departure to a high price close to departure. Long before departure, tickets have a lower price to create a baseload. Close to departure, the price is higher. Mostly at that time customers pay more because they have no choice. These customers are less flexible in the time period and must have that flight. The shorter the time before departure, the higher the price people accept to pay [5] (see Figure 1).

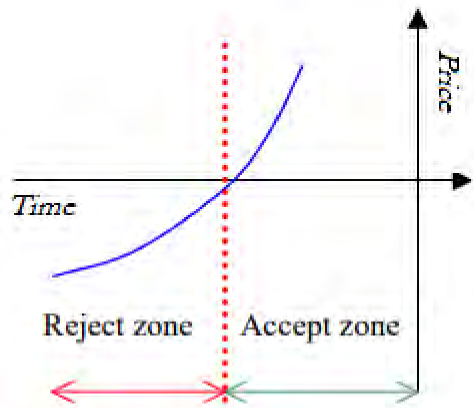

Figure 1: *Time before departure and the price people accept to pay*

The DPID and DPDS methods do not use the non-decreasing technique. If the revenue at a time point t is higher at a cheaper class than time point $t+1$, the downselling method sells the tickets in that cheaper fare class. The advantage of selling in a cheaper class is that the demand will increase and there will be more tickets sold, higher revenue at that point. But there are also disadvantages. If the price is lower than earlier in time, people will buy later in time because they think the price can decrease. Also, you miss the people who do want to buy that ticket for that high price later in time. The last disadvantage we mention is people who bought for a higher price earlier in time feel betrayed.

Hopman et al. [1] also researched this subject. They considered a downselling UP method (see Section 4.1). This method has a non-decreasing control, the price always goes up or stays the same. They compared this downselling UP method (DPDS↑) with the downselling model (DPDS). They looked at the waiting probability and also at the robustness of the methods. There are interesting results that are discussed in Section 2 of this paper. But what has not been investigated is the robustness of the two methods regarding wrong estimates of the demand. Therefore in this paper, we research the robustness regarding to wrong estimates of the demand for the downselling UP method comparing with the downselling model, the DPDS.

We are going to look how robust the DPDS↑ method is. In other words, how good should be the passenger demand forecast for these methods, to generate the most revenue? Forecasting is the biggest remaining challenge at Revenue Management. It does not only matter how many passengers will be on a flight, but the whole booking curve has to be forecasted. This depends on many factors such as seasonality, holidays, special events. There is not much literature about how to forecast.

3 Literature review

In this paper, we take into account single-resource problems. A single-resource problem is, for example, controlling the sale of different fare classes on a single flight leg of an airline [3]. Talluri and Van Ryzin [2] noticed that single-resource problems are important. Actually, in practice, many quantity-based revenue management problems are still frequently solved as a collection of single-resource problems. Moreover, single-resource models are useful as building blocks in heuristics for the network case.

Hopman et al. [1] also took into account single-resource revenue management. They figured out a downselling method and called it the DPDS model. This model takes downselling into account. They compared this method with the DPIP method and with the EMSRb heuristics as well. Both methods do not take downselling into account. The conclusion from [1] is that the DPDS method is much more able to sell the right fare to the right customer and this leads to significant revenue improvements. The mean revenues are between 20% and 30% higher than the other.

Further along in the paper Hopman et al. [1] studies a downselling UP method, the DPDS↑. This method does not sell a lower fare close to departure. The price increases from a low fare to a high fare. In Chapter 4.1 there is more explanation about this model. Hopman et al. [1] looked at the waiting probability of this method. The waiting probability of the DPDS↑ method is different from the waiting probability of the DPDS method. If people know that the price will never drop, people buy faster then when there is a change that the price will drop. [1] studies also the robustness of the DPDS method and the DPDS↑ method. They looked at the robustness of the DPDS↑ method with respect to under- and overforecasting demand. The results show that the DPDS↑ is about equal in its robustness compared to the DPDS method when it comes to overforecasting. What is lost in passengers made up in yield. But with underforecasting, it is less robust. This is a very interesting finding. What has not been investigated yet is what the point of changeover is. When is the DPDS↑ method better and when does the DPDS give a higher revenue? Which factors have an influence here?

In addition to determining which method will generate the highest revenue, there are more factors at stake. Talluri and Van Ryzin [2] write that ”customers who purchased early may get upset to see prices drop while they are still holding a reservation; indeed, many airlines give a price guarantee to refund the difference if there is a price drop (to encourage passengers to book early), making it costly for the firms to lower prices. And in the travel business, high-valuation high-uncertainty customers tend to purchase closer to the time of service. Hence, demand is less price-sensitive close to the time of service.”

Hopman et al.[1] write also that in case of the DPDS↑ method if this method is being used consistently, the customer responds differently. The customer will decide faster because the customer knows this is the lowest price for this ticket. So the numbers of the results are important, but the changing behavior must also be included.

4 Methods

4.1 Formulation

Earlier we said that Hopman et al.[1] consider a downselling method. They derived this method from the DPID formulation. In this section we show and explain the different methods. We use the following notation:

λ_j is the arrival rate of class j, j=1,..,J;
f_j the fare of product /class j, $f_1 \geq f_2 \geq .. \geq f_j$
x the remaining capacity
t the number of time units, t=1,..,T.

For simplicity, we assumed that the fare of a ticket is fixed over time and the fare are ordered downwards. So $f_1 \geq f_2 \geq .. \geq f_J$. The expected demand follows a Poisson process with mean λ_j. The time steps are so small that at most one arrival occurs. The function $V_t(x)$ is the revenue-to-go function, this function calculates the revenue to be earned having x seats and t units of time left. This function $V_t(x)$ is the number where it is all about. The method which gives the highest revenue-to-go is the best method to use.

The DPID equation is:

$$V_t(x) = max\left(\sum_{j=1}^{k} \lambda_j(t) \cdot (f_j + V_{t+1}(x-1)) + (1 - \sum_{j=1}^{k} \lambda_j(t))V_{t+1}(x))\right)$$

It is necessary to calculate this equation backward in time. It starts with $t = T$ and $x = 0$. This method first takes the fare plus the $t+1$ "revenue-to-go" (one seat more sold) times the arrival rate. The method does this for every class till k, k is the cheapest class available. Than plus one minus the sum of the arrival rate times the "revenue-to-go" now if you do not sell the seat. From this, we take the maximum.

They [1] use the dynamic programming formulation (DPID) to derive the downselling method. The equation for the DPDS method is:

$$V_t(x) = max\left(\sum_{j=1}^{k} \lambda_j(t) \cdot (f_k + V_{t+1}(x-1)) + (1 - \sum_{j=1}^{k} \lambda_j(t))V_{t+1}(x))\right)$$

The two equations look almost the same, but there is one important difference, the fare. The difference between these two methods is that the DPDS method assumes that arrivals of more expensive classes buy also cheaper classes. If the conditions are the same for every class, this is plausible. Everyone wants to buy a cheaper ticket. In the equation f_j is changed to f_k.

The downselling UP method (DPDS↑) that we mentioned before is derived from the DPDS method:

$$V_t(x,y) = max_{k\in\{1,..,y\}} \left(\sum_{j=1}^{k} \lambda_j(t)\cdot(f_k + V_{t+1}(x-1,k)) + (1 - \sum_{j=1}^{k} \lambda_j(t))V_{t+1}(x,k)) \right)$$

This method has the same conditions as the DPDS method. Only with this method, the price cannot decrease. There is a new variable added, variable y, that denotes the lowest class available. The price cannot drop over time because y stays the same or increases, but cannot decrease.

4.2 Data

We implement both methods, the DPDS and DPDS↑, and compare the methods with different forecasted and real-time demand. The next table shows the total forecasted demand and fares we use. These numbers are the same that is used in [1]. The data was adapted from real airline data and is scaled to high, medium and low demand factors. There are five fare classes. The given demand is the total demand per class. Table 1 shows the high demand data.

	Class					
Time point	1	2	3	4	5	Sum
1	0.0	0.0	0.0	0.0	5.1	5.1
2	0.0	0.0	0.2	1.4	3.8	5.4
3	0.0	0.2	1.0	3.4	2.6	7.2
4	0.0	0.5	1.2	3.8	1.3	6.8
5	0.4	0.5	1.2	1.0	0.0	3.1
6	0.8	1.6	0.4	0.0	0.0	2.8
7	1.2	0.5	0.0	0.0	0.0	1.7
8	1.6	0.5	0.0	0.0	0.0	2.1

Table 1: *Expected high demand data*

There are 8 time points. Because the arrival rate has to be smaller than one, we divide the demand by 100. Now we have 800 time points instead of 8. Table 2 gives the medium demand data and Table 3 the low demand data. Also with this data, we make 800 time point and divide the demand by 100.

Table 4 gives the total demand per fare class and the fare. Class 1 has the highest fare, 1800. Fare 5 is 400.

We implement a fixed model that remembers the choices of the fare classes from the forecasted demand model, hence we can compare forecasted demand with real-time demand. First, we run the two methods, downsell and DPDS↑, with the expected data. We remember the choices the models make concerning the fare classes and then we run the fixed models with the real-time data. These models use the choices of the fare classes which are stored. The fixed model that gives the most revenue is the best model for that situation. We assume there are 25 seats on the flight and as we mentioned we created 800 time points to sell the tickets.

	Class					
Time point	1	2	3	4	5	Sum
1	0.000	0.000	0.000	0.000	3.778	3.7778
2	0.000	0.000	0.1482	1.0370	2.8148	4.0000
3	0.000	0.1482	0.7407	2.5185	1.9259	5.3333
4	0.000	0.3704	0.8889	2.8148	0.9630	5.0370
5	0.2963	0.3704	0.8889	0.7407	0.000	2.2963
6	0.5926	1.1852	2.9630	0.0000	0.000	4.7408
7	0.8889	3.7037	0.000	0.000	0.000	4.5926
8	1.1852	3.7037	0.000	0.000	0.000	4.8889

Table 2: *Expected medium demand data*

	Class					
Time point	1	2	3	4	5	Sum
1	0.00000	0.00000	0.00000	0.00000	3.00000	3.00000
2	0.00000	0.00000	0.11765	0.82353	2.23530	3.17648
3	0.00000	0.11765	0.58824	2.00000	2.00000	4.23529
4	0.00000	0.29412	0.70588	2.23530	0.76471	4.00001
5	0.23529	0.29412	0.70588	0.58824	0.00000	1.82353
6	0.47059	0.94118	0.23529	0.00000	0.00000	1.64706
7	0.70588	0.29412	0.00000	0.00000	0.00000	1.00000
8	0.94118	0.29412	0.00000	0.00000	0.00000	1.23530

Table 3: *Expected low demand data*

	Class					
Demand	1	2	3	4	5	Sum
High	4	3.8	4	9.6	12.8	34.1
Med	3	2.8	3	7.1	9.1	25.3
Low	2.4	2.2	2.4	5.6	7.4	20.1
Fare	1800	1500	1000	800	400	

Table 4: *Total expected demand and fares*

5 Results

In this section we show the results. We look high over at the results of the two methods and then we zoom in on the point of changeover. We expect that the capacity influences the robustness of the methods, therefore we check also different capacities on both methods. Finally we try different fares to discover if this has influence on it.

5.1 High over

First we compare high, medium and low demand with each other, for example high forecasted demand with low real-time demand.

Forecasted demand	Real time demand	Rev DPDS	Rev DPDS↑	Percentage	Best result
High	High	5468.1	5438.6	99.39%	DPDS
High	Med	4318.7	4323.9	100.12%	DPDS↑
High	Low	3478.4	3490.6	100.35%	DPDS↑
Med	High	5260.3	5178.7	98.45%	DPDS
Med	Med	4428.2	4418.6	99.78%	DPDS
Med	Low	3603.7	3604.6	100.02%	DPDS↑
Low	High	4984.9	4886.1	98.02%	DPDS
Low	Med	4405.8	4390.2	99.65%	DPDS
Low	Low	3613.0	3611.7	99.96 %	DPDS

Table 5: *Results different demands*

In Table 5 can be seen the results of different forecasted and real-time demand. *Rev DPDS* and *Rev DPDS*↑ gives the revenue-to-go with 25 seats left at time point 800 before departure. *Percentage* gives the percentage of the revenue-to-go of DPDS↑ relative to the revenue-to-go of DPDS. Next to it is shown the method with the best result. Only at overforcasting the DPDS↑ method gives a better result than the downselling method. If the real demand is exactly the same as the forecast demand, the downselling method gives a higher revenue. For every comparison below forecasting this is also the case.

Why gives the DPDS↑ method a better result at overforecasting? With overforecasting there will be sold less tickets, but the tickets that are sold generates more revenue. If we look at the percentages, the results do not differ much. This is because the price does not fluctuate much. The demand per fare is slowly increasing. In Section 6 we will come back to this.

5.2 Point of changeover

Now we know that with overforecasting the DPDS↑ method gives a better result, but we are curious at what point the changeover takes place. Therefore we change the forecasted data to investigate. We begin by testing the medium booking data. The real-time demand is the medium data and we increase the forecasted demand. First, we increase by 5%, then 10%, and so on, until we find the point of changeover. "Med 5%" means the data of the medium demand curve plus 5% extra.

	Class						
Demand	1	2	3	4	5	Sum	Best method
Med	2.96	2.81	2,96	7.11	9.48	25.3	DPDS
Med 5%	3.11	2.96	3.11	7.47	9.96	26.6	DPDS
Med 10%	3.26	3.10	3.26	7.82	10.43	27.9	DPDS
Med 15%	3.41	3.24	3.41	8.18	10.90	29.1	DPDS
Med 16%	3.44	3.27	3.44	8.25	11.00	29.3	DPDS↑
Med 17%	3.47	3.29	3.47	8.32	11.09	29.6	DPDS↑
Med 20%	3.56	3.38	3.56	8.53	11.38	30.4	DPDS↑

Table 6: *Results medium demand overforecasted*

Table 6 shows the results of the overforecasted medium booking data. The point of changeover is between 15% and 16%. The next question is if this is the changeover point for all cases. Therefore we test also the high booking data and the low booking data.

	Class						
Demand	1	2	3	4	5	Sum	Best method
High	4	3.8	4	9.6	12.8	34.2	DPDS
High 15%	4.60	4.37	4.60	11.04	14.72	39.33	DPDS
High 20%	4.80	4.56	4.80	11.52	15.36	41.0	DPDS
High 22%	4.88	4.64	4.88	11.71	15.62	41.7	DPDS
High 23%	4.92	4.67	4.92	11.81	15.74	42.1	DPDS↑
High 25%	5.00	4.75	5.00	12.00	16.00	42.8	DPDS↑
Low	2.35	2.24	2.35	5.65	7.53	20.1	DPDS
Low 10%	2.59	2.46	2.59	6.21	8.28	22.1	DPDS
Low 12%	2.64	2.50	2.64	6.32	8.43	22.53	DPDS
Low 13%	2.66	2.53	2.66	6.38	8.51	22.73	DPDS
Low 14%	2.68	2.55	2.68	6.44	8.58	22.93	DPDS
Low 15%	2.71	2.57	2.71	6.49	8.66	23.1	DPDS↑
Low 16%	2.73	2.59	2.73	6.55	8.73	23.34	DPDS↑

Table 7: *Results high and low demand overforecasted*

Table 7 shows that the point of changeover is not the same in every case. For high demand, the changeover point lies higher, between 22% and 23%. For the low demand case is this point between 14% and 15%, lower than the other cases. So the conclusion is clear. The higher the demand, the higher the changeover point. That means that the higher the demand, the higher the overforecasting needs to be to let the DPDS↑ give the best result.

Data	Total demand	Changeover point	Demand changeover point	Number of over forecast	Capacity
Low	20.1	15%	23.1	3.0	25
Medium	25.3	16%	29.3	4.0	25
High	34.1	23%	42.1	8.0	25

Table 8: *Comparison changeover point*

Table 8 shows the data of the different cases. We know now that it depends on the forecasted demand. We think it could also depend on the capacity because if the capacity increases, for example, high demand could be low demand. We are going to test if the capacity is indeed a factor of influence. Once more we take the medium demand data for testing.

5.3 Capacity

	Capacity						
Demand	10	15	20	25	30	35	40
Med	DPDS	DPDS	DPDS	DPDS	DPDS	DPDS	DPDS
Med 5%	DPDS	DPDS	DPDS	DPDS	DPDS	DPDS	DPDS
Med 10%	DPDS	DPDS	DPDS	DPDS	DPDS	DPDS	DPDS
Med 15%	DPDS	DPDS	DPDS	DPDS	DPDS	DPDS	DPDS
Med 16%	DPDS↑	DPDS↑	DPDS↑	DPDS↑	DPDS↑	DPDS	DPDS
Med 17%	DPDS↑	DPDS↑	DPDS↑	DPDS↑	DPDS↑	DPDS	DPDS
Med 20%	DPDS↑	DPDS↑	DPDS↑	DPDS↑	DPDS↑	DPDS↑	DPDS
Med 30%	DPDS↑	DPDS↑	DPDS↑	DPDS↑	DPDS↑	DPDS↑	DPDS↑

Table 9: *Best method different capacity*

Table 9 shows the results of the comparisons with different capacities. We have tried less and more capacity than the 25 seats we used before. You can clearly see the differences. If there is more capacity, the downselling method gives a better result. This is logical. More capacity makes the overforecasting less big.

5.4 Fare

Another factor which may be of influence is the fare. The price gaps between fare classes are quite big. It is logical if the price differences between the classes are smaller, the point of changeover changes. In the next table, we show the different price levels we tested and the outcome of the point of changeover. We used the medium dataset with capacity 25.

	Class					Change side	Point of changeover
Fare	1	2	3	4	5		
Price 1	1800	1500	1000	800	400	→	15% - 16%
Price 2	180	150	100	80	40	→	15% - 16%
Price 3	2500	2000	1500	1000	500	↑	16% - 17%
Price 4	1000	900	800	700	600	↓	14% - 15%
Price 5	500	400	300	200	100	↓	13% - 14%
Price 6	150	120	100	70	50	↑	17% - 18%

Table 10: *Results different fares*

The fare levels indeed affect the changeover point, as can be seen in Table 10. We have listed the findings:

- Price range 2 is similar to price range 1, as it is equal to price range 1 divided by 10. You can see that the point of changeover is the same.
- The delta of price 3 is equal to every class. In comparison with the original fares, these fares are higher. The point of changeover is higher.

- The delta of price 4 and price 5 is 100, but the starting fares are different. If the fare is lower and the delta is the same, the point of changeover is lower.
- In addition, it depends on the demand per class. If the DPDS↑ method sells more in the most expensive class and the price difference between class 1 and 2 is small, the other method probably gives higher revenue.

6 Other data

The data we have used in this research is quite small and the price does not fluctuate much. Table 5 in Section 5.1 showed that the results of both methods do not differ much in revenue, about 1%. This is not much difference. The reason for this has to do with is the data. The data we have used in this paper is the data Hopman et al.[1] used. The demand per fare class is over time growing. Long before departure, the demand for the cheapest fare is the highest and short for departure the most expensive fare has the highest demand.

We have now a dataset from a Dutch airline. Figure 2 shows the booking curve and the fare class per sales date. The capacity of this flight is 189 seats. The yellow line is the price on the website at that moment. The blue line is the number of tickets that have been sold, the number of passengers. In the figure can be seen that the fare can decrease over time. In total there are 165 tickets sold. The capacity of this flight is almost eight times bigger than the data of [1] and there are 15 different fare classes. We do not know what the other arrival rates are of the different fare classes. This yield curve has more fluctuation than the data we used before. It should be mentioned that there are more extremely yield curves.

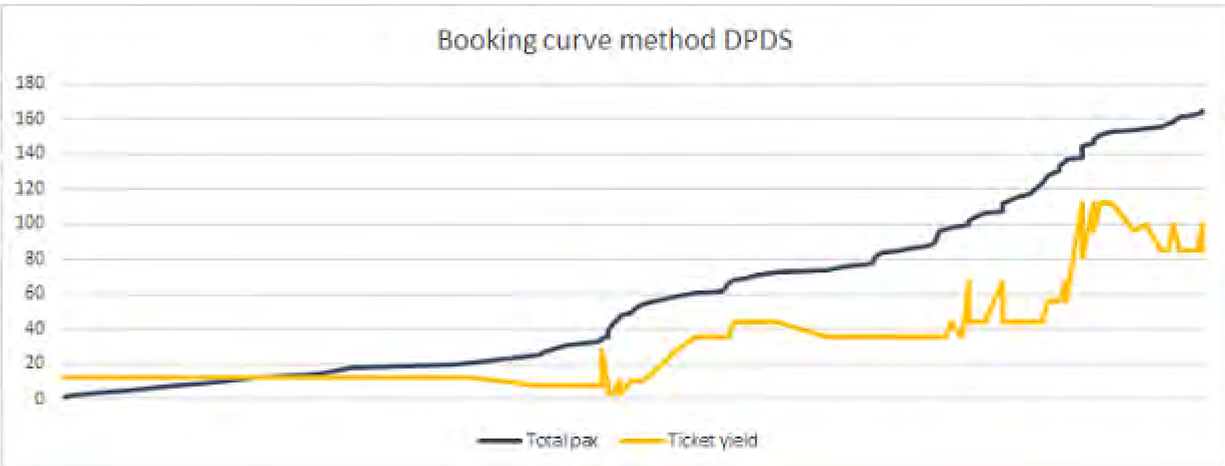

Figure 2: *Booking curve data Dutch airline*

The total revenue of this flight is €6722,10. As we noticed we do not know what the arrival rate should be if the DPDS↑ method was used. The arrival rate per fare class can be calculated with the likelihood formula. Assume there are two fare classes, a low fare class l and a high fare class h. The arrival rate can be calculated as follows:

λ_l is the arrival rate of the lower price class
λ_h is the arrival rate of the high price class
r_l is the realization of the lower price class
r_h is the realization of the higher price class
t the number of time units, t=1,..,T.

Estimate α and β to determine optimal policy

$$\lambda_h(t) = \beta_h \; exp^{\alpha_h t}$$
$$\lambda_l t) = \beta_l \; exp^{\alpha_l t}$$

Likelihood $L \; (\alpha_h, \beta_h, \alpha_l, \beta_l) =$

$$\frac{(\lambda_l(1) + \lambda_h(1))_l^r}{r_l!} \cdot exp^{-(\lambda_l(1)+\lambda_h(h))} \cdot \frac{\lambda_h(2)_h^r}{r_h!} \cdot exp^{\lambda_h(2)}$$

In our case, there are 15 different fare classes instead of 2. So the formula must be extended to 15 classes. This cost to much time for this research, therefore we have done a very simple manually estimate.

We take the data and start backward. If the fare class was already open, that arrival rate is taken. If the price is lower, the arrival rate increases or stays the same. The result is shown in Figure 3 below. The price is non-decreasing. The total revenue with this method is €6402,50. This differs €319,60 and in this case, that is about 5% less revenue. In total there are 175 tickets sold. So we can conclude that the DPDS method generates more revenue and the DPDS↑ method generates more passengers. The results are summed up in Table 11 below. In the tabel, *Percentage* gives the percentage of the revenue of model relative to the revenue of the DPDS model. These results are only if the forecasting arrival rate is right, and as we noticed is this very difficult for airlines.

Model	Revenue	Percentage
DPDS	6722.10	100%
DPDS↑	6402.50	95.26%

Table 11: *Results other data*

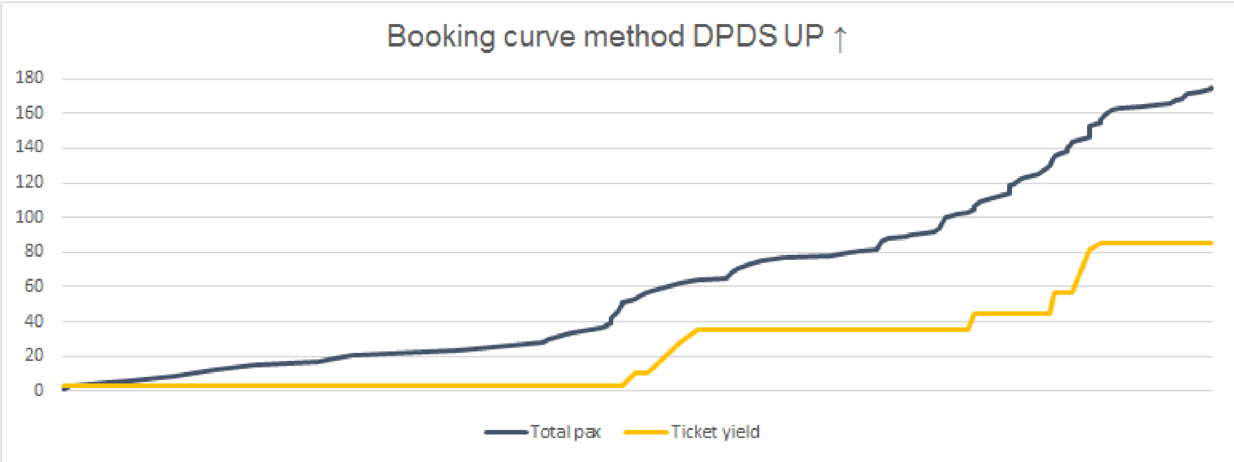

Figure 3: *Booking curve with the DPDS ↑ method*

We see in this example that the result can differ more than we saw in Section 5.1. We did not know what the arrival rate exactly was of the other fare classes, this is estimated. This seems realistic but we do not know if this happened if the DPDS↑ method was used. Further research is necessary to research what the arrival rate will have been and then calculate what the revenue differs is.

7 Discussion

The question of this paper is about the robustness of the downselling and downselling UP models. If the forecasted demand is well estimated, so the forecasted demand is equal to the real-time demand, the downselling model generates higher revenue. Only if there is overforecasting the DPDS↑ model could give a better result. With overforecasting there will be sold less tickets, but the tickets that are sold generates more revenue with the DPDS↑ method. There is a point of changeover. This point is not always the same, it depends on many factors. An important notice is that forecasting is the biggest remaining challenge at Revenue Management.

With the testing data, we could see that the higher the demand, the higher the overforecasting has to be to let the DPDS↑ give the best result. The capacity stays the same. If the forecasted demand stays the same and we change the capacity, the point of changeover also changes. Therefore we could say that the capacity has also influence.

The last factor we have tested is the fare. The fare of the classes is an important factor to determine the point of changeover. It also depends on the kind of data you put in the model. If the fare is distracted from another fare, the changeover point stays the same. It depends on the differences between the classes. If the DPDS↑ method sells more in the most expensive class and the difference between the most expensive class and the second expensive class is small, the downselling method gives a higher revenue.

The data that is used is quite small and the price does not fluctuate much. Therefore the results do not differ much. We tried other data and it showed that the difference is not that small all time. The revenue differs 5% in that case. This is when the forecasting is perfect. The arrival rate of the data is estimated quickly. Further research is necessary to research what the arrival rate will have been and then calculate what the revenue differs is.

These results are theoretical, but if the DPDS↑ is being used consistently, the customer will respond differently. The customer will decide faster and will assume a different booking behavior. This will work for the benefit of the DPDS↑ method.

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