

Dynamic Financial Analysis

Introduction to nonlife insurance decisions

BMI paper

Vrije Universiteit
Business Mathematics and Informatics
De Boelelaan 1081a
1081 HV Amsterdam
The Netherlands

Iris van Beusekom-Bastiaans

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Preface

This paper writes about a stochastic simulation model which can be used to measure risk against surplus. I have written this paper to make the subject Dynamic Financial Analysis more understandable and I have tried to answer questions such as why a Dynamic Financial Analysis would be useful, which variables are important and what are the effects of those variables.

My goal with this paper is that people with no real connection with the insurance business, but with a mathematical background understand this document and also learn something about the insurance business. Of course everybody that is interested in the subject can also read this paper.

This paper would not be the way it is now if I did not have such a good mentor who provided me ideas and kept me on the right track, thus thank you, Arjen Siegmans, for all your help.

I also want to thank my husband Daan and my family for their support and understanding, since I have been totally occupied with writing this paper.

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Chapter 1

Introduction

During my internship for a nonlife insurance corporation, Dynamic Financial Analysis would originally be the subject of the internship. Due to circumstances the subject changed, but my interest in Dynamic Financial Analysis (DFA) did not.

For my study business mathematics and informatics I had to do a literature study on a subject. Since I wanted to know more about DFA's, this report is about Dynamic Financial Analysis.

The former Dutch supervisor "Pensioen en verzekeringskamer" (PVK), whose tasks are now done by the DNB ("De Nederlandsche Bank"), made a consultation document to test the financial position and the solvency adequacy of pension funds and insurance corporations. In this document there are several proposals to test the financial position and the solvency adequacy: a small test, a standardised method and an internal method. All methods have requirements, but using a DFA to calculate the requested would fall within the boundaries of the internal method when talking about nonlife insurance corporations.

The consultation document, the "Financieel Toetsingskader", is the Dutch interpretation of a more elaborate solvency project that is planned to be finished in 2010, Solvency II. Solvency II will be the European standard to measure solvency for pension funds and insurance corporations.

The consultation document writes about a list of possible risks, which are stated on the next page:

1. Market risk
 - a. Interest risk
 - b. Exchange rate risk
 - c. Currency risk
 - d. Basic risk
 - e. Mismatch risk
 - f. Volatility risk
 - g. Reinvestment risk

2. Credit risk
 - a. Business related risk
 - b. Debt risk of investments
 - c. Political risk
 - d. Country risk

3. Liquidity risk
 - a. Catastrophe risk
 - b. Surrender risk
 - c. Migration risk
 - d. Publicity risk
 - e. Economic recession
 - f. Trust in the credit line
 - g. Access to the financial market

4. Insurance risk
 - a. Process risk
 - b. Premium risk
 - c. Product risk
 - d. Claim risk
 - e. Economic risk
 - f. Own retention risk
 - g. Policyholder risk
 - h. Reserving risk

5. Concentration risk

6. Operational risk

The standardised method of the consultation document only takes into account the interest risk, inflation risk, credit risk, stock risk, real-estate risk, raw material risk, currency risk, insurance-technical risk, concentration risk and the operational risk. An elaborate DFA which takes into account at least

those risks that are also used in the consultation document, would make the DFA model an intern model in the eyes of the supervisor.

The supervisor wants to know the financial position of all insurance companies with risks included and wants to know whether the companies are able to pay out in a bad year, when all the risks have a negative effect on the solvency.

Using Dynamic Financial Analysis to calculate the financial position and the solvency adequacy would fit in this picture, since a DFA gives a probability distribution of the surplus and the probability of losing $\alpha \cdot 100\%$ can be derived from the distribution of the surplus.

An insurance company has profits from a DFA as well, because with a DFA model simulations of the future are calculated which can imply that the current portfolio is not a good portfolio. The simulations can also imply that the premiums that are earned are not high enough. Furthermore the company will know which risks have an effect on the company and the DFA gives insight in these effects.

1.1 Problem description

The Casualty Actuarial Society has been active with the formulation and development of Dynamic Financial Analysis defined it as, (see John C. Burkett, Thomas McIntyre and Stephen M. Sonlin (2001)):

"A systematic approach to financial modelling in which financial results are projected under a variety of possible scenarios, showing how outcomes might be affected by changing internal and/or external conditions."

Reading this definition still leaves much open. There is no unique methodology for Dynamic Financial Analysis and there are several Dynamic Financial Analysis software products on the market, all using its own methodology.

Understanding a DFA model is not easy, because there are many things that have to be considered. Many models can be used to estimate variables such as the loss trends in premiums, underwriting cycles, etc., but it is more important to understand the basics of a DFA. Explaining the models that are used to calculate the variables is interesting as well, but there are many models that can be used, one better than the other. Choosing which model can be used best, is an art itself.

After understanding the basics of DFA, one could read more about the mod-

elling of certain variables, which can then be implemented in the basic DFA model. My goal is to make DFA more understandable, thus this paper. In order to make DFA more understandable, many variables that should be in a more elaborate DFA, are not put in this DFA model.

My goal with this paper is to make DFA more understandable and answer the questions why an insurance company would use a DFA, what can the company do with the DFA, what are the important variables and what changes can be made in the DFA.

Chapter 2

A DFA model

As written in the problem description, a DFA is an approach to financial modelling, with variables that affect the scenarios and thus the outcome.

In a DFA, two things are measured, the surplus and the risk. For each risk a surplus can be found, which is called a strategy. The main question in a DFA model is what is the best strategy? This chapter is about how to calculate the surplus and choosing the risk measure.

The surplus U_t for a year t is defined as the difference between the market value of the assets and the market value of the liabilities (derived from the discounted loss reserves and unearned premium reserves). Using the definition of the surplus, it would look like formula 2.1:

$$U_t = A_t - L_t. \quad (2.1)$$

with A_t = market value of the assets at year t and
 L_t = market value of the liabilities at year t .

An insurance company is affected by many things, such as inflation, market risk, credit risk, operational risk, etc.. All these risks can be implemented in the model, but to keep the model simple, these risks are not taken into account in this model.

The main variables in an insurance company that affect the assets and liabilities are the premiums, reserves, claimed losses, expenses and assets, such as stocks, bonds, real-estate, etc..

To keep the model simple and understandable, the earned premiums P_t , the market value of assets $\sum_i I_t(i)$, the expenses E_t , the losses paid Z_t and the

loss reserves R_t are taken into account. Other variables can also be inserted in the model, such as the issuing of new equity or taxes, but this would only make the model more difficult.

The following sections show how the assets, the liabilities, the earnings and the costs can be calculated.

2.1 Asset calculations

There are many sorts of assets, such as bonds, stocks, real-estate, cash, loans, etc.. New bonds can be emitted, stocks can be changed, etc.. To keep the model understandable, assume there are i sorts of assets in the portfolio, from which the market value are all calculated the same way, which will be explained in this section. Furthermore assume that there are no new assets obtained and no assets are lost, then A_t the value of an asset i in year t , can be calculated with formula 2.2:

$$A_t = \sum_i I_{t-1}(i) \cdot D_t(i) A_{t-1} + P_t - E_t - Z_t, \quad (2.2)$$

with $I_t(i)$ = the market value of asset i in year t ,
 $D_t(i)$ = the return on asset i in year t ,
 P_t = the earned premiums in year t ,
 E_t = the expenses in year t and
 Z_t = the losses paid in year t .

With formula 2.2 one could decide whether to keep the same portfolio as the year before or to change the portfolio, because the return on the portfolio is not as good as was presumed to be.

Of course the height of the earnings (earned premiums) and the costs (expenses and paid losses) also affect the amount of the assets. The pricing of the premiums, the composition of the expenses and the terms on which a claim will be paid all have effect on the asset amount of year t . What a company wants is to make profit in any way it can, this means that the premiums should be high enough to cover the expenses, the paid losses and a profit percentage.

When this is not the case, one should search the reason. In an unfortunately year, the reason could be a catastrophe. The possibility of a catastrophe should also be taken into account in the pricing of the premiums. Another

reason could be that the expenses that are made are too high, which can be solved by changing commission percentages or reorganise to work more efficiently and thus lowering the expenses. If the amount of paid losses is too high, then the company can think of refurbishing the policy.

The calculation of the premiums, the expenses and the paid losses will be discussed in sections 2.2, 2.3 and 2.5. The calculations of the market value of the assets will be discussed below.

The asset return $D_t(i)$ is dependent of the interest r_t in year t . A model is needed to calculate the asset return, which also keeps in mind the interest. There are several models which can be used, here the choice has fallen on a vector autoregressive model.

Suppose a vector Y_t , with

$$Y_t = \begin{pmatrix} D_t(i) \\ r_t \end{pmatrix}. \quad (2.3)$$

Then Y_t can be modelled with a vector autoregressive model:

$$Y_t = \mu^Y + \mathbf{A} \cdot Y_{t-1} + \epsilon_t^Y, \quad (2.4)$$

with $\mu^Y = \begin{pmatrix} \mu_{D_t(i)} \\ \mu_{r_t} \end{pmatrix}$, where $\mu_{D_t(i)}$ and μ_{r_t} are constants, which estimations can be based on historical data.

\mathbf{A} = a matrix with a Choleski decomposition of the covariance matrix of $D_t(i)$ and r_t , the covariance matrix is based on historical data,

$\epsilon_t^Y = \begin{pmatrix} \epsilon_{D_t(i)} \\ \epsilon_{r_t} \end{pmatrix}$, the error vector, with $\epsilon_{D_t(i)} \sim N(0, \sigma_{D_t(i)}^2)$ and $\epsilon_{r_t} \sim N(0, \sigma_{r_t}^2)$.

With the correct μ^Y , mean and covariance matrix (all based on historical data), the simulated scenarios will act the same way.

For each asset i the vector autoregressive model can be used, each having its own parameters.

The vector autoregressive model can also be modelled differently, taking along the whole portfolio in one model. In that case the correlation between the i sorts of asset are taken along as well. Depending the goal of the company, this second model can be used.

2.2 Premiums

In an insurance company, the written premiums are not the premiums that are really earned, the earned premiums. This is because a person does not have to take an insurance policy on the first of January, but can take a policy the whole year round. Another reason would be that a person only wants the policy for a short time (not a whole year) or does not like the policy. Many reasons can be thought of.

An insurance company also does not have to accept all the risk a client wants to insure. The part that an insurer does insure is called exposure. For written premiums this is the written exposure. There is also a percentage of written premiums that due to circumstances (think of people that do not pay) that is not earned. Therefore a company uses the earned premiums to calculate with. The earned premiums are a percentage of the written premiums.

For the written exposure a model is made. The model used here has three cases that can be distinguished:

- $j = 0$, exposure for new business,
- $j = 1$, exposure for renewal business, first renewal and
- $j = 2$, exposure for renewal business, second renewal and subsequent renewals.

There are three cases distinguished because the written exposure differs when a new product is thrown on the market. The second case and third case are modelled because the written exposure after the first renewal (thus for example the second year that the fairly new product is on the market) differs from the second and subsequent renewal. Perhaps more or less cases can be thought of (a product can take more years to be stable and have slightly different written exposures), but to keep the model simple, three cases are used.

It is logical that for each line of business k the premium and the written exposure are different. One could even consider that this is different for each product as well, but this is not taken into account in this model.

With the three cases for written exposure modelled, the written exposure itself ($w_t^j(k)$) in year t for line of business k with $j = 0, 1, 2$ can be modelled

with formula 2.5:

$$w_t^j(k) = (a^j(k) + b^j(k)w_{t-1}^j(k) + \epsilon_t^j(k))^+, \quad j = 0, 1, 2, \quad (2.5)$$

with $\epsilon_t^j(k) \sim N(0, \sigma^j(k)^2)$, $\epsilon_1^j(k), \epsilon_2^j(k), \dots$ i.i.d.
for line of business k and
 $a^j(k), b^j(k), \sigma^j(k)$ = parameters that can be estimated based
on historical data for line of business k .

The development of the exposure units follow an autoregressive process with order 1, AR(1). In order to keep the autoregressive process stationary, $b^j(k) < 1$ is assumed.

For the initial value $w_{t_0}^j(k)$ the current number of exposure units of each line of business k can be used.

In order to be able to calculate the earned premiums $P_t(k)$, the written premium $\tilde{P}_t^j(k)$ for each line of business k at time t has to be modelled first. Formula 2.6 is used to calculate the written premiums $\tilde{P}_t^j(k)$ for past years:

$$\tilde{P}_t^j(k) = \frac{w_t^j(k)}{w_{t-1}^j(k)} \tilde{P}_{t-1}^j(k), \quad j = 0, 1, 2, \quad (2.6)$$

with $w_t^0(k)$ = written exposure units for new business for line of business k ,
 $w_t^1(k)$ = written exposure units for renewal business, first renewal for
line of business k and
 $w_t^2(k)$ = written exposure units for renewal business, second renewal
and subsequent renewals for line of business k .

This model assumed there is no inflation and not taking into account the underwriting cycles.

In order to be able to calculate the written premiums $\tilde{P}_t^j(k)$ for the past years, an initial value of $\tilde{P}_t^j(k)$ has to be estimated. This is because the premium for an upcoming year in the future has to be determined before the year starts. The model for the written premium $P_t^j(k)$ that will be charged in year t for future simulated years is the following:

$$P_t^j(k) = \frac{\hat{w}_t^j(k)}{w_{t-1}^j(k)} \tilde{P}_{t-1}^j(k), \quad j = 0, 1, 2, \quad (2.7)$$

with $\hat{w}_t^j(k)$ calculated as in formula 2.5.

The initial value of $\tilde{P}_{t_0}^j(k)$ is the written premium of the year before the first projection year. This can be acquired with some calculations, but it is easier to use the value that is in the business plan of the company.

As written earlier, the earned premiums can be derived from the written premiums. The earned premium $P_t(k)$ can be calculated with formula 2.8:

$$P_t(k) = \sum_{j=0}^2 a_t^j(k) P_t^j(k) + (1 - a_{t-1}^j(k)) P_{t-1}^j(k), \quad (2.8)$$

with $a_t^j(k)$ = percentage of premium earned in year t , estimated based on historical data in renewal class j for line of business k .

Formula 2.8 is a formula for only one line of business. In formula 2.2 asks for the total of all the earned premiums, 2.9 can be used to calculate the total of earned premiums for all lines of business.

$$P_t = \sum_{k=1}^l \sum_{j=0}^2 a_t^j(k) P_t^j(k) + (1 - a_{t-1}^j(k)) P_{t-1}^j(k), \quad (2.9)$$

with k = the line of business and
 l = the number of lines of business.

2.3 Expenses

The expenses that are made by an insurance company lie in the administration, the selling, the processing of the claims, the commission costs, etc..

Assume that the expenses for a line of business k consist of a fixed amount of expenses $a^E(k)$ and a variable amount $b^E(k)$ that depends of the number of exposures $\sum_{j=0}^2 w_t^j(k)$, the expenses E_t can be calculated with formula 2.10:

$$E_t = \sum_{k=1}^l \left(a^E(k) + b^E(k) \sum_{j=0}^2 w_t^j(k) \right), \quad (2.10)$$

2.4 Liability calculations

The liabilities L_t can be given by formula 2.11:

$$L_t = R_t, \quad (2.11)$$

with R_t = loss reserves from year t .

Other reserves can also be taken into account. One can think of a reserve for unearned premiums or such. Before the loss reserve R_t can be calculated though, paid losses have to be calculated, because the paid losses are incorporated in the loss reserves. How to calculate the loss reserves are described in section 2.7.

2.5 Paid losses

Normally not all losses are paid out in the year that they happen (in the *accident year*), a part of the losses are paid out later. The losses of an accident year t_1 develop until all the claims of that accident year are paid (losses develop to the ultimate). A year that the losses of an accident year develop to another loss value is called a *development year* t_2 . Figure 2.1 shows this graphically.

An insurance company keeps in mind two sorts losses that can occur, losses from claims from normal business and losses from catastrophes. For past accident years the insurance company knows whether or not a catastrophic event has occurred, but for future years, which are going to be simulated, a catastrophe could happen. The simulation of the two sorts of losses can be done by modelling them together with one probability distribution or by modelling them separately. The latter has been chosen for this DFA model. How the modelling is done, will be described further in this section.

What the insurance company wants to know about the losses is how much the company has to pay in a year and what amount still has to be paid for that accident year. The total amount that has to be paid for an accident year is called the ultimate. This is the value an insurance company is interested in.

Suppose that current time is at the end of $t = t_0$. Then the paid losses for line of business k of accident year $t_1 = t_0$ and $t_2 = 0$ is known. For accident year $t_1 = t_0 - 1$ there are two values known, call them $Z_{t_1,0}(k)$ and

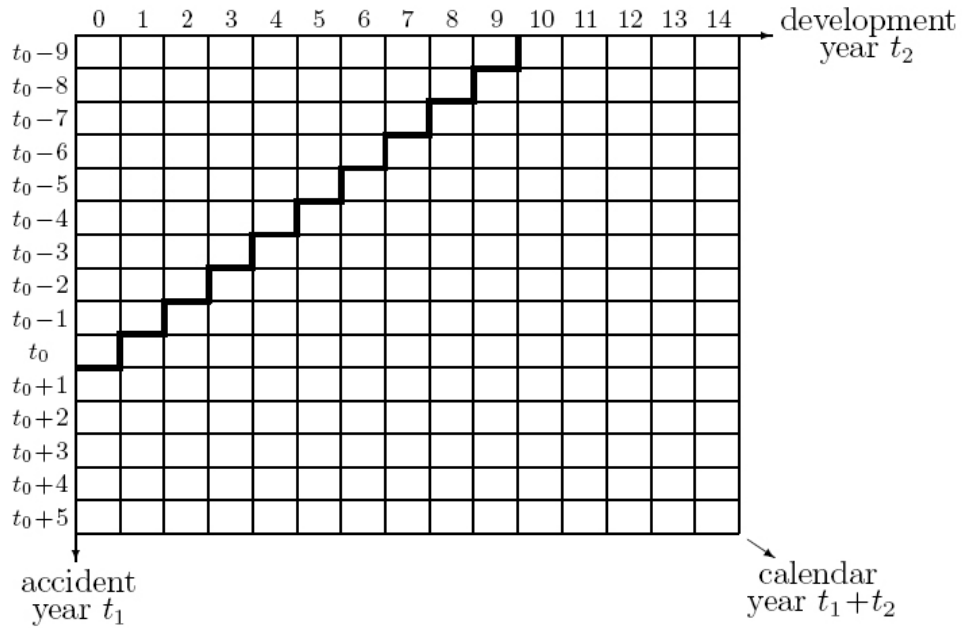


Figure 2.1: Paid losses

$Z_{t_1,1}(k)$. These two values are two payments that have been made for the same accident year (in this case for accident year $t_1 = t_0 - 1$, thus the two values should actually be given by $Z_{t_0-1,0}(k)$ and $Z_{t_0-1,1}(k)$). The two values apart are called incremental payments. When summing up both values, it is called the cumulative payment Z_{t_1} . This also means that for each accident year t_1 the cumulative payment can be given by:

$$Z_{t_1}(k) = \sum_{t_2} Z_{t_1,t_2}(k)$$

For past accident years ($t_1 \leq t_0$) there are incremental payments known (but not all). If those known incremental payments are put in a table, it is called an incremental loss triangle (because the values of the known payments have the shape of a triangle).

Suppose it takes $\tau(k)$ development years to reach the ultimate value of an accident year t_1 (in the insurance world the ultimate value is end-value, an accident year develops to the ultimate, thus there are no more payments for that accident year after that point). Also suppose that $\tau(k) > t_2$, then the incremental payments $Z_{t_1,t_2}(k)$ are known for the previous years ($t_1+t_2 \leq t_0$). For the future development years the incremental payments are not known. Define $d_{t_1,t_2}(k)$ as the loss development factor for line of business k , which is

given by:

$$d_{t_1, t_2}(k) = \frac{Z_{t_1, t_2}(k)}{\sum_{t=0}^{t_2-1} Z_{t_1, t}(k)}, \quad t_2 \geq 1. \quad (2.12)$$

The development factors $d_{t_1, t_2}(k)$ can be calculated for the previous years, but for the future values these development factors can not be calculated, thus have to be modelled. In this model a difference is made between development factors for past accident years ($t_1 \leq t_0$) and future accident years ($t_1 > t_0$), from which no payments are known yet.

Assume that $d_{t_1, t_2}(k)$ can be fitted with the lognormal distribution. Then for past accident years the incremental losses $Z_{t_1, t_2}(k)$ can be calculated with formula 2.13:

$$Z_{t_1, t_2}(k) = d_{t_1, t_2}(k) \sum_{t=0}^{t_2-1} Z_{t_1, t}(k), \quad (2.13)$$

with $d_{t_1, t_2}(k) \sim \text{lognormal}(\mu_{t_2}(k), \sigma_{t_2}(k)^2)$,

$\mu_{t_2}(k)$ = estimated logarithmic loss development factor for development year t_2 , based on historical data and

$\sigma_{t_2}(k)$ = estimated logarithmic standard deviation of loss development factor for development year t_2 , based on historical data.

Figure 2.2 shows which part is known (green), which part is calculated by the loss development factors $d_{t_1, t_2}(k)$ (blue) and which part is simulated in the DFA model (yellow).

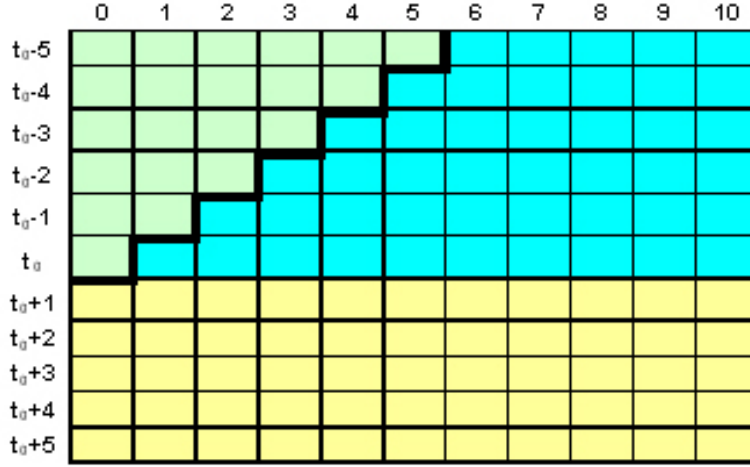


Figure 2.2: Calculation and simulation

With the incremental losses $Z_{t_1, t_2}(k)$ defined for past accident years $t_1 \leq t_0$, the ultimate loss amount $Z_{t_1}^{\text{ult}}(k)$ can be defined as:

$$Z_{t_1}^{\text{ult}}(k) = \sum_{t=0}^{\tau(k)} Z_{t_1, t}(k). \quad (2.14)$$

For future accident years $t_1 \geq t_0 + 1$ the ultimate loss amount $Z_{t_1}^{\text{ult}}(k)$ can be defined by formula 2.15

$$Z_{t_1}^{\text{ult}}(k) = \sum_{j=0}^2 N_{t_1}^j(k) X_{t_1}^j(k) + b_{t_1}(k) \sum_{i=1}^{M_{t_1}(k)} Y_{t_1, i}(k) - S_{t_1}(k), \quad (2.15)$$

with $N_{t_1}^j(k)$ = number of non-catastrophe losses in accident year t_1 for line of business k and renewal class j ,

$X_{t_1}^j(k)$ = severity of non-catastrophe losses in accident year t_1 for line of business k and renewal class j ,

$b_{t_1}(k)$ = market share of the company in year t_1 for line of business k ,

M_{t_1} = number of catastrophes in accident year t_1 ,

$Y_{t_1, i}^k$ = severity of catastrophe i in line of business k in accident year t_1 and

$S_{t_1}(k)$ = reinsurance recoverables.

Non-catastrophe losses can be calculated by the multiplication of the number of non-catastrophe losses $N_t^j(k)$ and the mean severity of the losses X_t^j for period t , renewal class j and line of business k , where $X_t^j(k)$ is given by formula 2.16:

$$\mathbb{E}X_t^j(k) = \frac{1}{N_t^j(k)} \sum_{i=1}^{N_t^j} X_t^j(i, k). \quad (2.16)$$

Loss numbers and the severity of the losses can be fitted to a distribution for each line of business, but assume that the number of non-catastrophe losses can be fitted with a negative binomial distribution and the severity of losses with a gamma distribution.

Let $\mu^{F,j}(k)$ and $\sigma^{F,j}(k)$ be the mean and standard deviation for the loss frequencies and $\mu^{X,j}(k)$ and $\sigma^{X,j}(k)$ the mean and standard deviation for the loss severities for renewal class j and line of business k .

The reason why loss frequencies are introduced is because loss numbers can be calculated by using loss frequencies and loss frequencies are more stable than loss numbers, thus better to use in modelling.

The loss numbers have a negative binomial distribution with parameters a and p , thus:

$$N_t^j(k) \sim NB(a(k), p(k)), \quad j = 0, 1, 2, \quad (2.17)$$

and $N_1^j(k)$, $N_2^j(k)$, ..., independent. Assume the existence of a mean $m_t^{N,j}(k)$ and variance $v_t^{N,j}(k)$. Then $a(k)$ and $p(k)$ can be calculated with formula 2.18 and formula 2.19.

$$m_t^{N,j}(k) = \mathbb{E}[N_t^j(k)] = \frac{a(k)(1-p(k))}{p(k)} \quad (2.18)$$

$$v_t^{N,j}(k) = \text{Var}(N_t^j(k)) = \frac{a(k)(1-p(k))}{p(k)^2} \quad (2.19)$$

with $m_t^{N,j}(k) = w_t^j(k)\mu^{F,j}(k)$,
 $v_t^{N,j}(k) = (w_t^j(k)\sigma^{F,j}(k))^2$,
 $w_t^j(k)$ = written exposure units (see formula 2.5),
 $\mu^{F,j}(k)$ = estimated loss frequency, based on historical data and
 $\sigma^{F,j}(k)$ = estimates standard deviation of loss frequency, based on historical data.

The values of $(N_t^j(k))$ are assumed realistic when $\text{Var}(N_t^j(k)) \geq \mathbb{E}[N_t^j(k)]$, because negative binomial distributions act this way.

The severity of losses is assumed to be Gamma distributed with parameters $\alpha(k)$ and $\theta(k)$, thus:

$$X_t^j(k) \sim \text{Gamma}(\alpha(k), \theta(k)), \quad j = 0, 1, 2, \quad (2.20)$$

with $X_1^j(k), X_2^j(k), \dots$ independent. Assume the existence of a mean $m_t^{X,j}(k)$ and variance $v_t^{X,j}(k)$. Then $\alpha(k)$ and $\theta(k)$ can be calculated with formula 2.21 and formula 2.22:

$$m_t^{X,j}(k) = \mathbb{E}[X_t^j] = \alpha\theta, \quad (2.21)$$

$$v_t^{X,j} = \text{Var}(X_t^j) = \alpha\theta^2, \quad (2.22)$$

with $m_t^{X,j}(k) = \mu^{X,j}(k)$,
 $v_t^{X,j}(k) = (\sigma^{X,j}(k))^2$,
 $\mu^{X,j}(k)$ = estimate mean severity, based on historical data,
 $\sigma^{X,j}(k)$ = estimate standard deviation of the severity, based on historical data.

The loss amount for non-catastrophe can be calculated by multiplying the number of losses with the severity, for line of business k this can be calculated with:

$$\sum_{j=0}^2 N_t^j(k) X_t^j(k). \quad (2.23)$$

Formula 2.23 is incorporated in formula 2.15.

Catastrophical losses are a result of catastrophes. Catastrophes happen thus have to be modelled as well. Assume the number of catastrophes $M_t(k)$ that

happen in year t for line of business k . Assume that $M_t(k)$ have no trends and have a distribution (negative binomial, poisson, binomial,...) with mean $m^M(k)$ and variance $v^M(k)$:

$$M_t(k) \sim \text{NB, Pois, Bin, ...} (m^M(k), v^M(k)), \quad (2.24)$$

with M_1, M_2, \dots i.d.d.

$m^M(k)$ = estimated number of catastrophes, based on historical data,
 $v^M(k)$ = estimated variance, based on historical data.

Let $Y_{t,i}$ be the total economic loss caused by catastrophe i in year t , with $i \in \{1, \dots, M_t\}$. Then the distribution of $Y_{t,i}$ can be given by:

$$Y_{t,i} \sim \text{lognormal, Pareto, ...} (\mu_t^Y, (\sigma_t^Y)^2), \quad (2.25)$$

with $Y_{t,1}, Y_{t,2}, \dots$ i.d.d.,

$Y_{t_1, i_1}, Y_{t_2, i_2}$ independent $\forall (t_1, i_1) \neq (t_2, i_2)$,
 μ^Y = estimate of the total economic loss, based on historical data,
 σ^Y = estimate standard deviation, based on historical data.

The percentage of total economic loss $Y_{t,1}$ that has effect on line of business k can now be calculated by introducing a variable $a_{t,i}(k)$ and using formula 2.26:

$$Y_{t,i}(k) = a_{t,i}(k) Y_{t,i} \quad (2.26)$$

with k = line of business,

l = total number of lines considered.

$a_{t,i}(k)$ can be simulated, but it is easier to estimate this percentage for each line of business, based on historical data.

In formula 2.15 the market share $b_t(k)$ of the company in year t for line of business k is also used. This variable is a percentage as well and is used to calculate the loss amount of a catastrophe the company has to pay. This means that with $a_{t,i}(k)$ the total economic loss is divided in k portions and with $b_{t_1}(k)$ the company's loss is calculated.

Formula 2.15 also considers the possibility that a reinsurer takes a part of the losses (non-catastrophical and catastrophical) for its account. Depending on what kind of reinsurance contracts the company has, for each line of business k and accident year t_1 , a formula for the reinsurance amount $S_{t_1}(k)$ can be made.

2.6 Discount rate

For long term interest rates one could use a constant interest rate or a use a yield curve to estimate the interest rate. To keep the model simple, this model assumes a constant discount rate, which is called $H_{t,T}$. Where $H_{t,T}$ is the T year discount rate at time t .

2.7 Loss Reserves

An insurance company keeps loss reserves in order to be able to pay out losses for accident years that have not yet reached its ultimate. That is why the insurance company keeps enough assets to be able to maintain this reserve. Reading the first line of this section would mean that the loss reserves can be calculated by the difference of the ultimate and the sum of the incremental payments that already have been paid. This is almost true, but not totally, because there are assets maintaining the balance, thus the reserve can be discounted with a risk free rate.

In order to calculate the discounted loss reserves, the incremental payments of the ultimate loss amounts have to be calculated. How to calculate the incremental payments of the ultimate loss amounts of calendar year $t_1 + t_2 \geq t_0 + 1$ for accident years $t_1 \leq t_0$ is explained in formula 2.13. The incremental payments for previous years $t_1 + t_2 \leq t_0$ are known. This leaves the calculation of the incremental payments for future ultimate loss amounts.

Assume that the incremental payments for future accident years $t_1 > t_0$ in development year t_2 can be calculated with formula 2.27:

$$Z_{t_1,t_2}(k) = A_{t_1,t_2}(k)Z_{t_1}^{\text{ult}}(k), \quad (2.27)$$

with $A_{t_1,t_2}(k)$ the incremental percentage of the future ultimate loss amount for line of business k . $A_{t_1,t_2}(k)$ can be simulated with a probability distribution with parameters based on payment patters of previous calendar years.

Roger Kaufmann, Andreas Gadmer and Ralf Klett (2001) chose the beta distribution to simulate $A_{t_1, t_2}(k)$:

$$A_{t_1, t_2}(k) = \begin{cases} B_{t_1, 0}(k), & \text{for } t_2 = 0, \\ B_{t_1, t_2}(k) \left(1 - \sum_{t=0}^{t_2-1} A_{t_1, t}(k)\right), & \text{for } t_2 \geq 1, \end{cases} \quad (2.28)$$

with $B_{t_1, t_2}(k) \sim \text{beta}(\alpha(k), \beta(k))$, with $\alpha(k) > -1$ and $\beta(k) > -1$,
 incremental loss payment factor due to accident year t_1 in
 development year t_2 for line of business k in relation to the
 sum of remaining incremental loss payments concerning the
 same accident year.

$\alpha(k)$ and $\beta(k)$ can be calculated by using formula 2.29 and formula 2.30:

$$m_{t_1, t_2}(k) = \mathbb{E}[B_{t_1, t_2}] = \frac{\alpha(k) + 1}{\alpha(k) + \beta(k) + 2} \quad (2.29)$$

$$v_{t_1, t_2}(k) = \text{Var}(B_{t_1, t_2}(k)) = \frac{(\alpha(k) + 1)(\beta(k) + 1)}{(\alpha(k) + \beta(k) + 2)^2(\alpha(k) + \beta(k) + 3)} \quad (2.30)$$

with $m_{t_1, t_2}(k)$ = estimated mean value of incremental loss payments due to
 accident year t_1 in development year t_2 for line of business
 k in relation to the sum of remaining incremental loss
 payments concerning the same accident year, based on

$$\frac{A_{t_1-1, t_2}}{\sum_{t=t_2}^{\tau(k)} A_{t_1-1, t}}, \frac{A_{t_1-2, t_2}}{\sum_{t=t_2}^{\tau(k)} A_{t_1-2, t}}, \dots,$$

$v_{t_1, t_2}(k)$ = estimated variance, based on the same historical data as
 the mean.

Sometimes no solution can be found where $\alpha(k) > -1$ and $\beta(k) > -1$. Then
 another probability distribution has to be used, which can be implemented
 as well.

For each accident year t_1 the estimated ultimate claim amount in each development year t_2 for line of business k can be calculated with formula 2.31:

$$\hat{Z}_{t_1, t_2}^{\text{ult}}(k) = \prod_{t=t_2+1}^{\tau(k)} (1 + e^{\mu_t(k)}) \sum_{t=0}^{t_2} Z_{t_1, t}(k), \quad (2.31)$$

with $\mu_t(k)$ = estimated logarithmic loss development factor for year t for line of business k , based on historical data,
 $Z_{t_1, t}(k)$ = simulated losses for accident year t_1 for line of business k , to be paid in development year t , see formula 2.13 and formula 2.27.

The reserves of accident year t_1 at the end of calendar year $t_1 + t_2$ can be calculated by the difference of the discounted ultimate claim amount $\hat{Z}_{t_1, t_2}^{\text{ult, disc}}(k)$ and the losses that are already paid for that accident year t_1 for each line of business k . $\hat{Z}_{t_1, t_2}^{\text{ult, disc}}(k)$ can be calculated with formula 2.32:

$$\hat{Z}_{t_1, t_2}^{\text{ult, disc}}(k) = \left(1 + e^{-H_{t_1+t_2, 1}} e^{\mu_{t_2}(k)+1} + \sum_{s=t_2+2}^{\tau(k)} e^{-H_{t_1+t_2, s-t_2}} e^{\mu_s(k)} \prod_{t=t_2+1}^{s-1} (1 + e^{\mu_t(k)}) \right) \cdot \sum_{t=0}^{t_2} Z_{t_1, t}(k), \quad (2.32)$$

with $H_{t, T}$ = the T year discount rate at time t , for see paragraph 2.6
 $\mu_t(k)$ = estimated logarithmic loss development factor for year t for line of business k , based on historical data,
 $Z_{t_1, t}(k)$ = simulated losses for accident year t_1 , paid in development year t for line of business k , see formula 2.13 and formula 2.27

With the discounted ultimate claim amount and the losses paid to date calculated, the total of loss reserves R_t can be given with formula 2.33:

$$R_t = \sum_{k=1}^l \sum_{t_2=0}^{\tau(k)} \left(\hat{Z}_{t-t_2, t_2}^{\text{ult, disc}}(k) - \sum_{s=0}^{t_2} Z_{t-t_2, s}(k) \right), \quad (2.33)$$

2.8 Risk measurement

At the beginning of this chapter two measurements were mentioned that are used in a DFA model, the surplus and the risk. What the insurance company wants to know is what return (surplus) can the company get at which risk? Take the expected surplus in year t as the return measure, thus $\mathbb{E}[U_t]$ and take $\mathbb{P}(U_t < 0)$ as the risk measure.

Having chosen the two measurements still leaves open which strategy is the best strategy.

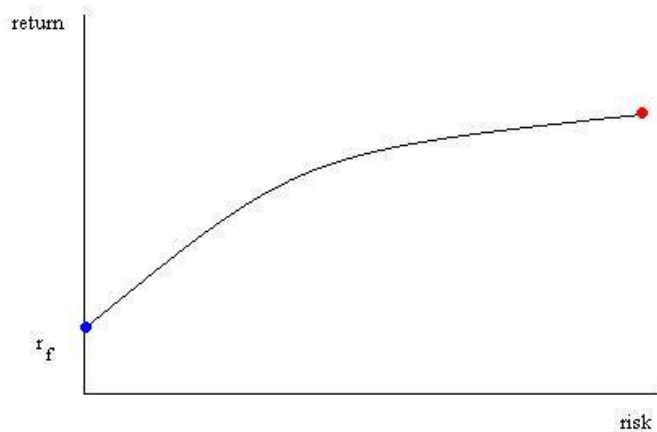


Figure 2.3: Return versus risk

When a company wants to have the highest return (the red dot in figure 2.3), then the best thing the company can do to acquire this return is to change the portfolio to a stock only portfolio. As shown in the figure, the highest return does have the most risk, thus this choice would be questionable.

The opposite of the wanting the highest return is wanting the lowest risk possible (the blue dot in figure 2.3). This can be acquired by reinsuring everything, but then no profit will be made and a company should ask itself why be an insurance company at all?

Simulating with the DFA will offer many strategies that can all be put in a

diagram such as in figure 2.4.

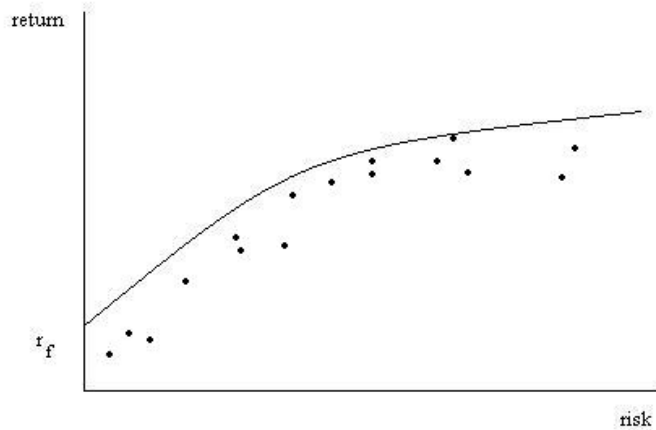


Figure 2.4: Strategies

A way to choose the optimal strategy is by setting a value θ for which the risk measure $\mathbb{P}(U_t < 0) \leq \theta$ and choosing the best return that fits with that probability.

Of course the company should also look what really happens in a strategy. An optimal strategy is only optimal when the variables within the strategy are realistic. The outcome of a strategy gives a portfolio combination with a certain return. It also gives an amount of claims that have fallen. Based on this outcome decisions are made, such as portfolio changes, changing the premium or changing the reinsurance contracts. This means that the choosing of the optimal strategy should be done with care.

2.9 Example calculation of the loss reserve

Assume the numbers in figures 2.5 and 2.6, that there is only one line of business and $\tau = 10$ and the discount rate is $H_{t,T} = 2\%$ for all T . What is the loss reserve for accident year $t_0 - 5$ when $t_2 = 5$ (thus the current time is t_0)?

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|-----------|-----|-----|----|----|----|----|---|---|---|---|----|
| t_{0-5} | 200 | 75 | 75 | 50 | 50 | 50 | | | | | |
| t_{0-4} | 197 | 95 | 79 | 47 | 53 | | | | | | |
| t_{0-3} | 204 | 103 | 85 | 43 | | | | | | | |
| t_{0-2} | 209 | 96 | 60 | | | | | | | | |
| t_{0-1} | 206 | 85 | | | | | | | | | |
| t_0 | 189 | | | | | | | | | | |
| t_{0+1} | | | | | | | | | | | |
| t_{0+2} | | | | | | | | | | | |
| t_{0+3} | | | | | | | | | | | |
| t_{0+4} | | | | | | | | | | | |
| t_{0+5} | | | | | | | | | | | |

Figure 2.5: A number example

| | $t=6$ | $t=7$ | $t=8$ | $t=9$ | $t=10$ |
|---------|--------|--------|--------|--------|--------|
| μ_t | -1.897 | -2.442 | -2.749 | -3.099 | -3.548 |

Figure 2.6: Loss development factor

$$\hat{Z}_{t_0-5,5}^{ult,disc} = \left(1 + e^{-H_{t_0,1}}e^{\mu_6} + \sum_{s=7}^{\tau} e^{-H_{t_0,s-5}}e^{\mu_s} \prod_{t=6}^{s-1} (1 + e^{\mu_t}) \right) \sum_{t=0}^5 Z_{t_0-5,t}$$

Rewriting this formula out gives:

$$\begin{aligned} \hat{Z}_{t_0-5,5}^{ult,disc} = & \left(1 + e^{-H_{t_0,1}}e^{\mu_6} + (e^{-H_{t_0,2}}e^{\mu_7}(1 + e^{\mu_6})(1 + e^{\mu_7})) \right. \\ & + (e^{-H_{t_0,3}}e^{\mu_8}(1 + e^{\mu_7})(1 + e^{\mu_8})) + (e^{-H_{t_0,4}}e^{\mu_9}(1 + e^{\mu_8})(1 + e^{\mu_9})) \\ & \left. + (e^{-H_{t_0,5}}e^{\mu_{10}}(1 + e^{\mu_9})(1 + e^{\mu_{10}})) \right) \sum_{t=0}^5 Z_{t_0-5,t} \end{aligned}$$

Calculating the parts give:

$$\sum_{t=0}^5 Z_{t_0-5,t} = 200 + 75 + 75 + 50 + 50 + 50 = 500,$$

$$1 + e^{-H_{t_0,1}}e^{\mu_6} = 1 + e^{-0.02}e^{-1.897} = 1.147,$$

$$e^{-H_{t_0,2}}e^{\mu_7}(1 + e^{\mu_6})(1 + e^{\mu_7}) = e^{-0.02}e^{-2.442}(1 + e^{-1.897})(1 + e^{-2.442}) = 0.107,$$

$$e^{-H_{t_0,3}}e^{\mu_8}(1 + e^{\mu_7})(1 + e^{\mu_8}) = e^{-0.02}e^{-2.749}(1 + e^{-2.442})(1 + e^{-2.749}) = 0.073,$$

$$e^{-H_{t_0,4}} e^{\mu_9} (1 + e^{\mu_8}) (1 + e^{\mu_9}) = e^{-0.02} e^{-3.099} (1 + e^{-2.749}) (1 + e^{-3.099}) = 0.049,$$

$$e^{-H_{t_0,5}} e^{\mu_{10}} (1 + e^{\mu_9}) (1 + e^{\mu_{10}}) = e^{-0.02} e^{-3.548} (1 + e^{-3.099}) (1 + e^{-3.548}) = 0.030.$$

Then $\hat{Z}_{t_0-5,5}^{\text{ult, disc}} = (1.147 + 0.107 + 0.073 + 0.049 + 0.030) \cdot 500 = 702, 81$.

The loss reserve in year t_0 for accident year $t_0 - 5$ is:

$$R_{t_0}(t_0 - 5) = 702, 55 - 500 = 202, 81$$

The remaining part of the loss reserve can be calculated the same way.

When the model had used a different discount rate for each year (following a term structure or another model), the loss reserve would have looked differently.

| | $H_{t_0, T}$ |
|-------|--------------|
| T = 1 | 2.0% |
| T = 2 | 2.2% |
| T = 3 | 2.4% |
| T = 4 | 2.6% |
| T = 5 | 2.8% |

Figure 2.7: Variable discount rates

When the discounting rates are as in figure 2.7, $\hat{Z}_{t_0-5,5}^{\text{ult, disc}} = 702.29$ and the loss reserve in year t_0 for accident year $t_0 - 5$ would be $R_{t_0}(t_0 - 5) = 202, 29$.

The difference between the two methods does not look substantial, but when there are thousands of claims in a year, then the discounted ultimates are in the millions and the reserves are considerable as well, thus the difference will be more sizable as well.

I chose to keep the model simple, but an insurance company that has more feeling with a DFA model should consider to implement a more elaborate discount rate model.

Chapter 3

Conclusion

This simple DFA model shows how a DFA works. Even though many variables and effects were not taken into account, it roughly gives an idea of the scope of what a DFA could do.

With the help of the outcome of the DFA model certain decisions can be made to change that outcome. The insurance company could decide to change the partition of the portfolio or change the height of the premiums based on the outcome of A_t . A decision can be made to change the reinsurance contracts, because the calculations the values of the future $Z_{t_1}^{\text{ult}}(k)$ indicate that in the future more (or less) reinsurance is needed than the current reinsurance contracts cover. An insurance company can even decide to change their business strategy after reading the outcome, because it indicates that a certain line of business has much more potential than considered. The table below shows the most important variables, their drivers and what decision can be made. It also shows that from the variables that are taken into account (the earned premiums, the market value of assets, the expenses, the losses paid and the loss reserves), the only variable with no real added value to the DFA model is the expenses.

| variable | drivers | decisions |
|-----------------|--|--|
| asset return | financial market | → change of portfolio |
| discount rate | financial market | → create a security margin on the loss reserve |
| claims | accident realisation & catastrophes | → change the premium → reinsurance |

This DFA model shows that it is better not to use a constant discount rate, because the discount rate affects the height of the loss reserves, thus the liabilities.

The model also gives a way to optimise the return against the risk. Even

though when taking other measurements for the return or risk, probably another way has to be used, it gives an idea of how to optimise these measures. When looking at the outcome of the risk and return, a company can search for certain points. A company can conclude that after many simulations a certain portfolio always comes out well within the strategies even when other variables, such as the discount rate, change. The company could then decide to take such a portfolio. Based on the return and the risk outcomes, the best strategy can affect a company's decision whether or not to change a portfolio or reinsurance contract or the premium policies.

This DFA does not take all risks into account that are required by the Dutch supervisor, but they can be modelled. With the probability distribution of the surplus, the solvency can be calculated. When $\mathbb{P}(U_t < 0)$, the company is considered insolvent. The supervisor also want to know the volatility of those risks.

This DFA misses the inflation risk, credit risk, raw material risk (if the company has raw materials), currency risk, insurance-technical risk, concentration risk and operational risk. When implementing these risks into the a DFA and the company can calculate the volatility of all the risks, then the supervisor would be satisfied.

Some remarks on DFA models in general:

- DFA models try to simulate the future, but as elaborate as the model is, a DFA can never simulate the real world with all its complexities.
- DFA is a model with assumptions. These assumptions can be wrong. Always validate the model.
- DFA requires calibration. Calibrating values takes time.
- A DFA can be made as elaborate as a company wants, but before expanding the model, be sure to understand the existing model and the effect the expanding has.

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