

# **COUNTERPARTY CREDIT RISK**

**BMI MASTERS THESIS**

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## **Preface**

This master's thesis is part of the BMI curriculum that is required to be delivered by the student in order to complete the program. A BMI thesis is basically a research project around a specific problem statement. The thesis is based on already available literature. However, the student can make use of computer generated data and simulation results. The thesis is written for an expert manager who has a general expertise in the subject area. It is assumed that the thesis has a practical benefit for the manager.

I would like to thank my supervisor, Erik Winands, for helping me choose the topic and guiding me throughout the writing of this thesis. I thank also Drs. Annemieke van Goor-Balk for providing me with necessary information and help facilitate my work.

## **Abstract**

One of the risks banks face is counterparty credit risk, which is the risk that results when a counterparty is unable or unwilling to meet agreed obligations. In particular, banks involved in over-the-counter (OTC) securities and derivatives transactions face this risk.

In light of the current global financial crisis, which resulted in the bankruptcy of large banks, it is of great importance to give more attention to methods that help mitigate counterparty credit risk as well as to the modeling, measuring and pricing of this risk. According to IMF's Global Financial Stability Report (2008), there is a persistent and increasing concern about counterparty credit risks (CCR). This risk has increased significantly threatening the existence of big banks in a chain reaction as a result of a default of a counterparty.

Financial institutions are required to have a minimum capital to shield against the default risk. Hence modeling CCR is important in order to determine the appropriate economic capital needed.

In this thesis I will discuss in brief recent works about the modeling and pricing of CCR. This includes bringing together different modeling and measuring methods both at counterparty as well as portfolio level. The thesis also discusses the minimum required capital when one engages in over-the-counter (OTC) derivative contracts and techniques used to reduce exposure to this risk.

The thesis has four main parts followed by a conclusion. In the first part Counterparty Credit Risk is described. Some OTC products will also be briefly discussed. Finally the risk measures used are defined. The second part introduces the general modeling and measuring of Counterparty Credit Risk and describes or analyzes the difference on the models used. Although it will not be in depth analysis models both at a counterparty levels and portfolio level will be presented. In addition to that a risk mitigating techniques in practice will be highlighted. In the third part I will focus on the credit derivative product called credit default swap (CDS). Here a recent model for CDS will be presented. Also pricing of the CDS using Monte Carlo simulation is discussed. The fourth part discusses the economic capital (EC), a measure of counterparty credit risk, followed by a brief summary of the Basel II treatment of OTC derivatives.



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# I. Counterparty Credit Risk

## I.1. Description

Financial institutions that are engaged in over-the-counter (OTC) securities and derivatives transactions face counterparty credit risk (CCR), which is the risk that results when a counterparty to a financial contract defaults before the contract expires. A counterparty is said to be in default if he is unable or unwilling to meet agreed up on obligations while the contract is having a positive value to the other party. It is therefore very important to measure this risk exposure and determine its impact on the firms.

Exposure (E) is measured either at a contract level or at counterparty (portfolio) level. If the counterparty to our firm defaults at time t in the future before the contract expires, our firm's exposure is given in general by (Pykhtin, 2008):

$$E_i(t) = \max[V_i(t), 0], \text{ for a contract level and}$$

$$E(t) = \sum_i \max[V_i(t), 0], \text{ for portfolio of the contracts with the counterparty,}$$

where:

i stands for the ith contract

V stands for the replacement cost of the contract (contracts value to our firm at t).

If the exposure is negative however, we have to pay this amount to the defaulting counterparty.

The future value of OTC derivatives portfolio is uncertain and changes as a function of market variables such as interest rates or exchange rates. This implies that the counterparties risk exposure varies over time for the same portfolio.<sup>1</sup>

Basically there are three types of firms that have this risk. Identifying to which group a firm belongs is important in modeling the counterparty risk exposure. Large derivative market maker engages in different types and positions of OTC derivative contracts with a large number of counterparties. Not every counterparty at a given time may have a positive exposure to the market maker. The other firm type enters one or a few derivative contracts (holding the same position) for the hedging purpose of a single market rate. In between these two types of firms are market participants. CCR also arises in Securities Financing Transactions (SFTs) such as repurchase agreements (repos), reverse repurchase agreements (reverse repos), securities borrowing and lending by these firms.<sup>2</sup> Repo consists of the borrowing and selling of government securities with the obligation to buy it back for a greater price in the future. The party at the other side of this agreement who agrees to resell the bought securities at a specific future date enters the reverse repo agreement. But this thesis discusses only CCR associated with OTC derivatives.

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<sup>1</sup> Measuring Counterparty Credit Exposure to a margined Counterparty, Michael S. Gibson

<sup>2</sup> Calculating and Hedging Exposure, Credit Value Adjustment and Economic Capital for Counterparty Credit risk, Evan Picoult

## **I.2. OTC securities<sup>3</sup>**

Over-the-counter (OTC) derivatives are derivatives whose transactions don't occur in a standard exchange facility. They are direct contracts between two parties. They are tailored in accordance with the wishes of the counterparties. The CCR associated with these contracts affects both parties to the contract. The OTC derivatives are constructed by applying either forwards, swaps or options on foreign currency exchanges, credit instruments, interest rates, equities and commodities.

### **Swaps**

In a swap contract counterparties exchange a series of cash flows based on the notional principal amount. The cash flows depend on random variables such as equity price or interest rate and the exchange can be tailored almost in any manner that suits the counterparties. A swap can be used for the purpose speculating on the direction of the market or hedging risks without liquidating the underlying asset or liability.

### **Forward Contracts**

A forward contract is an OTC instrument and is an agreement today between two parties to buy or sell an asset for a specified amount of forward price determined in the contract at a specific date in the future. The payoff to the parties, which is a premium for one party and discount for the other, will be the difference between the spot price at the settlement date and the forward price. Like swaps forward contracts can be used to hedge risk or speculate on the future value of the underlying asset.

### **OTC option**

An option contract gives one party to the contract the right but not the obligation to buy from or to sell to the other party the underlying instrument. The terms to the contract include the exercise price, expiry date and possible times of exercising the transactions. OTC options are more flexible than the normal in standard exchange facilities traded options and can be tailored to satisfy the counterparties needs in very different types of underliers.

### **Foreign exchange contracts**

Although the notional amount outstanding of these contracts is increasing, their share of the total OTC derivative market is declining. According to BIS report, the percentage shares were 30% in '98, 14% in 2004 and 11% in 2007. The report breaks down the FX derivatives in order of decreasing notional amounts outstanding into forwards and forex swaps, currency swaps and options.

Currency swap involves the exchange of cash flows denominated in different currencies based on interest rates to the underlying obligations. Both the principal and interest payments are exchanged.

### **Interest rate contracts**

These contracts include in order of market share forward rate agreements (FRA), Interest rate swaps and Options. According to Bank for International Settlements (BIS)

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<sup>3</sup> The discussions of the OTC derivatives is based among others on:  
<http://www.riskglossary.com/>, <http://www.investopedia.com/> and <http://en.wikipedia.org/>



report<sup>4</sup> interest rate contracts hold 67% of total notional amounts and 45% of total gross market positions in OTC derivatives market in June 2008. BIS explains this discrepancy as a result of longer maturities of these contracts. Notional amounts outstanding are the total nominal absolute value of all the OTC contracts that are not yet settled. They provide information about the cumulative amount of business activities and serve as reference to determine the contractual payments. But these amounts are generally not indicative of the value at risk. Gross market positions indicate the net position on the market of all positive and negative value contracts of the OTC product category on the date of the reporting. Here the value of the contract equals its replacing cost on the market. Therefore the gross market values are more accurate indicators of the scale of the value that is at risk.

FRA between counterparties is a forward contract that determines the rate of interest to be paid or received on an obligation in the future. Typically the agreement involves an exchange of a fixed rate with a variable reference rate. The final payments over a period are netted.

An interest rate swap contract involves the exchange of cash flows in the same currency in the future. The payment made by one of the counterparties is dependent on a fixed interest rate while the other party pays in relation to a floating rate based on a specified principal amount. In contrast with currency swaps, interest rate swap cash flows occurring on the same dates are netted.

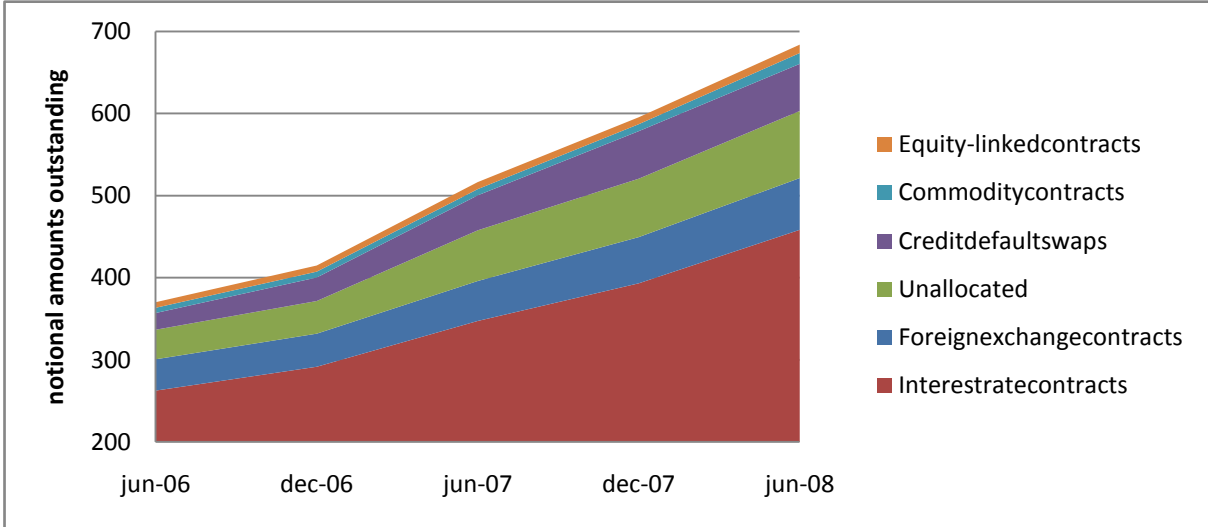


Figure 1: OTC derivatives notional amounts outstanding (in billions) based on BIS statistic

**Equity-linked contracts**

<sup>4</sup> <http://www.bis.org/statistics/otcder/dt1920a.pdf>

Equity derivatives are instruments with underlying assets based on equity securities. They include forwards and swaps, and OTC Options. The most common of these derivatives is equity OTC option. According to BIS report the notional amounts outstanding at the end of June 2008 of OTC equity derivatives more than doubled to \$10 trillion. The share of these options was at \$7.5 trillion while that of equity forwards and swaps amounted \$2.6 trillion.

### **Commodity contracts**

Commodity derivatives are linked to the price of the underlying commodity prices. These contracts include OTC derivatives on gold and Forwards & swaps and Options on other commodities, Although the relative importance of derivatives on gold decreased, the other contracts has increased significantly to a total notional amounts outstanding \$12.6 trillion at the end of June 2008 from 6 trillion in June 2006.

### **Credit derivatives**

Credit derivatives are contracts that depend on the default behavior of the counterparty. In other words, the underlying credit and the counterparty are positively correlated.<sup>5</sup> Credit default swaps (CDS) are by far the largest instrument in this category. The notional amounts outstanding in credit derivatives stood at \$57.3 trillion at end of June 2008.

## **I.3. Risk measures / indicators<sup>[6][7][8]</sup>**

The two main reasons for measuring CCR are the need to limit the risk level to the counterparties as well as the need to determine the proper amount of reserve capital to cushion the firm from potential danger in case the risk materializes. The potential future exposure (PFE), which gives the maximum counterparty exposure at a future date, is the most common exposure measure used to limit the risk level. The other important future counterparty risk measure is the expected positive exposure (EPE). It is the most common exposure measure used in calculating the economic capital. It gives time weighted average exposure of the counterparty within a given horizon time. In this section I will give a formal definition of these and other measures/indicators used to manage the risk exposure of the counterparties in an OTC derivative contract and discuss them briefly.

The first four measures are concerned about the extent of counterparty risk exposure. Knowing them is the first step in managing CCR. The other measures give the

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<sup>5</sup> Modeling counterparty credit exposure for credit default swaps, Christian t. hille, john ring, hideki shimamoto

<sup>6</sup> The discussions of the Risk measures/indicators in addition to the reports by BIS and Basel II is based on:  
<http://www.riskglossary.com/>, <http://www.investopedia.com/> and <http://en.wikipedia.org/>

<sup>7</sup> Counterparty Credit Risk Modeling: Risk Management, Pricing and Regulation, Edited by Michael Pykhatin, 2005

<sup>8</sup> Basel Committee on Banking Supervision, The Application of Basel II to Trading Activities and the Treatment of Double Default Effects, July 2005

distribution of default by the counterparties and quantify the real losses that the counterparties face based on the risk measures from above and the default distributions.

### Potential Future Exposure (PFE)

PFE quantifies the sensitivity of CCR to future changes in the market prices or rates as a percentile of the distribution of CCR exposure. The Basel Committee on Banking Supervision (BCBS) defines it as:

- the maximum positive exposure estimated to occur on a future date at a high level of statistical confidence. Banks often use PFE when measuring CCR exposure against counterparty credit limits.

The most important application of PFE is in OTC derivative contract's approval against CCR exposure limits and the determination of the EC.

### Expected Exposure (EE)

BCBS defines it as:

- the probability-weighted average exposure estimated to exist on a future date.
- Effective Expected Exposure at a specific date is the maximum expected exposure that occurs at that date or any prior date.

Note that only the positive exposures (E) on the given date are averaged as shown in the figure<sup>9</sup> below. The average of the negative exposures is the expected exposure of the other counterparty to the contract.

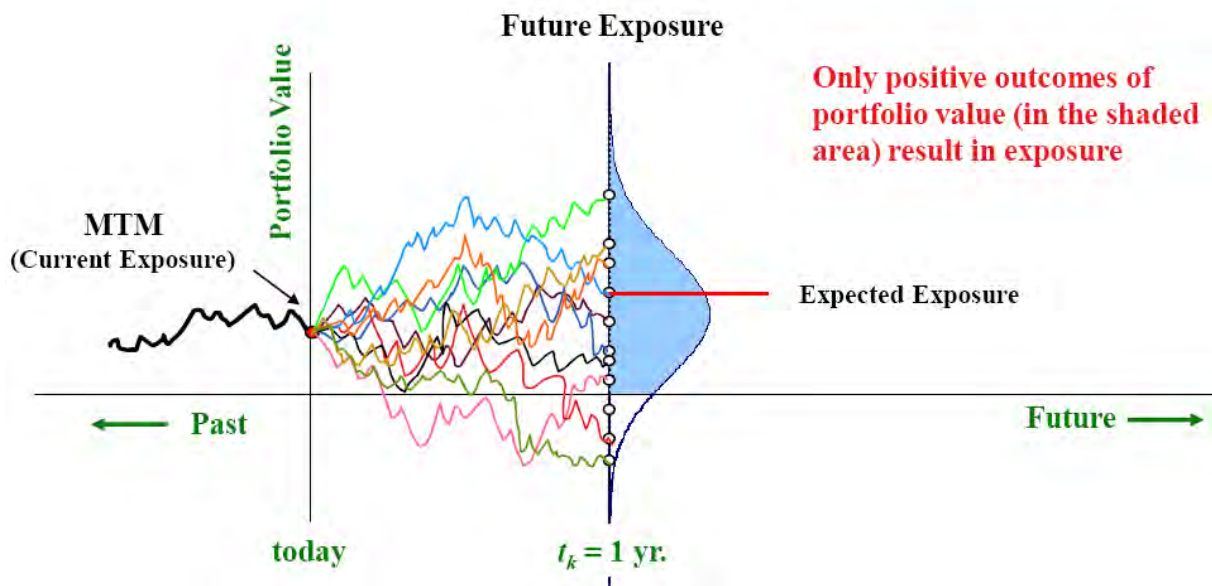


Figure 2: the figure shows that only the positive exposure is averaged over a given date to get EE

### Expected Positive Exposure (EPE)

BCBS defines it as:

<sup>9</sup> taken from: Pricing Counterparty Credit Risk for OTC Derivative Transactions, Michael Pykhtin, 2008

- The time-weighted average of individual expected exposures estimated for given forecasting horizons (e.g. one year)
- Effective EPE is the average of the effective EE over one year or until the maturity of the longest-maturity contract in the netting<sup>10</sup> set whichever is smaller.

EPE correctly gives the contribution of counterparty's portfolio to systematic risk.

Figure 2 shows how the above measures are related. All of them can be derived from the distribution of the exposure in future dates. For a given date in the future EE gives the expected value of the distribution on that specific date while the PFE gives the maximum value of the distribution for a given high confidence level. EPE is the same throughout the horizon time as is expected from the definition above.

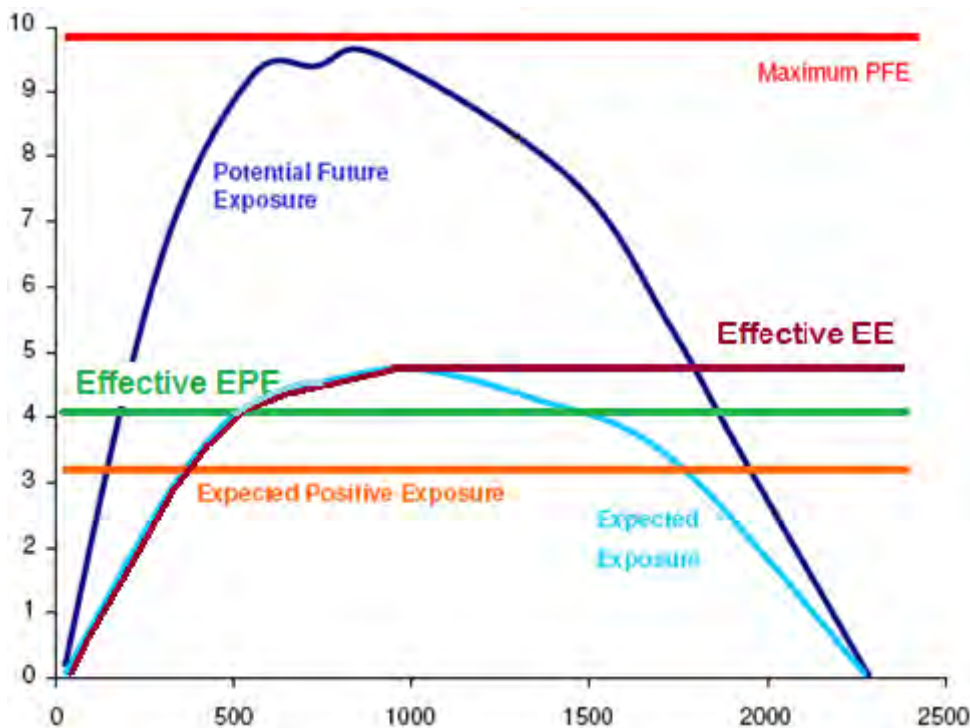


Figure 3: CCR exposure measures<sup>11</sup> in currency amounts in future dates

As it would be clear in the models in Part II the distribution of the CCR exposure in future dates is dependent on the market factors related to the OTC derivative contracts. OTC Counterparty exposure is the larger of zero and the market value of the portfolio of derivative positions with a counterparty that would be lost in the event of counterparty default. This exposure is usually only a small fraction of the total notional amount of trades with a counterparty.<sup>12</sup>

<sup>10</sup> refer Part II for netting set

<sup>11</sup> figure taken from (and modified): Counterparty Credit Risk, Amir Khwaja, February 7, 2008

<sup>12</sup> Measuring and marking counterparty risk, Eduardo Canabarro & Darrell Duffie  
( Extracted from Asset/Liability Management of Financial Institutions, Euromoney Books 2003, chapter 9)

The figure below (Amir Khwaja<sup>13</sup>) shows the PFE profile over time in relation to the daily mark to market values of an OTC contract.

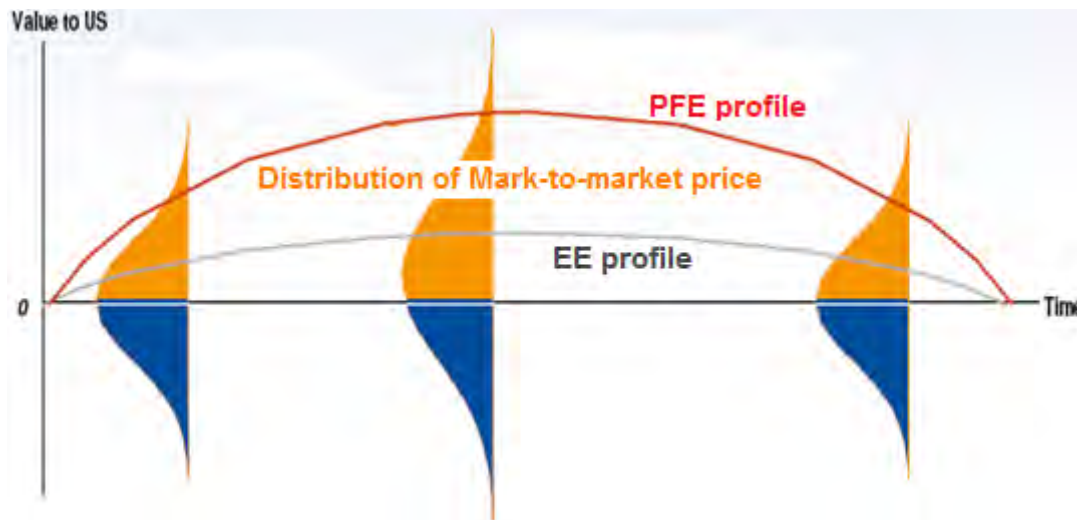


Figure 4: PFE and EE profiles through the life of the contract in relation to the distribution of MTM.

Mark-to-market (MTM) value at time  $t$  is the true value of the contract if it were to be sold on the market (cost of replacing) at that time and may differ from its original value at the contract date. All the CCR models make use of this value in their models. It is calculated from the simulated underlying market risk factors. The relevant market factors may differ per contract. For example interest rate swap contract (involving floating rates) is affected by the change in the reference rate like LIBOR<sup>14</sup>. The stock price of the underlying asset of an OTC equity option contract is another example of the market variable to be simulated. If the counterparty to a firm defaults at time  $t$ , then the recovery value of the counterparty is included in calculating the MTM.

### Current Exposure

BCBS defines it as:

- the larger of zero, or the market value of a transaction or portfolio of transactions within a netting set with a counterparty that would be lost upon the default of the counterparty, assuming no recovery on the value of those transactions in bankruptcy. Current exposure is also called Replacement Cost.

<sup>13</sup> modified

<sup>14</sup> „London Interbank Offered Rate (or LIBOR, pronounced /'laɪbɔːr/) is a daily reference rate based on the interest rates at which banks borrow unsecured funds from other banks in the London wholesale money market (or interbank market). It is roughly comparable to the U.S. Federal funds rate.”(Source wikipedia)

The following parameters are used in the calculation of economic capital for CCR under Basel II.

### **Exposure at default (EAD)**

EAD is the expected total amount in currency of the firms counterparty credit exposure in the event the counterparty defaults. It is often measured for a one year period or over the period until maturity if this is less than one year.

CCR generally refers to the bilateral credit risk of transactions with uncertain exposures that can vary over time with the movement of underlying market factors.

Basel II provides three alternative methods for calculating EAD that I will discuss briefly in part IV. However, EPE is generally regarded as the appropriate EAD measure to determine the EC for CCR.

### **Loss Given Default or LGD**

LGD is the loss a firm suffers as a result of the counterparty to an OTC derivative contract defaulting. It is therefore the fraction of EAD that will not be recovered following a default. Most banks calculate the LGD for an entire portfolio based on cumulative losses and exposure. Basel II requires that banks use an LGD of uncollateralized facility. A term usually used in the modeling of credit default swaps (see Part III) is recovery rate of default. It is one minus LGD.

LGD is assumed to stay constant over time in some industry sectors. However, LGD is in practice stochastic and is subject to both idiosyncratic (firm specific) and systematic risks (Gordy, 2003). For example, an LGD model by Moody predicts the potential interval of loss given default based on historical data of four main factors that are little correlated with each other. The factors include debt type and seniority, firm specific capital structure, industry and macroeconomic environment.

### **Probability of default (PD)**<sup>15</sup>

The probability of default gives the likelihood that a counterparty to the OTC derivative contract defaults. It is estimated for a single contract or a portfolio of OTC transactions depending on the credit quality (rating) of the counterparty. Unlike LGD, it doesn't depend on the transaction characteristics of the contract (example, collateral). Basel II requires that the PD be calculated over a one year horizon.

PD of a counterparty may vary systematically with macroeconomic conditions or the business cycle and therefore unlikely to be stable over time. Its value increases as the credit rating of the counterparty decreases.<sup>16</sup>

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<sup>15</sup> When calculated over a one year horizon PD is referred to as Expected Default Probability.

<sup>16</sup> Estimating Probabilities of Default, Til Schuermann, Samuel Hanson, July 2004

## II. Measuring and Modeling

### II.1. Introduction

The stochastic nature of CCR requires complex techniques to calculate the exposures to parties in an OTC contract. Some techniques focus on single contracts while others model the total portfolio exposure of a counterparty. There are two types of CCR exposure modeling methods, structural and reduced models. Structural models assume that default occurs when the value of the asset, which follows Merton's diffusion process, becomes lower than the debt. Reduced-form models on the other hand assume default as a Poisson event independent of the asset value of the firm (Joro and Na, 2003).<sup>17</sup> I will deal only with some structural models that have appeared in recent publications. These models aim at calculating the risk measures mentioned in Part II.

There are three different techniques employed in calculating counterparty exposures. These are add-on methods, analytical approximations and Monte Carlo (MC) simulation the last one being the most reliable given the fact that the future exposure is stochastic. In all the models the end goal is to approximate or simulate the future value of the OTC derivative positions held by the counterparties. This value can be positive or negative but the exposure at default for the counterparties is at least zero. The problem to be solved by the models is then how to compute for example the exposure over time (EPE) or at a future date (PFE) of the counterparties.

In the rest of this part of the thesis, I will discuss CCR reducing techniques, models for a portfolio of a single counterparty and a single position of a counterparty.

### II.2. Mitigating Counterparty credit risk

Firms that are engaged in OTC derivatives markets employ some techniques that help them reduce the counterparty credit risk exposure. These include netting, collateral and margin agreements.

#### **Netting Agreement**<sup>18</sup>

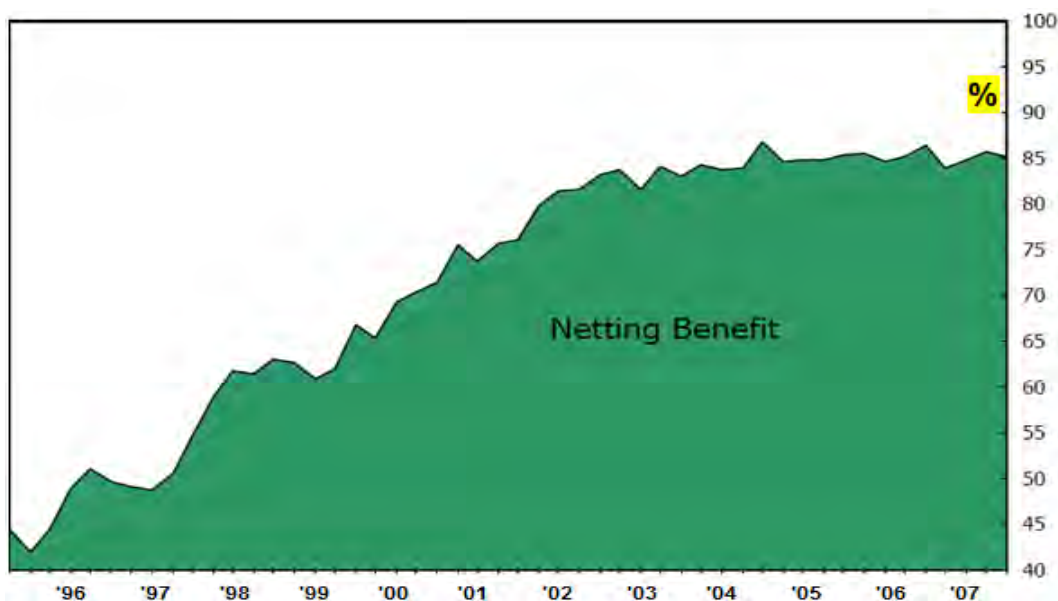
Netting agreements in an OTC derivatives contract are legally enforceable and allow counterparties to net off-setting obligations. This netting agreement that creates a single legal obligation of all the covered contracts between two parties is called bilateral netting. Hence in the event of default the counterparties receive the sum of all the positive and negative values of the contracts in the netting set. For the purpose of calculating economic capital however, Basel II allows banks to net group of transactions (netting set) that includes only OTC products and not across different product categories.

The inclusion of netting agreements effectively reduce CCR exposure significantly provided that the netting set is composed of oppositely positioned transactions or the underlying market factors are not perfectly correlated.

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<sup>17</sup> A simulation-based credit default swap pricing approach under jump-diffusion

<sup>18</sup> Comptroller of the Currency, OCC's Quarterly Report on Bank Trading and Derivatives Activities Second Quarter 2008, US department of treasury



**Figure 5: Percentage of Gross Exposure Eliminated Through Bilateral Netting, All Commercial Banks with Derivatives, 1996 Q1 - 2008 Q2.**

The 2008 2nd Quarterly Report on Bank Trading and Derivatives Activities by US administrator of national banks revealed that legally enforceable netting agreements allowed banks to reduce the gross credit exposure of \$2.8 trillion by 85.3% to \$406 billion in net current credit exposure (figure 5).

### **Collateral and Margin Agreement**

BCBS defines a margin Agreement as:

a contractual agreement or provisions to an agreement under which one counterparty must supply collateral to a second counterparty when an exposure of that second counterparty to the first counterparty exceeds a specified level.

The agreements between the counterparties defines the largest amount of exposure outstanding (threshold), where one of them calls for collateral depending on whose transactions are in-the-money.<sup>19</sup> There are also other terms besides threshold that are negotiated by the counterparties in the agreement.

The ISDA margin survey shows that 65 percent of OTC derivative credit exposure in 2007 is covered by collateral compared to 29 percent in 2003.

Like netting agreement, collateral and margin agreements reduce the risk exposure of the counterparties significantly. Gibson<sup>20</sup> shows as using both simulation and analytical methods the effects of a margin agreement in the figure below. The ratio of the expected positive exposure (EPE) with margin to EPE without margin is near to zero. This clearly implies that the exposure to the counterparty with margin agreement included is lower in comparison to the case where there was no margin.

<sup>19</sup> Prisco & Rosen

<sup>20</sup> Measuring Counterparty Credit Exposure to a Margined Counterparty, Gibson



The effect becomes even bigger as the MTM increases in value. Gibson's experiment showed up to 80% reduction in exposure for the counterparties.

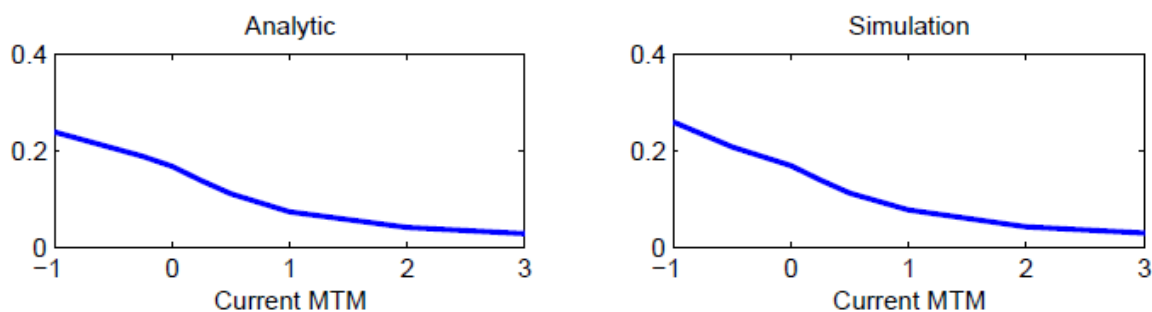


Figure 6: EPE with margin/EPE without margin

### II.3. Counterparty Contract

For firms with only few contracts and no netting agreements we can calculate the potential exposure of each contract separately. The potential exposure of the contracts is then the sum of the contracts current market value and an estimate of its potential increase over time with high level of confidence.

This method approximates the time-varying potential exposure of the contract by a single number such as the peak or the average of the contract's exposure profile over time calculated at some confidence level. However it doesn't give us accurate portfolio exposure of the firm if we sum up these individual contract exposures for many reasons. The possible presence of correlation between contracts and the different maturity times can be mentioned as one of the reasons.<sup>21</sup>

Following are some specific models for single contracts.

#### **Modeling Potential Future Exposure**<sup>22</sup>

The counterparty credit risk I discuss here is for a single position of a contract based on a paper by Prisco and Rosen.

Default can happen before or on the settlement date. The model primarily analyses the possible paths followed by the underlying in the future time sets in addition to the time duration since the contract is signed until the date for which the PFE is calculated.

The PFE value equals the cost of replacing the contract at the time of default provided this value is above zero. If the contract value to the counterparty is below zero then his PFE equals zero as given in the formula below. For a single contract position  $p$ , time sets  $\{t_0, \dots, t_k, \dots, t_N=T\}$  and  $t_0=0$  lets define  $S_j(t_k)$  as the state of the contract at time  $t_k$  along path  $j$ . Then PFE along path  $j$  and at time  $t_k$  is given by<sup>23</sup>:

<sup>21</sup> Economic capital for counterparty credit risk, RMA Journal, March, 2004 by Evan Picoult

<sup>22</sup> Modeling Stochastic Counterparty Credit Exposures for Derivatives Portfolios, Ben De Prisco & Dan Rosen

<sup>23</sup> Note that all the formulas in this paper are taken from the papers being discussed unless I explicitly mention otherwise. In some places I simplified some formulas or showed how they are derived.

$$PFE(p; S_j; t_k) = \max[0, V(p; S_j(t_k); t_k)] \quad (1)$$

Where the stochastic variable V is the mark-to-market value of the contract at time tk on path j.

The PFE value can be discounted with the appropriate factor to get the present value of the PFE.

$$PFE_{D(t)}(p; S_j; t_k) = PV_t(\max[0, V(p; S_j(t_k); t_k)])$$

The set of S<sub>j</sub>(t<sub>k</sub>) over the contract time until default or maturity contains all the market information in the given period.

The PFE of Prisco and Rosen is a bit different than the PFE defined in Part II in that it is the potential future changes in exposures during the contracts' lives. However, the equivalent to our PFE, the peak exposure, can be calculated from the formula in (1) as follows:

$$PFE^* = Q_\alpha^E(t_k) \quad \text{such that}$$

$$\Pr\{PFE(., t_k) \geq Q_\alpha^E(t_k)\} \leq 1 - \alpha \quad (2)$$

From the calculation of the PFE we can also easily derive many of the other relevant measures of the counterparty exposure such as expected exposure and expected positive exposure.

Suppose there are „q“ possible scenarios at time „tk“ given by

$$W_i, i=1, \dots, q;$$

$$\sum_i w_i = 1$$

Then the expected exposure is:

$$EE(t_k) = \sum_i PFE(S_i, t_k) \cdot w_i \quad (3)$$

We can use this result to calculate the EPE.

$$EPE(t_k) = \frac{1}{t_k - t_0} \sum_{l=1}^k EE(t_l)(t_l - t_{l-1}) \quad (4)$$

The PFE can be estimated either by using a Monte Carlo simulation or analytical methods. As I mentioned earlier, simulation is the most reliable way to model the stochastic behavior of PFE. Figure 1 from the paper by Prisco and Rosen shows a result of a Monte Carlo simulation of PFE and other measures derived from it.

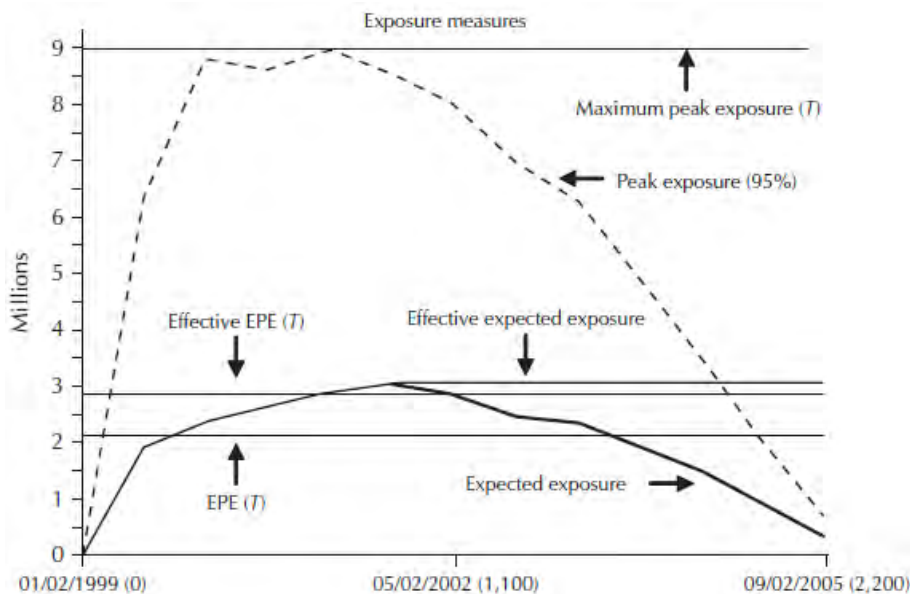


Figure 7: PFE and other derived exposure measures<sup>24</sup>

If the contract includes a collateral agreement, the amount of exposure of the firm to default risk is reduced by the collateral amount  $C$  the counterparty posts.

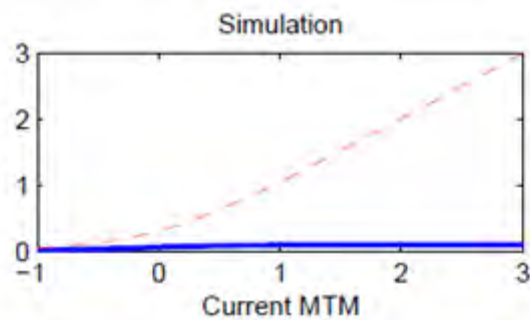
$$PFE(p_i; S_j, t_k) = \max[0, V(p_i; S_j(t_k); t_k) - C(p_i; S_j(t_k); t_k)] \quad (5)$$

### **EPE for a margined counterparty**

We already saw that we can derive the many exposure measures from a result of a Monte Carlo simulation. Gibson gives the simulation steps below to calculate the EPE of a counterparty who has included a margin agreement in his OTC derivatives contract.

The figure below shows a result of his simulation that compares the huge reduction in the calculated EPE when margin is taken into consideration.

<sup>24</sup> Prisco and Rosen



Figur 8: EPE for a margined counterparty (blue) & without margin (red strip) vs current MTM

### Simulation Steps

The first step implemented in measuring EPE is to simulate many paths in the future of the relevant market variables underlying the contract, such as bonds, equity or interest rates.

The second step involves the calculation of the mark to market value of the contract along each path. Here the contract value to the counterparty is priced based on the values of the market factors simulated above. Along the sample path at each time step, the model tests which margin rule to apply. If a margin call is made, the model follows the delivery of the collateral. The status of the delivery one day before is taken into account in the model when considering counterparty default.

Once the MTM values along the paths are known, the next step will be to calculate counterparty exposures at each time step along each path. The exposure equals zero or the MTM value whichever is greater.

Finally we calculate the average of the exposures across sample paths for all time steps (EE). The result is then averaged over time covering all the time steps to arrive at the EPE.

## II.4. Counterparty Portfolio

Many large financial institutes and other large market-makers have many positions in OTC derivative contracts on many underlying market factors. They usually also use risk mitigating techniques like netting agreements. Therefore, portfolio simulation gives the most accurate counterparty exposure profile at portfolio level than aggregating simulation of individual contracts in a portfolio and aggregating them. The following steps give the general steps to calculate the portfolio CCR exposure (Picoult).

1. Starting from the current market conditions, simulate thousands of scenarios of changes in all the market factors underlying the contracts in the portfolio over a set of future dates. These may include among others interest rates, stock prices, commodity prices, exchange rates and the likes.
2. Calculate the corresponding potential market values of each transaction at each future date of each simulated path. The simulated market value of the contracts at each future date will also depend on the number of remaining

unrealized cash flows of the contract, collateral and margin agreements and other terms and conditions of the contract.

3. For each simulated path and at each simulated future date aggregate the simulated market values of all the contracts to get the simulated exposure of the portfolio of transactions with the counterparty. Here enforce also netting agreements.
4. Finally, the current immediate exposure and future exposure profile of the counterparty calculated at some confidence level at a set of future dates. The future exposure profile includes the PFE calculated at a high confidence level, e.g., 99% as well as the expected positive exposure EPE.<sup>25</sup>

### **Modeling Potential Future Exposure**<sup>26</sup>

The PFE model of Prisco and Rosen also deals with a portfolio of a firms OTC derivative contracts with one or many counterparties. If there is no netting agreement with the counterparty, the firm's PFE is calculated as the gross sum of all the individual PFEs.<sup>27</sup>

$$PFE^G(P; S_j; t_k) = \sum_{i=1}^m \max[0, V(p_i; S_j(t_k); t_k)] \quad (6)$$

Many firms however make netting agreements. In that case the MTM values of the individual contracts are summed up, in contrast to equation (6), to arrive at the PFE.

$$PFE^N(P; S_j; t_k) = \max\left[0, \sum_{j=1}^m V(p_i; S_j(t_k); t_k)\right] = \max[0, V(P; S_j(t_k); t_k)] \quad (7)$$

P denotes a netting set of m positions.

The use of collateral by the counterparties reduces the risk exposure further. The PFE for a portfolio involving collaterals is

$$PFE^N(P; S_j; t_k) = \max[0, V(P; S_j(t_k); t_k) - C(P; S_j(t_k); t_k)] \quad (8)$$

where C stands for the posted collateral by the counterparty to the position P held by the firm.

Note that the same applies for a portfolio when netting is not allowed except that the PFE for each contract is separately calculated using each contracts collateral amount. Therefore, the PFE in this case will be the summation of the formula in (5) over all the contracts.

<sup>25</sup> Economic capital for counterparty credit risk, RMA Journal, March, 2004 by Evan Picoult

<sup>26</sup> Modelling Stochastic Counterparty Credit Exposures for Derivatives Portfolios, Ben De Prisco & Dan Rosen

<sup>27</sup> The variables in this section are as defined in the PFE section of II.3.

Large financial institutions may trade in a broad variety of OTC derivatives. They may have different netting agreements with different counterparties. Some of the agreements may allow cross netting of different product categories while others strictly limit the netting within the same group. The PFE of the portfolio will then be given by the sum of the individual PFEs of the netting sets and the PFEs of the other non-netting positions. This involves using the combination of the equations (6)-(8).

This model by Prisco and Rosen further assumes that the maximum exposure for a collateralized portfolio is the margin threshold. Hence, for one-sided collateral agreement and a margin threshold amount MT, the collateral amount posted by the counterparty against the position P of the firm is given by

$$C(P; S_j(t_k), t_k) = \max[0, V(P; S_j(t_k), t_k) - MT_{CP}] \quad (9)$$

For a netting portfolio, I simplify equation (8) as

$$PFE^N = \begin{cases} MT_{CP}, & V \geq MT_{CP} \\ \max[0, V(P; S_j(t_k), t_k)], & V < MT_{CP} \end{cases} \quad (10)$$

A risk of over collateralization arises when the firm calling the collateral has the right to re-use it for example to post collateral against a contract with another counterparty. In this case, the PFE for firm B in a two-way collateral agreement with a counterparty CP is given by substituting

$$C(P; S_j(t_k), t_k) = \max[0, V(P; S_j(t_k), t_k) - MT_{CP}] + \min[0, V(P; S_j(t_k), t_k) + MT_B] \quad (11)$$

in equation (8).

In the same way as for the case of a PFE for a single contract position by the same authors mentioned earlier, many other counterparty exposure measures can be derived. Therefore, equations (3) and (4) are also valid here.

### Monte Carlo simulation

The figure below shows simulated paths over time of the MTM values (left) and the corresponding exposures (right).

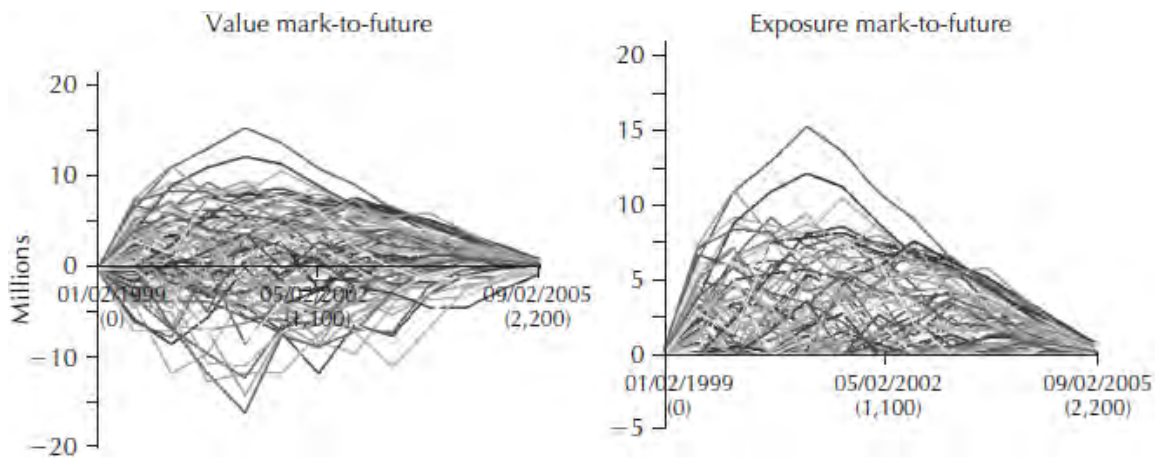


Figure 9: Counterparty exposure simulated over 2200 days

Prisco and Rosen describe in their paper the steps required to calculate PFE. The first step involves generating the joint evolution of all the relevant market factors affecting exposures and collateral. Next the MTM values of all the OTC instruments in the portfolio at each time point and for each scenario calculated. In the third step all the transactions are aggregated using the appropriate formulas among the equations above in order to arrive at the PFE. Finally all other relevant risk measures are derived from the PFE including peak exposure (PFE\* as defined in part I), EE and EPE.

### III. Credit Default Swaps

#### Description

A credit default swap (CDS) is a credit derivative contract between two counterparties. One party to the contract receives a periodic payment (seller) while the other receives a payment only if the underlying credit defaults (buyer). The buyer usually enters into CDS contract in order to hedge the risk he faces for holding a credit. If the obligator of the credit defaults, the buyer receives a payment from the seller. In other words the buyer of the CDS transfers the risk of credit to the seller of the contract. Until the underlying defaults or until the maturity of the contract, whichever is smaller, the seller receives a quarterly payment (premium legs).

The notional outstanding trading in CDS has increased exponentially in recent years as it can be seen in the graph below based on data made available by the ISDA.

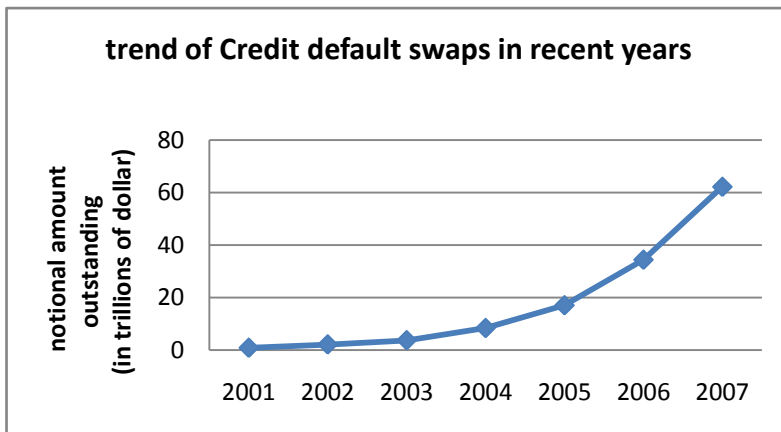


Figure 10: trend in CDS according to market survey by ISDA

Based on the reference entity, CDSs are divided into two groups. Single-name CDS has a single name underlying entity while Multi-name CDS has as reference many names like indices or portfolio of many entities. The share of the multi-name CDS is growing faster than the single name CDS although the later has still a greater share of the total CDS trading.

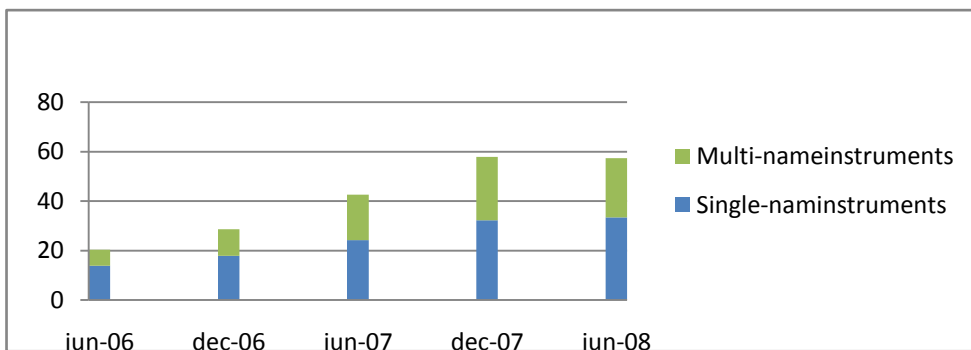


Figure 11: share of single name instruments in total CDS market (BIS data)



## Counterparty Risk

The importance of measuring and managing CCR associated with CDS is amplified with the recent credit crisis. For example the biggest American insurance company AIG was on a verge of collapse as a result of its big seller position in CDS contracts to big financial institutions. Many of these institutions demanded payment from AIG as collateral because its credit rating was down or because of the underlying credit defaults.

It is reasonable to assume that the counterparty is a firm with a high credit rating. The buyer of CDS contract intends to hedge the risk assuming that the counterparty will not default before the underlying. In bad macroeconomic situation however, in addition to the systematic risk, the counterparty is exposed to the risk of many underlying entities of CDS contract defaulting. This has the effect of increasing the CDS spread. Therefore, it is very important to take in to account the positive correlation between defaults by the counterparty and the underlying credit entity. CDS may be significantly overpriced if the default correlation between the protection seller and reference entity is ignored (Jarrow and Yu (2001)).

Remember from Part I that recovery rate of default equals one minus the loss given default (LGD). Let's now define  $R_c$  and  $R_u$  as the recovery rates of the counterparty and the underlying credit entity respectively. We will consider two cases: the potential future exposures to the buyer and the seller.

If the seller is in default and there is positive correlation between the protection seller and the reference entity, the buyer is exposed to a positive replacement cost (Hull and White (2001)), which is the excess premium required to enter a new contract. In general the buyer is exposed to the market value of the contract,  $\max\{0, V_t\}$ , multiplied by  $(1-R_c)$  when the counterparty defaults at time  $t$  before the contract expires.

The seller is exposed to the risk that the underlying firm defaults before the contract expires. In that case the exposure amount is  $(1-R_u)$  times the notional value of the credit ( $C$ ). In addition, the seller is exposed to default by the buyer in which case he may sell a new CDS contract at lower price. He might have already bought a protection against the original CDS and he may need to offset this position by the new selling.

In section III.1, I will discuss CDS risk valuation following the structural approach as it appears on the paper by Patras and Blanchet-Scalliet in July 2008. It will be a brief summary of the model. I will avoid mentioning complex formulas and their proof.

It is worth mentioning however that there are many reduced form models that recently appeared to measure the CDS counterparty risk and price the CDS spread. One model proved the importance of including spread volatility besides the default correlation between the underlying credit and the counterparty (Brigo, May 2008) in modeling CCR of CDS.

## CDS spread<sup>28</sup>

CDS spread is the percentage of the notional amount that the buyer of the contract should pay the seller annually until expiry or default. Usually the payment is done quarterly called premium legs. It is the contractual premium that the counterparty receives as compensation for providing protection to the buyer. The risk to the counterparty is losing the face value of the credit multiplied by [one minus the expected recovery rate] of the defaulted underlying credit before the contract expiry date. The spread is determined in such a way that the expected present value of the protection equals the total present value of the premium legs. O'Kane and Sen argue that the CDS spreads accurately reflect the market price of credit risk because the CDS market is relatively liquid.

## Marginal default window<sup>29</sup>

CDS contracts include a settlement period within which the seller of the contract is obliged to pay the buyer if the underlying defaults. Suppose the settlement period for a given CDS contract is 90 days. Then there is a chance of CCR exposure for the buyer if the counterparty defaults in less than 90 days after the default of the underlying. Hence it can be assumed that the loss to the buyer has a positive value in the interval [default\_timecp -90, default\_timecp].

# III.1. Example Structural CDS Model

## Introduction

This model derives the present value of the expected loss ( $D_c$ ), which the authors<sup>30</sup> refers to as the „counterparty default leg“, of a single name CDS as a result of the counterparty (seller) defaulting. While deriving the  $D_c$  the model considers the conditional distribution of the values of the CDS contract with respect to the default of the counterparty in addition to the joint distribution of the default times of the counterparty and the underlying firm.

As stated earlier the buyer's exposure when the counterparty defaults at time  $t$  before the contract expires equals the positive market value of the contract,  $\max\{0, V_t\}$ , multiplied by  $(1-R_c)$ . Before giving the exact formula of the  $D_c$ , let us introduce some notations.

The time variables  $\tau_1$  and  $\tau_2$  give the default times of the credit obligator and the counterparty respectively. Knowing the exact distribution of these default times is necessary for modeling the risk exposure of the counterparties.

The variable  $F_{\tau_2}$  gives the information until  $\tau_2$

- $C$  is the notional value of the underlying credit
- $r$  is the short term constant interest rate
- $T$  is date of maturity
- $s$  is the CDS spread

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<sup>28</sup> Credit Spreads Explained, O'Kane and Sen, March 2004

<sup>29</sup> Modeling Counterparty Credit Exposure for Credit Default Swaps, Hill, Ring and Shimamoto

<sup>30</sup> Patras and blanchet-scaillet

The present value of the  $D_c$  is the LGD fraction of the discounted expected future market value of the CDS contract at  $\tau_2$  as given in (1).

$$D_c = (1 - R_c) \cdot E\left[e^{-r\tau_2} \cdot \sup(0, p(V_1(\tau_2), \tau_2)) \mathbf{1}_{\tau_2 < \min(T, \tau_1)}\right]. \quad (1)$$

The variable  $p$  stands for the market price of the CDS contract at  $\tau_2$  (the default time of the counterparty). The condition here is that the underlying entity defaults (at  $\tau_1$ ) later than the counterparty. Note that the counterparty is labeled „2“, while the underlying firm labeled „1“.

At initiation of a CDS contract the payment by the seller ( $D_1$ ) in the event the underlying firm defaults is set equal to the sum of the premium legs ( $PR_1$ ). Then the market price ( $p$ ) of the CDS contract at default of the counterparty is given by the difference between  $D_1$  and  $PR_1$  at  $\tau_2$ .

To get  $D_1$  we discount the potential payment by the seller at  $\tau_1 < T$  to a present value at time  $\tau_2$  and calculate its expectation given the information about the two entities until  $\tau_2$ .

$$D_1(V_1(\tau_2), \tau_2) = E\left[C(1 - R_u)e^{-r(\tau_1 - \tau_2)} \mathbf{1}_{\tau_2 \leq \tau_1 \leq T} | F_{\tau_2}\right]. \quad (2)$$

The payments by the buyer are assumed to be continuous (rather than quarterly).

$$PR_1((V_1(\tau_2), \tau_2)) = \frac{sC}{r} \cdot E\left[(1 - e^{-r(\min(T, \tau_1) - \tau_2)}) \mathbf{1}_{\tau_1 \geq \tau_2} | F_{\tau_2}\right].$$

This premium payment by the buyer stops at time  $T$  or at  $\tau_1$  when the underlying firm defaults whichever is smaller. Therefore, we can split the above formula into these possibilities.

$$\begin{aligned} PR_1((V_1(\tau_2), \tau_2)) = & \left[ \frac{sC}{r} \mathbf{1}_{\tau_2 < \min(T, \tau_1)} - \frac{sC}{r} \cdot e^{-r(\tau_1 - \tau_2)} \mathbf{1}_{\tau_2 \leq \tau_1 < T} \right. \\ & \left. - \frac{sC}{r} \cdot e^{-r(T - \tau_2)} \mathbf{1}_{\tau_2 \leq T < \tau_1} | F_{\tau_2} \right] \quad (3) \end{aligned}$$

$$p(V_1(\tau_2), \tau_2) = D_1((V_1(\tau_2), \tau_2)) - PR_1((V_1(\tau_2), \tau_2)). \quad (4)$$

By subtracting (3) from (2) we get  $p$ .

$$\begin{aligned}
p(V_1(\tau_2), \tau_2) 1_{\tau_2 < \min(T, \tau_1)} &= E \left[ C(1 - R_u) e^{-r(\tau_1 - \tau_2)} 1_{\tau_2 \leq \tau_1 \leq T} - \frac{sC}{r} 1_{\tau_2 < \min(T, \tau_1)} \right. \\
&\quad \left. + \frac{sC}{r} \cdot e^{-r(\tau_1 - \tau_2)} 1_{\tau_2 \leq \tau_1 < T} + \frac{sC}{r} \cdot e^{-r(T - \tau_2)} 1_{\tau_2 \leq T < \tau_1} | F_{\tau_2} \right] \\
&= C 1_{\tau_2 < \min(T, \tau_1)} E \left[ \left( 1 - R_u + \frac{s}{r} \right) e^{-r(\tau_1 - \tau_2)} 1_{\tau_2 \leq \tau_1 \leq T} | F_{\tau_2} \right] \\
&\quad - E \left[ \frac{sC}{r} 1_{\tau_2 < \tau_1} | F_{\tau_2} \right] + E \left[ \frac{sC}{r} \cdot e^{-r(T - \tau_2)} 1_{\tau_2 \leq T < \tau_1} | F_{\tau_2} \right] \quad (5)
\end{aligned}$$

So substituting (5) in (1) and calculating it gives as  $D_c$ . The key to solving equation (5) lies in formulating the default probabilities needed to calculate the expectations. Before that, let us state the assumptions for default of a firm.

A firm is said to default when its value  $V_t$  falls below a time dependent continuous default barrier ( $v_i(t)$ ) as described by Black-Cox. This implies that default can happen anytime during the period until maturity<sup>31</sup>.

$$v_i(t) = K_i e^{Y_i t}, \quad (6)$$

Where:

$K_i$  is a positive constant number less than or equal to the debt

$Y$  is a fixed number and  $t$  is the time between zero and  $T$  (maturity)

Both the credit entity value ( $V_1$ ) and the counterparty value ( $V_2$ ) follow a random walk according to the formula (risk neutral process<sup>32</sup>)

$$\frac{dV_i(t)}{V_i(t)} = (r - k_i)dt + \sigma_i dB_i(t), \quad (7)$$

Where:

- $k_i \geq 0$  is the Payout ratio<sup>33</sup>

<sup>31</sup> This approach is called first passage time approach. Default can happen on or before maturity of the debt.

<sup>32</sup> All cash flows are discounted using the risk free rate as all investors are assumed risk averse. However, the result is also valid for risky assets. "A risk-neutral measure or Q-measure is a probability measure that results when one assumes that the current value of all financial assets is equal to the expected value of the future payoff of the asset discounted at the risk-free rate." (wikipedia)

- $r$  is the short term interest rate and is assumed non-random
- $B_i$  is the standard Brownian motion.

The model assumes as mentioned earlier a correlation between the values  $V_1$  and  $V_2$  implying that the corresponding Brownian motions are also correlated.

$$\text{cov}(B_1(t), B_2(t)) = \rho_t \quad (8)$$

The default times  $\tau_1$  and  $\tau_2$  should satisfy the equation

$$\tau_i = \inf\{t, V_i(t) \leq K_i e^{\gamma_i t}\}. \quad (9)$$

Rewriting right hand side we can convert equation (8) to a useful form.

$$\begin{aligned} V_i(t) \leq K_i e^{\gamma_i t} &\Rightarrow 1 \leq \frac{K_i e^{\gamma_i t}}{V_i(t)}, \\ &\Rightarrow \frac{V_i(0)}{K_i} \times 1 \leq \frac{V_i(0)}{K_i} \times \frac{K_i e^{\gamma_i t}}{V_i(t)}, \\ &\Rightarrow \ln\left(\frac{V_i(0)}{K_i}\right) \leq \ln\left(\frac{V_i(0) e^{\gamma_i t}}{V_i(t)}\right), \end{aligned} \quad (10)$$

Lets name the right hand side as  $W$  a function of  $V_t$ . We can now apply Itô's formula to  $W$  as follows:

$$\begin{aligned} dW_i &= \sigma_i V_i(t) \left(\frac{\partial W_i}{\partial V_i(t)}\right) dB_i(t) \\ &+ \left( (r - k) V_i(t) \left(\frac{\partial W_i}{\partial V_i(t)}\right) + \frac{1}{2} (\sigma_i V_i(t))^2 \left(\frac{\partial^2 W_i}{\partial V_i^2(t)}\right) + \left(\frac{\partial W_i}{\partial t}\right) \right) dt \end{aligned} \quad (11)$$

$$\frac{\partial W_i}{\partial V_i(t)} = \frac{V_i(t)}{V_i(0) e^{\gamma_i t}} \times \frac{-V_i(0) e^{\gamma_i t}}{V_i^2(t)} = \frac{-1}{V_i(t)} \quad (12)$$

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<sup>33</sup> The amount of earnings paid out in dividends to shareholders. Investors can use the payout ratio to determine what companies are doing with their earnings (= dividends per share/earnings per share). (<http://www.investopedia.com/terms/p/payoutratio.asp>)

$$\frac{\partial^2 W_i}{\partial V_i^2(t)} = \frac{1}{V_i^2(t)} \quad (13)$$

$$\frac{\partial W_i}{\partial t} = \frac{V_i(t)}{V_i(0)e^{\gamma_i t}} \times \frac{V_i(0)\gamma_i e^{\gamma_i t}}{V_i(t)} = \gamma_i \quad (14)$$

Now substituting equations (12)-(14) in to (11) we get:

$$dW_i = \sigma_i V_i(t) \left( \frac{-1}{V_i(t)} \right) dB_i(t) + \left( (r - k)V_i(t) \left( \frac{-1}{V_i(t)} \right) + \frac{1}{2} (\sigma_i V_i(t))^2 \left( \frac{1}{V_i^2(t)} \right) + \left( \gamma_i - \frac{dV_i(t)}{V_i(t)} \right) \right) dt$$

$$dW_i = -\sigma_i dB_i(t) + \left( -(r - k) + \frac{1}{2} \sigma_i^2 + \gamma_i \right) dt$$

$$W_i(t) = -\sigma_i B_i(t) + -(r - k)t + \frac{1}{2} \sigma_i^2 t + \gamma_i t \quad (15)$$

This is a Brownian motion with drift  $v_i = (r - k - \frac{1}{2} \sigma_i^2 - \gamma_i)$  and standard deviation  $\sigma_i$ .

The default probability (P) of the underlying firm conditional to the default of the counterparty ( $\tau_2 < \tau_1$ ) is then given by (Bielecki and M. Rutkowski):

$$P(\tau_1 \leq T | F_{\tau_2}) = N\left(\frac{-W_1(\tau_2) - v_1(T - \tau_2)}{\sigma_1 \sqrt{T - \tau_2}}\right) + e^{2v_1 \sigma_1^{-2} W_1(\tau_2)} N\left(\frac{-W_1(\tau_2) + v_1(T - \tau_2)}{\sigma_1 \sqrt{T - \tau_2}}\right) \quad (16)$$

Where:

$$v_1 = r - k_1 - \gamma_1 - \frac{1}{2} \sigma_1^2.$$

The first components of equation (5) is calculated by applying equation (16). I will give some of the steps taken by Patras and Blanchet-Scalliet.

$$\begin{aligned}
& E \left[ \left( 1 - R_u + \frac{s}{r} \right) e^{-r(\tau_1 - \tau_2)} \mathbf{1}_{\tau_2 \leq \tau_1 \leq T} | F_{\tau_2} \right] \\
&= \left( 1 - R_u + \frac{s}{r} \right) \cdot \int_{\tau_2}^T e^{-r(s - \tau_2)} dP(\tau_1 \leq s | F_{\tau_2}) \\
&= \left( 1 - R_u + \frac{s}{r} \right) \cdot \left[ e^{-\mu_{\tau_2}(\beta - \alpha)} N \left( \frac{-\mu_{\tau_2} - \alpha(T - \tau_2)}{\sqrt{T - \tau_2}} \right) + e^{-\mu_{\tau_2}(\beta + \alpha)} N \left( \frac{-\mu_{\tau_2} + \alpha(T - \tau_2)}{\sqrt{T - \tau_2}} \right) \right]
\end{aligned}$$

Where

$$\alpha = \sqrt{\frac{v_1^2}{\sigma_1^2} + 2r}, \quad \beta = \frac{v_1}{\sigma_1}, \quad \mu_{\tau_2} = \frac{W_1(\tau_2)}{\sigma_1}$$

In the same way

$$E \left[ \frac{sC}{r} \cdot e^{-r(T - \tau_2)} \mathbf{1}_{\tau_2 \leq T < \tau_1} | F_{\tau_2} \right]$$

would be calculated.

Note that the probability of default here is given by:

$$\begin{aligned}
P(\tau_1 > T | F_{\tau_2}) &= 1 - P(\tau_1 \leq T | F_{\tau_2}) \\
&= 1 - N \left( \frac{-W_1(\tau_2) - v_1(T - \tau_2)}{\sigma_1 \sqrt{T - \tau_2}} \right) + e^{2v_1 \sigma_1^{-2} W_1(\tau_2)} N \left( \frac{-W_1(\tau_2) + v_1(T - \tau_2)}{\sigma_1 \sqrt{T - \tau_2}} \right)
\end{aligned}$$

Finally the authors give the formula of  $D_c$  after calculating all the expectations in equation (5).

Without giving the proof, the present value of the expected loss in the event of default by the counterparty is<sup>34</sup>(Patras and Blanchet-Scalliet, 2008, refer their work for a detailed work out)

$$\begin{aligned}
D_c = & C(1 - R_c) \\
& * E \left[ \mathbf{1}_{\tau_2 < (T \cap \tau_1)} \left( e^{-r\tau_2} \left( 1 - R_u + \frac{s}{r} \right) \left( e^{-\mu\tau_2(\beta-\alpha)} N \left( \frac{-\mu\tau_2 - \alpha(T-\tau_2)}{\sqrt{T-\tau_2}} \right) \right. \right. \right. \\
& + e^{-\mu\tau_2(\beta+\alpha)} N \left( \frac{-\mu\tau_2 - \alpha(T-\tau_2)}{\sqrt{T-\tau_2}} \right) \left. \left. \left. - \frac{s}{r} \left( 1 - e^{-r(T-\tau_2)} \left( 1 - N \left( \frac{-\mu\tau_2 - \beta(T-\tau_2)}{\sqrt{T-\tau_2}} \right) \right) \right) \right. \right. \right. \\
& \left. \left. \left. - e^{-2\mu\tau_2\beta} N \left( \frac{-\mu\tau_2 - \beta(T-\tau_2)}{\sqrt{T-\tau_2}} \right) \right) \right) \right] \quad (17)
\end{aligned}$$

### III.2. Pricing CDS spread using Monte Carlo simulation

Monte Carlo simulation can be employed to duplicate the different future scenarios of both the assets of the counterparty and the underlying entity. Equation (6) can be simplified so that the future asset value can be calculated.

$$\int \frac{dV_i(t)}{V_i(t)} = \int [(r - k_i)dt + \sigma_i dB_i(t)]$$

$$\ln V_i(t) - \ln V_i(0) = (r - k_i)t + \sigma_i B_i(t)$$

$$V_i(t) = V_i(0) \exp((r - k_i)t + \sigma_i B_i(t))$$

This is a risk neutral lognormal process. The asset price increases with a constant drift amount  $r - k_i$  distorted by the volatility  $\sigma_i$  and the stochastic variable  $B$ .

<sup>34</sup> See chapter 2 and 3 of T. Bielecki and M. Rutkowski, Credit risk: modeling, valuation and hedging.



For example the following figure shows the possible paths followed by an asset with initial value 100, annual volatility of 25% and annual of drift 10%.

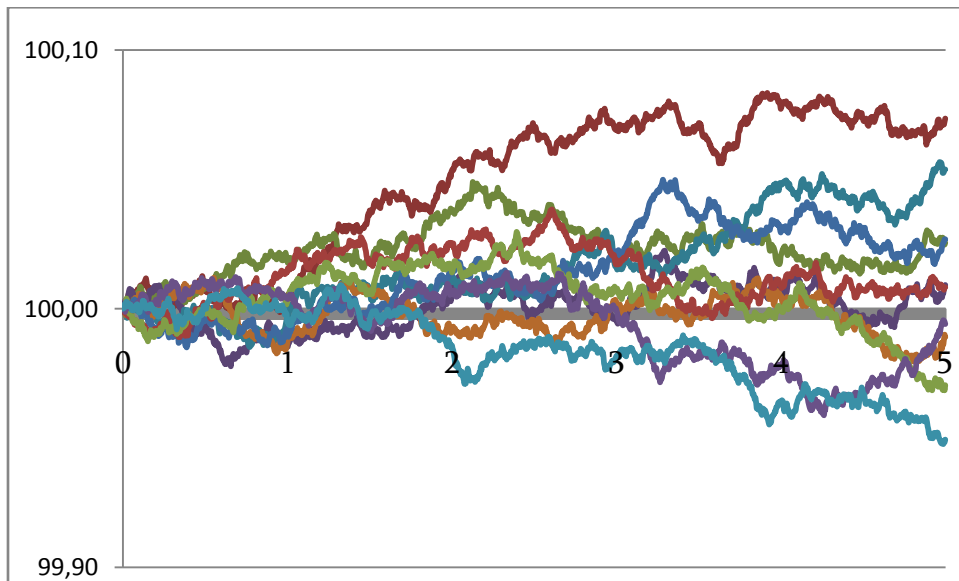


Figure 12: simulated random walk of an asset

After defining the formula to find the future possible paths of the two assets the next step is to determine the joint default probability or the default probability of the underlying conditional to the default of the counterparty.

$$P(\tau_1 \leq T | \tau_2 \leq \tau_1) = P(\min_{\tau_1 \leq t \leq T} (V_i(t)) \leq K_i e^{r_i t} | \tau_2 \leq \tau_1)$$

Both the default probabilities and the asset path can be simulated to find the risk exposure to the buyer as well as the CDS spread of the contract.

## IV. Basel II

### IV.1. Economic Capital<sup>[35],[36],[37]</sup>

In the previous parts I have discussed the first step in managing counterparty credit risk. We saw with help of some models how to model the risk exposure to a counterparty of an OTC derivatives contract. The next step will be to determine the adequate capital, called the economic capital, needed to buffer the counterparty from this exposure in case the other party to the contract defaults. In this part I will discuss about this important economic risk measure.

Economic capital (EC) is a measure of CCR expressed in currency amounts. It gives the potential future unexpected loss of economic value by the counterparty to a portfolio of contracts for a given horizon at a high confidence level. In contrast to other capital adequacy measures which relate capital levels to assets EC relates capital to risk. EC therefore serves as a protection against unexpected losses caused by the uncertain future risk exposure of counterparties.

Loss Distribution

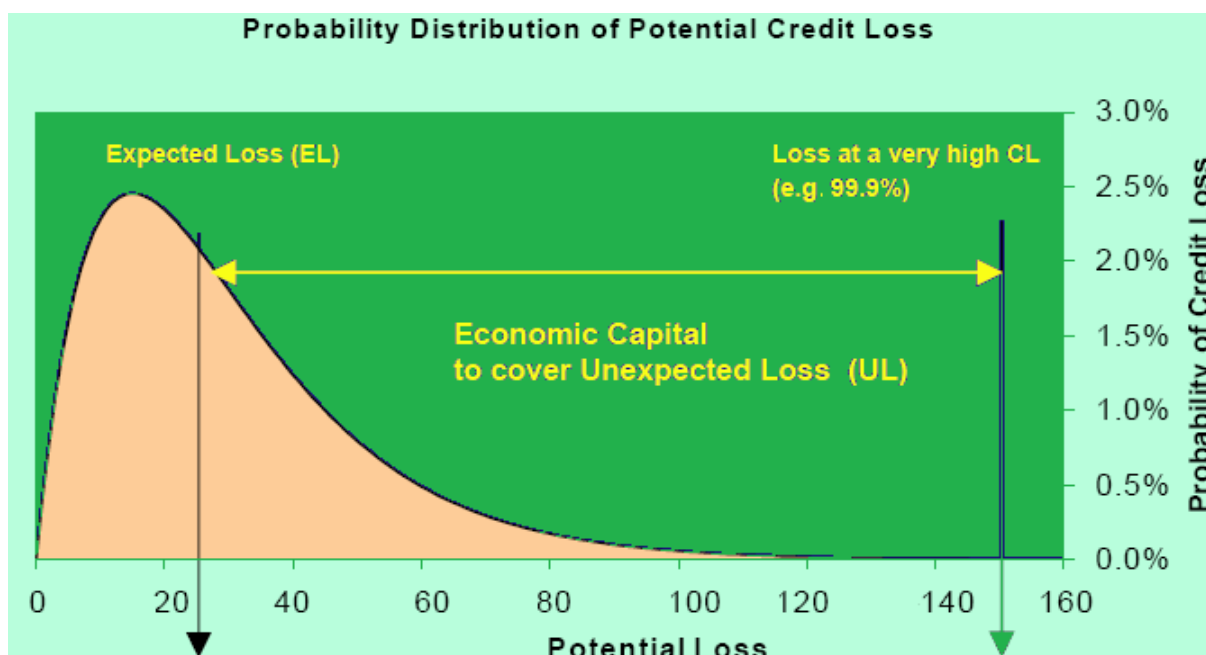


Figure 13: Loss distribution of a portfolio of OTC derivatives contracts

The probability distribution of potential loss over some time horizon can be used to derive Economic capital (EC) as shown in the figure 13. The loss distribution is characterized by the expected loss (EL) and the unexpected loss (UL) which is the

<sup>35</sup> Economic capital for counterparty credit risk, RMA Journal, March, 2004 by Evan Picoult

<sup>36</sup> [http://www.fdic.gov/regulations/examinations/supervisory/insights/siwin04/economic\\_capital.html](http://www.fdic.gov/regulations/examinations/supervisory/insights/siwin04/economic_capital.html)

<sup>37</sup> Counterparty Credit Risk Measurement Under Basel II, A presentation by ISDA, Asia 2007

potential to exceed the EL at a high confidence level (eg.,99.75). Economic capital equals the UL.

$$EL = PD \times LGD \times EAD$$

### **Calculating EC**

There are two perspectives in calculating EC. The simulation steps for both approaches can be summarized as follows.

#### **1. Default Only**

A full simulation that includes calculating a counterparty exposure can be used to determine the loss distribution of a portfolio and thereby the EC. Assuming we already simulated the portfolio exposure profile, the following steps can be followed to calculate the EC (Picoult).

- a. For each simulated path of the market factors underlying the counterparty portfolio, we simulate counterparty defaults at many future dates as follows. At each date we determine first the number of defaults by making a random draw, followed by another draw to determine which counterparties are allowed to default. We repeat the simulation if our firm had a negative exposure. Here we get a potential loss distribution from the thousands of simulations of defaults and recoveries.
- b. Repeat the simulation of potential defaults and recoveries for each stimulated path of the underlying market factors.
- c. Calculate the final loss distribution by appropriately aggregating all the potential loss distributions found above (weighted sum).
- d. Calculate EC by measuring its UL at the appropriate confidence level.

#### **2. Potential-loss-of-economic-value**

Assume that we can represent the potential exposure of each counterparty by a fixed exposure profile over time equal to the expected exposure (EE) profile. Then as alternative to the full simulation we can follow the following simulation steps (ISDA) to calculate EC.

- a. Simulate thousands of defaults and recoveries over time by considering the counterparty portfolio as loan portfolio.
- b. Construct the loss distribution and derive the EC.

## **IV.2. Basel II treatment of OTC CCR exposure<sup>38</sup>**

The banking rules and laws recommended by The Basel committee on Banking Supervision covers among others the CCR as a result of OTC derivatives. I will give a brief general look at the second of the Basel accords, Basel II, treatment of measuring CCR exposure and EC.

The Basel II consists of three pillars: the minimum capital requirements, supervisory review, and market discipline that set an international standard under which banks operate giving some freedom for extra laws and regulations national level supervisory bodies.

In the first pillar, two methodologies are provided for banks to model their minimum capital requirements for credit risk. The Standardized Approach measures credit risk based on externally provided credit assessments by eligible institutions. Under this approach OTC derivative contracts will be converted into credit exposure equivalents through the use of credit conversion factors (20% and 50% CCF for one year and more than one year maturity respectively). The alternative approach, the Internal Ratings-based Approach (IRB), on the other hand allows banks to use their internal rating systems. It relies on internal estimates of risk components such as PD, LGD and EAD in order to determine the capital requirement for a given exposure. EC is calculated based on measures of unexpected losses (UL) and expected losses (EL). Basel II states that EPE is generally accepted as the appropriate EAD measure to determine EC for CCR.

In both approaches to model minimum capital requirements, OTC derivatives Counterparty credit risk exposure can be calculated using anyone of the three methods recommended by the Basel accord. These methods are:

- internal model method (IMM)
- standardized method (SM)
- current exposure method (CEM)

When treated as a loan of the same amount, EPE gives correctly the counterparty's contribution to the systematic risk. However, it doesn't correctly measure the non-systematic risks in the portfolio<sup>39</sup>. In order to account for this problem the IMM and SM scale EPE using multipliers, termed "alpha" and "beta", when calculating the EAD for instruments with CCR. Both alpha and beta are set at 1.4, but supervisors have the flexibility to raise either parameter in appropriate situations.

### **Internal Modeling Method (IMM)**

The IMM can be implemented only if the supervisory body approves it. This method allows banks to use either the concept of EPE to estimate the EAD or adopt a more conservative measure based on peak exposure (PFE).

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<sup>38</sup>International Convergence of Capital Measurement and Capital Standards, Basel Committee on Banking Supervision, A Revised Framework Comprehensive Version, June 2006

<sup>39</sup> Analytic Methods for Portfolio Counterparty Credit Risk, Tom Wilde

The IMM model computes the firm's CCR exposure for a netting set taking margin agreements and collaterals into consideration. Since collaterals then will be included in the calculation of EAD, banks are required to use LGD that doesn't include collateral. LGD is determined either by the foundation or advanced IRB methodology. In the first case standard supervisory rules determine the level of LGD based on differentiation of the characteristics of the underlying transactions. In the advanced methodology, the bank itself determines the appropriate LGD based on analysis of robust data about the transaction characteristics (e.g. product type, wider range of collateral types) as well as borrower characteristics.

The effective EPE is required to be measured for one year time horizon just like the PD.

$$EAD = \alpha \times \text{Effective EPE}$$

Banks may be allowed to calculate their own alpha as the ratio of the EC, calculated using a joint simulation of market and credit risk factors, to the EC calculated when the counterparty exposures are assumed a constant amount equal to EPE.

### **Standardized Method (SM)**

The standardized method also employs the concept of EPE to estimate EAD. The standardized method can be used for OTC derivatives in case the use of IMM is not approved. The exposure amount or EAD is calculated separately for each netting set.

In order to calculate EAD, the assigned collateral is subtracted from the current market value of the portfolio of transactions within the netting set. His amount is then scaled by the multiplier beta.

In addition to conditioning the exposure amount or EAD on a "bad" state of the economy, Beta:

- addresses stochastic dependency of market values of exposures across counterparties,
- addresses estimation and modelling error
- grants appropriate incentive, for banks with diversified derivative transactions and risk areas, to choose the IMM over the standardised method
- and therefore recognition of a risk-sensitive treatment for banks that are actively using OTC derivative transactions.

### **The Current Exposure Method (CEM)**

Banks that don't qualify to use the IMM can use the CEM to calculate their capital requirements for CCR. But CEM is less risk sensitive in comparison with the standard method.

$$EAD = [(RC + \text{add-on}) - \text{volatility adjusted collateral}],$$

Where:

- Current Replacement Cost (RC) is the the larger of zero or the net replacement cost across all OTC derivative contracts in the netting set
- Add-on = the estimated amount of PFE
- Volatility adjusted collateral = the value of collateral

## V. Conclusion

The success of any financial institution in part depends on its ability to manage the different kinds of risks it faces effectively and efficiently. The extensive use of OTC derivatives by these firms requires the implementation of corresponding additional risk management models besides the well established handling of the traditional risks like loan credit risk, liquidity risk, market risk interest rate risk and the like.

The counterparty credit risk associated with OTC derivatives trading is characterized by the uncertain nature of the future value of the contracts which depend on the underlying market factors. As a result the mark-to-market value of the contract or portfolio of contracts may have switching positive and negative values until maturity exposing both parties to CCR. The counterparty with positive exposure at time of default by the other party faces an economic loss equivalent to the current replacement cost of the contract (or portfolio) in the market. This loss can have a tremendous negative effect on the financial stability of the exposed firm as well as to other firms doing business with it.

Central to managing CCR is to have in place models that efficiently measure the firm's future CCR exposures as a function of the relevant underlying market factors. This can be done on a single contract or portfolio level depending on the firm. The results of the models then can be used for the goals of the risk management: limiting the risk level and calculate the economic capital. If the models accurately estimate the future risk exposures, the firm will be able to limit itself to the total safe amount and positions of contracts it can engage in. The firm will also be able to put aside an economic risk capital to protect itself from unexpected losses that rarely occur due to bad macroeconomic or market conditions.

The use of risk mitigating techniques in OTC derivative contracts is very useful in reducing default risks. Therefore it should be part of the CCR management. In addition to mitigating risk, netting agreements provide efficient handling of cash flows by aggregating all the positive and negative valued contracts within a netting set. The other techniques, collateral and margin agreements, also has an additional advantage of serving as an early warning system of default by a counterparty. A firm with higher positive risk exposure may call for collateral. The other party cannot provide the collateral if it is in bad financial situation.

## References

Basel Committee on Banking Supervision, The Application of Basel II to Trading Activities and the Treatment of Double Default Effects, July 2005

BIS, Monetary and Economic Department, OTC derivatives market activity in the first half of 2008, November 2008

Blanchet-Scalliet C. & , Patras F., (July , 2008).Counterparty risk valuation for CDS,

Brigo D., Chourdakis K., (May, 2008). Counterparty Risk for Credit Default Swaps: Impact of Spread Volatility and Default Correlation, *Fitch Solutions Special global report*

Canabarro E. & Duffie D. Measuring and marking counterparty risk, *Extracted from Asset/Liability Management of Financial Institutions, Euromoney Books 2003, chapter 9*

Comptroller of the Currency, OCC's Quarterly Report on Bank Trading and Derivatives Activities, Second Quarter 2008, US department of treasury

Counterparty Credit Risk Measurement Under Basel II, A presentation by ISDA, Asia 2007

Hanson, S. (July 2004) Estimating Probabilities of Default, Til Schuermann,

International Swaps and Derivatives Association Market survey historical data, <http://www.isda.org/>

International Convergence of Capital Measurement and Capital Standards, Basel Committee on Banking Supervision, A Revised Framework Comprehensive Version, June 2006

Khwaja A., (Feb, 2008).Counterparty Credit Risk, *2 0 0 8 | Calypso Confidential. UNLEASH. PROFIT.*

O'Kane and Sen (March 2004). Credit Spreads Explained,

Picoult, E (March 2004). Counterparty credit risk, *RMA Journal*.

Pykhatin Michael (2005). Counterparty Credit Risk Modelling: Risk Management, Pricing and Regulation, Edited by

- Gibson, Michael S. Measuring Counterparty Credit Exposure to a margined Counterparty,
- Picoult, E. Calculating and Hedging Exposure, Credit Value Adjustment and Economic Capital for Counterparty Credit Risk.
- Prisco, B. & Rosen, D. Modeling Stochastic Counterparty Credit Exposures for Derivatives Portfolios,

## Referred websites

<http://www.riskglossary.com/>

<http://www.investopedia.com/>

<http://en.wikipedia.org/>

<http://www.bis.org/statistics/otcder/dt1920a.pdf>

[http://www.fdic.gov/regulations/examinations/supervisory/insights/siwin04 /economic\\_capital.html](http://www.fdic.gov/regulations/examinations/supervisory/insights/siwin04 /economic_capital.html)